Computer algebra independent integration tests

6-Hyperbolic-functions/6.1-Hyperbolic-sine/6.1.7-hyper^m-a+b-sinh^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [525]. This is test number [164].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (525)	% 0.00 (0)
Mathematica	% 95.43 (501)	% 4.57 (24)
Maple	% 92.95 (488)	% 7.05 (37)
Maxima	% 37.33 (196)	% 62.67 (329)
Fricas	% 69.71 (366)	% 30.29 (159)
Sympy	% 13.71 (72)	% 86.29 (453)
Giac	% 50.86 (267)	% 49.14 (258)
Mupad	% 47.05 (247)	% 52.95 (278)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

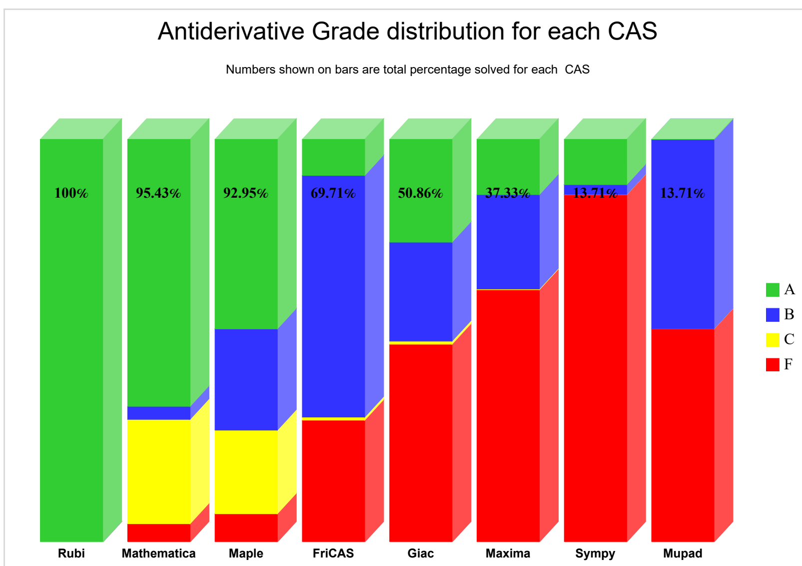
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

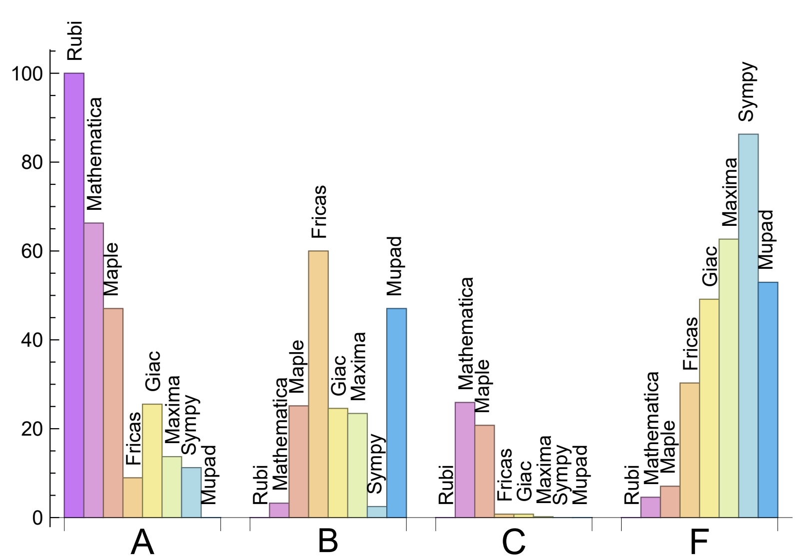
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	66.29	3.24	25.90	4.57
Maple	47.05	25.14	20.76	7.05
Maxima	13.71	23.43	0.19	62.67
Fricas	8.95	60.00	0.76	30.29
Sympy	11.24	2.48	0.00	86.29
Giac	25.52	24.57	0.76	49.14
Mupad	0.00	47.05	0.00	52.95

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	24	95.83 %	4.17 %	0.00 %
Maple	37	100.00 %	0.00 %	0.00 %
Maxima	329	89.06 %	0.91 %	10.03 %
Fricas	159	76.73 %	22.01 %	1.26 %
Sympy	453	37.97 %	62.03 %	0.00 %
Giac	258	25.58 %	0.39 %	74.03 %
Mupad	278	96.76 %	3.24 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

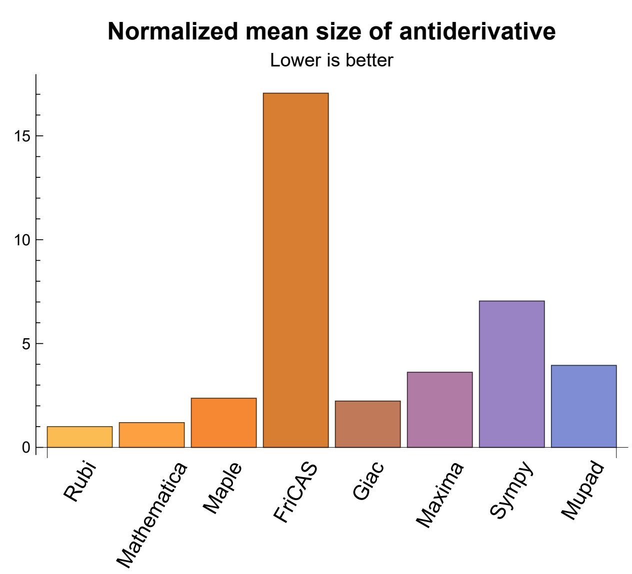
1.3 Performance

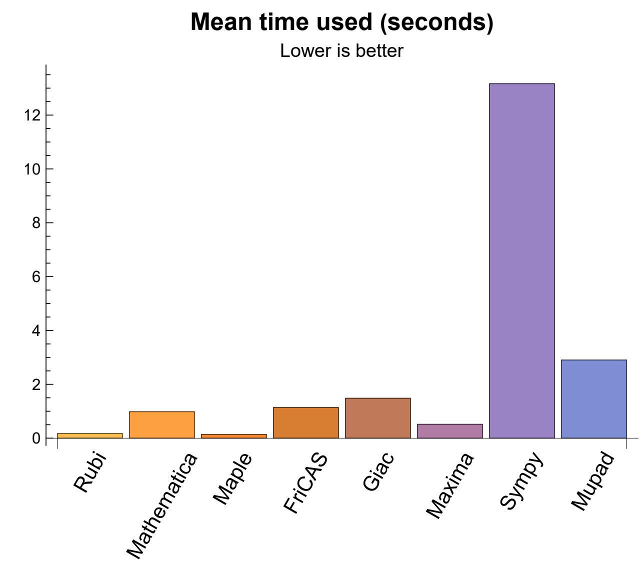
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	133.48	1.00	114.00	1.00
Mathematica	0.98	155.10	1.19	117.00	0.99
Maple	0.14	344.63	2.37	130.00	1.23
Maxima	0.51	332.44	3.62	188.50	2.35
Fricas	1.14	2067.33	17.05	1059.50	13.34
Sympy	13.17	401.53	7.05	210.50	2.64
Giac	1.48	308.01	2.23	163.00	1.78
Mupad	2.90	474.36	3.95	189.00	2.33

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {76, 116, 138, 247, 299, 301, 303, 310, 312, 314, 356, 367, 376, 384, 385, 394}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

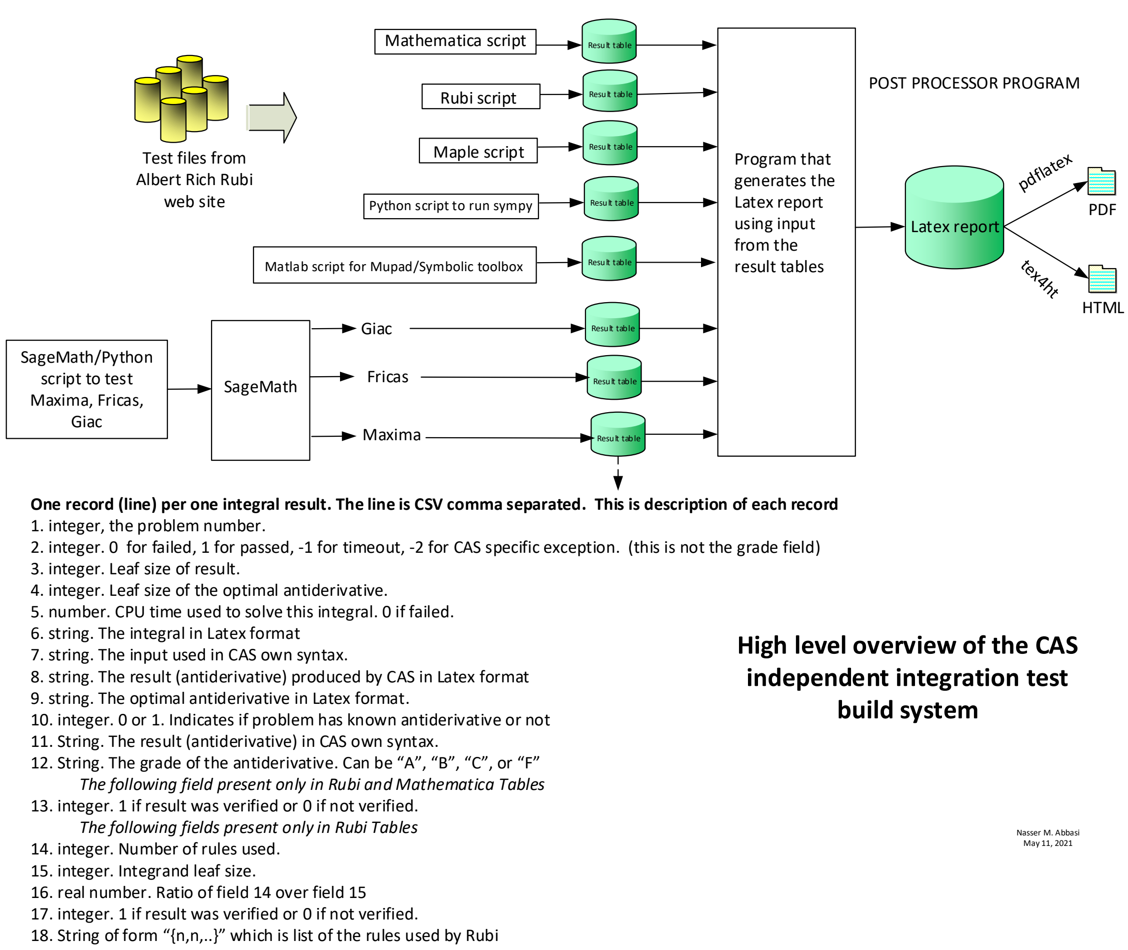
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 37, 39, 41, 42, 44, 46, 48, 50, 51, 53, 55, 57, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 72, 73, 76, 77, 78, 79, 80, 81, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 107, 108, 109, 110, 113, 114, 116, 117, 118, 119, 122, 123, 125, 126, 127, 128, 129, 133, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 265, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 302, 304, 305, 306, 307, 308, 309, 311, 313, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 359, 362, 363, 364, 365, 370, 373, 374, 375, 379, 382, 383, 389, 391, 392, 393, 398, 402, 403, 411, 412, 413, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 460, 461, 462, 465, 468, 469, 470, 471, 472, 473, 476, 479, 480, 481, 482, 483, 484, 487, 498, 509, 513, 514, 515, 516, 521, 522, 523, 524, 525 }

B grade: { 6, 8, 17, 18, 26, 138, 159, 193, 195, 206, 225, 226, 227, 228, 315, 345, 355 }

C grade: { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 71, 74, 75, 82, 85, 86, 102, 105, 106, 111, 112, 115, 120, 121, 124, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 229, 230, 231, 232, 233, 234, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 266, 267, 268, 269, 270, 271, 272, 273, 274, 299, 301, 303, 310, 312, 314, 356, 357, 358, 360, 361, 366, 367, 368, 369, 371, 372, 376, 377, 378, 380, 381, 384, 385, 386, 387, 388, 390, 394, 395, 396, 397, 399, 414, 418, 453, 463, 464, 466, 467, 474, 475, 477, 478, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 510, 511 }

F grade: { 63, 130, 131, 132, 134, 135, 136, 137, 139, 140, 400, 401, 404, 405, 406, 407, 408, 409, 410, 512, 517, 518, 519, 520 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 34, 36, 38, 40, 71, 72, 73, 74, 75, 82, 83, 84, 85, 86, 88, 89, 91, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 129, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 278, 280, 283, 284, 285, 286, 287, 288, 291, 292, 293, 294, 295, 296, 297, 298, 304, 306, 307, 308, 309, 311, 322, 334, 344, 353, 357, 358, 359, 360, 361, 363, 368, 369, 370, 371, 372, 374, 377, 378, 379, 380, 381, 383, 386, 387, 388, 389, 390, 393, 398, 428, 431, 432, 433, 434, 435, 436, 439, 442, 443, 444, 445, 446, 449, 452, 453, 454, 455, 456, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 485, 486, 487, 488, 489, 496, 497, 498, 499, 500, 507, 509, 510, 522, 523, 524, 525 }

B grade: { 28, 29, 30, 31, 32, 33, 35, 37, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 76, 77, 78, 79, 80, 81, 87, 90, 92, 97, 98, 99, 100, 101, 109, 110, 120, 121, 122, 128, 229, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 265, 276, 277, 281, 282, 289, 290, 299, 300, 301, 302, 303, 305, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 395, 396, 397, 399, 413, 417, 425, 508, 511 }

C grade: { 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 279, 352, 354, 355, 356, 362, 364, 365, 366, 367, 373, 375, 376, 382, 384, 385, 391, 392, 394, 411, 412, 414, 415, 416, 418, 426, 427, 429, 430, 437, 438, 440, 441, 447, 448, 450, 451, 457, 458, 459, 460, 461, 462, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 495, 501, 502, 503, 504, 505, 506, 521 }

F grade: { 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 419, 420, 421, 422, 423, 424, 512, 513, 514, 515, 516, 517, 518, 519, 520 }

2.1.4 Maxima

A grade: { 1, 3, 5, 6, 7, 10, 12, 14, 16, 19, 21, 23, 25, 27, 88, 93, 125, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 168, 184, 186, 188, 189, 190, 197, 199, 201, 203, 217, 218, 219, 220, 277, 279, 280, 283, 285, 286, 287, 293, 295, 296, 304, 306, 307, 428, 429, 430, 433, 439, 440, 441, 450, 451 }

B grade: { 2, 4, 8, 9, 11, 13, 15, 17, 18, 20, 22, 24, 26, 60, 61, 62, 63, 64, 65, 89, 94, 108, 117, 118, 148, 149, 156, 157, 158, 159, 160, 167, 169, 170, 185, 187, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 224, 225, 226, 227, 228, 265, 276, 278, 281, 282, 284, 288, 289, 290, 291, 292, 294, 297, 298, 299, 300, 301, 302, 303, 305, 308, 309, 310, 311, 312, 313, 314, 315, 349, 350, 351, 383, 392, 393, 425, 426, 427, 431, 432, 434, 435, 436, 437, 438, 442, 443, 444, 445, 446, 447, 448, 449, 452, 453, 454, 455, 456 }

C grade: { 128 }

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2.1.5 FriCAS

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C grade: { 89, 94, 128, 521 }

F grade: { 58, 59, 71, 72, 73, 74, 75, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97, 102, 103, 104, 105, 106, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 215, 216, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 268, 269, 347, 348, 357, 358, 359, 360, 361, 368, 369, 370, 371, 372, 377, 378, 379, 380, 381, 386, 387, 388, 389, 390, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 414, 418, 419, 420, 421, 422, 423, 424, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 485, 486, 487, 488, 489, 496, 497, 498, 499, 500, 501, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 141, 142, 143, 144, 145, 150, 151, 152, 153, 161, 162, 163, 184, 185, 186, 187, 188, 196, 197, 198, 199, 208, 209, 217, 218, 279, 280, 283, 284, 285, 286, 293, 294, 295, 296, 304, 305, 306, 307, 322, 334, 344, 413, 417 }

B grade: { 60, 61, 62, 63, 64, 65, 265, 276, 277, 278, 349, 350, 351 }

C grade: { }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 146, 147, 148, 149, 154, 155, 156, 157, 158, 159, 160, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 189, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 281, 282, 287, 288, 289, 290, 291, 292, 297, 298, 299, 300, 301, 302, 303, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

2.1.7 Giac

A grade: { 1, 3, 5, 6, 7, 9, 10, 12, 14, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 42, 44, 46, 48, 50, 57, 59, 61, 62, 65, 88, 93, 125, 141, 142, 143, 144, 145, 146, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 182, 183, 184, 186, 188, 189, 192, 197, 199, 201, 203, 204, 217, 218, 220, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 268, 269, 271, 272, 276, 277, 279, 280, 283, 285, 287, 288, 290, 292, 293, 295, 297, 304, 306, 319, 321, 324, 326, 331, 333, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 442, 443, 447, 448, 449, 452, 455, 456, 521 }

B grade: { 2, 4, 8, 11, 13, 15, 16, 17, 18, 20, 22, 24, 26, 41, 51, 53, 55, 60, 63, 64, 147, 148, 156, 157, 158, 166, 167, 168, 169, 170, 185, 187, 190, 191, 193, 194, 195, 196, 198, 200, 202, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 265, 270, 273, 274, 278, 281, 282, 284, 286, 289, 291, 294, 296, 298, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 328, 329, 336, 338, 339, 341, 343, 346, 348, 349, 350, 351, 480, 485, 486, 490, 491, 501, 502 }

C grade: { 89, 94, 128, 266 }

F grade: { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 235, 267, 275, 316, 318, 320, 322, 323, 325, 327, 330, 332, 334, 335, 337, 340, 342, 344, 345, 347, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 440, 441, 444, 445, 446, 450, 451, 453, 454, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 487, 488, 489, 492, 493, 494, 495, 496, 497, 498, 499, 500, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 60, 61, 62, 63, 64, 65, 88, 89, 108, 117, 118, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 265, 266, 268, 271, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 334, 344, 349, 350, 351, 353, 363, 374, 383, 392, 393, 403, 413, 417, 421, 424, 425, 426, 427, 428, 434, 435, 436, 437, 438, 439, 444, 445, 446, 447, 448, 449, 454, 455, 456, 521 }

C grade: { }

F grade: { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 269, 270, 272, 273, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 418, 419, 420, 422, 423, 429, 430, 431, 432, 433, 440, 441, 442, 443, 450, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	68	88	150	122	258	125	76
normalized size	1	1.00	0.76	0.99	1.69	1.37	2.90	1.40	0.85
time (sec)	N/A	0.055	0.108	0.035	0.654	0.473	3.241	0.131	0.784

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	77	56	141	102	105	112	57
normalized size	1	1.00	1.45	1.06	2.66	1.92	1.98	2.11	1.08
time (sec)	N/A	0.056	0.025	0.032	0.481	0.455	1.701	0.130	0.641

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	66	97	64	158	79	50
normalized size	1	1.00	0.77	1.08	1.59	1.05	2.59	1.30	0.82
time (sec)	N/A	0.043	0.086	0.031	0.316	0.661	0.952	0.129	0.109

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	53	34	67	48	56	70	34
normalized size	1	1.00	1.66	1.06	2.09	1.50	1.75	2.19	1.06
time (sec)	N/A	0.029	0.024	0.035	0.310	1.008	0.438	0.122	0.592

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	36	32	38	30	51	38	23
normalized size	1	1.00	1.20	1.07	1.27	1.00	1.70	1.27	0.77
time (sec)	N/A	0.016	0.029	0.020	0.333	0.454	0.216	0.136	0.074

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	62	24	43	126	0	50	66
normalized size	1	1.00	2.48	0.96	1.72	5.04	0.00	2.00	2.64
time (sec)	N/A	0.035	0.031	0.075	0.311	0.576	0.000	0.145	0.135

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	23	36	0	28	23
normalized size	1	1.00	1.00	1.38	1.44	2.25	0.00	1.75	1.44
time (sec)	N/A	0.027	0.019	0.059	0.426	0.519	0.000	0.146	0.566

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	99	40	125	484	0	96	131
normalized size	1	1.00	2.48	1.00	3.12	12.10	0.00	2.40	3.28
time (sec)	N/A	0.041	0.032	0.073	0.333	0.588	0.000	0.137	0.127

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	49	35	113	159	0	61	61
normalized size	1	1.00	1.14	0.81	2.63	3.70	0.00	1.42	1.42
time (sec)	N/A	0.039	0.029	0.071	0.451	0.412	0.000	0.137	0.602

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	133	150	267	238	490	215	149
normalized size	1	1.00	0.91	1.03	1.83	1.63	3.36	1.47	1.02
time (sec)	N/A	0.180	0.205	0.131	0.493	0.455	9.704	0.153	0.902

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	154	102	247	213	204	196	112
normalized size	1	1.00	1.81	1.20	2.91	2.51	2.40	2.31	1.32
time (sec)	N/A	0.097	0.039	0.035	0.385	0.637	5.450	0.155	0.234

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	117	99	118	189	149	332	159	108
normalized size	1	1.06	0.90	1.07	1.72	1.35	3.02	1.45	0.98
time (sec)	N/A	0.110	0.186	0.034	0.355	0.658	3.489	0.234	0.234

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	111	70	157	122	128	138	76
normalized size	1	1.00	1.95	1.23	2.75	2.14	2.25	2.42	1.33
time (sec)	N/A	0.060	0.037	0.033	0.409	0.652	1.748	0.159	0.647

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	79	105	80	168	101	67
normalized size	1	1.00	0.83	1.10	1.46	1.11	2.33	1.40	0.93
time (sec)	N/A	0.022	0.126	0.031	0.351	0.672	0.998	0.139	0.098

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	104	50	102	492	0	110	116
normalized size	1	1.00	2.00	0.96	1.96	9.46	0.00	2.12	2.23
time (sec)	N/A	0.066	0.034	0.071	0.602	0.474	0.000	0.170	0.163

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	64	56	52	63	89	0	135	67
normalized size	1	1.28	1.12	1.04	1.26	1.78	0.00	2.70	1.34
time (sec)	N/A	0.081	0.164	0.069	0.433	0.444	0.000	0.162	0.662

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	134	53	157	902	0	125	179
normalized size	1	1.00	2.39	0.95	2.80	16.11	0.00	2.23	3.20
time (sec)	N/A	0.088	0.063	0.085	0.591	2.068	0.000	0.163	0.666

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	85	47	121	174	0	81	166
normalized size	1	1.00	2.12	1.18	3.02	4.35	0.00	2.02	4.15
time (sec)	N/A	0.074	0.721	0.086	0.471	0.392	0.000	0.179	0.619

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	162	222	405	406	777	325	239
normalized size	1	1.00	0.62	0.85	1.55	1.56	2.98	1.25	0.92
time (sec)	N/A	0.430	0.420	0.133	0.370	0.475	24.458	0.207	1.203

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	127	158	376	373	330	296	185
normalized size	1	1.00	1.10	1.37	3.27	3.24	2.87	2.57	1.61
time (sec)	N/A	0.129	0.798	0.138	0.494	0.507	14.688	0.201	0.430

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	130	180	306	269	561	251	181
normalized size	1	1.00	0.72	0.99	1.69	1.49	3.10	1.39	1.00
time (sec)	N/A	0.193	0.291	0.040	0.356	1.027	9.849	0.179	0.967

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	94	116	263	234	221	222	129
normalized size	1	1.00	1.19	1.47	3.33	2.96	2.80	2.81	1.63
time (sec)	N/A	0.087	0.296	0.033	0.458	0.561	5.556	0.195	0.258

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	95	131	197	165	350	177	123
normalized size	1	1.00	0.74	1.02	1.54	1.29	2.73	1.38	0.96
time (sec)	N/A	0.100	0.258	0.034	0.330	0.467	3.631	0.145	0.750

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	86	193	1128	0	202	184
normalized size	1	1.00	1.00	1.04	2.33	13.59	0.00	2.43	2.22
time (sec)	N/A	0.087	0.215	0.074	0.799	0.561	0.000	0.209	0.298

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	113	94	130	169	0	177	121
normalized size	1	1.00	0.82	0.69	0.95	1.23	0.00	1.29	0.88
time (sec)	N/A	0.192	1.882	0.063	0.329	0.703	0.000	0.200	0.758

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	210	79	217	1814	0	174	229
normalized size	1	1.00	2.53	0.95	2.61	21.86	0.00	2.10	2.76
time (sec)	N/A	0.107	4.585	0.091	0.339	0.562	0.000	0.225	0.216

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	107	77	161	281	0	154	222
normalized size	1	1.00	0.95	0.68	1.42	2.49	0.00	1.36	1.96
time (sec)	N/A	0.141	2.448	0.075	0.339	0.593	0.000	0.201	0.138

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	165	448	0	3242	0	0	415
normalized size	1	1.00	1.51	4.11	0.00	29.74	0.00	0.00	3.81
time (sec)	N/A	0.149	0.885	0.085	0.000	0.630	0.000	0.000	1.641

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	97	670	0	1725	0	208	266
normalized size	1	1.00	0.80	5.54	0.00	14.26	0.00	1.72	2.20
time (sec)	N/A	0.235	0.478	0.108	0.000	0.628	0.000	4.389	1.070

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	134	227	0	1668	0	0	348
normalized size	1	1.00	1.70	2.87	0.00	21.11	0.00	0.00	4.41
time (sec)	N/A	0.108	0.432	0.067	0.000	0.657	0.000	0.000	1.381

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	454	0	859	0	126	216
normalized size	1	1.00	0.90	5.75	0.00	10.87	0.00	1.59	2.73
time (sec)	N/A	0.121	0.247	0.071	0.000	0.732	0.000	3.062	0.938

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	107	98	0	746	0	0	293
normalized size	1	1.00	1.91	1.75	0.00	13.32	0.00	0.00	5.23
time (sec)	N/A	0.085	0.242	0.067	0.000	0.578	0.000	0.000	1.174

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	312	0	464	0	64	473
normalized size	1	1.00	1.00	6.24	0.00	9.28	0.00	1.28	9.46
time (sec)	N/A	0.087	0.128	0.060	0.000	0.611	0.000	1.717	1.224

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	91	51	0	502	0	0	116
normalized size	1	1.00	2.28	1.28	0.00	12.55	0.00	0.00	2.90
time (sec)	N/A	0.047	0.129	0.038	0.000	0.721	0.000	0.000	0.987

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	267	0	430	0	47	146
normalized size	1	1.00	1.00	6.68	0.00	10.75	0.00	1.18	3.65
time (sec)	N/A	0.027	0.071	0.078	0.000	0.596	0.000	0.409	0.450

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	124	74	0	586	0	0	323
normalized size	1	1.00	2.07	1.23	0.00	9.77	0.00	0.00	5.38
time (sec)	N/A	0.074	0.216	0.097	0.000	0.560	0.000	0.000	1.059

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	319	0	675	0	72	176
normalized size	1	1.00	1.00	5.60	0.00	11.84	0.00	1.26	3.09
time (sec)	N/A	0.081	0.293	0.110	0.000	0.609	0.000	0.681	0.467

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	201	133	0	1837	0	0	571
normalized size	1	1.00	2.28	1.51	0.00	20.88	0.00	0.00	6.49
time (sec)	N/A	0.123	0.700	0.134	0.000	0.532	0.000	0.000	1.396

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	126	401	0	1972	0	118	350
normalized size	1	1.00	1.62	5.14	0.00	25.28	0.00	1.51	4.49
time (sec)	N/A	0.120	0.690	0.133	0.000	0.733	0.000	0.697	1.183

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	574	232	0	5809	0	0	1639
normalized size	1	1.00	4.42	1.78	0.00	44.68	0.00	0.00	12.61
time (sec)	N/A	0.204	6.284	0.129	0.000	0.650	0.000	0.000	5.659

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	155	519	0	4540	0	213	479
normalized size	1	1.00	1.41	4.72	0.00	41.27	0.00	1.94	4.35
time (sec)	N/A	0.136	1.480	0.142	0.000	0.615	0.000	0.702	1.209

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	99	798	0	1772	0	168	-1
normalized size	1	1.00	0.97	7.82	0.00	17.37	0.00	1.65	-0.01
time (sec)	N/A	0.169	0.878	0.083	0.000	1.047	0.000	5.593	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	141	341	0	1889	0	0	-1
normalized size	1	1.00	1.57	3.79	0.00	20.99	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.632	0.063	0.000	0.576	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	81	428	0	1523	0	135	-1
normalized size	1	1.00	0.96	5.10	0.00	18.13	0.00	1.61	-0.01
time (sec)	N/A	0.086	0.442	0.072	0.000	0.530	0.000	2.119	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	130	256	0	1628	0	0	-1
normalized size	1	1.00	1.60	3.16	0.00	20.10	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.340	0.049	0.000	0.600	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	96	749	0	1617	0	144	-1
normalized size	1	1.00	1.01	7.88	0.00	17.02	0.00	1.52	-0.01
time (sec)	N/A	0.069	0.300	0.086	0.000	0.756	0.000	0.424	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	176	350	0	2529	0	0	-1
normalized size	1	1.00	1.60	3.18	0.00	22.99	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.656	0.124	0.000	0.725	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	170	810	0	2988	0	229	-1
normalized size	1	1.00	1.20	5.70	0.00	21.04	0.00	1.61	-0.01
time (sec)	N/A	0.153	0.816	0.133	0.000	0.945	0.000	0.820	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	350	415	0	8059	0	0	-1
normalized size	1	1.00	2.17	2.58	0.00	50.06	0.00	0.00	-0.01
time (sec)	N/A	0.273	1.374	0.151	0.000	0.765	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	210	890	0	7110	0	220	-1
normalized size	1	1.00	1.21	5.11	0.00	40.86	0.00	1.26	-0.01
time (sec)	N/A	0.206	1.274	0.159	0.000	0.786	0.000	0.821	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	104	768	0	5186	0	282	-1
normalized size	1	1.00	0.84	6.19	0.00	41.82	0.00	2.27	-0.01
time (sec)	N/A	0.123	1.384	0.082	0.000	0.633	0.000	6.285	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	170	961	0	6087	0	0	-1
normalized size	1	1.00	1.26	7.12	0.00	45.09	0.00	0.00	-0.01
time (sec)	N/A	0.141	1.333	0.076	0.000	0.945	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	121	1408	0	5519	0	277	-1
normalized size	1	1.00	0.87	10.13	0.00	39.71	0.00	1.99	-0.01
time (sec)	N/A	0.157	1.358	0.081	0.000	0.825	0.000	3.546	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	149	964	0	5152	0	0	-1
normalized size	1	1.00	1.26	8.17	0.00	43.66	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.787	0.056	0.000	0.983	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	132	1768	0	5925	0	302	-1
normalized size	1	1.00	0.86	11.48	0.00	38.47	0.00	1.96	-0.01
time (sec)	N/A	0.155	1.227	0.109	0.000	0.605	0.000	0.905	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	237	1145	0	9815	0	0	-1
normalized size	1	1.00	1.43	6.90	0.00	59.13	0.00	0.00	-0.01
time (sec)	N/A	0.267	3.370	0.131	0.000	0.889	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	225	1850	0	9102	0	331	-1
normalized size	1	1.00	1.05	8.60	0.00	42.33	0.00	1.54	-0.00
time (sec)	N/A	0.287	1.814	0.151	0.000	0.959	0.000	1.621	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	419	1225	0	0	0	0	-1
normalized size	1	1.00	1.87	5.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.439	2.814	0.180	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	167	1930	0	0	0	378	-1
normalized size	1	1.00	0.64	7.45	0.00	0.00	0.00	1.46	-0.00
time (sec)	N/A	0.338	2.801	0.184	0.000	0.000	0.000	1.682	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	17	10	20	14	10	10
normalized size	1	1.00	1.00	8.50	5.00	10.00	7.00	5.00	5.00
time (sec)	N/A	0.016	0.004	0.038	0.311	1.045	0.542	0.119	0.039

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	17	36	49	84	104	18	18
normalized size	1	1.00	1.55	3.27	4.45	7.64	9.45	1.64	1.64
time (sec)	N/A	0.019	0.003	0.035	0.310	1.030	1.570	0.131	0.588

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	52	111	185	260	24	24
normalized size	1	1.00	1.42	2.74	5.84	9.74	13.68	1.26	1.26
time (sec)	N/A	0.020	0.003	0.036	0.322	2.197	3.617	0.121	0.609

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	B	B	B	B	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	0	40	61	66	209	37	50
normalized size	1	1.00	0.00	2.67	4.07	4.40	13.93	2.47	3.33
time (sec)	N/A	0.013	0.023	0.038	0.408	1.443	1.505	0.123	0.154

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	92	87	216	2052	62	77
normalized size	1	1.00	0.95	2.49	2.35	5.84	55.46	1.68	2.08
time (sec)	N/A	0.027	0.130	0.038	0.411	1.421	9.287	0.121	0.693

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	124	111	575	5666	74	112
normalized size	1	1.00	0.93	2.25	2.02	10.45	103.02	1.35	2.04
time (sec)	N/A	0.058	0.179	0.038	0.410	0.566	30.963	0.139	0.610

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	114	339	0	3037	0	0	-1
normalized size	1	1.00	0.88	2.61	0.00	23.36	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.668	0.171	0.000	1.371	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	97	200	0	2130	0	0	-1
normalized size	1	1.00	1.18	2.44	0.00	25.98	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.151	0.106	0.000	0.557	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	97	174	0	4423	0	0	-1
normalized size	1	1.00	1.15	2.07	0.00	52.65	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.125	0.160	0.000	1.246	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	104	230	0	1277	0	0	-1
normalized size	1	1.00	1.18	2.61	0.00	14.51	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.291	0.229	0.000	0.664	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	129	381	0	3395	0	0	-1
normalized size	1	1.00	0.90	2.65	0.00	23.58	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.539	0.178	0.000	3.405	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	210	512	0	0	0	0	-1
normalized size	1	1.00	0.70	1.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.327	1.378	0.139	0.000	3.120	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	170	341	0	0	0	0	-1
normalized size	1	1.00	0.96	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.844	0.119	0.000	4.955	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	0	0	0	-1
normalized size	1	1.00	1.15	2.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.096	0.097	0.000	0.762	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	151	161	0	0	0	0	-1
normalized size	1	1.00	0.76	0.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.608	0.122	0.000	2.079	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	208	436	0	0	0	0	-1
normalized size	1	1.00	0.75	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	2.988	0.157	0.000	2.875	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	151	483	0	4608	0	0	-1
normalized size	1	1.00	0.85	2.73	0.00	26.03	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.590	0.150	0.000	2.047	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	111	336	0	2977	0	0	-1
normalized size	1	1.00	0.92	2.78	0.00	24.60	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.324	0.103	0.000	4.829	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	136	268	0	5565	0	0	-1
normalized size	1	1.00	1.07	2.11	0.00	43.82	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.562	0.168	0.000	3.119	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	143	297	0	6622	0	0	-1
normalized size	1	1.00	1.10	2.28	0.00	50.94	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.712	0.188	0.000	4.081	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	123	379	0	3133	0	0	-1
normalized size	1	1.00	0.91	2.81	0.00	23.21	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.644	0.188	0.000	1.243	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	174	569	0	7369	0	0	-1
normalized size	1	1.00	0.87	2.86	0.00	37.03	0.00	0.00	-0.01
time (sec)	N/A	0.199	1.032	0.214	0.000	1.348	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	262	743	0	0	0	0	-1
normalized size	1	1.00	0.71	2.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	2.818	0.144	0.000	0.834	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	213	535	0	0	0	0	-1
normalized size	1	1.00	0.90	2.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.325	1.371	0.125	0.000	0.787	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	169	416	0	0	0	0	-1
normalized size	1	1.00	0.97	2.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.781	0.108	0.000	0.789	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	155	244	0	0	0	0	-1
normalized size	1	1.00	0.76	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.961	0.172	0.000	1.041	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	213	454	0	0	0	0	-1
normalized size	1	1.00	0.80	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	4.044	0.153	0.000	0.740	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	208	609	0	0	0	0	-1
normalized size	1	1.00	0.90	2.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	1.413	0.125	0.000	1.299	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	2	0	11	2
normalized size	1	1.00	1.00	1.27	1.00	0.18	0.00	1.00	0.18
time (sec)	N/A	0.024	0.006	0.075	0.413	3.466	0.000	0.111	0.066

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	25	14	0	11	5
normalized size	1	1.00	1.00	1.15	1.92	1.08	0.00	0.85	0.38
time (sec)	N/A	0.026	0.005	0.065	0.415	0.730	0.000	0.118	0.169

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	51	0	0	0	0	-1
normalized size	1	1.00	1.00	4.64	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.010	0.025	0.115	0.000	1.879	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	61	0	0	0	0	-1
normalized size	1	1.00	1.00	1.85	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.034	0.098	0.000	3.096	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	54	109	0	0	0	0	-1
normalized size	1	1.00	1.29	2.60	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.044	0.103	0.000	4.815	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	21	23	17	0	25	-1
normalized size	1	1.00	0.79	0.72	0.79	0.59	0.00	0.86	-0.03
time (sec)	N/A	0.026	0.021	0.072	0.416	2.912	0.000	0.138	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	25	21	53	26	0	25	-1
normalized size	1	1.00	0.76	0.64	1.61	0.79	0.00	0.76	-0.03
time (sec)	N/A	0.029	0.007	0.065	0.575	0.602	0.000	0.121	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	103	0	0	0	0	-1
normalized size	1	1.00	1.00	2.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.072	0.131	0.000	0.981	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	78	106	0	0	0	0	-1
normalized size	1	1.00	0.90	1.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.123	0.128	0.000	0.522	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	132	329	0	0	0	0	-1
normalized size	1	1.00	1.07	2.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.388	0.113	0.000	1.292	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	98	204	0	2116	0	0	-1
normalized size	1	1.00	1.18	2.46	0.00	25.49	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.280	0.128	0.000	2.925	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	49	108	0	1654	0	0	-1
normalized size	1	1.00	1.20	2.63	0.00	40.34	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.111	0.090	0.000	3.907	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	49	113	0	572	0	0	-1
normalized size	1	1.00	1.17	2.69	0.00	13.62	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.180	0.122	0.000	0.623	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	102	234	0	1285	0	0	-1
normalized size	1	1.00	1.15	2.63	0.00	14.44	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.341	0.158	0.000	1.090	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	168	344	0	0	0	0	-1
normalized size	1	1.00	0.73	1.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.957	0.109	0.000	1.892	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	89	113	0	0	0	0	-1
normalized size	1	1.00	0.70	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.250	0.105	0.000	0.531	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	0	0	0	-1
normalized size	1	1.00	1.13	1.43	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.083	0.079	0.000	2.434	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	150	189	0	0	0	0	-1
normalized size	1	1.00	1.12	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.532	0.135	0.000	0.835	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	201	456	0	0	0	0	-1
normalized size	1	1.00	0.75	1.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	3.540	0.158	0.000	2.086	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	98	146	0	3038	0	0	-1
normalized size	1	1.00	1.18	1.76	0.00	36.60	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.411	0.177	0.000	1.106	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	43	32	236	296	0	0	191
normalized size	1	1.00	1.19	0.89	6.56	8.22	0.00	0.00	5.31
time (sec)	N/A	0.053	0.149	0.082	0.447	0.979	0.000	0.000	0.921

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	98	154	0	1641	0	0	-1
normalized size	1	1.00	1.17	1.83	0.00	19.54	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.377	0.184	0.000	1.954	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	134	251	0	4441	0	0	-1
normalized size	1	1.00	0.96	1.81	0.00	31.95	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.715	0.213	0.000	3.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	211	500	0	0	0	0	-1
normalized size	1	1.00	0.62	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	1.242	0.159	0.000	2.926	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	156	313	0	0	0	0	-1
normalized size	1	1.00	0.61	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.235	1.047	0.126	0.000	1.967	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	151	127	0	0	0	0	-1
normalized size	1	1.00	0.87	0.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.486	0.126	0.000	2.731	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	253	0	0	0	0	-1
normalized size	1	1.00	0.87	2.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.179	0.126	0.000	0.748	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	185	284	0	0	0	0	-1
normalized size	1	1.00	0.64	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	1.294	0.171	0.000	3.194	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	130	230	0	7938	0	0	-1
normalized size	1	1.00	0.91	1.61	0.00	55.51	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.888	0.327	0.000	5.990	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	67	64	927	1214	0	0	148
normalized size	1	1.00	0.77	0.74	10.66	13.95	0.00	0.00	1.70
time (sec)	N/A	0.107	0.333	0.115	0.505	3.783	0.000	0.000	1.618

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	63	57	485	1186	0	0	133
normalized size	1	1.00	0.80	0.72	6.14	15.01	0.00	0.00	1.68
time (sec)	N/A	0.068	0.176	0.097	0.450	3.213	0.000	0.000	1.285

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	130	236	0	5342	0	0	-1
normalized size	1	1.00	0.96	1.74	0.00	39.28	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.794	0.248	0.000	5.094	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	207	868	0	0	0	0	-1
normalized size	1	1.00	0.60	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	2.089	0.247	0.000	1.045	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	198	659	0	0	0	0	-1
normalized size	1	1.00	0.81	2.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	1.670	0.171	0.000	0.892	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	187	598	0	0	0	0	-1
normalized size	1	1.00	0.78	2.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	1.440	0.165	0.000	1.967	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	0	0	0	-1
normalized size	1	1.00	0.76	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	1.375	0.349	0.000	2.949	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	234	747	0	0	0	0	-1
normalized size	1	1.00	0.61	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	2.400	0.213	0.000	0.631	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	19	15	5	8	0	5	-1
normalized size	1	1.00	1.36	1.07	0.36	0.57	0.00	0.36	-0.07
time (sec)	N/A	0.024	0.011	0.083	0.486	0.962	0.000	0.139	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	0	0	0	0	-1
normalized size	1	1.00	1.00	3.73	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.011	0.040	0.107	0.000	1.436	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	61	0	0	0	0	-1
normalized size	1	1.00	1.00	1.85	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.044	0.110	0.000	0.539	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	34	5	13	0	5	-1
normalized size	1	1.00	1.31	2.12	0.31	0.81	0.00	0.31	-0.06
time (sec)	N/A	0.024	0.009	0.072	0.591	1.025	0.000	0.135	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	63	0	0	0	0	-1
normalized size	1	1.00	1.26	1.50	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.053	0.092	0.000	0.620	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	9.649	0.901	0.000	0.816	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	11.804	0.952	0.000	1.987	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	15.958	0.658	0.000	1.146	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.217	0.469	0.000	0.898	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	4.469	0.394	0.000	0.975	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	131.290	0.427	0.000	1.105	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	180.001	0.396	0.000	0.944	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	11.003	0.698	0.000	0.788	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	250	0	0	0	0	0	-1
normalized size	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.785	0.527	0.000	2.125	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	5.814	0.379	0.000	0.983	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	10.863	0.369	0.000	0.656	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	81	82	164	188	192	182	85
normalized size	1	1.00	0.76	0.77	1.55	1.77	1.81	1.72	0.80
time (sec)	N/A	0.094	0.101	0.037	0.317	1.539	5.124	0.157	0.260

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	66	72	143	135	194	152	67
normalized size	1	1.00	0.67	0.73	1.44	1.36	1.96	1.54	0.68
time (sec)	N/A	0.108	0.129	0.034	0.313	0.866	2.942	0.136	0.454

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	79	60	120	105	117	122	55
normalized size	1	1.00	1.13	0.86	1.71	1.50	1.67	1.74	0.79
time (sec)	N/A	0.071	0.096	0.033	0.315	1.469	1.593	0.151	0.116

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	50	74	63	121	92	42
normalized size	1	1.00	0.75	0.83	1.23	1.05	2.02	1.53	0.70
time (sec)	N/A	0.068	0.129	0.035	0.302	1.822	0.852	0.146	0.198

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	28	59	47	41	59	29
normalized size	1	1.00	1.06	0.88	1.84	1.47	1.28	1.84	0.91
time (sec)	N/A	0.021	0.012	0.016	0.310	1.207	0.387	0.118	0.625

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	72	40	50	258	0	62	73
normalized size	1	1.00	1.80	1.00	1.25	6.45	0.00	1.55	1.82
time (sec)	N/A	0.056	0.053	0.093	0.318	2.184	0.000	0.130	0.691

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	23	47	40	0	59	47
normalized size	1	1.00	1.46	0.96	1.96	1.67	0.00	2.46	1.96
time (sec)	N/A	0.050	0.029	0.085	0.318	0.929	0.000	0.140	0.094

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	63	37	91	521	0	73	102
normalized size	1	1.00	1.62	0.95	2.33	13.36	0.00	1.87	2.62
time (sec)	N/A	0.063	0.013	0.098	0.317	0.623	0.000	0.149	0.105

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	76	36	131	652	0	62	110
normalized size	1	1.00	1.85	0.88	3.20	15.90	0.00	1.51	2.68
time (sec)	N/A	0.057	0.030	0.098	0.325	2.925	0.000	0.144	0.686

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	125	128	272	355	325	301	149
normalized size	1	1.00	0.65	0.67	1.42	1.85	1.69	1.57	0.78
time (sec)	N/A	0.179	0.683	0.042	0.324	0.702	13.678	0.186	0.932

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	133	122	237	274	340	260	126
normalized size	1	1.00	0.74	0.68	1.32	1.52	1.89	1.44	0.70
time (sec)	N/A	0.154	0.133	0.046	0.373	1.535	8.645	0.170	1.627

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	92	96	180	220	219	219	104
normalized size	1	1.00	0.71	0.74	1.38	1.69	1.68	1.68	0.80
time (sec)	N/A	0.119	0.415	0.043	0.326	0.469	5.159	0.202	0.247

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	94	85	151	160	212	178	85
normalized size	1	1.00	0.82	0.75	1.32	1.40	1.86	1.56	0.75
time (sec)	N/A	0.083	0.149	0.043	0.306	0.474	3.039	0.125	0.461

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	96	76	140	1052	0	154	177
normalized size	1	1.00	1.09	0.86	1.59	11.95	0.00	1.75	2.01
time (sec)	N/A	0.102	0.314	0.121	0.313	0.487	0.000	0.211	0.214

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	92	65	113	142	0	149	123
normalized size	1	1.00	1.12	0.79	1.38	1.73	0.00	1.82	1.50
time (sec)	N/A	0.098	0.275	0.105	0.326	0.447	0.000	0.198	0.738

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	105	63	152	1616	0	162	175
normalized size	1	1.00	1.36	0.82	1.97	20.99	0.00	2.10	2.27
time (sec)	N/A	0.105	0.031	0.133	0.329	0.679	0.000	0.204	0.705

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	81	65	170	1748	0	151	163
normalized size	1	1.00	1.07	0.86	2.24	23.00	0.00	1.99	2.14
time (sec)	N/A	0.091	0.407	0.135	0.327	0.566	0.000	0.202	0.133

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	149	66	188	2119	0	172	355
normalized size	1	1.00	1.66	0.73	2.09	23.54	0.00	1.91	3.94
time (sec)	N/A	0.132	0.040	0.177	0.326	0.533	0.000	0.235	0.143

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	197	73	303	2310	0	141	351
normalized size	1	1.00	2.24	0.83	3.44	26.25	0.00	1.60	3.99
time (sec)	N/A	0.103	0.939	0.204	0.325	0.663	0.000	0.214	0.652

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	235	90	316	3607	0	204	434
normalized size	1	1.00	1.77	0.68	2.38	27.12	0.00	1.53	3.26
time (sec)	N/A	0.175	0.054	0.184	0.329	0.523	0.000	0.253	0.157

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	194	188	387	568	498	431	231
normalized size	1	1.00	0.67	0.65	1.33	1.95	1.71	1.48	0.79
time (sec)	N/A	0.209	0.471	0.135	0.326	0.525	33.472	0.261	1.086

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	184	168	318	453	496	379	189
normalized size	1	1.00	0.69	0.63	1.19	1.70	1.86	1.42	0.71
time (sec)	N/A	0.222	0.448	0.049	0.329	0.555	21.919	0.249	2.953

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	159	141	280	380	340	327	164
normalized size	1	1.00	0.78	0.69	1.37	1.86	1.67	1.60	0.80
time (sec)	N/A	0.128	0.257	0.048	0.327	0.449	13.920	0.192	0.931

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	158	138	257	2609	0	279	315
normalized size	1	1.00	0.79	0.69	1.28	12.98	0.00	1.39	1.57
time (sec)	N/A	0.189	0.215	0.153	0.329	0.600	0.000	0.283	0.504

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	140	111	220	302	0	276	252
normalized size	1	1.00	0.92	0.73	1.45	1.99	0.00	1.82	1.66
time (sec)	N/A	0.138	1.135	0.138	0.324	0.442	0.000	0.300	0.396

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	150	115	244	3627	0	289	290
normalized size	1	1.00	0.96	0.74	1.56	23.25	0.00	1.85	1.86
time (sec)	N/A	0.169	3.441	0.145	0.328	0.630	0.000	0.331	0.929

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	169	101	260	3801	0	285	267
normalized size	1	1.00	1.31	0.78	2.02	29.47	0.00	2.21	2.07
time (sec)	N/A	0.120	0.437	0.174	0.332	0.579	0.000	0.336	0.880

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	218	108	255	4541	0	329	451
normalized size	1	1.00	1.47	0.73	1.72	30.68	0.00	2.22	3.05
time (sec)	N/A	0.173	6.136	0.191	0.337	0.619	0.000	0.386	0.868

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	225	99	365	4629	0	270	432
normalized size	1	1.00	1.72	0.76	2.79	35.34	0.00	2.06	3.30
time (sec)	N/A	0.125	1.872	0.184	0.723	0.796	0.000	0.336	0.807

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	236	119	355	6210	0	327	486
normalized size	1	1.00	1.42	0.72	2.14	37.41	0.00	1.97	2.93
time (sec)	N/A	0.197	1.546	0.186	0.432	0.668	0.000	0.376	0.294

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	168	259	0	0	0	0	1579
normalized size	1	1.00	0.51	0.79	0.00	0.00	0.00	0.00	4.81
time (sec)	N/A	0.739	0.366	0.122	0.000	0.000	0.000	0.000	10.805

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	299	207	0	0	0	0	1114
normalized size	1	1.00	1.01	0.70	0.00	0.00	0.00	0.00	3.78
time (sec)	N/A	0.559	0.332	0.111	0.000	0.000	0.000	0.000	11.481

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	214	128	0	0	0	0	906
normalized size	1	1.00	0.71	0.42	0.00	0.00	0.00	0.00	2.99
time (sec)	N/A	0.545	0.365	0.105	0.000	0.000	0.000	0.000	23.502

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	145	129	0	0	0	0	1498
normalized size	1	1.00	0.49	0.44	0.00	0.00	0.00	0.00	5.10
time (sec)	N/A	0.440	0.233	0.092	0.000	0.000	0.000	0.000	10.762

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	275	78	0	0	0	0	932
normalized size	1	1.00	1.05	0.30	0.00	0.00	0.00	0.00	3.56
time (sec)	N/A	0.271	0.183	0.085	0.000	0.000	0.000	0.000	11.126

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	199	82	0	0	0	0	857
normalized size	1	1.00	0.69	0.28	0.00	0.00	0.00	0.00	2.96
time (sec)	N/A	0.336	0.253	0.092	0.000	0.000	0.000	0.000	21.839

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	131	87	0	0	0	0	1261
normalized size	1	1.00	0.47	0.31	0.00	0.00	0.00	0.00	4.50
time (sec)	N/A	0.251	0.173	0.092	0.000	0.000	0.000	0.000	9.486

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	295	100	0	0	0	0	2970
normalized size	1	1.00	1.03	0.35	0.00	0.00	0.00	0.00	10.38
time (sec)	N/A	0.432	0.264	0.151	0.000	0.000	0.000	0.000	55.942

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	230	123	0	0	0	0	1293
normalized size	1	1.00	0.76	0.40	0.00	0.00	0.00	0.00	4.25
time (sec)	N/A	0.478	0.418	0.159	0.000	0.000	0.000	0.000	23.059

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	178	146	0	0	0	0	3605
normalized size	1	1.00	0.55	0.45	0.00	0.00	0.00	0.00	11.20
time (sec)	N/A	0.457	0.513	0.181	0.000	0.000	0.000	0.000	89.246

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	370	178	0	0	0	0	3086
normalized size	1	1.00	1.17	0.56	0.00	0.00	0.00	0.00	9.74
time (sec)	N/A	0.433	5.788	0.181	0.000	0.000	0.000	0.000	59.687

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	156	82	0	185	0	102	203
normalized size	1	1.00	1.12	0.59	0.00	1.33	0.00	0.73	1.46
time (sec)	N/A	0.188	1.505	0.056	0.000	0.900	0.000	0.134	1.772

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	156	80	0	180	0	106	225
normalized size	1	1.00	1.17	0.60	0.00	1.35	0.00	0.80	1.69
time (sec)	N/A	0.189	1.281	0.053	0.000	1.481	0.000	0.157	2.073

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	82	98	175	174	306	155	88
normalized size	1	1.00	0.74	0.88	1.58	1.57	2.76	1.40	0.79
time (sec)	N/A	0.143	0.137	0.037	0.319	1.075	8.335	0.173	0.916

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	93	66	157	155	128	142	66
normalized size	1	1.00	1.39	0.99	2.34	2.31	1.91	2.12	0.99
time (sec)	N/A	0.066	0.028	0.034	0.313	1.813	4.814	0.169	0.794

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	63	76	122	109	206	113	64
normalized size	1	1.00	0.76	0.92	1.47	1.31	2.48	1.36	0.77
time (sec)	N/A	0.098	0.079	0.038	0.316	3.300	2.948	0.163	0.159

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	69	44	97	91	80	100	46
normalized size	1	1.00	1.50	0.96	2.11	1.98	1.74	2.17	1.00
time (sec)	N/A	0.034	0.023	0.033	0.314	0.797	1.522	0.144	0.101

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	44	66	59	100	66	38
normalized size	1	1.00	0.94	0.85	1.27	1.13	1.92	1.27	0.73
time (sec)	N/A	0.034	0.058	0.016	0.316	0.652	0.805	0.145	0.700

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	70	36	71	395	0	78	96
normalized size	1	1.00	1.67	0.86	1.69	9.40	0.00	1.86	2.29
time (sec)	N/A	0.043	0.024	0.072	0.306	3.138	0.000	0.171	0.133

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	45	39	54	70	0	88	54
normalized size	1	1.00	1.15	1.00	1.38	1.79	0.00	2.26	1.38
time (sec)	N/A	0.052	0.118	0.063	0.318	0.803	0.000	0.177	0.723

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	82	38	115	690	0	107	126
normalized size	1	1.00	1.74	0.81	2.45	14.68	0.00	2.28	2.68
time (sec)	N/A	0.054	0.026	0.088	0.311	1.585	0.000	0.191	0.722

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	40	33	97	129	0	45	81
normalized size	1	1.00	1.29	1.06	3.13	4.16	0.00	1.45	2.61
time (sec)	N/A	0.050	0.016	0.082	0.319	0.733	0.000	0.162	0.702

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	139	54	174	1476	0	124	242
normalized size	1	1.00	2.17	0.84	2.72	23.06	0.00	1.94	3.78
time (sec)	N/A	0.063	0.035	0.092	0.336	0.579	0.000	0.179	0.743

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	45	228	333	0	97	337
normalized size	1	1.00	1.51	0.96	4.85	7.09	0.00	2.06	7.17
time (sec)	N/A	0.043	0.035	0.086	0.328	0.794	0.000	0.186	0.722

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	199	78	268	3115	0	207	472
normalized size	1	1.00	2.16	0.85	2.91	33.86	0.00	2.25	5.13
time (sec)	N/A	0.084	0.036	0.094	0.324	2.014	0.000	0.196	0.774

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	207	132	307	404	280	278	150
normalized size	1	1.00	1.72	1.10	2.56	3.37	2.33	2.32	1.25
time (sec)	N/A	0.129	0.060	0.044	0.330	0.405	31.999	0.206	0.357

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	139	148	260	305	484	241	149
normalized size	1	1.00	0.86	0.92	1.61	1.89	3.01	1.50	0.93
time (sec)	N/A	0.281	0.344	0.046	0.348	0.607	21.710	0.209	0.405

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	164	100	226	279	204	220	111
normalized size	1	1.00	1.78	1.09	2.46	3.03	2.22	2.39	1.21
time (sec)	N/A	0.086	0.039	0.042	0.334	1.667	13.083	0.230	0.883

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	92	111	183	205	332	183	108
normalized size	1	1.00	0.74	0.89	1.46	1.64	2.66	1.46	0.86
time (sec)	N/A	0.162	0.166	0.041	0.331	0.987	8.559	0.139	0.289

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	146	82	177	1575	0	196	198
normalized size	1	1.00	1.59	0.89	1.92	17.12	0.00	2.13	2.15
time (sec)	N/A	0.093	0.042	0.083	0.345	0.855	0.000	0.222	0.344

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	77	91	146	217	0	179	148
normalized size	1	1.00	0.75	0.88	1.42	2.11	0.00	1.74	1.44
time (sec)	N/A	0.194	0.305	0.075	0.357	0.994	0.000	0.245	0.232

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	74	204	2272	0	182	214
normalized size	1	1.00	1.57	0.80	2.22	24.70	0.00	1.98	2.33
time (sec)	N/A	0.135	0.051	0.099	0.339	3.951	0.000	0.234	0.834

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	75	165	300	0	142	164
normalized size	1	1.00	0.75	0.82	1.81	3.30	0.00	1.56	1.80
time (sec)	N/A	0.165	0.327	0.090	0.343	2.058	0.000	0.237	0.172

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	186	79	234	3356	0	179	328
normalized size	1	1.00	1.84	0.78	2.32	33.23	0.00	1.77	3.25
time (sec)	N/A	0.151	0.043	0.096	0.354	1.068	0.000	0.268	0.215

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	67	74	267	457	0	166	397
normalized size	1	1.00	0.80	0.88	3.18	5.44	0.00	1.98	4.73
time (sec)	N/A	0.146	0.831	0.097	0.339	1.314	0.000	0.260	0.746

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	240	92	299	4500	0	243	535
normalized size	1	1.00	2.16	0.83	2.69	40.54	0.00	2.19	4.82
time (sec)	N/A	0.174	0.042	0.095	0.348	1.068	0.000	0.282	0.818

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	288	258	600	1030	0	520	319
normalized size	1	1.00	1.31	1.17	2.73	4.68	0.00	2.36	1.45
time (sec)	N/A	0.218	2.278	0.169	0.341	0.853	0.000	0.393	1.762

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	185	218	501	795	484	446	266
normalized size	1	1.00	1.01	1.19	2.74	4.34	2.64	2.44	1.45
time (sec)	N/A	0.175	2.458	0.134	0.336	1.209	139.976	0.402	1.326

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	157	176	399	594	377	372	211
normalized size	1	1.00	1.10	1.23	2.79	4.15	2.64	2.60	1.48
time (sec)	N/A	0.157	0.905	0.050	0.337	1.372	69.185	0.347	1.069

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	139	148	327	3824	0	377	326
normalized size	1	1.00	0.88	0.94	2.07	24.20	0.00	2.39	2.06
time (sec)	N/A	0.135	0.370	0.081	0.339	0.974	0.000	0.415	0.652

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	155	130	334	4895	0	300	326
normalized size	1	1.00	1.05	0.88	2.26	33.07	0.00	2.03	2.20
time (sec)	N/A	0.209	0.349	0.102	0.329	1.941	0.000	0.437	1.189

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	173	125	340	6441	0	271	421
normalized size	1	1.00	1.22	0.88	2.39	45.36	0.00	1.91	2.96
time (sec)	N/A	0.273	0.389	0.101	0.337	1.711	0.000	0.486	1.154

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	223	128	390	8547	0	321	633
normalized size	1	1.00	1.43	0.82	2.50	54.79	0.00	2.06	4.06
time (sec)	N/A	0.304	0.697	0.098	0.344	2.521	0.000	0.483	1.135

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	219	143	463	10848	0	335	759
normalized size	1	1.00	1.28	0.84	2.71	63.44	0.00	1.96	4.44
time (sec)	N/A	0.332	1.679	0.142	0.334	1.999	0.000	0.482	1.119

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	265	166	573	0	0	477	1194
normalized size	1	1.00	1.40	0.88	3.03	0.00	0.00	2.52	6.32
time (sec)	N/A	0.369	2.461	0.155	0.339	0.000	0.000	0.532	1.114

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	246	202	720	0	0	537	1314
normalized size	1	1.00	1.12	0.92	3.27	0.00	0.00	2.44	5.97
time (sec)	N/A	0.398	2.039	0.141	0.342	0.000	0.000	0.524	1.103

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	189	240	442	627	877	401	393
normalized size	1	1.00	0.74	0.94	1.73	2.46	3.44	1.57	1.54
time (sec)	N/A	0.557	0.659	0.151	0.334	0.735	104.158	0.341	0.788

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	156	193	344	461	666	327	210
normalized size	1	1.00	0.74	0.91	1.63	2.18	3.16	1.55	1.00
time (sec)	N/A	0.387	0.390	0.128	0.326	1.054	49.877	0.143	0.563

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	134	163	284	474	0	355	265
normalized size	1	1.00	0.74	0.90	1.57	2.62	0.00	1.96	1.46
time (sec)	N/A	0.438	0.784	0.078	0.329	0.459	0.000	0.399	1.086

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	131	137	282	567	0	285	269
normalized size	1	1.00	0.81	0.85	1.75	3.52	0.00	1.77	1.67
time (sec)	N/A	0.384	0.779	0.095	0.337	0.993	0.000	0.452	1.069

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	110	126	359	768	0	286	511
normalized size	1	1.00	0.74	0.85	2.43	5.19	0.00	1.93	3.45
time (sec)	N/A	0.355	1.079	0.094	0.342	0.883	0.000	0.464	1.033

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	106	121	537	928	0	253	749
normalized size	1	1.00	0.80	0.91	4.04	6.98	0.00	1.90	5.63
time (sec)	N/A	0.289	0.722	0.154	0.353	0.734	0.000	0.467	1.032

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	130	842	1314	0	360	1500
normalized size	1	1.00	0.82	0.93	6.01	9.39	0.00	2.57	10.71
time (sec)	N/A	0.223	0.613	0.134	0.376	0.909	0.000	0.512	1.077

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	239	145	1291	1607	0	359	1955
normalized size	1	1.00	1.63	0.99	8.78	10.93	0.00	2.44	13.30
time (sec)	N/A	0.149	6.105	0.134	0.359	0.536	0.000	0.502	0.979

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	350	177	1916	2323	0	563	3138
normalized size	1	1.00	2.43	1.23	13.31	16.13	0.00	3.91	21.79
time (sec)	N/A	0.131	3.244	0.133	0.367	0.649	0.000	0.522	1.141

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	404	218	2731	2967	0	621	3823
normalized size	1	1.00	2.22	1.20	15.01	16.30	0.00	3.41	21.01
time (sec)	N/A	0.159	4.664	0.135	0.378	0.872	0.000	0.537	1.222

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	494	258	3719	3585	0	679	4490
normalized size	1	1.00	2.24	1.17	16.83	16.22	0.00	3.07	20.32
time (sec)	N/A	0.192	6.185	0.143	0.391	0.783	0.000	0.535	1.310

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	548	298	4883	4259	0	737	5190
normalized size	1	1.00	2.21	1.20	19.69	17.17	0.00	2.97	20.93
time (sec)	N/A	0.223	6.182	0.141	0.395	0.549	0.000	0.573	1.416

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	390	270	0	1617	0	553	1124
normalized size	1	1.00	2.64	1.82	0.00	10.93	0.00	3.74	7.59
time (sec)	N/A	0.254	0.347	0.127	0.000	0.745	0.000	0.532	9.796

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	235	175	0	1247	0	418	1046
normalized size	1	1.00	1.69	1.26	0.00	8.97	0.00	3.01	7.53
time (sec)	N/A	0.198	0.271	0.090	0.000	0.560	0.000	0.518	7.852

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	365	142	0	975	0	311	975
normalized size	1	1.00	3.17	1.23	0.00	8.48	0.00	2.70	8.48
time (sec)	N/A	0.115	0.175	0.063	0.000	1.138	0.000	0.372	6.158

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	221	126	0	979	0	329	1007
normalized size	1	1.00	1.77	1.01	0.00	7.83	0.00	2.63	8.06
time (sec)	N/A	0.104	0.170	0.095	0.000	0.936	0.000	0.306	8.151

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	385	159	0	1067	0	415	1243
normalized size	1	1.00	2.83	1.17	0.00	7.85	0.00	3.05	9.14
time (sec)	N/A	0.163	0.254	0.124	0.000	2.215	0.000	0.261	9.193

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	265	190	0	1954	0	463	1517
normalized size	1	1.00	1.44	1.03	0.00	10.62	0.00	2.52	8.24
time (sec)	N/A	0.201	0.389	0.142	0.000	1.023	0.000	0.275	12.211

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	158	223	0	1441	0	0	2191
normalized size	1	1.00	0.90	1.27	0.00	8.23	0.00	0.00	12.52
time (sec)	N/A	0.258	0.888	0.092	0.000	1.470	0.000	0.000	11.239

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	143	144	0	1009	0	13	1861
normalized size	1	1.00	1.13	1.13	0.00	7.94	0.00	0.10	14.65
time (sec)	N/A	0.206	0.473	0.068	0.000	1.172	0.000	0.532	11.048

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	127	94	0	975	0	1	1859
normalized size	1	1.00	1.02	0.75	0.00	7.80	0.00	0.01	14.87
time (sec)	N/A	0.121	0.356	0.076	0.000	1.020	0.000	0.349	12.901

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	128	102	0	975	0	1	1787
normalized size	1	1.00	1.11	0.89	0.00	8.48	0.00	0.01	15.54
time (sec)	N/A	0.098	0.249	0.081	0.000	3.681	0.000	0.161	10.105

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	143	135	0	1305	0	21	2128
normalized size	1	1.00	1.03	0.97	0.00	9.39	0.00	0.15	15.31
time (sec)	N/A	0.181	0.810	0.125	0.000	1.135	0.000	0.225	11.285

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	165	179	0	2206	0	34	2178
normalized size	1	1.00	1.11	1.21	0.00	14.91	0.00	0.23	14.72
time (sec)	N/A	0.211	2.278	0.156	0.000	3.494	0.000	0.215	12.333

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	615	1191	0	7664	0	1078	-1
normalized size	1	1.00	2.62	5.07	0.00	32.61	0.00	4.59	-0.00
time (sec)	N/A	0.486	0.973	0.129	0.000	0.931	0.000	1.338	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	737	1200	0	6266	0	1002	-1
normalized size	1	1.00	3.51	5.71	0.00	29.84	0.00	4.77	-0.00
time (sec)	N/A	0.370	0.698	0.266	0.000	0.913	0.000	1.151	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	597	1116	0	6250	0	1033	-1
normalized size	1	1.00	2.75	5.14	0.00	28.80	0.00	4.76	-0.00
time (sec)	N/A	0.294	0.579	0.110	0.000	0.838	0.000	0.971	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	422	767	0	5238	0	854	-1
normalized size	1	1.00	2.27	4.12	0.00	28.16	0.00	4.59	-0.01
time (sec)	N/A	0.205	0.512	0.098	0.000	0.638	0.000	0.733	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	597	1112	0	6018	0	1050	-1
normalized size	1	1.00	2.70	5.03	0.00	27.23	0.00	4.75	-0.00
time (sec)	N/A	0.299	0.384	0.135	0.000	0.877	0.000	0.544	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	761	966	0	7793	0	1112	-1
normalized size	1	1.00	2.34	2.97	0.00	23.98	0.00	3.42	-0.00
time (sec)	N/A	0.361	0.881	0.183	0.000	1.558	0.000	0.395	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	262	574	0	6944	0	149	-1
normalized size	1	1.00	0.82	1.79	0.00	21.70	0.00	0.47	-0.00
time (sec)	N/A	0.461	4.775	0.107	0.000	1.407	0.000	1.982	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	238	716	0	6045	0	153	-1
normalized size	1	1.00	1.02	3.07	0.00	25.94	0.00	0.66	-0.00
time (sec)	N/A	0.338	2.612	0.092	0.000	1.364	0.000	1.382	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	225	490	0	5658	0	128	-1
normalized size	1	1.00	1.15	2.51	0.00	29.02	0.00	0.66	-0.01
time (sec)	N/A	0.249	4.206	0.088	0.000	0.704	0.000	1.177	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	253	708	0	6525	0	152	-1
normalized size	1	1.00	1.15	3.22	0.00	29.66	0.00	0.69	-0.00
time (sec)	N/A	0.298	2.086	0.114	0.000	1.368	0.000	0.756	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	230	534	0	6522	0	127	-1
normalized size	1	1.00	1.10	2.54	0.00	31.06	0.00	0.60	-0.00
time (sec)	N/A	0.246	2.871	0.112	0.000	1.259	0.000	0.227	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	272	765	0	8824	0	238	-1
normalized size	1	1.00	1.15	3.23	0.00	37.23	0.00	1.00	-0.00
time (sec)	N/A	0.534	1.915	0.169	0.000	1.077	0.000	0.387	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	1021	3542	0	0	0	1089	-1
normalized size	1	1.00	3.24	11.24	0.00	0.00	0.00	3.46	-0.00
time (sec)	N/A	0.577	1.745	0.166	0.000	0.000	0.000	2.044	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	802	2563	0	0	0	1506	-1
normalized size	1	1.00	2.77	8.84	0.00	0.00	0.00	5.19	-0.00
time (sec)	N/A	0.467	1.330	0.157	0.000	0.000	0.000	1.637	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	1019	3511	0	0	0	1793	-1
normalized size	1	1.00	3.26	11.22	0.00	0.00	0.00	5.73	-0.00
time (sec)	N/A	0.498	1.961	0.171	0.000	0.000	0.000	1.427	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	802	2916	0	0	0	1576	-1
normalized size	1	1.00	2.78	10.12	0.00	0.00	0.00	5.47	-0.00
time (sec)	N/A	0.519	1.323	0.128	0.000	0.000	0.000	1.054	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	1018	3512	0	0	0	1125	-1
normalized size	1	1.00	3.25	11.22	0.00	0.00	0.00	3.59	-0.00
time (sec)	N/A	0.463	1.420	0.175	0.000	0.000	0.000	0.870	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	1189	3159	0	0	0	1781	-1
normalized size	1	1.00	1.93	5.12	0.00	0.00	0.00	2.89	-0.00
time (sec)	N/A	0.819	5.618	0.234	0.000	0.000	0.000	0.650	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	331	2236	0	0	0	389	-1
normalized size	1	1.00	1.04	7.01	0.00	0.00	0.00	1.22	-0.00
time (sec)	N/A	0.485	4.088	0.151	0.000	0.000	0.000	3.348	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	351	2681	0	0	0	451	-1
normalized size	1	1.00	1.02	7.77	0.00	0.00	0.00	1.31	-0.00
time (sec)	N/A	0.733	3.373	0.165	0.000	0.000	0.000	2.540	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	316	2214	0	0	0	362	-1
normalized size	1	1.00	1.01	7.05	0.00	0.00	0.00	1.15	-0.00
time (sec)	N/A	0.648	4.882	0.171	0.000	0.000	0.000	1.711	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	343	2670	0	0	0	449	-1
normalized size	1	1.00	0.99	7.67	0.00	0.00	0.00	1.29	-0.00
time (sec)	N/A	0.664	5.003	0.186	0.000	0.000	0.000	1.003	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	333	2290	0	0	0	391	-1
normalized size	1	1.00	1.04	7.16	0.00	0.00	0.00	1.22	-0.00
time (sec)	N/A	0.609	3.041	0.188	0.000	0.000	0.000	0.279	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	357	2747	0	0	0	486	-1
normalized size	1	1.00	0.99	7.65	0.00	0.00	0.00	1.35	-0.00
time (sec)	N/A	1.158	3.507	0.247	0.000	0.000	0.000	0.514	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	55	69	113	908	48	63
normalized size	1	1.00	0.96	2.20	2.76	4.52	36.32	1.92	2.52
time (sec)	N/A	0.018	0.104	0.041	0.448	0.608	5.781	0.122	0.156

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	45	44	0	596	0	281	205
normalized size	1	1.00	0.26	0.25	0.00	3.39	0.00	1.60	1.16
time (sec)	N/A	0.159	0.077	0.054	0.000	1.234	0.000	0.198	1.152

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	141	113	0	0	0	0	-1
normalized size	1	1.00	0.32	0.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.975	0.335	0.063	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	134	128	0	0	0	1	857
normalized size	1	1.00	0.77	0.73	0.00	0.00	0.00	0.01	4.90
time (sec)	N/A	0.279	0.176	0.057	0.000	0.000	0.000	0.352	58.564

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	160	162	0	0	0	1	-1
normalized size	1	1.00	0.65	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.517	0.258	0.063	0.000	0.000	0.000	0.797	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	439	124	0	3507	0	5246	-1
normalized size	1	1.00	1.81	0.51	0.00	14.49	0.00	21.68	-0.00
time (sec)	N/A	0.518	1.027	0.068	0.000	1.242	0.000	2.864	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	87	61	0	692	0	10	325
normalized size	1	1.00	1.23	0.86	0.00	9.75	0.00	0.14	4.58
time (sec)	N/A	0.138	0.217	0.062	0.000	1.080	0.000	0.117	4.209

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	127	64	0	3773	0	1	-1
normalized size	1	1.00	0.98	0.50	0.00	29.25	0.00	0.01	-0.01
time (sec)	N/A	0.198	0.125	0.068	0.000	2.760	0.000	0.140	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	437	124	0	3500	0	5248	-1
normalized size	1	1.00	1.92	0.54	0.00	15.35	0.00	23.02	-0.00
time (sec)	N/A	0.417	0.950	0.070	0.000	2.646	0.000	2.947	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	160	0	155	0	143	285
normalized size	1	1.00	0.84	1.93	0.00	1.87	0.00	1.72	3.43
time (sec)	N/A	0.100	0.450	0.056	0.000	0.947	0.000	0.160	2.707

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	99	0	708	0	0	273
normalized size	1	1.00	0.93	1.43	0.00	10.26	0.00	0.00	3.96
time (sec)	N/A	0.077	0.504	0.053	0.000	1.554	0.000	0.000	4.814

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	67	34	20	124	29	35
normalized size	1	1.00	1.06	3.72	1.89	1.11	6.89	1.61	1.94
time (sec)	N/A	0.052	0.004	0.049	0.334	2.569	6.088	2.016	1.364

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	78	25	12	153	28	25
normalized size	1	1.00	0.90	3.90	1.25	0.60	7.65	1.40	1.25
time (sec)	N/A	0.050	0.003	0.045	0.320	0.739	3.695	0.130	1.325

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	17	6	17	14	6
normalized size	1	1.00	1.00	1.17	2.83	1.00	2.83	2.33	1.00
time (sec)	N/A	0.044	0.002	0.030	0.332	1.505	2.125	0.117	1.287

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	11	5	5	2	5	5
normalized size	1	1.00	1.00	2.20	1.00	1.00	0.40	1.00	1.00
time (sec)	N/A	0.042	0.000	0.038	0.332	0.811	1.175	0.115	1.261

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	12	8	10	11	5	8	7
normalized size	1	1.00	1.71	1.14	1.43	1.57	0.71	1.14	1.00
time (sec)	N/A	0.028	0.003	0.025	0.447	0.781	0.202	0.130	0.072

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	50	40	151	0	52	54
normalized size	1	1.00	0.91	2.27	1.82	6.86	0.00	2.36	2.45
time (sec)	N/A	0.045	0.004	0.051	0.432	1.357	0.000	0.135	1.303

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	94	69	488	0	67	118
normalized size	1	1.00	0.97	2.69	1.97	13.94	0.00	1.91	3.37
time (sec)	N/A	0.060	0.005	0.060	0.412	1.550	0.000	0.124	1.288

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	63	95	152	117	250	121	76
normalized size	1	1.00	0.71	1.07	1.71	1.31	2.81	1.36	0.85
time (sec)	N/A	0.057	0.176	0.137	0.331	0.702	3.220	0.156	1.425

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	48	65	136	82	85	108	48
normalized size	1	1.00	1.04	1.41	2.96	1.78	1.85	2.35	1.04
time (sec)	N/A	0.042	0.140	0.128	0.345	2.507	1.610	0.132	1.340

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	43	70	76	59	150	71	38
normalized size	1	1.00	0.70	1.15	1.25	0.97	2.46	1.16	0.62
time (sec)	N/A	0.049	0.072	0.067	0.346	1.061	0.921	0.137	0.098

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	39	25	26	41	36	70	25
normalized size	1	1.00	1.39	0.89	0.93	1.46	1.29	2.50	0.89
time (sec)	N/A	0.022	0.012	0.033	0.332	2.025	0.416	0.138	0.091

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	37	39	56	101	0	40	88
normalized size	1	1.00	1.32	1.39	2.00	3.61	0.00	1.43	3.14
time (sec)	N/A	0.032	0.014	0.063	0.445	0.584	0.000	0.142	1.780

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	36	29	47	41	0	32	27
normalized size	1	1.00	1.89	1.53	2.47	2.16	0.00	1.68	1.42
time (sec)	N/A	0.034	0.013	0.148	0.347	0.893	0.000	0.134	0.797

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	71	82	136	324	0	105	127
normalized size	1	1.00	1.69	1.95	3.24	7.71	0.00	2.50	3.02
time (sec)	N/A	0.040	0.024	0.086	0.419	1.665	0.000	0.147	0.122

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	44	65	185	159	0	47	47
normalized size	1	1.00	1.38	2.03	5.78	4.97	0.00	1.47	1.47
time (sec)	N/A	0.034	0.008	0.098	0.327	1.460	0.000	0.158	0.821

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	124	228	1046	0	153	280
normalized size	1	1.00	0.86	1.77	3.26	14.94	0.00	2.19	4.00
time (sec)	N/A	0.047	0.157	0.097	0.427	1.368	0.000	0.154	0.827

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	102	85	486	343	0	83	298
normalized size	1	1.00	1.89	1.57	9.00	6.35	0.00	1.54	5.52
time (sec)	N/A	0.047	0.055	0.129	0.332	2.096	0.000	0.154	0.820

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	98	172	225	212	481	191	121
normalized size	1	1.00	0.62	1.08	1.42	1.33	3.03	1.20	0.76
time (sec)	N/A	0.174	0.321	0.082	0.342	0.524	9.781	0.157	0.359

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	128	242	188	136	196	80
normalized size	1	1.00	0.86	1.73	3.27	2.54	1.84	2.65	1.08
time (sec)	N/A	0.072	0.114	0.082	0.335	1.555	5.268	0.152	0.957

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	79	134	171	143	314	149	95
normalized size	1	1.00	0.66	1.13	1.44	1.20	2.64	1.25	0.80
time (sec)	N/A	0.143	0.292	0.076	0.338	2.223	3.544	0.161	0.217

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	41	45	106	58	134	42
normalized size	1	1.00	0.90	0.84	0.92	2.16	1.18	2.73	0.86
time (sec)	N/A	0.037	0.052	0.041	0.329	0.955	1.723	0.147	0.829

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	70	89	133	446	0	102	182
normalized size	1	1.00	1.27	1.62	2.42	8.11	0.00	1.85	3.31
time (sec)	N/A	0.058	0.293	0.086	0.673	0.906	0.000	0.157	0.172

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	71	119	97	0	131	75
normalized size	1	1.00	0.94	1.34	2.25	1.83	0.00	2.47	1.42
time (sec)	N/A	0.095	0.305	0.093	0.564	1.421	0.000	0.187	0.879

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	233	169	234	759	0	163	220
normalized size	1	1.00	3.64	2.64	3.66	11.86	0.00	2.55	3.44
time (sec)	N/A	0.080	6.841	0.105	0.453	1.915	0.000	0.173	0.885

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	57	96	267	200	0	98	194
normalized size	1	1.00	1.21	2.04	5.68	4.26	0.00	2.09	4.13
time (sec)	N/A	0.063	0.346	0.114	0.409	1.745	0.000	0.192	0.844

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	303	237	347	1472	0	218	327
normalized size	1	1.00	3.16	2.47	3.61	15.33	0.00	2.27	3.41
time (sec)	N/A	0.093	5.873	0.117	0.500	0.748	0.000	0.182	0.893

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	69	158	698	403	0	128	464
normalized size	1	1.00	1.21	2.77	12.25	7.07	0.00	2.25	8.14
time (sec)	N/A	0.060	0.392	0.120	0.344	0.593	0.000	0.176	0.863

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	715	302	483	2824	0	291	582
normalized size	1	1.00	5.46	2.31	3.69	21.56	0.00	2.22	4.44
time (sec)	N/A	0.133	10.226	0.180	0.537	2.176	0.000	0.221	0.922

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	144	267	363	376	774	293	209
normalized size	1	1.00	0.61	1.12	1.53	1.58	3.25	1.23	0.88
time (sec)	N/A	0.332	0.531	0.084	0.451	0.728	24.559	0.193	0.584

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	83	209	349	324	182	286	112
normalized size	1	1.00	0.85	2.13	3.56	3.31	1.86	2.92	1.14
time (sec)	N/A	0.090	0.240	0.086	0.406	0.578	13.837	0.193	0.294

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	120	216	287	257	559	231	166
normalized size	1	1.00	0.59	1.06	1.41	1.27	2.75	1.14	0.82
time (sec)	N/A	0.268	0.338	0.085	0.399	0.548	9.879	0.175	0.419

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	56	63	209	75	222	58
normalized size	1	1.00	0.88	0.84	0.94	3.12	1.12	3.31	0.87
time (sec)	N/A	0.045	0.114	0.041	0.300	1.397	4.965	0.194	0.154

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	100	155	233	1114	0	204	294
normalized size	1	1.00	1.16	1.80	2.71	12.95	0.00	2.37	3.42
time (sec)	N/A	0.076	0.549	0.105	0.443	1.469	0.000	0.206	1.041

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	78	131	215	178	0	197	141
normalized size	1	1.00	0.85	1.42	2.34	1.93	0.00	2.14	1.53
time (sec)	N/A	0.129	0.530	0.091	0.353	1.182	0.000	0.210	0.984

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	399	286	357	1679	0	247	308
normalized size	1	1.00	4.38	3.14	3.92	18.45	0.00	2.71	3.38
time (sec)	N/A	0.096	7.995	0.117	0.564	0.666	0.000	0.240	2.278

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	84	148	382	321	0	208	273
normalized size	1	1.00	1.02	1.80	4.66	3.91	0.00	2.54	3.33
time (sec)	N/A	0.108	0.764	0.125	0.378	0.738	0.000	0.229	0.160

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	472	376	489	2245	0	301	430
normalized size	1	1.00	4.58	3.65	4.75	21.80	0.00	2.92	4.17
time (sec)	N/A	0.132	10.067	0.140	0.494	1.034	0.000	0.232	0.186

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	86	199	824	530	0	213	563
normalized size	1	1.00	1.16	2.69	11.14	7.16	0.00	2.88	7.61
time (sec)	N/A	0.080	0.771	0.125	0.394	0.584	0.000	0.248	0.843

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	1192	467	646	3675	0	383	601
normalized size	1	1.00	7.74	3.03	4.19	23.86	0.00	2.49	3.90
time (sec)	N/A	0.153	14.010	0.140	0.497	0.712	0.000	0.236	0.196

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	163	289	1754	814	0	260	994
normalized size	1	1.00	2.04	3.61	21.92	10.18	0.00	3.25	12.42
time (sec)	N/A	0.074	0.597	0.165	0.434	0.638	0.000	0.242	0.875

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	117	1656	0	3066	0	0	954
normalized size	1	1.00	1.08	15.33	0.00	28.39	0.00	0.00	8.83
time (sec)	N/A	0.113	0.488	0.137	0.000	0.983	0.000	0.000	1.514

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	1497	0	1817	0	226	264
normalized size	1	1.00	0.88	12.37	0.00	15.02	0.00	1.87	2.18
time (sec)	N/A	0.221	0.335	0.115	0.000	1.078	0.000	4.142	1.372

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	79	1148	0	1490	0	0	668
normalized size	1	1.00	1.03	14.91	0.00	19.35	0.00	0.00	8.68
time (sec)	N/A	0.093	0.252	0.111	0.000	0.538	0.000	0.000	1.283

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	993	0	875	0	138	300
normalized size	1	1.00	0.99	12.26	0.00	10.80	0.00	1.70	3.70
time (sec)	N/A	0.133	0.159	0.098	0.000	0.579	0.000	2.933	1.612

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	732	0	659	0	0	426
normalized size	1	1.00	0.96	14.08	0.00	12.67	0.00	0.00	8.19
time (sec)	N/A	0.070	0.046	0.101	0.000	0.592	0.000	0.000	1.144

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	577	0	443	0	68	166
normalized size	1	1.00	1.00	11.54	0.00	8.86	0.00	1.36	3.32
time (sec)	N/A	0.082	0.087	0.094	0.000	0.492	0.000	1.566	0.483

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	459	128	0	23
normalized size	1	1.00	1.00	0.75	0.00	14.34	4.00	0.00	0.72
time (sec)	N/A	0.037	0.010	0.025	0.000	0.588	4.504	0.000	0.870

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	481	0	511	0	0	648
normalized size	1	1.00	0.92	8.15	0.00	8.66	0.00	0.00	10.98
time (sec)	N/A	0.067	0.133	0.131	0.000	0.892	0.000	0.000	1.358

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	335	0	709	0	80	265
normalized size	1	1.00	1.00	5.58	0.00	11.82	0.00	1.33	4.42
time (sec)	N/A	0.079	0.162	0.119	0.000	0.590	0.000	0.719	1.471

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	91	662	0	1644	0	0	2797
normalized size	1	1.00	0.99	7.20	0.00	17.87	0.00	0.00	30.40
time (sec)	N/A	0.105	0.239	0.148	0.000	0.714	0.000	0.000	6.137

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	84	535	0	2444	0	138	710
normalized size	1	1.00	0.95	6.08	0.00	27.77	0.00	1.57	8.07
time (sec)	N/A	0.105	0.535	0.148	0.000	0.572	0.000	0.677	2.456

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	139	1023	0	5500	0	0	6237
normalized size	1	1.00	1.01	7.41	0.00	39.86	0.00	0.00	45.20
time (sec)	N/A	0.162	0.546	0.153	0.000	0.643	0.000	0.000	12.047

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	119	907	0	6046	0	253	1152
normalized size	1	1.00	0.94	7.20	0.00	47.98	0.00	2.01	9.14
time (sec)	N/A	0.147	0.931	0.168	0.000	0.827	0.000	0.704	2.891

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	118	1659	0	3629	0	305	-1
normalized size	1	1.00	0.75	10.50	0.00	22.97	0.00	1.93	-0.01
time (sec)	N/A	0.258	0.596	0.138	0.000	0.555	0.000	7.190	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	106	1403	0	2739	0	0	-1
normalized size	1	1.00	1.02	13.49	0.00	26.34	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.300	0.121	0.000	0.747	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	108	1116	0	1527	0	178	-1
normalized size	1	1.00	1.08	11.16	0.00	15.27	0.00	1.78	-0.01
time (sec)	N/A	0.138	0.617	0.117	0.000	0.499	0.000	5.396	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	75	808	0	1615	0	0	-1
normalized size	1	1.00	0.97	10.49	0.00	20.97	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.350	0.120	0.000	0.561	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	78	404	0	1421	0	126	-1
normalized size	1	1.00	0.99	5.11	0.00	17.99	0.00	1.59	-0.01
time (sec)	N/A	0.077	0.208	0.109	0.000	0.684	0.000	2.159	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	57	0	1320	428	0	54
normalized size	1	1.00	0.97	0.86	0.00	20.00	6.48	0.00	0.82
time (sec)	N/A	0.047	0.045	0.033	0.000	0.711	22.635	0.000	0.914

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	174	1080	0	2143	0	0	-1
normalized size	1	1.00	1.64	10.19	0.00	20.22	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.409	0.150	0.000	0.651	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	105	798	0	3147	0	221	-1
normalized size	1	1.00	0.92	7.00	0.00	27.61	0.00	1.94	-0.01
time (sec)	N/A	0.179	0.922	0.145	0.000	0.612	0.000	0.820	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	230	1265	0	6548	0	0	-1
normalized size	1	1.00	1.46	8.06	0.00	41.71	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.930	0.179	0.000	0.774	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	130	998	0	7894	0	270	-1
normalized size	1	1.00	0.91	6.98	0.00	55.20	0.00	1.89	-0.01
time (sec)	N/A	0.207	1.981	0.177	0.000	0.749	0.000	0.811	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	164	2048	0	5511	0	353	-1
normalized size	1	1.00	1.02	12.80	0.00	34.44	0.00	2.21	-0.01
time (sec)	N/A	0.236	1.423	0.139	0.000	0.573	0.000	8.405	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	149	1884	0	5844	0	0	-1
normalized size	1	1.00	1.12	14.17	0.00	43.94	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.371	0.122	0.000	0.687	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	102	664	0	4486	0	244	-1
normalized size	1	1.00	0.89	5.82	0.00	39.35	0.00	2.14	-0.01
time (sec)	N/A	0.095	0.692	0.123	0.000	0.589	0.000	5.485	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	114	1348	0	4907	0	0	-1
normalized size	1	1.00	0.97	11.52	0.00	41.94	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.684	0.131	0.000	0.786	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	124	1322	0	5183	0	269	-1
normalized size	1	1.00	0.87	9.24	0.00	36.24	0.00	1.88	-0.01
time (sec)	N/A	0.126	0.922	0.121	0.000	0.840	0.000	3.744	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	79	85	0	3934	915	0	87
normalized size	1	1.00	0.82	0.89	0.00	40.98	9.53	0.00	0.91
time (sec)	N/A	0.057	0.180	0.029	0.000	0.564	89.111	0.000	0.944

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	321	2118	0	8083	0	0	-1
normalized size	1	1.00	2.02	13.32	0.00	50.84	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.781	0.172	0.000	0.742	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	165	1694	0	9442	0	367	-1
normalized size	1	1.00	0.96	9.85	0.00	54.90	0.00	2.13	-0.01
time (sec)	N/A	0.265	1.141	0.163	0.000	0.828	0.000	1.775	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	222	2307	0	0	0	0	-1
normalized size	1	1.00	1.02	10.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	1.881	0.214	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	169	1892	0	0	0	436	-1
normalized size	1	1.00	0.83	9.32	0.00	0.00	0.00	2.15	-0.00
time (sec)	N/A	0.341	2.311	0.198	0.000	0.000	0.000	1.777	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	54	64	70	238	41	56
normalized size	1	1.00	1.26	2.84	3.37	3.68	12.53	2.16	2.95
time (sec)	N/A	0.045	0.078	0.058	0.487	0.629	6.525	0.145	0.142

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	14	50	39	71	129	37	39
normalized size	1	1.00	1.40	5.00	3.90	7.10	12.90	3.70	3.90
time (sec)	N/A	0.039	0.010	0.053	0.459	1.008	1.609	0.141	0.058

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	98	75	163	2431	61	66
normalized size	1	1.00	1.07	3.27	2.50	5.43	81.03	2.03	2.20
time (sec)	N/A	0.059	0.072	0.061	0.615	0.600	17.301	0.148	0.836

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	124	52	0	3281	0	0	-1
normalized size	1	1.00	1.06	0.44	0.00	28.04	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.685	0.141	0.000	0.813	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	96	62	0	2419	0	0	61
normalized size	1	1.00	1.33	0.86	0.00	33.60	0.00	0.00	0.85
time (sec)	N/A	0.056	0.259	0.031	0.000	0.686	0.000	0.000	0.961

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	130	51	0	5139	0	0	-1
normalized size	1	1.00	1.53	0.60	0.00	60.46	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.748	0.130	0.000	0.809	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	175	35	0	1327	0	0	-1
normalized size	1	1.00	2.03	0.41	0.00	15.43	0.00	0.00	-0.01
time (sec)	N/A	0.096	1.557	0.135	0.000	0.734	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	684	35	0	3727	0	0	-1
normalized size	1	1.00	4.53	0.23	0.00	24.68	0.00	0.00	-0.01
time (sec)	N/A	0.143	11.684	0.155	0.000	0.984	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	211	521	0	0	0	0	-1
normalized size	1	1.00	0.70	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	1.412	0.190	0.000	0.571	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	168	339	0	0	0	0	-1
normalized size	1	1.00	0.75	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.816	0.173	0.000	0.834	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	0	0	0	-1
normalized size	1	1.00	1.15	2.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.087	0.000	0.000	0.655	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	148	177	0	0	0	0	-1
normalized size	1	1.00	2.11	2.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.489	0.185	0.000	0.453	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	204	318	0	0	0	0	-1
normalized size	1	1.00	0.99	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	3.008	0.246	0.000	0.487	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	149	77	0	4603	0	0	-1
normalized size	1	1.00	0.95	0.49	0.00	29.32	0.00	0.00	-0.01
time (sec)	N/A	0.129	1.249	0.168	0.000	1.028	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	90	0	3161	0	0	60
normalized size	1	1.00	0.89	0.87	0.00	30.39	0.00	0.00	0.58
time (sec)	N/A	0.069	0.489	0.034	0.000	1.038	0.000	0.000	1.024

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	142	63	0	6337	0	0	-1
normalized size	1	1.00	1.14	0.50	0.00	50.70	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.474	0.130	0.000	1.187	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	150	63	0	7350	0	0	-1
normalized size	1	1.00	1.13	0.47	0.00	55.26	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.787	0.159	0.000	1.112	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	66	63	0	3089	0	0	-1
normalized size	1	1.00	0.52	0.50	0.00	24.52	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.125	0.175	0.000	1.021	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	959	63	0	7633	0	0	-1
normalized size	1	1.00	4.68	0.31	0.00	37.23	0.00	0.00	-0.00
time (sec)	N/A	0.172	15.161	0.178	0.000	2.707	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	256	730	0	0	0	0	-1
normalized size	1	1.00	0.72	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.393	2.606	0.197	0.000	0.694	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	213	535	0	0	0	0	-1
normalized size	1	1.00	0.71	1.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.291	1.381	0.188	0.000	0.625	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	169	416	0	0	0	0	-1
normalized size	1	1.00	0.97	2.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.763	0.000	0.000	0.743	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	160	334	0	0	0	0	-1
normalized size	1	1.00	0.76	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.201	0.970	0.170	0.000	0.492	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	197	328	0	0	0	0	-1
normalized size	1	1.00	1.02	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	2.021	0.192	0.000	0.721	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	77	35	0	2479	0	0	-1
normalized size	1	1.00	0.97	0.44	0.00	31.38	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.108	0.117	0.000	1.003	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	34	0	1990	0	0	33
normalized size	1	1.00	1.00	0.89	0.00	52.37	0.00	0.00	0.87
time (sec)	N/A	0.047	0.017	0.030	0.000	0.807	0.000	0.000	1.016

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	0	598	0	0	-1
normalized size	1	1.00	1.00	0.76	0.00	13.00	0.00	0.00	-0.02
time (sec)	N/A	0.069	0.032	0.118	0.000	0.510	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	443	35	0	1503	0	0	-1
normalized size	1	1.00	4.57	0.36	0.00	15.49	0.00	0.00	-0.01
time (sec)	N/A	0.107	9.368	0.151	0.000	0.668	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	179	344	0	0	0	0	-1
normalized size	1	1.00	0.74	1.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	0.855	0.178	0.000	0.654	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	95	86	0	0	0	0	-1
normalized size	1	1.00	0.54	0.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.238	0.125	0.000	0.448	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	0	0	0	-1
normalized size	1	1.00	1.13	1.43	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.074	0.000	0.000	0.721	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	159	133	0	0	0	0	-1
normalized size	1	1.00	0.99	0.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.646	0.203	0.000	0.736	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	219	343	0	0	0	0	-1
normalized size	1	1.00	1.00	1.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	2.227	0.283	0.000	0.493	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	89	35	0	3126	0	0	-1
normalized size	1	1.00	1.16	0.45	0.00	40.60	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.172	0.116	0.000	1.175	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	236	245	0	0	191
normalized size	1	1.00	1.00	0.97	8.14	8.45	0.00	0.00	6.59
time (sec)	N/A	0.044	0.029	0.026	0.832	0.623	0.000	0.000	1.133

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	315	101	0	1717	0	0	-1
normalized size	1	1.00	3.71	1.19	0.00	20.20	0.00	0.00	-0.01
time (sec)	N/A	0.097	7.577	0.172	0.000	0.768	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	231	95	0	4845	0	0	-1
normalized size	1	1.00	1.63	0.67	0.00	34.12	0.00	0.00	-0.01
time (sec)	N/A	0.165	5.209	0.227	0.000	1.277	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	196	498	0	0	0	0	-1
normalized size	1	1.00	0.60	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	1.112	0.184	0.000	0.517	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	155	322	0	0	0	0	-1
normalized size	1	1.00	0.64	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.621	0.194	0.000	0.563	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	143	181	0	0	0	0	-1
normalized size	1	1.00	1.57	1.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.320	0.175	0.000	0.609	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	253	0	0	0	0	-1
normalized size	1	1.00	0.87	2.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.156	0.000	0.000	0.988	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	178	345	0	0	0	0	-1
normalized size	1	1.00	0.82	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	1.219	0.211	0.000	0.725	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	126	65	0	6774	0	0	-1
normalized size	1	1.00	0.94	0.49	0.00	50.55	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.888	0.144	0.000	2.140	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	50	65	927	945	0	0	144
normalized size	1	1.00	0.68	0.89	12.70	12.95	0.00	0.00	1.97
time (sec)	N/A	0.093	0.101	0.138	0.517	0.990	0.000	0.000	1.831

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	47	56	485	912	0	0	129
normalized size	1	1.00	0.72	0.86	7.46	14.03	0.00	0.00	1.98
time (sec)	N/A	0.058	0.046	0.025	0.468	2.053	0.000	0.000	1.486

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	1331	169	0	5396	0	0	-1
normalized size	1	1.00	9.93	1.26	0.00	40.27	0.00	0.00	-0.01
time (sec)	N/A	0.149	9.366	0.206	0.000	1.237	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	206	812	0	0	0	0	-1
normalized size	1	1.00	0.62	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	2.245	0.219	0.000	0.475	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	178	597	0	0	0	0	-1
normalized size	1	1.00	0.80	2.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	1.435	0.195	0.000	0.637	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	193	662	0	0	0	0	-1
normalized size	1	1.00	0.85	2.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	1.439	0.224	0.000	2.880	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	0	0	0	-1
normalized size	1	1.00	0.76	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	1.301	0.002	0.000	1.480	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	260	1002	0	0	0	0	-1
normalized size	1	1.00	0.89	3.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	3.457	0.242	0.000	1.944	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	9.198	0.724	0.000	2.829	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	10.762	0.967	0.000	0.759	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	119	120	0	0	0	0	0	-1
normalized size	1	0.95	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.202	0.687	0.000	0.485	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	64
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.96
time (sec)	N/A	0.046	0.026	0.487	0.000	1.495	0.000	0.000	1.503

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	3.509	0.405	0.000	0.579	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	5.719	0.449	0.000	0.717	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	10.017	0.717	0.000	0.647	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	10.596	0.582	0.000	0.830	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.893	0.203	0.000	1.686	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	4.434	0.399	0.000	0.851	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	8.400	0.392	0.000	0.858	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	220	780	0	2595	0	0	-1
normalized size	1	1.00	0.85	3.01	0.00	10.02	0.00	0.00	-0.00
time (sec)	N/A	0.296	0.457	0.215	0.000	3.911	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	117	359	0	879	0	0	-1
normalized size	1	1.00	0.86	2.64	0.00	6.46	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.149	0.197	0.000	2.074	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	89	0	225	68	0	39
normalized size	1	1.00	0.95	2.07	0.00	5.23	1.58	0.00	0.91
time (sec)	N/A	0.050	0.028	0.028	0.000	2.306	1.818	0.000	1.008

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	229	206	0	0	0	0	-1
normalized size	1	1.00	0.80	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.482	0.251	0.192	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	288	955	0	5181	0	0	-1
normalized size	1	1.00	1.07	3.54	0.00	19.19	0.00	0.00	-0.00
time (sec)	N/A	0.323	0.610	0.392	0.000	2.135	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	123	481	0	2137	0	0	-1
normalized size	1	1.00	0.87	3.39	0.00	15.05	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.472	0.374	0.000	1.975	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	144	0	564	151	0	45
normalized size	1	1.00	0.86	2.94	0.00	11.51	3.08	0.00	0.92
time (sec)	N/A	0.055	0.061	0.034	0.000	0.528	4.360	0.000	1.406

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	280	567	0	0	0	0	-1
normalized size	1	1.00	0.73	1.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.631	0.719	0.478	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.144	0.351	0.000	0.710	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.056	0.293	0.000	1.500	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	38
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.03
time (sec)	N/A	0.043	0.009	0.290	0.000	2.385	0.000	0.000	1.130

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.117	5.816	0.000	0.479	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.054	5.585	0.000	2.914	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	38
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.03
time (sec)	N/A	0.044	0.009	6.026	0.000	2.035	0.000	0.000	0.948

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	23	41	45	47	0	25	27
normalized size	1	1.00	1.35	2.41	2.65	2.76	0.00	1.47	1.59
time (sec)	N/A	0.032	0.014	0.090	0.330	1.035	0.000	0.130	0.084

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	51	42	292	875	0	80	252
normalized size	1	1.00	0.81	0.67	4.63	13.89	0.00	1.27	4.00
time (sec)	N/A	0.128	0.100	0.237	0.881	2.953	0.000	0.205	0.940

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	42	106	311	0	53	67
normalized size	1	1.00	0.76	1.11	2.79	8.18	0.00	1.39	1.76
time (sec)	N/A	0.113	0.086	0.219	1.187	0.700	0.000	0.190	0.910

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	32	139	0	26	18
normalized size	1	1.00	1.00	1.06	1.78	7.72	0.00	1.44	1.00
time (sec)	N/A	0.069	0.040	0.076	1.760	1.021	0.000	0.136	0.922

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	42	68	200	0	51	-1
normalized size	1	1.00	0.84	0.84	1.36	4.00	0.00	1.02	-0.02
time (sec)	N/A	0.098	0.064	0.180	2.340	2.386	0.000	0.137	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	54	126	764	0	89	-1
normalized size	1	1.00	0.89	0.62	1.45	8.78	0.00	1.02	-0.01
time (sec)	N/A	0.142	0.262	0.200	1.606	1.386	0.000	0.150	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	75	85	891	1645	0	99	-1
normalized size	1	1.00	0.62	0.71	7.42	13.71	0.00	0.82	-0.01
time (sec)	N/A	0.130	0.360	0.265	0.956	0.573	0.000	0.232	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	69	387	742	0	74	-1
normalized size	1	1.00	0.60	0.76	4.25	8.15	0.00	0.81	-0.01
time (sec)	N/A	0.125	0.200	0.209	1.217	2.119	0.000	0.201	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	40	41	50	182	0	38	-1
normalized size	1	1.00	0.70	0.72	0.88	3.19	0.00	0.67	-0.02
time (sec)	N/A	0.109	0.054	0.203	1.416	0.920	0.000	0.142	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	35	42	125	317	0	57	67
normalized size	1	1.00	0.62	0.75	2.23	5.66	0.00	1.02	1.20
time (sec)	N/A	0.114	0.075	0.237	1.478	1.125	0.000	0.155	0.927

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	47	55	487	885	0	80	281
normalized size	1	1.00	0.52	0.60	5.35	9.73	0.00	0.88	3.09
time (sec)	N/A	0.122	0.078	0.204	1.109	0.681	0.000	0.166	0.942

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	67	65	1050	1696	0	104	427
normalized size	1	1.00	0.54	0.52	8.47	13.68	0.00	0.84	3.44
time (sec)	N/A	0.125	0.201	0.212	0.739	0.624	0.000	0.201	0.906

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	43	41	446	1387	0	95	381
normalized size	1	1.00	0.65	0.62	6.76	21.02	0.00	1.44	5.77
time (sec)	N/A	0.129	0.095	0.232	1.970	0.494	0.000	0.399	0.888

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	31	41	184	641	0	65	82
normalized size	1	1.00	0.74	0.98	4.38	15.26	0.00	1.55	1.95
time (sec)	N/A	0.118	0.076	0.189	1.890	0.428	0.000	0.314	0.118

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	33	168	0	29	30
normalized size	1	1.00	1.00	1.05	1.74	8.84	0.00	1.53	1.58
time (sec)	N/A	0.070	0.032	0.089	2.602	0.516	0.000	0.198	0.847

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	49	33	40	174	0	0	-1
normalized size	1	1.00	1.58	1.06	1.29	5.61	0.00	0.00	-0.03
time (sec)	N/A	0.086	0.054	0.160	3.001	0.533	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	42	100	529	0	0	-1
normalized size	1	1.00	0.98	0.64	1.52	8.02	0.00	0.00	-0.02
time (sec)	N/A	0.129	0.123	0.211	2.619	0.921	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	68	628	1328	0	96	-1
normalized size	1	1.00	0.73	0.75	6.90	14.59	0.00	1.05	-0.01
time (sec)	N/A	0.144	0.139	0.235	0.725	0.468	0.000	0.359	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	44	51	217	504	0	63	-1
normalized size	1	1.00	0.71	0.82	3.50	8.13	0.00	1.02	-0.02
time (sec)	N/A	0.121	0.058	0.215	0.495	0.577	0.000	0.268	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	101	170	0	0	76
normalized size	1	1.00	1.00	1.28	4.04	6.80	0.00	0.00	3.04
time (sec)	N/A	0.104	0.040	0.162	0.527	0.511	0.000	0.000	0.112

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	37	44	556	647	0	0	95
normalized size	1	1.00	0.61	0.72	9.11	10.61	0.00	0.00	1.56
time (sec)	N/A	0.115	0.065	0.209	0.731	0.433	0.000	0.000	0.889

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	49	54	1231	1399	0	0	381
normalized size	1	1.00	0.51	0.56	12.82	14.57	0.00	0.00	3.97
time (sec)	N/A	0.123	0.092	0.249	0.651	0.532	0.000	0.000	0.899

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	44	586	2507	0	98	457
normalized size	1	1.00	0.75	0.65	8.62	36.87	0.00	1.44	6.72
time (sec)	N/A	0.141	0.118	0.288	0.700	0.560	0.000	0.536	0.163

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	34	44	268	1400	0	68	305
normalized size	1	1.00	0.77	1.00	6.09	31.82	0.00	1.55	6.93
time (sec)	N/A	0.126	0.127	0.245	0.715	0.452	0.000	0.441	0.926

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	61	608	0	32	58
normalized size	1	1.00	1.00	0.95	2.90	28.95	0.00	1.52	2.76
time (sec)	N/A	0.077	0.040	0.081	0.603	0.498	0.000	0.296	0.877

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	41	44	76	271	0	0	-1
normalized size	1	1.00	0.77	0.83	1.43	5.11	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.064	0.158	1.025	0.466	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	36	100	565	0	0	-1
normalized size	1	1.00	1.02	0.55	1.52	8.56	0.00	0.00	-0.02
time (sec)	N/A	0.139	0.134	0.184	0.861	0.524	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	58	69	369	1423	0	93	-1
normalized size	1	1.00	0.55	0.65	3.48	13.42	0.00	0.88	-0.01
time (sec)	N/A	0.160	0.103	0.217	0.565	0.528	0.000	0.369	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	46	51	321	254	0	0	-1
normalized size	1	1.00	0.72	0.80	5.02	3.97	0.00	0.00	-0.02
time (sec)	N/A	0.132	0.084	0.212	0.518	0.559	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	823	612	0	0	71
normalized size	1	1.00	0.76	0.92	21.66	16.11	0.00	0.00	1.87
time (sec)	N/A	0.126	0.051	0.188	0.797	0.601	0.000	0.000	0.899

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	41	67	1531	1410	0	68	305
normalized size	1	1.00	0.53	0.87	19.88	18.31	0.00	0.88	3.96
time (sec)	N/A	0.144	0.109	0.290	0.622	0.496	0.000	0.592	0.158

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	51	57	2216	2511	0	98	457
normalized size	1	1.00	0.44	0.50	19.27	21.83	0.00	0.85	3.97
time (sec)	N/A	0.150	0.142	0.258	0.684	0.565	0.000	0.665	0.950

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	151	43	0	4704	0	0	-1
normalized size	1	1.00	0.81	0.23	0.00	25.16	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.564	0.247	0.000	1.607	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	88	43	0	1670	0	0	-1
normalized size	1	1.00	0.70	0.34	0.00	13.25	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.422	0.217	0.000	1.558	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	65	41	0	624	0	0	-1
normalized size	1	1.00	1.05	0.66	0.00	10.06	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.054	0.170	0.000	1.285	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	46	0	605	0	0	-1
normalized size	1	1.00	0.98	0.85	0.00	11.20	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.050	0.156	0.000	0.822	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	69	58	0	1445	0	0	-1
normalized size	1	1.00	0.65	0.55	0.00	13.63	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.379	0.225	0.000	0.916	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	102	80	0	3880	0	0	-1
normalized size	1	1.00	0.61	0.48	0.00	23.23	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.580	0.242	0.000	1.357	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	214	369	0	0	0	0	-1
normalized size	1	1.00	0.73	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.306	2.029	0.314	0.000	0.611	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	150	233	0	0	0	0	-1
normalized size	1	1.00	0.89	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.516	0.290	0.000	0.579	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	0	0	0	-1
normalized size	1	1.00	1.15	2.33	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.089	0.218	0.000	0.519	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	154	215	0	0	0	0	-1
normalized size	1	1.00	0.76	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.584	0.271	0.000	0.627	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	210	522	0	0	0	0	-1
normalized size	1	1.00	0.78	1.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	3.099	0.307	0.000	0.661	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	169	71	0	6380	0	0	-1
normalized size	1	1.00	0.73	0.31	0.00	27.50	0.00	0.00	-0.00
time (sec)	N/A	0.290	1.644	0.255	0.000	1.294	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	122	71	0	2454	0	0	-1
normalized size	1	1.00	0.78	0.46	0.00	15.73	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.561	0.233	0.000	1.176	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	86	69	0	1052	0	0	-1
normalized size	1	1.00	0.96	0.77	0.00	11.69	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.148	0.195	0.000	1.109	0.000	0.000	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	62	0	1000	0	0	-1
normalized size	1	1.00	0.88	0.79	0.00	12.82	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.137	0.170	0.000	0.748	0.000	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	90	84	0	2406	0	0	-1
normalized size	1	1.00	0.64	0.60	0.00	17.19	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.454	0.214	0.000	1.021	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	199	123	113	0	5509	0	0	-1
normalized size	1	0.98	0.61	0.56	0.00	27.14	0.00	0.00	-0.00
time (sec)	N/A	0.232	0.808	0.251	0.000	1.208	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	224	385	0	0	0	0	-1
normalized size	1	1.00	0.73	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	2.809	0.309	0.000	0.478	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	188	414	0	0	0	0	-1
normalized size	1	1.00	0.72	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	2.908	0.280	0.000	0.642	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	169	416	0	0	0	0	-1
normalized size	1	1.00	0.97	2.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.762	0.242	0.000	0.427	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	184	330	0	0	0	0	-1
normalized size	1	1.00	0.72	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	2.154	0.296	0.000	0.538	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	229	540	0	0	0	0	-1
normalized size	1	1.00	0.75	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	4.715	0.303	0.000	0.692	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	116	43	0	4100	0	0	-1
normalized size	1	1.00	0.82	0.30	0.00	28.87	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.474	0.251	0.000	0.999	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	43	0	1320	0	754	-1
normalized size	1	1.00	0.96	0.48	0.00	14.83	0.00	8.47	-0.01
time (sec)	N/A	0.112	0.119	0.219	0.000	0.648	0.000	5.147	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	41	0	433	0	0	-1
normalized size	1	1.00	1.07	1.00	0.00	10.56	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.041	0.183	0.000	0.827	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	0	410	0	0	-1
normalized size	1	1.00	1.00	1.06	0.00	12.42	0.00	0.00	-0.03
time (sec)	N/A	0.071	0.038	0.149	0.000	0.545	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	72	44	0	1144	0	0	-1
normalized size	1	1.00	0.94	0.57	0.00	14.86	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.204	0.204	0.000	0.801	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	100	54	0	3086	0	0	-1
normalized size	1	1.00	0.79	0.43	0.00	24.49	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.372	0.221	0.000	0.775	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	206	366	0	0	0	1320	-1
normalized size	1	1.00	0.94	1.67	0.00	0.00	0.00	6.03	-0.00
time (sec)	N/A	0.196	2.134	0.330	0.000	0.542	0.000	9.516	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	109	239	0	0	0	374	-1
normalized size	1	1.00	0.70	1.53	0.00	0.00	0.00	2.40	-0.01
time (sec)	N/A	0.181	0.415	0.298	0.000	0.467	0.000	2.664	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	0	0	0	-1
normalized size	1	1.00	1.13	1.43	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.071	0.158	0.000	0.745	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	105	217	0	0	0	0	-1
normalized size	1	1.00	0.51	1.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.399	0.274	0.000	0.494	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	208	522	0	0	0	0	-1
normalized size	1	1.00	0.73	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	3.543	0.335	0.000	0.510	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	113	103	0	10168	0	2634	-1
normalized size	1	1.00	0.60	0.55	0.00	54.37	0.00	14.09	-0.01
time (sec)	N/A	0.251	0.473	0.399	0.000	1.147	0.000	51.395	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	79	103	0	4050	0	975	-1
normalized size	1	1.00	0.65	0.84	0.00	33.20	0.00	7.99	-0.01
time (sec)	N/A	0.140	0.119	0.404	0.000	0.738	0.000	15.037	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	93	0	1370	0	0	-1
normalized size	1	1.00	0.84	1.35	0.00	19.86	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.077	0.234	0.000	0.657	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	35	0	1137	0	0	-1
normalized size	1	1.00	0.81	0.61	0.00	19.95	0.00	0.00	-0.02
time (sec)	N/A	0.084	0.058	0.138	0.000	0.548	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	69	43	0	3228	0	0	-1
normalized size	1	1.00	0.63	0.39	0.00	29.35	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.107	0.188	0.000	0.680	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	94	43	0	7562	0	0	-1
normalized size	1	1.00	0.56	0.26	0.00	45.28	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.331	0.222	0.000	0.869	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	212	354	0	0	0	0	-1
normalized size	1	1.00	0.77	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	2.254	0.360	0.000	0.477	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	158	256	0	0	0	0	-1
normalized size	1	1.00	0.73	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	1.329	0.322	0.000	0.481	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	252	0	0	0	0	-1
normalized size	1	1.00	0.87	2.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.162	0.256	0.000	0.528	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	153	219	0	0	0	0	-1
normalized size	1	1.00	0.65	0.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.270	0.802	0.294	0.000	0.463	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	214	522	0	0	0	0	-1
normalized size	1	1.00	0.63	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.408	3.290	0.340	0.000	0.502	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	114	213	0	0	0	3972	-1
normalized size	1	1.00	0.49	0.92	0.00	0.00	0.00	17.12	-0.00
time (sec)	N/A	0.316	0.598	0.454	0.000	0.000	0.000	104.390	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	82	213	0	10506	0	2132	-1
normalized size	1	1.00	0.50	1.31	0.00	64.45	0.00	13.08	-0.01
time (sec)	N/A	0.166	0.127	0.388	0.000	1.390	0.000	40.897	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	60	173	0	4200	0	0	-1
normalized size	1	1.00	0.61	1.75	0.00	42.42	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.087	0.319	0.000	0.698	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	49	65	0	3084	0	0	-1
normalized size	1	1.00	0.59	0.78	0.00	37.16	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.065	0.172	0.000	0.625	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	69	73	0	7594	0	0	-1
normalized size	1	1.00	0.48	0.51	0.00	53.10	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.306	0.219	0.000	0.813	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	117	73	0	0	0	0	-1
normalized size	1	1.00	0.56	0.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.428	0.265	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	252	663	0	0	0	0	-1
normalized size	1	1.00	0.76	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	3.461	0.395	0.000	0.562	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	215	799	0	0	0	0	-1
normalized size	1	1.00	0.78	2.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	2.707	0.362	0.000	0.514	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	0	0	0	-1
normalized size	1	1.00	0.76	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.962	0.580	0.000	0.539	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	226	642	0	0	0	0	-1
normalized size	1	1.00	0.64	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	2.704	0.381	0.000	0.704	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	247	923	0	0	0	0	-1
normalized size	1	1.00	0.64	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.562	2.888	0.374	0.000	0.586	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	10.592	0.451	0.000	1.184	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	90	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.263	0.584	0.000	2.044	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.072	0.440	0.000	2.064	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	0.061	0.449	0.000	0.594	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	71	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.385	0.472	0.000	1.518	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	36.301	0.422	0.000	0.604	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	5.649	0.411	0.000	0.644	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	6.707	0.409	0.000	0.788	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	38.399	0.405	0.000	1.245	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	136	132	0	1115	0	209	1129
normalized size	1	1.00	0.89	0.87	0.00	7.34	0.00	1.38	7.43
time (sec)	N/A	0.252	0.355	0.151	0.000	4.050	0.000	0.172	0.917

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	0	0	0	-1
normalized size	1	1.00	1.00	0.75	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	0.025	0.579	0.000	0.000	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	0	1663	0	0	-1
normalized size	1	1.00	1.00	0.76	0.00	36.96	0.00	0.00	-0.02
time (sec)	N/A	0.082	0.023	0.125	0.000	6.285	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	113	0	0	-1
normalized size	1	1.00	1.00	0.83	0.00	3.90	0.00	0.00	-0.03
time (sec)	N/A	0.087	0.027	0.066	0.000	2.247	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	38	0	156	0	0	-1
normalized size	1	1.00	0.96	0.81	0.00	3.32	0.00	0.00	-0.02
time (sec)	N/A	0.092	0.023	0.046	0.000	0.480	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [182] had the largest ratio of [1.000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	21	0.143
2	A	3	2	1.00	21	0.095
3	A	3	3	1.00	21	0.143
4	A	2	1	1.00	19	0.053
5	A	3	2	1.00	12	0.167
6	A	2	2	1.00	19	0.105
7	A	2	2	1.00	21	0.095
8	A	2	2	1.00	21	0.095
9	A	3	3	1.00	21	0.143
10	A	6	6	1.00	23	0.261
11	A	3	2	1.00	23	0.087
12	A	2	2	1.06	23	0.087
13	A	3	2	1.00	21	0.095
14	A	1	1	1.00	14	0.071
15	A	4	3	1.00	21	0.143
16	A	4	4	1.28	23	0.174
17	A	5	4	1.00	23	0.174
18	A	4	3	1.00	23	0.130
19	A	7	6	1.00	23	0.261
20	A	3	2	1.00	23	0.087
21	A	3	2	1.00	23	0.087
22	A	3	2	1.00	21	0.095
23	A	2	2	1.00	14	0.143
24	A	4	3	1.00	21	0.143
25	A	5	5	1.00	23	0.217
26	A	5	4	1.00	23	0.174
27	A	5	4	1.00	23	0.174
28	A	4	3	1.00	23	0.130
29	A	6	6	1.00	23	0.261
30	A	4	3	1.00	23	0.130
31	A	5	5	1.00	23	0.217
32	A	3	3	1.00	23	0.130
33	A	3	3	1.00	23	0.130
34	A	2	2	1.00	21	0.095
35	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	4	4	1.00	21	0.190
37	A	3	3	1.00	23	0.130
38	A	5	5	1.00	23	0.217
39	A	4	3	1.00	23	0.130
40	A	6	6	1.00	23	0.261
41	A	4	3	1.00	23	0.130
42	A	5	5	1.00	23	0.217
43	A	3	3	1.00	23	0.130
44	A	4	4	1.00	23	0.174
45	A	3	3	1.00	21	0.143
46	A	4	4	1.00	14	0.286
47	A	5	5	1.00	21	0.238
48	A	4	4	1.00	23	0.174
49	A	6	6	1.00	23	0.261
50	A	5	4	1.00	23	0.174
51	A	4	3	1.00	23	0.130
52	A	4	4	1.00	23	0.174
53	A	5	4	1.00	23	0.174
54	A	4	3	1.00	21	0.143
55	A	5	5	1.00	14	0.357
56	A	6	6	1.00	21	0.286
57	A	5	5	1.00	23	0.217
58	A	7	6	1.00	23	0.261
59	A	6	5	1.00	23	0.217
60	A	3	3	1.00	8	0.375
61	A	3	2	1.00	8	0.250
62	A	3	2	1.00	8	0.250
63	A	2	2	1.00	10	0.200
64	A	4	4	1.00	10	0.400
65	A	5	5	1.00	10	0.500
66	A	5	5	1.00	25	0.200
67	A	4	4	1.00	23	0.174
68	A	6	5	1.00	23	0.217
69	A	4	4	1.00	25	0.160
70	A	5	5	1.00	25	0.200
71	A	7	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
72	A	6	6	1.00	25	0.240
73	A	2	2	1.00	16	0.125
74	A	7	7	1.00	25	0.280
75	A	7	7	1.00	25	0.280
76	A	6	5	1.00	25	0.200
77	A	5	4	1.00	23	0.174
78	A	7	6	1.00	23	0.261
79	A	7	6	1.00	25	0.240
80	A	5	4	1.00	25	0.160
81	A	6	5	1.00	25	0.200
82	A	8	7	1.00	25	0.280
83	A	7	6	1.00	25	0.240
84	A	6	6	1.00	16	0.375
85	A	6	6	1.00	25	0.240
86	A	7	7	1.00	25	0.280
87	A	7	7	1.00	16	0.438
88	A	3	3	1.00	10	0.300
89	A	3	3	1.00	12	0.250
90	A	1	1	1.00	12	0.083
91	A	2	2	1.00	10	0.200
92	A	2	2	1.00	12	0.167
93	A	4	4	1.00	10	0.400
94	A	4	4	1.00	12	0.333
95	A	4	4	1.00	12	0.333
96	A	6	6	1.00	10	0.600
97	A	6	6	1.00	12	0.500
98	A	4	4	1.00	25	0.160
99	A	3	3	1.00	23	0.130
100	A	3	3	1.00	23	0.130
101	A	4	4	1.00	25	0.160
102	A	6	6	1.00	25	0.240
103	A	5	5	1.00	25	0.200
104	A	2	2	1.00	16	0.125
105	A	5	5	1.00	25	0.200
106	A	7	7	1.00	25	0.280
107	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	2	2	1.00	23	0.087
109	A	4	4	1.00	23	0.174
110	A	6	6	1.00	25	0.240
111	A	7	7	1.00	25	0.280
112	A	6	6	1.00	25	0.240
113	A	6	6	1.00	25	0.240
114	A	4	4	1.00	16	0.250
115	A	7	7	1.00	25	0.280
116	A	5	5	1.00	25	0.200
117	A	3	3	1.00	25	0.120
118	A	3	3	1.00	23	0.130
119	A	6	6	1.00	23	0.261
120	A	7	7	1.00	25	0.280
121	A	5	5	1.00	25	0.200
122	A	7	6	1.00	25	0.240
123	A	7	7	1.00	16	0.438
124	A	8	8	1.00	25	0.320
125	A	3	3	1.00	10	0.300
126	A	1	1	1.00	12	0.083
127	A	2	2	1.00	10	0.200
128	A	3	3	1.00	12	0.250
129	A	2	2	1.00	12	0.167
130	A	3	3	1.00	25	0.120
131	A	5	5	1.00	23	0.217
132	A	4	4	1.00	23	0.174
133	A	3	3	1.00	21	0.143
134	A	3	3	1.00	21	0.143
135	A	3	3	1.00	23	0.130
136	A	3	3	1.00	23	0.130
137	A	3	3	1.00	23	0.130
138	A	3	3	1.00	23	0.130
139	A	3	3	1.00	23	0.130
140	A	3	3	1.00	23	0.130
141	A	7	4	1.00	21	0.190
142	A	8	4	1.00	21	0.190
143	A	6	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
144	A	6	4	1.00	19	0.210
145	A	3	1	1.00	12	0.083
146	A	5	4	1.00	19	0.210
147	A	5	4	1.00	21	0.190
148	A	4	3	1.00	21	0.143
149	A	5	3	1.00	21	0.143
150	A	10	4	1.00	23	0.174
151	A	11	4	1.00	23	0.174
152	A	8	5	1.00	21	0.238
153	A	8	4	1.00	14	0.286
154	A	7	5	1.00	21	0.238
155	A	8	5	1.00	23	0.217
156	A	6	4	1.00	23	0.174
157	A	7	5	1.00	23	0.217
158	A	8	6	1.00	23	0.261
159	A	6	4	1.00	23	0.174
160	A	9	4	1.00	23	0.174
161	A	13	4	1.00	23	0.174
162	A	14	5	1.00	21	0.238
163	A	10	4	1.00	14	0.286
164	A	12	5	1.00	21	0.238
165	A	10	6	1.00	23	0.261
166	A	10	6	1.00	23	0.261
167	A	9	6	1.00	23	0.261
168	A	11	7	1.00	23	0.304
169	A	8	5	1.00	23	0.217
170	A	11	6	1.00	23	0.261
171	A	15	6	1.00	23	0.261
172	A	15	7	1.00	23	0.304
173	A	14	5	1.00	23	0.217
174	A	13	5	1.00	23	0.217
175	A	11	5	1.00	23	0.217
176	A	11	4	1.00	21	0.190
177	A	11	4	1.00	14	0.286
178	A	14	6	1.00	21	0.286
179	A	15	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	15	7	1.00	23	0.304
181	A	16	7	1.00	23	0.304
182	A	12	8	1.00	8	1.000
183	A	12	8	1.00	10	0.800
184	A	6	6	1.00	21	0.286
185	A	3	2	1.00	21	0.095
186	A	5	5	1.00	21	0.238
187	A	2	1	1.00	19	0.053
188	A	4	2	1.00	12	0.167
189	A	4	3	1.00	19	0.158
190	A	4	4	1.00	21	0.190
191	A	4	4	1.00	21	0.190
192	A	4	3	1.00	21	0.143
193	A	4	4	1.00	21	0.190
194	A	3	2	1.00	21	0.095
195	A	5	5	1.00	21	0.238
196	A	3	2	1.00	23	0.087
197	A	7	6	1.00	23	0.261
198	A	3	2	1.00	21	0.095
199	A	6	5	1.00	14	0.357
200	A	4	3	1.00	21	0.143
201	A	6	5	1.00	23	0.217
202	A	5	4	1.00	23	0.174
203	A	6	5	1.00	23	0.217
204	A	6	5	1.00	23	0.217
205	A	5	4	1.00	23	0.174
206	A	6	5	1.00	23	0.217
207	A	3	2	1.00	23	0.087
208	A	3	2	1.00	23	0.087
209	A	3	2	1.00	21	0.095
210	A	4	3	1.00	21	0.143
211	A	5	4	1.00	23	0.174
212	A	6	5	1.00	23	0.217
213	A	7	5	1.00	23	0.217
214	A	8	5	1.00	23	0.217
215	A	8	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	8	5	1.00	23	0.217
217	A	9	6	1.00	23	0.261
218	A	8	5	1.00	14	0.357
219	A	8	5	1.00	23	0.217
220	A	8	5	1.00	23	0.217
221	A	7	5	1.00	23	0.217
222	A	6	5	1.00	23	0.217
223	A	5	4	1.00	23	0.174
224	A	4	3	1.00	23	0.130
225	A	3	2	1.00	23	0.087
226	A	3	2	1.00	23	0.087
227	A	3	2	1.00	23	0.087
228	A	3	2	1.00	23	0.087
229	A	6	5	1.00	24	0.208
230	A	6	5	1.00	24	0.208
231	A	4	4	1.00	24	0.167
232	A	4	4	1.00	22	0.182
233	A	7	6	1.00	22	0.273
234	A	7	6	1.00	24	0.250
235	A	7	5	1.00	24	0.208
236	A	7	5	1.00	24	0.208
237	A	4	3	1.00	24	0.125
238	A	4	3	1.00	15	0.200
239	A	6	4	1.00	24	0.167
240	A	6	4	1.00	24	0.167
241	A	7	6	1.00	24	0.250
242	A	5	5	1.00	24	0.208
243	A	5	5	1.00	24	0.208
244	A	5	5	1.00	24	0.208
245	A	5	5	1.00	22	0.227
246	A	11	7	1.00	22	0.318
247	A	14	9	1.00	24	0.375
248	A	6	5	1.00	24	0.208
249	A	7	6	1.00	24	0.250
250	A	5	4	1.00	24	0.167
251	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
252	A	7	5	1.00	24	0.208
253	A	6	6	1.00	24	0.250
254	A	6	6	1.00	24	0.250
255	A	6	6	1.00	24	0.250
256	A	6	5	1.00	24	0.208
257	A	6	6	1.00	22	0.273
258	A	16	7	1.00	22	0.318
259	A	9	7	1.00	24	0.292
260	A	6	5	1.00	24	0.208
261	A	6	5	1.00	24	0.208
262	A	6	5	1.00	24	0.208
263	A	6	5	1.00	15	0.333
264	A	8	6	1.00	24	0.250
265	A	3	3	1.00	10	0.300
266	A	10	6	1.00	8	0.750
267	A	17	5	1.00	10	0.500
268	A	7	3	1.00	10	0.300
269	A	9	3	1.00	10	0.300
270	A	17	6	1.00	8	0.750
271	A	8	6	1.00	8	0.750
272	A	9	3	1.00	8	0.375
273	A	17	6	1.00	10	0.600
274	A	7	3	1.00	10	0.300
275	A	10	6	1.00	10	0.600
276	A	3	2	1.00	15	0.133
277	A	3	3	1.00	15	0.200
278	A	2	2	1.00	15	0.133
279	A	2	2	1.00	15	0.133
280	A	2	2	1.00	13	0.154
281	A	3	3	1.00	13	0.231
282	A	4	3	1.00	15	0.200
283	A	5	4	1.00	21	0.190
284	A	3	2	1.00	21	0.095
285	A	4	4	1.00	21	0.190
286	A	2	1	1.00	19	0.053
287	A	3	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	3	3	1.00	21	0.143
289	A	3	3	1.00	21	0.143
290	A	2	1	1.00	21	0.048
291	A	4	4	1.00	21	0.190
292	A	3	2	1.00	21	0.095
293	A	6	5	1.00	23	0.217
294	A	3	2	1.00	23	0.087
295	A	5	5	1.00	23	0.217
296	A	3	2	1.00	21	0.095
297	A	4	3	1.00	21	0.143
298	A	5	4	1.00	23	0.174
299	A	5	4	1.00	23	0.174
300	A	4	3	1.00	23	0.130
301	A	4	4	1.00	23	0.174
302	A	3	2	1.00	23	0.087
303	A	5	5	1.00	23	0.217
304	A	7	6	1.00	23	0.261
305	A	3	2	1.00	23	0.087
306	A	6	6	1.00	23	0.261
307	A	3	2	1.00	21	0.095
308	A	4	3	1.00	21	0.143
309	A	6	5	1.00	23	0.217
310	A	5	4	1.00	23	0.174
311	A	5	4	1.00	23	0.174
312	A	6	5	1.00	23	0.217
313	A	4	3	1.00	23	0.130
314	A	5	5	1.00	23	0.217
315	A	3	2	1.00	23	0.087
316	A	4	3	1.00	23	0.130
317	A	6	6	1.00	23	0.261
318	A	4	3	1.00	23	0.130
319	A	5	5	1.00	23	0.217
320	A	3	3	1.00	23	0.130
321	A	4	4	1.00	23	0.174
322	A	2	2	1.00	21	0.095
323	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
324	A	3	3	1.00	23	0.130
325	A	5	5	1.00	23	0.217
326	A	4	3	1.00	23	0.130
327	A	6	6	1.00	23	0.261
328	A	4	3	1.00	23	0.130
329	A	6	6	1.00	23	0.261
330	A	5	4	1.00	23	0.174
331	A	5	5	1.00	23	0.217
332	A	3	3	1.00	23	0.130
333	A	3	3	1.00	23	0.130
334	A	3	3	1.00	21	0.143
335	A	5	5	1.00	21	0.238
336	A	5	4	1.00	23	0.174
337	A	6	6	1.00	23	0.261
338	A	5	4	1.00	23	0.174
339	A	6	6	1.00	23	0.261
340	A	4	4	1.00	23	0.174
341	A	4	3	1.00	23	0.130
342	A	4	4	1.00	23	0.174
343	A	4	4	1.00	23	0.174
344	A	4	3	1.00	21	0.143
345	A	6	6	1.00	21	0.286
346	A	6	5	1.00	23	0.217
347	A	7	6	1.00	23	0.261
348	A	6	5	1.00	23	0.217
349	A	4	3	1.00	15	0.200
350	A	3	3	1.00	15	0.200
351	A	5	4	1.00	15	0.267
352	A	5	5	1.00	25	0.200
353	A	4	4	1.00	23	0.174
354	A	6	6	1.00	23	0.261
355	A	4	4	1.00	25	0.160
356	A	5	5	1.00	25	0.200
357	A	7	7	1.00	25	0.280
358	A	6	6	1.00	25	0.240
359	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
360	A	2	2	1.00	25	0.080
361	A	5	5	1.00	25	0.200
362	A	6	5	1.00	25	0.200
363	A	5	4	1.00	23	0.174
364	A	7	7	1.00	23	0.304
365	A	7	7	1.00	25	0.280
366	A	5	4	1.00	25	0.160
367	A	6	5	1.00	25	0.200
368	A	8	7	1.00	25	0.280
369	A	7	7	1.00	25	0.280
370	A	6	6	1.00	16	0.375
371	A	6	6	1.00	25	0.240
372	A	5	5	1.00	25	0.200
373	A	4	4	1.00	25	0.160
374	A	3	3	1.00	23	0.130
375	A	3	3	1.00	23	0.130
376	A	4	4	1.00	25	0.160
377	A	6	6	1.00	25	0.240
378	A	5	5	1.00	25	0.200
379	A	2	2	1.00	16	0.125
380	A	7	7	1.00	25	0.280
381	A	5	5	1.00	25	0.200
382	A	4	4	1.00	25	0.160
383	A	2	2	1.00	23	0.087
384	A	4	4	1.00	23	0.174
385	A	6	6	1.00	25	0.240
386	A	7	7	1.00	25	0.280
387	A	6	6	1.00	25	0.240
388	A	2	2	1.00	25	0.080
389	A	4	4	1.00	16	0.250
390	A	5	5	1.00	25	0.200
391	A	5	5	1.00	25	0.200
392	A	3	3	1.00	25	0.120
393	A	3	3	1.00	23	0.130
394	A	6	6	1.00	23	0.261
395	A	7	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	5	5	1.00	25	0.200
397	A	5	5	1.00	25	0.200
398	A	7	7	1.00	16	0.438
399	A	6	6	1.00	25	0.240
400	A	3	3	1.00	25	0.120
401	A	5	5	1.00	23	0.217
402	A	4	4	0.95	23	0.174
403	A	3	3	1.00	21	0.143
404	A	3	3	1.00	21	0.143
405	A	3	3	1.00	23	0.130
406	A	3	3	1.00	23	0.130
407	A	3	3	1.00	23	0.130
408	A	3	3	1.00	14	0.214
409	A	3	3	1.00	23	0.130
410	A	3	3	1.00	23	0.130
411	A	4	3	1.00	25	0.120
412	A	4	3	1.00	25	0.120
413	A	4	3	1.00	23	0.130
414	A	19	13	1.00	23	0.565
415	A	4	3	1.00	25	0.120
416	A	4	3	1.00	25	0.120
417	A	4	3	1.00	23	0.130
418	A	19	13	1.00	23	0.565
419	A	6	4	1.00	23	0.174
420	A	5	4	1.00	23	0.174
421	A	2	2	1.00	21	0.095
422	A	6	4	1.00	23	0.174
423	A	5	4	1.00	23	0.174
424	A	2	2	1.00	21	0.095
425	A	4	4	1.00	13	0.308
426	A	5	4	1.00	25	0.160
427	A	5	4	1.00	25	0.160
428	A	4	4	1.00	23	0.174
429	A	5	5	1.00	23	0.217
430	A	7	7	1.00	25	0.280
431	A	7	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	6	6	1.00	25	0.240
433	A	5	5	1.00	25	0.200
434	A	5	4	1.00	25	0.160
435	A	5	4	1.00	25	0.160
436	A	5	4	1.00	25	0.160
437	A	5	4	1.00	25	0.160
438	A	5	4	1.00	25	0.160
439	A	4	4	1.00	23	0.174
440	A	4	4	1.00	23	0.174
441	A	6	6	1.00	25	0.240
442	A	5	4	1.00	25	0.160
443	A	4	4	1.00	25	0.160
444	A	4	4	1.00	25	0.160
445	A	4	3	1.00	25	0.120
446	A	5	4	1.00	25	0.160
447	A	5	4	1.00	25	0.160
448	A	5	4	1.00	25	0.160
449	A	4	4	1.00	23	0.174
450	A	5	5	1.00	23	0.217
451	A	6	6	1.00	25	0.240
452	A	5	5	1.00	25	0.200
453	A	5	5	1.00	25	0.200
454	A	4	4	1.00	25	0.160
455	A	5	4	1.00	25	0.160
456	A	5	4	1.00	25	0.160
457	A	6	6	1.00	25	0.240
458	A	5	5	1.00	25	0.200
459	A	4	4	1.00	23	0.174
460	A	4	4	1.00	23	0.174
461	A	5	5	1.00	25	0.200
462	A	6	6	1.00	25	0.240
463	A	7	7	1.00	25	0.280
464	A	6	6	1.00	25	0.240
465	A	2	2	1.00	16	0.125
466	A	6	6	1.00	25	0.240
467	A	7	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
468	A	7	6	1.00	25	0.240
469	A	6	5	1.00	25	0.200
470	A	5	4	1.00	23	0.174
471	A	5	4	1.00	23	0.174
472	A	6	5	1.00	25	0.200
473	A	7	6	0.98	25	0.240
474	A	8	8	1.00	25	0.320
475	A	7	7	1.00	25	0.280
476	A	6	6	1.00	16	0.375
477	A	7	7	1.00	25	0.280
478	A	8	8	1.00	25	0.320
479	A	5	5	1.00	25	0.200
480	A	4	4	1.00	25	0.160
481	A	3	3	1.00	23	0.130
482	A	3	3	1.00	23	0.130
483	A	4	4	1.00	25	0.160
484	A	5	5	1.00	25	0.200
485	A	5	5	1.00	25	0.200
486	A	6	6	1.00	25	0.240
487	A	2	2	1.00	16	0.125
488	A	6	6	1.00	25	0.240
489	A	7	7	1.00	25	0.280
490	A	6	6	1.00	25	0.240
491	A	5	5	1.00	25	0.200
492	A	4	4	1.00	23	0.174
493	A	4	4	1.00	23	0.174
494	A	5	5	1.00	25	0.200
495	A	6	6	1.00	25	0.240
496	A	6	6	1.00	25	0.240
497	A	5	5	1.00	25	0.200
498	A	4	4	1.00	16	0.250
499	A	7	7	1.00	25	0.280
500	A	8	7	1.00	25	0.280
501	A	7	6	1.00	25	0.240
502	A	6	5	1.00	25	0.200
503	A	5	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
504	A	5	4	1.00	23	0.174
505	A	6	5	1.00	25	0.200
506	A	7	6	1.00	25	0.240
507	A	7	6	1.00	25	0.240
508	A	6	6	1.00	25	0.240
509	A	7	7	1.00	16	0.438
510	A	8	8	1.00	25	0.320
511	A	9	8	1.00	25	0.320
512	A	3	3	1.00	25	0.120
513	A	3	3	1.00	23	0.130
514	A	2	2	1.00	21	0.095
515	A	2	2	1.00	21	0.095
516	A	3	3	1.00	23	0.130
517	A	3	3	1.00	23	0.130
518	A	3	3	1.00	23	0.130
519	A	3	3	1.00	23	0.130
520	A	3	3	1.00	23	0.130
521	A	12	11	1.00	15	0.733
522	A	4	4	1.00	15	0.267
523	A	5	5	1.00	15	0.333
524	A	4	4	1.00	15	0.267
525	A	5	5	1.00	15	0.333

Chapter 3

Listing of integrals

3.1 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=89

$$\frac{(6a-5b)\sinh^3(c+dx)\cosh(c+dx)}{24d}-\frac{(6a-5b)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x(6a-5b)+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}$$

```
[Out] 1/16*(6*a-5*b)*x-1/16*(6*a-5*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/24*(6*a-5*b)*co
sh(d*x+c)*sinh(d*x+c)^3/d+1/6*b*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3014, 2635, 8}

$$\frac{(6a-5b)\sinh^3(c+dx)\cosh(c+dx)}{24d}-\frac{(6a-5b)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x(6a-5b)+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((6*a - 5*b)*x)/16 - ((6*a - 5*b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) + ((6
*a - 5*b)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c +
 d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)dx = \frac{b\cosh(c+dx)\sinh^5(c+dx)}{6d} + \frac{1}{6}(6a-5b)\int \sinh^4(c+dx)\,dx$$
$$= \frac{(6a-5b)\cosh(c+dx)\sinh^3(c+dx)}{24d} + \frac{b\cosh(c+dx)\sinh^5(c+dx)}{6d}$$
$$= -\frac{(6a-5b)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(6a-5b)\cosh(c+dx)\sinh^3}{24d}$$
$$= \frac{1}{16}(6a-5b)x - \frac{(6a-5b)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(6a-5b)\cosh}{}$$

Mathematica [A] time = 0.11, size = 68, normalized size = 0.76

$$\frac{(45b-48a)\sinh(2(c+dx)) + (6a-9b)\sinh(4(c+dx)) + 72ac + 72adx + b\sinh(6(c+dx)) - 60bc - 60bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (72*a*c - 60*b*c + 72*a*d*x - 60*b*d*x + (-48*a + 45*b)*Sinh[2*(c + d*x)] +
 (6*a - 9*b)*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.47, size = 122, normalized size = 1.37

$$\frac{3\,b\cosh(dx+c)\sinh(dx+c)^5 + 2\left(5\,b\cosh(dx+c)^3 + 3\,(2\,a-3\,b)\cosh(dx+c)\right)\sinh(dx+c)^3 + 6\,(6\,a-5\,b)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 + 3*(2*a -
 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(6*a - 5*b)*d*x + 3*(b*cosh(d*x +
c)^5 + 2*(2*a - 3*b)*cosh(d*x + c)^3 - (16*a - 15*b)*cosh(d*x + c))*sinh(d*
x + c))/d
```

giac [A] time = 0.13, size = 125, normalized size = 1.40

$$\frac{1}{16}(6\,a-5\,b)x + \frac{be^{(6\,dx+6\,c)}}{384\,d} + \frac{(2\,a-3\,b)e^{(4\,dx+4\,c)}}{128\,d} - \frac{(16\,a-15\,b)e^{(2\,dx+2\,c)}}{128\,d} + \frac{(16\,a-15\,b)e^{(-2\,dx-2\,c)}}{128\,d} - \frac{(2\,a-3\,b)e^{(-4\,dx-4}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*a - 5*b)*x + 1/384*b*e^(6*d*x + 6*c)/d + 1/128*(2*a - 3*b)*e^(4*d*x
 + 4*c)/d - 1/128*(16*a - 15*b)*e^(2*d*x + 2*c)/d + 1/128*(16*a - 15*b)*e^(
-2*d*x - 2*c)/d - 1/128*(2*a - 3*b)*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x
- 6*c)/d
```

maple [A] time = 0.04, size = 88, normalized size = 0.99

$$\frac{b\left(\left(\frac{(\sinh^5(dx+c))}{6} - \frac{5(\sinh^3(dx+c))}{24} + \frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c) - \frac{5dx}{16} - \frac{5c}{16}\right) + a\left(\left(\frac{(\sinh^3(dx+c))}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)
-5/16*d*x-5/16*c)+a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*
x+3/8*c))
```

maxima [A] time = 0.65, size = 150, normalized size = 1.69

$$\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{384}b\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/384*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*
c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-
4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

mupad [B] time = 0.78, size = 76, normalized size = 0.85

$$\frac{\frac{3a\sinh(4c+4dx)}{2}-12a\sinh(2c+2dx)+\frac{45b\sinh(2c+2dx)}{4}-\frac{9b\sinh(4c+4dx)}{4}+\frac{b\sinh(6c+6dx)}{4}+18adx-15bdx}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] ((3*a*sinh(4*c + 4*d*x))/2 - 12*a*sinh(2*c + 2*d*x) + (45*b*sinh(2*c + 2*d*
x))/4 - (9*b*sinh(4*c + 4*d*x))/4 + (b*sinh(6*c + 6*d*x))/4 + 18*a*d*x - 15
*b*d*x)/(48*d)
```

sympy [A] time = 3.24, size = 258, normalized size = 2.90

$$\begin{cases} \frac{3ax\sinh^4(c+dx)}{8}-\frac{3ax\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3ax\cosh^4(c+dx)}{8}+\frac{5a\sinh^3(c+dx)\cosh(c+dx)}{8d}-\frac{3a\sinh(c+dx)\cosh^3(c+dx)}{8d}+\frac{5bx}{} \\ x\left(a+b\sinh^2(c)\right)\sinh^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*b*x*sinh(c + d*x)**6/16 - 15
*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cosh(c
+ d*x)**4/16 - 5*b*x*cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(c + d
*x)/(16*d) - 5*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c + d*x
)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c)**4, Tru
e))
```

3.2 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=53

$$\frac{(a-2b)\cosh^3(c+dx)}{3d}-\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^5(c+dx)}{5d}$$

```
[Out] -(a-b)*cosh(d*x+c)/d+1/3*(a-2*b)*cosh(d*x+c)^3/d+1/5*b*cosh(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3013, 373}

$$\frac{(a-2b)\cosh^3(c+dx)}{3d}-\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(((a - b)*Cosh[c + d*x])/d) + ((a - 2*b)*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[
c + d*x]^5)/(5*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a-b+bx^2\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int\left(a\left(1-\frac{b}{a}\right)-(a-2b)x^2-bx^4\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{(a-b)\cosh(c+dx)}{d}+\frac{(a-2b)\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 1.45

$$-\frac{3a\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}+\frac{5b\cosh(c+dx)}{8d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) + (5*b*Cosh[c + d*x])/(8*d) + (a*Cosh[3*(c + d*x
)])/(12*d) - (5*b*Cosh[3*(c + d*x)])/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d)
```

fricas [B] time = 0.46, size = 102, normalized size = 1.92

$$\frac{3\,b\cosh\left(dx+c\right)^{5}+15\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}+5\,(4\,a-5\,b)\cosh\left(dx+c\right)^{3}+15\,\left(2\,b\cosh\left(dx+c\right)^{3}+(\cdots\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 + 5*(4*a -
5*b)*cosh(d*x + c)^3 + 15*(2*b*cosh(d*x + c)^3 + (4*a - 5*b)*cosh(d*x + c))
*sinh(d*x + c)^2 - 30*(6*a - 5*b)*cosh(d*x + c))/d
```

giac [B] time = 0.13, size = 112, normalized size = 2.11

$$\frac{be^{(5\,dx+5\,c)}}{160\,d}+\frac{(4\,a-5\,b)e^{(3\,dx+3\,c)}}{96\,d}-\frac{(6\,a-5\,b)e^{(dx+c)}}{16\,d}-\frac{(6\,a-5\,b)e^{(-dx-c)}}{16\,d}+\frac{(4\,a-5\,b)e^{(-3\,dx-3\,c)}}{96\,d}+\frac{be^{(-5\,dx-5\,c)}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/160*b*e^(5*d*x + 5*c)/d + 1/96*(4*a - 5*b)*e^(3*d*x + 3*c)/d - 1/16*(6*a
- 5*b)*e^(d*x + c)/d - 1/16*(6*a - 5*b)*e^(-d*x - c)/d + 1/96*(4*a - 5*b)*e
^(-3*d*x - 3*c)/d + 1/160*b*e^(-5*d*x - 5*c)/d
```

maple [A] time = 0.03, size = 56, normalized size = 1.06

$$\frac{b\left(\frac{8}{15}+\frac{\left(\sinh^{4}(dx+c)\right)}{5}-\frac{4\left(\sinh^{2}(dx+c)\right)}{15}\right)\cosh\left(dx+c\right)+a\left(-\frac{2}{3}+\frac{\left(\sinh^{2}(dx+c)\right)}{3}\right)\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a*(-2/3+1/3*
sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.48, size = 141, normalized size = 2.66

$$\frac{1}{480}\,b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 1
50*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/24*a*
(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/
d)
```

mupad [B] time = 0.64, size = 57, normalized size = 1.08

$$\frac{15\,b\cosh\left(c+d\,x\right)-15\,a\cosh\left(c+d\,x\right)+5\,a\cosh\left(c+d\,x\right)^{3}-10\,b\cosh\left(c+d\,x\right)^{3}+3\,b\cosh\left(c+d\,x\right)^{5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (15*b*cosh(c + d*x) - 15*a*cosh(c + d*x) + 5*a*cosh(c + d*x)^3 - 10*b*cosh(
c + d*x)^3 + 3*b*cosh(c + d*x)^5)/(15*d)
```

sympy [A] time = 1.70, size = 105, normalized size = 1.98

$$\begin{cases} \frac{a\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a\cosh^3(c+dx)}{3d} + \frac{b\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4b\sinh^2(c+dx)\cosh^3(c+dx)}{3d} + \frac{8b\cosh^5(c+dx)}{15d} & \text{for } d \neq 0 \\ x\left(a+b\sinh^2(c)\right)\sinh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
+ b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c + d*x)**2*cosh(c + d*x)**
3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sin
h(c)**3, True))
```

3.3 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(4a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{1}{8}x(4a-3b)+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

```
[Out] -1/8*(4*a-3*b)*x+1/8*(4*a-3*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)*
sinh(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3014, 2635, 8}

$$\frac{(4a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{1}{8}x(4a-3b)+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((4*a - 3*b)*x)/8 + ((4*a - 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*C
osh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)dx=\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}(-4a+3b)\int \sinh^2(c+dx)\,dx$$
$$=\frac{(4a-3b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+d$$
$$=-\frac{1}{8}(4a-3b)x+\frac{(4a-3b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+$$

Mathematica [A] time = 0.09, size = 47, normalized size = 0.77

$$\frac{-4(4a-3b)(c+dx)+8(a-b)\sinh(2(c+dx))+b\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-4*(4*a - 3*b)*(c + d*x) + 8*(a - b)*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x
)])/(32*d)
```

fricas [A] time = 0.66, size = 64, normalized size = 1.05

$$\frac{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}-\left(4\,a-3\,b\right)dx+\left(b\cosh\left(dx+c\right)^{3}+4\,\left(a-b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 - (4*a - 3*b)*d*x + (b*cosh(d*x + c)^3
 + 4*(a - b)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.13, size = 79, normalized size = 1.30

$$-\frac{1}{8}\left(4\,a-3\,b\right)x+\frac{be^{\left(4\,dx+4\,c\right)}}{64\,d}+\frac{\left(a-b\right)e^{\left(2\,dx+2\,c\right)}}{8\,d}-\frac{\left(a-b\right)e^{\left(-2\,dx-2\,c\right)}}{8\,d}-\frac{be^{\left(-4\,dx-4\,c\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*a - 3*b)*x + 1/64*b*e^(4*d*x + 4*c)/d + 1/8*(a - b)*e^(2*d*x + 2*c)
/d - 1/8*(a - b)*e^(-2*d*x - 2*c)/d - 1/64*b*e^(-4*d*x - 4*c)/d
```

maple [A] time = 0.03, size = 66, normalized size = 1.08

$$\frac{b\left(\left(\frac{\left(\sinh^{3}(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh\left(dx+c\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a*(1
/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [A] time = 0.32, size = 97, normalized size = 1.59

$$\frac{1}{64}\,b\left(24\,x+\frac{e^{\left(4\,dx+4\,c\right)}}{d}-\frac{8\,e^{\left(2\,dx+2\,c\right)}}{d}+\frac{8\,e^{\left(-2\,dx-2\,c\right)}}{d}-\frac{e^{\left(-4\,dx-4\,c\right)}}{d}\right)-\frac{1}{8}\,a\left(4\,x-\frac{e^{\left(2\,dx+2\,c\right)}}{d}+\frac{e^{\left(-2\,dx-2\,c\right)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c
)/d)
```

mupad [B] time = 0.11, size = 50, normalized size = 0.82

$$\frac{\frac{a\sinh(2\,c+2\,d\,x)}{4}-\frac{b\sinh(2\,c+2\,d\,x)}{4}+\frac{b\sinh(4\,c+4\,d\,x)}{32}}{d}-\frac{a\,x}{2}+\frac{3\,b\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] ((a*sinh(2*c + 2*d*x))/4 - (b*sinh(2*c + 2*d*x))/4 + (b*sinh(4*c + 4*d*x))/
32)/d - (a*x)/2 + (3*b*x)/8
```

sympy [A] time = 0.95, size = 158, normalized size = 2.59

$$\begin{cases} \frac{ax\sinh^2(c+dx)}{2} - \frac{ax\cosh^2(c+dx)}{2} + \frac{a\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{3bx\sinh^4(c+dx)}{8} - \frac{3bx\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3bx\cosh^4(c+dx)}{8} + \\ x\left(a + b\sinh^2(c)\right)\sinh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x
)*cosh(c + d*x)/(2*d) + 3*b*x*sinh(c + d*x)**4/8 - 3*b*x*sinh(c + d*x)**2*c
osh(c + d*x)**2/4 + 3*b*x*cosh(c + d*x)**4/8 + 5*b*sinh(c + d*x)**3*cosh(c
+ d*x)/(8*d) - 3*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a +
 b*sinh(c)**2)*sinh(c)**2, True))
```

3.4 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

```
[Out] (a-b)*cosh(d*x+c)/d+1/3*b*cosh(d*x+c)^3/d
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3013}

$$\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
[Out] ((a - b)*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int\left(a-b+bx^2\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.66

$$\frac{a\sinh(c)\sinh(dx)}{d}+\frac{a\cosh(c)\cosh(dx)}{d}-\frac{3b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
[Out] (a*Cosh[c]*Cosh[d*x])/d - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])
/(12*d) + (a*Sinh[c]*Sinh[d*x])/d
```

fricas [A] time = 1.01, size = 48, normalized size = 1.50

$$\frac{b\cosh\left(dx+c\right)^3+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+3\,(4\,a-3\,b)\cosh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
[Out] 1/12*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + 3*(4*a - 3*b)
*cosh(d*x + c))/d
```

giac [B] time = 0.12, size = 70, normalized size = 2.19

$$\frac{be^{(3\,dx+3\,c)}}{24\,d}+\frac{(4\,a-3\,b)e^{(dx+c)}}{8\,d}+\frac{(4\,a-3\,b)e^{(-dx-c)}}{8\,d}+\frac{be^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*b*e^(3*d*x + 3*c)/d + 1/8*(4*a - 3*b)*e^(d*x + c)/d + 1/8*(4*a - 3*b)*
e^(-d*x - c)/d + 1/24*b*e^(-3*d*x - 3*c)/d
```

maple [A] time = 0.04, size = 34, normalized size = 1.06

$$\frac{b\left(-\frac{2}{3}+\frac{\left(\sinh^{2}(dx+c)\right)}{3}\right)\cosh\left(dx+c\right)+a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a*cosh(d*x+c))
```

maxima [B] time = 0.31, size = 67, normalized size = 2.09

$$\frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
- 3*c)/d) + a*cosh(d*x + c)/d
```

mupad [B] time = 0.59, size = 34, normalized size = 1.06

$$\frac{3\,a\cosh\left(c+d\,x\right)-3\,b\cosh\left(c+d\,x\right)+b\cosh\left(c+d\,x\right)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (3*a*cosh(c + d*x) - 3*b*cosh(c + d*x) + b*cosh(c + d*x)^3)/(3*d)
```

sympy [A] time = 0.44, size = 56, normalized size = 1.75

$$\begin{cases} \frac{a\cosh\left(c+dx\right)}{d}+\frac{b\sinh^{2}\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2b\cosh^{3}\left(c+dx\right)}{3d} & \text{for}\ d\neq 0\\ x\left(a+b\sinh^{2}\left(c\right)\right)\sinh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*cosh(c + d*x)/d + b*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*b*cos
h(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c), True))
```

3.5 $\int \left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$ax+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

```
[Out] a*x-1/2*b*x+1/2*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2635, 8}

$$ax+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x]^2,x]
```

```
[Out] a*x - (b*x)/2 + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^2(c+dx)\right)\,dx &= ax+b\int \sinh^2(c+dx)\,dx\\ &= ax+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{1}{2}b\int 1\,dx\\ &= ax-\frac{bx}{2}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 1.20

$$ax+\frac{b(-c-dx)}{2d}+\frac{b\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x]^2,x]
```

```
[Out] a*x + (b*(-c - d*x))/(2*d) + (b*Sinh[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.45, size = 30, normalized size = 1.00

$$\frac{(2\,a-b)dx+b\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a - b)*d*x + b*cosh(d*x + c)*sinh(d*x + c))/d
```

giac [A] time = 0.14, size = 38, normalized size = 1.27

$$-\frac{1}{8}\, b\left(4\, x - \frac{e^{(2\, dx+2\, c)}}{d} + \frac{e^{(-2\, dx-2\, c)}}{d}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*x
```

maple [A] time = 0.02, size = 32, normalized size = 1.07

$$ax + \frac{b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c)^2,x)
```

```
[Out] a*x+b/d*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)
```

maxima [A] time = 0.33, size = 38, normalized size = 1.27

$$-\frac{1}{8}\, b\left(4\, x - \frac{e^{(2\, dx+2\, c)}}{d} + \frac{e^{(-2\, dx-2\, c)}}{d}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*x
```

mupad [B] time = 0.07, size = 23, normalized size = 0.77

$$a\, x - \frac{b\, x}{2} + \frac{b\sinh\left(2\, c + 2\, d\, x\right)}{4\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sinh(c + d*x)^2,x)
```

```
[Out] a*x - (b*x)/2 + (b*sinh(2*c + 2*d*x))/(4*d)
```

sympy [A] time = 0.22, size = 51, normalized size = 1.70

$$ax + b\left(\begin{cases} \frac{x\sinh^2(c+dx)}{2} - \frac{x\cosh^2(c+dx)}{2} + \frac{\sinh(c+dx)\cosh(c+dx)}{2d} & \text{for}\ d \neq 0 \\ x\sinh^2(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**2/2 + sinh(c + d
*x)*cosh(c + d*x)/(2*d), Ne(d, 0)), (x*sinh(c)**2, True))
```

3.6 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=25

$$\frac{b\cosh(c+dx)}{d}-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}$$

```
[Out] -a*arctanh(cosh(d*x+c))/d+b*cosh(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3014, 3770}

$$\frac{b\cosh(c+dx)}{d}-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) + (b*Cosh[c + d*x])/d
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{b\cosh(c+dx)}{d}+a\int \operatorname{csch}(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.03, size = 62, normalized size = 2.48

$$\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\sinh(c)\sinh(dx)}{d}+\frac{b\cosh(c)\cosh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[Sinh[c/2
+ (d*x)/2]])/d + (b*Sinh[c]*Sinh[d*x])/d
```

fricas [B] time = 0.58, size = 126, normalized size = 5.04

$$\frac{b\cosh(dx+c)^2+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2-2\,(a\cosh(dx+c)+a\sinh(dx+c))\log(\mathrm{co}}{2\,(d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^
2 - 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
 + 1) + 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
+ c) - 1) + b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

giac [A] time = 0.15, size = 50, normalized size = 2.00

$$\frac{be^{(dx+c)} + be^{(-dx-c)} - 2\,a\log\left(e^{(dx+c)} + 1\right) + 2\,a\log\left(\left|e^{(dx+c)} - 1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(b*e^(d*x + c) + b*e^(-d*x - c) - 2*a*log(e^(d*x + c) + 1) + 2*a*log(ab
s(e^(d*x + c) - 1)))/d
```

maple [A] time = 0.08, size = 24, normalized size = 0.96

$$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right) + b\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*cosh(d*x+c))
```

maxima [A] time = 0.31, size = 43, normalized size = 1.72

$$\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) + \frac{a\log\left(\tanh\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

mupad [B] time = 0.14, size = 66, normalized size = 2.64

$$\frac{b\,\mathrm{e}^{-c-d\,x}}{2\,d} + \frac{b\,\mathrm{e}^{c+d\,x}}{2\,d} - \frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/sinh(c + d*x),x)
```

```
[Out] (b*exp(- c - d*x))/(2*d) + (b*exp(c + d*x))/(2*d) - (2*atan((a*exp(d*x)*exp
(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^{2}\left(c + dx\right)\right)\operatorname{csch}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*csch(c + d*x), x)
```

3.7 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=16

$$bx-\frac{a\coth(c+dx)}{d}$$

```
[Out] b*x-a*coth(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3012, 8}

$$bx-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] b*x - (a*Coth[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx&=-\frac{a\coth(c+dx)}{d}+b\int 1\,dx\\&=bx-\frac{a\coth(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$bx-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] b*x - (a*Coth[c + d*x])/d
```

fricas [B] time = 0.52, size = 36, normalized size = 2.25

$$-\frac{a\cosh(dx+c)-(bdx+a)\sinh(dx+c)}{d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*cosh(d*x + c) - (b*d*x + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

giac [A] time = 0.15, size = 28, normalized size = 1.75

$$\frac{(dx+c)b-\frac{2a}{e^{(2dx+2c)}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
[Out] ((d*x + c)*b - 2*a/(e^(2*d*x + 2*c) - 1))/d
```

maple [A] time = 0.06, size = 22, normalized size = 1.38

$$\frac{-\coth(dx+c)\,a+(dx+c)\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
[Out] 1/d*(-coth(d*x+c)*a+(d*x+c)*b)
```

maxima [A] time = 0.43, size = 23, normalized size = 1.44

$$bx+\frac{2a}{d\left(e^{(-2dx-2c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
[Out] b*x + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.57, size = 23, normalized size = 1.44

$$b\,x-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/sinh(c + d*x)^2,x)
[Out] b*x - (2*a)/(d*(exp(2*c + 2*d*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^{2}(c+dx)\right)\operatorname{csch}^{2}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
[Out] Integral((a + b*sinh(c + d*x)**2)*csch(c + d*x)**2, x)
```

3.8 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

```
[Out] 1/2*(a-2*b)*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3012, 3770}

$$\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - 2*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/
(2*d)
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx&=-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{1}{2}(a-2b)\int \operatorname{csch}(c+dx)\,dx\\&=\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\end{aligned}$$

Mathematica [B] time = 0.03, size = 99, normalized size = 2.48

$$-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/8*(a*Csch[(c + d*x)/2]^2)/d - (b*Log[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Si
nh[c/2 + (d*x)/2]])/d - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x
)/2]^2)/(8*d)
```

fricas [B] time = 0.59, size = 484, normalized size = 12.10

$$-\frac{2\,a\cosh\left(dx+c\right)^3+6\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+2\,a\sinh\left(dx+c\right)^3+2\,a\cosh\left(dx+c\right)-\left((a-2\,b)\cosh\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d*
x + c)^3 + 2*a*cosh(d*x + c) - ((a - 2*b)*cosh(d*x + c)^4 + 4*(a - 2*b)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a - 2*b)*sinh(d*x + c)^4 - 2*(a - 2*b)*cosh(d
*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^2 + 4*(
(a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*
b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a - 2*b)*cosh(d*x + c)^4 + 4*
(a - 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - 2*b)*sinh(d*x + c)^4 - 2*(a
- 2*b)*cosh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x
 + c)^2 + 4*((a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a - 2*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x +
c)^2 + a)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x +
c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d
)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) +
 d)
```

giac [B] time = 0.14, size = 96, normalized size = 2.40

$$\frac{(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((a - 2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a - 2*b)*log(e^(d*x +
 c) + e^(-d*x - c) - 2) - 4*a*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c) +
e^(-d*x - c))^2 - 4))/d
```

maple [A] time = 0.07, size = 40, normalized size = 1.00

$$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)-2b\operatorname{arctanh}\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-2*b*arctanh(exp(d
*x+c)))
```

maxima [B] time = 0.33, size = 125, normalized size = 3.12

$$\frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c)
+ e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b*(l
og(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

mupad [B] time = 0.13, size = 131, normalized size = 3.28

$$\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a\,\sqrt{-d^2}-2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^2-4\,a\,b+4\,b^2}}\right)\,\sqrt{a^2-4\,a\,b+4\,b^2}}{\sqrt{-d^2}}-\frac{a\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) - 2*b*(-d^2)^(1/2)))/(d*(a^2 - 4*a*b
 + 4*b^2)^(1/2)))*(a^2 - 4*a*b + 4*b^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c + d*
x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x) -
2*exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^2\left(c + dx\right)\right)\operatorname{csch}^3\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*csch(c + d*x)**3, x)
```

3.9 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=43

$$\frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

```
[Out] 1/3*(2*a-3*b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)*csch(d*x+c)^2/d
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3012, 3767, 8}

$$\frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((2*a - 3*b)*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= -\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{1}{3}(-2a+3b)\int \operatorname{csch}^2(c+dx)\,dx\\ &= -\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{(i(2a-3b))\operatorname{Subst}(\int 1\,dx,x,-i\operatorname{c}\\ &= \frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 1.14

$$\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{b\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c +
 d*x]^2)/(3*d)
```

fricas [B] time = 0.41, size = 159, normalized size = 3.70

$$\frac{4\left((a-3\,b)\cosh\left(dx+c\right)^{2}-2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(a\right.}{3\left(d\cosh\left(dx+c\right)^{4}+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+d\sinh\left(dx+c\right)^{4}-4\,d\cosh\left(dx+c\right)^{2}+2\left(3\,d\cosh\left(dx+c\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 4/3*((a - 3*b)*cosh(d*x + c)^2 - 2*a*cosh(d*x + c)*sinh(d*x + c) + (a - 3*b
)*sinh(d*x + c)^2 - 3*a + 3*b)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(
d*x + c)^3 + d*sinh(d*x + c)^4 - 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)
^2 - 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*
x + c) + 3*d)
```

giac [A] time = 0.14, size = 61, normalized size = 1.42

$$-\frac{2\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-2\,a+3\,b\right)}{3\,d\left(e^{(2\,dx+2\,c)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) - 2*a
 + 3*b)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

maple [A] time = 0.07, size = 35, normalized size = 0.81

$$\frac{a\left(\frac{2}{3}-\frac{\mathrm{csch}(dx+c)^{2}}{3}\right)\coth\left(dx+c\right)-b\coth\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-b*coth(d*x+c))
```

maxima [B] time = 0.45, size = 113, normalized size = 2.63

$$\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)+\frac{2\,b}{d\left(e^{(-2\,dx-2\,c)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.60, size = 61, normalized size = 1.42

$$-\frac{2\,\left(3\,b-2\,a+6\,a\,{\mathrm{e}}^{2\,c+2\,d\,x}-6\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}\right)}{3\,d\,{\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/sinh(c + d*x)^4,x)
```

```
[Out] -(2*(3*b - 2*a + 6*a*exp(2*c + 2*d*x) - 6*b*exp(2*c + 2*d*x) + 3*b*exp(4*c
+ 4*d*x)))/(3*d*(exp(2*c + 2*d*x) - 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.10 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=146

$$\frac{\left(48a^2-208ab+139b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{\left(80a^2-176ab+93b^2\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(4\right.$$

```
[Out] 1/128*(48*a^2-80*a*b+35*b^2)*x-1/128*(80*a^2-176*a*b+93*b^2)*cosh(d*x+c)*si
nh(d*x+c)/d+1/192*(48*a^2-208*a*b+139*b^2)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/48
*(16*a-13*b)*b*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b^2*cosh(d*x+c)^3*sinh(d*x+c
)^5/d
```

Rubi [A] time = 0.18, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = $\{3187, 463, 455, 1157, 385, 206\}$

$$\frac{\left(48a^2-208ab+139b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{\left(80a^2-176ab+93b^2\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(4\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((48*a^2 - 80*a*b + 35*b^2)*x)/128 - ((80*a^2 - 176*a*b + 93*b^2)*Cosh[c +
d*x]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 208*a*b + 139*b^2)*Cosh[c + d*x]^3
*Sinh[c + d*x])/(192*d) + ((16*a - 13*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(
48*d) + (b^2*Cosh[c + d*x]^3*Sinh[c + d*x]^5)/(8*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
```

```
IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh^3(c+dx)\sinh^5(c+dx)}{8d} - \frac{\text{Subst}\left(\int \frac{x^4\left(-8a^2+5b^2+8(a-b)^2x^2\right)}{\left(1-x^2\right)^4}\,d\right.}{8d}$$

$$= \frac{(16a-13b)b\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^2\cosh^3(c+dx)\sinh}{8d}$$

$$= \frac{\left(48a^2-208ab+139b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{192d} + \frac{(16a-13}{}$$

$$= -\frac{\left(80a^2-176ab+93b^2\right)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{\left(48a^2-2\right.}{}$$

$$= \frac{1}{128}\left(48a^2-80ab+35b^2\right)x - \frac{\left(80a^2-176ab+93b^2\right)\cosh(c+d}{128d}$$

Mathematica [A] time = 0.20, size = 133, normalized size = 0.91

$$\frac{-96\left(8a^2-15ab+7b^2\right)\sinh(2(c+dx))+24\left(4a^2-12ab+7b^2\right)\sinh(4(c+dx))+1152a^2c+1152a^2dx+32a}{3072}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (1152*a^2*c - 1920*a*b*c + 840*b^2*c + 1152*a^2*d*x - 1920*a*b*d*x + 840*b^
2*d*x - 96*(8*a^2 - 15*a*b + 7*b^2)*Sinh[2*(c + d*x)] + 24*(4*a^2 - 12*a*b
+ 7*b^2)*Sinh[4*(c + d*x)] + 32*a*b*Sinh[6*(c + d*x)] - 32*b^2*Sinh[6*(c +
d*x)] + 3*b^2*Sinh[8*(c + d*x)])/(3072*d)
```

fricas [A] time = 0.46, size = 238, normalized size = 1.63

$$\frac{3\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+3\left(7\,b^2\cosh\left(dx+c\right)^3+8\left(ab-b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+\left(21\,b^2\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 + 8*(
a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 + 80*(a
*b - b^2)*cosh(d*x + c)^3 + 12*(4*a^2 - 12*a*b + 7*b^2)*cosh(d*x + c))*sinh
(d*x + c)^3 + 3*(48*a^2 - 80*a*b + 35*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 + 8
*(a*b - b^2)*cosh(d*x + c)^5 + 4*(4*a^2 - 12*a*b + 7*b^2)*cosh(d*x + c)^3 -
 8*(8*a^2 - 15*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.15, size = 215, normalized size = 1.47

$$\frac{1}{128}\left(48\,a^2-80\,ab+35\,b^2\right)x+\frac{b^2e^{(8\,dx+8\,c)}}{2048\,d}-\frac{b^2e^{(-8\,dx-8\,c)}}{2048\,d}+\frac{\left(ab-b^2\right)e^{(6\,dx+6\,c)}}{192\,d}+\frac{\left(4\,a^2-12\,ab+7\,b^2\right)e^{(4\,dx+4\,c)}}{256\,d}-\frac{\left(8\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/128*(48*a^2 - 80*a*b + 35*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d - 1/2048*
b^2*e^(-8*d*x - 8*c)/d + 1/192*(a*b - b^2)*e^(6*d*x + 6*c)/d + 1/256*(4*a^2
 - 12*a*b + 7*b^2)*e^(4*d*x + 4*c)/d - 1/64*(8*a^2 - 15*a*b + 7*b^2)*e^(2*d
*x + 2*c)/d + 1/64*(8*a^2 - 15*a*b + 7*b^2)*e^(-2*d*x - 2*c)/d - 1/256*(4*a
^2 - 12*a*b + 7*b^2)*e^(-4*d*x - 4*c)/d - 1/192*(a*b - b^2)*e^(-6*d*x - 6*c
)/d
```

maple [A] time = 0.13, size = 150, normalized size = 1.03

$$\frac{b^2\left(\left(\frac{\left(\sinh^7(dx+c)\right)}{8}-\frac{7\left(\sinh^5(dx+c)\right)}{48}+\frac{35\left(\sinh^3(dx+c)\right)}{192}-\frac{35\sinh(dx+c)}{128}\right)\cosh\left(dx+c\right)+\frac{35dx}{128}+\frac{35c}{128}\right)+2ab\left(\left(\frac{\left(\sinh^5(dx+c)\right)}{6}-\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128
*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+2*a*b*((1/6*sinh(d*x+c)^5-5/
24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a^2*((1/4*s
inh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.49, size = 267, normalized size = 1.83

$$\frac{1}{64}\,a^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{6144}\,b^2\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/6144*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d
*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d
- (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^
(-8*d*x - 8*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) -
```

```
 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c))/d)
```

mupad [B] time = 0.90, size = 149, normalized size = 1.02

$$\frac{12\,a^2\,\mathrm{sinh}\left(4\,c+4\,d\,x\right)-96\,a^2\,\mathrm{sinh}\left(2\,c+2\,d\,x\right)-84\,b^2\,\mathrm{sinh}\left(2\,c+2\,d\,x\right)+21\,b^2\,\mathrm{sinh}\left(4\,c+4\,d\,x\right)-4\,b^2\,\mathrm{sinh}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (12*a^2*sinh(4*c + 4*d*x) - 96*a^2*sinh(2*c + 2*d*x) - 84*b^2*sinh(2*c + 2*
d*x) + 21*b^2*sinh(4*c + 4*d*x) - 4*b^2*sinh(6*c + 6*d*x) + (3*b^2*sinh(8*c
 + 8*d*x))/8 + 180*a*b*sinh(2*c + 2*d*x) - 36*a*b*sinh(4*c + 4*d*x) + 4*a*b
*sinh(6*c + 6*d*x) + 144*a^2*d*x + 105*b^2*d*x - 240*a*b*d*x)/(384*d)
```

sympy [A] time = 9.70, size = 490, normalized size = 3.36

$$\begin{cases} \frac{3a^2x\sinh^4(c+dx)}{8} - \frac{3a^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3a^2x\cosh^4(c+dx)}{8} + \frac{5a^2\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{3a^2\sinh(c+dx)\cosh^3(c+dx)}{8d} + \\ x\left(a+b\sinh^2(c)\right)^2\sinh^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*a**2*x*sinh(c + d*x)**4/8 - 3*a**2*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**2*x*cosh(c + d*x)**4/8 + 5*a**2*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) - 3*a**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*a*b*x*sinh(c
+ d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*sinh(
c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*sinh(c
 + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3
*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + 35*b**2*x*sinh(c + d*x)*
*8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**2*x*sinh(c
 + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**
6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c + d*x)**7*cosh(c + d
*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**2
*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*sinh(c + d*x)*cosh(c +
 d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**4, True))
```

3.11 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=85

$$\frac{b(2a-3b)\cosh^5(c+dx)}{5d}+\frac{(a-3b)(a-b)\cosh^3(c+dx)}{3d}-\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^7(c+dx)}{7d}$$

```
[Out] -(a-b)^2*cosh(d*x+c)/d+1/3*(a-3*b)*(a-b)*cosh(d*x+c)^3/d+1/5*(2*a-3*b)*b*co
sh(d*x+c)^5/d+1/7*b^2*cosh(d*x+c)^7/d
```

Rubi [A] time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3186, 373}

$$\frac{b(2a-3b)\cosh^5(c+dx)}{5d}+\frac{(a-3b)(a-b)\cosh^3(c+dx)}{3d}-\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Cosh[c + d*x])/d) + ((a - 3*b)*(a - b)*Cosh[c + d*x]^3)/(3*d)
+ ((2*a - 3*b)*b*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= -\frac{\operatorname{Subst}\left(\int\left(1-x^2\right)\left(a-b+bx^2\right)^2\,dx, x, \cosh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int\left((a-b)^2+(a-3b)(-a+b)x^2-(2a-3b)bx^4-b^2x^6\right)\,dx\right.}{d}\\ &= -\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{(a-3b)(a-b)\cosh^3(c+dx)}{3d}+\frac{(2a-3b)}{}\end{aligned}$$

Mathematica [A] time = 0.04, size = 154, normalized size = 1.81

$$-\frac{3a^2\cosh(c+dx)}{4d}+\frac{a^2\cosh(3(c+dx))}{12d}+\frac{5ab\cosh(c+dx)}{4d}-\frac{5ab\cosh(3(c+dx))}{24d}+\frac{ab\cosh(5(c+dx))}{40d}-\frac{35b^2\cosh}{64}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-3*a^2*Cosh[c + d*x])/(4*d) + (5*a*b*Cosh[c + d*x])/(4*d) - (35*b^2*Cosh[c
 + d*x])/(64*d) + (a^2*Cosh[3*(c + d*x)])/(12*d) - (5*a*b*Cosh[3*(c + d*x)]
```

```
)/(24*d) + (7*b^2*Cosh[3*(c + d*x)])/(64*d) + (a*b*Cosh[5*(c + d*x)])/(40*d
) - (7*b^2*Cosh[5*(c + d*x)])/(320*d) + (b^2*Cosh[7*(c + d*x)])/(448*d)
```

fricas [B] time = 0.64, size = 213, normalized size = 2.51

$$\frac{15\,b^2\cosh\left(dx+c\right)^7+105\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+21\left(8\,ab-7\,b^2\right)\cosh\left(dx+c\right)^5+105\left(5\,b^2\cosh\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(15*b^2*cosh(d*x + c)^7 + 105*b^2*cosh(d*x + c)*sinh(d*x + c)^6 + 21
*(8*a*b - 7*b^2)*cosh(d*x + c)^5 + 105*(5*b^2*cosh(d*x + c)^3 + (8*a*b - 7*
b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 35*(16*a^2 - 40*a*b + 21*b^2)*cosh(d*
x + c)^3 + 105*(3*b^2*cosh(d*x + c)^5 + 2*(8*a*b - 7*b^2)*cosh(d*x + c)^3 +
 (16*a^2 - 40*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 105*(48*a^2 -
80*a*b + 35*b^2)*cosh(d*x + c))/d
```

giac [B] time = 0.15, size = 196, normalized size = 2.31

$$\frac{b^2e^{(7\,dx+7\,c)}}{896\,d}+\frac{b^2e^{(-7\,dx-7\,c)}}{896\,d}+\frac{\left(8\,ab-7\,b^2\right)e^{(5\,dx+5\,c)}}{640\,d}+\frac{\left(16\,a^2-40\,ab+21\,b^2\right)e^{(3\,dx+3\,c)}}{384\,d}-\frac{\left(48\,a^2-80\,ab+35\,b^2\right)e^{(dx-}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/896*b^2*e^(7*d*x + 7*c)/d + 1/896*b^2*e^(-7*d*x - 7*c)/d + 1/640*(8*a*b -
 7*b^2)*e^(5*d*x + 5*c)/d + 1/384*(16*a^2 - 40*a*b + 21*b^2)*e^(3*d*x + 3*c
)/d - 1/128*(48*a^2 - 80*a*b + 35*b^2)*e^(d*x + c)/d - 1/128*(48*a^2 - 80*a
*b + 35*b^2)*e^(-d*x - c)/d + 1/384*(16*a^2 - 40*a*b + 21*b^2)*e^(-3*d*x -
3*c)/d + 1/640*(8*a*b - 7*b^2)*e^(-5*d*x - 5*c)/d
```

maple [A] time = 0.04, size = 102, normalized size = 1.20

$$\frac{b^2\left(-\frac{16}{35}+\frac{\left(\sinh^6(dx+c)\right)}{7}-\frac{6\left(\sinh^4(dx+c)\right)}{35}+\frac{8\left(\sinh^2(dx+c)\right)}{35}\right)\cosh\left(dx+c\right)+2ab\left(\frac{8}{15}+\frac{\left(\sinh^4(dx+c)\right)}{5}-\frac{4\left(\sinh^2(dx+c)\right)}{15}\right)\cos}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*c
osh(d*x+c)+2*a*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^
2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.38, size = 247, normalized size = 2.91

$$-\frac{1}{4480}\,b^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5*c)
/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(
```

```
-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e
^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 0.23, size = 112, normalized size = 1.32

$$\frac{\frac{a^2\cosh(c+d\,x)^3}{3}-a^2\cosh\left(c+d\,x\right)+\frac{2\,a\,b\cosh(c+d\,x)^5}{5}-\frac{4\,a\,b\cosh(c+d\,x)^3}{3}+2\,a\,b\cosh\left(c+d\,x\right)+\frac{b^2\cosh(c+d\,x)^7}{7}-\frac{3\,b^2\cosh(c+}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] ((a^2*cosh(c + d*x)^3)/3 - b^2*cosh(c + d*x) - a^2*cosh(c + d*x) + b^2*cosh
(c + d*x)^3 - (3*b^2*cosh(c + d*x)^5)/5 + (b^2*cosh(c + d*x)^7)/7 + 2*a*b*c
osh(c + d*x) - (4*a*b*cosh(c + d*x)^3)/3 + (2*a*b*cosh(c + d*x)^5)/5)/d
```

sympy [A] time = 5.45, size = 204, normalized size = 2.40

$$\begin{cases} \frac{a^2\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{2a^2\cosh^3(c+dx)}{3d}+\frac{2ab\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{8ab\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{16ab\cosh^5(c+dx)}{15d}+\frac{b^2\sin}{} \\ x\left(a+b\sinh^2(c)\right)^2\sinh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/
(3*d) + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 8*a*b*sinh(c + d*x)**2*cos
h(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15*d) + b**2*sinh(c + d*x)**
6*cosh(c + d*x)/d - 2*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b**2*sin
h(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**2*cosh(c + d*x)**7/(35*d), Ne(
d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**3, True))
```

3.12 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=110

$$\frac{\left(8a^2-20ab+11b^2\right)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}x\left(8a^2-12ab+5b^2\right)+\frac{b(4a-3b)\sinh(c+dx)\cosh^3(c+d$$

```
[Out] -1/16*(8*a^2-12*a*b+5*b^2)*x+1/16*(8*a^2-20*a*b+11*b^2)*cosh(d*x+c)*sinh(d*
x+c)/d+1/8*(4*a-3*b)*b*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b^2*cosh(d*x+c)^3*si
nh(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3170, 3169}

$$\frac{\left(16a^2-36ab+15b^2\right)\sinh(c+dx)\cosh(c+dx)}{48d}-\frac{1}{16}x\left(8a^2-12ab+5b^2\right)+\frac{b(4a-5b)\sinh^3(c+dx)\cosh(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((8*a^2 - 12*a*b + 5*b^2)*x)/16 + ((16*a^2 - 36*a*b + 15*b^2)*Cosh[c + d*x
]*Sinh[c + d*x])/(48*d) + ((4*a - 5*b)*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24
*d) + (Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(6*d)
```

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}-\frac{1}{6}\int\left(a-(4a\right.$$

$$=-\frac{1}{16}\left(8a^2-12ab+5b^2\right)x+\frac{\left(16a^2-36ab+15b^2\right)\cosh(c+dx)\text{s}}{48d}$$

Mathematica [A] time = 0.19, size = 99, normalized size = 0.90

$$\frac{\left(48a^2-96ab+45b^2\right)\sinh(2(c+dx))-96a^2c-96a^2dx+3b(4a-3b)\sinh(4(c+dx))+144abc+144abdx+}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-96*a^2*c + 144*a*b*c - 60*b^2*c - 96*a^2*d*x + 144*a*b*d*x - 60*b^2*d*x +
 (48*a^2 - 96*a*b + 45*b^2)*Sinh[2*(c + d*x)] + 3*(4*a - 3*b)*b*Sinh[4*(c +
 d*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.66, size = 149, normalized size = 1.35

$$3\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+2\left(5\,b^2\cosh\left(dx+c\right)^3+3\left(4\,ab-3\,b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3-6\left(8\,a^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^2*cosh(d*x + c)^3 + 3*(4
*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(8*a^2 - 12*a*b + 5*b^2)*d
*x + 3*(b^2*cosh(d*x + c)^5 + 2*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + (16*a^2 -
 32*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.23, size = 159, normalized size = 1.45

$$-\frac{1}{16}\left(8\,a^2-12\,ab+5\,b^2\right)x+\frac{b^2e^{(6\,dx+6\,c)}}{384\,d}-\frac{b^2e^{(-6\,dx-6\,c)}}{384\,d}+\frac{\left(4\,ab-3\,b^2\right)e^{(4\,dx+4\,c)}}{128\,d}+\frac{\left(16\,a^2-32\,ab+15\,b^2\right)e^{(2\,dx+2\,c)}}{128\,d}-\frac{(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/16*(8*a^2 - 12*a*b + 5*b^2)*x + 1/384*b^2*e^(6*d*x + 6*c)/d - 1/384*b^2*
e^(-6*d*x - 6*c)/d + 1/128*(4*a*b - 3*b^2)*e^(4*d*x + 4*c)/d + 1/128*(16*a^
2 - 32*a*b + 15*b^2)*e^(2*d*x + 2*c)/d - 1/128*(16*a^2 - 32*a*b + 15*b^2)*e
^(-2*d*x - 2*c)/d - 1/128*(4*a*b - 3*b^2)*e^(-4*d*x - 4*c)/d
```

maple [A] time = 0.03, size = 118, normalized size = 1.07

$$\frac{b^2\left(\left(\frac{\left(\sinh^5(dx+c)\right)}{6}-\frac{5\left(\sinh^3(dx+c)\right)}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh\left(dx+c\right)-\frac{5dx}{16}-\frac{5c}{16}\right)+2ab\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh\left(d\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+
c)-5/16*d*x-5/16*c)+2*a*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+
3/8*d*x+3/8*c)+a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [A] time = 0.35, size = 189, normalized size = 1.72

$$\frac{1}{32}\,ab\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{8}\,a^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{384}\,b^2\left(\frac{(9\,e}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x -
 2*c)/d) - 1/384*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d
*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c))/d)
```

mupad [B] time = 0.23, size = 108, normalized size = 0.98

$$\frac{12\,a^2\sinh\left(2\,c+2\,d\,x\right)+\frac{45\,b^2\sinh(2\,c+2\,d\,x)}{4}-\frac{9\,b^2\sinh(4\,c+4\,d\,x)}{4}+\frac{b^2\sinh(6\,c+6\,d\,x)}{4}-24\,a\,b\sinh\left(2\,c+2\,d\,x\right)+3\,a\,b\,\mathrm{si}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (12*a^2*sinh(2*c + 2*d*x) + (45*b^2*sinh(2*c + 2*d*x))/4 - (9*b^2*sinh(4*c
+ 4*d*x))/4 + (b^2*sinh(6*c + 6*d*x))/4 - 24*a*b*sinh(2*c + 2*d*x) + 3*a*b*
sinh(4*c + 4*d*x) - 24*a^2*d*x - 15*b^2*d*x + 36*a*b*d*x)/(48*d)
```

sympy [A] time = 3.49, size = 332, normalized size = 3.02

$$\begin{cases} \frac{a^2x\sinh^2(c+dx)}{2}-\frac{a^2x\cosh^2(c+dx)}{2}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{3abx\sinh^4(c+dx)}{4}-\frac{3abx\sinh^2(c+dx)\cosh^2(c+dx)}{2}+\frac{3abx\cosh^4(c\cdot}{4} \\ x\left(a+b\sinh^2(c)\right)^2\sinh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh(
c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c
+ d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d)
+ 5*b**2*x*sinh(c + d*x)**6/16 - 15*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**
2/16 + 15*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b**2*x*cosh(c + d
*x)**6/16 + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b**2*sinh(c +
 d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16
*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**2, True))
```

3.13 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=57

$$\frac{2b(a-b)\cosh^3(c+dx)}{3d}+\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^5(c+dx)}{5d}$$

```
[Out] (a-b)^2*cosh(d*x+c)/d+2/3*(a-b)*b*cosh(d*x+c)^3/d+1/5*b^2*cosh(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 194}

$$\frac{2b(a-b)\cosh^3(c+dx)}{3d}+\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)^2*Cosh[c + d*x])/d + (2*(a - b)*b*Cosh[c + d*x]^3)/(3*d) + (b^2*Co
sh[c + d*x]^5)/(5*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int\left(a-b+bx^2\right)^2\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^2\left(1+\frac{b(-2a+b)}{a^2}\right)+2ab\left(1-\frac{b}{a}\right)x^2+b^2x^4\right)\,dx,x,\cosh(c+\right.}{d}\\ &= \frac{(a-b)^2\cosh(c+dx)}{d}+\frac{2(a-b)b\cosh^3(c+dx)}{3d}+\frac{b^2\cosh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 111, normalized size = 1.95

$$\frac{a^2\sinh(c)\sinh(dx)}{d}+\frac{a^2\cosh(c)\cosh(dx)}{d}-\frac{3ab\cosh(c+dx)}{2d}+\frac{ab\cosh(3(c+dx))}{6d}+\frac{5b^2\cosh(c+dx)}{8d}-\frac{5b^2\cosh(3}{48}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Cosh[c]*Cosh[d*x])/d - (3*a*b*Cosh[c + d*x])/(2*d) + (5*b^2*Cosh[c + d
*x])/(8*d) + (a*b*Cosh[3*(c + d*x)])/(6*d) - (5*b^2*Cosh[3*(c + d*x)])/(48*
d) + (b^2*Cosh[5*(c + d*x)])/(80*d) + (a^2*Sinh[c]*Sinh[d*x])/d
```

fricas [B] time = 0.65, size = 122, normalized size = 2.14

$$\frac{3\,b^2\cosh\left(dx+c\right)^5+15\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+5\left(8\,ab-5\,b^2\right)\cosh\left(dx+c\right)^3+15\left(2\,b^2\cosh\left(dx+c\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^2*cosh(d*x + c)^5 + 15*b^2*cosh(d*x + c)*sinh(d*x + c)^4 + 5*(8*
a*b - 5*b^2)*cosh(d*x + c)^3 + 15*(2*b^2*cosh(d*x + c)^3 + (8*a*b - 5*b^2)*
cosh(d*x + c))*sinh(d*x + c)^2 + 30*(8*a^2 - 12*a*b + 5*b^2)*cosh(d*x + c))
/d
```

giac [B] time = 0.16, size = 138, normalized size = 2.42

$$\frac{b^2e^{(5\,dx+5\,c)}}{160\,d}+\frac{b^2e^{(-5\,dx-5\,c)}}{160\,d}+\frac{\left(8\,ab-5\,b^2\right)e^{(3\,dx+3\,c)}}{96\,d}+\frac{\left(8\,a^2-12\,ab+5\,b^2\right)e^{(dx+c)}}{16\,d}+\frac{\left(8\,a^2-12\,ab+5\,b^2\right)e^{(-dx-c)}}{16\,d}+\frac{(8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/160*b^2*e^(5*d*x + 5*c)/d + 1/160*b^2*e^(-5*d*x - 5*c)/d + 1/96*(8*a*b -
5*b^2)*e^(3*d*x + 3*c)/d + 1/16*(8*a^2 - 12*a*b + 5*b^2)*e^(d*x + c)/d + 1/
16*(8*a^2 - 12*a*b + 5*b^2)*e^(-d*x - c)/d + 1/96*(8*a*b - 5*b^2)*e^(-3*d*x
 - 3*c)/d
```

maple [A] time = 0.03, size = 70, normalized size = 1.23

$$\frac{b^2\left(\frac{8}{15}+\frac{\left(\sinh^4(dx+c)\right)}{5}-\frac{4\left(\sinh^2(dx+c)\right)}{15}\right)\cosh\left(dx+c\right)+2ab\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh\left(dx+c\right)+a^2\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(-2/
3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*cosh(d*x+c))
```

maxima [B] time = 0.41, size = 157, normalized size = 2.75

$$\frac{1}{480}\,b^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{1}{12}\,ab\left(\frac{e^{(3\,dx+3\,c)}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/480*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/12*
a*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3
*c)/d) + a^2*cosh(d*x + c)/d
```

mupad [B] time = 0.65, size = 76, normalized size = 1.33

$$\frac{15\,a^2\cosh\left(c+d\,x\right)+10\,a\,b\cosh\left(c+d\,x\right)^3-30\,a\,b\cosh\left(c+d\,x\right)+3\,b^2\cosh\left(c+d\,x\right)^5-10\,b^2\cosh\left(c+d\,x\right)^3}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^2,x)
```

[Out] (15*a^2*cosh(c + d*x) + 15*b^2*cosh(c + d*x) - 10*b^2*cosh(c + d*x)^3 + 3*b
^2*cosh(c + d*x)^5 - 30*a*b*cosh(c + d*x) + 10*a*b*cosh(c + d*x)^3)/(15*d)

sympy [A] time = 1.75, size = 128, normalized size = 2.25

$$\begin{cases} \frac{a^2 \cosh(c+dx)}{d} + \frac{2ab \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{4ab \cosh^3(c+dx)}{3d} + \frac{b^2 \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{4b^2 \sinh^2(c+dx) \cosh^3(c+dx)}{3d} + \frac{8b^2 \cosh \\ x\left(a + b \sinh^2(c)\right)^2 \sinh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Piecewise((a**2*cosh(c + d*x)/d + 2*a*b*sinh(c + d*x)**2*cosh(c + d*x)/d -
4*a*b*cosh(c + d*x)**3/(3*d) + b**2*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*
*2*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b**2*cosh(c + d*x)**5/(15*d)
, Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c), True))

3.14 $\int \left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=72

$$\frac{1}{8}x\left(8a^2-8ab+3b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

```
[Out] 1/8*(8*a^2-8*a*b+3*b^2)*x+1/8*(8*a-3*b)*b*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b^2
*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3179}

$$\frac{1}{8}x\left(8a^2-8ab+3b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*x)/8 + ((8*a - 3*b)*b*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 3179

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[((8*a^2 +
 8*a*b + 3*b^2)*x)/8, x] + (-Simp[(b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f),
x] - Simp[(b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{1}{8}\left(8a^2-8ab+3b^2\right)x+\frac{(8a-3b)b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^2\cosh(c+d\ldots}{\ldots}$$

Mathematica [A] time = 0.13, size = 60, normalized size = 0.83

$$\frac{4\left(8a^2-8ab+3b^2\right)(c+dx)+8b(2a-b)\sinh(2(c+dx))+b^2\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (4*(8*a^2 - 8*a*b + 3*b^2)*(c + d*x) + 8*(2*a - b)*b*Sinh[2*(c + d*x)] + b^
2*Sinh[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.67, size = 80, normalized size = 1.11

$$\frac{b^2\cosh(dx+c)\sinh(dx+c)^3+\left(8a^2-8ab+3b^2\right)dx+\left(b^2\cosh(dx+c)^3+4\left(2ab-b^2\right)\cosh(dx+c)\right)\sinh\ldots}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)*sinh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*d*x + (b^2
*cosh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.14, size = 101, normalized size = 1.40

$$\frac{1}{8}\left(8a^2-8ab+3b^2\right)x+\frac{b^2e^{(4dx+4c)}}{64d}-\frac{b^2e^{(-4dx-4c)}}{64d}+\frac{\left(2ab-b^2\right)e^{(2dx+2c)}}{8d}-\frac{\left(2ab-b^2\right)e^{(-2dx-2c)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 - 8*a*b + 3*b^2)*x + 1/64*b^2*e^(4*d*x + 4*c)/d - 1/64*b^2*e^(-4
*d*x - 4*c)/d + 1/8*(2*a*b - b^2)*e^(2*d*x + 2*c)/d - 1/8*(2*a*b - b^2)*e^(
-2*d*x - 2*c)/d
```

maple [A] time = 0.03, size = 79, normalized size = 1.10

$$\frac{b^2\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+a^2(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a^2*(d*x+c))
```

maxima [A] time = 0.35, size = 105, normalized size = 1.46

$$\frac{1}{64}b^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{4}ab\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x -
 2*c)/d) + a^2*x
```

mupad [B] time = 0.10, size = 67, normalized size = 0.93

$$a^2x+\frac{3b^2x}{8}-abx-\frac{b^2\sinh(2c+2dx)}{4d}+\frac{b^2\sinh(4c+4dx)}{32d}+\frac{ab\sinh(2c+2dx)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] a^2*x + (3*b^2*x)/8 - a*b*x - (b^2*sinh(2*c + 2*d*x))/(4*d) + (b^2*sinh(4*c
 + 4*d*x))/(32*d) + (a*b*sinh(2*c + 2*d*x))/(2*d)
```

sympy [A] time = 1.00, size = 168, normalized size = 2.33

$$\begin{cases} a^2x+abx\sinh^2(c+dx)-abx\cosh^2(c+dx)+\frac{ab\sinh(c+dx)\cosh(c+dx)}{d}+\frac{3b^2x\sinh^4(c+dx)}{8}-\frac{3b^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} \\ x\left(a+b\sinh^2(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + a*b*x*sinh(c + d*x)**2 - a*b*x*cosh(c + d*x)**2 + a*b*s
inh(c + d*x)*cosh(c + d*x)/d + 3*b**2*x*sinh(c + d*x)**4/8 - 3*b**2*x*sinh(
c + d*x)**2*cosh(c + d*x)**2/4 + 3*b**2*x*cosh(c + d*x)**4/8 + 5*b**2*sinh(
c + d*x)**3*cosh(c + d*x)/(8*d) - 3*b**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*
d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2, True))
```

3.15 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=52

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a-b)\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}$$

```
[Out] -a^2*arctanh(cosh(d*x+c))/d+(2*a-b)*b*cosh(d*x+c)/d+1/3*b^2*cosh(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 390, 206}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a-b)\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) + ((2*a - b)*b*Cosh[c + d*x])/d + (b^2*Co
sh[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx&=-\frac{\operatorname{Subst}\left(\int\frac{\left(a-b+bx^2\right)^2}{1-x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\&=-\frac{\operatorname{Subst}\left(\int\left(-(2a-b)b-b^2x^2+\frac{a^2}{1-x^2}\right)dx,x,\cosh(c+dx)\right)}{d}\\&=\frac{(2a-b)b\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{a^2\operatorname{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\right.}{d}\\&=-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{(2a-b)b\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+}{3d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 104, normalized size = 2.00

$$\frac{a^2\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a^2\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{2ab\sinh(c)\sinh(dx)}{d}+\frac{2ab\cosh(c)\cosh(dx)}{d}-\frac{3b^2\cosh(c+d}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*a*b*Cosh[c]*Cosh[d*x])/d - (3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c
+ d*x)])/(12*d) - (a^2*Log[Cosh[c/2 + (d*x)/2]])/d + (a^2*Log[Sinh[c/2 + (d
*x)/2]])/d + (2*a*b*Sinh[c]*Sinh[d*x])/d
```

fricas [B] time = 0.47, size = 492, normalized size = 9.46

$$\frac{b^2\cosh\left(dx+c\right)^6+6\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+b^2\sinh\left(dx+c\right)^6+3\left(8\,ab-3\,b^2\right)\cosh\left(dx+c\right)^4+3\left(5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(
d*x + c)^6 + 3*(8*a*b - 3*b^2)*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 +
 8*a*b - 3*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + 3*(8*a*b - 3*b
^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(8*a*b - 3*b^2)*cosh(d*x + c)^2 + 3*
(5*b^2*cosh(d*x + c)^4 + 6*(8*a*b - 3*b^2)*cosh(d*x + c)^2 + 8*a*b - 3*b^2)
*sinh(d*x + c)^2 + b^2 - 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)^2*si
nh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c)^3)*lo
g(cosh(d*x + c) + sinh(d*x + c) + 1) + 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh
(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(
d*x + c)^3)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(b^2*cosh(d*x + c)^5
 + 2*(8*a*b - 3*b^2)*cosh(d*x + c)^3 + (8*a*b - 3*b^2)*cosh(d*x + c))*sinh(
d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh
(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

giac [B] time = 0.17, size = 110, normalized size = 2.12

$$\frac{b^2e^{(3\,dx+3\,c)}+24\,abe^{(dx+c)}-9\,b^2e^{(dx+c)}-24\,a^2\log\left(e^{(dx+c)}+1\right)+24\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)+\left(24\,abe^{(2\,dx+2\,c)}-9\,b\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*d*x + 3*c) + 24*a*b*e^(d*x + c) - 9*b^2*e^(d*x + c) - 24*a^2
*log(e^(d*x + c) + 1) + 24*a^2*log(abs(e^(d*x + c) - 1)) + (24*a*b*e^(2*d*x
 + 2*c) - 9*b^2*e^(2*d*x + 2*c) + b^2)*e^(-3*d*x - 3*c))/d
```

maple [A] time = 0.07, size = 50, normalized size = 0.96

$$\frac{-2a^2\operatorname{arctanh}\left(e^{dx+c}\right)+2ab\cosh\left(dx+c\right)+b^2\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*cosh(d*x+c)+b^2*(-2/3+1/3*sinh(d*x+c)
^2)*cosh(d*x+c))
```

maxima [B] time = 0.60, size = 102, normalized size = 1.96

$$\frac{1}{24}\,b^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+ab\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + a*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a^2*log(tanh(1/2*d*x +
 1/2*c))/d
```

mupad [B] time = 0.16, size = 116, normalized size = 2.23

$$\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}+\frac{b\,\mathrm{e}^{-c-d\,x}\,\left(8\,a-3\,b\right)}{8\,d}+\frac{b\,\mathrm{e}^{c+d\,x}\,\left(8\,a-3\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x),x)
```

```
[Out] (b^2*exp(- 3*c - 3*d*x))/(24*d) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2)
)/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^2)^(1/2) + (b^2*exp(3*c + 3*d*x))/(24*d
) + (b*exp(- c - d*x)*(8*a - 3*b))/(8*d) + (b*exp(c + d*x)*(8*a - 3*b))/(8*
d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^2\left(c+dx\right)\right)^2\operatorname{csch}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)**2*csch(c + d*x), x)
```

3.16 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=50

$$-\frac{a^2\coth(c+dx)}{d}+\frac{1}{2}bx(4a-b)+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] 1/2*(4*a-b)*b*x-a^2*coth(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3187, 462, 385, 206}

$$\frac{\left(2a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d}+\frac{1}{2}bx(4a-b)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - b)*b*x)/2 - (a^2*Cosh[c + d*x]^2*Coth[c + d*x])/d + ((2*a^2 + b^2)*
Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \text{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{x^2\left(1-x^2\right)^2}dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{a(a+2b)+(a-b)^2x^2}{\left(1-x^2\right)^2}dx, x, \text{tar}\right.}{d}$$

$$= -\frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d} + \frac{\left(2a^2+b^2\right)\cosh(c+dx)\sinh(c+}{2d}$$

$$= \frac{1}{2}(4a-b)bx - \frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d} + \frac{\left(2a^2+b^2\right)\cosh(c+}{2d}$$

Mathematica [A] time = 0.16, size = 56, normalized size = 1.12

$$-\frac{a^2\coth(c+dx)}{d} + 2abx + \frac{b^2(-c-dx)}{2d} + \frac{b^2\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (b^2*(-c - d*x))/(2*d) - (a^2*Coth[c + d*x])/d + (b^2*Sinh[2*(c +
 d*x)])/(4*d)
```

fricas [A] time = 0.44, size = 89, normalized size = 1.78

$$\frac{b^2\cosh(dx+c)^3 + 3b^2\cosh(dx+c)\sinh(dx+c)^2 - \left(8a^2+b^2\right)\cosh(dx+c) + 4\left(\left(4ab-b^2\right)dx + 2a^2\right)\sinh(d}{8d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 - (8*a^2 + b
^2)*cosh(d*x + c) + 4*((4*a*b - b^2)*d*x + 2*a^2)*sinh(d*x + c))/(d*sinh(d*
x + c))
```

giac [B] time = 0.16, size = 135, normalized size = 2.70

$$\frac{b^2e^{(2dx+2c)} + 4\left(4ab-b^2\right)(dx+c) - \frac{4abe^{(4dx+4c)} - b^2e^{(4dx+4c)} + 16a^2e^{(2dx+2c)} - 4abe^{(2dx+2c)} + 2b^2e^{(2dx+2c)} - b^2}{e^{(4dx+4c)} - e^{(2dx+2c)}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(b^2*e^(2*d*x + 2*c) + 4*(4*a*b - b^2)*(d*x + c) - (4*a*b*e^(4*d*x + 4*
c) - b^2*e^(4*d*x + 4*c) + 16*a^2*e^(2*d*x + 2*c) - 4*a*b*e^(2*d*x + 2*c) +
 2*b^2*e^(2*d*x + 2*c) - b^2)/(e^(4*d*x + 4*c) - e^(2*d*x + 2*c)))/d
```

maple [A] time = 0.07, size = 52, normalized size = 1.04

$$\frac{-a^2\coth(dx+c) + 2ab(dx+c) + b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*(d*x+c)+b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*
x-1/2*c))
```

maxima [A] time = 0.43, size = 63, normalized size = 1.26

$$-\frac{1}{8}\,b^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+2\,abx+\frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a*b*x + 2*a^2/(
d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.66, size = 67, normalized size = 1.34

$$\frac{b\,x\,\left(4\,a-b\right)}{2}-\frac{2\,a^2}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{b^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}+\frac{b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x)^2,x)
```

```
[Out] (b*x*(4*a - b))/2 - (2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) - (b^2*exp(- 2*c - 2
*d*x))/(8*d) + (b^2*exp(2*c + 2*d*x))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.17 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=56

$$-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2\cosh(c+dx)}{d}$$

```
[Out] 1/2*a*(a-4*b)*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-1/2*a^2*coth(d*x+c)*
csch(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 390, 385, 206}

$$-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + (b^2*Cosh[c + d*x])/d - (a^2*C
oth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^2}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a(a-2b)+2abx^2}{\left(1-x^2\right)^2}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{b^2\cosh(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{a(a-2b)+2abx^2}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{b^2\cosh(c+dx)}{d} - \frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{(a(a-4b))\,\text{Su}}{}$$
$$= \frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b^2\cosh(c+dx)}{d} - \frac{a^2\coth(c+}{}$$

Mathematica [B] time = 0.06, size = 134, normalized size = 2.39

$$-\frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{2ab\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{2ab\log\left(\text{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (a^2*Csch[(c + d*x)/2]^2)/(8*d) - (2*a*b*Log[Co
sh[c/2 + (d*x)/2]])/d + (2*a*b*Log[Sinh[c/2 + (d*x)/2]])/d - (a^2*Log[Tanh[
(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d) + (b^2*Sinh[c]*Sinh[
d*x])/d
```

fricas [B] time = 2.07, size = 902, normalized size = 16.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d
*x + c)^6 - (2*a^2 + b^2)*cosh(d*x + c)^4 + (15*b^2*cosh(d*x + c)^2 - 2*a^2
 - b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x + c)^2 + (15*b^2*cosh(d*x +
 c)^4 - 6*(2*a^2 + b^2)*cosh(d*x + c)^2 - 2*a^2 - b^2)*sinh(d*x + c)^2 + b^
2 + ((a^2 - 4*a*b)*cosh(d*x + c)^5 + 5*(a^2 - 4*a*b)*cosh(d*x + c)*sinh(d*x
 + c)^4 + (a^2 - 4*a*b)*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^3 +
 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^3 + 2*(5*(
a^2 - 4*a*b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)
^2 + (a^2 - 4*a*b)*cosh(d*x + c) + (5*(a^2 - 4*a*b)*cosh(d*x + c)^4 - 6*(a^
2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c))*log(cosh(d*x + c)
+ sinh(d*x + c) + 1) - ((a^2 - 4*a*b)*cosh(d*x + c)^5 + 5*(a^2 - 4*a*b)*cos
h(d*x + c)*sinh(d*x + c)^4 + (a^2 - 4*a*b)*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b
)*cosh(d*x + c)^3 + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(
d*x + c)^3 + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x
+ c))*sinh(d*x + c)^2 + (a^2 - 4*a*b)*cosh(d*x + c) + (5*(a^2 - 4*a*b)*cosh
(d*x + c)^4 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c))
*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*b^2*cosh(d*x + c)^5 - 2*(2*a
^2 + b^2)*cosh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*
```

```
cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 - 2
*d*cosh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 2*(5*d*c
osh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*
d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

giac [B] time = 0.16, size = 125, normalized size = 2.23

$$\frac{2\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{4\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}+\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^2*(e^(d*x + c) + e^(-d*x - c)) - 4*a^2*(e^(d*x + c) + e^(-d*x - c)
)/((e^(d*x + c) + e^(-d*x - c))^2 - 4) + (a^2 - 4*a*b)*log(e^(d*x + c) + e^
(-d*x - c) + 2) - (a^2 - 4*a*b)*log(e^(d*x + c) + e^(-d*x - c) - 2))/d
```

maple [A] time = 0.08, size = 53, normalized size = 0.95

$$\frac{a^2\left(-\frac{\mathrm{csch}(dx+c)\coth(dx+c)}{2}+\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)-4ab\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b^2\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-4*a*b*arctanh(e
xp(d*x+c))+b^2*cosh(d*x+c))
```

maxima [B] time = 0.59, size = 157, normalized size = 2.80

$$\frac{1}{2}\,b^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)-2\,ab\left(\frac{\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d
 - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*a*b*(log(e^(-d*x - c) + 1)/d - l
og(e^(-d*x - c) - 1)/d)
```

mupad [B] time = 0.67, size = 179, normalized size = 3.20

$$\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^2\,\sqrt{-d^2}-4\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}\right)\,\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}{\sqrt{-d^2}}+\frac{b^2\,\mathrm{e}^{c+d\,x}}{2\,d}+\frac{b^2\,\mathrm{e}^{-c-d\,x}}{2\,d}-\frac{a^2\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a^2\,\mathrm{e}^{c+d}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) - 4*a*b*(-d^2)^(1/2)))/(d*(a^4 - 8
*a^3*b + 16*a^2*b^2)^(1/2)))*(a^4 - 8*a^3*b + 16*a^2*b^2)^(1/2))/(-d^2)^(1/
2) + (b^2*exp(c + d*x))/(2*d) + (b^2*exp(- c - d*x))/(2*d) - (a^2*exp(c + d
*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^2*exp(c + d*x))/(d*(exp(4*c + 4*d*x)
 - 2*exp(2*c + 2*d*x) + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
[Out] Timed out
```

3.18 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=40

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a(a-2b)\coth(c+dx)}{d}+b^2x$$

```
[Out] b^2*x+a*(a-2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3187, 461, 207}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a(a-2b)\coth(c+dx)}{d}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] b^2*x + (a*(a - 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{x^4\left(1-x^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2}{x^4}-\frac{a(a-2b)}{x^2}-\frac{b^2}{-1+x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{b^2\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x\right.}{d}\\
&= b^2x+\frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [B] time = 0.72, size = 85, normalized size = 2.12

$$\frac{4\sinh^4(c+dx)\left(a\operatorname{csch}^2(c+dx)+b\right)^2\left(3b^2(c+dx)-a\coth(c+dx)\left(a\operatorname{csch}^2(c+dx)-2a+6b\right)\right)}{3d(2a+b\cosh(2(c+dx))-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (4*(b + a*Csch[c + d*x]^2)^2*(3*b^2*(c + d*x) - a*Coth[c + d*x]*(-2*a + 6*b
 + a*Csch[c + d*x]^2))*Sinh[c + d*x]^4)/(3*d*(2*a - b + b*Cosh[2*(c + d*x)]
)^2)
```

fricas [B] time = 0.39, size = 174, normalized size = 4.35

$$\frac{2\left(a^2-3\,ab\right)\cosh\left(dx+c\right)^3+6\left(a^2-3\,ab\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+\left(3\,b^2dx-2\,a^2+6\,ab\right)\sinh\left(dx+c\right)^3}{3\left(d\sinh\left(dx+c\right)^3+3\left(d\cosh\left(\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(a^2 - 3*a*b)*cosh(d*x + c)^3 + 6*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d
*x + c)^2 + (3*b^2*d*x - 2*a^2 + 6*a*b)*sinh(d*x + c)^3 - 6*(a^2 - a*b)*cos
h(d*x + c) - 3*(3*b^2*d*x - (3*b^2*d*x - 2*a^2 + 6*a*b)*cosh(d*x + c)^2 - 2
*a^2 + 6*a*b)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)
*sinh(d*x + c))
```

giac [B] time = 0.18, size = 81, normalized size = 2.02

$$\frac{3\left(dx+c\right)b^2-\frac{4\left(3\,abe^{(4\,dx+4\,c)}+3\,a^2e^{(2\,dx+2\,c)}-6\,abe^{(2\,dx+2\,c)}-a^2+3\,ab\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*b^2 - 4*(3*a*b*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - 6
*a*b*e^(2*d*x + 2*c) - a^2 + 3*a*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [A] time = 0.09, size = 47, normalized size = 1.18

$$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth\left(dx+c\right)-2ab\coth\left(dx+c\right)+b^2\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*a*b*coth(d*x+c)+b^2*(d*x+c))
```

maxima [B] time = 0.47, size = 121, normalized size = 3.02

$$b^2x+\frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)+\frac{}{d\left(e^{(-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*x + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.62, size = 166, normalized size = 4.15

$$b^2\,x-\frac{\frac{4\,a\,b}{3\,d}-\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b-a^2\right)}{3\,d}+\frac{4\,a\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}+\frac{\frac{4\,\left(a\,b-a^2\right)}{3\,d}-\frac{4\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{3\,d}}{{\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1}-\frac{4\,a\,b}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/sinh(c + d*x)^4,x)
```

```
[Out] b^2*x - ((4*a*b)/(3*d) - (8*exp(2*c + 2*d*x)*(a*b - a^2))/(3*d) + (4*a*b*ex
p(4*c + 4*d*x))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
 6*d*x) - 1) + ((4*(a*b - a^2))/(3*d) - (4*a*b*exp(2*c + 2*d*x))/(3*d))/(ex
p(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (4*a*b)/(3*d*(exp(2*c + 2*d*x) -
 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.19 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=261

$$\frac{3}{256}x(4a-3b)\left(8a^2-14ab+7b^2\right)+\frac{\left(48a^3-272a^2b+314ab^2-105b^3\right)\sinh(c+dx)\cosh^3(c+dx)}{640d}-\frac{\left(576a^3-17\right.}{}$$

```
[Out] 3/256*(4*a-3*b)*(8*a^2-14*a*b+7*b^2)*x-1/1280*(576*a^3-1744*a^2*b+1678*a*b^
2-525*b^3)*cosh(d*x+c)*sinh(d*x+c)/d+1/640*(48*a^3-272*a^2*b+314*a*b^2-105*
b^3)*cosh(d*x+c)^3*sinh(d*x+c)/d+3/80*(2*a-3*b)*cosh(d*x+c)^5*sinh(d*x+c)^3
*(a-(a-b)*tanh(d*x+c)^2)^2/d+1/10*cosh(d*x+c)^7*sinh(d*x+c)^3*(a-(a-b)*tanh
(d*x+c)^2)^3/d-1/160*b*cosh(d*x+c)^3*sinh(d*x+c)^3*(a*(14*a-9*b)-(22*a-21*b
)*(a-b)*tanh(d*x+c)^2)/d
```

Rubi [A] time = 0.43, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3187, 467, 577, 455, 385, 206}

$$\frac{\left(-272a^2b+48a^3+314ab^2-105b^3\right)\sinh(c+dx)\cosh^3(c+dx)}{640d}-\frac{\left(-1744a^2b+576a^3+1678ab^2-525b^3\right)\sinh}{1280d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*x)/256 - ((576*a^3 - 1744*a^2*b + 1
678*a*b^2 - 525*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(1280*d) + ((48*a^3 - 272
*a^2*b + 314*a*b^2 - 105*b^3)*Cosh[c + d*x]^3*Sinh[c + d*x])/(640*d) + (3*(
2*a - 3*b)*Cosh[c + d*x]^5*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^2)
/(80*d) + (Cosh[c + d*x]^7*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^3)
/(10*d) - (b*Cosh[c + d*x]^3*Sinh[c + d*x]^3*(a*(14*a - 9*b) - (22*a - 21*b
)*(a - b)*Tanh[c + d*x]^2))/(160*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int\frac{x^4\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^6}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\cosh^7(c+dx)\sinh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10d}-\frac{\text{Subst}\Big(}{}$$

$$=\frac{3(2a-3b)\cosh^5(c+dx)\sinh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{80d}$$

$$=\frac{3(2a-3b)\cosh^5(c+dx)\sinh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{80d}$$

$$=\frac{\left(48a^3-272a^2b+314ab^2-105b^3\right)\cosh^3(c+dx)\sinh(c+dx)}{640d}+\frac{3(}{}$$

$$=-\frac{\left(576a^3-1744a^2b+1678ab^2-525b^3\right)\cosh(c+dx)\sinh(c+dx)}{1280d}$$

$$=\frac{3}{256}(4a-3b)\left(8a^2-14ab+7b^2\right)x-\frac{\left(576a^3-1744a^2b+1678ab^2-\right.}{1}$$

Mathematica [A] time = 0.42, size = 162, normalized size = 0.62

$$\frac{120(4a-3b)\left(8a^2-14ab+7b^2\right)(c+dx)+10b\left(16a^2-32ab+15b^2\right)\sinh(6(c+dx))-20\left(128a^3-360a^2b+336\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (120*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*(c + d*x) - 20*(128*a^3 - 360*a^2
*b + 336*a*b^2 - 105*b^3)*Sinh[2*(c + d*x)] + 40*(8*a^3 - 36*a^2*b + 42*a*b
^2 - 15*b^3)*Sinh[4*(c + d*x)] + 10*b*(16*a^2 - 32*a*b + 15*b^2)*Sinh[6*(c
+ d*x)] + 5*(6*a - 5*b)*b^2*Sinh[8*(c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(
10240*d)
```

fricas [A] time = 0.48, size = 406, normalized size = 1.56

$$\frac{5\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^9+10\left(6\,b^3\cosh\left(dx+c\right)^3+\left(6\,ab^2-5\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^7+\left(126\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2560*(5*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 10*(6*b^3*cosh(d*x + c)^3 + (
6*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (126*b^3*cosh(d*x + c)^5
+ 70*(6*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 15*(16*a^2*b - 32*a*b^2 + 15*b^3)*
cosh(d*x + c))*sinh(d*x + c)^5 + 10*(6*b^3*cosh(d*x + c)^7 + 7*(6*a*b^2 - 5
*b^3)*cosh(d*x + c)^5 + 5*(16*a^2*b - 32*a*b^2 + 15*b^3)*cosh(d*x + c)^3 +
4*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3
0*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*d*x + 5*(b^3*cosh(d*x + c)^9 + 2*
(6*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 3*(16*a^2*b - 32*a*b^2 + 15*b^3)*cosh(d
*x + c)^5 + 8*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*cosh(d*x + c)^3 - 2*(1
28*a^3 - 360*a^2*b + 336*a*b^2 - 105*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.21, size = 325, normalized size = 1.25

$$\frac{b^3e^{(10\,dx+10\,c)}}{10240\,d}-\frac{b^3e^{(-10\,dx-10\,c)}}{10240\,d}+\frac{3}{256}\left(32\,a^3-80\,a^2b+70\,ab^2-21\,b^3\right)x+\frac{\left(6\,ab^2-5\,b^3\right)e^{(8\,dx+8\,c)}}{4096\,d}+\frac{\left(16\,a^2b-32\,ab\right.}{2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10240*b^3*e^(10*d*x + 10*c)/d - 1/10240*b^3*e^(-10*d*x - 10*c)/d + 3/256*
(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*x + 1/4096*(6*a*b^2 - 5*b^3)*e^(8*d
*x + 8*c)/d + 1/2048*(16*a^2*b - 32*a*b^2 + 15*b^3)*e^(6*d*x + 6*c)/d + 1/5
12*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*e^(4*d*x + 4*c)/d - 1/1024*(128*a
^3 - 360*a^2*b + 336*a*b^2 - 105*b^3)*e^(2*d*x + 2*c)/d + 1/1024*(128*a^3 -
 360*a^2*b + 336*a*b^2 - 105*b^3)*e^(-2*d*x - 2*c)/d - 1/512*(8*a^3 - 36*a^
2*b + 42*a*b^2 - 15*b^3)*e^(-4*d*x - 4*c)/d - 1/2048*(16*a^2*b - 32*a*b^2 +
 15*b^3)*e^(-6*d*x - 6*c)/d - 1/4096*(6*a*b^2 - 5*b^3)*e^(-8*d*x - 8*c)/d
```

maple [A] time = 0.13, size = 222, normalized size = 0.85

$$\frac{b^3\left(\left(\frac{\left(\sinh^9(dx+c)\right)}{10}-\frac{9\left(\sinh^7(dx+c)\right)}{80}+\frac{21\left(\sinh^5(dx+c)\right)}{160}-\frac{21\left(\sinh^3(dx+c)\right)}{128}+\frac{63\sinh(dx+c)}{256}\right)\cosh\left(dx+c\right)-\frac{63dx}{256}-\frac{63c}{256}\right)+3a\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/12
8*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+3*a*b^
2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d
*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+3*a^2*b*((1/6*sinh(d*x+c)^5-5/24*si
```

```
nh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a^3*((1/4*sinh(d
*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.37, size = 405, normalized size = 1.55

$$\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{20480}b^3\left(\frac{\left(25e^{(-2dx-2c)}-150e^{(-4dx-4c)}+600\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((25*e^(-2*d*x - 2*c) - 150*e^(-4*
d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x +
10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c)
+ 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 1
/2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x -
6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 16
8*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) - 1/128*a
^2*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 12
0*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
))/d)
```

mupad [B] time = 1.20, size = 239, normalized size = 0.92

$$\frac{40a^3\sinh(4c+4dx)-320a^3\sinh(2c+2dx)+\frac{525b^3\sinh(2c+2dx)}{2}-75b^3\sinh(4c+4dx)+\frac{75b^3\sinh(6c+6dx)}{4}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (40*a^3*sinh(4*c + 4*d*x) - 320*a^3*sinh(2*c + 2*d*x) + (525*b^3*sinh(2*c +
 2*d*x))/2 - 75*b^3*sinh(4*c + 4*d*x) + (75*b^3*sinh(6*c + 6*d*x))/4 - (25*
b^3*sinh(8*c + 8*d*x))/8 + (b^3*sinh(10*c + 10*d*x))/4 - 840*a*b^2*sinh(2*c
 + 2*d*x) + 900*a^2*b*sinh(2*c + 2*d*x) + 210*a*b^2*sinh(4*c + 4*d*x) - 180
*a^2*b*sinh(4*c + 4*d*x) - 40*a*b^2*sinh(6*c + 6*d*x) + 20*a^2*b*sinh(6*c +
 6*d*x) + (15*a*b^2*sinh(8*c + 8*d*x))/4 + 480*a^3*d*x - 315*b^3*d*x + 1050
*a*b^2*d*x - 1200*a^2*b*d*x)/(1280*d)
```

sympy [A] time = 24.46, size = 777, normalized size = 2.98

$$\begin{cases}\frac{3a^3x\sinh^4(c+dx)}{8}-\frac{3a^3x\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3a^3x\cosh^4(c+dx)}{8}+\frac{5a^3\sinh^3(c+dx)\cosh(c+dx)}{8d}-\frac{3a^3\sinh(c+dx)\cosh^3(c+dx)}{8d}+ \\ x\left(a+b\sinh^2(c)\right)^3\sinh^4(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((3*a**3*x*sinh(c + d*x)**4/8 - 3*a**3*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**3*x*cosh(c + d*x)**4/8 + 5*a**3*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) - 3*a**3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 15*a**2*b*x*sin
h(c + d*x)**6/16 - 45*a**2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 45*a*
*2*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 15*a**2*b*x*cosh(c + d*x)**6/
16 + 33*a**2*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*a**2*b*sinh(c + d*
x)**3*cosh(c + d*x)**3/(2*d) + 15*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16
*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*cos
h(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 105*
```

```
a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x)*
*8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*sin
h(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh(c
 + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 63
*b**3*x*sinh(c + d*x)**10/256 - 315*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**
2/256 + 315*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 315*b**3*x*sinh(
c + d*x)**4*cosh(c + d*x)**6/128 + 315*b**3*x*sinh(c + d*x)**2*cosh(c + d*x
)**8/256 - 63*b**3*x*cosh(c + d*x)**10/256 + 193*b**3*sinh(c + d*x)**9*cosh
(c + d*x)/(256*d) - 237*b**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 21
*b**3*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 147*b**3*sinh(c + d*x)**3*
cosh(c + d*x)**7/(128*d) + 63*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d),
Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**4, True))
```

3.20 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=115

$$\frac{b^2(3a-4b)\cosh^7(c+dx)}{7d}+\frac{3b(a-2b)(a-b)\cosh^5(c+dx)}{5d}+\frac{(a-4b)(a-b)^2\cosh^3(c+dx)}{3d}-\frac{(a-b)^3\cosh(c+dx)}{d}$$

```
[Out] -(a-b)^3*cosh(d*x+c)/d+1/3*(a-4*b)*(a-b)^2*cosh(d*x+c)^3/d+3/5*(a-2*b)*(a-b
)*b*cosh(d*x+c)^5/d+1/7*(3*a-4*b)*b^2*cosh(d*x+c)^7/d+1/9*b^3*cosh(d*x+c)^9
/d
```

Rubi [A] time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3186, 373}

$$\frac{b^2(3a-4b)\cosh^7(c+dx)}{7d}+\frac{3b(a-2b)(a-b)\cosh^5(c+dx)}{5d}+\frac{(a-4b)(a-b)^2\cosh^3(c+dx)}{3d}-\frac{(a-b)^3\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -(((a - b)^3*Cosh[c + d*x])/d) + ((a - 4*b)*(a - b)^2*Cosh[c + d*x]^3)/(3*d
) + (3*(a - 2*b)*(a - b)*b*Cosh[c + d*x]^5)/(5*d) + ((3*a - 4*b)*b^2*Cosh[c
 + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(9*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a-b+bx^2\right)^3\,dx,x,\cosh(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left((a-b)^3-(a-4b)(a-b)^2x^2+3(a-2b)b(-a+b)x^4-\right.\right.}{d}\\
&= -\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{(a-4b)(a-b)^2\cosh^3(c+dx)}{3d}+\frac{3(a-2b}{}
\end{aligned}$$

Mathematica [A] time = 0.80, size = 127, normalized size = 1.10

$$\frac{-1890(4a-3b)\left(8a^2-14ab+7b^2\right)\cosh(c+dx)+420\left(16a^3-60a^2b+63ab^2-21b^3\right)\cosh(3(c+dx))+135b^2(4}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-1890*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*Cosh[c + d*x] + 420*(16*a^3 - 6
0*a^2*b + 63*a*b^2 - 21*b^3)*Cosh[3*(c + d*x)] + 756*(4*a - 3*b)*(a - b)*b*
Cosh[5*(c + d*x)] + 135*(4*a - 3*b)*b^2*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(
c + d*x)])/(80640*d)
```

fricas [B] time = 0.51, size = 373, normalized size = 3.24

$$35\,b^3\cosh\left(dx+c\right)^9+315\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8+135\left(4\,ab^2-3\,b^3\right)\cosh\left(dx+c\right)^7+105\left(28\,b^3\cosh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^3*cosh(d*x + c)^9 + 315*b^3*cosh(d*x + c)*sinh(d*x + c)^8 + 1
35*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^7 + 105*(28*b^3*cosh(d*x + c)^3 + 9*(4*a
*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 756*(4*a^2*b - 7*a*b^2 + 3*b
^3)*cosh(d*x + c)^5 + 315*(14*b^3*cosh(d*x + c)^5 + 15*(4*a*b^2 - 3*b^3)*co
sh(d*x + c)^3 + 12*(4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(d*x + c))*sinh(d*x + c)
^4 + 420*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*cosh(d*x + c)^3 + 315*(4*b
^3*cosh(d*x + c)^7 + 9*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + 24*(4*a^2*b - 7*
a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*
cosh(d*x + c))*sinh(d*x + c)^2 - 1890*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^
3)*cosh(d*x + c))/d
```

giac [B] time = 0.20, size = 296, normalized size = 2.57

$$\frac{b^3e^{(9\,dx+9\,c)}}{4608\,d}+\frac{b^3e^{(-9\,dx-9\,c)}}{4608\,d}+\frac{3\left(4\,ab^2-3\,b^3\right)e^{(7\,dx+7\,c)}}{3584\,d}+\frac{3\left(4\,a^2b-7\,ab^2+3\,b^3\right)e^{(5\,dx+5\,c)}}{640\,d}+\frac{\left(16\,a^3-60\,a^2b+63\,ab^2\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4608*b^3*e^(9*d*x + 9*c)/d + 1/4608*b^3*e^(-9*d*x - 9*c)/d + 3/3584*(4*a*
b^2 - 3*b^3)*e^(7*d*x + 7*c)/d + 3/640*(4*a^2*b - 7*a*b^2 + 3*b^3)*e^(5*d*x
 + 5*c)/d + 1/384*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*e^(3*d*x + 3*c)/d
 - 3/256*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*e^(d*x + c)/d - 3/256*(32*
a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*e^(-d*x - c)/d + 1/384*(16*a^3 - 60*a^2
*b + 63*a*b^2 - 21*b^3)*e^(-3*d*x - 3*c)/d + 3/640*(4*a^2*b - 7*a*b^2 + 3*b
^3)*e^(-5*d*x - 5*c)/d + 3/3584*(4*a*b^2 - 3*b^3)*e^(-7*d*x - 7*c)/d
```

maple [A] time = 0.14, size = 158, normalized size = 1.37

$$b^3\left(\frac{128}{315}+\frac{\left(\sinh^8(dx+c)\right)}{9}-\frac{8\left(\sinh^6(dx+c)\right)}{63}+\frac{16\left(\sinh^4(dx+c)\right)}{105}-\frac{64\left(\sinh^2(dx+c)\right)}{315}\right)\cosh\left(dx+c\right)+3a\,b^2\left(-\frac{16}{35}+\frac{\left(\sinh^6(dx+c)\right)}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4
-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*s
inh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(8/15+1/5*sinh(d*x+c)^
4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.49, size = 376, normalized size = 3.27

$$-\frac{1}{161280}\,b^3\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\frac{39690\,e^{(-d}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-
4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x
 - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d)
 + 1/160*a^2*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c
)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) +
1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d)
```

mupad [B] time = 0.43, size = 185, normalized size = 1.61

$$\frac{\frac{a^3 \cosh(c+d\,x)^3}{3} - a^3 \cosh\left(c + d\,x\right) + \frac{3\,a^2\,b \cosh(c+d\,x)^5}{5} - 2\,a^2\,b \cosh\left(c + d\,x\right)^3 + 3\,a^2\,b \cosh\left(c + d\,x\right) + \frac{3\,a\,b^2 \cosh(c+d\,x)^7}{7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (b^3*cosh(c + d*x) - a^3*cosh(c + d*x) + (a^3*cosh(c + d*x)^3)/3 - (4*b^3*c
osh(c + d*x)^3)/3 + (6*b^3*cosh(c + d*x)^5)/5 - (4*b^3*cosh(c + d*x)^7)/7 +
 (b^3*cosh(c + d*x)^9)/9 + 3*a*b^2*cosh(c + d*x)^3 - 2*a^2*b*cosh(c + d*x)^
3 - (9*a*b^2*cosh(c + d*x)^5)/5 + (3*a^2*b*cosh(c + d*x)^5)/5 + (3*a*b^2*co
sh(c + d*x)^7)/7 - 3*a*b^2*cosh(c + d*x) + 3*a^2*b*cosh(c + d*x))/d
```

sympy [A] time = 14.69, size = 330, normalized size = 2.87

$$\begin{cases} \frac{a^3 \sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^3\cosh^3(c+dx)}{3d} + \frac{3a^2 b \sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4a^2 b \sinh^2(c+dx)\cosh^3(c+dx)}{d} + \frac{8a^2 b\cosh^5(c+dx)}{5d} + \frac{3ab}{} \\ x\left(a + b\sinh^2(c)\right)^3 \sinh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**3*cosh(c + d*x)**3/
(3*d) + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*a**2*b*sinh(c + d*x)*
*2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5/(5*d) + 3*a*b**2*sinh(c +
 d*x)**6*cosh(c + d*x)/d - 6*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 2
4*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 48*a*b**2*cosh(c + d*x)*
*7/(35*d) + b**3*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**3*sinh(c + d*x)**6
*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) -
 64*b**3*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**3*cosh(c + d*x)*
*9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**3, True))
```

3.21 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=181

$$\frac{b\left(24a^2-64ab+35b^2\right)\sinh^3(c+dx)\cosh(c+dx)}{192d}+\frac{\left(96a^3-376a^2b+360ab^2-105b^3\right)\sinh(c+dx)\cosh(c+$$

```
[Out] -1/128*(64*a^3-144*a^2*b+120*a*b^2-35*b^3)*x+1/384*(96*a^3-376*a^2*b+360*a*
b^2-105*b^3)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*b*(24*a^2-64*a*b+35*b^2)*cosh(
d*x+c)*sinh(d*x+c)^3/d+1/48*(6*a-7*b)*cosh(d*x+c)*sinh(d*x+c)*(a+b*sinh(d*x
+c)^2)^2/d+1/8*cosh(d*x+c)*sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3/d
```

Rubi [A] time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3170, 3169}

$$\frac{b\left(24a^2-64ab+35b^2\right)\sinh^3(c+dx)\cosh(c+dx)}{192d}+\frac{\left(-376a^2b+96a^3+360ab^2-105b^3\right)\sinh(c+dx)\cosh(c$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -((64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*x)/128 + ((96*a^3 - 376*a^2*b +
 360*a*b^2 - 105*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(384*d) + (b*(24*a^2 - 6
4*a*b + 35*b^2)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) + ((6*a - 7*b)*Cosh[
c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(48*d) + (Cosh[c + d*x]*S
inh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3)/(8*d)
```

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3}{8d}-\frac{1}{8}\int\left(a-(6a\right.$$

$$=\frac{(6a-7b)\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{48d}+\frac{\cos}{}$$

$$=-\frac{1}{128}\left(64a^3-144a^2b+120ab^2-35b^3\right)x+\frac{\left(96a^3-376a^2b+36\right.}{}$$

Mathematica [A] time = 0.29, size = 130, normalized size = 0.72

$$\frac{24b\left(12a^2-18ab+7b^2\right)\sinh(4(c+dx))-24\left(64a^3-144a^2b+120ab^2-35b^3\right)(c+dx)+48\left(16a^3-48a^2b+45a\right.}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-24*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*(c + d*x) + 48*(16*a^3 - 48*
a^2*b + 45*a*b^2 - 14*b^3)*Sinh[2*(c + d*x)] + 24*b*(12*a^2 - 18*a*b + 7*b^
2)*Sinh[4*(c + d*x)] + 16*(3*a - 2*b)*b^2*Sinh[6*(c + d*x)] + 3*b^3*Sinh[8*
(c + d*x)])/(3072*d)
```

fricas [A] time = 1.03, size = 269, normalized size = 1.49

$$3\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+3\left(7\,b^3\cosh\left(dx+c\right)^3+4\left(3\,ab^2-2\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+\left(21\,b^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^3*cosh(d*x + c)^3 + 4*(
3*a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^3*cosh(d*x + c)^5 +
 40*(3*a*b^2 - 2*b^3)*cosh(d*x + c)^3 + 12*(12*a^2*b - 18*a*b^2 + 7*b^3)*co
sh(d*x + c))*sinh(d*x + c)^3 - 3*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*
d*x + 3*(b^3*cosh(d*x + c)^7 + 4*(3*a*b^2 - 2*b^3)*cosh(d*x + c)^5 + 4*(12*
a^2*b - 18*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 48*a^2*b + 45*a*b^2
 - 14*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.18, size = 251, normalized size = 1.39

$$\frac{b^3e^{(8\,dx+8\,c)}}{2048\,d}-\frac{b^3e^{(-8\,dx-8\,c)}}{2048\,d}-\frac{1}{128}\left(64\,a^3-144\,a^2b+120\,ab^2-35\,b^3\right)x+\frac{\left(3\,ab^2-2\,b^3\right)e^{(6\,dx+6\,c)}}{384\,d}+\frac{\left(12\,a^2b-18\,ab^2+7\right.}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2048*b^3*e^(8*d*x + 8*c)/d - 1/2048*b^3*e^(-8*d*x - 8*c)/d - 1/128*(64*a^
3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*x + 1/384*(3*a*b^2 - 2*b^3)*e^(6*d*x +
6*c)/d + 1/256*(12*a^2*b - 18*a*b^2 + 7*b^3)*e^(4*d*x + 4*c)/d + 1/128*(16*
a^3 - 48*a^2*b + 45*a*b^2 - 14*b^3)*e^(2*d*x + 2*c)/d - 1/128*(16*a^3 - 48*
a^2*b + 45*a*b^2 - 14*b^3)*e^(-2*d*x - 2*c)/d - 1/256*(12*a^2*b - 18*a*b^2
+ 7*b^3)*e^(-4*d*x - 4*c)/d - 1/384*(3*a*b^2 - 2*b^3)*e^(-6*d*x - 6*c)/d
```

maple [A] time = 0.04, size = 180, normalized size = 0.99

$$b^3\left(\left(\frac{\left(\sinh^7(dx+c)\right)}{8}-\frac{7\left(\sinh^5(dx+c)\right)}{48}+\frac{35\left(\sinh^3(dx+c)\right)}{192}-\frac{35\sinh(dx+c)}{128}\right)\cosh\left(dx+c\right)+\frac{35dx}{128}+\frac{35c}{128}\right)+3a\,b^2\left(\left(\frac{\left(\sinh^5(dx+c)\right)}{6}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128
*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+3*a*b^2*((1/6*sinh(d*x+c)^5-
5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+3*a^2*b*(
(1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^3*(1/2*cos
h(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [A] time = 0.36, size = 306, normalized size = 1.69

$$\frac{3}{64}\,a^2b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{8}\,a^3\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{6144}\,b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x
 - 2*c)/d) - 1/6144*b^3*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*
e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x
 - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/
d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x +
 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c))/d)
```

mupad [B] time = 0.97, size = 181, normalized size = 1.00

$$\frac{96\,a^3\sinh\left(2\,c+2\,d\,x\right)-84\,b^3\sinh\left(2\,c+2\,d\,x\right)+21\,b^3\sinh\left(4\,c+4\,d\,x\right)-4\,b^3\sinh\left(6\,c+6\,d\,x\right)+\frac{3\,b^3\sinh(8\,c+8}{8}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (96*a^3*sinh(2*c + 2*d*x) - 84*b^3*sinh(2*c + 2*d*x) + 21*b^3*sinh(4*c + 4*
d*x) - 4*b^3*sinh(6*c + 6*d*x) + (3*b^3*sinh(8*c + 8*d*x))/8 + 270*a*b^2*si
nh(2*c + 2*d*x) - 288*a^2*b*sinh(2*c + 2*d*x) - 54*a*b^2*sinh(4*c + 4*d*x)
+ 36*a^2*b*sinh(4*c + 4*d*x) + 6*a*b^2*sinh(6*c + 6*d*x) - 192*a^3*d*x + 10
5*b^3*d*x - 360*a*b^2*d*x + 432*a^2*b*d*x)/(384*d)
```

sympy [A] time = 9.85, size = 561, normalized size = 3.10

$$\begin{cases} \frac{a^3x\sinh^2(c+dx)}{2}-\frac{a^3x\cosh^2(c+dx)}{2}+\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{9a^2bx\sinh^4(c+dx)}{8}-\frac{9a^2bx\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{9a^2bx\cosh^4}{8} \\ x\left(a+b\sinh^2(c)\right)^3\sinh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*x
*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*a
**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c
+ d*x)**3/(8*d) + 15*a*b**2*x*sinh(c + d*x)**6/16 - 45*a*b**2*x*sinh(c + d*
x)**4*cosh(c + d*x)**2/16 + 45*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/1
6 - 15*a*b**2*x*cosh(c + d*x)**6/16 + 33*a*b**2*sinh(c + d*x)**5*cosh(c + d
*x)/(16*d) - 5*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a*b**2*s
inh(c + d*x)*cosh(c + d*x)**5/(16*d) + 35*b**3*x*sinh(c + d*x)**8/128 - 35*
b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**3*x*sinh(c + d*x)**4*c
osh(c + d*x)**4/64 - 35*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b*
*3*x*cosh(c + d*x)**8/128 + 93*b**3*sinh(c + d*x)**7*cosh(c + d*x)/(128*d)
- 511*b**3*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**3*sinh(c + d*
x)**3*cosh(c + d*x)**5/(384*d) - 35*b**3*sinh(c + d*x)*cosh(c + d*x)**7/(12
8*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**2, True))
```

3.22 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=79

$$\frac{3b^2(a-b)\cosh^5(c+dx)}{5d}+\frac{b(a-b)^2\cosh^3(c+dx)}{d}+\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{b^3\cosh^7(c+dx)}{7d}$$

```
[Out] (a-b)^3*cosh(d*x+c)/d+(a-b)^2*b*cosh(d*x+c)^3/d+3/5*(a-b)*b^2*cosh(d*x+c)^5
/d+1/7*b^3*cosh(d*x+c)^7/d
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 194}

$$\frac{3b^2(a-b)\cosh^5(c+dx)}{5d}+\frac{b(a-b)^2\cosh^3(c+dx)}{d}+\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{b^3\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - b)^3*Cosh[c + d*x])/d + ((a - b)^2*b*Cosh[c + d*x]^3)/d + (3*(a - b)*
b^2*Cosh[c + d*x]^5)/(5*d) + (b^3*Cosh[c + d*x]^7)/(7*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int\left(a-b+bx^2\right)^3\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^3\left(1-\frac{b\left(3a^2-3ab+b^2\right)}{a^3}\right)+3a^2b\left(1+\frac{b(-2a+b)}{a^2}\right)x^2+3ab^2\left(1-\right.\right.}{d}\\ &= \frac{(a-b)^3\cosh(c+dx)}{d}+\frac{(a-b)^2b\cosh^3(c+dx)}{d}+\frac{3(a-b)b^2\cosh^5(}{5d}\end{aligned}$$

Mathematica [A] time = 0.30, size = 94, normalized size = 1.19

$$\frac{\cosh(c+dx)\left(1120a^3+b\left(560a^2-784ab+299b^2\right)\cosh(2(c+dx))-2800a^2b+6b^2(14a-9b)\cosh(4(c+dx))+\right.}{1120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Cosh[c + d*x]*(1120*a^3 - 2800*a^2*b + 2492*a*b^2 - 762*b^3 + b*(560*a^2 -
 784*a*b + 299*b^2)*Cosh[2*(c + d*x)] + 6*(14*a - 9*b)*b^2*Cosh[4*(c + d*x)
] + 5*b^3*Cosh[6*(c + d*x)]))/(1120*d)
```

fricas [B] time = 0.56, size = 234, normalized size = 2.96

$$\frac{5\,b^3\cosh\left(dx+c\right)^7+35\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+7\left(12\,ab^2-7\,b^3\right)\cosh\left(dx+c\right)^5+35\left(5\,b^3\cosh\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b^3*cosh(d*x + c)^7 + 35*b^3*cosh(d*x + c)*sinh(d*x + c)^6 + 7*(1
2*a*b^2 - 7*b^3)*cosh(d*x + c)^5 + 35*(5*b^3*cosh(d*x + c)^3 + (12*a*b^2 -
7*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 35*(16*a^2*b - 20*a*b^2 + 7*b^3)*co
sh(d*x + c)^3 + 35*(3*b^3*cosh(d*x + c)^5 + 2*(12*a*b^2 - 7*b^3)*cosh(d*x +
 c)^3 + 3*(16*a^2*b - 20*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 35
*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*cosh(d*x + c))/d
```

giac [B] time = 0.19, size = 222, normalized size = 2.81

$$\frac{b^3e^{(7\,dx+7\,c)}}{896\,d}+\frac{b^3e^{(-7\,dx-7\,c)}}{896\,d}+\frac{\left(12\,ab^2-7\,b^3\right)e^{(5\,dx+5\,c)}}{640\,d}+\frac{\left(16\,a^2b-20\,ab^2+7\,b^3\right)e^{(3\,dx+3\,c)}}{128\,d}+\frac{\left(64\,a^3-144\,a^2b+120\,\right.}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/896*b^3*e^(7*d*x + 7*c)/d + 1/896*b^3*e^(-7*d*x - 7*c)/d + 1/640*(12*a*b^
2 - 7*b^3)*e^(5*d*x + 5*c)/d + 1/128*(16*a^2*b - 20*a*b^2 + 7*b^3)*e^(3*d*x
 + 3*c)/d + 1/128*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*e^(d*x + c)/d +
 1/128*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*e^(-d*x - c)/d + 1/128*(16
*a^2*b - 20*a*b^2 + 7*b^3)*e^(-3*d*x - 3*c)/d + 1/640*(12*a*b^2 - 7*b^3)*e^
(-5*d*x - 5*c)/d
```

maple [A] time = 0.03, size = 116, normalized size = 1.47

$$\frac{b^3\left(-\frac{16}{35}+\frac{\left(\sinh^6(dx+c)\right)}{7}-\frac{6\left(\sinh^4(dx+c)\right)}{35}+\frac{8\left(\sinh^2(dx+c)\right)}{35}\right)\cosh\left(dx+c\right)+3a\,b^2\left(\frac{8}{15}+\frac{\left(\sinh^4(dx+c)\right)}{5}-\frac{4\left(\sinh^2(dx+c)\right)}{15}\right)\mathrm{co}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*c
osh(d*x+c)+3*a*b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+
3*a^2*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*cosh(d*x+c))
```

maxima [B] time = 0.46, size = 263, normalized size = 3.33

$$-\frac{1}{4480}\,b^3\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/160*a*b^2*(3*e^(5*d*x + 5*
```

```
c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e
^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/8*a^2*b*(e^(3*d*x + 3*c)/d -
9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a^3*cosh(d*x + c
)/d
```

mupad [B] time = 0.26, size = 129, normalized size = 1.63

$$\frac{a^3 \cosh(c+d\,x) + a^2\,b\cosh(c+d\,x)^3 - 3\,a^2\,b\cosh(c+d\,x) + \frac{3\,a\,b^2\cosh(c+d\,x)^5}{5} - 2\,a\,b^2\cosh(c+d\,x)^3 + 3\,a\,b^2\cos}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (a^3*cosh(c + d*x) - b^3*cosh(c + d*x) + b^3*cosh(c + d*x)^3 - (3*b^3*cosh(
c + d*x)^5)/5 + (b^3*cosh(c + d*x)^7)/7 - 2*a*b^2*cosh(c + d*x)^3 + a^2*b*c
osh(c + d*x)^3 + (3*a*b^2*cosh(c + d*x)^5)/5 + 3*a*b^2*cosh(c + d*x) - 3*a^
2*b*cosh(c + d*x))/d
```

sympy [A] time = 5.56, size = 221, normalized size = 2.80

$$\begin{cases} \frac{a^3\cosh(c+dx)}{d} + \frac{3a^2b\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^2b\cosh^3(c+dx)}{d} + \frac{3ab^2\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4ab^2\sinh^2(c+dx)\cosh^3(c+dx)}{d} + \frac{8ab}{} \\ x\left(a+b\sinh^2(c)\right)^3\sinh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 3*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)/d
 - 2*a**2*b*cosh(c + d*x)**3/d + 3*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)/d
- 4*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a*b**2*cosh(c + d*x)**5/
(5*d) + b**3*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**3*sinh(c + d*x)**4*cos
h(c + d*x)**3/d + 8*b**3*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**3*
cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c), True)
)
```

3.23 $\int \left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=128

$$\frac{b\left(64a^2-54ab+15b^2\right)\sinh(c+dx)\cosh(c+dx)}{48d}+\frac{1}{16}x(2a-b)\left(8a^2-8ab+5b^2\right)+\frac{5b^2(2a-b)\sinh^3(c+dx)\cosh(c+dx)}{24d}$$

```
[Out] 1/16*(2*a-b)*(8*a^2-8*a*b+5*b^2)*x+1/48*b*(64*a^2-54*a*b+15*b^2)*cosh(d*x+c
)*sinh(d*x+c)/d+5/24*(2*a-b)*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b*cosh(d*x
+c)*sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2/d
```

Rubi [A] time = 0.10, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3180, 3169}

$$\frac{b\left(64a^2-54ab+15b^2\right)\sinh(c+dx)\cosh(c+dx)}{48d}+\frac{1}{16}x(2a-b)\left(8a^2-8ab+5b^2\right)+\frac{5b^2(2a-b)\sinh^3(c+dx)\cosh(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*x)/16 + (b*(64*a^2 - 54*a*b + 15*b^2)*Co
sh[c + d*x]*Sinh[c + d*x])/(48*d) + (5*(2*a - b)*b^2*Cosh[c + d*x]*Sinh[c +
 d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)
/(6*d)
```

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{b\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}+\frac{1}{6}\int \left(a+b\sinh^2(c+dx)\right)\ldots \\ &= \frac{1}{16}(2a-b)\left(8a^2-8ab+5b^2\right)x+\frac{b\left(64a^2-54ab+15b^2\right)\cosh(c+dx)\sinh(c+\ldots}{48d}\end{aligned}$$

Mathematica [A] time = 0.26, size = 95, normalized size = 0.74

$$\frac{12(2a-b)\left(8a^2-8ab+5b^2\right)(c+dx)+9b\left(16a^2-16ab+5b^2\right)\sinh(2(c+dx))+9b^2(2a-b)\sinh(4(c+dx))\ldots}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (12*(2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*(c + d*x) + 9*b*(16*a^2 - 16*a*b + 5*
b^2)*Sinh[2*(c + d*x)] + 9*(2*a - b)*b^2*Sinh[4*(c + d*x)] + b^3*Sinh[6*(c
+ d*x)])/(192*d)
```

fricas [A] time = 0.47, size = 165, normalized size = 1.29

$$\frac{3 b^3 \cosh (dx+c) \sinh (dx+c)^5 + 2 \left(5 b^3 \cosh (dx+c)^3 + 9 \left(2 ab^2 - b^3\right) \cosh (dx+c)\right) \sinh (dx+c)^3 + 6 \left(16 a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^3*cosh(d*x + c)^3 + 9*(2
*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(16*a^3 - 24*a^2*b + 18*a*
b^2 - 5*b^3)*d*x + 3*(b^3*cosh(d*x + c)^5 + 6*(2*a*b^2 - b^3)*cosh(d*x + c)
^3 + 3*(16*a^2*b - 16*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.14, size = 177, normalized size = 1.38

$$\frac{b^3 e^{(6\,dx+6\,c)}}{384\,d} - \frac{b^3 e^{(-6\,dx-6\,c)}}{384\,d} + \frac{1}{16}\left(16\,a^3 - 24\,a^2 b + 18\,ab^2 - 5\,b^3\right)x + \frac{3\left(2\,ab^2 - b^3\right)e^{(4\,dx+4\,c)}}{128\,d} + \frac{3\left(16\,a^2 b - 16\,ab^2 + 5\,b^3\right.}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/384*b^3*e^(6*d*x + 6*c)/d - 1/384*b^3*e^(-6*d*x - 6*c)/d + 1/16*(16*a^3 -
 24*a^2*b + 18*a*b^2 - 5*b^3)*x + 3/128*(2*a*b^2 - b^3)*e^(4*d*x + 4*c)/d +
 3/128*(16*a^2*b - 16*a*b^2 + 5*b^3)*e^(2*d*x + 2*c)/d - 3/128*(16*a^2*b -
16*a*b^2 + 5*b^3)*e^(-2*d*x - 2*c)/d - 3/128*(2*a*b^2 - b^3)*e^(-4*d*x - 4*
c)/d
```

maple [A] time = 0.03, size = 131, normalized size = 1.02

$$\frac{b^3 \left(\left(\frac{\left(\sinh^5(dx+c)\right)}{6} - \frac{5\left(\sinh^3(dx+c)\right)}{24} + \frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c) - \frac{5dx}{16} - \frac{5c}{16}\right) + 3a\,b^2 \left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+
c)-5/16*d*x-5/16*c)+3*a*b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c
)+3/8*d*x+3/8*c)+3*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a^3*(d
*x+c))
```

maxima [A] time = 0.33, size = 197, normalized size = 1.54

$$\frac{3}{64}\,ab^2\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) - \frac{3}{8}\,a^2 b\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + a^3 x - \frac{1}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d
*x - 2*c)/d) + a^3*x - 1/384*b^3*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c)
```

```
 - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

mupad [B] time = 0.75, size = 123, normalized size = 0.96

$$\frac{\frac{45\,b^3\,\sinh(2\,c+2\,d\,x)}{4}-\frac{9\,b^3\,\sinh(4\,c+4\,d\,x)}{4}+\frac{b^3\,\sinh(6\,c+6\,d\,x)}{4}-36\,a\,b^2\,\sinh\left(2\,c+2\,d\,x\right)+36\,a^2\,b\,\sinh\left(2\,c+2\,d\,x\right)+\frac{9\,a\,b^2}{}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] ((45*b^3*sinh(2*c + 2*d*x))/4 - (9*b^3*sinh(4*c + 4*d*x))/4 + (b^3*sinh(6*c
 + 6*d*x))/4 - 36*a*b^2*sinh(2*c + 2*d*x) + 36*a^2*b*sinh(2*c + 2*d*x) + (9
*a*b^2*sinh(4*c + 4*d*x))/2 + 48*a^3*d*x - 15*b^3*d*x + 54*a*b^2*d*x - 72*a
^2*b*d*x)/(48*d)
```

sympy [A] time = 3.63, size = 350, normalized size = 2.73

$$\begin{cases} a^3x+\frac{3a^2bx\sinh^2(c+dx)}{2}-\frac{3a^2bx\cosh^2(c+dx)}{2}+\frac{3a^2b\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{9ab^2x\sinh^4(c+dx)}{8}-\frac{9ab^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} \\ x\left(a+b\sinh^2(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x*sinh(c + d*x)**2/2 - 3*a**2*b*x*cosh(c + d*x
)**2/2 + 3*a**2*b*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 9*a*b**2*x*sinh(c + d
*x)**4/8 - 9*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a*b**2*x*cosh
(c + d*x)**4/8 + 15*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a*b**2*
sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*b**3*x*sinh(c + d*x)**6/16 - 15*b*
*3*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b**3*x*sinh(c + d*x)**2*cosh
(c + d*x)**4/16 - 5*b**3*x*cosh(c + d*x)**6/16 + 11*b**3*sinh(c + d*x)**5*c
osh(c + d*x)/(16*d) - 5*b**3*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*
*3*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*
*3, True))
```

3.24 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\left(3a^2-3ab+b^2\right)\cosh(c+dx)}{d}+\frac{b^2(3a-2b)\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^5(c+dx)}{5d}$$

```
[Out] -a^3*arctanh(cosh(d*x+c))/d+b*(3*a^2-3*a*b+b^2)*cosh(d*x+c)/d+1/3*(3*a-2*b)
*b^2*cosh(d*x+c)^3/d+1/5*b^3*cosh(d*x+c)^5/d
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 390, 206}

$$\frac{b\left(3a^2-3ab+b^2\right)\cosh(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2(3a-2b)\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) + (b*(3*a^2 - 3*a*b + b^2)*Cosh[c + d*x])
/d + ((3*a - 2*b)*b^2*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^3}{1-x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int \left(-b\left(3a^2-3ab+b^2\right)-(3a-2b)b^2x^2-b^3x^4+\frac{a^3}{1-x^2}\right)dx,\right.}{d}\\ &= \frac{b\left(3a^2-3ab+b^2\right)\cosh(c+dx)}{d}+\frac{(3a-2b)b^2\cosh^3(c+dx)}{3d}+\frac{b^3 \mathrm{c}}{}\\ &= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\left(3a^2-3ab+b^2\right)\cosh(c+dx)}{d}+\frac{(3a}{}\end{aligned}$$

Mathematica [A] time = 0.22, size = 83, normalized size = 1.00

$$\frac{3\left(80a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+b^3\cosh(5(c+dx))\right)+30b\left(24a^2-18ab+5b^2\right)\cosh(c+dx)+5b^2(12a-5b)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (30*b*(24*a^2 - 18*a*b + 5*b^2)*Cosh[c + d*x] + 5*(12*a - 5*b)*b^2*Cosh[3*(
c + d*x)] + 3*(b^3*Cosh[5*(c + d*x)] + 80*a^3*Log[Tanh[(c + d*x)/2]]))/(240
*d)
```

fricas [B] time = 0.56, size = 1128, normalized size = 13.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^10 + 30*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^
3*sinh(d*x + c)^10 + 5*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^8 + 5*(27*b^3*cosh(
d*x + c)^2 + 12*a*b^2 - 5*b^3)*sinh(d*x + c)^8 + 40*(9*b^3*cosh(d*x + c)^3
+ (12*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(24*a^2*b - 18*a*b
^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(63*b^3*cosh(d*x + c)^4 + 72*a^2*b - 54*a*
b^2 + 15*b^3 + 14*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(
189*b^3*cosh(d*x + c)^5 + 70*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 45*(24*a^
2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(24*a^2*b - 18*
a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 10*(63*b^3*cosh(d*x + c)^6 + 35*(12*a*b^2
- 5*b^3)*cosh(d*x + c)^4 + 72*a^2*b - 54*a*b^2 + 15*b^3 + 45*(24*a^2*b - 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(9*b^3*cosh(d*x + c)^
7 + 7*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 15*(24*a^2*b - 18*a*b^2 + 5*b^3)
*cosh(d*x + c)^3 + 3*(24*a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*b^3 + 5*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2 + 5*(27*b^3*cosh(d*x
+ c)^8 + 28*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 90*(24*a^2*b - 18*a*b^2 +
5*b^3)*cosh(d*x + c)^4 + 12*a*b^2 - 5*b^3 + 36*(24*a^2*b - 18*a*b^2 + 5*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(
d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*
cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh(d*x + c)^4 + a^3
*sinh(d*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 480*(a^3*cosh(d*
x + c)^5 + 5*a^3*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sin
h(d*x + c)^2 + 10*a^3*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)
*sinh(d*x + c)^4 + a^3*sinh(d*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) -
 1) + 10*(3*b^3*cosh(d*x + c)^9 + 4*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 18
*(24*a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 12*(24*a^2*b - 18*a*b^2 +
5*b^3)*cosh(d*x + c)^3 + (12*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(
d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x + c)^
3*sinh(d*x + c)^2 + 10*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*x + c
)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5)
```

giac [B] time = 0.21, size = 202, normalized size = 2.43

$$\frac{3\,b^3e^{(5\,dx+5\,c)}+60\,ab^2e^{(3\,dx+3\,c)}-25\,b^3e^{(3\,dx+3\,c)}+720\,a^2be^{(dx+c)}-540\,ab^2e^{(dx+c)}+150\,b^3e^{(dx+c)}-480\,a^3\log\left(e^{(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^3*e^(5*d*x + 5*c) + 60*a*b^2*e^(3*d*x + 3*c) - 25*b^3*e^(3*d*x +
 3*c) + 720*a^2*b*e^(d*x + c) - 540*a*b^2*e^(d*x + c) + 150*b^3*e^(d*x + c)
```

```
- 480*a^3*log(e^(d*x + c) + 1) + 480*a^3*log(abs(e^(d*x + c) - 1)) + (720*
a^2*b*e^(4*d*x + 4*c) - 540*a*b^2*e^(4*d*x + 4*c) + 150*b^3*e^(4*d*x + 4*c)
 + 60*a*b^2*e^(2*d*x + 2*c) - 25*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-5*d*x - 5
*c))/d
```

maple [A] time = 0.07, size = 86, normalized size = 1.04

$$\frac{-2a^3 \operatorname{arctanh}\left(e^{dx+c}\right) + 3a^2 b \cosh(dx+c) + 3a b^2 \left(-\frac{2}{3} + \frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh(dx+c) + b^3\left(\frac{8}{15} + \frac{\left(\sinh^4(dx+c)\right)}{5} - \frac{4(\sin}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*cosh(d*x+c)+3*a*b^2*(-2/3+1/3*sinh(
d*x+c)^2)*cosh(d*x+c)+b^3*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(
d*x+c))
```

maxima [B] time = 0.80, size = 193, normalized size = 2.33

$$\frac{1}{480} b^3\left(\frac{3 e^{(5 dx+5 c)}}{d} - \frac{25 e^{(3 dx+3 c)}}{d} + \frac{150 e^{(dx+c)}}{d} + \frac{150 e^{(-dx-c)}}{d} - \frac{25 e^{(-3 dx-3 c)}}{d} + \frac{3 e^{(-5 dx-5 c)}}{d}\right) + \frac{1}{8} ab^2\left(\frac{e^{(3 dx+3 c)}}{d} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/480*b^3*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/8*a
*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x -
3*c)/d) + 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a^3*log(tanh(1/2*d*x
 + 1/2*c))/d
```

mupad [B] time = 0.30, size = 184, normalized size = 2.22

$$\frac{e^{c+dx}\left(24\,a^2\,b - 18\,a\,b^2 + 5\,b^3\right)}{16\,d} - \frac{2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\sqrt{a^6}}{\sqrt{-d^2}} + \frac{e^{-c-d\,x}\left(24\,a^2\,b - 18\,a\,b^2 + 5\,b^3\right)}{16\,d} + \frac{b^3\,e^{-5\,c-5\,d\,x}}{160\,d} + \frac{b^3\,e}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x),x)
```

```
[Out] (exp(c + d*x)*(24*a^2*b - 18*a*b^2 + 5*b^3))/(16*d) - (2*atan((a^3*exp(d*x)
*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2) + (exp(- c
 - d*x)*(24*a^2*b - 18*a*b^2 + 5*b^3))/(16*d) + (b^3*exp(- 5*c - 5*d*x))/(1
60*d) + (b^3*exp(5*c + 5*d*x))/(160*d) + (b^2*exp(- 3*c - 3*d*x)*(12*a - 5*
b))/(96*d) + (b^2*exp(3*c + 3*d*x)*(12*a - 5*b))/(96*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.25 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=137

$$\frac{3}{8}bx\left(8a^2-4ab+b^2\right)-\frac{a(2a+b)(4a+b)\coth(c+dx)}{8d}+\frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx\right.}{4d}$$

```
[Out] 3/8*b*(8*a^2-4*a*b+b^2)*x-1/8*a*(2*a+b)*(4*a+b)*coth(d*x+c)/d+1/4*b*cosh(d*
x+c)^4*coth(d*x+c)*(a-(a-b)*tanh(d*x+c)^2)^2/d+1/8*b*cosh(d*x+c)^2*coth(d*x
+c)*(a*(4*a+b)-(4*a-3*b)*(a-b)*tanh(d*x+c)^2)/d
```

Rubi [A] time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 468, 577, 453, 206}

$$\frac{3}{8}bx\left(8a^2-4ab+b^2\right)-\frac{a(2a+b)(4a+b)\coth(c+dx)}{8d}+\frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*b*(8*a^2 - 4*a*b + b^2)*x)/8 - (a*(2*a + b)*(4*a + b)*Coth[c + d*x])/(8*
d) + (b*Cosh[c + d*x]^4*Coth[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(4*d
) + (b*Cosh[c + d*x]^2*Coth[c + d*x]*(a*(4*a + b) - (4*a - 3*b)*(a - b)*Tan
h[c + d*x]^2))/(8*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
```

```
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{x^2\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{4d} + \frac{\operatorname{Subst}(\ldots}{}$$

$$= \frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{4d} + \frac{b\cosh\ldots}{}$$

$$= -\frac{a(2a+b)(4a+b)\coth(c+dx)}{8d} + \frac{b\cosh^4(c+dx)\coth(c+dx)\left(a\ldots\right.}{4d}$$

$$= \frac{3}{8}b\left(8a^2-4ab+b^2\right)x - \frac{a(2a+b)(4a+b)\coth(c+dx)}{8d} + \frac{b\cosh^4(c\ldots}{}$$

Mathematica [A] time = 1.88, size = 113, normalized size = 0.82

$$\frac{\sinh^6(c+dx)\left(a\operatorname{csch}^2(c+dx)+b\right)^3\left(-32a^3\coth(c+dx)+12b\left(8a^2-4ab+b^2\right)(c+dx)+8b^2(3a-b)\sinh(2(c\ldots\right.}{4d(2a+b\cosh(2(c+dx))-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((b + a*Csch[c + d*x]^2)^3*Sinh[c + d*x]^6*(12*b*(8*a^2 - 4*a*b + b^2)*(c +
 d*x) - 32*a^3*Coth[c + d*x] + 8*(3*a - b)*b^2*Sinh[2*(c + d*x)] + b^3*Sinh
[4*(c + d*x)]))/(4*d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

fricas [A] time = 0.70, size = 169, normalized size = 1.23

$$\frac{b^3\cosh(dx+c)^5 + 5b^3\cosh(dx+c)\sinh(dx+c)^4 + 3\left(8ab^2-3b^3\right)\cosh(dx+c)^3 + \left(10b^3\cosh(dx+c)^3 + 9\ldots\right.}{6\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)*sinh(d*x + c)^4 + 3*(8*a*b^
2 - 3*b^3)*cosh(d*x + c)^3 + (10*b^3*cosh(d*x + c)^3 + 9*(8*a*b^2 - 3*b^3)*
cosh(d*x + c))*sinh(d*x + c)^2 - 8*(8*a^3 + 3*a*b^2 - b^3)*cosh(d*x + c) +
8*(8*a^3 + 3*(8*a^2*b - 4*a*b^2 + b^3)*d*x)*sinh(d*x + c))/(d*sinh(d*x + c)
)
```

giac [A] time = 0.20, size = 177, normalized size = 1.29

$$\frac{b^3e^{(4\,dx+4\,c)} + 24\,ab^2e^{(2\,dx+2\,c)} - 8\,b^3e^{(2\,dx+2\,c)} + 24\left(8\,a^2b - 4\,ab^2 + b^3\right)(dx+c) - \frac{128\,a^3}{e^{(2\,dx+2\,c)}-1} - \left(144\,a^2be^{(4\,dx+4\,c)} - \right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*c
) + 24*(8*a^2*b - 4*a*b^2 + b^3)*(d*x + c) - 128*a^3/(e^(2*d*x + 2*c) - 1)
- (144*a^2*b*e^(4*d*x + 4*c) - 72*a*b^2*e^(4*d*x + 4*c) + 18*b^3*e^(4*d*x +
 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x -
 4*c))/d
```

maple [A] time = 0.06, size = 94, normalized size = 0.69

$$\frac{-a^3\coth(dx+c) + 3a^2b\,(dx+c) + 3a\,b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + b^3\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*(d*x+c)+3*a*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-
1/2*d*x-1/2*c)+b^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x
+3/8*c))
```

maxima [A] time = 0.33, size = 130, normalized size = 0.95

$$\frac{1}{64}\,b^3\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) - \frac{3}{8}\,ab^2\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + 3\,a^2bx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x
 - 2*c)/d) + 3*a^2*b*x + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.76, size = 121, normalized size = 0.88

$$\frac{3\,b\,x\,\left(8\,a^2 - 4\,a\,b + b^2\right)}{8} - \frac{2\,a^3}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x} - 1\right)} - \frac{b^3\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d} + \frac{b^3\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d} - \frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}\,(3\,a-b)}{8\,d} + \frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}\,(3\,a-b)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x)^2,x)
```

```
[Out] (3*b*x*(8*a^2 - 4*a*b + b^2))/8 - (2*a^3)/(d*(exp(2*c + 2*d*x) - 1)) - (b^3
*exp(- 4*c - 4*d*x))/(64*d) + (b^3*exp(4*c + 4*d*x))/(64*d) - (b^2*exp(- 2*
c - 2*d*x)*(3*a - b))/(8*d) + (b^2*exp(2*c + 2*d*x)*(3*a - b))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.26 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2(3a-b)\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}$$

```
[Out] 1/2*a^2*(a-6*b)*arctanh(cosh(d*x+c))/d+(3*a-b)*b^2*cosh(d*x+c)/d+1/3*b^3*co
sh(d*x+c)^3/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 390, 385, 206}

$$\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b^2(3a-b)\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + ((3*a - b)*b^2*Cosh[c + d*x]
)/d + (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^3}{\left(1-x^2\right)^2}dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left((3a-b)b^2+b^3x^2+\frac{a^2(a-3b)+3a^2bx^2}{\left(1-x^2\right)^2}\right)dx, x, \cosh(c+dx\right.}{d}$$

$$= \frac{(3a-b)b^2\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}+\frac{\text{Subst}\left(\int \frac{a^2(a-3b)+}{(1-x^2}\right.}{}$$

$$= \frac{(3a-b)b^2\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}-\frac{a^3\coth(c+dx)\operatorname{cs}}{2d}$$

$$= \frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{(3a-b)b^2\cosh(c+dx)}{d}+\frac{b^3}{}$$

Mathematica [B] time = 4.59, size = 210, normalized size = 2.53

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^3\left(3a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+3a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+12a^3\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)-12a^3\text{lo}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -1/3*((-18*(4*a - b)*b^2*Cosh[c]*Cosh[d*x] - 2*b^3*Cosh[3*c]*Cosh[3*d*x] +
3*a^3*Csch[(c + d*x)/2]^2 - 12*a^3*Log[Cosh[(c + d*x)/2]] + 72*a^2*b*Log[Co
sh[(c + d*x)/2]] + 12*a^3*Log[Sinh[(c + d*x)/2]] - 72*a^2*b*Log[Sinh[(c + d
*x)/2]] + 3*a^3*Sech[(c + d*x)/2]^2 - 72*a*b^2*Sinh[c]*Sinh[d*x] + 18*b^3*S
inh[c]*Sinh[d*x] - 2*b^3*Sinh[3*c]*Sinh[3*d*x])*(a + b*Sinh[c + d*x]^2)^3)/
(d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.56, size = 1814, normalized size = 21.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sin
h(d*x + c)^10 + (36*a*b^2 - 11*b^3)*cosh(d*x + c)^8 + (45*b^3*cosh(d*x + c)
^2 + 36*a*b^2 - 11*b^3)*sinh(d*x + c)^8 + 8*(15*b^3*cosh(d*x + c)^3 + (36*a
*b^2 - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(12*a^3 + 18*a*b^2 - 5*b^
3)*cosh(d*x + c)^6 + 2*(105*b^3*cosh(d*x + c)^4 - 12*a^3 - 18*a*b^2 + 5*b^3
 + 14*(36*a*b^2 - 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*b^3*cosh
(d*x + c)^5 + 14*(36*a*b^2 - 11*b^3)*cosh(d*x + c)^3 - 3*(12*a^3 + 18*a*b^2
 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(12*a^3 + 18*a*b^2 - 5*b^3)*co
sh(d*x + c)^4 + 2*(105*b^3*cosh(d*x + c)^6 + 35*(36*a*b^2 - 11*b^3)*cosh(d*
x + c)^4 - 12*a^3 - 18*a*b^2 + 5*b^3 - 15*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*b^3*cosh(d*x + c)^7 + 7*(36*a*b^2 - 11*
b^3)*cosh(d*x + c)^5 - 5*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - (12*
a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + (36*a*b^2 -
11*b^3)*cosh(d*x + c)^2 + (45*b^3*cosh(d*x + c)^8 + 28*(36*a*b^2 - 11*b^3)*
cosh(d*x + c)^6 - 30*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 36*a*b^2
 - 11*b^3 - 12*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 12*((a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 - 6*a^2*b)*cosh(d*x + c)*si
```

```
nh(d*x + c)^6 + (a^3 - 6*a^2*b)*sinh(d*x + c)^7 - 2*(a^3 - 6*a^2*b)*cosh(d*
x + c)^5 - (2*a^3 - 12*a^2*b - 21*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 5*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 2*(a^3 - 6*a^2*b)*cosh(d*x
 + c))*sinh(d*x + c)^4 + (a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (35*(a^3 - 6*a^2
*b)*cosh(d*x + c)^4 + a^3 - 6*a^2*b - 20*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + (21*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 - 20*(a^3 - 6*a^2*b)*c
osh(d*x + c)^3 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(a^3
 - 6*a^2*b)*cosh(d*x + c)^6 - 10*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 3*(a^3 -
 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
 + 1) - 12*((a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 - 6*a^2*b)*cosh(d*x +
c)*sinh(d*x + c)^6 + (a^3 - 6*a^2*b)*sinh(d*x + c)^7 - 2*(a^3 - 6*a^2*b)*co
sh(d*x + c)^5 - (2*a^3 - 12*a^2*b - 21*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sin
h(d*x + c)^5 + 5*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 2*(a^3 - 6*a^2*b)*cos
h(d*x + c))*sinh(d*x + c)^4 + (a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (35*(a^3 -
6*a^2*b)*cosh(d*x + c)^4 + a^3 - 6*a^2*b - 20*(a^3 - 6*a^2*b)*cosh(d*x + c)
^2)*sinh(d*x + c)^3 + (21*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 - 20*(a^3 - 6*a^2
*b)*cosh(d*x + c)^3 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7
*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 - 10*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 3*(
a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) - 1) + 2*(5*b^3*cosh(d*x + c)^9 + 4*(36*a*b^2 - 11*b^3)*cosh(d*x + c)
^7 - 6*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^5 - 4*(12*a^3 + 18*a*b^2 -
 5*b^3)*cosh(d*x + c)^3 + (36*a*b^2 - 11*b^3)*cosh(d*x + c))*sinh(d*x + c))
/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7
 - 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^5 + 5*(
7*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^
3 + (35*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21
*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c
)^2 + (7*d*cosh(d*x + c)^6 - 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*si
nh(d*x + c))
```

giac [B] time = 0.23, size = 174, normalized size = 2.10

$$\frac{b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+36\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-12\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-\frac{24\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}+6\left(a^3-6\,a^2b\right)\mathrm{lo}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 36*a*b^2*(e^(d*x + c) + e^(-d*x
- c)) - 12*b^3*(e^(d*x + c) + e^(-d*x - c)) - 24*a^3*(e^(d*x + c) + e^(-d*x
 - c))/((e^(d*x + c) + e^(-d*x - c))^2 - 4) + 6*(a^3 - 6*a^2*b)*log(e^(d*x
+ c) + e^(-d*x - c) + 2) - 6*(a^3 - 6*a^2*b)*log(e^(d*x + c) + e^(-d*x - c)
 - 2))/d
```

maple [A] time = 0.09, size = 79, normalized size = 0.95

$$\frac{a^3\left(-\frac{\mathrm{csch}(dx+c)\coth(dx+c)}{2}+\mathrm{arctanh}\left(e^{dx+c}\right)\right)-6a^2b\,\mathrm{arctanh}\left(e^{dx+c}\right)+3a\,b^2\cosh(dx+c)+b^3\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-6*a^2*b*arctanh
(exp(d*x+c))+3*a*b^2*cosh(d*x+c)+b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.34, size = 217, normalized size = 2.61

$$\frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3}{2}\,ab^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + 3/2*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^3*(log(e^(
-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x -
3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 3*a^2*b*(log(e^(-d
*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

mupad [B] time = 0.22, size = 229, normalized size = 2.76

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(a^3\sqrt{-d^2}-6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6-12\,a^5\,b+36\,a^4\,b^2}}\right)\sqrt{a^6-12\,a^5\,b+36\,a^4\,b^2}}{\sqrt{-d^2}}+\frac{b^3\,e^{-3\,c-3\,d\,x}}{24\,d}+\frac{b^3\,e^{3\,c+3\,d\,x}}{24\,d}+\frac{3\,b^2\,e^{c+d\,x}\,(4\,a-b)}{8\,d}+\frac{3\,b^2\,\dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(a^6 -
 12*a^5*b + 36*a^4*b^2)^(1/2)))*(a^6 - 12*a^5*b + 36*a^4*b^2)^(1/2))/(-d^2)
^(1/2) + (b^3*exp(- 3*c - 3*d*x))/(24*d) + (b^3*exp(3*c + 3*d*x))/(24*d) +
(3*b^2*exp(c + d*x)*(4*a - b))/(8*d) + (3*b^2*exp(- c - d*x)*(4*a - b))/(8*
d) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d*x))/(
d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.27 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=113

$$\frac{a\left(2a^2-5ab-2b^2\right)\coth(c+dx)}{2d}-\frac{a^2(2a+3b)\coth^3(c+dx)}{6d}+\frac{1}{2}b^2x(6a-b)+\frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a\right.}{2d}$$

```
[Out] 1/2*(6*a-b)*b^2*x+1/2*a*(2*a^2-5*a*b-2*b^2)*coth(d*x+c)/d-1/6*a^2*(2*a+3*b)
*coth(d*x+c)^3/d+1/2*b*cosh(d*x+c)^2*coth(d*x+c)^3*(a-(a-b)*tanh(d*x+c)^2)^
2/d
```

Rubi [A] time = 0.14, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3187, 468, 570, 207}

$$\frac{a\left(2a^2-5ab-2b^2\right)\coth(c+dx)}{2d}-\frac{a^2(2a+3b)\coth^3(c+dx)}{6d}+\frac{1}{2}b^2x(6a-b)+\frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((6*a - b)*b^2*x)/2 + (a*(2*a^2 - 5*a*b - 2*b^2)*Coth[c + d*x])/(2*d) - (a^
2*(2*a + 3*b)*Coth[c + d*x]^3)/(6*d) + (b*Cosh[c + d*x]^2*Coth[c + d*x]^3*(
a - (a - b)*Tanh[c + d*x]^2)^2)/(2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{x^4\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{2d} + \text{Su}$$

$$= \frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{2d} + \text{Su}$$

$$= \frac{a\left(2a^2-5ab-2b^2\right)\coth(c+dx)}{2d} - \frac{a^2(2a+3b)\coth^3(c+dx)}{6d} +$$

$$= \frac{1}{2}(6a-b)b^2x + \frac{a\left(2a^2-5ab-2b^2\right)\coth(c+dx)}{2d} - \frac{a^2(2a+3b)\,\mathrm{c}}{6}$$

Mathematica [A] time = 2.45, size = 107, normalized size = 0.95

$$\frac{2\sinh^6(c+dx)\left(a\operatorname{csch}^2(c+dx)+b\right)^3\left(3b^2(2(6a-b)(c+dx)+b\sinh(2(c+dx)))-4a^2\coth(c+dx)\left(a\operatorname{csch}^2(\right.\right.}{3d(2a+b\cosh(2(c+dx))-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (2*(b + a*Csch[c + d*x]^2)^3*Sinh[c + d*x]^6*(-4*a^2*Coth[c + d*x]*(-2*a +
9*b + a*Csch[c + d*x]^2) + 3*b^2*(2*(6*a - b)*(c + d*x) + b*Sinh[2*(c + d*x
)])))/(3*d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

fricas [B] time = 0.59, size = 281, normalized size = 2.49

$$\frac{3\,b^3\cosh(dx+c)^5 + 15\,b^3\cosh(dx+c)\sinh(dx+c)^4 + \left(16\,a^3-72\,a^2b-9\,b^3\right)\cosh(dx+c)^3 - 4\left(4\,a^3-18\,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^3*cosh(d*x + c)^5 + 15*b^3*cosh(d*x + c)*sinh(d*x + c)^4 + (16*a^
3 - 72*a^2*b - 9*b^3)*cosh(d*x + c)^3 - 4*(4*a^3 - 18*a^2*b - 3*(6*a*b^2 -
b^3)*d*x)*sinh(d*x + c)^3 + 3*(10*b^3*cosh(d*x + c)^3 + (16*a^3 - 72*a^2*b
- 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 6*(8*a^3 - 12*a^2*b - b^3)*cosh(d
*x + c) + 12*(4*a^3 - 18*a^2*b - 3*(6*a*b^2 - b^3)*d*x - (4*a^3 - 18*a^2*b
- 3*(6*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3
 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

giac [A] time = 0.20, size = 154, normalized size = 1.36

$$\frac{3\,b^3e^{(2\,dx+2\,c)} + 12\left(6\,ab^2-b^3\right)(dx+c) - 3\left(12\,ab^2e^{(2\,dx+2\,c)} - 2\,b^3e^{(2\,dx+2\,c)} + b^3\right)e^{(-2\,dx-2\,c)} - \frac{16\left(9\,a^2be^{(4\,dx+4\,c)}+6\,a^3e^{(}\right.}{(}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*e^(2*d*x + 2*c) + 12*(6*a*b^2 - b^3)*(d*x + c) - 3*(12*a*b^2*e^
(2*d*x + 2*c) - 2*b^3*e^(2*d*x + 2*c) + b^3)*e^(-2*d*x - 2*c) - 16*(9*a^2*b
*e^(4*d*x + 4*c) + 6*a^3*e^(2*d*x + 2*c) - 18*a^2*b*e^(2*d*x + 2*c) - 2*a^3
 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [A] time = 0.08, size = 77, normalized size = 0.68

$$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)-3a^2b\coth(dx+c)+3a\,b^2\,(dx+c)+b^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-3*a^2*b*coth(d*x+c)+3*a*b^2*(d
*x+c)+b^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [A] time = 0.34, size = 161, normalized size = 1.42

$$-\frac{1}{8}\,b^3\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+3\,ab^2x+\frac{4}{3}\,a^3\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{}{d\left(3\,e^{(-2\,dx-2\,c)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 3*a*b^2*x + 4/3*a
^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.14, size = 222, normalized size = 1.96

$$\frac{\frac{2\left(3\,a^2\,b-2\,a^3\right)}{3\,d}-\frac{2\,a^2\,b\,e^{2\,c+2\,d\,x}}{d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1}-\frac{\frac{2\,a^2\,b}{d}-\frac{4\,e^{2\,c+2\,d\,x}\left(3\,a^2\,b-2\,a^3\right)}{3\,d}+\frac{2\,a^2\,b\,e^{4\,c+4\,d\,x}}{d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1}+\frac{b^2\,x\,\left(6\,a-b\right)}{2}-\frac{b^3\,e^{-2\,c-2\,d\,x}}{8\,d}+\frac{b^3\,e^{2\,c+2\,d\,x}}{8\,d}-\frac{}{d\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/sinh(c + d*x)^4,x)
```

```
[Out] ((2*(3*a^2*b - 2*a^3))/(3*d) - (2*a^2*b*exp(2*c + 2*d*x))/d)/(exp(4*c + 4*d
*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*a^2*b)/d - (4*exp(2*c + 2*d*x)*(3*a^2*b
 - 2*a^3))/(3*d) + (2*a^2*b*exp(4*c + 4*d*x))/d)/(3*exp(2*c + 2*d*x) - 3*ex
p(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (b^2*x*(6*a - b))/2 - (b^3*exp(- 2
*c - 2*d*x))/(8*d) + (b^3*exp(2*c + 2*d*x))/(8*d) - (2*a^2*b)/(d*(exp(2*c +
 2*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.28 $\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{7/2}d\sqrt{a-b}}+\frac{\left(a^2+ab+b^2\right)\cosh(c+dx)}{b^3d}-\frac{(a+2b)\cosh^3(c+dx)}{3b^2d}+\frac{\cosh^5(c+dx)}{5bd}$$

```
[Out] (a^2+a*b+b^2)*cosh(d*x+c)/b^3/d-1/3*(a+2*b)*cosh(d*x+c)^3/b^2/d+1/5*cosh(d*
x+c)^5/b/d-a^3*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/b^(7/2)/d/(a-b)^(1/2
)
```

Rubi [A] time = 0.15, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 390, 205}

$$\frac{\left(a^2+ab+b^2\right)\cosh(c+dx)}{b^3d}-\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{7/2}d\sqrt{a-b}}-\frac{(a+2b)\cosh^3(c+dx)}{3b^2d}+\frac{\cosh^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((a^3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(7/2)*d)
) + ((a^2 + a*b + b^2)*Cosh[c + d*x])/(b^3*d) - ((a + 2*b)*Cosh[c + d*x]^3)
/(3*b^2*d) + Cosh[c + d*x]^5/(5*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{a^2+ab+b^2}{b^3} + \frac{(a+2b)x^2}{b^2} - \frac{x^4}{b} + \frac{a^3}{b^3(a-b+bx^2)}\right) dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d} - \frac{(a+2b)\cosh^3(c+dx)}{3b^2d} + \frac{\cosh^5(c+dx)}{5bd} - \frac{a^3\,\text{Subst}\big(}{}$$

$$= -\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{7/2}d} + \frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d} - \frac{(a+2b)\cosh^3(c+dx)}{3b^2d} +$$

Mathematica [C] time = 0.89, size = 165, normalized size = 1.51

$$\frac{-\frac{240a^3\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}} + 30\sqrt{b}\left(8a^2+6ab+5b^2\right)\cosh(c+dx) - 5b^{3/2}(4a+5b)\,\text{c}}{240b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((-240*a^3*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + A
rcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b] +
30*Sqrt[b]*(8*a^2 + 6*a*b + 5*b^2)*Cosh[c + d*x] - 5*b^(3/2)*(4*a + 5*b)*Co
sh[3*(c + d*x)] + 3*b^(5/2)*Cosh[5*(c + d*x)])/(240*b^(7/2)*d)
```

fricas [B] time = 0.63, size = 3242, normalized size = 29.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/480*(3*(a*b^3 - b^4)*cosh(d*x + c)^10 + 30*(a*b^3 - b^4)*cosh(d*x + c)*s
inh(d*x + c)^9 + 3*(a*b^3 - b^4)*sinh(d*x + c)^10 - 5*(4*a^2*b^2 + a*b^3 -
5*b^4)*cosh(d*x + c)^8 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4 - 27*(a*b^3 - b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^8 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^3 - (4*a
^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(8*a^3*b - 2*a^
2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)
^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b^3 - 15*b^4 - 14*(4*a^2*b^2 + a*b^3 - 5*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*(a*b^3 - b^4)*cosh(d*x + c)^5
- 70*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^3 + 45*(8*a^3*b - 2*a^2*b^2
- a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(8*a^3*b - 2*a^2*b^2 -
 a*b^3 - 5*b^4)*cosh(d*x + c)^4 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^6 - 35
*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b
^3 - 15*b^4 + 45*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 3*a*b^3 - 3*b^4 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^7 - 7*(4
*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 15*(8*a^3*b - 2*a^2*b^2 - a*b^3
 - 5*b^4)*cosh(d*x + c)^3 + 3*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*
x + c))*sinh(d*x + c)^3 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2 + 5
*(27*(a*b^3 - b^4)*cosh(d*x + c)^8 - 28*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*
x + c)^6 + 90*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 4*a^2
*b^2 - a*b^3 + 5*b^4 + 36*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x +
```

```
c)^2)*sinh(d*x + c)^2 - 240*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^4*si
nh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*cosh(d*x + c)
^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh(d*x + c)^4 + a^3*sinh(d*x + c
)^5)*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
 c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
 + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*c
osh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x
 + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*
x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d
*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d
*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + b)) + 10*(3*(a*b^3 - b^4)*cosh(d*x + c)^9 - 4*(
4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^7 + 18*(8*a^3*b - 2*a^2*b^2 - a*b^
3 - 5*b^4)*cosh(d*x + c)^5 + 12*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(
d*x + c)^3 - (4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a*
b^4 - b^5)*d*cosh(d*x + c)^5 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)^4*sinh(d*x +
 c) + 10*(a*b^4 - b^5)*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*(a*b^4 - b^5)
*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)*sinh(d
*x + c)^4 + (a*b^4 - b^5)*d*sinh(d*x + c)^5), 1/480*(3*(a*b^3 - b^4)*cosh(d
*x + c)^10 + 30*(a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + 3*(a*b^3 - b^
4)*sinh(d*x + c)^10 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^8 - 5*(4*
a^2*b^2 + a*b^3 - 5*b^4 - 27*(a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8
 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^3 - (4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d
*x + c))*sinh(d*x + c)^7 + 30*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*
x + c)^6 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*
a*b^3 - 15*b^4 - 14*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x +
 c)^6 + 4*(189*(a*b^3 - b^4)*cosh(d*x + c)^5 - 70*(4*a^2*b^2 + a*b^3 - 5*b^
4)*cosh(d*x + c)^3 + 45*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 30*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^
4 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^6 - 35*(4*a^2*b^2 + a*b^3 - 5*b^4)*c
osh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b^3 - 15*b^4 + 45*(8*a^3*b - 2*
a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a*b^3 - 3*b^4
 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^7 - 7*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh
(d*x + c)^5 + 15*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^3 + 3*
(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*(4
*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2 + 5*(27*(a*b^3 - b^4)*cosh(d*x +
c)^8 - 28*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 90*(8*a^3*b - 2*a^2
*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 4*a^2*b^2 - a*b^3 + 5*b^4 + 36*(8*a
^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*(a
^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x
+ c)^3*sinh(d*x + c)^2 + 10*a^3*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cos
h(d*x + c)*sinh(d*x + c)^4 + a^3*sinh(d*x + c)^5)*sqrt(a*b - b^2)*arctan(-1
/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)
^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x
 + c))/sqrt(a*b - b^2)) + 480*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^4*
sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*cosh(d*x +
c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh(d*x + c)^4 + a^3*sinh(d*x +
 c)^5)*sqrt(a*b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*
x + c))/(a - b)) + 10*(3*(a*b^3 - b^4)*cosh(d*x + c)^9 - 4*(4*a^2*b^2 + a*b
^3 - 5*b^4)*cosh(d*x + c)^7 + 18*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh
(d*x + c)^5 + 12*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^3 - (4
*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a*b^4 - b^5)*d*co
sh(d*x + c)^5 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*(a*b^4
 - b^5)*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*(a*b^4 - b^5)*d*cosh(d*x + c
)^2*sinh(d*x + c)^3 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a*
b^4 - b^5)*d*sinh(d*x + c)^5)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[22,73]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[-10,75]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.The choice was done assuming [a,b]=[-4,-35]Warning, need to choose a bra
nch for the root of a polynomial with parameters. This might be wrong.The c
hoice was done assuming [a,b]=[-34,-61]Warning, need to choose a branch for
 the root of a polynomial with parameters. This might be wrong.The choice w
as done assuming [a,b]=[-40,7]Warning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done a
ssuming [a,b]=[-85,96]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[69,-9]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[43
,41]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[80,58]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[3,43]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[51,61]Warning, need to choose
a branch for the root of a polynomial with parameters. This might be wrong.
The choice was done assuming [a,b]=[-97,-57]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.The cho
ice was done assuming [a,b]=[38,-97]Warning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming [a,b]=[-86,85]Undef/Unsigned Inf encountered in limitEvaluati
on time: 2.64Limit: Max order reached or unable to make series expansion Er
ror: Bad Argument Value
```

maple [B] time = 0.08, size = 448, normalized size = 4.11

$$-\frac{1}{5db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}-\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{a}{2d\,b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/5/d/b/(tanh(1/2*d*x+1/2*c)-1)^5-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/
b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a+3/8/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/3/d/b^2
/(tanh(1/2*d*x+1/2*c)-1)^3*a-1/12/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/d/b^3/(ta
nh(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-3/8/d/b/(tanh(
1/2*d*x+1/2*c)-1)-1/d*a^3/b^3/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/
2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))+1/5/d/b/(tanh(1/2*d*x+1/2*c)+1)^5-1/2/d/
```

```
b/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a+3/8/d/b/(
tanh(1/2*d*x+1/2*c)+1)^2-1/3/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^3*a+1/12/d/b/(ta
nh(1/2*d*x+1/2*c)+1)^3+1/d/b^3/(tanh(1/2*d*x+1/2*c)+1)*a^2+1/2/d/b^2/(tanh(
1/2*d*x+1/2*c)+1)*a+3/8/d/b/(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\,b^2e^{(10\,dx+10\,c)} + 3\,b^2 - 5\left(4\,abe^{(8\,c)} + 5\,b^2e^{(8\,c)}\right)e^{(8\,dx)} + 30\left(8\,a^2e^{(6\,c)} + 6\,abe^{(6\,c)} + 5\,b^2e^{(6\,c)}\right)e^{(6\,dx)} + 30\left(8\,a^2e^{(4\,c}\right.}{480\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(3*b^2*e^(10*d*x + 10*c) + 3*b^2 - 5*(4*a*b*e^(8*c) + 5*b^2*e^(8*c))*
e^(8*d*x) + 30*(8*a^2*e^(6*c) + 6*a*b*e^(6*c) + 5*b^2*e^(6*c))*e^(6*d*x) +
30*(8*a^2*e^(4*c) + 6*a*b*e^(4*c) + 5*b^2*e^(4*c))*e^(4*d*x) - 5*(4*a*b*e^(
2*c) + 5*b^2*e^(2*c))*e^(2*d*x))*e^(-5*d*x - 5*c)/(b^3*d) - 1/128*integrate
(256*(a^3*e^(3*d*x + 3*c) - a^3*e^(d*x + c))/(b^4*e^(4*d*x + 4*c) + b^4 + 2
*(2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.64, size = 415, normalized size = 3.81

$$\frac{e^{-5\,c-5\,d\,x}}{160\,b\,d}-\frac{\sqrt{a^6}\left(2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^{c}\,\sqrt{b^7\,d^2\,(a-b)}}{2\,b^3\,d\,(a-b)\,\sqrt{a^6}}\right)+2\,\mathrm{atan}\left(\frac{\left(e^{d\,x}\,e^{c}\left(\frac{2\,a^7}{b^{11}\,d\,(a-b)^2\,\sqrt{a^6}}-\frac{4\left(2\,a^4\,b^4\,d\,\sqrt{a^6}-2\,a^5\,b^3\,d\,\sqrt{a^6}\right)}{a^3\,b^8\,(a-b)\,\sqrt{a\,b^7\,d^2-b^8\,d^2}\,\sqrt{b^7\,d^2\,(a-b)}}\right)+\frac{2\,a^7\,e^{3\,c}\,e^{3}}{b^{11}\,d\,(a-b)^2}\right.}{4\,a^4}\right.\right.}{2\,\sqrt{a\,b^7\,d^2-b^8\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^7/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(- 5*c - 5*d*x)/(160*b*d) - ((a^6)^(1/2)*(2*atan((a^3*exp(d*x)*exp(c)*(b
^7*d^2*(a - b))^(1/2))/(2*b^3*d*(a - b)*(a^6)^(1/2))) + 2*atan(((exp(d*x)*e
xp(c)*((2*a^7)/(b^11*d*(a - b)^2*(a^6)^(1/2)) - (4*(2*a^4*b^4*d*(a^6)^(1/2)
 - 2*a^5*b^3*d*(a^6)^(1/2)))/(a^3*b^8*(a - b)*(a*b^7*d^2 - b^8*d^2)^(1/2)*(
b^7*d^2*(a - b))^(1/2))) + (2*a^7*exp(3*c)*exp(3*d*x))/(b^11*d*(a - b)^2*(a
^6)^(1/2)))*(b^9*(a*b^7*d^2 - b^8*d^2)^(1/2) - a*b^8*(a*b^7*d^2 - b^8*d^2)^
(1/2)))/(4*a^4))))/(2*(a*b^7*d^2 - b^8*d^2)^(1/2)) + exp(5*c + 5*d*x)/(160*
b*d) + (exp(c + d*x)*(6*a*b + 8*a^2 + 5*b^2))/(16*b^3*d) + (exp(- c - d*x)*
(6*a*b + 8*a^2 + 5*b^2))/(16*b^3*d) - (exp(- 3*c - 3*d*x)*(4*a + 5*b))/(96*
b^2*d) - (exp(3*c + 3*d*x)*(4*a + 5*b))/(96*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.29 $\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=121

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^3d\sqrt{a-b}}+\frac{x\left(8a^2+4ab+3b^2\right)}{8b^3}-\frac{(4a+3b)\sinh(c+dx)\cosh(c+dx)}{8b^2d}+\frac{\sinh^3(c+dx)\cosh(c}{4bd}$$

```
[Out] 1/8*(8*a^2+4*a*b+3*b^2)*x/b^3-1/8*(4*a+3*b)*cosh(d*x+c)*sinh(d*x+c)/b^2/d+1
/4*cosh(d*x+c)*sinh(d*x+c)^3/b/d-a^(5/2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^
(1/2))/b^3/d/(a-b)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3187, 470, 578, 522, 206, 208}

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^3d\sqrt{a-b}}+\frac{x\left(8a^2+4ab+3b^2\right)}{8b^3}-\frac{(4a+3b)\sinh(c+dx)\cosh(c+dx)}{8b^2d}+\frac{\sinh^3(c+dx)\cosh(c}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((8*a^2 + 4*a*b + 3*b^2)*x)/(8*b^3) - (a^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c
+ d*x])/Sqrt[a]])/(Sqrt[a - b]*b^3*d) - ((4*a + 3*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*b*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3\left(a-(a-b)x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(a+3b)x^2\right)}{\left(1-x^2\right)^2\left(a+(-a+b)x^2\right)}\,dx, x, \tanh(c+dx)\right)}{4bd}$$

$$= -\frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{a^3\,\text{Subst}\left(\right.}{}$$

$$= \frac{\left(8a^2+4ab+3b^2\right)x}{8b^3} - \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,b^3d} - \frac{(4a+3b)\cosh(c+dx)\sinh}{8b^2d}$$

Mathematica [A] time = 0.48, size = 97, normalized size = 0.80

$$\frac{-\frac{32a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + 4\left(8a^2+4ab+3b^2\right)(c+dx) - 8b(a+b)\sinh(2(c+dx)) + b^2\sinh(4(c+dx))}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (4*(8*a^2 + 4*a*b + 3*b^2)*(c + d*x) - (32*a^(5/2)*ArcTanh[(Sqrt[a - b]*Tan
h[c + d*x])/Sqrt[a]])/Sqrt[a - b] - 8*b*(a + b)*Sinh[2*(c + d*x)] + b^2*Sin
h[4*(c + d*x)])/(32*b^3*d)
```

fricas [B] time = 0.63, size = 1725, normalized size = 14.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh
(d*x + c)^8 + 8*(8*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b + b^2)
*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b - 2*b^2)*sinh(d*x + c)^
6 + 8*(7*b^2*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(35*b^2*cosh(d*x + c)^4 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*x - 60*(a*b + b^
2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 +
 4*a*b + 3*b^2)*d*x*cosh(d*x + c) - 20*(a*b + b^2)*cosh(d*x + c)^3)*sinh(d*
x + c)^3 + 8*(a*b + b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8
*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b + b^2)*cosh(d*x + c)^4
+ 2*a*b + 2*b^2)*sinh(d*x + c)^2 + 32*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x
 + c)^3*sinh(d*x + c) + 6*a^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^2*cosh(
d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4)*sqrt(a/(a - b))*log((b^2*co
sh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 +
 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*
sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b
^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b
- b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 -
3*a*b + b^2)*sqrt(a/(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d
*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d
*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + b)) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 +
 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c)^5 + 2*(a*
b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cos
h(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^
3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4), 1/64*(b^2*cosh(
d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 8*
(8*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b + b^2)*cosh(d*x + c)^6
 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b - 2*b^2)*sinh(d*x + c)^6 + 8*(7*b^2*cos
h(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cos
h(d*x + c)^4 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*x - 60*(a*b + b^2)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 + 4*a*b + 3*b^2)*
d*x*cosh(d*x + c) - 20*(a*b + b^2)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a*
b + b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8*a^2 + 4*a*b + 3
*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b + b^2)*cosh(d*x + c)^4 + 2*a*b + 2*b^2)
*sinh(d*x + c)^2 - 64*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^3*sinh(d*x
 + c) + 6*a^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^2*cosh(d*x + c)*sinh(d*
x + c)^3 + a^2*sinh(d*x + c)^4)*sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-
a/(a - b))/a) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*
x*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c)^5 + 2*(a*b + b^2)*cosh(d*x
+ c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^3*d*cosh(d*x + c)*
sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4)]
```

giac [A] time = 4.39, size = 208, normalized size = 1.72

$$-\frac{\frac{64\,a^3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b^3}-\frac{8\left(8\,a^2+4\,ab+3\,b^2\right)(dx+c)}{b^3}-\frac{be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-8\,be^{(2\,dx+2\,c)}}{b^2}+\frac{\left(48\,a^2e^{(4\,dx+4\,c)}+24\,abe^{(4\,dx+4\,c)}+18\,b^2e^{(4\,d}}{}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*a^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(s
qrt(-a^2 + a*b)*b^3) - 8*(8*a^2 + 4*a*b + 3*b^2)*(d*x + c)/b^3 - (b*e^(4*d*
x + 4*c) - 8*a*e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c))/b^2 + (48*a^2*e^(4*d*
x + 4*c) + 24*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) - 8*a*b*e^(2*d*x
 + 2*c) - 8*b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/b^3)/d
```

maple [B] time = 0.11, size = 670, normalized size = 5.54

$$\frac{1}{4db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a}{2d\,b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{8db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/4/d/b/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/b
^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-1/8/d/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/b^2/
(tanh(1/2*d*x+1/2*c)-1)*a-3/8/d/b/(tanh(1/2*d*x+1/2*c)-1)-1/d/b^3*ln(tanh(1
/2*d*x+1/2*c)-1)*a^2-1/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-3/8/d/b*ln(tanh(
1/2*d*x+1/2*c)-1)+1/d*a^3/b^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a^3/b^2/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*a^3/b^3/((2*(-b*(a-b))^(1/2)+
a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a
)^(1/2))+1/d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/4/d/b
/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b^2/(tan
h(1/2*d*x+1/2*c)+1)^2*a+1/8/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/b^2/(tanh(1
/2*d*x+1/2*c)+1)*a-3/8/d/b/(tanh(1/2*d*x+1/2*c)+1)+1/d/b^3*ln(tanh(1/2*d*x+
1/2*c)+1)*a^2+1/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+3/8/d/b*ln(tanh(1/2*d*x
+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 1.07, size = 266, normalized size = 2.20

$$\frac{x\left(8\,a^2+4\,a\,b+3\,b^2\right)}{8\,b^3}-\frac{e^{-4\,c-4\,d\,x}}{64\,b\,d}+\frac{e^{4\,c+4\,d\,x}}{64\,b\,d}+\frac{e^{-2\,c-2\,d\,x}\,\left(a+b\right)}{8\,b^2\,d}-\frac{e^{2\,c+2\,d\,x}\,\left(a+b\right)}{8\,b^2\,d}+\frac{a^{5/2}\,\ln\left(\frac{4\,a^3\,e^{2\,c+2\,d\,x}}{b^4}-\frac{2\,a^{5/2}\,\left(b\,d+\right.}{}\right.}{2\,b^3\,d\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^6/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (x*(4*a*b + 8*a^2 + 3*b^2))/(8*b^3) - exp(- 4*c - 4*d*x)/(64*b*d) + exp(4*c
 + 4*d*x)/(64*b*d) + (exp(- 2*c - 2*d*x)*(a + b))/(8*b^2*d) - (exp(2*c + 2*
d*x)*(a + b))/(8*b^2*d) + (a^(5/2)*log((4*a^3*exp(2*c + 2*d*x))/b^4 - (2*a^
(5/2)*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(b^4*d*(a - b)
^(1/2))))/(2*b^3*d*(a - b)^(1/2)) - (a^(5/2)*log((4*a^3*exp(2*c + 2*d*x))/b
^4 + (2*a^(5/2)*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(b^4
*d*(a - b)^(1/2))))/(2*b^3*d*(a - b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.30 $\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{5/2}d\sqrt{a-b}}-\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}$$

```
[Out] -(a+b)*cosh(d*x+c)/b^2/d+1/3*cosh(d*x+c)^3/b/d+a^2*arctan(cosh(d*x+c)*b^(1/
2)/(a-b)^(1/2))/b^(5/2)/d/(a-b)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 390, 205}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{5/2}d\sqrt{a-b}}-\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a^2*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(5/2)*d) -
 ((a + b)*Cosh[c + d*x])/(b^2*d) + Cosh[c + d*x]^3/(3*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a+b}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a-b+bx^2)}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{b^2d}$$
$$= \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{5/2}d}-\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}$$

Mathematica [C] time = 0.43, size = 134, normalized size = 1.70

$$\frac{\frac{12a^2\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}}-3\sqrt{b}\,(4a+3b)\cosh(c+dx)+b^{3/2}\cosh(3(c+dx))}{12b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((12*a^2*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + Arc
Tan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b] - 3*
Sqrt[b]*(4*a + 3*b)*Cosh[c + d*x] + b^(3/2)*Cosh[3*(c + d*x)])/(12*b^(5/2)*
d)
```

fricas [B] time = 0.66, size = 1668, normalized size = 21.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(a*b^2 - b^3)*cosh(d*x + c)*sinh(d
*x + c)^5 + (a*b^2 - b^3)*sinh(d*x + c)^6 - 3*(4*a^2*b - a*b^2 - 3*b^3)*cos
h(d*x + c)^4 - 3*(4*a^2*b - a*b^2 - 3*b^3 - 5*(a*b^2 - b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 4*(5*(a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^2*b - a*b^2
 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 - b^3 - 3*(4*a^2*b - a*b^2
 - 3*b^3)*cosh(d*x + c)^2 + 3*(5*(a*b^2 - b^3)*cosh(d*x + c)^4 - 4*a^2*b +
a*b^2 + 3*b^3 - 6*(4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - 12*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*c
osh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c)^3)*sqrt(-a*b + b^2)*log((b
*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 -
2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*
x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)
- 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 +
(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b
)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^
4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*
x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) +
b)) + 6*((a*b^2 - b^3)*cosh(d*x + c)^5 - 2*(4*a^2*b - a*b^2 - 3*b^3)*cosh(d
*x + c)^3 - (4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3
 - b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 - b^4)*d*cosh(d*x + c)^2*sinh(d*x + c)
```

```
 + 3*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d*sinh(d
*x + c)^3), 1/24*((a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(a*b^2 - b^3)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a*b^2 - b^3)*sinh(d*x + c)^6 - 3*(4*a^2*b - a*b^2 -
 3*b^3)*cosh(d*x + c)^4 - 3*(4*a^2*b - a*b^2 - 3*b^3 - 5*(a*b^2 - b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^
2*b - a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 - b^3 - 3*(4*a^
2*b - a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 3*(5*(a*b^2 - b^3)*cosh(d*x + c)^4 -
 4*a^2*b + a*b^2 + 3*b^3 - 6*(4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)^2*sinh(d*x + c
) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c)^3)*sqrt(a*b - b
^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*
sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a -
3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh
(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(
d*x + c)^3)*sqrt(a*b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + si
nh(d*x + c))/(a - b)) + 6*((a*b^2 - b^3)*cosh(d*x + c)^5 - 2*(4*a^2*b - a*b
^2 - 3*b^3)*cosh(d*x + c)^3 - (4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c))*sinh
(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 - b^4)*d*cosh(d*x +
c)^2*sinh(d*x + c) + 3*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a*b
^3 - b^4)*d*sinh(d*x + c)^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[22,73]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[-10,75]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.The choice was done assuming [a,b]=[-1,84]Warning, need to choose a bran
ch for the root of a polynomial with parameters. This might be wrong.The ch
oice was done assuming [a,b]=[-91,-60]Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done assuming [a,b]=[-33,-40]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-18,-85]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[1,-81]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
70,33]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[14,-81]Wa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming [a,b]=[39,-90]Undef/Unsi
gned Inf encountered in limitEvaluation time: 2.08Limit: Max order reached
or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.07, size = 227, normalized size = 2.87

$$-\frac{1}{3db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{a}{d\,b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a^{2}\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/3/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/d/b^
2/(tanh(1/2*d*x+1/2*c)-1)*a+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a^2/b^2/(a*
b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2)
)+1/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/d/b
^2/(tanh(1/2*d*x+1/2*c)+1)*a-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3\left(4\,ae^{(4\,c)}+3\,be^{(4\,c)}\right)e^{(4\,dx)}+3\left(4\,ae^{(2\,c)}+3\,be^{(2\,c)}\right)e^{(2\,dx)}-be^{(6\,dx+6\,c)}-b\right)e^{(-3\,dx-3\,c)}}{24\,b^{2}d}+\frac{1}{32}\int\frac{64\left(a^{2}e^{(3\,dx}\right.}{b^{3}e^{(4\,dx+4\,c)}+b^{3}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(4*a*e^(4*c) + 3*b*e^(4*c))*e^(4*d*x) + 3*(4*a*e^(2*c) + 3*b*e^(2*
c))*e^(2*d*x) - b*e^(6*d*x + 6*c) - b)*e^(-3*d*x - 3*c)/(b^2*d) + 1/32*inte
grate(64*(a^2*e^(3*d*x + 3*c) - a^2*e^(d*x + c))/(b^3*e^(4*d*x + 4*c) + b^3
 + 2*(2*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.38, size = 348, normalized size = 4.41

$$\frac{\left(2\,\mathrm{atan}\left(\frac{a^{2}\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{b^{5}\,d^{2}\,(a-b)}}{2\,b^{2}\,d\,(a-b)\,\sqrt{a^{4}}}\right)+2\,\mathrm{atan}\left(\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\left(\frac{2\,a^{2}}{b^{8}\,d\,(a-b)^{2}\,\sqrt{a^{4}}}-\frac{4\left(2\,a^{3}\,b^{3}\,d\,\sqrt{a^{4}}-2\,a^{4}\,b^{2}\,d\,\sqrt{a^{4}}\right)}{a^{5}\,b^{6}\,(a-b)\,\sqrt{a\,b^{5}\,d^{2}-b^{6}\,d^{2}}\,\sqrt{b^{5}\,d^{2}\,(a-b)}}\right)+\frac{2\,a^{2}\,\mathrm{e}^{3\,c}\,\mathrm{e}^{3\,d\,x}}{b^{8}\,d\,(a-b)^{2}\,\sqrt{a^{4}}}\right)\left(b\right.}{2\,\sqrt{a\,b^{5}\,d^{2}-b^{6}\,d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] ((2*atan((a^2*exp(d*x)*exp(c)*(b^5*d^2*(a - b))^(1/2))/(2*b^2*d*(a - b)*(a^
4)^(1/2))) + 2*atan((exp(d*x)*exp(c)*((2*a^2)/(b^8*d*(a - b)^2*(a^4)^(1/2))
 - (4*(2*a^3*b^3*d*(a^4)^(1/2) - 2*a^4*b^2*d*(a^4)^(1/2)))/(a^5*b^6*(a - b)
*(a*b^5*d^2 - b^6*d^2)^(1/2)*(b^5*d^2*(a - b))^(1/2))) + (2*a^2*exp(3*c)*ex
p(3*d*x))/(b^8*d*(a - b)^2*(a^4)^(1/2)))*((b^7*(a*b^5*d^2 - b^6*d^2)^(1/2))
/4 - (a*b^6*(a*b^5*d^2 - b^6*d^2)^(1/2))/4)))*(a^4)^(1/2))/(2*(a*b^5*d^2 -
b^6*d^2)^(1/2)) + exp(- 3*c - 3*d*x)/(24*b*d) + exp(3*c + 3*d*x)/(24*b*d) -
 (exp(c + d*x)*(4*a + 3*b))/(8*b^2*d) - (exp(- c - d*x)*(4*a + 3*b))/(8*b^2
*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.31 $\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{b^2d\sqrt{a-b}}-\frac{x(2a+b)}{2b^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a+b)*x/b^2+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d+a^(3/2)*arctanh((a-b)^(1
/2)*tanh(d*x+c)/a^(1/2))/b^2/d/(a-b)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = $\{3187, 470, 522, 206, 208\}$

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{b^2d\sqrt{a-b}}-\frac{x(2a+b)}{2b^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((2*a + b)*x)/(2*b^2) + (a^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[
a]])/(Sqrt[a - b]*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
```

```
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x, \tanh(c+dx)\right)}{2bd}$$
$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{b^2d} - \frac{(2a+b)\,\text{S}}{}$$
$$= -\frac{(2a+b)x}{2b^2} + \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

Mathematica [A] time = 0.25, size = 71, normalized size = 0.90

$$\frac{\frac{4a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - 2(2a+b)(c+dx) + b\sinh(2(c+dx))}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-2*(2*a + b)*(c + d*x) + (4*a^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sq
rt[a]])/Sqrt[a - b] + b*Sinh[2*(c + d*x)])/(4*b^2*d)
```

fricas [B] time = 0.73, size = 859, normalized size = 10.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a + b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a + b)*d*x - 3*b*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*
x + c) + a*sinh(d*x + c)^2)*sqrt(a/(a - b))*log((b^2*cosh(d*x + c)^4 + 4*b^
2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cos
h(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c) - 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)
*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/
(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d
*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x
 + c) + b)) + 4*(2*(2*a + b)*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*
x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) +
b^2*d*sinh(d*x + c)^2), -1/8*(4*(2*a + b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x
+ c)^4 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a
+ b)*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 8*(a*cosh(d*x + c)^2 + 2*
```

```
a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*sqrt(-a/(a - b))*arctan(
1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^
2 + 2*a - b)*sqrt(-a/(a - b))/a) + 4*(2*(2*a + b)*d*x*cosh(d*x + c) - b*cos
h(d*x + c)^3)*sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x
+ c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)]
```

giac [A] time = 3.06, size = 126, normalized size = 1.59

$$\frac{\frac{8\,a^2\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b^2}-\frac{4\,(dx+c)(2\,a+b)}{b^2}+\frac{e^{(2\,dx+2\,c)}}{b}+\frac{\left(4\,ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}-b\right)e^{(-2\,dx-2\,c)}}{b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt
(-a^2 + a*b)*b^2) - 4*(d*x + c)*(2*a + b)/b^2 + e^(2*d*x + 2*c)/b + (4*a*e^
(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c) - b)*e^(-2*d*x - 2*c)/b^2)/d
```

maple [B] time = 0.07, size = 454, normalized size = 5.75

$$\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,b^2}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2db}-\frac{a^2\,\mathrm{ar}}{d\,b^2\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2
*ln(tanh(1/2*d*x+1/2*c)-1)+1/2/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-1/d*a^2/b^2/((
2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2))-1/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2))+1/d*a^2/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*a^2/b/(-b*(a-b))^(1/2
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(
tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/d/b*ln(tanh(
1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 0.94, size = 216, normalized size = 2.73

$$\frac{e^{2\,c+2\,d\,x}}{8\,b\,d}-\frac{e^{-2\,c-2\,d\,x}}{8\,b\,d}-\frac{x\,(2\,a+b)}{2\,b^2}+\frac{a^{3/2}\,\ln\left(-\frac{4\,a^2\,e^{2\,c+2\,d\,x}}{b^3}-\frac{2\,a^{3/2}\,\left(b\,d+2\,a\,d\,e^{2\,c+2\,d\,x}-b\,d\,e^{2\,c+2\,d\,x}\right)}{b^3\,d\,\sqrt{a-b}}\right)}{2\,b^2\,d\,\sqrt{a-b}}-\frac{a^{3/2}\,\ln\left(\frac{2\,a^{3/2}\,\left(b\,d+2\,a\,d\,e\right.}{b^3\,d}\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(2*c + 2*d*x)/(8*b*d) - exp(- 2*c - 2*d*x)/(8*b*d) - (x*(2*a + b))/(2*b^
2) + (a^(3/2)*log(- (4*a^2*exp(2*c + 2*d*x))/b^3 - (2*a^(3/2)*(b*d + 2*a*d*
exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(b^3*d*(a - b)^(1/2))))/(2*b^2*d*
(a - b)^(1/2)) - (a^(3/2)*log((2*a^(3/2)*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*
d*exp(2*c + 2*d*x)))/(b^3*d*(a - b)^(1/2)) - (4*a^2*exp(2*c + 2*d*x))/b^3))
/(2*b^2*d*(a - b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.32 $\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{\cosh(c+dx)}{bd}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{3/2}d\sqrt{a-b}}$$

```
[Out] cosh(d*x+c)/b/d-a*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/b^(3/2)/d/(a-b)^(
1/2)
```

Rubi [A] time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 388, 205}

$$\frac{\cosh(c+dx)}{bd}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{3/2}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(3/2)*d))
+ Cosh[c + d*x]/(b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{a-b+bx^2}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\cosh(c+dx)}{bd}-\frac{a\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx,x,\cosh(c+dx)\right)}{bd}\\ &= -\frac{a\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{3/2}d}+\frac{\cosh(c+dx)}{bd}\end{aligned}$$

Mathematica [C] time = 0.24, size = 107, normalized size = 1.91

$$\frac{\sqrt{b}\cosh(c+dx)-\frac{a\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}}}{b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-((a*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan
[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b]) + Sqrt
[b]*Cosh[c + d*x])/(b^(3/2)*d)
```

fricas [B] time = 0.58, size = 746, normalized size = 13.32

$$\left[\frac{\left(ab-b^2\right)\cosh\left(dx+c\right)^2+2\left(ab-b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\left(ab-b^2\right)\sinh\left(dx+c\right)^2-\sqrt{-ab+b^2}\left(a\cosh\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x +
c) + (a*b - b^2)*sinh(d*x + c)^2 - sqrt(-a*b + b^2)*(a*cosh(d*x + c) + a*si
nh(d*x + c))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 +
 sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*s
qrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3
 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c
))*sinh(d*x + c) + b)) + a*b - b^2)/((a*b^2 - b^3)*d*cosh(d*x + c) + (a*b^2
 - b^3)*d*sinh(d*x + c)), 1/2*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*
cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 - 2*sqrt(a*b - b^
2)*(a*cosh(d*x + c) + a*sinh(d*x + c))*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b
*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x +
 c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) + 2
*sqrt(a*b - b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c))*arctan(-1/2*sqrt(a*b -
 b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + a*b - b^2)/((a*b^2 - b^3)*
d*cosh(d*x + c) + (a*b^2 - b^3)*d*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
```

```
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[-18,-81]Warning, ne
ed to choose a branch for the root of a polynomial with parameters. This mi
ght be wrong.The choice was done assuming [a,b]=[-10,75]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a,b]=[4,51]Warning, need to choose a bra
nch for the root of a polynomial with parameters. This might be wrong.The c
hoice was done assuming [a,b]=[44,-86]Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done assuming [a,b]=[34,-93]Warning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done a
ssuming [a,b]=[80,-1]Undef/Unsigned Inf encountered in limitEvaluation time
: 1.5Limit: Max order reached or unable to make series expansion Error: Bad
 Argument Value
```

maple [B] time = 0.07, size = 98, normalized size = 1.75

$$-\frac{a\arctan\left(\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2a+4b}{4\sqrt{ab-b^2}}\right)}{db\sqrt{ab-b^2}}+\frac{1}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d*a/b/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*
b-b^2)^(1/2))+1/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d/b/(tanh(1/2*d*x+1/2*c)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2\,dx+2\,c)}+1\right)e^{(-dx-c)}}{2\,bd}-\frac{1}{8}\int\frac{16\left(ae^{(3\,dx+3\,c)}-ae^{(dx+c)}\right)}{b^2e^{(4\,dx+4\,c)}+b^2+2\left(2\,abe^{(2\,c)}-b^2e^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/8*integrate(16*(a*e^(3*d*x
 + 3*c) - a*e^(d*x + c))/(b^2*e^(4*d*x + 4*c) + b^2 + 2*(2*a*b*e^(2*c) - b^
2*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.17, size = 293, normalized size = 5.23

$$\frac{e^{c+d\,x}}{2\,b\,d}-\frac{\left(2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^c\,\sqrt{b^3\,d^2\,(a-b)}}{2\,b\,d\,(a-b)\,\left(a^2\right)^{3/2}}\right)+2\,\mathrm{atan}\left(\left(\left(e^{d\,x}\,e^c\,\left(\frac{2\,a^3}{b^5\,d\,(a-b)^2\,\left(a^2\right)^{3/2}}-\frac{4\left(2\,b^2\,d\,\left(a^2\right)^{3/2}-2\,a\,b\,d\,\left(a^2\right)^{3/2}\right)}{a^3\,b^4\,(a-b)\,\sqrt{a\,b^3\,d^2-b^4\,d^2}\,\sqrt{b^3\,d^2\,(a-b)}}\right)+\frac{2\,a^3}{b^5\,d\,(a}\right.\right.\right.}{2\,\sqrt{a\,b^3\,d^2-b^4\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(c + d*x)/(2*b*d) - ((2*atan((a^3*exp(d*x)*exp(c)*(b^3*d^2*(a - b))^(1/2
))/(2*b*d*(a - b)*(a^2)^(3/2))) + 2*atan((exp(d*x)*exp(c)*((2*a^3)/(b^5*d*(
a - b)^2*(a^2)^(3/2)) - (4*(2*b^2*d*(a^2)^(3/2) - 2*a*b*d*(a^2)^(3/2)))/(a^
3*b^4*(a - b)*(a*b^3*d^2 - b^4*d^2)^(1/2)*(b^3*d^2*(a - b))^(1/2))) + (2*a^
3*exp(3*c)*exp(3*d*x))/(b^5*d*(a - b)^2*(a^2)^(3/2)))*((b^5*(a*b^3*d^2 - b^
```

```
4*d^2)^(1/2))/4 - (a*b^4*(a*b^3*d^2 - b^4*d^2)^(1/2))/4)))*(a^2)^(1/2))/(2*
(a*b^3*d^2 - b^4*d^2)^(1/2)) + exp(- c - d*x)/(2*b*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.33 $\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{b}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a-b}}$$

```
[Out] x/b-arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/b/d/(a-b)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3171, 3181, 208}

$$\frac{x}{b}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*b
*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sinh^2(c+dx)}\,dx}{b}\\ &= \frac{x}{b}-\frac{a\,\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx,x,\tanh(c+dx)\right)}{bd}\\ &= \frac{x}{b}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,bd}\end{aligned}$$

Mathematica [A] time = 0.13, size = 50, normalized size = 1.00

$$\frac{-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}+c+dx}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (c + d*x - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a -
b])/(b*d)
```

fricas [B] time = 0.61, size = 464, normalized size = 9.28

$$\left[\frac{2\,dx + \sqrt{\frac{a}{a-b}}\,\log\left(\frac{b^2\cosh(dx+c)^4 + 4\,b^2\cosh(dx+c)\sinh(dx+c)^3 + b^2\sinh(dx+c)^4 + 2\left(2\,ab-b^2\right)\cosh(dx+c)^2 + 2\left(3\,b^2\cosh(dx+c)^2 + 2\,ab-b^2\right)\sinh(\ldots}{b\cosh(dx+c)^4 + 4\,b\cosh(dx+c)\sinh(dx+c)^3 + b\sinh(dx+c)^4 + \ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(a/(a - b))*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c
)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) +
 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)
 + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b)))/(b*c
osh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*
(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)
^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/(
b*d), (d*x - sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a/(a - b))/a))/(b*d
)]
```

giac [A] time = 1.72, size = 64, normalized size = 1.28

$$-\frac{\frac{a\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b} - \frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
 a*b)*b) - (d*x + c)/b)/d
```

maple [B] time = 0.06, size = 312, normalized size = 6.24

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{db} + \frac{a\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{db\sqrt{\left(2\sqrt{-b\,(a-b)}-a+2b\right)a}} + \frac{a\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\sqrt{-b\,(a-b)}\,\sqrt{\left(2\sqrt{-b\,(a-b)}-a+2b\right)a}} - \frac{a\,\mathrm{arctanh}\ldots}{db\sqrt{\left(2\sqrt{-\ldots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/d*a/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*a/b/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*
```

```
a)^(1/2))+1/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arcta
nh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b*ln(tan
h(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 1.22, size = 473, normalized size = 9.46

$$\frac{x}{b}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\left(b^5\,\sqrt{b^3\,d^2-a\,b^2\,d^2}-a\,b^4\,\sqrt{b^3\,d^2-a\,b^2\,d^2}\right)\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\left(\frac{2\left(8\,a^2-8\,a\,b+b^2\right)\left(8\,a^{5/2}\,\sqrt{b^3\,d^2-a\,b^2\,d^2}-8\,a^{3/2}\,b\,\sqrt{b^3\,d^2-a\,b^2\,d^2}+\sqrt{a}\,b^2\,\sqrt{b^3\,d^2-a\,b^2\,d^2}\cdots\right)}{b^8\,d\,(a-b)^2\,\sqrt{b^3\,d^2-a\,b^2\,d^2}}\cdots\right.\right.}{\cdots}\right)}{\sqrt{b^3\,d^2-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] x/b - (a^(1/2)*atan(((b^5*(b^3*d^2 - a*b^2*d^2)^(1/2) - a*b^4*(b^3*d^2 - a*
b^2*d^2)^(1/2))*(exp(2*c)*exp(2*d*x)*((2*(8*a^2 - 8*a*b + b^2)*(8*a^(5/2)*(
b^3*d^2 - a*b^2*d^2)^(1/2) - 8*a^(3/2)*b*(b^3*d^2 - a*b^2*d^2)^(1/2) + a^(1
/2)*b^2*(b^3*d^2 - a*b^2*d^2)^(1/2)))/(b^8*d*(a - b)^2*(b^3*d^2 - a*b^2*d^2
)^(1/2)) + (4*a^(1/2)*(4*a - 2*b)*(4*a*b^3*d - 12*a^2*b^2*d + 8*a^3*b*d))/(
b^7*(a - b)*(b^3*d^2 - a*b^2*d^2)^(1/2)*(-b^2*d^2*(a - b))^(1/2))) + (2*(2*
a^(3/2)*b*(b^3*d^2 - a*b^2*d^2)^(1/2) - a^(1/2)*b^2*(b^3*d^2 - a*b^2*d^2)^(
1/2))*(8*a^2 - 8*a*b + b^2))/(b^8*d*(a - b)^2*(b^3*d^2 - a*b^2*d^2)^(1/2))
+ (4*a^(1/2)*(2*a^2*b^2*d - 2*a*b^3*d)*(4*a - 2*b))/(b^7*(a - b)*(b^3*d^2 -
 a*b^2*d^2)^(1/2)*(-b^2*d^2*(a - b))^(1/2))))/(4*a)))/(b^3*d^2 - a*b^2*d^2)
^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.34 $\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{b}\,d\sqrt{a-b}}$$

[Out] arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)/b^(1/2)

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{b}\,d\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]

[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(Sqrt[a - b]*Sqrt[b]*d)

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,\sqrt{b}\,d}\end{aligned}$$

Mathematica [C] time = 0.13, size = 91, normalized size = 2.28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{b}\,d\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]

[Out] (ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqrt
[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt[b]*d)

fricas [B] time = 0.72, size = 502, normalized size = 12.55

$$\left[-\frac{\sqrt{-ab+b^2}\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a-3\,b)\cosh(dx+c)^2+2\,\left(3\,b\cosh(dx+c)^2-2\,a+3\,b\right)\sinh(dx}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*
x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d
*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b))/((a*b - b^2)*d), (sqrt(a*b - b^2)*arctan(-
1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c
)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*
x + c))/sqrt(a*b - b^2)) - sqrt(a*b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cos
h(d*x + c) + sinh(d*x + c))/(a - b)))/((a*b - b^2)*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[-18,-81]Warning, ne
ed to choose a branch for the root of a polynomial with parameters. This mi
ght be wrong.The choice was done assuming [a,b]=[-10,75]Undef/Unsigned Inf
encountered in limitEvaluation time: 0.8Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.04, size = 51, normalized size = 1.28

$$\frac{\arctan\left(\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2a+4b}{4\sqrt{ab-b^2}}\right)}{d\sqrt{ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(dx+c\right)}{b\sinh\left(dx+c\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^2 + a), x)
```

mupad [B] time = 0.99, size = 116, normalized size = 2.90

$$\frac{\ln\left(-\frac{4\left(a+a\,e^{2\,c+2\,d\,x}\right)}{b^2\,(a-b)}-\frac{8\,a\,e^{c+d\,x}}{(-b)^{5/2}\,\sqrt{a-b}}\right)-\ln\left(\frac{8\,a\,e^{c+d\,x}}{(-b)^{5/2}\,\sqrt{a-b}}-\frac{4\left(a+a\,e^{2\,c+2\,d\,x}\right)}{b^2\,(a-b)}\right)}{2\,\sqrt{-b}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (log(- (4*(a + a*exp(2*c + 2*d*x)))/(b^2*(a - b)) - (8*a*exp(c + d*x))/((-b
)^(5/2)*(a - b)^(1/2))) - log((8*a*exp(c + d*x))/((-b)^(5/2)*(a - b)^(1/2))
 - (4*(a + a*exp(2*c + 2*d*x)))/(b^2*(a - b))))/(2*(-b)^(1/2)*d*(a - b)^(1/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.35 $\int \frac{1}{a+b \sinh^2(c+dx)} \, dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\, d\sqrt{a-b}}$$

```
[Out] arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/d/a^(1/2)/(a-b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3181, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\, d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b\sinh^2(c+dx)}\, dx &= \frac{\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\, dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}\, d} \end{aligned}$$

Mathematica [A] time = 0.07, size = 40, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\, d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

fricas [B] time = 0.60, size = 430, normalized size = 10.75

$$\left[\frac{\log\left(\frac{b^{2}\cosh(dx+c)^{4}+4\,b^{2}\cosh(dx+c)\sinh(dx+c)^{3}+b^{2}\sinh(dx+c)^{4}+2\left(2\,ab-b^{2}\right)\cosh(dx+c)^{2}+2\left(3\,b^{2}\cosh(dx+c)^{2}+2\,ab-b^{2}\right)\sinh(dx+c)^{2}+8\,a^{2}-8\,ab}{b\cosh(dx+c)^{4}+4\,b\cosh(dx+c)\sinh(dx+c)^{3}+b\sinh(dx+c)^{4}+2\,(2\,a-b)\cosh(dx+c)^{2}+2\left(3\,b\cosh\right.}\right.}{2\sqrt{a^{2}-a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*s
inh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2
 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c
)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2
*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*
b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c)
+ b))/(sqrt(a^2 - a*b)*d), -sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2
 + a*b)/(a^2 - a*b))/((a^2 - a*b)*d)]
```

giac [A] time = 0.41, size = 47, normalized size = 1.18

$$\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b
)*d)
```

maple [B] time = 0.08, size = 267, normalized size = 6.68

$$-\frac{\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\sqrt{\left(2\sqrt{-b\,(a-b)}\,-a+2b\right)a}}-\frac{\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)b}{d\sqrt{-b\,(a-b)}\,\sqrt{\left(2\sqrt{-b\,(a-b)}\,-a+2b\right)a}}+\frac{\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)}{d\sqrt{\left(2\sqrt{-b\,(a-b)}\,+a-2b\right)a}}-\frac{}{d\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)
-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a
)^(1/2))*b+1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(-b*(a-b))^(1/2)/((2*(-b*(
a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**B**] time = 0.45, size = 146, normalized size = 3.65

$$\frac{\ln\left(-\frac{4\,e^{2\,c+2\,d\,x}}{b}-\frac{2\left(b\,d+2\,a\,d\,e^{2\,c+2\,d\,x}-b\,d\,e^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b\,d\,\sqrt{a-b}}\right)-\ln\left(\frac{2\left(b\,d+2\,a\,d\,e^{2\,c+2\,d\,x}-b\,d\,e^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b\,d\,\sqrt{a-b}}-\frac{4\,e^{2\,c+2\,d\,x}}{b}\right)}{2\,\sqrt{a}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (log(- (4*exp(2*c + 2*d*x))/b - (2*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp(
2*c + 2*d*x)))/(a^(1/2)*b*d*(a - b)^(1/2))) - log((2*(b*d + 2*a*d*exp(2*c +
 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(1/2)*b*d*(a - b)^(1/2)) - (4*exp(2*c +
 2*d*x))/b))/(2*a^(1/2)*d*(a - b)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.36 $\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{ad\sqrt{a-b}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d-arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a
/d/(a-b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3186, 391, 206, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{ad\sqrt{a-b}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*Sqrt[a - b]*d))
- ArcTanh[Cosh[c + d*x]]/(a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{ad} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{ad}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Mathematica [C] time = 0.22, size = 124, normalized size = 2.07

$$\frac{\sqrt{a-b}\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) - \sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{ad\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-(Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]) - S
qrt[b]*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + Sqrt[a
 - b]*Log[Tanh[(c + d*x)/2]])/(a*Sqrt[a - b]*d)
```

fricas [B] time = 0.56, size = 586, normalized size = 9.77

$$\left[\frac{\sqrt{-\frac{b}{a-b}}\log\left(\frac{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2(2a-3b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c)^2-2a+3b\right)\sinh(dx+c)^2+\cdots}{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(d\cdots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*
x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(
d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c)
+ (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c) + sinh(d
*x + c) - 1))/(a*d), -(sqrt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x
 + c) + sinh(d*x + c))) - sqrt(b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3
*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x
 + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b)
 + log(cosh(d*x + c) + sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh(d*x +
c) - 1))/(a*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Undef/Unsigned Inf encountered in limitLimit: Max order reache
d or unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.10, size = 74, normalized size = 1.23

$$-\frac{b\arctan\left(\frac{2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2a+4b}{4\sqrt{ab-b^{2}}}\right)}{da\sqrt{ab-b^{2}}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d/a*b/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*
b-b^2)^(1/2))+1/d/a*ln(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad}+\frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad}-2\int\frac{be^{(3\,dx+3\,c)}-be^{(dx+c)}}{abe^{(4\,dx+4\,c)}+ab+2\left(2\,a^{2}e^{(2\,c)}-abe^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a*b*e^(4*d*x + 4*c) + a*
b + 2*(2*a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.06, size = 323, normalized size = 5.38

$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(16\,a^2\,\sqrt{-a^2\,d^2}+9\,b^2\,\sqrt{-a^2\,d^2}-24\,a\,b\,\sqrt{-a^2\,d^2}\right)}{16\,d\,a^3-24\,d\,a^2\,b+9\,d\,a\,b^2}\right)}{\sqrt{-a^2\,d^2}}-\frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{a^2\,d^2\,(a-b)}}{2\,a\,d\,(a-b)}\right)+2\,\mathrm{atan}\left(\frac{4\,a^4\,d^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+4\,a^2\,b^2 \cdots}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] - (2*atan((exp(d*x)*exp(c)*(16*a^2*(-a^2*d^2)^(1/2) + 9*b^2*(-a^2*d^2)^(1/2
) - 24*a*b*(-a^2*d^2)^(1/2)))/(16*a^3*d + 9*a*b^2*d - 24*a^2*b*d)))/(-a^2*d
^2)^(1/2) - (b^(1/2)*(2*atan((b^(1/2)*exp(d*x)*exp(c)*(a^2*d^2*(a - b))^(1/
2))/(2*a*d*(a - b))) + 2*atan((4*a^4*d^2*exp(d*x)*exp(c) + 4*a^2*b^2*d^2*ex
p(d*x)*exp(c) + b*exp(3*c)*exp(3*d*x)*(a^3*d^2 - a^2*b*d^2)^(1/2)*(a^2*d^2*
(a - b))^(1/2) - 8*a^3*b*d^2*exp(d*x)*exp(c) + b*exp(d*x)*exp(c)*(a^3*d^2 -
 a^2*b*d^2)^(1/2)*(a^2*d^2*(a - b))^(1/2))/(b^(1/2)*d*(2*a*b - 2*a^2)*(a^2*
d^2*(a - b))^(1/2)))))/(2*(a^3*d^2 - a^2*b*d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}\left(c+dx\right)}{a+b\sinh^{2}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sinh(c + d*x)**2), x)
```

3.37 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d\sqrt{a-b}}-\frac{\coth(c+dx)}{ad}$$

```
[Out] -coth(d*x+c)/a/d-b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/d/(a-b)
^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3187, 453, 208}

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d\sqrt{a-b}}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a - b]*d))
 - Coth[c + d*x]/(a*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^2\left(a-(a-b)x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\coth(c+dx)}{ad} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{ad}$$
$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}\,d} - \frac{\coth(c+dx)}{ad}$$

Mathematica [A] time = 0.29, size = 57, normalized size = 1.00

$$\frac{-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - \sqrt{a}\coth(c+dx)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b]) - Sqrt[a]*
Coth[c + d*x])/(a^(3/2)*d)
```

fricas [B] time = 0.61, size = 675, normalized size = 11.84

$$\left[\frac{\left(b\cosh(dx+c)^2 + 2b\cosh(dx+c)\sinh(dx+c) + b\sinh(dx+c)^2 - b\right)\sqrt{a^2-ab}\,\log\left(\frac{b^2\cosh(dx+c)^4+4b^2\cosh(\ldots}{\ldots}\right.}{2\left(\left(a^3-a^2\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
)^2 - b)*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sin
h(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3
*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 +
 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b
*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*
a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
+ c))*sinh(d*x + c) + b)) - 4*a^2 + 4*a*b)/((a^3 - a^2*b)*d*cosh(d*x + c)^2
 + 2*(a^3 - a^2*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - a^2*b)*d*sinh(d*x
 + c)^2 - (a^3 - a^2*b)*d), ((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*
x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(
-a^2 + a*b)/(a^2 - a*b)) - 2*a^2 + 2*a*b)/((a^3 - a^2*b)*d*cosh(d*x + c)^2
+ 2*(a^3 - a^2*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - a^2*b)*d*sinh(d*x
+ c)^2 - (a^3 - a^2*b)*d)]
```

giac [A] time = 0.68, size = 72, normalized size = 1.26

$$-\frac{\frac{b\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a} + \frac{2}{a\left(e^{(2dx+2c)}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
 a*b)*a) + 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

maple [B] time = 0.11, size = 319, normalized size = 5.60

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{b\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{da\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}+\frac{\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)b^2}{da\sqrt{-b\left(a-b\right)}\,\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}-\frac{b\,\mathrm{arctanh}\left(\frac{a}{\sqrt{(2\sqrt{}}}\right.}{da\sqrt{\left(2\sqrt{-b\left(a-\right.}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)+1/d*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/a/(-b
*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2-1/d*b/a/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*
a)^(1/2))+1/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arcta
nh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-1/2/d/a/
tanh(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 0.47, size = 176, normalized size = 3.09

$$\frac{b\,\ln\left(\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}}{a}-\frac{2\,\left(b\,d+2\,a\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{a^{3/2}\,d\,\sqrt{a-b}}\right)}{2\,a^{3/2}\,d\,\sqrt{a-b}}-\frac{2}{a\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{b\,\ln\left(\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}}{a}+\frac{2\,\left(b\,d+2\,a\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{a^{3/2}\,d\,\sqrt{a-b}}\right)}{2\,a^{3/2}\,d\,\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (b*log((4*exp(2*c + 2*d*x))/a - (2*(b*d + 2*a*d*exp(2*c + 2*d*x) - b*d*exp(
2*c + 2*d*x)))/(a^(3/2)*d*(a - b)^(1/2))))/(2*a^(3/2)*d*(a - b)^(1/2)) - 2/
(a*d*(exp(2*c + 2*d*x) - 1)) - (b*log((4*exp(2*c + 2*d*x))/a + (2*(b*d + 2*
a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(3/2)*d*(a - b)^(1/2))))/(
2*a^(3/2)*d*(a - b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{2}{\left(c + dx \right)}}{a + b\sinh^{2}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
[Out] Integral(csch(c + d*x)**2/(a + b*sinh(c + d*x)**2), x)
```

3.38 $\int \frac{\text{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2d\sqrt{a-b}}+\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

```
[Out] 1/2*(a+2*b)*arctanh(cosh(d*x+c))/a^2/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d+b^(3
/2)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/a^2/d/(a-b)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3186, 414, 522, 206, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2d\sqrt{a-b}}+\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^2*Sqrt[a - b]*d) +
 ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Coth[c + d*x]*Csch[c + d*x
])/(2*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
```

```
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-b+bx^2\right)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{\text{Subst}\left(\int \frac{a+b+bx^2}{\left(1-x^2\right)\left(a-b+bx^2\right)}\,dx, x, \cosh(c+dx)\right)}{2ad}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{a^2 d} + \frac{(a+2b)}{}$$

$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2\sqrt{a-b}\,d} + \frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2 d} - \frac{\coth(c+dx)\operatorname{csch}(c}{2ad}$$

Mathematica [C] time = 0.70, size = 201, normalized size = 2.28

$$-\frac{\operatorname{csch}^4(c+dx)(2a+b\cosh(2(c+dx))-b)\left(2a\sqrt{a-b}\,\cosh(c+dx)-2\sinh^2(c+dx)\left(2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan}{\sqrt{a-}}\right.\right.\right.}{8a^2 d\sqrt{a-b}\,\left(a\operatorname{csch}^2(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/8*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^4*(2*a*Sqrt[a - b]*Cosh
[c + d*x] - 2*(2*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2]])/Sqr
t[a - b]] + 2*b^(3/2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2]])/Sqrt[a
 - b]] - Sqrt[a - b]*(a + 2*b)*Log[Tanh[(c + d*x)/2]])*Sinh[c + d*x]^2))/(a
^2*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))
```

fricas [B] time = 0.53, size = 1837, normalized size = 20.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d
*x + c)^3 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh
(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x +
c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-b/(
a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh
(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a
+ 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*
sinh(d*x + c) + 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d
*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*co
sh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c
)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*c
osh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*c
osh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 2*a*cosh(d*
x + c) - ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a +
```

```
2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c
)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) +
 sinh(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2
 + 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)
*cosh(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(co
sh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c
))/(a^2*d*cosh(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*s
inh(d*x + c)^4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)
^2 - a^2*d)*sinh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c
))*sinh(d*x + c)), -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x +
 c)^2 + 2*a*sinh(d*x + c)^3 - 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x
 + c) + b)*sqrt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh
(d*x + c))) + 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*
sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*
x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(
b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^
2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 +
4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b) + 2*a*cosh(d*x + c) - ((a + 2*
b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2*b)*
sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x + c
)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3 - (a + 2*b)*c
osh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sinh(d*x + c) +
1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)
*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3
- (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sin
h(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(a^2*d*cosh(d*
x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(d*x + c)^4 -
2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(
d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*sinh(d*x + c))
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Unde
f/Unsigned Inf encountered in limitEvaluation time: 0.63Limit: Max order re
ached or unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.13, size = 133, normalized size = 1.51

$$\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{b^2\arctan\left(\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2a+4b}{4\sqrt{ab-b^2}}\right)}{d\,a^2\sqrt{ab-b^2}}-\frac{1}{8da\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2+1/d*b^2/a^2/(a*b-b^2)^(1/2)*arctan(1/4*(2*tan
h(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))-1/8/d/a/tanh(1/2*d*x+1/2*c)^
2-1/2/d/a*ln(tanh(1/2*d*x+1/2*c))-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{e^{(3\,dx+3\,c)}+e^{(dx+c)}}{ade^{(4\,dx+4\,c)}-2\,ade^{(2\,dx+2\,c)}+ad}+\frac{(a+2\,b)\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{2\,a^2d}-\frac{(a+2\,b)\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{2\,a^2d}+8\int\frac{}{4\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*
c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a +
2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*(b^2*e^(3*d*x
+ 3*c) - b^2*e^(d*x + c))/(a^2*b*e^(4*d*x + 4*c) + a^2*b + 2*(2*a^3*e^(2*c)
 - a^2*b*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.40, size = 571, normalized size = 6.49

$$\frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a^7\,\sqrt{-a^4\,d^2}+18\,b^7\,\sqrt{-a^4\,d^2}-36\,a^2\,b^5\,\sqrt{-a^4\,d^2}-30\,a^3\,b^4\,\sqrt{-a^4\,d^2}+12\,a^4\,b^3\,\sqrt{-a^4\,d^2}+21\,a^5\,b^2\,\sqrt{-a^4\,d^2}+9\,a\,b^6\,\sqrt{-a^4\,d^2}+8\,a^6\,b\right.}{a^8\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+9\,a^2\,b^6\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}-18\,a^4\,b^4\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}-6\,a^5\,b^3\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+9\,a^6\,b^2\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+6\,a^7\,b\,d\,\sqrt{}}\right.}{\sqrt{-a^4\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a^7*(-a^4*d^2)^(1/2) + 18*b^7*(-a^4*d^2)^(1/2) - 36
*a^2*b^5*(-a^4*d^2)^(1/2) - 30*a^3*b^4*(-a^4*d^2)^(1/2) + 12*a^4*b^3*(-a^4*
d^2)^(1/2) + 21*a^5*b^2*(-a^4*d^2)^(1/2) + 9*a*b^6*(-a^4*d^2)^(1/2) + 8*a^6
*b*(-a^4*d^2)^(1/2)))/(a^8*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 9*a^2*b^6*d*(4*a
*b + a^2 + 4*b^2)^(1/2) - 18*a^4*b^4*d*(4*a*b + a^2 + 4*b^2)^(1/2) - 6*a^5*
b^3*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 9*a^6*b^2*d*(4*a*b + a^2 + 4*b^2)^(1/2)
 + 6*a^7*b*d*(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 + 4*b^2)^(1/2))/(-a
^4*d^2)^(1/2) - exp(c + d*x)/(a*d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x)
)/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - ((-b)^(3/2)*log((64*(
exp(2*c + 2*d*x) + 1)*(3*a^2*b + a^3 - 3*b^3))/(a^5*(a - b)^2) - (128*exp(c
 + d*x)*(3*a^2*b + a^3 - 3*b^3))/(a^5*(-b)^(1/2)*(a - b)^(3/2))))/(2*a^2*d*
(a - b)^(1/2)) + ((-b)^(3/2)*log((64*(exp(2*c + 2*d*x) + 1)*(3*a^2*b + a^3
- 3*b^3))/(a^5*(a - b)^2) + (128*exp(c + d*x)*(3*a^2*b + a^3 - 3*b^3))/(a^5
*(-b)^(1/2)*(a - b)^(3/2))))/(2*a^2*d*(a - b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*sinh(c + d*x)**2), x)
```

3.39 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d\sqrt{a-b}}+\frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

```
[Out] (a+b)*coth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d+b^2*arctanh((a-b)^(1/2)*tanh(
d*x+c)/a^(1/2))/a^(5/2)/d/(a-b)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3187, 461, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d\sqrt{a-b}}+\frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]*d)
+ ((a + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^4(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{-a-b}{a^2x^2}+\frac{b^2}{a^2(a-(a-b)x^2)}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d}$$
$$= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}\,d} + \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.69, size = 126, normalized size = 1.62

$$-\frac{\operatorname{csch}^2(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\sqrt{a}\sqrt{a-b}\coth(c+dx)\left(a\operatorname{csch}^2(c+dx)-2a-3b\right)-3b^2\tanh^{-1}\right.}{6a^{5/2}d\sqrt{a-b}\left(a\operatorname{csch}^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/6*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(-3*b^2*ArcTanh[(Sqrt
[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a - b]*Coth[c + d*x]*(-2*a -
 3*b + a*Csch[c + d*x]^2)))/(a^(5/2)*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))
```

fricas [B] time = 0.73, size = 1972, normalized size = 25.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b - a*b^2)*sinh(d*x + c)^4 + 8*a^3 + 4*a^2*b - 1
2*a*b^2 - 24*(a^3 - a*b^2)*cosh(d*x + c)^2 - 24*(a^3 - a*b^2 - 3*(a^2*b - a
*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cos
h(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 +
3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4
*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*c
osh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^
2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c
))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*c
osh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^
2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)
*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 +
b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 +
2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*
sinh(d*x + c) + b)) + 48*((a^2*b - a*b^2)*cosh(d*x + c)^3 - (a^3 - a*b^2)*c
osh(d*x + c))*sinh(d*x + c))/((a^4 - a^3*b)*d*cosh(d*x + c)^6 + 6*(a^4 - a^
3*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - a^3*b)*d*sinh(d*x + c)^6 - 3*
(a^4 - a^3*b)*d*cosh(d*x + c)^4 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^2 - (a
^4 - a^3*b)*d)*sinh(d*x + c)^4 + 3*(a^4 - a^3*b)*d*cosh(d*x + c)^2 + 4*(5*(
```

```
a^4 - a^3*b)*d*cosh(d*x + c)^3 - 3*(a^4 - a^3*b)*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^4 - 6*(a^4 - a^3*b)*d*cosh(d*x
+ c)^2 + (a^4 - a^3*b)*d)*sinh(d*x + c)^2 - (a^4 - a^3*b)*d + 6*((a^4 - a^3
*b)*d*cosh(d*x + c)^5 - 2*(a^4 - a^3*b)*d*cosh(d*x + c)^3 + (a^4 - a^3*b)*d
*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 24
*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b - a*b^2)*sinh(d*x
 + c)^4 + 4*a^3 + 2*a^2*b - 6*a*b^2 - 12*(a^3 - a*b^2)*cosh(d*x + c)^2 - 12
*(a^3 - a*b^2 - 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(b^2
*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^
6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4
 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*s
inh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2
*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-
a^2 + a*b)/(a^2 - a*b)) + 24*((a^2*b - a*b^2)*cosh(d*x + c)^3 - (a^3 - a*b^
2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 - a^3*b)*d*cosh(d*x + c)^6 + 6*(a^4
- a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - a^3*b)*d*sinh(d*x + c)^6
- 3*(a^4 - a^3*b)*d*cosh(d*x + c)^4 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^2
- (a^4 - a^3*b)*d)*sinh(d*x + c)^4 + 3*(a^4 - a^3*b)*d*cosh(d*x + c)^2 + 4*
(5*(a^4 - a^3*b)*d*cosh(d*x + c)^3 - 3*(a^4 - a^3*b)*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^4 - 6*(a^4 - a^3*b)*d*cosh(
d*x + c)^2 + (a^4 - a^3*b)*d)*sinh(d*x + c)^2 - (a^4 - a^3*b)*d + 6*((a^4 -
 a^3*b)*d*cosh(d*x + c)^5 - 2*(a^4 - a^3*b)*d*cosh(d*x + c)^3 + (a^4 - a^3*
b)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [A] time = 0.70, size = 118, normalized size = 1.51

$$\frac{\frac{3\,b^{2}\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}\,a^{2}}+\frac{2\left(3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+2\,a+3\,b\right)}{a^{2}\left(e^{(2\,dx+2\,c)}-1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt
(-a^2 + a*b)*a^2) + 2*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 6*b*e^(2
*d*x + 2*c) + 2*a + 3*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

maple [B] time = 0.13, size = 401, normalized size = 5.14

$$-\frac{\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}+\frac{3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)b}{2d\,a^{2}}-\frac{b^{2}\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\,a^{2}\sqrt{\left(2\sqrt{-b\,(a-b)}\,-a+2b\right)a}}-\frac{b^{3}\arctan\left(\frac{a\,\mathrm{ta}}{\sqrt{(2\sqrt{-}}}\right.}{d\,a^{2}\sqrt{-b\,(a-b)}\,\sqrt{(2\sqrt{-}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)+1/2/d/a^2*tanh(
1/2*d*x+1/2*c)*b-1/d*b^2/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*b^3/a^2/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b^2/a^2/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2))-1/d*b^3/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/24/d/a
```

```
/tanh(1/2*d*x+1/2*c)^3+3/8/d/a/tanh(1/2*d*x+1/2*c)+1/2/d*b/a^2/tanh(1/2*d*x
+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 1.18, size = 350, normalized size = 4.49

$$\frac{2\,b}{a^2\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{8}{3\,a\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{4}{a\,d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{b^2\,\ln\left(\frac{4\,b^2\,(2\,a}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (2*b)/(a^2*d*(exp(2*c + 2*d*x) - 1)) - 8/(3*a*d*(3*exp(2*c + 2*d*x) - 3*exp
(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - 4/(a*d*(exp(4*c + 4*d*x) - 2*exp(2
*c + 2*d*x) + 1)) - (b^2*log((4*b^2*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) +
 b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a^5*(a - b)) - (8*b^2*(b
+ 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(9/2)*(a - b)^(1/2))))/(
2*a^(5/2)*d*(a - b)^(1/2)) + (b^2*log((4*b^2*(2*a*b - b^2 + 8*a^2*exp(2*c +
 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a^5*(a - b)) + (
8*b^2*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(9/2)*(a - b)^(
1/2))))/(2*a^(5/2)*d*(a - b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.40 $\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=130

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3d\sqrt{a-b}}+\frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d}-\frac{\left(3a^2+4ab+8b^2\right)\tanh^{-1}(\cosh(c+dx))}{8a^3d}-\frac{\coth(}{}$$

```
[Out] -1/8*(3*a^2+4*a*b+8*b^2)*arctanh(cosh(d*x+c))/a^3/d+1/8*(3*a+4*b)*coth(d*x+
c)*csch(d*x+c)/a^2/d-1/4*coth(d*x+c)*csch(d*x+c)^3/a/d-b^(5/2)*arctan(cosh(
d*x+c)*b^(1/2)/(a-b)^(1/2))/a^3/d/(a-b)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 522, 206, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3d\sqrt{a-b}}-\frac{\left(3a^2+4ab+8b^2\right)\tanh^{-1}(\cosh(c+dx))}{8a^3d}+\frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d}-\frac{\coth(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^3*Sqrt[a - b]*d)
) - ((3*a^2 + 4*a*b + 8*b^2)*ArcTanh[Cosh[c + d*x]])/(8*a^3*d) + ((3*a + 4*
b)*Coth[c + d*x]*Csch[c + d*x])/(8*a^2*d) - (Coth[c + d*x]*Csch[c + d*x]^3)
/(4*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a-b+bx^2\right)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}^3(c+dx)}{4ad} - \frac{\operatorname{Subst}\left(\int \frac{3a+b+3bx^2}{\left(1-x^2\right)^2\left(a-b+bx^2\right)}\,dx, x, \cosh(c+dx)\right)}{4ad}$$

$$= \frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d} - \frac{\coth(c+dx)\operatorname{csch}^3(c+dx)}{4ad} - \frac{\operatorname{Subst}\left(\int \frac{3a^2+\cdots}{(\cdots}\right.}{\cdots}$$

$$= \frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d} - \frac{\coth(c+dx)\operatorname{csch}^3(c+dx)}{4ad} - \frac{b^3\operatorname{Subst}\left(\int \frac{\cdots}{a\cdots}\right.}{\cdots}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3\sqrt{a-b}\,d} - \frac{\left(3a^2+4ab+8b^2\right)\tanh^{-1}(\cosh(c+dx))}{8a^3d} + \frac{(3a+4b\cdots}{\cdots}$$

Mathematica [C] time = 6.28, size = 574, normalized size = 4.42

$$-\frac{b^{5/2}\operatorname{csch}^2(c+dx)(2a+b\cosh(2(c+dx))-b)\tan^{-1}\left(\frac{\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{b}\cosh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a}\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a-b}}\right)}{2a^3d\sqrt{a-b}\left(a\operatorname{csch}^2(c+dx)+b\right)} - \frac{b^{5/2}\operatorname{csch}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/2*(b^(5/2)*ArcTan[(Sech[(c + d*x)/2]*(Sqrt[b]*Cosh[(c + d*x)/2] - I*Sqrt
[a]*Sinh[(c + d*x)/2]))/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c
 + d*x]^2)/(a^3*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2)) - (b^(5/2)*ArcTan[(S
ech[(c + d*x)/2]*(Sqrt[b]*Cosh[(c + d*x)/2] + I*Sqrt[a]*Sinh[(c + d*x)/2]))
/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2)/(2*a^3*Sqrt[
a - b]*d*(b + a*Csch[c + d*x]^2)) + ((3*a + 4*b)*(2*a - b + b*Cosh[2*(c + d
*x)])*Csch[(c + d*x)/2]^2*Csch[c + d*x]^2)/(64*a^2*d*(b + a*Csch[c + d*x]^2
)) - ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[(c + d*x)/2]^4*Csch[c + d*x]^2)/
(128*a*d*(b + a*Csch[c + d*x]^2)) + ((3*a^2 + 4*a*b + 8*b^2)*(2*a - b + b*C
osh[2*(c + d*x)])*Csch[c + d*x]^2*Log[Tanh[(c + d*x)/2]])/(16*a^3*d*(b + a*
Csch[c + d*x]^2)) + ((3*a + 4*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d
*x]^2*Sech[(c + d*x)/2]^2)/(64*a^2*d*(b + a*Csch[c + d*x]^2)) + ((2*a - b +
```

```
b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*Sech[(c + d*x)/2]^4)/(128*a*d*(b + a*
Csch[c + d*x]^2))
```

fricas [B] time = 0.65, size = 5809, normalized size = 44.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(3*a^2 + 4*a*b)*cosh(d*x + c)^7 + 14*(3*a^2 + 4*a*b)*cosh(d*x + c)*
sinh(d*x + c)^6 + 2*(3*a^2 + 4*a*b)*sinh(d*x + c)^7 - 2*(11*a^2 + 4*a*b)*co
sh(d*x + c)^5 + 2*(21*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 - 11*a^2 - 4*a*b)*sin
h(d*x + c)^5 + 10*(7*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 - (11*a^2 + 4*a*b)*cos
h(d*x + c))*sinh(d*x + c)^4 - 2*(11*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(35*(3
*a^2 + 4*a*b)*cosh(d*x + c)^4 - 10*(11*a^2 + 4*a*b)*cosh(d*x + c)^2 - 11*a^
2 - 4*a*b)*sinh(d*x + c)^3 + 2*(21*(3*a^2 + 4*a*b)*cosh(d*x + c)^5 - 10*(11
*a^2 + 4*a*b)*cosh(d*x + c)^3 - 3*(11*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x
+ c)^2 + 4*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2
*sinh(d*x + c)^8 - 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - b^2)*
sinh(d*x + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 - 3*b^2*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 - 30*b^2*cosh(d*
x + c)^2 + 3*b^2)*sinh(d*x + c)^4 - 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d
*x + c)^5 - 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*(7*b^2*cosh(d*x + c)^6 - 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2
- b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 - 3*b^2*cosh(d*x + c)
^5 + 3*b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a -
 b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*
x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3
*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sin
h(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x
+ c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(
d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh
(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 2*(3*a^2 + 4*a
*b)*cosh(d*x + c) - ((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 4
*a*b + 8*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 + 4*a*b + 8*b^2)*sinh(
d*x + c)^8 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 + 4*a*
b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^6 + 8*(7*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*
x + c))*sinh(d*x + c)^5 + 6*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35
*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 - 30*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c)^2 + 9*a^2 + 12*a*b + 24*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^5 - 10*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a^2 + 4*a*b +
 8*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 - 15
*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 9*(3*a^2 + 4*a*b + 8*b^2)*cosh(d
*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + 8*b^2
+ 8*((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^7 - 3*(3*a^2 + 4*a*b + 8*b^2)*co
sh(d*x + c)^5 + 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - (3*a^2 + 4*a*b
+ 8*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
1) + ((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 4*a*b + 8*b^2)*c
osh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c)^8 - 4*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh
(d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 4*a*b
+ 8*b^2)*cosh(d*x + c)^3 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + 6*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^4 - 30*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 9*
a^2 + 12*a*b + 24*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(
```

```
d*x + c)^5 - 10*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 4*a*b
+ 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*
x + c)^2 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 - 15*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^4 + 9*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a
^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + 8*b^2 + 8*((3*a^2 + 4
*a*b + 8*b^2)*cosh(d*x + c)^7 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 +
 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - (3*a^2 + 4*a*b + 8*b^2)*cosh(d
*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(3*a^
2 + 4*a*b)*cosh(d*x + c)^6 - 5*(11*a^2 + 4*a*b)*cosh(d*x + c)^4 - 3*(11*a^2
 + 4*a*b)*cosh(d*x + c)^2 + 3*a^2 + 4*a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x +
 c)^8 + 8*a^3*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d*sinh(d*x + c)^8 - 4*a
^3*d*cosh(d*x + c)^6 + 6*a^3*d*cosh(d*x + c)^4 + 4*(7*a^3*d*cosh(d*x + c)^2
 - a^3*d)*sinh(d*x + c)^6 - 4*a^3*d*cosh(d*x + c)^2 + 8*(7*a^3*d*cosh(d*x +
 c)^3 - 3*a^3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^3*d*cosh(d*x + c)^
4 - 30*a^3*d*cosh(d*x + c)^2 + 3*a^3*d)*sinh(d*x + c)^4 + a^3*d + 8*(7*a^3*
d*cosh(d*x + c)^5 - 10*a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(7*a^3*d*cosh(d*x + c)^6 - 15*a^3*d*cosh(d*x + c)^4 + 9*a^3*
d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 8*(a^3*d*cosh(d*x + c)^7 - 3*a
^3*d*cosh(d*x + c)^5 + 3*a^3*d*cosh(d*x + c)^3 - a^3*d*cosh(d*x + c))*sinh(
d*x + c)), 1/8*(2*(3*a^2 + 4*a*b)*cosh(d*x + c)^7 + 14*(3*a^2 + 4*a*b)*cosh
(d*x + c)*sinh(d*x + c)^6 + 2*(3*a^2 + 4*a*b)*sinh(d*x + c)^7 - 2*(11*a^2 +
 4*a*b)*cosh(d*x + c)^5 + 2*(21*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 - 11*a^2 -
4*a*b)*sinh(d*x + c)^5 + 10*(7*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 - (11*a^2 +
4*a*b)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(11*a^2 + 4*a*b)*cosh(d*x + c)^3
+ 2*(35*(3*a^2 + 4*a*b)*cosh(d*x + c)^4 - 10*(11*a^2 + 4*a*b)*cosh(d*x + c)
^2 - 11*a^2 - 4*a*b)*sinh(d*x + c)^3 + 2*(21*(3*a^2 + 4*a*b)*cosh(d*x + c)^
5 - 10*(11*a^2 + 4*a*b)*cosh(d*x + c)^3 - 3*(11*a^2 + 4*a*b)*cosh(d*x + c))
*sinh(d*x + c)^2 - 8*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x +
c)^7 + b^2*sinh(d*x + c)^8 - 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)
^2 - b^2)*sinh(d*x + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^
3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 - 30*b
^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^4 - 4*b^2*cosh(d*x + c)^2 + 8*(7*
b^2*cosh(d*x + c)^5 - 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*
x + c)^3 + 4*(7*b^2*cosh(d*x + c)^6 - 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d
*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 - 3*b^2*cos
h(d*x + c)^5 + 3*b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*sq
rt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh(d*x + c))) +
 8*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*
x + c)^8 - 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x
 + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 - 30*b^2*cosh(d*x + c)^2
 + 3*b^2)*sinh(d*x + c)^4 - 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d*x + c)^
5 - 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^
2*cosh(d*x + c)^6 - 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2 - b^2)*s
inh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 - 3*b^2*cosh(d*x + c)^5 + 3*b
^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a - b))*arct
an(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x
+ c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh
(d*x + c))*sqrt(b/(a - b))/b) + 2*(3*a^2 + 4*a*b)*cosh(d*x + c) - ((3*a^2 +
 4*a*b + 8*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)*s
inh(d*x + c)^7 + (3*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c)^8 - 4*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 -
3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c)^3 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(
3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 4*a*b + 8*b^2)*cosh
(d*x + c)^4 - 30*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 9*a^2 + 12*a*b +
 24*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 - 1
0*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c))*sinh(d*x + c)^3 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 4*(
```

```
7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 - 15*(3*a^2 + 4*a*b + 8*b^2)*cosh
(d*x + c)^4 + 9*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 4*a*b - 8
*b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + 8*b^2 + 8*((3*a^2 + 4*a*b + 8*b^2)*
cosh(d*x + c)^7 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 + 4*
a*b + 8*b^2)*cosh(d*x + c)^3 - (3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(
d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((3*a^2 + 4*a*b + 8*b^2)
*cosh(d*x + c)^8 + 8*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)*sinh(d*x + c)^7
+ (3*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c)^8 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c)^6 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 4*a*b -
 8*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - 3*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2 + 4*a*b +
 8*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 - 3
0*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 9*a^2 + 12*a*b + 24*b^2)*sinh(d
*x + c)^4 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 - 10*(3*a^2 + 4*a*
b + 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(
d*x + c)^3 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 4*a*
b + 8*b^2)*cosh(d*x + c)^6 - 15*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 9
*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x
+ c)^2 + 3*a^2 + 4*a*b + 8*b^2 + 8*((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^7
 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 + 4*a*b + 8*b^2)*co
sh(d*x + c)^3 - (3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(c
osh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(3*a^2 + 4*a*b)*cosh(d*x + c)^6 -
5*(11*a^2 + 4*a*b)*cosh(d*x + c)^4 - 3*(11*a^2 + 4*a*b)*cosh(d*x + c)^2 + 3
*a^2 + 4*a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^8 + 8*a^3*d*cosh(d*x + c)
*sinh(d*x + c)^7 + a^3*d*sinh(d*x + c)^8 - 4*a^3*d*cosh(d*x + c)^6 + 6*a^3*
d*cosh(d*x + c)^4 + 4*(7*a^3*d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^6 - 4
*a^3*d*cosh(d*x + c)^2 + 8*(7*a^3*d*cosh(d*x + c)^3 - 3*a^3*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(35*a^3*d*cosh(d*x + c)^4 - 30*a^3*d*cosh(d*x + c)^2
+ 3*a^3*d)*sinh(d*x + c)^4 + a^3*d + 8*(7*a^3*d*cosh(d*x + c)^5 - 10*a^3*d*
cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*d*cosh(
d*x + c)^6 - 15*a^3*d*cosh(d*x + c)^4 + 9*a^3*d*cosh(d*x + c)^2 - a^3*d)*si
nh(d*x + c)^2 + 8*(a^3*d*cosh(d*x + c)^7 - 3*a^3*d*cosh(d*x + c)^5 + 3*a^3*
d*cosh(d*x + c)^3 - a^3*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Unde
f/Unsigned Inf encountered in limitEvaluation time: 0.62Limit: Max order re
ached or unable to make series expansion Error: Bad Argument Value
```

maple [**A**] time = 0.13, size = 232, normalized size = 1.78

$$\frac{\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64da}-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{8d\,a^2}-\frac{b^3\arctan\left(\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2a+4b}{4\sqrt{ab-b^2}}\right)}{d\,a^3\sqrt{ab-b^2}}-\frac{1}{64da\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+-\ 8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/64/d/a*tanh(1/2*d*x+1/2*c)^4-1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/8/d/a^2*tanh
(1/2*d*x+1/2*c)^2*b-1/d/a^3*b^3/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+
1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))-1/64/d/a/tanh(1/2*d*x+1/2*c)^4+1/8/d/a
/tanh(1/2*d*x+1/2*c)^2+1/8/d/a^2/tanh(1/2*d*x+1/2*c)^2*b+3/8/d/a*ln(tanh(1/
2*d*x+1/2*c))+1/2/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))+1/d/a^3*ln(tanh(1/2*d*x+1
/2*c))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\,ae^{(7\,c)} + 4\,be^{(7\,c)}\right)e^{(7\,dx)} - \left(11\,ae^{(5\,c)} + 4\,be^{(5\,c)}\right)e^{(5\,dx)} - \left(11\,ae^{(3\,c)} + 4\,be^{(3\,c)}\right)e^{(3\,dx)} + \left(3\,ae^{c} + 4\,be^{c}\right)e^{(dx)}}{4\left(a^2de^{(8\,dx+8\,c)} - 4\,a^2de^{(6\,dx+6\,c)} + 6\,a^2de^{(4\,dx+4\,c)} - 4\,a^2de^{(2\,dx+2\,c)} + a^2d\right)} - \frac{\left(3\,a^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*((3*a*e^(7*c) + 4*b*e^(7*c))*e^(7*d*x) - (11*a*e^(5*c) + 4*b*e^(5*c))*e
^(5*d*x) - (11*a*e^(3*c) + 4*b*e^(3*c))*e^(3*d*x) + (3*a*e^c + 4*b*e^c)*e^(
d*x))/(a^2*d*e^(8*d*x + 8*c) - 4*a^2*d*e^(6*d*x + 6*c) + 6*a^2*d*e^(4*d*x +
 4*c) - 4*a^2*d*e^(2*d*x + 2*c) + a^2*d) - 1/8*(3*a^2 + 4*a*b + 8*b^2)*log(
(e^(d*x + c) + 1)*e^(-c))/(a^3*d) + 1/8*(3*a^2 + 4*a*b + 8*b^2)*log((e^(d*x
 + c) - 1)*e^(-c))/(a^3*d) - 32*integrate(1/16*(b^3*e^(3*d*x + 3*c) - b^3*e
^(d*x + c))/(a^3*b*e^(4*d*x + 4*c) + a^3*b + 2*(2*a^4*e^(2*c) - a^3*b*e^(2*
c))*e^(2*d*x)), x)
```

mupad [B] time = 5.66, size = 1639, normalized size = 12.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^5*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (exp(c + d*x)*(4*a*b + 3*a^2))/(4*a^3*d*(exp(2*c + 2*d*x) - 1)) - (atan((ex
p(d*x)*exp(c)*(243*a^12*(-a^6*d^2)^(1/2) + 18432*b^12*(-a^6*d^2)^(1/2) + 69
12*a^2*b^10*(-a^6*d^2)^(1/2) - 30720*a^3*b^9*(-a^6*d^2)^(1/2) - 26880*a^4*b
^8*(-a^6*d^2)^(1/2) - 24192*a^5*b^7*(-a^6*d^2)^(1/2) + 5024*a^6*b^6*(-a^6*d
^2)^(1/2) + 13408*a^7*b^5*(-a^6*d^2)^(1/2) + 17160*a^8*b^4*(-a^6*d^2)^(1/2)
 + 9540*a^9*b^3*(-a^6*d^2)^(1/2) + 4563*a^10*b^2*(-a^6*d^2)^(1/2) + 9216*a*
b^11*(-a^6*d^2)^(1/2) + 1134*a^11*b*(-a^6*d^2)^(1/2)))/(81*a^13*d*(64*a*b^3
 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 2304*a^3*b^10*d*(64*a*b^
3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) - 3840*a^6*b^7*d*(64*a*b^
3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) - 1440*a^7*b^6*d*(64*a*b^
3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) - 864*a^8*b^5*d*(64*a*b^3
 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 1600*a^9*b^4*d*(64*a*b^3
 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 1200*a^10*b^3*d*(64*a*b^
3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 945*a^11*b^2*d*(64*a*b^
3 + 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2) + 270*a^12*b*d*(64*a*b^3
+ 24*a^3*b + 9*a^4 + 64*b^4 + 64*a^2*b^2)^(1/2)))*(64*a*b^3 + 24*a^3*b + 9*
a^4 + 64*b^4 + 64*a^2*b^2)^(1/2))/(4*(-a^6*d^2)^(1/2)) - (6*exp(c + d*x))/(
a*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*
exp(c + d*x))/(a*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6
*d*x) + exp(8*c + 8*d*x) + 1)) - ((2*atan((b^3*exp(d*x)*exp(c)*(a^6*d^2*(a
- b))^(1/2)*(15*a^4*b + 9*a^5 - 48*b^5 + 40*a^3*b^2))/(2*a^3*d*(b^5)^(1/2)*
(6*a^5*b - 48*a*b^5 + 9*a^6 + 48*b^6 - 40*a^3*b^3 + 25*a^4*b^2))) + 2*atan(
(exp(d*x)*exp(c)*((4*(18*a^9*d*(b^5)^(1/2) - 96*a^4*d*(b^5)^(3/2) + 96*a^3*
b*d*(b^5)^(3/2) + 12*a^8*b*d*(b^5)^(1/2) - 80*a^6*b^3*d*(b^5)^(1/2) + 50*a^
```

```
7*b^2*d*(b^5)^(1/2)))/(a^8*b^4*(a - b)*(a^7*d^2 - a^6*b*d^2)^(1/2)*(a*b - a
^2)*(a^6*d^2*(a - b))^(1/2)*(15*a^4*b + 9*a^5 - 48*b^5 + 40*a^3*b^2)) + (2*
(40*a^3*b^5*(a^7*d^2 - a^6*b*d^2)^(1/2) - 48*b^8*(a^7*d^2 - a^6*b*d^2)^(1/2
) + 15*a^4*b^4*(a^7*d^2 - a^6*b*d^2)^(1/2) + 9*a^5*b^3*(a^7*d^2 - a^6*b*d^2
)^(1/2)))/(a^11*b*d*(a - b)*(a^7*d^2 - a^6*b*d^2)^(1/2)*(a*b - a^2)*(b^5)^(
1/2)*(6*a^5*b - 48*a*b^5 + 9*a^6 + 48*b^6 - 40*a^3*b^3 + 25*a^4*b^2))) + (2
*exp(3*c)*exp(3*d*x)*(40*a^3*b^5*(a^7*d^2 - a^6*b*d^2)^(1/2) - 48*b^8*(a^7*
d^2 - a^6*b*d^2)^(1/2) + 15*a^4*b^4*(a^7*d^2 - a^6*b*d^2)^(1/2) + 9*a^5*b^3
*(a^7*d^2 - a^6*b*d^2)^(1/2)))/(a^11*b*d*(a - b)*(a^7*d^2 - a^6*b*d^2)^(1/2
)*(a*b - a^2)*(b^5)^(1/2)*(6*a^5*b - 48*a*b^5 + 9*a^6 + 48*b^6 - 40*a^3*b^3
 + 25*a^4*b^2)))*((a^11*b*(a^7*d^2 - a^6*b*d^2)^(1/2))/4 + (a^9*b^3*(a^7*d^
2 - a^6*b*d^2)^(1/2))/4 - (a^10*b^2*(a^7*d^2 - a^6*b*d^2)^(1/2))/2)))*(b^5)
^(1/2))/(2*(a^7*d^2 - a^6*b*d^2)^(1/2)) - (exp(c + d*x)*(a - 4*b))/(2*a^2*d
*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.41 $\int \frac{\text{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d\sqrt{a-b}}+\frac{(2a+b)\coth^3(c+dx)}{3a^2d}-\frac{\left(a^2+ab+b^2\right)\coth(c+dx)}{a^3d}-\frac{\coth^5(c+dx)}{5ad}$$

```
[Out] -(a^2+a*b+b^2)*coth(d*x+c)/a^3/d+1/3*(2*a+b)*coth(d*x+c)^3/a^2/d-1/5*coth(d
*x+c)^5/a/d-b^3*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)/d/(a-b)^(1
/2)
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3187, 461, 208}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d\sqrt{a-b}}-\frac{\left(a^2+ab+b^2\right)\coth(c+dx)}{a^3d}+\frac{(2a+b)\coth^3(c+dx)}{3a^2d}-\frac{\coth^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[a - b]*d
)) - ((a^2 + a*b + b^2)*Coth[c + d*x])/(a^3*d) + ((2*a + b)*Coth[c + d*x]^3
)/(3*a^2*d) - Coth[c + d*x]^5/(5*a*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6} + \frac{-2a-b}{a^2x^4} + \frac{a^2+ab+b^2}{a^3x^2} + \frac{b^3}{a^3(-a+(a-b)x^2)}\right) dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a^2+ab+b^2)\coth(c+dx)}{a^3d} + \frac{(2a+b)\coth^3(c+dx)}{3a^2d} - \frac{\coth^5(c+dx)}{5ad} + \frac{b^3\,\text{Subst}\left(\right.}{}$$
$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a-b}\,d} - \frac{(a^2+ab+b^2)\coth(c+dx)}{a^3d} + \frac{(2a+b)\coth^3(c+dx)}{3a^2d}$$

Mathematica [**A**] time = 1.48, size = 155, normalized size = 1.41

$$-\frac{\text{csch}^2(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\sqrt{a}\sqrt{a-b}\coth(c+dx)\left(3a^2\text{csch}^4(c+dx)+8a^2-a(4a+5b)\text{csch}^2(\right.\right.}{30a^{7/2}d\sqrt{a-b}\left(a\,\text{csch}^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -1/30*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(15*b^3*ArcTanh[(Sqr
t[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a - b]*Coth[c + d*x]*(8*a^2
 + 10*a*b + 15*b^2 - a*(4*a + 5*b)*Csch[c + d*x]^2 + 3*a^2*Csch[c + d*x]^4)
))/(a^(7/2)*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))
```

fricas [**B**] time = 0.61, size = 4540, normalized size = 41.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(60*(a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 480*(a^2*b^2 - a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^7 + 60*(a^2*b^2 - a*b^3)*sinh(d*x + c)^8 - 120*(a^3*b
 + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^6 - 120*(a^3*b + a^2*b^2 - 2*a*b^3 - 14
*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 240*(14*(a^2*b^2 - a*
b^3)*cosh(d*x + c)^3 - 3*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c))*sinh(d*
x + c)^5 + 40*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4 + 40*(1
05*(a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3
- 45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*a^4
+ 8*a^3*b + 20*a^2*b^2 - 60*a*b^3 + 160*(21*(a^2*b^2 - a*b^3)*cosh(d*x + c)
^5 - 15*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^3 + (8*a^4 - a^3*b + 2*a^
2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 40*(4*a^4 + a^3*b + a^2*b
^2 - 6*a*b^3)*cosh(d*x + c)^2 + 40*(42*(a^2*b^2 - a*b^3)*cosh(d*x + c)^6 -
45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^4 - 4*a^4 - a^3*b - a^2*b^2 +
6*a*b^3 + 6*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 - 15*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 +
 b^3*sinh(d*x + c)^10 - 5*b^3*cosh(d*x + c)^8 + 10*b^3*cosh(d*x + c)^6 + 5*
(9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^8 + 40*(3*b^3*cosh(d*x + c)^3 -
 b^3*cosh(d*x + c))*sinh(d*x + c)^7 - 10*b^3*cosh(d*x + c)^4 + 10*(21*b^3*c
osh(d*x + c)^4 - 14*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^6 + 4*(63*b^3*
cosh(d*x + c)^5 - 70*b^3*cosh(d*x + c)^3 + 15*b^3*cosh(d*x + c))*sinh(d*x +
 c)^5 + 5*b^3*cosh(d*x + c)^2 + 10*(21*b^3*cosh(d*x + c)^6 - 35*b^3*cosh(d*
```

```
x + c)^4 + 15*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^4 + 40*(3*b^3*cosh(d
*x + c)^7 - 7*b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)^3 - b^3*cosh(d*x +
c))*sinh(d*x + c)^3 - b^3 + 5*(9*b^3*cosh(d*x + c)^8 - 28*b^3*cosh(d*x + c)
^6 + 30*b^3*cosh(d*x + c)^4 - 12*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2
 + 10*(b^3*cosh(d*x + c)^9 - 4*b^3*cosh(d*x + c)^7 + 6*b^3*cosh(d*x + c)^5
- 4*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)
*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(
d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2
*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3
+ (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
) + 80*(6*(a^2*b^2 - a*b^3)*cosh(d*x + c)^7 - 9*(a^3*b + a^2*b^2 - 2*a*b^3)
*cosh(d*x + c)^5 + 2*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^3
- (4*a^4 + a^3*b + a^2*b^2 - 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 -
 a^4*b)*d*cosh(d*x + c)^10 + 10*(a^5 - a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)
^9 + (a^5 - a^4*b)*d*sinh(d*x + c)^10 - 5*(a^5 - a^4*b)*d*cosh(d*x + c)^8 +
 5*(9*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5 - a^4*b)*d)*sinh(d*x + c)^8 +
10*(a^5 - a^4*b)*d*cosh(d*x + c)^6 + 40*(3*(a^5 - a^4*b)*d*cosh(d*x + c)^3
- (a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*(a^5 - a^4*b)*d*c
osh(d*x + c)^4 - 14*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + (a^5 - a^4*b)*d)*sinh
(d*x + c)^6 - 10*(a^5 - a^4*b)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - a^4*b)*d*co
sh(d*x + c)^5 - 70*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + 15*(a^5 - a^4*b)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - a^4*b)*d*cosh(d*x + c)^6 - 35*(
a^5 - a^4*b)*d*cosh(d*x + c)^4 + 15*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5
- a^4*b)*d)*sinh(d*x + c)^4 + 5*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + 40*(3*(a^
5 - a^4*b)*d*cosh(d*x + c)^7 - 7*(a^5 - a^4*b)*d*cosh(d*x + c)^5 + 5*(a^5 -
 a^4*b)*d*cosh(d*x + c)^3 - (a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ 5*(9*(a^5 - a^4*b)*d*cosh(d*x + c)^8 - 28*(a^5 - a^4*b)*d*cosh(d*x + c)^6
 + 30*(a^5 - a^4*b)*d*cosh(d*x + c)^4 - 12*(a^5 - a^4*b)*d*cosh(d*x + c)^2
+ (a^5 - a^4*b)*d)*sinh(d*x + c)^2 - (a^5 - a^4*b)*d + 10*((a^5 - a^4*b)*d*
cosh(d*x + c)^9 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^7 + 6*(a^5 - a^4*b)*d*cos
h(d*x + c)^5 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + (a^5 - a^4*b)*d*cosh(d*x
 + c))*sinh(d*x + c)), -1/15*(30*(a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 240*(a
^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + 30*(a^2*b^2 - a*b^3)*sinh(d
*x + c)^8 - 60*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^6 - 60*(a^3*b + a^
2*b^2 - 2*a*b^3 - 14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 1
20*(14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^3 - 3*(a^3*b + a^2*b^2 - 2*a*b^3)*co
sh(d*x + c))*sinh(d*x + c)^5 + 20*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cos
h(d*x + c)^4 + 20*(105*(a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*a^4 - a^3*b +
2*a^2*b^2 - 9*a*b^3 - 45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 16*a^4 + 4*a^3*b + 10*a^2*b^2 - 30*a*b^3 + 80*(21*(a^2*b^2 - a
*b^3)*cosh(d*x + c)^5 - 15*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^3 + (8
*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 20*(4*
a^4 + a^3*b + a^2*b^2 - 6*a*b^3)*cosh(d*x + c)^2 + 20*(42*(a^2*b^2 - a*b^3)
*cosh(d*x + c)^6 - 45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^4 - 4*a^4 -
 a^3*b - a^2*b^2 + 6*a*b^3 + 6*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 - 15*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c
)*sinh(d*x + c)^9 + b^3*sinh(d*x + c)^10 - 5*b^3*cosh(d*x + c)^8 + 10*b^3*c
osh(d*x + c)^6 + 5*(9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^8 + 40*(3*b^
3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c)^7 - 10*b^3*cosh(d*x +
c)^4 + 10*(21*b^3*cosh(d*x + c)^4 - 14*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x
+ c)^6 + 4*(63*b^3*cosh(d*x + c)^5 - 70*b^3*cosh(d*x + c)^3 + 15*b^3*cosh(d
*x + c))*sinh(d*x + c)^5 + 5*b^3*cosh(d*x + c)^2 + 10*(21*b^3*cosh(d*x + c)
^6 - 35*b^3*cosh(d*x + c)^4 + 15*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^4
 + 40*(3*b^3*cosh(d*x + c)^7 - 7*b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)^
3 - b^3*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 + 5*(9*b^3*cosh(d*x + c)^8 - 2
```

```
8*b^3*cosh(d*x + c)^6 + 30*b^3*cosh(d*x + c)^4 - 12*b^3*cosh(d*x + c)^2 + b
^3)*sinh(d*x + c)^2 + 10*(b^3*cosh(d*x + c)^9 - 4*b^3*cosh(d*x + c)^7 + 6*b
^3*cosh(d*x + c)^5 - 4*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x +
c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 4
0*(6*(a^2*b^2 - a*b^3)*cosh(d*x + c)^7 - 9*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh
(d*x + c)^5 + 2*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^3 - (4*
a^4 + a^3*b + a^2*b^2 - 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 - a^4*
b)*d*cosh(d*x + c)^10 + 10*(a^5 - a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)^9 +
(a^5 - a^4*b)*d*sinh(d*x + c)^10 - 5*(a^5 - a^4*b)*d*cosh(d*x + c)^8 + 5*(9
*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5 - a^4*b)*d)*sinh(d*x + c)^8 + 10*(a
^5 - a^4*b)*d*cosh(d*x + c)^6 + 40*(3*(a^5 - a^4*b)*d*cosh(d*x + c)^3 - (a^
5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*(a^5 - a^4*b)*d*cosh(d
*x + c)^4 - 14*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + (a^5 - a^4*b)*d)*sinh(d*x
+ c)^6 - 10*(a^5 - a^4*b)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - a^4*b)*d*cosh(d*
x + c)^5 - 70*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + 15*(a^5 - a^4*b)*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - a^4*b)*d*cosh(d*x + c)^6 - 35*(a^5 -
 a^4*b)*d*cosh(d*x + c)^4 + 15*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5 - a^4
*b)*d)*sinh(d*x + c)^4 + 5*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + 40*(3*(a^5 - a
^4*b)*d*cosh(d*x + c)^7 - 7*(a^5 - a^4*b)*d*cosh(d*x + c)^5 + 5*(a^5 - a^4*
b)*d*cosh(d*x + c)^3 - (a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(
9*(a^5 - a^4*b)*d*cosh(d*x + c)^8 - 28*(a^5 - a^4*b)*d*cosh(d*x + c)^6 + 30
*(a^5 - a^4*b)*d*cosh(d*x + c)^4 - 12*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + (a^
5 - a^4*b)*d)*sinh(d*x + c)^2 - (a^5 - a^4*b)*d + 10*((a^5 - a^4*b)*d*cosh(
d*x + c)^9 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^7 + 6*(a^5 - a^4*b)*d*cosh(d*x
 + c)^5 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + (a^5 - a^4*b)*d*cosh(d*x + c)
)*sinh(d*x + c))]
```

giac [B] time = 0.70, size = 213, normalized size = 1.94

$$-\frac{\frac{15\, b^{3} \arctan\left(\frac{b e^{(2\, dx+2\, c)}+2\, a-b}{2\, \sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}\, a^{3}} + \frac{2\left(15\, b^{2} e^{(8\, dx+8\, c)}-30\, ab e^{(6\, dx+6\, c)}-60\, b^{2} e^{(6\, dx+6\, c)}+80\, a^{2} e^{(4\, dx+4\, c)}+70\, ab e^{(4\, dx+4\, c)}+90\, b^{2} e^{(4\, dx+4\, c)}-40\, a^{2} e^{(2\, dx+2\, c)}- \cdots}{a^{3}\left(e^{(2\, dx+2\, c)}-1\right)^{5}}}{15\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*b^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(s
qrt(-a^2 + a*b)*a^3) + 2*(15*b^2*e^(8*d*x + 8*c) - 30*a*b*e^(6*d*x + 6*c) -
 60*b^2*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x + 4*c) + 70*a*b*e^(4*d*x + 4*c) +
 90*b^2*e^(4*d*x + 4*c) - 40*a^2*e^(2*d*x + 2*c) - 50*a*b*e^(2*d*x + 2*c) -
 60*b^2*e^(2*d*x + 2*c) + 8*a^2 + 10*a*b + 15*b^2)/(a^3*(e^(2*d*x + 2*c) -
1)^5))/d
```

maple [B] time = 0.14, size = 519, normalized size = 4.72

$$-\frac{\tanh^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{160 da}+\frac{5\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96 da}+\frac{\left(\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right) b}{24 d\, a^{2}}-\frac{5 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16 da}-\frac{3 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right) b}{8 d\, a^{2}}-\frac{b^{2} \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2 d\, a^{3}} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/160/d/a*tanh(1/2*d*x+1/2*c)^5+5/96/d/a*tanh(1/2*d*x+1/2*c)^3+1/24/d/a^2*
tanh(1/2*d*x+1/2*c)^3*b-5/16/d/a*tanh(1/2*d*x+1/2*c)-3/8/d/a^2*tanh(1/2*d*x
+1/2*c)*b-1/2/d/a^3*b^2*tanh(1/2*d*x+1/2*c)+1/d/a^3*b^3/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*
```

```
a)^(1/2))+1/d/a^3*b^4/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/a^3
*b^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/a^3*b^4/(-b*(a-b))^(1/2)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)+a-2*b)*a)^(1/2))-1/160/d/a/tanh(1/2*d*x+1/2*c)^5+5/96/d/a/tanh(1/2*d*x+1/
2*c)^3+1/24/d/a^2/tanh(1/2*d*x+1/2*c)^3*b-5/16/d/a/tanh(1/2*d*x+1/2*c)-3/8/
d*b/a^2/tanh(1/2*d*x+1/2*c)-1/2/d/a^3/tanh(1/2*d*x+1/2*c)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 1.21, size = 479, normalized size = 4.35

$$\frac{4\,b}{a^2\,d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{32}{5\,a\,d\,\left(5\,{\mathrm{e}}^{2\,c+2\,d\,x}-10\,{\mathrm{e}}^{4\,c+4\,d\,x}+10\,{\mathrm{e}}^{6\,c+6\,d\,x}-5\,{\mathrm{e}}^{8\,c+8\,d\,x}+{\mathrm{e}}^{10\,c+10\,d\,x}-1\right)}-\frac{}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^6*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (4*b)/(a^2*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - 32/(5*a*d*(5*ex
p(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*
d*x) + exp(10*c + 10*d*x) - 1)) - (2*b^2)/(a^3*d*(exp(2*c + 2*d*x) - 1)) -
(8*(4*a - b))/(3*a^2*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c +
 6*d*x) - 1)) - 16/(a*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*
c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (b^3*log((4*b^4*(2*a*b - b^2 + 8*a^2*
exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a^7*(a
- b)) - (8*b^4*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(13/2)
*(a - b)^(1/2))))/(2*a^(7/2)*d*(a - b)^(1/2)) - (b^3*log((4*b^4*(2*a*b - b^
2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x))
)/(a^7*(a - b)) + (8*b^4*(b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))
/(a^(13/2)*(a - b)^(1/2))))/(2*a^(7/2)*d*(a - b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.42 $\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^2d(a-b)^{3/2}}-\frac{a\tanh(c+dx)}{2bd(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{x}{b^2}$$

```
[Out] x/b^2-1/2*(2*a-3*b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/(a-b)^
(3/2)/b^2/d-1/2*a*tanh(d*x+c)/(a-b)/b/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.17, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 470, 522, 206, 208}

$$-\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^2d(a-b)^{3/2}}-\frac{a\tanh(c+dx)}{2bd(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] x/b^2 - (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/
(2*(a - b)^(3/2)*b^2*d) - (a*Tanh[c + d*x])/(2*(a - b)*b*d*(a - (a - b)*Tan
h[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\tanh(c+dx)}{2(a-b)bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+(a-2b)x^2}{(1-x^2)\left(a+(-a+b)x^2\right)}\,dx, x, \tan\right.}{2(a-b)bd}$$

$$= -\frac{a\tanh(c+dx)}{2(a-b)bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{b^2d} -$$

$$= \frac{x}{b^2} - \frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2(a-b)^{3/2}b^2d} - \frac{a\tanh(c+dx)}{2(a-b)bd\left(a-(a-b)\tanh^2(c+\right.}$$

Mathematica [A] time = 0.88, size = 99, normalized size = 0.97

$$-\frac{\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}} + \frac{ab\sinh(2(c+dx))}{(a-b)(2a+b\cosh(2(c+dx))-b)} - 2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/2*(-2*(c + d*x) + (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x
])/Sqrt[a]])/(a - b)^(3/2) + (a*b*Sinh[2*(c + d*x)])/((a - b)*(2*a - b + b*
Cosh[2*(c + d*x)])))/(b^2*d)
```

fricas [B] time = 1.05, size = 1772, normalized size = 17.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b - b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b - b^2)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a*b - b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b - b^2)*d*x + 4
*(2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x + c)^2 + 4*(6*(a*b -
b^2)*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*sinh(
d*x + c)^2 + ((2*a*b - 3*b^2)*cosh(d*x + c)^4 + 4*(2*a*b - 3*b^2)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (2*a*b - 3*b^2)*sinh(d*x + c)^4 + 2*(4*a^2 - 8*a*b +
 3*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a^2 - 8*
a*b + 3*b^2)*sinh(d*x + c)^2 + 2*a*b - 3*b^2 + 4*((2*a*b - 3*b^2)*cosh(d*x
+ c)^3 + (4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a -
b))*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*si
nh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2
+ 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)
```

```
^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a*b - b^2)*cosh(d*x +
 c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x +
c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(
d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2
+ 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3
+ (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*a*b + 8*(2*(a*b - b^2)*d
*x*cosh(d*x + c)^3 + (2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x +
 c))*sinh(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^4 + 4*(a*b^3 - b^4)*d*co
sh(d*x + c)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*sinh(d*x + c)^4 + 2*(2*a^2*b^
2 - 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^2
 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d + 4*((a
*b^3 - b^4)*d*cosh(d*x + c)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d*cosh(d*x + c)
)*sinh(d*x + c)), 1/2*(2*(a*b - b^2)*d*x*cosh(d*x + c)^4 + 8*(a*b - b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a*b - b^2)*d*x*sinh(d*x + c)^4 + 2*(a*
b - b^2)*d*x + 2*(2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x + c)^
2 + 2*(6*(a*b - b^2)*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - 3*a*b + b^2)*d*x + 2*
a^2 - a*b)*sinh(d*x + c)^2 - ((2*a*b - 3*b^2)*cosh(d*x + c)^4 + 4*(2*a*b -
3*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b - 3*b^2)*sinh(d*x + c)^4 + 2*
(4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - 3*b^2)*cosh(d*x + c
)^2 + 4*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^2 + 2*a*b - 3*b^2 + 4*((2*a*b -
3*b^2)*cosh(d*x + c)^3 + (4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x +
c))*sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh
(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a/(a - b))/a) + 2*a*b + 4*(2
*(a*b - b^2)*d*x*cosh(d*x + c)^3 + (2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a
*b)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^4 + 4*(a*b
^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*sinh(d*x + c)^4
 + 2*(2*a^2*b^2 - 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a*b^3 - b^4)*d*c
osh(d*x + c)^2 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 -
b^4)*d + 4*((a*b^3 - b^4)*d*cosh(d*x + c)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

giac [**A**] time = 5.59, size = 168, normalized size = 1.65

$$-\frac{\frac{\left(2a^2-3ab\right)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\left(ab^2-b^3\right)\sqrt{-a^2+ab}}-\frac{2\left(2a^2e^{(2dx+2c)}-abe^{(2dx+2c)}+ab\right)}{\left(ab^2-b^3\right)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{(2dx+2c)}+b\right)}-\frac{2(dx+c)}{b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - 3*a*b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 +
a*b))/((a*b^2 - b^3)*sqrt(-a^2 + a*b)) - 2*(2*a^2*e^(2*d*x + 2*c) - a*b*e^(
2*d*x + 2*c) + a*b)/((a*b^2 - b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c)
 - 2*b*e^(2*d*x + 2*c) + b)) - 2*(d*x + c)/b^2)/d
```

maple [**B**] time = 0.08, size = 798, normalized size = 7.82

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,b^2}-\frac{a\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)(a-b)}-\frac{\cdots}{db\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d*a/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3
-1/d*a/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+
```

```
1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)+1/d*a^2/b^2/(a-b)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2))+1/d*a^2/b/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1
/d*a^2/b^2/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a^2/b/(a-b)/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/2/d*a/b/(a-b)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2))-3/2/d*a/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*ar
ctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/2/d*a/b/
(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((
2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/2/d*a/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2))+1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(c+d\,x\right)^4}{{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.43 $\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2b^{3/2}d(a-b)^{3/2}}-\frac{a\cosh(c+dx)}{2bd(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

```
[Out] 1/2*(a-2*b)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/b^(3/2)/d-1
/2*a*cosh(d*x+c)/(a-b)/b/d/(a-b+b*cosh(d*x+c)^2)
```

Rubi [A] time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 385, 205}

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2b^{3/2}d(a-b)^{3/2}}-\frac{a\cosh(c+dx)}{2bd(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*(a - b)^(3/2)*b^
(3/2)*d) - (a*Cosh[c + d*x])/(2*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\cosh(c+dx)}{2(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(\right.}{2(a-b)bd}$$

$$= \frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}b^{3/2}d} - \frac{a\cosh(c+dx)}{2(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)}$$

Mathematica [C] time = 0.63, size = 141, normalized size = 1.57

$$\frac{-\frac{2a\sqrt{b}\cosh(c+dx)}{(a-b)(2a+b\cosh(2(c+dx))-b)} + \frac{(a-2b)\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{3/2}}}{2b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (((a - 2*b)*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] +
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/(a - b)^(3/2)
 - (2*a*Sqrt[b]*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(
2*b^(3/2)*d)
```

fricas [B] time = 0.58, size = 1889, normalized size = 20.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b - a*b^2)*sinh(d*x + c)^3 + ((a*b - 2*b^2)*cosh(
d*x + c)^4 + 4*(a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b - 2*b^2)*
sinh(d*x + c)^4 + 2*(2*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a*b - 2
*b^2)*cosh(d*x + c)^2 + 2*a^2 - 5*a*b + 2*b^2)*sinh(d*x + c)^2 + a*b - 2*b^
2 + 4*((a*b - 2*b^2)*cosh(d*x + c)^3 + (2*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*
(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 -
(2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x
+ c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x +
 c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*
x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 +
2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
(2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(a^2*b - a*b^2)*cosh(d*x +
 c) + 4*(a^2*b - a*b^2 + 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/
((a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 4*(a^2*b^3 - 2*a*b^4 + b^5)*
d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^3 - 2*a*b^4 + b^5)*d*sinh(d*x + c)
^4 + 2*(2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^
2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4
 - b^5)*d)*sinh(d*x + c)^2 + (a^2*b^3 - 2*a*b^4 + b^5)*d + 4*((a^2*b^3 - 2*
a*b^4 + b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*
```

```
cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 6*
(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2*b - a*b^2)*sinh(d*x
+ c)^3 - ((a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(a*b - 2*b^2)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 5*a*b + 2*b^2)*co
sh(d*x + c)^2 + 2*(3*(a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a^2 - 5*a*b + 2*b^2)
*sinh(d*x + c)^2 + a*b - 2*b^2 + 4*((a*b - 2*b^2)*cosh(d*x + c)^3 + (2*a^2
- 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2)*arctan(-1/2*
(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
+ (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x +
c))/sqrt(a*b - b^2)) + ((a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(a*b - 2*b^2)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 5*a
*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a^2 -
5*a*b + 2*b^2)*sinh(d*x + c)^2 + a*b - 2*b^2 + 4*((a*b - 2*b^2)*cosh(d*x +
c)^3 + (2*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2
)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + 2*
(a^2*b - a*b^2)*cosh(d*x + c) + 2*(a^2*b - a*b^2 + 3*(a^2*b - a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c))/((a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 4*
(a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^3 - 2*a*
b^4 + b^5)*d*sinh(d*x + c)^4 + 2*(2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*
cosh(d*x + c)^2 + 2*(3*(a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (2*a^3
*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d)*sinh(d*x + c)^2 + (a^2*b^3 - 2*a*b^4 +
 b^5)*d + 4*((a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^2 - 5*a
^2*b^3 + 4*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[66,-29]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[-
21,2]Warning, need to choose a branch for the root of a polynomial with par
ameters. This might be wrong.The choice was done assuming [a,b]=[15,2]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-92,94]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[44,-86]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.The choice was done assuming [a,b]=[-27,-68]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [a,b]=[-70,50]Warning, need to choose a branch for
 the root of a polynomial with parameters. This might be wrong.The choice w
as done assuming [a,b]=[-63,-1]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[91,-7]Undef/Unsigned Inf encountered in limitEvaluation tim
e: 1.77Limit: Max order reached or unable to make series expansion Error: B
ad Argument Value
```

maple [B] time = 0.06, size = 341, normalized size = 3.79

$$\frac{16a\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(16ab-16b^2\right)\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)}-\frac{}{d\left(16ab-16b^2\right)\left(\left(\tanh^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 16/d/(16*a*b-16*b^2)/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*t
anh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)^2-32/d/(16*a*b-16*b^2)/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
*tanh(1/2*d*x+1/2*c)^2*b-16/d/(16*a*b-16*b^2)/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a+8/d/(16*a*b-16*b^2)/(a
*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2
))*a-16/d/(16*a*b-16*b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)
^2*a-2*a+4*b)/(a*b-b^2)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ae^{(3\,dx+3\,c)}+ae^{(dx+c)}}{ab^2d-b^3d+\left(ab^2de^{(4\,c)}-b^3de^{(4\,c)}\right)e^{(4\,dx)}+2\left(2\,a^2bde^{(2\,c)}-3\,ab^2de^{(2\,c)}+b^3de^{(2\,c)}\right)e^{(2\,dx)}}+\frac{1}{8}\int\frac{}{ab^2-b^3+\left(ab^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(a*e^(3*d*x + 3*c) + a*e^(d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(4*c) -
b^3*d*e^(4*c))*e^(4*d*x) + 2*(2*a^2*b*d*e^(2*c) - 3*a*b^2*d*e^(2*c) + b^3*d
*e^(2*c))*e^(2*d*x)) + 1/8*integrate(8*((a*e^(3*c) - 2*b*e^(3*c))*e^(3*d*x)
 - (a*e^c - 2*b*e^c)*e^(d*x))/(a*b^2 - b^3 + (a*b^2*e^(4*c) - b^3*e^(4*c))*
e^(4*d*x) + 2*(2*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) + b^3*e^(2*c))*e^(2*d*x)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(c+d\,x\right)^3}{{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.44 $\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=84

$$\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,d(a-b)^{3/2}}$$

```
[Out] 1/2*cosh(d*x+c)*sinh(d*x+c)/(a-b)/d/(a+b*sinh(d*x+c)^2)-1/2*arctanh((a-b)^(
1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(3/2)/d/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3173, 12, 3181, 208}

$$\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a - b)^(3/2)*d) +
 (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\int \frac{a}{a+b\sinh^2(c+dx)}\,dx}{2a(a-b)}$$
$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\int \frac{1}{a+b\sinh^2(c+dx)}\,dx}{2(a-b)}$$
$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2(a-b)d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a-b)^{3/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.44, size = 81, normalized size = 0.96

$$\frac{\frac{\sinh(2(c+dx))}{(a-b)(2a+b\cosh(2(c+dx))-b)} - \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{3/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-(ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a - b)^(3/2))) +
Sinh[2*(c + d*x)]/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*d)
```

fricas [B] time = 0.53, size = 1523, normalized size = 18.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*
(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b
+ a*b^2)*sinh(d*x + c)^2 + (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(
d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b
^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x +
 c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^
2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)
^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b
^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(
d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
 - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/((a^3
*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - 2*a^2*b^3 + a*b^
4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*sinh(d
*x + c)^4 + 2*(2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 +
 2*(3*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (2*a^4*b - 5*a^3*b^
2 + 4*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*d
 + 4*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (2*a^4*b - 5*a^3*b^
2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b - 2*a
*b^2 + 2*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a
```

```
*b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x +
c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sin
h(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 +
 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(
-a^2 + a*b)/(a^2 - a*b)))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4
+ 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^
2 - 2*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(2*a^4*b - 5*a^3*b^2 + 4*a^2*b
^3 - a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d
*x + c)^2 + (2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 +
(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d + 4*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d
*x + c)^3 + (2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh
(d*x + c))]
```

giac [A] time = 2.12, size = 135, normalized size = 1.61

$$-\frac{\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,(a-b)}+\frac{2\left(2\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b\right)}{\left(ab-b^2\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
 + a*b)*(a - b)) + 2*(2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)/((a*b -
b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)))/
d
```

maple [B] time = 0.07, size = 428, normalized size = 5.10

$$\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)(a-b)}+\frac{}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)+1/2
/d/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2))*b-1/2/d/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/(a-b
)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*
d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(c+d\,x\right)^2}{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.45 $\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=81

$$\frac{\cosh(c+dx)}{2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2\sqrt{b}\,d(a-b)^{3/2}}$$

```
[Out] 1/2*cosh(d*x+c)/(a-b)/d/(a-b+b*cosh(d*x+c)^2)+1/2*arctan(cosh(d*x+c)*b^(1/2
)/(a-b)^(1/2))/(a-b)^(3/2)/d/b^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 199, 205}

$$\frac{\cosh(c+dx)}{2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2\sqrt{b}\,d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*(a - b)^(3/2)*Sqrt[b]*d) + C
osh[c + d*x]/(2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)}{2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{2(a-b)d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}\sqrt{b}\,d} + \frac{\cosh(c+dx)}{2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)}$$

Mathematica [C] time = 0.34, size = 130, normalized size = 1.60

$$\frac{\frac{2\cosh(c+dx)}{(a-b)(2a+b\cosh(2(c+dx))-b)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{b}\,(a-b)^{3/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqr
t[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/((a - b)^(3/2)*Sqrt[b]) +
 (2*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*d)
```

fricas [B] time = 0.60, size = 1628, normalized size = 20.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b - b^2)*cosh(d*x + c)^3 + 12*(a*b - b^2)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a*b - b^2)*sinh(d*x + c)^3 + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2
*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (
2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-a*b + b^2)*log((b*cosh(d*x
 + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a -
3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh
(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d
*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh
(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*
a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(
a*b - b^2)*cosh(d*x + c) + 4*(3*(a*b - b^2)*cosh(d*x + c)^2 + a*b - b^2)*si
nh(d*x + c))/((a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 - 2*
a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d*
sinh(d*x + c)^4 + 2*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*d*cosh(d*x + c)^2
 + 2*(3*(a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (2*a^3*b - 5*a^2*b^2
+ 4*a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*d + 4*((a^2
*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a*b - b^2)*cosh(d*x + c)^3 +
6*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a*b - b^2)*sinh(d*x + c)^3
 + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)
```

```
^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d
*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) +
 b)*sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh
(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x
 + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - (b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d
*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d
*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(a*b - b^2)*arc
tan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + 2*(a*b
- b^2)*cosh(d*x + c) + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + a*b - b^2)*sinh(d
*x + c))/((a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 - 2*a*b^
3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d*sinh
(d*x + c)^4 + 2*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*d*cosh(d*x + c)^2 + 2
*(3*(a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (2*a^3*b - 5*a^2*b^2 + 4*
a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*d + 4*((a^2*b^2
 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)
*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[66,-29]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[-
21,2]Warning, need to choose a branch for the root of a polynomial with par
ameters. This might be wrong.The choice was done assuming [a,b]=[15,2]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-45,5]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[89,-20]Undef/Unsigned Inf enc
ountered in limitEvaluation time: 1.08Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

maple [B] time = 0.05, size = 256, normalized size = 3.16

$$-\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)(a-b)}+\frac{}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\text{ta}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2+2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)/a*tanh(1/2*d*x+1/2*c)^
2*b+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)/(a-b)+1/2/d/(a-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1
/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{(3\,dx+3\,c)}+e^{(dx+c)}}{abd-b^2d+\left(abde^{(4\,c)}-b^2de^{(4\,c)}\right)e^{(4\,dx)}+2\left(2\,a^2de^{(2\,c)}-3\,abde^{(2\,c)}+b^2de^{(2\,c)}\right)e^{(2\,dx)}}+\frac{1}{2}\int\frac{}{ab-b^2+\left(abe^{(4\,c)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) + e^(d*x + c))/(a*b*d - b^2*d + (a*b*d*e^(4*c) - b^2*d*e^(
4*c))*e^(4*d*x) + 2*(2*a^2*d*e^(2*c) - 3*a*b*d*e^(2*c) + b^2*d*e^(2*c))*e^(
2*d*x)) + 1/2*integrate(2*(e^(3*d*x + 3*c) - e^(d*x + c))/(a*b - b^2 + (a*b
*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^2*e^(2*c) - 3*a*b*e^(2*c) + b^2*
e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(c+d\,x\right)}{{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.46 $\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*(2*a-b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(3/2)/d-
1/2*b*cosh(d*x+c)*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3184, 12, 3181, 208}

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(-2),x]
```

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)
^(3/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*a*(a - b)*d*(a + b*Sinh[c +
d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\int \frac{-2a+b}{a+b\sinh^2(c+dx)}\,dx}{2a(a-b)}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{(2a-b)\int \frac{1}{a+b\sinh^2(c+dx)}\,dx}{2a(a-b)}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{(2a-b)\,\mathrm{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+\right.}{2a(a-b)d}$$

$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d} - \frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)}$$

Mathematica [**A**] time = 0.30, size = 96, normalized size = 1.01

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}} - \frac{b\sinh(2(c+dx))}{2ad(a-b)(2a+b\cosh(2(c+dx))-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-2),x]
```

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)
^(3/2)*d) - (b*Sinh[2*(c + d*x)])/(2*a*(a - b)*d*(2*a - b + b*Cosh[2*(c + d
*x)]))
```

fricas [**B**] time = 0.76, size = 1617, normalized size = 17.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*(
2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b +
 a*b^2)*sinh(d*x + c)^2 + ((2*a*b - b^2)*cosh(d*x + c)^4 + 4*(2*a*b - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b - b^2)*sinh(d*x + c)^4 + 2*(4*a^2 -
4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*a^2 -
 4*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(d*x + c
)^3 + (4*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*l
og((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*
x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a
*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 +
(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b
*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 +
2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x +
c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))
/((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b - 2*a^3*b^2 +
a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*d*
sinh(d*x + c)^4 + 2*(2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)
^2 + 2*(3*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (2*a^5 - 5*a^4*
b + 4*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)
*d + 4*((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^5 - 5*a^4*b
+ 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a^2*b - 2*a*
```

```
b^2 + 2*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a*
b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c
)^2 - ((2*a*b - b^2)*cosh(d*x + c)^4 + 4*(2*a*b - b^2)*cosh(d*x + c)*sinh(d
*x + c)^3 + (2*a*b - b^2)*sinh(d*x + c)^4 + 2*(4*a^2 - 4*a*b + b^2)*cosh(d*
x + c)^2 + 2*(3*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*a*b + b^2)*sinh(d
*x + c)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(d*x + c)^3 + (4*a^2 - 4*a*b
 + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)
*sqrt(-a^2 + a*b)/(a^2 - a*b)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x +
 c)^4 + 4*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^4*b - 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^5 - 5*a^4*b + 4*a^3
*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*co
sh(d*x + c)^2 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2
+ (a^4*b - 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh
(d*x + c)^3 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh
(d*x + c))]
```

giac [A] time = 0.42, size = 144, normalized size = 1.52

$$\frac{\frac{(2a-b)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{(a^2-ab)\sqrt{-a^2+ab}}+\frac{2\left(2ae^{(2dx+2c)}-be^{(2dx+2c)}+b\right)}{(a^2-ab)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{(2dx+2c)}+b\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(
(a^2 - a*b)*sqrt(-a^2 + a*b)) + 2*(2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c)
+ b)/((a^2 - a*b)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x +
 2*c) + b)))/d
```

maple [B] time = 0.09, size = 749, normalized size = 7.88

$$-\frac{b\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)a\left(a-b\right)}-\frac{}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\mathrm{t}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)*b/a/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*b/a/(a-b)*tanh(1/2*d*x+1
/2*c)-1/d/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/(a-b)/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2))*b+1/d/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(a-
b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b+1/2/d/(a-b)*b/a/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2))+1/2/d/(a-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^
(1/2))*b^2-1/2/d/(a-b)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/(a-b)/a/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(1/(a + b*sinh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.47 $\int \frac{\text{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{b}\,(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2d(a-b)^{3/2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{b\cosh(c+dx)}{2ad(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

```
[Out] -arctanh(cosh(d*x+c))/a^2/d-1/2*b*cosh(d*x+c)/a/(a-b)/d/(a-b+b*cosh(d*x+c)^
2)-1/2*(3*a-2*b)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^2/(a-b)^
(3/2)/d
```

Rubi [A] time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3186, 414, 522, 206, 205}

$$-\frac{\sqrt{b}\,(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2d(a-b)^{3/2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{b\cosh(c+dx)}{2ad(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*a^2*(
a - b)^(3/2)*d) - ArcTanh[Cosh[c + d*x]]/(a^2*d) - (b*Cosh[c + d*x])/(2*a*(
a - b)*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh(c+dx)}{2a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{-2a+b+bx^2}{(1-x^2)\left(a-b+bx^2\right)}\,dx, x, \cosh(c$$

$$= -\frac{b\cosh(c+dx)}{2a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{a^2d} -$$

$$= -\frac{(3a-2b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{b\cos}{2a(a-b)d\left(a-\right.}$$

Mathematica [C] time = 0.66, size = 176, normalized size = 1.60

$$\frac{-\frac{2ab\cosh(c+dx)}{(a-b)(2a+b\cosh(2(c+dx))-b)} + \frac{\sqrt{b}(2b-3a)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{b}(2b-3a)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + 2\log\left(\tanh\left(\frac{1}{2}\right.\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((Sqrt[b]*(-3*a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2]])/Sqrt[
a - b]])/(a - b)^(3/2) + (Sqrt[b]*(-3*a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*
Tanh[(c + d*x)/2]])/Sqrt[a - b]])/(a - b)^(3/2) - (2*a*b*Cosh[c + d*x])/((a
- b)*(2*a - b + b*Cosh[2*(c + d*x)])) + 2*Log[Tanh[(c + d*x)/2]])/(2*a^2*d)
```

fricas [B] time = 0.72, size = 2529, normalized size = 22.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b
*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) - ((3*a*b - 2*b^2)*cosh(d*x + c)^4 +
 4*(3*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - 2*b^2)*sinh(d*x
 + c)^4 + 2*(6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*
cosh(d*x + c)^2 + 6*a^2 - 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a*b - 2*b^2 +
4*((3*a*b - 2*b^2)*cosh(d*x + c)^3 + (6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)
*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2
*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a
- b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh
(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a -
 b)) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d
```

```
*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x
 + c) + b)) + 4*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cos
h(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh
(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b +
 b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
- 4*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^2
 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2
+ a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b + b^2)*cosh(d
*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*a*b*c
osh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 +
 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b - a^2*b^2)*d*
sinh(d*x + c)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a
^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + (2*a^4 - 3*a^3*b + a^2*b^2)*d)*sinh(d*x
 + c)^2 + (a^3*b - a^2*b^2)*d + 4*((a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 + (2
*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a*b*cosh
(d*x + c)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 +
 2*a*b*cosh(d*x + c) + ((3*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(3*a*b - 2*b^2)
*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(6*a^2
 - 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*cosh(d*x + c)^2 +
6*a^2 - 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a*b - 2*b^2 + 4*((3*a*b - 2*b^2)
*cosh(d*x + c)^3 + (6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh(d*x + c))) -
 ((3*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(3*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (3*a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(6*a^2 - 7*a*b + 2*b^2)*cosh
(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*cosh(d*x + c)^2 + 6*a^2 - 7*a*b + 2*b^2)
*sinh(d*x + c)^2 + 3*a*b - 2*b^2 + 4*((3*a*b - 2*b^2)*cosh(d*x + c)^3 + (6*
a^2 - 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a - b))*arctan(1
/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)
^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x
 + c))*sqrt(b/(a - b))/b) + 2*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*
a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a
*b + b^2)*sinh(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2
*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(
d*x + c) + 1) - 2*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*c
osh(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*si
nh(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b
 + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1
) + 2*(3*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^3*b - a^2*b^2)*d*cos
h(d*x + c)^4 + 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b
 - a^2*b^2)*d*sinh(d*x + c)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x +
c)^2 + 2*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + (2*a^4 - 3*a^3*b + a^2*b^
2)*d)*sinh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 4*((a^3*b - a^2*b^2)*d*cosh(d
*x + c)^3 + (2*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
```

```
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[66,-29]Undef/Unsigned Inf encountered in limitLimit: Max order reach
ed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.12, size = 350, normalized size = 3.18

$$\frac{\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)(a-b)\,a}-\frac{}{d\,a^2\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)/(a-b)/a*tanh(1/2*d*x+1/2*c)^2*b-2/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4
*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*
x+1/2*c)^2-1/d/a*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)/(a-b)-3/2/d/a*b/(a-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2
*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))+1/d/a^2*b^2/(a-b)/(a*b-b
^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))+1
/d/a^2*ln(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{be^{(3\,dx+3\,c)}+be^{(dx+c)}}{a^2bd-ab^2d+\left(a^2bde^{(4\,c)}-ab^2de^{(4\,c)}\right)e^{(4\,dx)}+2\left(2\,a^3de^{(2\,c)}-3\,a^2bde^{(2\,c)}+ab^2de^{(2\,c)}\right)e^{(2\,dx)}}-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{\cdots}\right.}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(4*c)
- a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(2*a^3*d*e^(2*c) - 3*a^2*b*d*e^(2*c) + a*b
^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log((e^(
d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) - 2*b^2*e^(
3*c))*e^(3*d*x) - (3*a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^3*b - a^2*b^2 + (a^3*
b*e^(4*c) - a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 3*a^3*b*e^(2*c)
 + a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(c+d\,x\right)\,{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.48 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=142

$$-\frac{b(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a-b)^{3/2}}+\frac{\left(2a^2-4ab+3b^2\right)\tanh(c+dx)}{2a^2d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

```
[Out] -1/2*(4*a-3*b)*b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a-b)^(3/
2)/d-coth(d*x+c)/a/d/(a-(a-b)*tanh(d*x+c)^2)+1/2*(2*a^2-4*a*b+3*b^2)*tanh(d
*x+c)/a^2/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3187, 462, 385, 208}

$$\frac{\left(2a^2-4ab+3b^2\right)\tanh(c+dx)}{2a^2d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{b(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a-b)^{3/2}}-\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((4*a - 3*b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(5/2)*(a
 - b)^(3/2)*d) - Coth[c + d*x]/(a*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((2*a^
2 - 4*a*b + 3*b^2)*Tanh[c + d*x])/(2*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d*
x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a-3b+ax^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{ad}$$

$$= -\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\left(2a^2-4ab+3b^2\right)\tanh(c+dx)}{2a^2(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{((4}{}$$

$$= -\frac{(4a-3b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a-b)^{3/2}d} - \frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{(2}{2a^2(a}$$

Mathematica [A] time = 0.82, size = 170, normalized size = 1.20

$$-\frac{\text{csch}^5(c+dx)(2a+b\cosh(2(c+dx))-b)\left(2\sqrt{a}\,\sqrt{a-b}\,\cosh(c+dx)\left(4a^2+b(2a-3b)\cosh(2(c+dx))-6a\right.\right.}{16a^{5/2}d(a-b)^{3/2}\left(a\,\text{csch}^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/16*((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^5*(2*Sqrt[a]*Sqrt[a -
b]*Cosh[c + d*x]*(4*a^2 - 6*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*(c + d*x)])
- 2*b*(-4*a + 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b +
b*Cosh[2*(c + d*x)])*Sinh[c + d*x]))/(a^(5/2)*(a - b)^(3/2)*d*(b + a*Csch[c
 + d*x]^2)^2)
```

fricas [B] time = 0.95, size = 2988, normalized size = 21.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 16*(4*a^3*b - 7*
a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(4*a^3*b - 7*a^2*b^2 +
 3*a*b^3)*sinh(d*x + c)^4 + 8*a^3*b - 20*a^2*b^2 + 12*a*b^3 + 8*(4*a^4 - 11
*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d*x + c)^2 + 8*(4*a^4 - 11*a^3*b + 10*a
^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 - ((4*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 6*(4*a*b^2 - 3*b^3)*cosh(
d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - 3*b^3)*sinh(d*x + c)^6 + (16*a^2*b -
24*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + (16*a^2*b - 24*a*b^2 + 9*b^3 + 15*(4*a*
b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*a*b^2 - 3*b^3)*cosh
(d*x + c)^3 + (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
- 4*a*b^2 + 3*b^3 - (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2 + (15*(4*
a*b^2 - 3*b^3)*cosh(d*x + c)^4 - 16*a^2*b + 24*a*b^2 - 9*b^3 + 6*(16*a^2*b
- 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - 3*b^
3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^3 - (16*
a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log
((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x
```

```
+ c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
 - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2
*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(
d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*c
osh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*
(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)
^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) +
16*((4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 11*a^3*b + 1
0*a^2*b^2 - 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 2*a^4*b^2 + a^
3*b^3)*d*cosh(d*x + c)^6 + 6*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^5 + (a^5*b - 2*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^6 + (4*a^6
- 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^4 + (15*(a^5*b - 2*a^4
*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*
b^3)*d)*sinh(d*x + c)^4 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cos
h(d*x + c)^2 + 4*(5*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 + (4*a^
6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (
15*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 + 6*(4*a^6 - 11*a^5*b +
10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^2 - (4*a^6 - 11*a^5*b + 10*a^4*b^2
- 3*a^3*b^3)*d)*sinh(d*x + c)^2 - (a^5*b - 2*a^4*b^2 + a^3*b^3)*d + 2*(3*(a
^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 + 2*(4*a^6 - 11*a^5*b + 10*a^
4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^3 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a
^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(4*a^3*b - 7*a^2*b^2 + 3*a
*b^3)*cosh(d*x + c)^4 + 8*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sin
h(d*x + c)^3 + 2*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^4 + 4*a^3*b
- 10*a^2*b^2 + 6*a*b^3 + 4*(4*a^4 - 11*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d
*x + c)^2 + 4*(4*a^4 - 11*a^3*b + 10*a^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 7*a^2
*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((4*a*b^2 - 3*b^3)*cosh(
d*x + c)^6 + 6*(4*a*b^2 - 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 -
 3*b^3)*sinh(d*x + c)^6 + (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + (
16*a^2*b - 24*a*b^2 + 9*b^3 + 15*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(5*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^3 + (16*a^2*b - 24*a*b^2 +
9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a*b^2 + 3*b^3 - (16*a^2*b - 24*a*
b^2 + 9*b^3)*cosh(d*x + c)^2 + (15*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^4 - 16*a
^2*b + 24*a*b^2 - 9*b^3 + 6*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 24
*a*b^2 + 9*b^3)*cosh(d*x + c)^3 - (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*co
sh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(
a^2 - a*b)) + 8*((4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 + (4*a^4 -
 11*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 2
*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 6*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*c
osh(d*x + c)*sinh(d*x + c)^5 + (a^5*b - 2*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c
)^6 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^4 + (15*(
a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (4*a^6 - 11*a^5*b + 10*a^4
*b^2 - 3*a^3*b^3)*d)*sinh(d*x + c)^4 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a
^3*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x +
 c)^3 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d
*x + c)^3 + (15*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 + 6*(4*a^6
- 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^2 - (4*a^6 - 11*a^5*b
+ 10*a^4*b^2 - 3*a^3*b^3)*d)*sinh(d*x + c)^2 - (a^5*b - 2*a^4*b^2 + a^3*b^3
)*d + 2*(3*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 + 2*(4*a^6 - 11*
a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^3 - (4*a^6 - 11*a^5*b + 10*
a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [A] time = 0.82, size = 229, normalized size = 1.61

$$-\frac{\frac{\left(4\,ab-3\,b^{2}\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\left(a^{3}-a^{2}b\right)\sqrt{-a^{2}+ab}}+\frac{2\left(4\,abe^{(4\,dx+4\,c)}-3\,b^{2}e^{(4\,dx+4\,c)}+8\,a^{2}e^{(2\,dx+2\,c)}-14\,abe^{(2\,dx+2\,c)}+6\,b^{2}e^{(2\,dx+2\,c)}+2\,ab-3\,b^{2}\right)}{\left(a^{3}-a^{2}b\right)\left(be^{(6\,dx+6\,c)}+4\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-4\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}-b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((4*a*b - 3*b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 +
a*b))/((a^3 - a^2*b)*sqrt(-a^2 + a*b)) + 2*(4*a*b*e^(4*d*x + 4*c) - 3*b^2*e
^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 14*a*b*e^(2*d*x + 2*c) + 6*b^2*e^(
2*d*x + 2*c) + 2*a*b - 3*b^2)/((a^3 - a^2*b)*(b*e^(6*d*x + 6*c) + 4*a*e^(4*
d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - 4*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c
) - b)))/d
```

maple [B] time = 0.13, size = 810, normalized size = 5.70

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}+\frac{b^2\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)(a-b)}+\frac{}{d\,a^2\left(\left(\mathrm{t}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/a^2*tanh(1/2*d*x+1/2*c)+1/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3
+1/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)+2/d/(a-b)*b/a/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2))+2/d/(a-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2-2
/d/(a-b)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d/(a-b)/a/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2))*b^2-3/2/d/a^2*b^2/(a-b)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2))-3/2/d/a^2*b^3/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/2/
d/a^2*b^2/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/2/d/a^2*b^3/(a-b)/(-b*(a-b)
)^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a^2/tanh(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(c+d\,x\right)^2\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.49 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2} dx$

Optimal. Leaf size=161

$$\frac{b^{3/2}(5a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3d(a-b)^{3/2}}+\frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{b(a-2b)\cosh(c+dx)}{2a^2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)}-\frac{c}{2a}$$

```
[Out] 1/2*(5*a-4*b)*b^(3/2)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/a^3/(a-b)^(3/
2)/d+1/2*(a+4*b)*arctanh(cosh(d*x+c))/a^3/d-1/2*(a-2*b)*b*cosh(d*x+c)/a^2/(
a-b)/d/(a-b+b*cosh(d*x+c)^2)-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a-b+b*cosh(d*
x+c)^2)
```

Rubi [A] time = 0.27, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}(5a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3d(a-b)^{3/2}}-\frac{b(a-2b)\cosh(c+dx)}{2a^2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)}+\frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{c}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*a^3*(a
 - b)^(3/2)*d) + ((a + 4*b)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) - ((a - 2*b)*
b*Cosh[c + d*x])/(2*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)) - (Coth[c +
d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+bx^2)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{a+b+3bx^2}{(1-x^2)(a-b+bx^2)^2}\,dx, x, \cosh(c+dx)\right)}{2ad}$$

$$= -\frac{(a-2b)b\cosh(c+dx)}{2a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{(a-2b)b\cosh(c+dx)}{2a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)} + \frac{((5a-}{}$$

$$= \frac{(5a-4b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3(a-b)^{3/2}d} + \frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d} - \frac{(a-}{2a^2(a-b)d}$$

Mathematica [C] time = 1.37, size = 350, normalized size = 2.17

$$\frac{\operatorname{csch}^3(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\frac{4b^{3/2}(5a-4b)\operatorname{csch}(c+dx)(2a+b\cosh(2(c+dx))-b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{4b^{3/2}(5a-}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^3*((8*a*b^2*Coth[c + d*x])/(
a - b) + (4*(5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/
2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x])/(a - b)^(3/
2) + (4*(5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/
Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x])/(a - b)^(3/2) -
 a*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[(c + d*x)/2]^2*Csch[c + d*x] - 4*(a
 + 4*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Log[Tanh[(c + d*x)/2]
```

```
] - a*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Sech[(c + d*x)/2]^2))/(
32*a^3*d*(b + a*Csch[c + d*x]^2)^2)
```

fricas [B] time = 0.76, size = 8059, normalized size = 50.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b - 2*a*b^2)*cosh(d*x + c)^7 + 28*(a^2*b - 2*a*b^2)*cosh(d*x
+ c)*sinh(d*x + c)^6 + 4*(a^2*b - 2*a*b^2)*sinh(d*x + c)^7 + 4*(4*a^3 - 5*a
^2*b + 2*a*b^2)*cosh(d*x + c)^5 + 4*(4*a^3 - 5*a^2*b + 2*a*b^2 + 21*(a^2*b
- 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b - 2*a*b^2)*cosh(
d*x + c)^3 + (4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 4
*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 4*(35*(a^2*b - 2*a*b^2)*cosh
(d*x + c)^4 + 4*a^3 - 5*a^2*b + 2*a*b^2 + 10*(4*a^3 - 5*a^2*b + 2*a*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(a^2*b - 2*a*b^2)*cosh(d*x + c)^5 +
10*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 - 5*a^2*b + 2*a*b
^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((5*a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8
*(5*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2 - 4*b^3)*sinh(d
*x + c)^8 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 4*(5*a^2*b - 9*
a*b^2 + 4*b^3 + 7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7
*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^5 - 2*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^4 +
 2*(35*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 20*a^2*b + 31*a*b^2 - 12*b^3 + 3
0*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*
b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)
^3 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a*b^
2 - 4*b^3 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(5*a*b^2 -
 4*b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 +
5*a^2*b - 9*a*b^2 + 4*b^3 - 3*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*((5*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - 9*
a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x +
c)^3 + (5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a
 - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a +
 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*s
inh(d*x + c) + 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*
x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cos
h(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*co
sh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*co
sh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(a^2*b - 2
*a*b^2)*cosh(d*x + c) - 2*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8*(a
^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b + 3*a*b^2 -
4*b^3)*sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^
6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^3
 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(
4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^4 + 2*(35*(a^2*b + 3*a*b
^2 - 4*b^3)*cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b^2 - 12*b^3 + 30*(a^3
 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2*
b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)
*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c))*sin
h(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3
)*cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(a^
3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + a^3 + 2*a^2*b - 7*a*b^2 +
4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(a^3 + 2*a^2*b - 7
```

*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cos
h(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*((a^2*b + 3*a*b^2 - 4*b^3)*c
osh(d*x + c)^8 + 8*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7
+ (a^2*b + 3*a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 +
4*b^3)*cosh(d*x + c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*
a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4
*b^3)*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 - 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^4 +
 2*(35*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b
^2 - 12*b^3 + 30*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(a^3 + 2*a^2
*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^
3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^
2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cos
h(d*x + c)^6 + 15*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + a^3 +
 2*a^2*b - 7*a*b^2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^7 +
 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b - 2
5*a*b^2 + 12*b^3)*cosh(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(
d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(7*(a^2
*b - 2*a*b^2)*cosh(d*x + c)^6 + 5*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)
^4 + a^2*b - 2*a*b^2 + 3*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(
d*x + c))/((a^4*b - a^3*b^2)*d*cosh(d*x + c)^8 + 8*(a^4*b - a^3*b^2)*d*cosh
(d*x + c)*sinh(d*x + c)^7 + (a^4*b - a^3*b^2)*d*sinh(d*x + c)^8 + 4*(a^5 -
2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b - a^3*b^2)*d*cosh(d*x +
c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*sinh(d*x + c)^6 - 2*(4*a^5 - 7*a^4*b +
3*a^3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c)^3 + 3
*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b
- a^3*b^2)*d*cosh(d*x + c)^4 + 30*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)
^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(a^5 - 2*a^4*b +
a^3*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c)^5 + 10*
(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c)^
6 + 15*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 - 3*(4*a^5 - 7*a^4*b + 3
*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*sinh(d*x + c)^2
+ (a^4*b - a^3*b^2)*d + 8*((a^4*b - a^3*b^2)*d*cosh(d*x + c)^7 + 3*(a^5 - 2
*a^4*b + a^3*b^2)*d*cosh(d*x + c)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d*cosh(
d*x + c)^3 + (a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), -1/
2*(2*(a^2*b - 2*a*b^2)*cosh(d*x + c)^7 + 14*(a^2*b - 2*a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^6 + 2*(a^2*b - 2*a*b^2)*sinh(d*x + c)^7 + 2*(4*a^3 - 5*a^2*b
 + 2*a*b^2)*cosh(d*x + c)^5 + 2*(4*a^3 - 5*a^2*b + 2*a*b^2 + 21*(a^2*b - 2*
a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(a^2*b - 2*a*b^2)*cosh(d*x
+ c)^3 + (4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(4*
a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 2*(35*(a^2*b - 2*a*b^2)*cosh(d*x
 + c)^4 + 4*a^3 - 5*a^2*b + 2*a*b^2 + 10*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(a^2*b - 2*a*b^2)*cosh(d*x + c)^5 + 10*(
4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 - 5*a^2*b + 2*a*b^2)*
cosh(d*x + c))*sinh(d*x + c)^2 - ((5*a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8*(5*
a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2 - 4*b^3)*sinh(d*x +
 c)^8 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 4*(5*a^2*b - 9*a*b^
2 + 4*b^3 + 7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(5*
a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^4 + 2*(
35*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 20*a^2*b + 31*a*b^2 - 12*b^3 + 30*(5
*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*b^2
- 4*b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^3 -
 (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a*b^2 -
4*b^3 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(5*a*b^2 - 4*b

```
^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 5*a^
2*b - 9*a*b^2 + 4*b^3 - 3*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 + 8*((5*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - 9*a*b^
2 + 4*b^3)*cosh(d*x + c)^5 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^3
 + (5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a - b)
)*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh(d*x + c))) + ((5*a*b^2 -
 4*b^3)*cosh(d*x + c)^8 + 8*(5*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7
 + (5*a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d
*x + c)^6 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3 + 7*(5*a*b^2 - 4*b^3)*cosh(d*x + c
)^2)*sinh(d*x + c)^6 + 8*(7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b
- 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(20*a^2*b - 31*a*b^2
+ 12*b^3)*cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 20*a^
2*b + 31*a*b^2 - 12*b^3 + 30*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^4 + 8*(7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - 9*a
*b^2 + 4*b^3)*cosh(d*x + c)^3 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c
))*sinh(d*x + c)^3 + 5*a*b^2 - 4*b^3 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d
*x + c)^2 + 4*(7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - 9*a*b^2
+ 4*b^3)*cosh(d*x + c)^4 + 5*a^2*b - 9*a*b^2 + 4*b^3 - 3*(20*a^2*b - 31*a*b
^2 + 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a*b^2 - 4*b^3)*cosh(d
*x + c)^7 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (20*a^2*b - 31*
a*b^2 + 12*b^3)*cosh(d*x + c)^3 + (5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)
)*sinh(d*x + c))*sqrt(b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d
*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (
3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b) + 2*(a^2
*b - 2*a*b^2)*cosh(d*x + c) - ((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^8 +
8*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b + 3*a*b^
2 - 4*b^3)*sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x +
 c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*a*b^2 - 4*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x +
c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 -
 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^4 + 2*(35*(a^2*b + 3
*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b^2 - 12*b^3 + 30*
(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(
a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(a^3 + 2*a^2*b - 7*a*b^2 + 4*
b^3)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c))
*sinh(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4
*b^3)*cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15
*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + a^3 + 2*a^2*b - 7*a*b^
2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(a^3 + 2*a^2*b
 - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)
*cosh(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^2*b + 3*a*b^2 - 4*b^3)
*cosh(d*x + c)^8 + 8*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^
7 + (a^2*b + 3*a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2
+ 4*b^3)*cosh(d*x + c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b +
3*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 -
 4*b^3)*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^5 - 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^4
 + 2*(35*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a
*b^2 - 12*b^3 + 30*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(a^3 + 2*a
^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*
b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*
a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*c
osh(d*x + c)^6 + 15*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + a^3
 + 2*a^2*b - 7*a*b^2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^7
 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b -
```

```
 25*a*b^2 + 12*b^3)*cosh(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cos
h(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(a
^2*b - 2*a*b^2)*cosh(d*x + c)^6 + 5*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x +
c)^4 + a^2*b - 2*a*b^2 + 3*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sin
h(d*x + c))/((a^4*b - a^3*b^2)*d*cosh(d*x + c)^8 + 8*(a^4*b - a^3*b^2)*d*co
sh(d*x + c)*sinh(d*x + c)^7 + (a^4*b - a^3*b^2)*d*sinh(d*x + c)^8 + 4*(a^5
- 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b - a^3*b^2)*d*cosh(d*x
+ c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*sinh(d*x + c)^6 - 2*(4*a^5 - 7*a^4*b
+ 3*a^3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c)^3 +
 3*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*
b - a^3*b^2)*d*cosh(d*x + c)^4 + 30*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x +
c)^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(a^5 - 2*a^4*b
+ a^3*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c)^5 + 1
0*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 - (4*a^5 - 7*a^4*b + 3*a^3*b^
2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c
)^6 + 15*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 - 3*(4*a^5 - 7*a^4*b +
 3*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*sinh(d*x + c)^
2 + (a^4*b - a^3*b^2)*d + 8*((a^4*b - a^3*b^2)*d*cosh(d*x + c)^7 + 3*(a^5 -
 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d*cos
h(d*x + c)^3 + (a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[66,-29]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[-
21,2]Warning, need to choose a branch for the root of a polynomial with par
ameters. This might be wrong.The choice was done assuming [a,b]=[15,2]Undef
/Unsigned Inf encountered in limitEvaluation time: 0.72Limit: Max order rea
ched or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.15, size = 415, normalized size = 2.58

$$\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^2}-\frac{b^2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)(a-b)}+\frac{}{d\,a^3\left(\left(\tanh\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/a^2-1/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^
2+2/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2+1/d*b^2/a^2/(tanh(1/2*d*x+1/
2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)+5/2/d
/a^2*b^2/(a-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*
b)/(a*b-b^2)^(1/2))-2/d*b^3/a^3/(a-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/
2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))-1/8/d/a^2/tanh(1/2*d*x+1/2*c)^2-
1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c))-2/d/a^3*ln(tanh(1/2*d*x+1/2*c))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(abe^{(7c)} - 2b^2e^{(7c)}\right)e^{(7dx)} + \left(4a^2e^{(5c)} - 5abe^{(5c)} + 2b^2e^{(5c)}\right)e^{(5dx)} + \left(4a^2e^{(3c)} - 5\right.}{a^3bd - a^2b^2d + \left(a^3bde^{(8c)} - a^2b^2de^{(8c)}\right)e^{(8dx)} + 4\left(a^4de^{(6c)} - 2a^3bde^{(6c)} + a^2b^2de^{(6c)}\right)e^{(6dx)} - 2\left(4a^4de^{(4c)} - 7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -((a*b*e^(7*c) - 2*b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) - 5*a*b*e^(5*c)
+ 2*b^2*e^(5*c))*e^(5*d*x) + (4*a^2*e^(3*c) - 5*a*b*e^(3*c) + 2*b^2*e^(3*c)
)*e^(3*d*x) + (a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^3*b*d - a^2*b^2*d + (a^3*b*
d*e^(8*c) - a^2*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^4*d*e^(6*c) - 2*a^3*b*d*e^(
6*c) + a^2*b^2*d*e^(6*c))*e^(6*d*x) - 2*(4*a^4*d*e^(4*c) - 7*a^3*b*d*e^(4*c
) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x) + 4*(a^4*d*e^(2*c) - 2*a^3*b*d*e^(2*c) +
 a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*(a + 4*b)*log((e^(d*x + c) + 1)*e^(-c)
)/(a^3*d) - 1/2*(a + 4*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) + 8*integra
te(1/8*((5*a*b^2*e^(3*c) - 4*b^3*e^(3*c))*e^(3*d*x) - (5*a*b^2*e^c - 4*b^3*
e^c)*e^(d*x))/(a^4*b - a^3*b^2 + (a^4*b*e^(4*c) - a^3*b^2*e^(4*c))*e^(4*d*x
) + 2*(2*a^5*e^(2*c) - 3*a^4*b*e^(2*c) + a^3*b^2*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c + d\,x)^3\,\left(b\sinh(c + d\,x)^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.50 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{b^2(6a-5b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a-b)^{3/2}}-\frac{(2a-5b)\coth^3(c+dx)}{6a^2d(a-b)}+\frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{2a^3d(a-b)}-\frac{b\operatorname{csch}^3(c+d\cdots}{2ad(a-b)\left(a-(a\cdots\right.}$$

```
[Out] 1/2*(6*a-5*b)*b^2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)/(a-b)^(3
/2)/d+1/2*(2*a^2+a*b-5*b^2)*coth(d*x+c)/a^3/(a-b)/d-1/6*(2*a-5*b)*coth(d*x+
c)^3/a^2/(a-b)/d-1/2*b*csch(d*x+c)^3*sech(d*x+c)/a/(a-b)/d/(a-(a-b)*tanh(d*
x+c)^2)
```

Rubi [A] time = 0.21, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3187, 468, 570, 208}

$$\frac{b^2(6a-5b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a-b)^{3/2}}+\frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{2a^3d(a-b)}-\frac{(2a-5b)\coth^3(c+dx)}{6a^2d(a-b)}-\frac{b\operatorname{csch}^3(c+d\cdots}{2ad(a-b)\left(a-(a\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((6*a - 5*b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(
a - b)^(3/2)*d) + ((2*a^2 + a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(a - b)*d) -
 ((2*a - 5*b)*Coth[c + d*x]^3)/(6*a^2*(a - b)*d) - (b*Csch[c + d*x]^3*Sech[
c + d*x])/(2*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
```

IntegerQ[p]

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{2a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)\left(2a-5b+(-2a+b)x^2\right)}{x^4\left(a+(-a+b)x^2\right)}\,dx, x\right.}{2a(a-b)d}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{2a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \left(\frac{2a-5b}{ax^4} + \frac{-2a^2-ab+5b^2}{a^2x^2} + \frac{(6}{a^2(a}\right.\right.}{2a(a-b}$$

$$= \frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{2a^3(a-b)d} - \frac{(2a-5b)\coth^3(c+dx)}{6a^2(a-b)d} - \frac{b\operatorname{csch}^3(c+dx}{2a(a-b)d\left(a-(a-\right.}$$

$$= \frac{(6a-5b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^{3/2}d} + \frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{2a^3(a-b)d} - \frac{(2a-5}{6}$$

Mathematica [A] time = 1.27, size = 210, normalized size = 1.21

$$\frac{\operatorname{csch}^4(c+dx)(2a+b\cosh(2(c+dx))-b)\left(-2a^{3/2}\coth(c+dx)\operatorname{csch}^2(c+dx)(2a+b\cosh(2(c+dx))-b) - \frac{3\sqrt{}}{}\right.}{24a^{7/2}d\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^4*((3*(6*a - 5*b)*b^2*ArcTan
h[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cosh[2*(c + d*x)]))/(a
- b)^(3/2) + 4*Sqrt[a]*(a + 3*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d
*x] - 2*a^(3/2)*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csch[c + d*x]
^2 - (3*Sqrt[a]*b^3*Sinh[2*(c + d*x)])/(a - b)))/(24*a^(7/2)*d*(b + a*Csch[
c + d*x]^2)^2)
```

fricas [B] time = 0.79, size = 7110, normalized size = 40.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^8 + 96*(6*a^3*b^
2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + 12*(6*a^3*b^2 - 1
1*a^2*b^3 + 5*a*b^4)*sinh(d*x + c)^8 + 24*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^
3 - 10*a*b^4)*cosh(d*x + c)^6 + 24*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*
a*b^4 + 14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 48*(14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^3 + 3*(6*a^4*
b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 16
```

*a^4*b + 16*a^3*b^2 - 92*a^2*b^3 + 60*a*b^4 - 8*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^4 - 8*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4 - 105*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^4 - 45*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^5 + 15*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^3 - (24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^4)*cosh(d*x + c)^2 + 8*(42*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^6 + 8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^4 + 45*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^4 - 6*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((6*a*b^3 - 5*b^4)*cosh(d*x + c)^10 + 10*(6*a*b^3 - 5*b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (6*a*b^3 - 5*b^4)*sinh(d*x + c)^10 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^8 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4 + 45*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^3 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^6 + 2*(105*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 36*a^2*b^2 + 60*a*b^3 - 25*b^4 + 14*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 14*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^3 - 3*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 2*(105*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 35*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 60*a*b^3 + 25*b^4 - 15*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 6*a*b^3 + 5*b^4 + 8*(15*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^7 + 7*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^5 - 5*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^3 + (36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^2 + (45*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^8 + 28*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^6 - 30*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^4 - 24*a^2*b^2 + 50*a*b^3 - 25*b^4 + 12*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^9 + 4*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^7 - 6*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^5 + 4*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^3 - (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*(6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^7 + 9*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^5 - 2*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^3 + (8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^10 + 10*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^6*b - 2*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^10 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^8 + (45*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^8 - 2*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^6 + 8*(15*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 14*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 - (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 + 4*(63*(a^6*b -

2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 14*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^3 - 3*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b ^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*b^3) *d*cosh(d*x + c)^6 + 35*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 - 15*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c) ^2 + (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^4 - (4*a^ 7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 + 8*(15*(a^6*b - 2 *a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 7*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^5 - 5*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^3 + (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d* x + c))*sinh(d*x + c)^3 + (45*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c) ^8 + 28*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^6 - 30* (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 + 12*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 - (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^2 - (a^6*b - 2*a^5*b^2 + a^4*b^3) *d + 2*(5*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^9 + 4*(4*a^7 - 13*a ^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^7 - 6*(6*a^7 - 17*a^6*b + 16 *a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^5 + 4*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^3 - (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/6*(6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^8 + 48*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)*sin h(d*x + c)^7 + 6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*sinh(d*x + c)^8 + 12*(6 *a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^6 + 12*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4 + 14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b ^4)*cosh(d*x + c)^3 + 3*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh (d*x + c))*sinh(d*x + c)^5 + 8*a^4*b + 8*a^3*b^2 - 46*a^2*b^3 + 30*a*b^4 - 4*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^4 - 4*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4 - 105*(6*a^3* b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^4 - 45*(6*a^4*b - 23*a^3*b^2 + 27 *a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(21*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^5 + 15*(6*a^4*b - 23*a^3*b^2 + 27*a^2* b^3 - 10*a*b^4)*cosh(d*x + c)^3 - (24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2 *b^3 - 45*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^5 - 2*a^4*b - 47*a ^3*b^2 + 71*a^2*b^3 - 30*a*b^4)*cosh(d*x + c)^2 + 4*(42*(6*a^3*b^2 - 11*a^2 *b^3 + 5*a*b^4)*cosh(d*x + c)^6 + 8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^4 + 45*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^4 - 6*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((6*a*b^3 - 5*b^4)*cosh(d*x + c)^10 + 10*(6*a* b^3 - 5*b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (6*a*b^3 - 5*b^4)*sinh(d*x + c)^10 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^8 + (24*a^2*b^2 - 50* a*b^3 + 25*b^4 + 45*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8* (15*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^3 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*co sh(d*x + c))*sinh(d*x + c)^7 - 2*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^6 + 2*(105*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 36*a^2*b^2 + 60*a*b^3 - 25*b^4 + 14*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 14*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^3 - 3*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 2* (105*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 35*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 60*a*b^3 + 25*b^4 - 15*(36*a^2*b^2 - 60*a* b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 6*a*b^3 + 5*b^4 + 8*(15*(6 *a*b^3 - 5*b^4)*cosh(d*x + c)^7 + 7*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d *x + c)^5 - 5*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^3 + (36*a^2*b^ 2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (24*a^2*b^2 - 50*a* b^3 + 25*b^4)*cosh(d*x + c)^2 + (45*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^8 + 28* (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^6 - 30*(36*a^2*b^2 - 60*a*b^ 3 + 25*b^4)*cosh(d*x + c)^4 - 24*a^2*b^2 + 50*a*b^3 - 25*b^4 + 12*(36*a^2*b ^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(6*a*b^3 -

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5*b^4)*cosh(d*x + c)^9 + 4*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^7
 - 6*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^5 + 4*(36*a^2*b^2 - 60*
a*b^3 + 25*b^4)*cosh(d*x + c)^3 - (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x
 + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b
)/(a^2 - a*b)) + 8*(6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^7 +
9*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^5 - 2*(24*a^
5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^3 + (8*a^
5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^4)*cosh(d*x + c))*sinh(d*x +
 c))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^10 + 10*(a^6*b - 2*a^5*
b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^6*b - 2*a^5*b^2 + a^4*b
^3)*d*sinh(d*x + c)^10 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh
(d*x + c)^8 + (45*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (4*a^7
- 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^8 - 2*(6*a^7 - 17*a^6
*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^6 + 8*(15*(a^6*b - 2*a^5*b^2 +
 a^4*b^3)*d*cosh(d*x + c)^3 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d
*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*co
sh(d*x + c)^4 + 14*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x +
 c)^2 - (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*
(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 + 4*(63*(a^6*
b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 14*(4*a^7 - 13*a^6*b + 14*a^5*
b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^3 - 3*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a
^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*
b^3)*d*cosh(d*x + c)^6 + 35*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*c
osh(d*x + c)^4 - 15*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x
+ c)^2 + (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^4 - (
4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 + 8*(15*(a^6*b
 - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 7*(4*a^7 - 13*a^6*b + 14*a^5*b^
2 - 5*a^4*b^3)*d*cosh(d*x + c)^5 - 5*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4
*b^3)*d*cosh(d*x + c)^3 + (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cos
h(d*x + c))*sinh(d*x + c)^3 + (45*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x
+ c)^8 + 28*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^6 -
 30*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 + 12*(6*a
^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 - (4*a^7 - 13*a^6
*b + 14*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^2 - (a^6*b - 2*a^5*b^2 + a^4*
b^3)*d + 2*(5*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^9 + 4*(4*a^7 -
13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^7 - 6*(6*a^7 - 17*a^6*b
+ 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^5 + 4*(6*a^7 - 17*a^6*b + 16*a^5*
b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^3 - (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4
*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [A] time = 0.82, size = 220, normalized size = 1.26

$$\frac{\frac{3\left(6\,ab^2-5\,b^3\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^4-a^3b\right)\sqrt{-a^2+ab}}+\frac{6\left(2\,ab^2e^{(2\,dx+2\,c)}-b^3e^{(2\,dx+2\,c)}+b^3\right)}{\left(a^4-a^3b\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)}+\frac{8\left(3\,be^{(4\,dx+4\,c)}-3\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+a+3\,b\right)}{a^3\left(e^{(2\,dx+2\,c)}-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(6*a*b^2 - 5*b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2
 + a*b))/((a^4 - a^3*b)*sqrt(-a^2 + a*b)) + 6*(2*a*b^2*e^(2*d*x + 2*c) - b^
3*e^(2*d*x + 2*c) + b^3)/((a^4 - a^3*b)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x +
 2*c) - 2*b*e^(2*d*x + 2*c) + b)) + 8*(3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x +
 2*c) - 6*b*e^(2*d*x + 2*c) + a + 3*b)/(a^3*(e^(2*d*x + 2*c) - 1)^3))/d
```

maple [B] time = 0.16, size = 890, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/24/d/a^2*tanh(1/2*d*x+1/2*c)^3+3/8/d/a^2*tanh(1/2*d*x+1/2*c)+1/d/a^3*tan
h(1/2*d*x+1/2*c)*b-1/d/a^3*b^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*
c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/d/a^3*b^3
/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)-3/d/a^2*b^2/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2))-3/d/a^2*b^3/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/d/a^2
*b^2/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d/a^2*b^3/(a-b)/(-b*(a-b))^(1/2)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2))+5/2/d/a^3*b^3/(a-b)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2))+5/2/d/a^3*b^4/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/2/
d/a^3*b^3/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+5/2/d/a^3*b^4/(a-b)/(-b*(a-b)
)^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/24/d/a^2/tanh(1/2*d*x+1/2*c)^3+3/8/
d/a^2/tanh(1/2*d*x+1/2*c)+1/d/a^3/tanh(1/2*d*x+1/2*c)*b
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(c+d\,x\right)^4\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.51 $\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=124

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,d(a-b)^{5/2}}-\frac{3\tanh(c+dx)}{8d(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh^3(c+dx)}{4d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

```
[Out] 3/8*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(5/2)/d/a^(1/2)+1/4*tanh
(d*x+c)^3/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)^2-3/8*tanh(d*x+c)/(a-b)^2/d/(a-(a
-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{3187, 288, 208\}$

$$\frac{\tanh^3(c+dx)}{4d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}-\frac{3\tanh(c+dx)}{8d(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,d(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a - b)^(5/2)*d
) + Tanh[c + d*x]^3/(4*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - (3*Tanh
[c + d*x])/(8*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+\right.}{4(a-b)d}$$

$$= \frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{3\tanh(c+dx)}{8(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a-b)^{5/2}d} + \frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{}{8(a-b)^2}$$

Mathematica [**A**] time = 1.38, size = 104, normalized size = 0.84

$$\frac{\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{5/2}} + \frac{\sinh(2(c+dx))((2a-5b)\cosh(2(c+dx))-8a+5b)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)) +
 ((-8*a + 5*b + (2*a - 5*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b)^
2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

fricas [**B**] time = 0.63, size = 5186, normalized size = 41.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)^6 + 2
4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)*sinh(d*x + c)
^5 + 4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*sinh(d*x + c)^6 + 8*a^
3*b^2 - 28*a^2*b^3 + 20*a*b^4 + 4*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2
*b^3 + 15*a*b^4)*cosh(d*x + c)^4 + 4*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*
a^2*b^3 + 15*a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*
a*b^4)*cosh(d*x + c)^3 + (16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15
*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4*b - 40*a^3*b^2 + 47*a^2*b
^3 - 15*a*b^4)*cosh(d*x + c)^2 + 4*(8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15*
a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)^4 +
6*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 - 3*(b^4*cosh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x +
 c)^7 + b^4*sinh(d*x + c)^8 + 4*(2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(7*b^4*
cosh(d*x + c)^2 + 2*a*b^3 - b^4)*sinh(d*x + c)^6 + 8*(7*b^4*cosh(d*x + c)^3
 + 3*(2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2*b^2 - 8*a*b^
3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*b^4*cosh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^
3 + 3*b^4 + 30*(2*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + b^4 + 8*(
7*b^4*cosh(d*x + c)^5 + 10*(2*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^2*b^2 - 8
```

```
*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b^3 - b^4)*cosh(d*x
 + c)^2 + 4*(7*b^4*cosh(d*x + c)^6 + 15*(2*a*b^3 - b^4)*cosh(d*x + c)^4 + 2
*a*b^3 - b^4 + 3*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 8*(b^4*cosh(d*x + c)^7 + 3*(2*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^2*
b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a*b^3 - b^4)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*
sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2
*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^
2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4
*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 +
 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
+ c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d
*x + c))*sinh(d*x + c) + b)) + 8*(3*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*
a*b^4)*cosh(d*x + c)^5 + 2*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 +
15*a*b^4)*cosh(d*x + c)^3 + (8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15*a*b^4)*
cosh(d*x + c))*sinh(d*x + c))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c)^8 + 8*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*sinh(d*x +
 c)^8 + 4*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*
x + c)^6 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^2
 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d)*sinh(d*x + c)
^6 + 2*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a
*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6
 - a*b^7)*d*cosh(d*x + c)^4 + 30*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*
b^6 + a*b^7)*d*cosh(d*x + c)^2 + (8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*
a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d)*sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 7*a^4*b
^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - 3*a
^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^5 + 10*(2*a^5*b^3 - 7*a^4*b^4 +
 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^3 + (8*a^6*b^2 - 32*a^5*b^3
 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^6
 + 15*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x +
c)^4 + 3*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3
*a*b^7)*d*cosh(d*x + c)^2 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6
 + a*b^7)*d)*sinh(d*x + c)^2 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d +
 8*((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^7 + 3*(2*a^5*
b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^5 + (8*a^6
*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*cosh(
d*x + c)^3 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh
(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a
*b^4)*cosh(d*x + c)^6 + 12*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*co
sh(d*x + c)*sinh(d*x + c)^5 + 2*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^
4)*sinh(d*x + c)^6 + 4*a^3*b^2 - 14*a^2*b^3 + 10*a*b^4 + 2*(16*a^5 - 72*a^4
*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*cosh(d*x + c)^4 + 2*(16*a^5 - 72*
a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21
*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(8*a^4*b - 24*a
^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)^3 + (16*a^5 - 72*a^4*b + 102*a
^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^4*b
 - 40*a^3*b^2 + 47*a^2*b^3 - 15*a*b^4)*cosh(d*x + c)^2 + 2*(8*a^4*b - 40*a^
3*b^2 + 47*a^2*b^3 - 15*a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a
*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 1
5*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^4*cosh(d*x + c)^8 + 8*b^4*
cosh(d*x + c)*sinh(d*x + c)^7 + b^4*sinh(d*x + c)^8 + 4*(2*a*b^3 - b^4)*cos
h(d*x + c)^6 + 4*(7*b^4*cosh(d*x + c)^2 + 2*a*b^3 - b^4)*sinh(d*x + c)^6 +
8*(7*b^4*cosh(d*x + c)^3 + 3*(2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*b^4*cosh(d*x + c
```

```
)^4 + 8*a^2*b^2 - 8*a*b^3 + 3*b^4 + 30*(2*a*b^3 - b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + b^4 + 8*(7*b^4*cosh(d*x + c)^5 + 10*(2*a*b^3 - b^4)*cosh(d*x
 + c)^3 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*
(2*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*b^4*cosh(d*x + c)^6 + 15*(2*a*b^3 -
b^4)*cosh(d*x + c)^4 + 2*a*b^3 - b^4 + 3*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b^4*cosh(d*x + c)^7 + 3*(2*a*b^3 - b^4)*c
osh(d*x + c)^5 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a*b^3 -
 b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*s
qrt(-a^2 + a*b)/(a^2 - a*b)) + 4*(3*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*
a*b^4)*cosh(d*x + c)^5 + 2*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 +
15*a*b^4)*cosh(d*x + c)^3 + (8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15*a*b^4)*
cosh(d*x + c))*sinh(d*x + c))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c)^8 + 8*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*sinh(d*x +
 c)^8 + 4*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*
x + c)^6 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^2
 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d)*sinh(d*x + c)
^6 + 2*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a
*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6
- a*b^7)*d*cosh(d*x + c)^4 + 30*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*
b^6 + a*b^7)*d*cosh(d*x + c)^2 + (8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*
a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d)*sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 7*a^4*b
^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - 3*a
^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^5 + 10*(2*a^5*b^3 - 7*a^4*b^4 +
 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^3 + (8*a^6*b^2 - 32*a^5*b^3
 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^6
 + 15*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x +
c)^4 + 3*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3
*a*b^7)*d*cosh(d*x + c)^2 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6
+ a*b^7)*d)*sinh(d*x + c)^2 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d +
 8*((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^7 + 3*(2*a^5*
b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^5 + (8*a^6
*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*cosh(
d*x + c)^3 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh
(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 6.28, size = 282, normalized size = 2.27

$$\frac{\frac{3\,\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^2-2\,ab+b^2\right)\sqrt{-a^2+ab}}-\frac{2\left(8\,a^2be^{(6\,dx+6\,c)}-16\,ab^2e^{(6\,dx+6\,c)}+5\,b^3e^{(6\,dx+6\,c)}+16\,a^3e^{(4\,dx+4\,c)}-56\,a^2be^{(4\,dx+4\,c)}+46\,ab^2e^{(4\,dx+4\,c)}-15\,b^3e^{(4\,dx+4\,c)}\ldots}{\left(a^2b^2-2\,ab^3+b^4\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+\ldots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^2 - 2
*a*b + b^2)*sqrt(-a^2 + a*b)) - 2*(8*a^2*b*e^(6*d*x + 6*c) - 16*a*b^2*e^(6*
d*x + 6*c) + 5*b^3*e^(6*d*x + 6*c) + 16*a^3*e^(4*d*x + 4*c) - 56*a^2*b*e^(4
*d*x + 4*c) + 46*a*b^2*e^(4*d*x + 4*c) - 15*b^3*e^(4*d*x + 4*c) + 8*a^2*b*e
^(2*d*x + 2*c) - 32*a*b^2*e^(2*d*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) + 2*a*b^
2 - 5*b^3)/((a^2*b^2 - 2*a*b^3 + b^4)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2
*c) - 2*b*e^(2*d*x + 2*c) + b)^2))/d
```

maple [B] time = 0.08, size = 768, normalized size = 6.19

$$-\frac{3a\left(\tanh^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\left(\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)^{2}\left(a^{2}-2ab+b^{2}\right)}+\frac{}{4d\left(\left(\tanh^{4}\left(\frac{dx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+11/4/d/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a-5/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^5*b+11/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*a-5/d/(tanh(1/
2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b-3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(
1/2*d*x+1/2*c)-3/8/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d/(a^
2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b+3/8/d/(a^2-2*a
*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(c+d\,x\right)^4}{{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.52 $\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=135

$$\frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8b^{3/2}d(a-b)^{5/2}}+\frac{(a-4b)\cosh(c+dx)}{8bd(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{a\cosh(c+dx)}{4bd(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}$$

```
[Out] 1/8*(a-4*b)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/b^(3/2)/d-1
/4*a*cosh(d*x+c)/(a-b)/b/d/(a-b+b*cosh(d*x+c)^2)^2+1/8*(a-4*b)*cosh(d*x+c)/
(a-b)^2/b/d/(a-b+b*cosh(d*x+c)^2)
```

Rubi [A] time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 385, 199, 205}

$$\frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8b^{3/2}d(a-b)^{5/2}}+\frac{(a-4b)\cosh(c+dx)}{8bd(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{a\cosh(c+dx)}{4bd(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - 4*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*b^
(3/2)*d) - (a*Cosh[c + d*x])/(4*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2)^2)
+ ((a - 4*b)*Cosh[c + d*x])/(8*(a - b)^2*b*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+bx^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{(a-4b)\,\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^2}\,dx, x, \cosh(\right.}{4(a-b)bd}$$

$$= -\frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{(a-4b)\cosh(c+dx)}{8(a-b)^2bd\left(a-b+b\cosh^2(c+dx)\right)} +$$

$$= \frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}b^{3/2}d} - \frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{(a}{8(a-b)^2}$$

Mathematica [C] time = 1.33, size = 170, normalized size = 1.26

$$\frac{\frac{2\sqrt{b}\cosh(c+dx)\left(-2a^2+b(a-4b)\cosh(2(c+dx))-5ab+4b^2\right)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} + \frac{(a-4b)\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{5/2}}}{8b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (((a - 4*b)*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] +
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/(a - b)^(5/2)
 + (2*Sqrt[b]*Cosh[c + d*x]*(-2*a^2 - 5*a*b + 4*b^2 + (a - 4*b)*b*Cosh[2*(c
 + d*x)]))/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*b^(3/2)*d)
```

fricas [B] time = 0.95, size = 6087, normalized size = 45.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^7 + 28*(a^2*b^2 - 5*a*b^
3 + 4*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(a^2*b^2 - 5*a*b^3 + 4*b^4)*si
nh(d*x + c)^7 - 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^5
- 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4 - 21*(a^2*b^2 - 5*a*b^3 + 4*b^4
)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh
(d*x + c)^3 - (4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(
d*x + c)^4 - 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + 4
*(35*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^4 - 4*a^3*b - 5*a^2*b^2 + 13
*a*b^3 - 4*b^4 - 10*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 4*(21*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^5 - 10
*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 - 3*(4*a^3*b + 5*
a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((a*b^2 - 4*b^
3)*cosh(d*x + c)^8 + 8*(a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b
^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6
 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3 + 7*(a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(2*a^2*b - 9*a*b^2 + 4
*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 - 40*a^2*b + 35*a*b^2 - 12*
```

b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 - 4*b^3)*cosh(d*x + c)^4 + 8*a^3 - 40*a
^2*b + 35*a*b^2 - 12*b^3 + 30*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b - 9*a*
b^2 + 4*b^3)*cosh(d*x + c)^3 + (8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + a*b^2 - 4*b^3 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*c
osh(d*x + c)^2 + 4*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b - 9*a*b
^2 + 4*b^3)*cosh(d*x + c)^4 + 2*a^2*b - 9*a*b^2 + 4*b^3 + 3*(8*a^3 - 40*a^2
*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 - 4*b^
3)*cosh(d*x + c)^7 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + (8*a^3
 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^3 + (2*a^2*b - 9*a*b^2 + 4*b
^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*co
sh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)
^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2
 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*co
sh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*co
sh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(a^2*b^2 -
 5*a*b^3 + 4*b^4)*cosh(d*x + c) + 4*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x
 + c)^6 - 5*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^4 + a^2*
b^2 - 5*a*b^3 + 4*b^4 - 3*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x +
 c)^8 + 8*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)*sinh(d*x +
c)^7 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*sinh(d*x + c)^8 + 4*(2*a^4*b
^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b
^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4
+ 9*a^2*b^5 - 5*a*b^6 + b^7)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^2 - 32*a^4*b^3
 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^
3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^3 + 3*(2*a^4*b^3 - 7*a^3
*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*
(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 - 7
*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + (8*a^5*b^2 - 32*a
^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d)*sinh(d*x + c)^4 + 4
*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + 8*
(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^5 + 10*(2*a^4*b^3
 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^3 + (8*a^5*b^2 - 3
2*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^
6 + 15*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^
4 + 3*(8*a^5*b^2 - 32*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)
*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d)
*sinh(d*x + c)^2 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d + 8*((a^3*b^4 -
3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^7 + 3*(2*a^4*b^3 - 7*a^3*b^4 + 9
*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^5 + (8*a^5*b^2 - 32*a^4*b^3 + 51*
a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^3 + (2*a^4*b^3 - 7
*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*
(2*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^7 + 14*(a^2*b^2 - 5*a*b^3 + 4*
b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2*b^2 - 5*a*b^3 + 4*b^4)*sinh(d*x
 + c)^7 - 2*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^5 - 2*(4
*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4 - 21*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x +
 c)^3 - (4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x +
c)^4 - 2*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + 2*(35*(
a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^4 - 4*a^3*b - 5*a^2*b^2 + 13*a*b^3
 - 4*b^4 - 10*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^3 + 2*(21*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^5 - 10*(4*a^
3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 - 3*(4*a^3*b + 5*a^2*b^
2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((a*b^2 - 4*b^3)*cos

```
h(d*x + c)^8 + 8*(a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - 4
*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 4*(
2*a^2*b - 9*a*b^2 + 4*b^3 + 7*(a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*c
osh(d*x + c)^4 + 2*(35*(a*b^2 - 4*b^3)*cosh(d*x + c)^4 + 8*a^3 - 40*a^2*b +
 35*a*b^2 - 12*b^3 + 30*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^4 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b - 9*a*b^2 +
4*b^3)*cosh(d*x + c)^3 + (8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x +
c))*sinh(d*x + c)^3 + a*b^2 - 4*b^3 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*
x + c)^2 + 4*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b - 9*a*b^2 + 4
*b^3)*cosh(d*x + c)^4 + 2*a^2*b - 9*a*b^2 + 4*b^3 + 3*(8*a^3 - 40*a^2*b + 3
5*a*b^2 - 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 - 4*b^3)*cos
h(d*x + c)^7 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + (8*a^3 - 40*
a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^3 + (2*a^2*b - 9*a*b^2 + 4*b^3)*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3
+ 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(
d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)
) - ((a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8*(a*b^2 - 4*b^3)*cosh(d*x + c)*sinh
(d*x + c)^7 + (a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b - 9*a*b^2 + 4*b^
3)*cosh(d*x + c)^6 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3 + 7*(a*b^2 - 4*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(2*a
^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 - 40*a^2*
b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 - 4*b^3)*cosh(d*x + c
)^4 + 8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3 + 30*(2*a^2*b - 9*a*b^2 + 4*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^5 +
10*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + (8*a^3 - 40*a^2*b + 35*a*b
^2 - 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 - 4*b^3 + 4*(2*a^2*b -
9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 1
5*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 2*a^2*b - 9*a*b^2 + 4*b^3 +
 3*(8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 8*((a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d
*x + c)^5 + (8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^3 + (2*a^2
*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2)*arctan(
-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + 2*(a^2*b^2
- 5*a*b^3 + 4*b^4)*cosh(d*x + c) + 2*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*
x + c)^6 - 5*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^4 + a^2
*b^2 - 5*a*b^3 + 4*b^4 - 3*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x
+ c)^8 + 8*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)*sinh(d*x +
 c)^7 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*sinh(d*x + c)^8 + 4*(2*a^4*
b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^3*
b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4
 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^2 - 32*a^4*b^
3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a
^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^3 + 3*(2*a^4*b^3 - 7*a^
3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35
*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 -
7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + (8*a^5*b^2 - 32*
a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d)*sinh(d*x + c)^4 +
4*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + 8
*(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^5 + 10*(2*a^4*b^3
 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^3 + (8*a^5*b^2 -
32*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c))*s
inh(d*x + c)^3 + 4*(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)
^6 + 15*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)
^4 + 3*(8*a^5*b^2 - 32*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7
)*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d
)*sinh(d*x + c)^2 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d + 8*((a^3*b^4 -
```

```
3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^7 + 3*(2*a^4*b^3 - 7*a^3*b^4 +
9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^5 + (8*a^5*b^2 - 32*a^4*b^3 + 51
*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^3 + (2*a^4*b^3 -
7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-85,-18]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[33,-80]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-98,-18]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-57,-1
0]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.The choice was done assuming [a,b]=[-57,-3]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [a,b]=[-53,60]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[80,-1]schur row 3 -6.9034e-07
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[-51,-3]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-78,38]Warning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.The choice was done assuming [a,b]=[-75,-16]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.The choice was done assuming [a,b]=[-64,-88]Warning, need to choose a bran
ch for the root of a polynomial with parameters. This might be wrong.The ch
oice was done assuming [a,b]=[82,-14]Warning, need to choose a branch for t
he root of a polynomial with parameters. This might be wrong.The choice was
 done assuming [a,b]=[42,-23]Warning, need to choose a branch for the root
of a polynomial with parameters. This might be wrong.The choice was done as
suming [a,b]=[90,-28]Undef/Unsigned Inf encountered in limitEvaluation time
: 2.85Limit: Max order reached or unable to make series expansion Error: Ba
d Argument Value
```

maple [B] time = 0.08, size = 961, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/d/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^4+1/2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*t
anh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+2/d/(ta
nh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
```

```
)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*b^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^4+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
2*a^2+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*a-4/d/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a^2/b/(a^2-2*a*b+b^2)-1/2/d/(
tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)^2*a/(a^2-2*a*b+b^2)+1/8/d/b/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*
(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))*a-1/2/d/(a^2-2*a*b+b^2
)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(abe^{(7\,c)} - 4\,b^2e^{(7\,c)}\right)e^{(7\,dx)} - \left(4\,a^2e^{(5\,c)} + 9\right.}{4\left(a^2b^3d - 2\,ab^4d + b^5d + \left(a^2b^3de^{(8\,c)} - 2\,ab^4de^{(8\,c)} + b^5de^{(8\,c)}\right)e^{(8\,dx)} + 4\left(2\,a^3b^2de^{(6\,c)} - 5\,a^2b^3de^{(6\,c)} + 4\,ab^4de^{(6\,c)} -\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((a*b*e^(7*c) - 4*b^2*e^(7*c))*e^(7*d*x) - (4*a^2*e^(5*c) + 9*a*b*e^(5*
c) - 4*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) + 9*a*b*e^(3*c) - 4*b^2*e^(3
*c))*e^(3*d*x) + (a*b*e^c - 4*b^2*e^c)*e^(d*x))/(a^2*b^3*d - 2*a*b^4*d + b^
5*d + (a^2*b^3*d*e^(8*c) - 2*a*b^4*d*e^(8*c) + b^5*d*e^(8*c))*e^(8*d*x) + 4
*(2*a^3*b^2*d*e^(6*c) - 5*a^2*b^3*d*e^(6*c) + 4*a*b^4*d*e^(6*c) - b^5*d*e^(
6*c))*e^(6*d*x) + 2*(8*a^4*b*d*e^(4*c) - 24*a^3*b^2*d*e^(4*c) + 27*a^2*b^3*
d*e^(4*c) - 14*a*b^4*d*e^(4*c) + 3*b^5*d*e^(4*c))*e^(4*d*x) + 4*(2*a^3*b^2*
d*e^(2*c) - 5*a^2*b^3*d*e^(2*c) + 4*a*b^4*d*e^(2*c) - b^5*d*e^(2*c))*e^(2*d
*x)) + 1/8*integrate(2*((a*e^(3*c) - 4*b*e^(3*c))*e^(3*d*x) - (a*e^c - 4*b*
e^c)*e^(d*x))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2*e^(4*c) - 2*a*b^3*e^(4*c)
 + b^4*e^(4*c))*e^(4*d*x) + 2*(2*a^3*b*e^(2*c) - 5*a^2*b^2*e^(2*c) + 4*a*b^
3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(c + d\,x\right)^3}{\left(b\sinh\left(c + d\,x\right)^2 + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.53 $\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3} dx$

Optimal. Leaf size=139

$$-\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a-b)^{5/2}}+\frac{(2a+b)\sinh(c+dx)\cosh(c+dx)}{8ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] -1/8*(4*a-b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(5/2)/d
+1/4*cosh(d*x+c)*sinh(d*x+c)/(a-b)/d/(a+b*sinh(d*x+c)^2)^2+1/8*(2*a+b)*cosh
(d*x+c)*sinh(d*x+c)/a/(a-b)^2/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A] time = 0.16, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3173, 12, 3181, 208}

$$-\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a-b)^{5/2}}+\frac{(2a+b)\sinh(c+dx)\cosh(c+dx)}{8ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -((4*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a - b
)^(5/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(4*(a - b)*d*(a + b*Sinh[c + d*x
]^2)^2) + ((2*a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*a*(a - b)^2*d*(a + b*S
inh[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{\int \frac{a-2a\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx}{4a(a-b)}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} - \frac{\int \frac{a(4a}{a+b\sinh}}{8a^2(a}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} - \frac{(4a-b)}{8}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} - \frac{(4a-b)}{}$$

$$= -\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^{5/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(2a+b)c}{8a(a-b)}$$

Mathematica [A] time = 1.36, size = 121, normalized size = 0.87

$$\frac{\frac{\sinh(2(c+dx))\left(8a^2+b(2a+b)\cosh(2(c+dx))-4ab-b^2\right)}{a(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} - \frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{5/2}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-(((4*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a - b
)^(5/2))) + ((8*a^2 - 4*a*b - b^2 + b*(2*a + b)*Cosh[2*(c + d*x)])*Sinh[2*(
c + d*x)])/(a*(a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

fricas [B] time = 0.82, size = 5519, normalized size = 39.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 + 24*(4*a^3*b^2 -
 5*a^2*b^3 + a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(4*a^3*b^2 - 5*a^2*b^
3 + a*b^4)*sinh(d*x + c)^6 + 8*a^3*b^2 - 4*a^2*b^3 - 4*a*b^4 + 4*(16*a^5 -
24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 4*(16*a^5 - 2
4*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4 + 15*(4*a^3*b^2 - 5*a^2*b^3 + a*b
^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4
)*cosh(d*x + c)^3 + (16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^
4)*cosh(d*x + c)^2 + 4*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^4 + 15*(4*a
^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 24*a^4*b + 6*a^3*
b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a*b^3 - b
^4)*cosh(d*x + c)^8 + 8*(4*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*
a*b^3 - b^4)*sinh(d*x + c)^8 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^
6 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4 + 7*(4*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^6 + 8*(7*(4*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 6*a*b^3
+ b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(32*a^3*b - 40*a^2*b^2 + 20*a*b^3
```

- 3*b^4)*cosh(d*x + c)^4 + 2*(35*(4*a*b^3 - b^4)*cosh(d*x + c)^4 + 32*a^3*
b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4 + 30*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x
+ c)^2)*sinh(d*x + c)^4 + 4*a*b^3 - b^4 + 8*(7*(4*a*b^3 - b^4)*cosh(d*x +
c)^5 + 10*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^3 + (32*a^3*b - 40*a^2*
b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 6*a
*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - b^4)*cosh(d*x + c)^6 + 15*(8*
a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^2*b^2 - 6*a*b^3 + b^4 + 3*(3
2*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
8*((4*a*b^3 - b^4)*cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*
x + c)^5 + (32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (8*
a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log(
(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x +
c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
- b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*
a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*co
sh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(
2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^
2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 8
(3(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 + 2*(16*a^5 - 24*a^4*b
+ 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (16*a^4*b - 20*a^3*b^2
+ a^2*b^3 + 3*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 - 3*a^4*b^4 +
3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^
5 - a^2*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3
*b^5 - a^2*b^6)*d*sinh(d*x + c)^8 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 -
5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*
b^5 - a^2*b^6)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a
^3*b^5 + a^2*b^6)*d)*sinh(d*x + c)^6 + 2*(8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3
- 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 -
3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 7*a^5*
b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2
(35(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^4 + 30*(2*
a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^2 +
(8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d
)*sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*
b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d
*cosh(d*x + c)^5 + 10*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*
b^6)*d*cosh(d*x + c)^3 + (8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 +
17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 -
3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^6 + 15*(2*a^6*b^2 - 7*a^5*
b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4 + 3*(8*a^7*b - 32*
a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)
^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d)*sinh(d*x
+ c)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d + 8*((a^5*b^3 - 3*a^
4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^7 + 3*(2*a^6*b^2 - 7*a^5*b^3 +
9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^5 + (8*a^7*b - 32*a^6*b^2
+ 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^3 + (2
*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*si
nh(d*x + c)), -1/8*(2*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 + 12*
(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(4*a^3*b^
2 - 5*a^2*b^3 + a*b^4)*sinh(d*x + c)^6 + 4*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 +
2*(16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 2
*(16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4 + 15*(4*a^3*b^2 - 5*a
^2*b^3 + a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(4*a^3*b^2 - 5*a^2*
b^3 + a*b^4)*cosh(d*x + c)^3 + (16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 -
3*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(16*a^4*b - 20*a^3*b^2 + a^2*b
^3 + 3*a*b^4)*cosh(d*x + c)^2 + 2*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^
4 + 15*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 24*a^4
*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((

```
4*a*b^3 - b^4)*cosh(d*x + c)^8 + 8*(4*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x +
 c)^7 + (4*a*b^3 - b^4)*sinh(d*x + c)^8 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4)*cos
h(d*x + c)^6 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4 + 7*(4*a*b^3 - b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(8*a^2*b^2
 - 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(32*a^3*b - 40*a^2*b^2
 + 20*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 2*(35*(4*a*b^3 - b^4)*cosh(d*x + c)^
4 + 32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4 + 30*(8*a^2*b^2 - 6*a*b^3 + b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*a*b^3 - b^4 + 8*(7*(4*a*b^3 - b^4)*
cosh(d*x + c)^5 + 10*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^3 + (32*a^3*
b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^
2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - b^4)*cosh(d*x + c)
^6 + 15*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^2*b^2 - 6*a*b^3 +
 b^4 + 3*(32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + 8*((4*a*b^3 - b^4)*cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 6*a*b^3 + b
^4)*cosh(d*x + c)^5 + (32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x +
 c)^3 + (8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2
 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) +
b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 4*(3*(4*a^3*b^
2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 + 2*(16*a^5 - 24*a^4*b + 6*a^3*b^2 +
 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (16*a^4*b - 20*a^3*b^2 + a^2*b^3 +
3*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 -
a^2*b^6)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*
d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^
6)*d*sinh(d*x + c)^8 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a
^2*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6
)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*
b^6)*d)*sinh(d*x + c)^6 + 2*(8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4
 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 - 3*a^4*b^4 +
3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b
^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^3
 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^4 + 30*(2*a^6*b^2 - 7*a
^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^2 + (8*a^7*b - 32
*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d)*sinh(d*x +
c)^4 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d
*x + c)^2 + 8*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c
)^5 + 10*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d
*x + c)^3 + (8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 -
3*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 - 3*a^4*b^4 + 3
*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^6 + 15*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b
^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4 + 3*(8*a^7*b - 32*a^6*b^2 + 51*
a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^2 + (2*a^6*b
^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d)*sinh(d*x + c)^2 + (a^5
*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d + 8*((a^5*b^3 - 3*a^4*b^4 + 3*a^3
*b^5 - a^2*b^6)*d*cosh(d*x + c)^7 + 3*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 -
5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^5 + (8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3
 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^3 + (2*a^6*b^2 - 7*
a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 3.55, size = 277, normalized size = 1.99

$$-\frac{\frac{(4\,a-b)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^3-2\,a^2b+ab^2\right)\sqrt{-a^2+ab}}+\frac{2\left(4\,ab^2e^{(6\,dx+6\,c)}-b^3e^{(6\,dx+6\,c)}+16\,a^3e^{(4\,dx+4\,c)}-8\,a^2be^{(4\,dx+4\,c)}-2\,ab^2e^{(4\,dx+4\,c)}+3\,b^3e^{(4\,dx+4\,c)}+16\,a^2be^{(2\,dx+2\,c)}-4\right.}{\left(a^3b-2\,a^2b^2+ab^3\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((4*a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/
((a^3 - 2*a^2*b + a*b^2)*sqrt(-a^2 + a*b)) + 2*(4*a*b^2*e^(6*d*x + 6*c) - b
```

```
^3*e^(6*d*x + 6*c) + 16*a^3*e^(4*d*x + 4*c) - 8*a^2*b*e^(4*d*x + 4*c) - 2*a
*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 16*a^2*b*e^(2*d*x + 2*c) - 4
*a*b^2*e^(2*d*x + 2*c) - 3*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + b^3)/((a^3*b - 2
*a^2*b^2 + a*b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x +
 2*c) + b)^2))/d
```

maple [B] time = 0.08, size = 1408, normalized size = 10.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/4/d/(tanh(1/2*d*x+1/2*
c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+
b^2)*tanh(1/2*d*x+1/2*c)^7*b-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)
^5*a+9/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b+1/d/(tanh(1/2*d*x
+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b^2-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3*a+9/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b+1/d/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b^2+1/d/(tanh(1/2*d*x+1/2*c)^4*a
-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+1/2
/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/(a^2-2*a*b+b^2)/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b-1/2/d/(a^2-2*a*b+b^2)/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)+a-2*b)*a)^(1/2))+1/2/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2))*b-1/8/d/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/8/
d/(a^2-2*a*b+b^2)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2+1/8/d
/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/8/d/(a^2-2*a*b+b^2)/a/
(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(c+d\,x\right)^2}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3,x)
[Out] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
[Out] Timed out
```

3.54 $\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=118

$$\frac{3\cosh(c+dx)}{8d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}+\frac{\cosh(c+dx)}{4d(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8\sqrt{b}\,d(a-b)^{5/2}}$$

```
[Out] 1/4*cosh(d*x+c)/(a-b)/d/(a-b+b*cosh(d*x+c)^2)^2+3/8*cosh(d*x+c)/(a-b)^2/d/(
a-b+b*cosh(d*x+c)^2)+3/8*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/(a-b)^(5/2
)/d/b^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 199, 205}

$$\frac{3\cosh(c+dx)}{8d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}+\frac{\cosh(c+dx)}{4d(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8\sqrt{b}\,d(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*Sqrt[b]*d)
 + Cosh[c + d*x]/(4*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) + (3*Cosh[c +
d*x])/(8*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{4(a-b)d}$$

$$= \frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{3\cosh(c+dx)}{8(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)} + \text{3 S}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}\sqrt{b}\,d} + \frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{\text{3 cosh}}{8(a-b)^2d\left(a-b\right.}$$

Mathematica [C] time = 0.79, size = 149, normalized size = 1.26

$$\frac{\frac{2\cosh(c+dx)(10a+3b\cosh(2(c+dx))-7b)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} + \frac{3\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{b}\,(a-b)^{5/2}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(
Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/((a - b)^(5/2)*Sqrt[b
]) + (2*Cosh[c + d*x]*(10*a - 7*b + 3*b*Cosh[2*(c + d*x)]))/((a - b)^2*(2*a
 - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

fricas [B] time = 0.98, size = 5152, normalized size = 43.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*(a*b^2 - b^3)*cosh(d*x + c)^7 + 84*(a*b^2 - b^3)*cosh(d*x + c)*si
nh(d*x + c)^6 + 12*(a*b^2 - b^3)*sinh(d*x + c)^7 + 4*(20*a^2*b - 31*a*b^2 +
 11*b^3)*cosh(d*x + c)^5 + 4*(20*a^2*b - 31*a*b^2 + 11*b^3 + 63*(a*b^2 - b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(21*(a*b^2 - b^3)*cosh(d*x + c)^3
+ (20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(20*a^2
*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 4*(105*(a*b^2 - b^3)*cosh(d*x + c
)^4 + 20*a^2*b - 31*a*b^2 + 11*b^3 + 10*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^3 + 4*(63*(a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(20
*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 3*(20*a^2*b - 31*a*b^2 + 11*b
^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*
x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x + c
)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*co
sh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2 -
 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b - b
^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*cos
h(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2*
```

```
cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*b
^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x
+ c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x
 + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b + b^2)*log(
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
- 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c
) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3
+ (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) +
 b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c)
+ b)) + 12*(a*b^2 - b^3)*cosh(d*x + c) + 4*(21*(a*b^2 - b^3)*cosh(d*x + c)^
6 + 5*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 3*a*b^2 - 3*b^3 + 3*
(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^3*b^3 -
3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^8 + 8*(a^3*b^3 - 3*a^2*b^4 + 3*a
*b^5 - b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b^3 - 3*a^2*b^4 + 3*a*b^
5 - b^6)*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5
 + b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*co
sh(d*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh
(d*x + c)^6 + 2*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5
- 3*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*c
osh(d*x + c)^3 + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*
d*cosh(d*x + c)^4 + 30*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*
d*cosh(d*x + c)^2 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*
b^5 - 3*b^6)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*
a*b^5 + b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)
*d*cosh(d*x + c)^5 + 10*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)
*d*cosh(d*x + c)^3 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a
*b^5 - 3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b^3 - 3*a^2*b^4
+ 3*a*b^5 - b^6)*d*cosh(d*x + c)^6 + 15*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4
- 5*a*b^5 + b^6)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 -
 41*a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cosh(d*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3
+ 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 - 3*a^2*b^4 + 3*
a*b^5 - b^6)*d + 8*((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^7
 + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d*x + c)^5
+ (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cos
h(d*x + c)^3 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d
*x + c))*sinh(d*x + c)), 1/8*(6*(a*b^2 - b^3)*cosh(d*x + c)^7 + 42*(a*b^2 -
 b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 6*(a*b^2 - b^3)*sinh(d*x + c)^7 + 2*(
20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 2*(20*a^2*b - 31*a*b^2 + 11
*b^3 + 63*(a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(21*(a*b^2 -
b^3)*cosh(d*x + c)^3 + (20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d
*x + c)^4 + 2*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 2*(105*(a*b^
2 - b^3)*cosh(d*x + c)^4 + 20*a^2*b - 31*a*b^2 + 11*b^3 + 10*(20*a^2*b - 31
*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*(a*b^2 - b^3)*cos
h(d*x + c)^5 + 10*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 3*(20*a^
2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x +
 c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*
b - b^2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x
 + c)^6 + 8*(7*b^2*cosh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x +
 c)^4 + 30*(2*a*b - b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x
+ c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a
^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d
*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3
*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b
^2 + 8*(b^2*cosh(d*x + c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*
```

```
a*b + 3*b^2)*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x
 + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c
)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - 3*(b^2*cosh(d*x + c)^8 +
 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2
)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6
 + 8*(7*b^2*cosh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 +
 30*(2*a*b - b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4
+ 8*(7*b^2*cosh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*
a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)
^2 + 4*(7*b^2*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2
 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*
(b^2*cosh(d*x + c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3
*b^2)*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*
b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b
)) + 6*(a*b^2 - b^3)*cosh(d*x + c) + 2*(21*(a*b^2 - b^3)*cosh(d*x + c)^6 +
5*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 3*a*b^2 - 3*b^3 + 3*(20*
a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^3*b^3 - 3*a^
2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^8 + 8*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5
 - b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 -
b^6)*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b
^6)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d
*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh(d*x
 + c)^6 + 2*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 - 3*
b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(
d*x + c)^3 + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*co
sh(d*x + c)^4 + 30*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*co
sh(d*x + c)^2 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5
- 3*b^6)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^
5 + b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*c
osh(d*x + c)^5 + 10*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*c
osh(d*x + c)^3 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5
 - 3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b^3 - 3*a^2*b^4 + 3*
a*b^5 - b^6)*d*cosh(d*x + c)^6 + 15*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*
a*b^5 + b^6)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*
a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cosh(d*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3 + 9*
a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 - 3*a^2*b^4 + 3*a*b^
5 - b^6)*d + 8*((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^7 + 3
*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d*x + c)^5 + (8
*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cosh(d*
x + c)^3 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d*x +
 c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-85,-18]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[33,-80]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-98,-18]Warning, need to choose a branch for the root of a polynomial with
```

```
parameters. This might be wrong.The choice was done assuming [a,b]=[-57,-1
0]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.The choice was done assuming [a,b]=[-57,-3]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [a,b]=[-53,60]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[80,-1]schur row 3 -6.9034e-07
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[-51,-3]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-64,-74]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[-56,-37]Undef/Unsigned Inf enc
ountered in limitEvaluation time: 2.12Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

maple [B] time = 0.06, size = 964, normalized size = 8.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+4/d/(tanh(1/2*d*x+1/2
*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b
+b^2)*tanh(1/2*d*x+1/2*c)^6*b-2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1
/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2
*c)^6*b^2+15/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-23/2/d/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+14/d/(tanh(1/2*d*x+1/2*c)^4*a-2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*b^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^4-4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^
2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
4*b^3-15/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*a+8/d/(tanh(1/2*d
*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a
^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2
*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^2*b^2+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)-1/2/d/(tanh(1/2*d*x+1/2*c)^4
*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)
*b+3/8/d/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^
2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(20\, ae^{(5\, c)} - 11\, be^{(5}\right.}{4\left(a^2b^2d - 2\, ab^3d + b^4d + \left(a^2b^2de^{(8\, c)} - 2\, ab^3de^{(8\, c)} + b^4de^{(8\, c)}\right)e^{(8\, dx)} + 4\left(2\, a^3bde^{(6\, c)} - 5\, a^2b^2de^{(6\, c)} + 4\, ab^3de^{(6\, c}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((20*a*e^(5*c) - 11*b*e^(5*c))*e^(5*d*x) + (20*a*e^(3*c) - 11*b*e^(3*c)
)*e^(3*d*x) + 3*b*e^(7*d*x + 7*c) + 3*b*e^(d*x + c))/(a^2*b^2*d - 2*a*b^3*d
 + b^4*d + (a^2*b^2*d*e^(8*c) - 2*a*b^3*d*e^(8*c) + b^4*d*e^(8*c))*e^(8*d*x
) + 4*(2*a^3*b*d*e^(6*c) - 5*a^2*b^2*d*e^(6*c) + 4*a*b^3*d*e^(6*c) - b^4*d*
e^(6*c))*e^(6*d*x) + 2*(8*a^4*d*e^(4*c) - 24*a^3*b*d*e^(4*c) + 27*a^2*b^2*d
```

```
*e^(4*c) - 14*a*b^3*d*e^(4*c) + 3*b^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^3*b*d*e
^(2*c) - 5*a^2*b^2*d*e^(2*c) + 4*a*b^3*d*e^(2*c) - b^4*d*e^(2*c))*e^(2*d*x)
) + 1/2*integrate(3/2*(e^(3*d*x + 3*c) - e^(d*x + c))/(a^2*b - 2*a*b^2 + b^
3 + (a^2*b*e^(4*c) - 2*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^3*e^
(2*c) - 5*a^2*b*e^(2*c) + 4*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c+d\,x)}{\left(b\sinh(c+d\,x)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^3,x)
[Out] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
[Out] Timed out
```

3.55 $\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$-\frac{3b(2a-b)\sinh(c+dx)\cosh(c+dx)}{8a^2d(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{5/2}}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] 1/8*(8*a^2-8*a*b+3*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a
-b)^(5/2)/d-1/4*b*cosh(d*x+c)*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)^2)^2-3
/8*(2*a-b)*b*cosh(d*x+c)*sinh(d*x+c)/a^2/(a-b)^2/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A] time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3184, 3173, 12, 3181, 208}

$$\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{5/2}}-\frac{3b(2a-b)\sinh(c+dx)\cosh(c+dx)}{8a^2d(a-b)^2\left(a+b\sinh^2(c+dx)\right)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(-3),x]
```

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a
^(5/2)*(a - b)^(5/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(4*a*(a - b)*d*(a
 + b*Sinh[c + d*x]^2)^2) - (3*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*a
^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
```

```
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{\int \frac{-4a+3b+2b\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx}{4a(a-b)}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a-b)b\cosh(c+dx)\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} - \frac{\int \frac{-}{a-}}{8}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a-b)b\cosh(c+dx)\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{(8a}{}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a-b)b\cosh(c+dx)\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{(8a}{}$$

$$= \frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{5/2}d} - \frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} -$$

Mathematica [A] time = 1.23, size = 132, normalized size = 0.86

$$\frac{\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{a}\,b\sinh(2(c+dx))\left(-16a^2+3b(b-2a)\cosh(2(c+dx))+16ab-3b^2\right)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-3),x]
```

```
[Out] (((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a
- b)^(5/2) + (Sqrt[a]*b*(-16*a^2 + 16*a*b - 3*b^2 + 3*b*(-2*a + b)*Cosh[2*(
c + d*x)])*Sinh[2*(c + d*x)])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2)
)/(8*a^(5/2)*d)
```

fricas [B] time = 0.60, size = 5925, normalized size = 38.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 24
*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^
5 + 4*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 24*a^
3*b^2 - 36*a^2*b^3 + 12*a*b^4 + 12*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2
*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 12*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a
^2*b^3 + 3*a*b^4 + 5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 16*(5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b
^4)*cosh(d*x + c)^3 + 3*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*
b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3
 - 9*a*b^4)*cosh(d*x + c)^2 + 4*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b
```

^4 + 15*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 18*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4 + 30*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^3 + 3*b^4 + 8*(7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4 + 3*(64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 8*(3*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 6*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + (40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 30*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + 10*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^2 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d + 8*((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^3 + (2*a^7*b - 7*a^6*b^2 +

```
 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(2*(
8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 12*(8*a^4*b
 - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(8*a
^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 12*a^3*b^2 - 18
*a^2*b^3 + 6*a*b^4 + 6*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b
^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*
b^4 + 5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^4 + 8*(5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x +
 c)^3 + 3*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 2*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b^4)*co
sh(d*x + c)^2 + 2*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b^4 + 15*(8*a^4
*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 18*(16*a^5 - 40*a
^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 - ((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^2*b^2 - 8*a*b^3 +
 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*sinh(
d*x + c)^8 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^6 +
 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 - 8*a*b^3 + 3*b
^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*c
osh(d*x + c)^3 + 3*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)
*cosh(d*x + c)^4 + 2*(35*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 64
*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4 + 30*(16*a^3*b - 24*a^2*b
^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b
^3 + 3*b^4 + 8*(7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(16*a^
3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (64*a^4 - 128*a^3*b
 + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^
3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(8*a^2*b^2 - 8*
a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b
^4)*cosh(d*x + c)^4 + 16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4 + 3*(64*a^4
 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(16*a^3*b - 24*a
^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (64*a^4 - 128*a^3*b + 112*a^2*
b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (16*a^3*b - 24*a^2*b^2 + 14*a*b^3
 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cos
h(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a -
b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 4*(3*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3
 - 3*a*b^4)*cosh(d*x + c)^5 + 6*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3
 + 3*a*b^4)*cosh(d*x + c)^3 + (40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b^4
)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5
)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*si
nh(d*x + c)^8 + 4*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(
d*x + c)^2 + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d)*sin
h(d*x + c)^6 + 2*(8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 -
 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3
*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 +
a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 - 3*a^5*b^3 + 3*
a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 30*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3
 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^8 - 32*a^7*b + 51*a^6*b^2 -
 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^7*b - 7*a
^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2
 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + 10*(2*a^7*b - 7*a^6
*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^8 - 32*a^7
*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x
+ c)^6 + 15*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(
d*x + c)^4 + 3*(8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3
*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4
```

```
+ a^3*b^5)*d)*sinh(d*x + c)^2 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)
*d + 8*((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^7 + 3*(
2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 +
(8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cos
h(d*x + c)^3 + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*co
sh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 0.91, size = 302, normalized size = 1.96

$$\frac{\frac{\left(8a^2-8ab+3b^2\right)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\left(a^4-2a^3b+a^2b^2\right)\sqrt{-a^2+ab}}+\frac{2\left(8a^2be^{(6dx+6c)}-8ab^2e^{(6dx+6c)}+3b^3e^{(6dx+6c)}+48a^3e^{(4dx+4c)}-72a^2be^{(4dx+4c)}+42ab^2e^{(4dx+4c)}-9\right.}{\left(a^4-2a^3b+a^2b^2\right)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-\right.}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((8*a^2 - 8*a*b + 3*b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(
-a^2 + a*b))/((a^4 - 2*a^3*b + a^2*b^2)*sqrt(-a^2 + a*b)) + 2*(8*a^2*b*e^(6
*d*x + 6*c) - 8*a*b^2*e^(6*d*x + 6*c) + 3*b^3*e^(6*d*x + 6*c) + 48*a^3*e^(4
*d*x + 4*c) - 72*a^2*b*e^(4*d*x + 4*c) + 42*a*b^2*e^(4*d*x + 4*c) - 9*b^3*e
^(4*d*x + 4*c) + 40*a^2*b*e^(2*d*x + 2*c) - 40*a*b^2*e^(2*d*x + 2*c) + 9*b^
3*e^(2*d*x + 2*c) + 6*a*b^2 - 3*b^3)/((a^4 - 2*a^3*b + a^2*b^2)*(b*e^(4*d*x
 + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2))/d
```

maple [B] time = 0.11, size = 1768, normalized size = 11.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*b+5/4/d/(tanh(1/2*d*x+1/2
*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^5*b-29/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b^2+3/d/(ta
nh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+2/d/(tanh(1/2*d*x+1/2*c)^
4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^3*b-29/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^3*b^2+3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-2/d/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)-1/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/(a
^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d/a^2/(a^2-2*a*b+b^2)/((
2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2))*b^2-1/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2))*b+1/d/(a^2-2*a*b+b^2)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2))*b^2-3/8/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
```

```
-a+2*b)*a)^(1/2))*b^3+1/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d
/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d/a^2/(a^2-2*a*b+b^2
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-1/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((
2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2))*b+1/d/(a^2-2*a*b+b^2)/a/(-b*(a-b))^(1/2)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2))*b^2-3/8/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2))*b^3
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\sinh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(1/(a + b*sinh(c + d*x)^2)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.56 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=166

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}-\frac{b(7a-4b)\cosh(c+dx)}{8a^2d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3d(a-b)^{5/2}}$$

```
[Out] -arctanh(cosh(d*x+c))/a^3/d-1/4*b*cosh(d*x+c)/a/(a-b)/d/(a-b+b*cosh(d*x+c)^
2)^2-1/8*(7*a-4*b)*b*cosh(d*x+c)/a^2/(a-b)^2/d/(a-b+b*cosh(d*x+c)^2)-1/8*(1
5*a^2-20*a*b+8*b^2)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))*b^(1/2)/a^3/(a-
b)^(5/2)/d
```

Rubi [A] time = 0.27, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3186, 414, 527, 522, 206, 205}

$$-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3d(a-b)^{5/2}}-\frac{b(7a-4b)\cosh(c+dx)}{8a^2d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -(Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a -
 b]])/(8*a^3*(a - b)^(5/2)*d) - ArcTanh[Cosh[c + d*x]]/(a^3*d) - (b*Cosh[c
+ d*x])/(4*a*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) - ((7*a - 4*b)*b*Cosh
[c + d*x])/(8*a^2*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+bx^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{-4a+b+3bx^2}{(1-x^2)\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+\right.}{4a(a-b)d}$$

$$= -\frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(7a-4b)b\cosh(c+dx)}{8a^2(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)}$$

$$= -\frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(7a-4b)b\cosh(c+dx)}{8a^2(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)}$$

$$= -\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3d} - \frac{}{4a(a-}$$

Mathematica [C] time = 3.37, size = 237, normalized size = 1.43

$$-\frac{\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{8a^2b\cosh(c+dx)}{(a-b)(2a+b\cosh(2(c+dx))-b)^2} + \cdots}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*((Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(
c + d*x)/2])/Sqrt[a - b]])/(a - b)^(5/2) + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^
2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(5/
2) + (8*a^2*b*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])^2) +
(2*a*(7*a - 4*b)*b*Cosh[c + d*x])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)]
)) - 8*Log[Tanh[(c + d*x)/2]])/(a^3*d)
```

fricas [B] time = 0.89, size = 9815, normalized size = 59.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^7 + 28*(7*a^2*b^2 - 4*a*b^3)*
cosh(d*x + c)*sinh(d*x + c)^6 + 4*(7*a^2*b^2 - 4*a*b^3)*sinh(d*x + c)^7 + 4
*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^5 + 4*(36*a^3*b - 31*a^2*b
^2 + 4*a*b^3 + 21*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 +
20*(7*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + (36*a^3*b - 31*a^2*b^2 + 4*a*
b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*c
osh(d*x + c)^3 + 4*(35*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 + 36*a^3*b - 3
1*a^2*b^2 + 4*a*b^3 + 10*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 4*(21*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 10*(36*a^3
*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a
*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cos
h(d*x + c)^8 + 8*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c
)^7 + (15*a^2*b^2 - 20*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 55*a^
2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a
*b^3 - 8*b^4 + 7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 8*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(30*a^3*b
 - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(120*a
^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*
(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 120*a^4 - 280*a^3*b + 269
*a^2*b^2 - 124*a*b^3 + 24*b^4 + 30*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 20*a*b^3 + 8*b^4 + 8*(7*
(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 55*a^2*b^2
 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^3 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 -
 124*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 55*a^2*
b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b
^4)*cosh(d*x + c)^6 + 15*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*
x + c)^4 + 30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 3*(120*a^4 - 280*a^3*
b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 55*a^2*b^2
 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 -
 124*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (30*a^3*b - 55*a^2*b^2 + 36*a*b^3 -
8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a - b))*log((b*cosh(d*x + c)^
4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*c
osh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b
*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*co
sh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x
+ c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d
*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b)) + 4*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c) +
16*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)
*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(d*x + c)^8
+ 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(2*a^3*b - 5*
a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b -
 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 24*
a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*(a^2*b^2 - 2
*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 +
3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x
+ c)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^4
- 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*(a^2*b^2 -
 2*a*b^3 + b^4)*cosh(d*x + c)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*
cosh(d*x + c)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b
```

+ 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 16*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(d*x + c)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(7*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^6 + 5*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^4 + 7*a^2*b^2 - 4*a*b^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^8 + 8*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*sinh(d*x + c)^8 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^3 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^4 + 30*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^4 + 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d + 8*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^5 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^7 + 14*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(7*a^2*b^2 - 4*a*b^3)*sinh(d*x + c)^7 + 2*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^5 + 2*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3 + 21*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + (36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + 2*(35*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 + 36*a^3*b - 31*a^2*b^2 + 4*a*b^3 + 10*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 10*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x +

c))*sinh(d*x + c)^2 + ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^8 + 8*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^2*b^2 - 20*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4 + 30*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 20*a*b^3 + 8*b^4 + 8*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^3 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^6 + 15*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 3*(120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh(d*x + c))) - ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^8 + 8*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^2*b^2 - 20*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4 + 30*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 20*a*b^3 + 8*b^4 + 8*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^3 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^6 + 15*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 3*(120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b) + 2*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c) + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(d*x + c)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b

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^4)*cosh(d*x + c)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^6 + 15*(
2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a^3*b - 5*a^2*b^2
+ 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^7
 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^4 - 24*a^
3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2
 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*
x + c) + 1) - 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a^2*b^2 - 2
*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sin
h(d*x + c)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*
(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^3 +
 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35
*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2
- 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b
^4)*cosh(d*x + c)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c
)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*
(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*
a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a
^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*
b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d
*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(7*a^
2*b^2 - 4*a*b^3)*cosh(d*x + c)^6 + 5*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh
(d*x + c)^4 + 7*a^2*b^2 - 4*a*b^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x +
c)^8 + 8*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 +
(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*sinh(d*x + c)^8 + 4*(2*a^6*b - 5*a^5*b^2
+ 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*
b^4)*d*cosh(d*x + c)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*sin
h(d*x + c)^6 + 2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d
*cosh(d*x + c)^4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^3 +
 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x +
c)^5 + 2*(35*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^4 + 30*(2*a^6*
b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + (8*a^7 - 24*a^6*b
+ 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b - 5*
a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^
3 + a^3*b^4)*d*cosh(d*x + c)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*
b^4)*d*cosh(d*x + c)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^
3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b
^4)*d*cosh(d*x + c)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*co
sh(d*x + c)^4 + 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*
d*cosh(d*x + c)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*sinh(d*x
 + c)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d + 8*((a^5*b^2 - 2*a^4*b^3 + a^3
*b^4)*d*cosh(d*x + c)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*c
osh(d*x + c)^5 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d
*cosh(d*x + c)^3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x +
 c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-85,-18]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[33,-80]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-98,-18]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-57,-1
0]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.The choice was done assuming [a,b]=[-57,-3]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [a,b]=[-53,60]Undef/Unsigned
 Inf encountered in limitEvaluation time: 1Limit: Max order reached or unab
le to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.13, size = 1145, normalized size = 6.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 9/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*b-7/d/(tanh(1/2*d*x+1/2*
c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^6*b^2+4/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^6-27/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+45/2/d
/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-30/d/(tanh(1/2*d*x+1/2*
c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^4*b^3+12/d/a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a-2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh
(1/2*d*x+1/2*c)^4+27/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-17/
d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^
2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*b^2+8/d/a^2*b^3/(tanh(1/2*
d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-9/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*b+3/2/d/a*b
^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)
^2*b+a)^2/(a^2-2*a*b+b^2)-15/8/d/a*b/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan
(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))+5/2/d/a^2*b^2/(a^
2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)
/(a*b-b^2)^(1/2))-1/d/a^3*b^3/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2
*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))+1/d/a^3*ln(tanh(1/2*d*x+
1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(7\,ab^2e^{(7\,c)} - 4\,b^3e^{(7\,c)}\right)e^{(7\,dx)} + \left(36\,a^2b\cdots\right.}{4\left(a^4b^2d - 2\,a^3b^3d + a^2b^4d + \left(a^4b^2de^{(8\,c)} - 2\,a^3b^3de^{(8\,c)} + a^2b^4de^{(8\,c)}\right)e^{(8\,dx)} + 4\left(2\,a^5bde^{(6\,c)} - 5\,a^4b^2de^{(6\,c)} + 4\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((7*a*b^2*e^(7*c) - 4*b^3*e^(7*c))*e^(7*d*x) + (36*a^2*b*e^(5*c) - 31*
a*b^2*e^(5*c) + 4*b^3*e^(5*c))*e^(5*d*x) + (36*a^2*b*e^(3*c) - 31*a*b^2*e^(
```

```
3*c) + 4*b^3*e^(3*c))*e^(3*d*x) + (7*a*b^2*e^c - 4*b^3*e^c)*e^(d*x))/(a^4*b
^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) - 2*a^3*b^3*d*e^(8*c) +
 a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(2*a^5*b*d*e^(6*c) - 5*a^4*b^2*d*e^(6*c)
+ 4*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(8*a^6*d*e^(4*c) -
 24*a^5*b*d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) - 14*a^3*b^3*d*e^(4*c) + 3*a^2*b
^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^5*b*d*e^(2*c) - 5*a^4*b^2*d*e^(2*c) + 4*a^
3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(
-c))/(a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/8*((15
*a^2*b*e^(3*c) - 20*a*b^2*e^(3*c) + 8*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^
c - 20*a*b^2*e^c + 8*b^3*e^c)*e^(d*x))/(a^5*b - 2*a^4*b^2 + a^3*b^3 + (a^5*
b*e^(4*c) - 2*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^6*e^(2*
c) - 5*a^5*b*e^(2*c) + 4*a^4*b^2*e^(2*c) - a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh (c + d\,x)\,\left(b\sinh (c + d\,x)^2 + a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.57 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3} dx$

Optimal. Leaf size=215

$$-\frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3d(a-b)^2}-\frac{b\coth(c+dx)\left(-\left((4a-b)\tanh^2(c+dx)\right)+4a-5b\right)}{8a^2d(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{3b\left(8a^2-12ab+5b\right.}{8a^{7/2}}$$

```
[Out] -3/8*b*(8*a^2-12*a*b+5*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2
)/(a-b)^(5/2)/d-1/8*(4*a-5*b)*(2*a-3*b)*coth(d*x+c)/a^3/(a-b)^2/d-1/4*b*csc
h(d*x+c)*sech(d*x+c)^3/a/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)^2-1/8*b*coth(d*x+c
)*(4*a-5*b-(4*a-b)*tanh(d*x+c)^2)/a^2/(a-b)^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.29, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 468, 577, 453, 208}

$$-\frac{3b\left(8a^2-12ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a-b)^{5/2}}-\frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3d(a-b)^2}-\frac{b\coth(c+dx)\left(-(4a-b)\operatorname{ta}\right.}{8a^2d(a-b)^2\left(a-(a-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 - 12*a*b + 5*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(8*a^(7/2)*(a - b)^(5/2)*d) - ((4*a - 5*b)*(2*a - 3*b)*Coth[c + d*x])/(8*
a^3*(a - b)^2*d) - (b*Csch[c + d*x]*Sech[c + d*x]^3)/(4*a*(a - b)*d*(a - (a
 - b)*Tanh[c + d*x]^2)^2) - (b*Coth[c + d*x]*(4*a - 5*b - (4*a - b)*Tanh[c
+ d*x]^2))/(8*a^2*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\text{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^2\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\,\text{csch}(c+dx)\text{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)\left(4a-5b+(-4a+b)x^2\right)}{x^2\left(a+(-a+b)x^2\right)^2}\,dx, x, \text{t}\right.}{4a(a-b)d}$$

$$= -\frac{b\,\text{csch}(c+dx)\text{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{b\coth(c+dx)\left(4a-5b-(4a-b)\tanh\right.}{8a^2(a-b)^2d\left(a-(a-b)\tanh^2(c\right.}$$

$$= -\frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3(a-b)^2d} - \frac{b\,\text{csch}(c+dx)\text{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{b\,\text{co}}{}$$

$$= -\frac{3b\left(8a^2-12ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^{5/2}d} - \frac{(4a-5b)(2a-3b)\coth(c+dx}{8a^3(a-b)^2d}$$

Mathematica [A] time = 1.81, size = 225, normalized size = 1.05

$$\frac{\text{csch}^6(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\frac{4a^{3/2}b^2\sinh(2(c+dx))}{a-b} - \frac{3b\left(8a^2-12ab+5b^2\right)(2a+b\cosh(2(c+dx))-b)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+d}{\sqrt{a}}\right.}{(a-b)^{5/2}}\right.}{64a^{7/2}d\left(a\,\text{csch}^2(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^6*((-3*b*(8*a^2 - 12*a*b + 5
*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cosh[2*(c +
 d*x)])^2)/(a - b)^(5/2) - 8*Sqrt[a]*(2*a - b + b*Cosh[2*(c + d*x)])^2*Coth
[c + d*x] + (4*a^(3/2)*b^2*Sinh[2*(c + d*x)])/(a - b) + (Sqrt[a]*(10*a - 7*
b)*b^2*(2*a - b + b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a - b)^2))/(64*a
^(7/2)*d*(b + a*Csch[c + d*x]^2)^3)
```

fricas [B] time = 0.96, size = 9102, normalized size = 42.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(12*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^8
+ 96*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)*sinh(d*x
 + c)^7 + 12*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*sinh(d*x + c)^
8 + 24*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*
x + c)^6 + 24*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5 +
 14*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 32*a^4*b^2 - 136*a^3*b^3 + 164*a^2*b^4 - 60*a*b^5 + 48*(14*(8*a
^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^3 + 3*(24*a^5*b -
 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 8*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 4
5*a*b^5)*cosh(d*x + c)^4 + 8*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^
3 + 238*a^2*b^4 - 45*a*b^5 + 105*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a
*b^5)*cosh(d*x + c)^4 + 45*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4
 + 10*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(8*a^4*b^2 - 20*a^3*
b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^5 + 15*(24*a^5*b - 76*a^4*b^2 + 9
1*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^3 + (64*a^6 - 296*a^5*b +
548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c))*sinh(d*x
 + c)^3 + 8*(32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b^5)
*cosh(d*x + c)^2 + 8*(42*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*co
sh(d*x + c)^6 + 32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b
^5 + 45*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d
*x + c)^4 + 6*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4
 - 45*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b^3 - 12*a*b^4 +
5*b^5)*cosh(d*x + c)^10 + 10*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)*s
inh(d*x + c)^9 + (8*a^2*b^3 - 12*a*b^4 + 5*b^5)*sinh(d*x + c)^10 + (64*a^3*
b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^8 + (64*a^3*b^2 - 136
*a^2*b^3 + 100*a*b^4 - 25*b^5 + 45*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 + 8*(15*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c
)^3 + (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c))*sinh(d
*x + c)^7 + 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*c
osh(d*x + c)^6 + 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b
^5 + 105*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^4 + 14*(64*a^3*b^2 -
136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*
(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^5 + 14*(64*a^3*b^2 - 136*a^2*b
^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^3 + 3*(64*a^4*b - 192*a^3*b^2 + 224*
a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*a^2*b^3 +
12*a*b^4 - 5*b^5 - 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25
*b^5)*cosh(d*x + c)^4 + 2*(105*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)
^6 - 64*a^4*b + 192*a^3*b^2 - 224*a^2*b^3 + 120*a*b^4 - 25*b^5 + 35*(64*a^3
*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^4 + 15*(64*a^4*b - 1
92*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(15*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^7 + 7*(64*a^3*b^2
 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^5 + 5*(64*a^4*b - 192*a^
3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^3 - (64*a^4*b - 192
*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c))*sinh(d*x + c)^3
 - (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^2 + (45*(8
*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 136*a^2*b^3
 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^6 - 64*a^3*b^2 + 136*a^2*b^3 - 100*a*b
^4 + 25*b^5 + 30*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5
)*cosh(d*x + c)^4 - 12*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 +
25*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(8*a^2*b^3 - 12*a*b^4 + 5*b
^5)*cosh(d*x + c)^9 + 4*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cos
```

```
h(d*x + c)^7 + 6*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5
)*cosh(d*x + c)^5 - 4*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 2
5*b^5)*cosh(d*x + c)^3 - (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^
2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cos
h(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*
x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 +
2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
(2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*(6*(8*a^4*b^2 - 20*a^3*b^
3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^7 + 9*(24*a^5*b - 76*a^4*b^2 + 91*a
^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^5 + 2*(64*a^6 - 296*a^5*b + 5
48*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c)^3 + (32*a^
5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b^5)*cosh(d*x + c))*si
nh(d*x + c))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^1
0 + 10*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)*sinh(d*x
 + c)^9 + (a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*sinh(d*x + c)^10 +
(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c
)^8 + (45*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^2 + (
8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d)*sinh(d*x + c
)^8 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^
5)*d*cosh(d*x + c)^6 + 8*(15*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*
cosh(d*x + c)^3 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b
^5)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*
b^4 - a^4*b^5)*d*cosh(d*x + c)^4 + 14*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 -
23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2
- 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d)*sinh(d*x + c)^6 - 2*(8*a^9 - 36*a
^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^4
+ 4*(63*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^5 + 14*
(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c
)^3 + 3*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^
5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b
^4 - a^4*b^5)*d*cosh(d*x + c)^6 + 35*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 2
3*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^4 + 15*(8*a^9 - 36*a^8*b + 65*a^7*b^
2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^2 - (8*a^9 - 36*a^
8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d)*sinh(d*x + c)^4
- (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x +
 c)^2 + 8*(15*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^7
 + 7*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*
x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a
^4*b^5)*d*cosh(d*x + c)^3 - (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 2
7*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^7*b^2 - 3*
a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^8 + 28*(8*a^8*b - 29*a^7*b^2
 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^6 + 30*(8*a^9 - 36*
a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^4
 - 12*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)
*d*cosh(d*x + c)^2 - (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^
4*b^5)*d)*sinh(d*x + c)^2 - (a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d +
 2*(5*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^9 + 4*(8*
a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^7
 + 6*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*
d*cosh(d*x + c)^5 - 4*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*
b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^3 - (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 2
3*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(6*(8*a^4*b^2
- 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^8 + 48*(8*a^4*b^2 - 20*a
^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + 6*(8*a^4*b^2
 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*sinh(d*x + c)^8 + 12*(24*a^5*b - 76*a
```

```
^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^6 + 12*(24*a^5*b
 - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5 + 14*(8*a^4*b^2 - 20*a^3
*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*a^4*b^2
- 68*a^3*b^3 + 82*a^2*b^4 - 30*a*b^5 + 24*(14*(8*a^4*b^2 - 20*a^3*b^3 + 17*
a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^3 + 3*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3
- 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(64*a^6 - 296*a
^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c)^4
+ 4*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^
5 + 105*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^4 + 4
5*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c
)^2)*sinh(d*x + c)^4 + 16*(21*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^
5)*cosh(d*x + c)^5 + 15*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 +
10*a*b^5)*cosh(d*x + c)^3 + (64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3
 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(32*a^5*b - 1
44*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b^5)*cosh(d*x + c)^2 + 4*(42*
(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^6 + 32*a^5*b
- 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b^5 + 45*(24*a^5*b - 76*a^
4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^4 + 6*(64*a^6 - 2
96*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c
)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^10 +
 10*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)*sinh(d*x + c)^9 + (8*a^2*b
^3 - 12*a*b^4 + 5*b^5)*sinh(d*x + c)^10 + (64*a^3*b^2 - 136*a^2*b^3 + 100*a
*b^4 - 25*b^5)*cosh(d*x + c)^8 + (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25
*b^5 + 45*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
 8*(15*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^3 + (64*a^3*b^2 - 136*a
^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(64*a^4*b -
 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^6 + 2*(64*a^
4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5 + 105*(8*a^2*b^3 - 12*
a*b^4 + 5*b^5)*cosh(d*x + c)^4 + 14*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 -
 25*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(8*a^2*b^3 - 12*a*b^4 + 5
*b^5)*cosh(d*x + c)^5 + 14*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*
cosh(d*x + c)^3 + 3*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*
b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*a^2*b^3 + 12*a*b^4 - 5*b^5 - 2*(64*
a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^4 + 2
*(105*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^6 - 64*a^4*b + 192*a^3*b
^2 - 224*a^2*b^3 + 120*a*b^4 - 25*b^5 + 35*(64*a^3*b^2 - 136*a^2*b^3 + 100*
a*b^4 - 25*b^5)*cosh(d*x + c)^4 + 15*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3
- 120*a*b^4 + 25*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(8*a^2*b^3 -
 12*a*b^4 + 5*b^5)*cosh(d*x + c)^7 + 7*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^
4 - 25*b^5)*cosh(d*x + c)^5 + 5*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120
*a*b^4 + 25*b^5)*cosh(d*x + c)^3 - (64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 -
120*a*b^4 + 25*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - (64*a^3*b^2 - 136*a^2*
b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^2 + (45*(8*a^2*b^3 - 12*a*b^4 + 5*b
^5)*cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*co
sh(d*x + c)^6 - 64*a^3*b^2 + 136*a^2*b^3 - 100*a*b^4 + 25*b^5 + 30*(64*a^4*
b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^4 - 12*(6
4*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 2*(5*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^9 + 4*(
64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^7 + 6*(64*a^4*
b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^5 - 4*(64
*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^3 -
(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c))*sinh(d*x + c
))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh
(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 8*
(6*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^7 + 9*(24*
a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^5 +
2*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)
*cosh(d*x + c)^3 + (32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30
*a*b^5)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a
```

```
^4*b^5)*d*cosh(d*x + c)^10 + 10*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)
*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b
^5)*d*sinh(d*x + c)^10 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 +
5*a^4*b^5)*d*cosh(d*x + c)^8 + (45*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b
^5)*d*cosh(d*x + c)^2 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5
*a^4*b^5)*d)*sinh(d*x + c)^8 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^
3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^7*b^2 - 3*a^6*b^3
+ 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^3 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b
^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7
*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^4 + 14*(8*a^8*b - 2
9*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^2 + (8*a^9
 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d)*sinh(d*x
 + c)^6 - 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^
4*b^5)*d*cosh(d*x + c)^4 + 4*(63*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5
)*d*cosh(d*x + c)^5 + 14*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 +
5*a^4*b^5)*d*cosh(d*x + c)^3 + 3*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^
3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7*
b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^6 + 35*(8*a^8*b - 29
*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^4 + 15*(8*a
^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*
x + c)^2 - (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4
*b^5)*d)*sinh(d*x + c)^4 - (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4
+ 5*a^4*b^5)*d*cosh(d*x + c)^2 + 8*(15*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a
^4*b^5)*d*cosh(d*x + c)^7 + 7*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b
^4 + 5*a^4*b^5)*d*cosh(d*x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a
^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^3 - (8*a^9 - 36*a^8*b + 65
*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x +
 c)^3 + (45*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^8 +
 28*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x
 + c)^6 + 30*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a
^4*b^5)*d*cosh(d*x + c)^4 - 12*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3
+ 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^2 - (8*a^8*b - 29*a^7*b^2 + 39*a^
6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d)*sinh(d*x + c)^2 - (a^7*b^2 - 3*a^6*b^3 +
 3*a^5*b^4 - a^4*b^5)*d + 2*(5*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*
d*cosh(d*x + c)^9 + 4*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a
^4*b^5)*d*cosh(d*x + c)^7 + 6*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 +
 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^5 - 4*(8*a^9 - 36*a^8*b + 65*a^7*b
^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^3 - (8*a^8*b - 29
*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x +
 c))]
```

giac [A] time = 1.62, size = 331, normalized size = 1.54

$$-\frac{\frac{3\left(8a^2b-12ab^2+5b^3\right)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\left(a^5-2a^4b+a^3b^2\right)\sqrt{-a^2+ab}}+\frac{2\left(16a^2b^2e^{(6dx+6c)}-20ab^3e^{(6dx+6c)}+7b^4e^{(6dx+6c)}+80a^3be^{(4dx+4c)}-136a^2b^2e^{(4dx+4c)}+86ab^3e^{(\cdots}\right.}{\left(a^5-2a^4b+a^3b^2\right)\left(be^{(4dx+4c)}+4ae^{(\cdots}\right.}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(8*a^2*b - 12*a*b^2 + 5*b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a -
b)/sqrt(-a^2 + a*b))/((a^5 - 2*a^4*b + a^3*b^2)*sqrt(-a^2 + a*b)) + 2*(16*a
^2*b^2*e^(6*d*x + 6*c) - 20*a*b^3*e^(6*d*x + 6*c) + 7*b^4*e^(6*d*x + 6*c) +
 80*a^3*b*e^(4*d*x + 4*c) - 136*a^2*b^2*e^(4*d*x + 4*c) + 86*a*b^3*e^(4*d*x
 + 4*c) - 21*b^4*e^(4*d*x + 4*c) + 64*a^2*b^2*e^(2*d*x + 2*c) - 76*a*b^3*e^
(2*d*x + 2*c) + 21*b^4*e^(2*d*x + 2*c) + 10*a*b^3 - 7*b^4)/((a^5 - 2*a^4*b
+ a^3*b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) +
 b)^2) + 16/(a^3*(e^(2*d*x + 2*c) - 1)))/d
```

maple [B] time = 0.15, size = 1850, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/a^3*tanh(1/2*d*x+1/2*c)+3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+
1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*
x+1/2*c)^7-9/4/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-3/d/(
tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b^2+49/4/d/(tanh(1/2*d*x+1/2*
c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-7/d/a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a-2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(
1/2*d*x+1/2*c)^5-3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*t
anh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b^2+49/
4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-7/d/a^3*b^4/(tanh(
1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2
/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)-9/4/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*
c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+3
/d/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-9/2/d/a^2/(a^2-2*a*b+b^
2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2+15/8/d/a^3*b^3/(a^2-2*a*b+b^2)/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2))+3/d/(a^2-2*a*b+b^2)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2))*b^2-9/2/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2))*b^3+15/8/d/a^3*b^4/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2))-3/d/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+
9/2/d/a^2/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-15/8/d/a^3*b^3/
(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d/(a^2-2*a*b+b^2)/a/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-9/2/d/a^2/(a^2-2*a*b+b^2)/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^3+15/8/d/a^3*b^4/(a^2-2*a*b+b^2
)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*
d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a^3/tanh(1/2*d*x+1/2
*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(c+d\,x\right)^2\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.58 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=224

$$\frac{(a+6b)\tanh^{-1}(\cosh(c+dx))}{2a^4d}-\frac{b(a-4b)(4a-3b)\cosh(c+dx)}{8a^3d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{b(2a-3b)\cosh(c+dx)}{4a^2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}$$

```
[Out] 1/8*b^(3/2)*(35*a^2-56*a*b+24*b^2)*arctan(cosh(d*x+c)*b^(1/2)/(a-b)^(1/2))/
a^4/(a-b)^(5/2)/d+1/2*(a+6*b)*arctanh(cosh(d*x+c))/a^4/d-1/4*(2*a-3*b)*b*co
sh(d*x+c)/a^2/(a-b)/d/(a-b+b*cosh(d*x+c)^2)^2-1/8*(a-4*b)*(4*a-3*b)*b*cosh(
d*x+c)/a^3/(a-b)^2/d/(a-b+b*cosh(d*x+c)^2)-1/2*coth(d*x+c)*csch(d*x+c)/a/d/
(a-b+b*cosh(d*x+c)^2)^2
```

Rubi [A] time = 0.44, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}\left(35a^2-56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^4d(a-b)^{5/2}}-\frac{b(a-4b)(4a-3b)\cosh(c+dx)}{8a^3d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{b(2a-3b)\cos}{4a^2d(a-b)\left(a+b\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a -
 b]])/(8*a^4*(a - b)^(5/2)*d) + ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d
) - ((2*a - 3*b)*b*Cosh[c + d*x])/(4*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]
^2)^2) - ((a - 4*b)*(4*a - 3*b)*b*Cosh[c + d*x])/(8*a^3*(a - b)^2*d*(a - b
+ b*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cos
h[c + d*x]^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+bx^2)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+b+5bx^2}{(1-x^2)(a-b+bx^2)^3}\,dx, x, \cosh(c+dx)\right)}{2ad}$$

$$= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{\text{Subs}}{}$$

$$= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(a-4b)(4a-3b)b\cosh(c+dx)}{8a^3(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)}$$

$$= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(a-4b)(4a-3b)b\cosh(c+dx)}{8a^3(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)}$$

$$= \frac{b^{3/2}\left(35a^2-56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{5/2}d} + \frac{(a+6b)\tanh^{-1}(\cosh(c+dx))}{2a^4d}$$

Mathematica [C] time = 2.81, size = 419, normalized size = 1.87

$$\frac{\operatorname{csch}^5(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\frac{8a^2b^2\coth(c+dx)}{a-b} + \frac{b^{3/2}\left(35a^2-56ab+24b^2\right)\operatorname{csch}(c+dx)(2a+b\cosh(2(c+dx))-b)^2\tan^{-1}\left(\frac{\sqrt{b}}{}\right.}{(a-b)^{5/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^5*((8*a^2*b^2*Coth[c + d*x])
/(a - b) + (2*a*(11*a - 8*b)*b^2*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d
*x])/(a - b)^2 + (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b] - I*Sq
rt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])^2*Csc
h[c + d*x])/(a - b)^(5/2) + (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqr
t[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d
*x)])^2*Csch[c + d*x])/(a - b)^(5/2) - a*(2*a - b + b*Cosh[2*(c + d*x)])^2*
Csch[(c + d*x)/2]^2*Csch[c + d*x] - 4*(a + 6*b)*(2*a - b + b*Cosh[2*(c + d*
x)])^2*Csch[c + d*x]*Log[Tanh[(c + d*x)/2]] - a*(2*a - b + b*Cosh[2*(c + d*
x)])^2*Csch[c + d*x]*Sech[(c + d*x)/2]^2))/(64*a^4*d*(b + a*Csch[c + d*x]^2
)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-85,-18]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[33,-80]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-98,-18]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-57,-1
0]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.The choice was done assuming [a,b]=[-57,-3]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [a,b]=[-53,60]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[80,-1]schur row 3 -6.9034e-07
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[-51,-3]Undef/Un
signed Inf encountered in limitEvaluation time: 1.52Limit: Max order reache
d or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.18, size = 1225, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/a^3-13/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*
x+1/2*c)^6*b^2+10/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^
```

```
2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-6/
d/a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+39/4/d/(tanh(1/2*d*x
+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*b^2/
(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-67/2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^4*b^3+46/d/a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x
+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2
*c)^4-20/d/a^4*b^5/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-39/4/d/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*b^2+26/d/a^2*b^3/(tanh(1/2*d*x+1
/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a
*b+b^2)*tanh(1/2*d*x+1/2*c)^2-14/d/a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*
d*x+1/2*c)^2+13/4/d/a*b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)-5/2/d/a^2*b^3/(tanh(1/2*d*
x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-
2*a*b+b^2)+35/8/d/a^2*b^2/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tan
h(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))-7/d/a^3*b^3/(a^2-2*a*b+b^2)/
(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1
/2))+3/d/a^4*b^4/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x
+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))-1/8/d/a^3/tanh(1/2*d*x+1/2*c)^2-1/2/d
/a^3*ln(tanh(1/2*d*x+1/2*c))-3/d/a^4*ln(tanh(1/2*d*x+1/2*c))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((4*a^2*b^2*e^(11*c) - 19*a*b^3*e^(11*c) + 12*b^4*e^(11*c))*e^(11*d*x)
 + (32*a^3*b*e^(9*c) - 128*a^2*b^2*e^(9*c) + 129*a*b^3*e^(9*c) - 36*b^4*e^(
9*c))*e^(9*d*x) + 2*(32*a^4*e^(7*c) - 80*a^3*b*e^(7*c) + 94*a^2*b^2*e^(7*c)
 - 55*a*b^3*e^(7*c) + 12*b^4*e^(7*c))*e^(7*d*x) + 2*(32*a^4*e^(5*c) - 80*a^
3*b*e^(5*c) + 94*a^2*b^2*e^(5*c) - 55*a*b^3*e^(5*c) + 12*b^4*e^(5*c))*e^(5*
d*x) + (32*a^3*b*e^(3*c) - 128*a^2*b^2*e^(3*c) + 129*a*b^3*e^(3*c) - 36*b^4
*e^(3*c))*e^(3*d*x) + (4*a^2*b^2*e^c - 19*a*b^3*e^c + 12*b^4*e^c)*e^(d*x))/
(a^5*b^2*d - 2*a^4*b^3*d + a^3*b^4*d + (a^5*b^2*d*e^(12*c) - 2*a^4*b^3*d*e^
(12*c) + a^3*b^4*d*e^(12*c))*e^(12*d*x) + 2*(4*a^6*b*d*e^(10*c) - 11*a^5*b^
2*d*e^(10*c) + 10*a^4*b^3*d*e^(10*c) - 3*a^3*b^4*d*e^(10*c))*e^(10*d*x) + (
16*a^7*d*e^(8*c) - 64*a^6*b*d*e^(8*c) + 95*a^5*b^2*d*e^(8*c) - 62*a^4*b^3*d
*e^(8*c) + 15*a^3*b^4*d*e^(8*c))*e^(8*d*x) - 4*(8*a^7*d*e^(6*c) - 28*a^6*b*
d*e^(6*c) + 37*a^5*b^2*d*e^(6*c) - 22*a^4*b^3*d*e^(6*c) + 5*a^3*b^4*d*e^(6*
c))*e^(6*d*x) + (16*a^7*d*e^(4*c) - 64*a^6*b*d*e^(4*c) + 95*a^5*b^2*d*e^(4*
c) - 62*a^4*b^3*d*e^(4*c) + 15*a^3*b^4*d*e^(4*c))*e^(4*d*x) + 2*(4*a^6*b*d*
e^(2*c) - 11*a^5*b^2*d*e^(2*c) + 10*a^4*b^3*d*e^(2*c) - 3*a^3*b^4*d*e^(2*c)
)*e^(2*d*x)) + 1/2*(a + 6*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^4*d) - 1/2*(a
 + 6*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^4*d) + 8*integrate(1/32*((35*a^2*b
^2*e^(3*c) - 56*a*b^3*e^(3*c) + 24*b^4*e^(3*c))*e^(3*d*x) - (35*a^2*b^2*e^c
 - 56*a*b^3*e^c + 24*b^4*e^c)*e^(d*x))/(a^6*b - 2*a^5*b^2 + a^4*b^3 + (a^6*
b*e^(4*c) - 2*a^5*b^2*e^(4*c) + a^4*b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^7*e^(2*
c) - 5*a^6*b*e^(2*c) + 4*a^5*b^2*e^(2*c) - a^4*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh(c+d\,x)^3\,{\left(b\,\sinh(c+d\,x)^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.59 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=259

$$-\frac{b(10a-7b)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{8a^2d(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{b^2\left(48a^2-80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a-b)^{5/2}}-\frac{\left(8a^2-52ab+35b^2\right.}{24a^3d(a}$$

```
[Out] 1/8*b^2*(48*a^2-80*a*b+35*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(
9/2)/(a-b)^(5/2)/d+1/8*(8*a^3-4*a^2*b-45*a*b^2+35*b^3)*coth(d*x+c)/a^4/(a-b
)^2/d-1/24*(8*a^2-52*a*b+35*b^2)*coth(d*x+c)^3/a^3/(a-b)^2/d-1/4*b*csch(d*x
+c)^3*sech(d*x+c)^3/a/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)^2-1/8*(10*a-7*b)*b*cs
ch(d*x+c)^3*sech(d*x+c)/a^2/(a-b)^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.34, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3187, 468, 577, 570, 208}

$$\frac{b^2\left(48a^2-80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a-b)^{5/2}}-\frac{\left(8a^2-52ab+35b^2\right)\coth^3(c+dx)}{24a^3d(a-b)^2}+\frac{\left(-4a^2b+8a^3-45ab^2+3\right.}{8a^4d(a-b}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]
])/(8*a^(9/2)*(a - b)^(5/2)*d) + ((8*a^3 - 4*a^2*b - 45*a*b^2 + 35*b^3)*Cot
h[c + d*x])/(8*a^4*(a - b)^2*d) - ((8*a^2 - 52*a*b + 35*b^2)*Coth[c + d*x]^
3)/(24*a^3*(a - b)^2*d) - (b*Csch[c + d*x]^3*Sech[c + d*x]^3)/(4*a*(a - b)*
d*(a - (a - b)*Tanh[c + d*x]^2)^2) - ((10*a - 7*b)*b*Csch[c + d*x]^3*Sech[c
 + d*x])/(8*a^2*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^4\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(4a-7b+(-4a+b)x^2\right)}{x^4\left(a+(-a+b)x^2\right)^2}\,dx,\right.}{4a(a-b)d}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(10a-7b)b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+}{8a^2(a-b)^2d\left(a-(a-b)\tanh^2(c+\right.}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(10a-7b)b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+}{8a^2(a-b)^2d\left(a-(a-b)\tanh^2(c+\right.}$$

$$= \frac{\left(8a^3-4a^2b-45ab^2+35b^3\right)\coth(c+dx)}{8a^4(a-b)^2d} - \frac{\left(8a^2-52ab+35b^2\right)\coth^3(c+dx}{24a^3(a-b)^2d}$$

$$= \frac{b^2\left(48a^2-80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^{5/2}d} + \frac{\left(8a^3-4a^2b-45ab^2+35b\right.}{8a^4(a-b)^2c}$$

Mathematica [A] time = 2.80, size = 167, normalized size = 0.64

$$\frac{\frac{3b^2\left(48a^2-80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}} + \sqrt{a}\left(\frac{3b^3\sinh(2(c+dx))\left(-32a^2+b(11b-14a)\cosh(2(c+dx))+40ab-11b^2\right)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} - 8\coth(c+d\right.}{24a^{9/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt
[a]])/(a - b)^(5/2) + Sqrt[a]*(-8*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c + d*
x]^2) + (3*b^3*(-32*a^2 + 40*a*b - 11*b^2 + b*(-14*a + 11*b)*Cosh[2*(c + d*
x)])*Sinh[2*(c + d*x)])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2)))/(24
*a^(9/2)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.68, size = 378, normalized size = 1.46

$$\frac{\frac{3\left(48\,a^2b^2-80\,ab^3+35\,b^4\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^6-2\,a^5b+a^4b^2\right)\sqrt{-a^2+ab}}+\frac{6\left(24\,a^2b^3e^{(6\,dx+6\,c)}-32\,ab^4e^{(6\,dx+6\,c)}+11\,b^5e^{(6\,dx+6\,c)}+112\,a^3b^2e^{(4\,dx+4\,c)}-200\,a^2b^3e^{(4\,dx+4\,c)}+13\cdots\right.}{\left(a^6-2\,a^5b+a^4b^2\right)\left(be^{(4\,dx+4\,c)}\cdots\right.}}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(48*a^2*b^2 - 80*a*b^3 + 35*b^4)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*
a - b)/sqrt(-a^2 + a*b))/((a^6 - 2*a^5*b + a^4*b^2)*sqrt(-a^2 + a*b)) + 6*(
24*a^2*b^3*e^(6*d*x + 6*c) - 32*a*b^4*e^(6*d*x + 6*c) + 11*b^5*e^(6*d*x + 6
*c) + 112*a^3*b^2*e^(4*d*x + 4*c) - 200*a^2*b^3*e^(4*d*x + 4*c) + 130*a*b^4
*e^(4*d*x + 4*c) - 33*b^5*e^(4*d*x + 4*c) + 88*a^2*b^3*e^(2*d*x + 2*c) - 11
2*a*b^4*e^(2*d*x + 2*c) + 33*b^5*e^(2*d*x + 2*c) + 14*a*b^4 - 11*b^5)/((a^6
 - 2*a^5*b + a^4*b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d
*x + 2*c) + b)^2) + 16*(9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 18*b*e^
(2*d*x + 2*c) + 2*a + 9*b)/(a^4*(e^(2*d*x + 2*c) - 1)^3))/d
```

maple [B] time = 0.18, size = 1930, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -6/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^3-
6/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/
2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^3-
35/8/d/a^4*b^5/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
))-35/8/d/a^4*b^5/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2))+10/d/a^3*b^4/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2))+10/d/a^3*b^4/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(
1/2))+4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+4/d/(tanh(1/
2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a
^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-6/d/a^2/(a^2-2*a*b+b^2)/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2))*b^2+6/d/a^2/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(
1/2))*b^2+13/4/d/a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+11/d/
a^4*b^5/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+11/d/a^4*b^5/(tanh(1/2
```

```
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a
^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+13/4/d/a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a
-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*t
anh(1/2*d*x+1/2*c)-10/d/a^3*b^3/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
))-4/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/
2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-69/4/d/a^3*b^4/
(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*
b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-69/4/d/a^3*b^4/(tanh(1/2*d*x+1
/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a
*b+b^2)*tanh(1/2*d*x+1/2*c)^3-4/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)+10/d/a^3*b^3/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+35/8/d/
a^4*b^4/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-35/8/d/a^4*b^4/(a^2-2
*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d/a^3*tanh(1/2*d*x+1/2*c)+3/8/d/
a^3/tanh(1/2*d*x+1/2*c)-1/24/d/a^3/tanh(1/2*d*x+1/2*c)^3-1/24/d/a^3*tanh(1/
2*d*x+1/2*c)^3+3/2/d/a^4*tanh(1/2*d*x+1/2*c)*b+3/2/d/a^4/tanh(1/2*d*x+1/2*c
)*b
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh(c+d\,x)^4\,\left(b\sinh(c+d\,x)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.60 $\int \frac{1}{1+\sinh^2(x)}\,dx$

Optimal. Leaf size=2

$$\tanh(x)$$

[Out] tanh(x)

Rubi [A] time = 0.02, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3175, 3767, 8}

$$\tanh(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^2)^(-1),x]

[Out] Tanh[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sinh^2(x)}\,dx &= \int \operatorname{sech}^2(x)\,dx\\ &= i\,\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)\\ &= \tanh(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\tanh(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^2)^(-1),x]

[Out] Tanh[x]

fricas [B] time = 1.05, size = 20, normalized size = 10.00

$$-\frac{2}{\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2),x, algorithm="fricas")
[Out] -2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

giac [B] time = 0.12, size = 10, normalized size = 5.00

$$-\frac{2}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2),x, algorithm="giac")
[Out] -2/(e^(2*x) + 1)
```

maple [B] time = 0.04, size = 17, normalized size = 8.50

$$\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2),x)
[Out] 2*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

maxima [B] time = 0.31, size = 10, normalized size = 5.00

$$\frac{2}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2),x, algorithm="maxima")
[Out] 2/(e^(-2*x) + 1)
```

mupad [B] time = 0.04, size = 10, normalized size = 5.00

$$-\frac{2}{e^{2x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 + 1),x)
[Out] -2/(exp(2*x) + 1)
```

sympy [B] time = 0.54, size = 14, normalized size = 7.00

$$\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2),x)
[Out] 2*tanh(x/2)/(tanh(x/2)**2 + 1)
```

3.61 $\int \frac{1}{\left(1+\sinh^2(x)\right)^2} \, dx$

Optimal. Leaf size=11

$$\tanh(x)-\frac{\tanh^3(x)}{3}$$

```
[Out] tanh(x)-1/3*tanh(x)^3
```

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3175, 3767}

$$\tanh(x)-\frac{\tanh^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)^(-2),x]
```

```
[Out] Tanh[x] - Tanh[x]^3/3
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1+\sinh^2(x)\right)^2} \, dx &= \int \operatorname{sech}^4(x) \, dx \\
&= i \operatorname{Subst}\left(\int \left(1+x^2\right) \, dx, x, -i \tanh(x)\right) \\
&= \tanh(x)-\frac{\tanh^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.55

$$\frac{2 \tanh(x)}{3}+\frac{1}{3} \tanh(x) \operatorname{sech}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)^(-2),x]
```

```
[Out] (2*Tanh[x])/3 + (Sech[x]^2*Tanh[x])/3
```

fricas [B] time = 1.03, size = 84, normalized size = 7.64

$$-\frac{8\,(2\,\cosh(x)+\sinh(x))}{3\left(\cosh(x)^5+5\,\cosh(x)\sinh(x)^4+\sinh(x)^5+\left(10\,\cosh(x)^2+3\right)\sinh(x)^3+3\,\cosh(x)^3+\left(10\,\cosh(x)^3+9\,\text{cc}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/3*(2*cosh(x) + sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + (
10*cosh(x)^2 + 3)*sinh(x)^3 + 3*cosh(x)^3 + (10*cosh(x)^3 + 9*cosh(x))*sinh
(x)^2 + (5*cosh(x)^4 + 9*cosh(x)^2 + 2)*sinh(x) + 4*cosh(x))
```

giac [A] time = 0.13, size = 18, normalized size = 1.64

$$-\frac{4\left(3\,e^{(2\,x)}+1\right)}{3\left(e^{(2\,x)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*x) + 1)/(e^(2*x) + 1)^3
```

maple [B] time = 0.04, size = 36, normalized size = 3.27

$$-\frac{2\left(-\left(\tanh^{5}\left(\frac{x}{2}\right)\right)-\frac{2\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}-\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2)^2,x)
```

```
[Out] -2*(-tanh(1/2*x)^5-2/3*tanh(1/2*x)^3-tanh(1/2*x))/(tanh(1/2*x)^2+1)^3
```

maxima [B] time = 0.31, size = 49, normalized size = 4.45

$$\frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}+\frac{4}{3\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e
^(-4*x) + e^(-6*x) + 1)
```

mupad [B] time = 0.59, size = 18, normalized size = 1.64

$$-\frac{4\left(3\,\mathrm{e}^{2\,x}+1\right)}{3\left(\mathrm{e}^{2\,x}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 + 1)^2,x)
```

```
[Out] -(4*(3*exp(2*x) + 1))/(3*(exp(2*x) + 1)^3)
```

sympy [B] time = 1.57, size = 104, normalized size = 9.45

$$\frac{6\tanh^{5}\left(\frac{x}{2}\right)}{3\tanh^{6}\left(\frac{x}{2}\right)+9\tanh^{4}\left(\frac{x}{2}\right)+9\tanh^{2}\left(\frac{x}{2}\right)+3}+\frac{4\tanh^{3}\left(\frac{x}{2}\right)}{3\tanh^{6}\left(\frac{x}{2}\right)+9\tanh^{4}\left(\frac{x}{2}\right)+9\tanh^{2}\left(\frac{x}{2}\right)+3}+\frac{6}{3\tanh^{6}\left(\frac{x}{2}\right)+9\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2)**2,x)
```

```
[Out] 6*tanh(x/2)**5/(3*tanh(x/2)**6 + 9*tanh(x/2)**4 + 9*tanh(x/2)**2 + 3) + 4*t
anh(x/2)**3/(3*tanh(x/2)**6 + 9*tanh(x/2)**4 + 9*tanh(x/2)**2 + 3) + 6*tanh
(x/2)/(3*tanh(x/2)**6 + 9*tanh(x/2)**4 + 9*tanh(x/2)**2 + 3)
```

3.62 $\int \frac{1}{\left(1+\sinh^2(x)\right)^3} dx$

Optimal. Leaf size=19

$$\frac{\tanh^5(x)}{5} - \frac{2\tanh^3(x)}{3} + \tanh(x)$$

```
[Out] tanh(x)-2/3*tanh(x)^3+1/5*tanh(x)^5
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3175, 3767}

$$\frac{\tanh^5(x)}{5} - \frac{2\tanh^3(x)}{3} + \tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)^(-3),x]
[Out] Tanh[x] - (2*Tanh[x]^3)/3 + Tanh[x]^5/5
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1+\sinh^2(x)\right)^3} dx &= \int \operatorname{sech}^6(x)\, dx \\
&= i \operatorname{Subst}\left(\int \left(1+2x^2+x^4\right) dx, x, -i\tanh(x)\right) \\
&= \tanh(x) - \frac{2\tanh^3(x)}{3} + \frac{\tanh^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.42

$$\frac{8\tanh(x)}{15} + \frac{1}{5}\tanh(x)\operatorname{sech}^4(x) + \frac{4}{15}\tanh(x)\operatorname{sech}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)^(-3),x]
[Out] (8*Tanh[x])/15 + (4*Sech[x]^2*Tanh[x])/15 + (Sech[x]^4*Tanh[x])/5
```

fricas [B] time = 2.20, size = 185, normalized size = 9.74

$$-\frac{}{15\left(\cosh(x)^8 + 8\cosh(x)\sinh(x)^7 + \sinh(x)^8 + \left(28\cosh(x)^2+5\right)\sinh(x)^6 + 5\cosh(x)^6 + 2\left(28\cosh(x)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(x)^2 + 18*cosh(x)*sinh(x) + 11*sinh(x)^2 + 5)/(cosh(x)^8 +
8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 + 5)*sinh(x)^6 + 5*cosh(x)^
6 + 2*(28*cosh(x)^3 + 15*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 + 15*cosh(x)^
2 + 2)*sinh(x)^4 + 10*cosh(x)^4 + 4*(14*cosh(x)^5 + 25*cosh(x)^3 + 10*cosh(
x))*sinh(x)^3 + (28*cosh(x)^6 + 75*cosh(x)^4 + 60*cosh(x)^2 + 11)*sinh(x)^2
 + 11*cosh(x)^2 + 2*(4*cosh(x)^7 + 15*cosh(x)^5 + 20*cosh(x)^3 + 9*cosh(x))
*sinh(x) + 5)
```

giac [A] time = 0.12, size = 24, normalized size = 1.26

$$-\frac{16\left(10\,e^{(4\,x)}+5\,e^{(2\,x)}+1\right)}{15\left(e^{(2\,x)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*x) + 5*e^(2*x) + 1)/(e^(2*x) + 1)^5
```

maple [B] time = 0.04, size = 52, normalized size = 2.74

$$-\frac{2\left(-\left(\tanh^{9}\left(\frac{x}{2}\right)\right)-\frac{4\left(\tanh^{7}\left(\frac{x}{2}\right)\right)}{3}-\frac{58\left(\tanh^{5}\left(\frac{x}{2}\right)\right)}{15}-\frac{4\left(\tanh^{3}\left(\frac{x}{2}\right)\right)}{3}-\tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh^{2}\left(\frac{x}{2}\right)+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2)^3,x)
```

```
[Out] -2*(-tanh(1/2*x)^9-4/3*tanh(1/2*x)^7-58/15*tanh(1/2*x)^5-4/3*tanh(1/2*x)^3-
tanh(1/2*x))/(tanh(1/2*x)^2+1)^5
```

maxima [B] time = 0.32, size = 111, normalized size = 5.84

$$\frac{16\,e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)}+\frac{32\,e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10
*x) + 1) + 32/3*e^(-4*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*
x) + e^(-10*x) + 1) + 16/15/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(
-8*x) + e^(-10*x) + 1)
```

mupad [B] time = 0.61, size = 24, normalized size = 1.26

$$-\frac{16\left(5\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{4\,x}+1\right)}{15\left(\mathrm{e}^{2\,x}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 + 1)^3,x)
```

```
[Out] -(16*(5*exp(2*x) + 10*exp(4*x) + 1))/(15*(exp(2*x) + 1)^5)
```

sympy [B] time = 3.62, size = 260, normalized size = 13.68

$$\frac{30\tanh^{9}\left(\frac{x}{2}\right)}{15\tanh^{10}\left(\frac{x}{2}\right)+75\tanh^{8}\left(\frac{x}{2}\right)+150\tanh^{6}\left(\frac{x}{2}\right)+150\tanh^{4}\left(\frac{x}{2}\right)+75\tanh^{2}\left(\frac{x}{2}\right)+15}+\frac{}{15\tanh^{10}\left(\frac{x}{2}\right)+75\tanh^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2)**3,x)
```

```
[Out] 30*tanh(x/2)**9/(15*tanh(x/2)**10 + 75*tanh(x/2)**8 + 150*tanh(x/2)**6 + 15
0*tanh(x/2)**4 + 75*tanh(x/2)**2 + 15) + 40*tanh(x/2)**7/(15*tanh(x/2)**10
+ 75*tanh(x/2)**8 + 150*tanh(x/2)**6 + 150*tanh(x/2)**4 + 75*tanh(x/2)**2 +
 15) + 116*tanh(x/2)**5/(15*tanh(x/2)**10 + 75*tanh(x/2)**8 + 150*tanh(x/2)
**6 + 150*tanh(x/2)**4 + 75*tanh(x/2)**2 + 15) + 40*tanh(x/2)**3/(15*tanh(x
/2)**10 + 75*tanh(x/2)**8 + 150*tanh(x/2)**6 + 150*tanh(x/2)**4 + 75*tanh(x
/2)**2 + 15) + 30*tanh(x/2)/(15*tanh(x/2)**10 + 75*tanh(x/2)**8 + 150*tanh(
x/2)**6 + 150*tanh(x/2)**4 + 75*tanh(x/2)**2 + 15)
```

3.63 $\int \frac{1}{1-\sinh^2(x)} \, dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3181, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1-\sinh^2(x)} \, dx &= \text{Subst}\left(\int \frac{1}{1-2x^2} \, dx, x, \tanh(x)\right) \\ &= \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{1-\sinh^2(x)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - Sinh[x]^2)^(-1),x]
```

```
[Out] Integrate[(1 - Sinh[x]^2)^(-1), x]
```

fricas [B] time = 1.44, size = 66, normalized size = 4.40

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2-2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3))
```

giac [B] time = 0.12, size = 37, normalized size = 2.47

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6))
```

maple [B] time = 0.04, size = 40, normalized size = 2.67

$$\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{2}+\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2),x)
```

```
[Out] 1/2*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+1/2*2^(1/2)*arctanh(1/4*
(2*tanh(1/2*x)+2)*2^(1/2))
```

maxima [B] time = 0.41, size = 61, normalized size = 4.07

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/4*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1))
```

mupad [B] time = 0.15, size = 50, normalized size = 3.33

$$-\frac{\sqrt{2}\ \left(\ln\left(4\,e^{2\,x}-\frac{\sqrt{2}\ \left(12\,e^{2\,x}-4\right)}{4}\right)-\ln\left(4\,e^{2\,x}+\frac{\sqrt{2}\ \left(12\,e^{2\,x}-4\right)}{4}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^2 - 1),x)
```

```
[Out] -(2^(1/2)*(log(4*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/4) - log(4*exp(2*x)
 + (2^(1/2)*(12*exp(2*x) - 4))/4)))/4
```

sympy [B] time = 1.51, size = 209, normalized size = 13.93

$$\frac{816\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{1632\sqrt{2}+2308}+\frac{577\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{1632\sqrt{2}+2308}+\frac{816\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{1632\sqrt{2}+2308}+\frac{577\sqrt{2}\log\left(\right.}{1632}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2),x)
```

```
[Out] 816*log(tanh(x/2) - 1 + sqrt(2))/(1632*sqrt(2) + 2308) + 577*sqrt(2)*log(ta
nh(x/2) - 1 + sqrt(2))/(1632*sqrt(2) + 2308) + 816*log(tanh(x/2) + 1 + sqrt
(2))/(1632*sqrt(2) + 2308) + 577*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(1632
*sqrt(2) + 2308) - 577*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(1632*sqrt(2) +
 2308) - 816*log(tanh(x/2) - sqrt(2) - 1)/(1632*sqrt(2) + 2308) - 577*sqrt(
2)*log(tanh(x/2) - sqrt(2) + 1)/(1632*sqrt(2) + 2308) - 816*log(tanh(x/2) -
 sqrt(2) + 1)/(1632*sqrt(2) + 2308)
```

3.64 $\int \frac{1}{\left(1-\sinh^2(x)\right)^2} dx$

Optimal. Leaf size=37

$$\frac{3\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\sinh(x)\cosh(x)}{4\left(1-\sinh^2(x)\right)}$$

```
[Out] 1/4*cosh(x)*sinh(x)/(1-sinh(x)^2)+3/8*arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3184, 12, 3181, 206}

$$\frac{3\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\sinh(x)\cosh(x)}{4\left(1-\sinh^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(-2),x]
```

```
[Out] (3*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) + (Cosh[x]*Sinh[x])/(4*(1 - Sinh[x
]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(1-\sinh^2(x)\right)^2}\,dx = \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)} - \frac{1}{4}\int -\frac{3}{1-\sinh^2(x)}\,dx$$
$$= \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)} + \frac{3}{4}\int \frac{1}{1-\sinh^2(x)}\,dx$$
$$= \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{3\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}} + \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)}$$

Mathematica [A] time = 0.13, size = 35, normalized size = 0.95

$$\frac{3\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}} - \frac{\sinh(2x)}{4(\cosh(2x)-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(-2),x]
[Out] (3*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) - Sinh[2*x]/(4*(-3 + Cosh[2*x]))
```

fricas [B] time = 1.42, size = 216, normalized size = 5.84

$$-\frac{24\cosh(x)^2 - 3\left(\sqrt{2}\cosh(x)^4 + 4\sqrt{2}\cosh(x)\sinh(x)^3 + \sqrt{2}\sinh(x)^4 + 6\left(\sqrt{2}\cosh(x)^2 - \sqrt{2}\right)\sinh(x)^2 - 6\sqrt{}\cdots}{16\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^2,x, algorithm="fricas")
[Out] -1/16*(24*cosh(x)^2 - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 +
sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^2 - 6*sqrt(2)*c
osh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log
(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*s
qrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) + 48*co
sh(x)*sinh(x) + 24*sinh(x)^2 - 8)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x
)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))
*sinh(x) + 1)
```

giac [B] time = 0.12, size = 62, normalized size = 1.68

$$-\frac{3}{16}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2} + 2e^{(2x)} - 6\right|}{\left|4\sqrt{2} + 2e^{(2x)} - 6\right|}\right) - \frac{3e^{(2x)} - 1}{2\left(e^{(4x)} - 6e^{(2x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^2,x, algorithm="giac")
[Out] -3/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - 1/2*(3*e^(2*x) - 1)/(e^(4*x) - 6*e^(2*x) + 1)
```

maple [B] time = 0.04, size = 92, normalized size = 2.49

$$-\frac{-\frac{\tanh\left(\frac{x}{2}\right)}{4} - \frac{1}{4}}{\tanh^2\left(\frac{x}{2}\right) - 2\tanh\left(\frac{x}{2}\right) - 1} + \frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{8} - \frac{-\frac{\tanh\left(\frac{x}{2}\right)}{4} + \frac{1}{4}}{\tanh^2\left(\frac{x}{2}\right) + 2\tanh\left(\frac{x}{2}\right) - 1} + \frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2\tanh\cdots}{\cdots}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2)^2,x)
```

```
[Out] -(-1/4*tanh(1/2*x)-1/4)/(tanh(1/2*x)^2-2*tanh(1/2*x)-1)+3/8*2^(1/2)*arctanh
(1/4*(2*tanh(1/2*x)-2)*2^(1/2))-(-1/4*tanh(1/2*x)+1/4)/(tanh(1/2*x)^2+2*tan
h(1/2*x)-1)+3/8*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

maxima [B] time = 0.41, size = 87, normalized size = 2.35

$$\frac{3}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{3}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-\frac{3\,e^{(-2\,x)}-1}{2\left(6\,e^{(-2\,x)}-e^{(-4\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 3/16*sqr
t(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 1/2*(3*e^(-2*x)
- 1)/(6*e^(-2*x) - e^(-4*x) - 1)
```

mupad [B] time = 0.69, size = 77, normalized size = 2.08

$$\frac{3\sqrt{2}\,\ln\left(3\,e^{2\,x}+\frac{3\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{16}\right)}{16}-\frac{3\sqrt{2}\,\ln\left(3\,e^{2\,x}-\frac{3\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{16}\right)}{16}-\frac{\frac{3\,e^{2\,x}}{2}-\frac{1}{2}}{e^{4\,x}-6\,e^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 - 1)^2,x)
```

```
[Out] (3*2^(1/2)*log(3*exp(2*x) + (3*2^(1/2)*(12*exp(2*x) - 4))/16))/16 - (3*2^(1
/2)*log(3*exp(2*x) - (3*2^(1/2)*(12*exp(2*x) - 4))/16))/16 - ((3*exp(2*x))/
2 - 1/2)/(exp(4*x) - 6*exp(2*x) + 1)
```

sympy [B] time = 9.29, size = 2052, normalized size = 55.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2)**2,x)
```

```
[Out] 525888*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x/2)
**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x
/2)**2 + 1402368*sqrt(2) + 1983248) + 371859*sqrt(2)*log(tanh(x/2) - 1 + sq
rt(2))*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 -
11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 19
83248) - 2231154*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(1402368
*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414
208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 3155328*log(tanh(x/
2) - 1 + sqrt(2))*tanh(x/2)**2/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh
(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*s
qrt(2) + 1983248) + 525888*log(tanh(x/2) - 1 + sqrt(2))/(1402368*sqrt(2)*ta
nh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)
*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) + 371859*sqrt(2)*log(tanh(x/2) -
 1 + sqrt(2))/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 118994
88*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248)
 + 525888*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x
/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tan
h(x/2)**2 + 1402368*sqrt(2) + 1983248) + 371859*sqrt(2)*log(tanh(x/2) + 1 +
 sqrt(2))*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4
```

```
 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) +
 1983248) - 2231154*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1402
368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8
414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 3155328*log(tanh
(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*t
anh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 140236
8*sqrt(2) + 1983248) + 525888*log(tanh(x/2) + 1 + sqrt(2))/(1402368*sqrt(2)
*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt
(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) + 371859*sqrt(2)*log(tanh(x/2
) + 1 + sqrt(2))/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 118
99488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 19832
48) - 371859*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**4/(1402368*sqr
t(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*
sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 525888*log(tanh(x/2) -
sqrt(2) - 1)*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)
**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2
) + 1983248) + 3155328*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**2/(1402368*s
qrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 841420
8*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) + 2231154*sqrt(2)*log(t
anh(x/2) - sqrt(2) - 1)*tanh(x/2)**2/(1402368*sqrt(2)*tanh(x/2)**4 + 198324
8*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 140
2368*sqrt(2) + 1983248) - 371859*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(1402
368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8
414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 525888*log(tanh(
x/2) - sqrt(2) - 1)/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 -
11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 19
83248) - 371859*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**4/(1402368*
sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 84142
08*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 525888*log(tanh(x/2)
 - sqrt(2) + 1)*tanh(x/2)**4/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x
/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqr
t(2) + 1983248) + 3155328*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(140236
8*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 841
4208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) + 2231154*sqrt(2)*lo
g(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(1402368*sqrt(2)*tanh(x/2)**4 + 198
3248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 +
1402368*sqrt(2) + 1983248) - 371859*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)/(1
402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2
- 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) - 525888*log(ta
nh(x/2) - sqrt(2) + 1)/(1402368*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4
 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) +
 1983248) + 701184*sqrt(2)*tanh(x/2)**3/(1402368*sqrt(2)*tanh(x/2)**4 + 198
3248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x/2)**2 +
1402368*sqrt(2) + 1983248) + 991624*tanh(x/2)**3/(1402368*sqrt(2)*tanh(x/2)
**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*sqrt(2)*tanh(x
/2)**2 + 1402368*sqrt(2) + 1983248) + 701184*sqrt(2)*tanh(x/2)/(1402368*sqr
t(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 8414208*
sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248) + 991624*tanh(x/2)/(14023
68*sqrt(2)*tanh(x/2)**4 + 1983248*tanh(x/2)**4 - 11899488*tanh(x/2)**2 - 84
14208*sqrt(2)*tanh(x/2)**2 + 1402368*sqrt(2) + 1983248)
```

3.65 $\int \frac{1}{\left(1-\sinh^2(x)\right)^3} \, dx$

Optimal. Leaf size=55

$$\frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}}+\frac{9\sinh(x)\cosh(x)}{32\left(1-\sinh^2(x)\right)}+\frac{\sinh(x)\cosh(x)}{8\left(1-\sinh^2(x)\right)^2}$$

```
[Out] 1/8*cosh(x)*sinh(x)/(1-sinh(x)^2)^2+9/32*cosh(x)*sinh(x)/(1-sinh(x)^2)+19/6
4*arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3184, 3173, 12, 3181, 206}

$$\frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}}+\frac{9\sinh(x)\cosh(x)}{32\left(1-\sinh^2(x)\right)}+\frac{\sinh(x)\cosh(x)}{8\left(1-\sinh^2(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(-3),x]
```

```
[Out] (19*ArcTanh[Sqrt[2]*Tanh[x]])/(32*Sqrt[2]) + (Cosh[x]*Sinh[x])/(8*(1 - Sinh
[x]^2)^2) + (9*Cosh[x]*Sinh[x])/(32*(1 - Sinh[x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-\sinh^2(x)\right)^3}\,dx &= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2} - \frac{1}{8}\int \frac{-7-2\sinh^2(x)}{\left(1-\sinh^2(x)\right)^2}\,dx \\
&= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2} + \frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)} - \frac{1}{32}\int -\frac{19}{1-\sinh^2(x)}\,dx \\
&= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2} + \frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)} + \frac{19}{32}\int \frac{1}{1-\sinh^2(x)}\,dx \\
&= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2} + \frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)} + \frac{19}{32}\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right) \\
&= \frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}} + \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2} + \frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 51, normalized size = 0.93

$$\frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}} - \frac{9\sinh(2x)}{32(\cosh(2x)-3)} + \frac{\sinh(2x)}{4(\cosh(2x)-3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(-3),x]
```

```
[Out] (19*ArcTanh[Sqrt[2]*Tanh[x]])/(32*Sqrt[2]) + Sinh[2*x]/(4*(-3 + Cosh[2*x])^
2) - (9*Sinh[2*x])/(32*(-3 + Cosh[2*x]))
```

fricas [B] time = 0.57, size = 575, normalized size = 10.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(152*cosh(x)^6 + 912*cosh(x)*sinh(x)^5 + 152*sinh(x)^6 + 456*(5*cosh
(x)^2 - 3)*sinh(x)^4 - 1368*cosh(x)^4 + 608*(5*cosh(x)^3 - 9*cosh(x))*sinh(
x)^3 + 8*(285*cosh(x)^4 - 1026*cosh(x)^2 + 89)*sinh(x)^2 + 712*cosh(x)^2 -
19*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4
*(7*sqrt(2)*cosh(x)^2 - 3*sqrt(2))*sinh(x)^6 - 12*sqrt(2)*cosh(x)^6 + 8*(7*
sqrt(2)*cosh(x)^3 - 9*sqrt(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4
- 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2))*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(
7*sqrt(2)*cosh(x)^5 - 30*sqrt(2)*cosh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3
+ 4*(7*sqrt(2)*cosh(x)^6 - 45*sqrt(2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 - 3*
sqrt(2))*sinh(x)^2 - 12*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 - 9*sqrt(2
)*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*
log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(
2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) + 16
*(57*cosh(x)^5 - 342*cosh(x)^3 + 89*cosh(x))*sinh(x) - 72)/(cosh(x)^8 + 8*c
osh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*sinh(x)^6 - 12*cosh(x)^6
 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 90*cosh(x)^2 +
 19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 - 30*cosh(x)^3 + 19*cosh(x))
*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 + 57*cosh(x)^2 - 3)*sinh(x)^2 -
12*cosh(x)^2 + 8*(cosh(x)^7 - 9*cosh(x)^5 + 19*cosh(x)^3 - 3*cosh(x))*sinh(
x) + 1)
```

giac [A] time = 0.14, size = 74, normalized size = 1.35

$$-\frac{19}{128}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-\frac{19e^{(6x)}-171e^{(4x)}+89e^{(2x)}-9}{16\left(e^{(4x)}-6e^{(2x)}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -19/128*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*
x) - 6)) - 1/16*(19*e^(6*x) - 171*e^(4*x) + 89*e^(2*x) - 9)/(e^(4*x) - 6*e^
(2*x) + 1)^2
```

maple [B] time = 0.04, size = 124, normalized size = 2.25

$$-\frac{-\frac{13\left(\tanh^3\left(\frac{x}{2}\right)\right)}{8}+\frac{11\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8}+\frac{31\tanh\left(\frac{x}{2}\right)}{8}+\frac{11}{8}}{4\left(\tanh^2\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{19\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{64}-\frac{-\frac{13\left(\tanh^3\left(\frac{x}{2}\right)\right)}{8}-\frac{11\left(\tanh^2\left(\frac{x}{2}\right)\right)}{8}+}{4\left(\tanh^2\left(\frac{x}{2}\right)+2\tanh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2)^3,x)
```

```
[Out] -1/4*(-13/8*tanh(1/2*x)^3+11/8*tanh(1/2*x)^2+31/8*tanh(1/2*x)+11/8)/(tanh(1
/2*x)^2-2*tanh(1/2*x)-1)^2+19/64*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1
/2))-1/4*(-13/8*tanh(1/2*x)^3-11/8*tanh(1/2*x)^2+31/8*tanh(1/2*x)-11/8)/(ta
nh(1/2*x)^2+2*tanh(1/2*x)-1)^2+19/64*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*
2^(1/2))
```

maxima [B] time = 0.41, size = 111, normalized size = 2.02

$$\frac{19}{128}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{19}{128}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-\frac{89e^{(-2x)}-171e^{(-4x)}+19e^{(-6x)}-9}{16\left(12e^{(-2x)}-38e^{(-4x)}+12e^{(-6x)}-e^{(-8x)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 19/128*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 19/128
*sqrt(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 1/16*(89*e^(
-2*x) - 171*e^(-4*x) + 19*e^(-6*x) - 9)/(12*e^(-2*x) - 38*e^(-4*x) + 12*e^(
-6*x) - e^(-8*x) - 1)
```

mupad [B] time = 0.61, size = 112, normalized size = 2.04

$$\frac{17e^{2x}-3}{38e^{4x}-12e^{2x}-12e^{6x}+e^{8x}+1}-\frac{19\sqrt{2}\ \ln\left(\frac{19e^{2x}}{8}-\frac{19\sqrt{2}\left(12e^{2x}-4\right)}{128}\right)}{128}+\frac{19\sqrt{2}\ \ln\left(\frac{19e^{2x}}{8}+\frac{19\sqrt{2}\left(12e^{2x}-4\right)}{128}\right)}{128}-\frac{}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^2 - 1)^3,x)
```

```
[Out] (17*exp(2*x) - 3)/(38*exp(4*x) - 12*exp(2*x) - 12*exp(6*x) + exp(8*x) + 1)
- (19*2^(1/2)*log((19*exp(2*x))/8 - (19*2^(1/2)*(12*exp(2*x) - 4))/128))/12
8 + (19*2^(1/2)*log((19*exp(2*x))/8 + (19*2^(1/2)*(12*exp(2*x) - 4))/128))/
128 - ((19*exp(2*x))/16 - 57/16)/(exp(4*x) - 6*exp(2*x) + 1)
```

sympy [B] time = 30.96, size = 5666, normalized size = 103.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2)**3,x)
```

```
[Out] 10001001174720*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**8/(33687582904320*sq
rt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2
)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh
(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 4
04250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 476414366270
72) + 7071775749331*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**8/(3368
7582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 57169723952
4864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160
*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tan
h(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) +
 47641436627072) - 84861308991972*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh
(x/2)**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8
 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 12
80128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 57169
7239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 3368758290
4320*sqrt(2) + 47641436627072) - 120012014096640*log(tanh(x/2) - 1 + sqrt(2
))*tanh(x/2)**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(
x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)*
*6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4
- 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 336
87582904320*sqrt(2) + 47641436627072) + 380038044639360*log(tanh(x/2) - 1 +
 sqrt(2))*tanh(x/2)**4/(33687582904320*sqrt(2)*tanh(x/2)**8 + 4764143662707
2*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tan
h(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x
/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**
2 + 33687582904320*sqrt(2) + 47641436627072) + 268727478474578*sqrt(2)*log(
tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**4/(33687582904320*sqrt(2)*tanh(x/2)**8
+ 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851
840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 18103745
91828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt
(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) - 8486130899197
2*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(33687582904320*sqrt(2)
*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6
- 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)
**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250
994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) -
 120012014096640*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(33687582904320*
sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x
/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*ta
nh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 -
 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 4764143662
7072) + 10001001174720*log(tanh(x/2) - 1 + sqrt(2))/(33687582904320*sqrt(2)
*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6
- 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)
**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250
994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) +
 7071775749331*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))/(33687582904320*sqrt(2)
*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6
- 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)
**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250
994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) +
 10001001174720*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**8/(33687582904320*s
qrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/
2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tan
h(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 -
404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627
```

```
072) + 7071775749331*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**8/(336
87582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 5716972395
24864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 128012815036416
0*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*ta
nh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2)
+ 47641436627072) - 84861308991972*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tan
h(x/2)**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**
8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1
280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 5716
97239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 336875829
04320*sqrt(2) + 47641436627072) - 120012014096640*log(tanh(x/2) + 1 + sqrt(
2))*tanh(x/2)**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh
(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)
**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4
 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33
687582904320*sqrt(2) + 47641436627072) + 380038044639360*log(tanh(x/2) + 1
+ sqrt(2))*tanh(x/2)**4/(33687582904320*sqrt(2)*tanh(x/2)**8 + 476414366270
72*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*ta
nh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(
x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)*
*2 + 33687582904320*sqrt(2) + 47641436627072) + 268727478474578*sqrt(2)*log
(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**4/(33687582904320*sqrt(2)*tanh(x/2)**8
 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 40425099485
1840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374
591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqr
t(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) - 848613089919
72*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(33687582904320*sqrt(2
)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6
 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2
)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 40425
0994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072)
- 120012014096640*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(33687582904320
*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(
x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*t
anh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2
- 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 476414366
27072) + 10001001174720*log(tanh(x/2) + 1 + sqrt(2))/(33687582904320*sqrt(2
)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6
 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2
)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 40425
0994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072)
+ 7071775749331*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(33687582904320*sqrt(2
)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6
 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2
)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 40425
0994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072)
- 7071775749331*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**8/(33687582
904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864
*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqr
t(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/
2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 476
41436627072) - 10001001174720*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**8/(33
687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239
524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 12801281503641
60*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*t
anh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2)
 + 47641436627072) + 120012014096640*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)
**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 57
1697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128
```

```
150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 5716972395
24864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*
sqrt(2) + 47641436627072) + 84861308991972*sqrt(2)*log(tanh(x/2) - sqrt(2)
- 1)*tanh(x/2)**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tan
h(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2
)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**
4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 3
3687582904320*sqrt(2) + 47641436627072) - 268727478474578*sqrt(2)*log(tanh(
x/2) - sqrt(2) - 1)*tanh(x/2)**4/(33687582904320*sqrt(2)*tanh(x/2)**8 + 476
41436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*s
qrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828
736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*t
anh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) - 380038044639360*lo
g(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**4/(33687582904320*sqrt(2)*tanh(x/2)**
8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 4042509948
51840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 181037
4591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sq
rt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) + 12001201409
6640*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**2/(33687582904320*sqrt(2)*tanh
(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404
250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 +
 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 40425099485
1840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) + 8486
1308991972*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**2/(3368758290432
0*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh
(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*
tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2
 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436
627072) - 7071775749331*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(3368758290432
0*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh
(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*
tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2
 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436
627072) - 10001001174720*log(tanh(x/2) - sqrt(2) - 1)/(33687582904320*sqrt(
2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**
6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/
2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 4042
50994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072)
 - 7071775749331*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**8/(3368758
2904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 57169723952486
4*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sq
rt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x
/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47
641436627072) - 10001001174720*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**8/(3
3687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 57169723
9524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364
160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*
tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2
) + 47641436627072) + 120012014096640*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2
)**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 5
71697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 128012
8150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239
524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320
*sqrt(2) + 47641436627072) + 84861308991972*sqrt(2)*log(tanh(x/2) - sqrt(2)
 + 1)*tanh(x/2)**6/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*ta
nh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/
2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)*
*4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 +
33687582904320*sqrt(2) + 47641436627072) - 268727478474578*sqrt(2)*log(tanh
```

```
(x/2) - sqrt(2) + 1)*tanh(x/2)**4/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47
641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*
sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 181037459182
8736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*
tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) - 380038044639360*l
og(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**4/(33687582904320*sqrt(2)*tanh(x/2)*
*8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994
851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 18103
74591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*s
qrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) + 1200120140
96640*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(33687582904320*sqrt(2)*tan
h(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 40
4250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4
 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 4042509948
51840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) + 848
61308991972*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(336875829043
20*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tan
h(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)
*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**
2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 4764143
6627072) - 7071775749331*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)/(336875829043
20*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tan
h(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)
*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**
2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 4764143
6627072) - 10001001174720*log(tanh(x/2) - sqrt(2) + 1)/(33687582904320*sqrt
(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)*
*6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x
/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404
250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072
) + 27371161109760*sqrt(2)*tanh(x/2)**7/(33687582904320*sqrt(2)*tanh(x/2)**
8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 4042509948
51840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 181037
4591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sq
rt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) + 38708667259
496*tanh(x/2)**7/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh
(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)
**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4
 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33
687582904320*sqrt(2) + 47641436627072) - 145901899670408*tanh(x/2)**5/(3368
7582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 57169723952
4864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160
*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tan
h(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) +
 47641436627072) - 103168222644480*sqrt(2)*tanh(x/2)**5/(33687582904320*sqr
t(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)
**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(
x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 40
4250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 4764143662707
2) - 145901899670408*tanh(x/2)**3/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47
641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*
sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 181037459182
8736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*
tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072) - 103168222644480*s
qrt(2)*tanh(x/2)**3/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*t
anh(x/2)**8 - 571697239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x
/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)
**4 - 571697239524864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 +
 33687582904320*sqrt(2) + 47641436627072) + 27371161109760*sqrt(2)*tanh(x/2
```

```
)/(33687582904320*sqrt(2)*tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 5716
97239524864*tanh(x/2)**6 - 404250994851840*sqrt(2)*tanh(x/2)**6 + 128012815
0364160*sqrt(2)*tanh(x/2)**4 + 1810374591828736*tanh(x/2)**4 - 571697239524
864*tanh(x/2)**2 - 404250994851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sq
rt(2) + 47641436627072) + 38708667259496*tanh(x/2)/(33687582904320*sqrt(2)*
tanh(x/2)**8 + 47641436627072*tanh(x/2)**8 - 571697239524864*tanh(x/2)**6 -
 404250994851840*sqrt(2)*tanh(x/2)**6 + 1280128150364160*sqrt(2)*tanh(x/2)*
*4 + 1810374591828736*tanh(x/2)**4 - 571697239524864*tanh(x/2)**2 - 4042509
94851840*sqrt(2)*tanh(x/2)**2 + 33687582904320*sqrt(2) + 47641436627072)
```

3.66 $\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=130

$$-\frac{(a-b)(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8b^{3/2}f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4bf}-\frac{(a+3b)\cosh(e+fx}{}$$

```
[Out] -1/8*(a-b)*(a+3*b)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))
/b^(3/2)/f+1/4*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(3/2)/b/f-1/8*(a+3*b)*cosh
(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 388, 195, 217, 206}

$$-\frac{(a-b)(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8b^{3/2}f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4bf}-\frac{(a+3b)\cosh(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((a - b)*(a + 3*b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e +
 f*x]^2]])/(8*b^(3/2)*f) - ((a + 3*b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e +
 f*x]^2])/(8*b*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*b*
f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int\left(1-x^2\right)\sqrt{a-b+bx^2}\,dx,x,\cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{4bf} - \frac{(a+3b)\,\text{Subst}\left(\int\sqrt{a-\right.}{}$$

$$= -\frac{(a+3b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8bf} + \frac{\cosh(e+fx)\left(a\right.}{}$$

$$= -\frac{(a+3b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8bf} + \frac{\cosh(e+fx)\left(a\right.}{}$$

$$= -\frac{(a-b)(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a+3b)\cosh(e+fx)}{}$$

Mathematica [A] time = 0.67, size = 114, normalized size = 0.88

$$\frac{\frac{(b-a)(a+3b)\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{b^{3/2}}+\frac{\cosh(e+fx)\sqrt{4a+2b\cosh(2(e+fx))-2b}\,(a+b\cosh(2(e+fx))-4b)}{2b}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((Cosh[e + f*x]*(a - 4*b + b*Cosh[2*(e + f*x)])*Sqrt[4*a - 2*b + 2*b*Cosh[2
*(e + f*x)]])/(2*b) + ((-a + b)*(a + 3*b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x]
 + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])/b^(3/2))/(8*f)
```

fricas [B] time = 1.37, size = 3037, normalized size = 23.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(2*((a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*
cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sin
h(f*x + e)^2 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2
 + 2*a*b - 3*b^2)*sinh(f*x + e)^4)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a
^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b
)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e
)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x
 + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x +
e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x
 + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 +
(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2
 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*co
sh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2
```

```
)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh
(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f
*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*
x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x
+ e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3
*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))
*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) +
4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e
))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*
sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*s
inh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 2*((
a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e
)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2
 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 + 2*a*b - 3
*b^2)*sinh(f*x + e)^4)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*
sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*co
sh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)
^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))
*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2)) - sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x
 + e)^5 + b^2*sinh(f*x + e)^6 + (2*a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b^2*c
osh(f*x + e)^2 + 2*a*b - 7*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3
+ (2*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + (2*a*b - 7*b^2)*cosh(f*x
 + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(2*a*b - 7*b^2)*cosh(f*x + e)^2 + 2*a
*b - 7*b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(2*a*b - 7
*b^2)*cosh(f*x + e)^3 + (2*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^4 + 4*b^2
*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^2
+ 4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + b^2*f*sinh(f*x + e)^4), 1/64*(4*(
(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x +
e)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sinh(f*x + e)^
2 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 + 2*a*b -
3*b^2)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*co
sh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e
)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (
6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(
f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 4*((a^2 + 2*a*b
 - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^3*sinh(f*
x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 +
 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 + 2*a*b - 3*b^2)*sinh(
f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*
sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*
a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*si
nh(f*x + e) + b)) + sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh
(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (2*a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b
^2*cosh(f*x + e)^2 + 2*a*b - 7*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e
)^3 + (2*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + (2*a*b - 7*b^2)*cosh
(f*x + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(2*a*b - 7*b^2)*cosh(f*x + e)^2 +
 2*a*b - 7*b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(2*a*b
```

```
 - 7*b^2)*cosh(f*x + e)^3 + (2*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^4 + 4
*b^2*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b^2*f*cosh(f*x + e)^2*sinh(f*x + e
)^2 + 4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + b^2*f*sinh(f*x + e)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.17, size = 339, normalized size = 2.61

$$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(4\sqrt{b\left(\cosh^4\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^2\left(fx+e\right)\right)}\,b^{\frac{5}{2}}\left(\cosh^2\left(fx+e\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(4*(b*cosh(f*x+e)^4+(a-b)*co
sh(f*x+e)^2)^(1/2)*b^(5/2)*cosh(f*x+e)^2-10*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x
+e)^2)^(1/2)*b^(5/2)+2*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(3/2
)-ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b
^(1/2)+a-b)/b^(1/2))*a^2*b-2*a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4
+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*b^2+3*b^3*ln(1/2*(2*b*cos
h(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/
2)))/b^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a}\, \sinh\left(fx+e\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e+f\,x\right)^3\, \sqrt{b\sinh\left(e+f\,x\right)^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.67 $\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{b}f}$$

```
[Out] 1/2*(a-b)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/b^(1/2
)+1/2*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 195, 217, 206}

$$\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*Sqrt[b]*f) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a-b+bx^2}\,dx, x, \cosh(e+fx)\right)}{f}$$
$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-b+b x^2}}$$
$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,d$$
$$= \frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2\sqrt{b}\,f} + \frac{\cosh(e+fx)\sqrt{a-b+b\cosh$$

Mathematica [A] time = 0.15, size = 97, normalized size = 1.18

$$\frac{\cosh(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}}{2\sqrt{2}f} + \frac{(a-b)\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/(2*Sqrt[2]*f) + ((a - b
)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])
/(2*Sqrt[b]*f)
```

fricas [B] time = 0.56, size = 2130, normalized size = 25.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (
a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f
*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x
+ e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14
*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (
9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^
2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4
*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b -
4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cos
h(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)
^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x
+ e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5
 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 +
 a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sin
h(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18
*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f
*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*
cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^
3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*
```

```
x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x +
e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e
)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a - b)*cosh(f
*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2
)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*s
inh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b
)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(
2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2
 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^
2 + 2*b*f*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2), -1/8*(2*((a -
 b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(
f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*
sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x +
 e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f
*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 +
 (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((a - b)*cosh(f*x + e)^2
+ 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2)*sqrt(-b)
*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*c
osh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*
(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)
^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) -
sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x +
 e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x
+ e)^2 + 2*b*f*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.11, size = 200, normalized size = 2.44

$$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(2\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^{2}\left(fx+e\right)\right)}\sqrt{b}+a\ln\left(\frac{2b(\cosh}{}\right.\right.}{4\sqrt{b}\cosh\left(fx+e\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)*b^(1/2)+a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a
-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-b*ln(1/2*(2*b*cosh(f*x+e)^2+
```

```
2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/b^(1/2
)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a}\,\sinh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e+f\,x\right)\,\sqrt{b\,{\sinh\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh^{2}{\left(e + fx \right)}} \sinh{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sinh(e + f*x), x)
```

3.68 $\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}$$

```
[Out] -arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))*a^(1/2)/f+arctanh
(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3186, 402, 217, 206, 377}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x
]^2]])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{1-x^2}\,dx, x, \cosh(e+fx)\right)}{f}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{}{\sqrt{a-l}}\right.}{}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,\right.}{}$$
$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.13, size = 97, normalized size = 1.15

$$\frac{\sqrt{b}\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]]]) + Sqrt[b]*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b +
b*Cosh[2*(e + f*x)]]])/f
```

fricas [B] time = 1.25, size = 4423, normalized size = 52.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x +
 e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2
*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x +
e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2
+ b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^
3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*
x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cos
h(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(
14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3
 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(
a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x +
e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e
)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*
a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)
^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*
cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 +
3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3
```

```
 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(
f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x
 + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x +
 e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 2*sqrt(a)*log(-((a + b)*cosh(f*x
+ e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4
+ 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sin
h(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) +
 a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^
4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh
(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(b)*log(-(b*cosh(f*x
 + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)
*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2
)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*s
qrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e
)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/f, 1/4*(4*sqrt(-a)*arctan(sqrt(2
)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*s
qrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4
 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh
(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh
(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(b)*log((a
^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f
*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 +
 a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*
b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)
^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)
*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 +
 a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x
 + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x +
e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*
b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6
+ 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*
x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(
f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x
+ e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*si
nh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4
*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh
(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*c
osh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x
+ e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e
)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5
+ sinh(f*x + e)^6)) + sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cos
h(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*
sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)))/f, -1/2*(sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cos
h(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)
```

```
*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6
*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f
*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + sqrt(-b)*arctan(
sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
- 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(
f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x
+ e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b
)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(
b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(a)*
log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (
a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*
x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*
x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
+ e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*co
sh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)))/f,
 1/2*(2*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh
(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f
*x + e) + b)) - sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x +
 e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f
*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*co
sh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)
^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) - sqrt(-b)*arctan(sqrt(2)
*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sq
rt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4
+ 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(
f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(
f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.16, size = 174, normalized size = 2.07

$$-\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\,\left(\sqrt{a}\,\ln\left(\frac{(a+b)\left(\cosh^{2}(fx+e)\right)+2\sqrt{a}\,\sqrt{b\left(\cosh^{4}(fx+e)\right)+(a-b)\left(\cosh^{2}(fx+e)\right)}+a-}{\sinh(fx+e)^{2}}\right.\right.}{2\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(a^(1/2)*ln(((a+b)*cosh(f*x+
e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)
```

```
^2)-b^(1/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^
2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \operatorname{csch}\left(fx+e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b \sinh\left(e+f\,x\right)^2 + a}}{\sinh\left(e+f\,x\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh^2\left(e+fx\right)} \operatorname{csch}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*csch(e + f*x), x)
```

3.69 $\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=88

$$\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{a}f}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}$$

```
[Out] 1/2*(a-b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/a^(1/2
)-1/2*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3186, 378, 377, 206}

$$\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{a}f}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*Sqrt[a]*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x
])/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{(1-x^2)^2}dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2f} + \frac{(a-b)\operatorname{Sub}}{}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2f} + \frac{(a-b)\operatorname{Sub}}{}$$

$$= \frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a-b+b\cosh^2(e+fx)}\operatorname{coth}}{2f}$$

Mathematica [A] time = 0.29, size = 104, normalized size = 1.18

$$\frac{2(a-b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)-\sqrt{2}\sqrt{a}\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}}{4\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (2*(a - b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e +
 f*x]*Csch[e + f*x])/(4*Sqrt[a]*f)
```

fricas [B] time = 0.66, size = 1277, normalized size = 14.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a - b)*sinh(f*x + e)^4 - 2*(a - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*
x + e)^2 - a + b)*sinh(f*x + e)^2 + 4*((a - b)*cosh(f*x + e)^3 - (a - b)*co
sh(f*x + e))*sinh(f*x + e) + a - b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 +
 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a
 - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x +
e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*
((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/
(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3
*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e
)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^2 + 2
*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^4 + 4*a*f*cosh(f*x + e)*s
inh(f*x + e)^3 + a*f*sinh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 + 2*(3*a*f*cos
h(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 4*(a*f*cosh(f*x + e)^3 - a*f*co
sh(f*x + e))*sinh(f*x + e)), -1/2*(((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cos
h(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f*x + e)^4 - 2*(a - b)*cosh(f*x +
```

```
 e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 - a + b)*sinh(f*x + e)^2 + 4*((a - b)*
cosh(f*x + e)^3 - (a - b)*cosh(f*x + e))*sinh(f*x + e) + a - b)*sqrt(-a)*ar
ctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh
(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*
a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2
+ 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqr
t(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)
^2 + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e
)^4 + 4*a*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*f*sinh(f*x + e)^4 - 2*a*f*cos
h(f*x + e)^2 + 2*(3*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 4*(a
*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.23, size = 230, normalized size = 2.61

$$-\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(-a\ln\left(\frac{(a+b)\left(\cosh^{2}\left(fx+e\right)\right)+2\sqrt{a}\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+(a-b)\left(\cosh^{2}\left(fx+e\right)\right)}+a-b}{\sinh\left(fx+e\right)^{2}}\right.\right.}{4\sqrt{a}\ \mathrm{sir}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-a*ln(((a+b)*cosh(f*x+e)^2+
2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*s
inh(f*x+e)^2+b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2+2*a^(1/2)*((a+b*sinh(f*
x+e)^2)*cosh(f*x+e)^2)^(1/2))/a^(1/2)/sinh(f*x+e)^2/cosh(f*x+e)/(a+b*sinh(f
*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^{2}+a}\ \operatorname{csch}\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b \sinh\left(e+f\,x\right)^{2}+a}}{\sinh\left(e+f\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^3,x)
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.70 $\int \mathrm{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=144

$$-\frac{(a-b)(3a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8a^{3/2}f}-\frac{\coth(e+fx)\mathrm{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4af}+(3a+b$$

```
[Out] -1/8*(a-b)*(3*a+b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))
/a^(3/2)/f-1/4*(a-b+b*cosh(f*x+e)^2)^(3/2)*coth(f*x+e)*csch(f*x+e)^3/a/f+1/
8*(3*a+b)*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 382, 378, 377, 206}

$$-\frac{(a-b)(3a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8a^{3/2}f}-\frac{\coth(e+fx)\mathrm{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4af}+(3a+b$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((a - b)*(3*a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e +
 f*x]^2]])/(8*a^(3/2)*f) + ((3*a + b)*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[
e + f*x]*Csch[e + f*x])/(8*a*f) - ((a - b + b*Cosh[e + f*x]^2)^(3/2)*Coth[e
 + f*x]*Csch[e + f*x]^3)/(4*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{\left(1-x^2\right)^3}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\operatorname{csch}^3(e+fx)}{4af} - \frac{(3a+b)}{}$$

$$= \frac{(3a+b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{8af} - \frac{\left(a-\right.}{}$$

$$= \frac{(3a+b)\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{8af} - \frac{\left(a-\right.}{}$$

$$= -\frac{(a-b)(3a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{(3a+b)\sqrt{a-b+b\,\mathrm{cc}}}{}$$

Mathematica [A] time = 0.54, size = 129, normalized size = 0.90

$$\frac{\left(-6a^2+4ab+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)-\sqrt{2}\sqrt{a}\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}}{16a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-6*a^2 + 4*a*b + 2*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a
- b + b*Cosh[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e +
f*x)]]*Coth[e + f*x]*Csch[e + f*x]*(-3*a + b + 2*a*Csch[e + f*x]^2))/(16*a^
(3/2)*f)
```

fricas [B] time = 3.41, size = 3395, normalized size = 23.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^8 + 8*(3*a^2 - 2*a*b - b^2)*co
sh(f*x + e)*sinh(f*x + e)^7 + (3*a^2 - 2*a*b - b^2)*sinh(f*x + e)^8 - 4*(3*
a^2 - 2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x +
e)^2 - 3*a^2 + 2*a*b + b^2)*sinh(f*x + e)^6 + 8*(7*(3*a^2 - 2*a*b - b^2)*co
sh(f*x + e)^3 - 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*
(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(3*a^2 - 2*a*b - b^2)*cosh(f*
x + e)^4 - 30*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 + 9*a^2 - 6*a*b - 3*b^2
```

)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^5 - 10*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 + 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^6 - 15*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 + 9*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 - 3*a^2 + 2*a*b + b^2)*sinh(f*x + e)^2 + 3*a^2 - 2*a*b - b^2 + 8*((3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^7 - 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^5 + 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 - (3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((3*a^2 - a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - a*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - a*b)*sinh(f*x + e)^6 - (11*a^2 - a*b)*cosh(f*x + e)^4 + (15*(3*a^2 - a*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*sinh(f*x + e)^4 + 4*(5*(3*a^2 - a*b)*cosh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f*x + e)^2 + (15*(3*a^2 - a*b)*cosh(f*x + e)^4 - 6*(11*a^2 - a*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*sinh(f*x + e)^2 + 3*a^2 - a*b + 2*(3*(3*a^2 - a*b)*cosh(f*x + e)^5 - 2*(11*a^2 - a*b)*cosh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^8 + 8*a^2*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*f*sinh(f*x + e)^8 - 4*a^2*f*cosh(f*x + e)^6 + 6*a^2*f*cosh(f*x + e)^4 + 4*(7*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^6 + 8*(7*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^5 - 4*a^2*f*cosh(f*x + e)^2 + 2*(35*a^2*f*cosh(f*x + e)^4 - 30*a^2*f*cosh(f*x + e)^2 + 3*a^2*f)*sinh(f*x + e)^4 + 8*(7*a^2*f*cosh(f*x + e)^5 - 10*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*f + 4*(7*a^2*f*cosh(f*x + e)^6 - 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 8*(a^2*f*cosh(f*x + e)^7 - 3*a^2*f*cosh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^8 + 8*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (3*a^2 - 2*a*b - b^2)*sinh(f*x + e)^8 - 4*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 - 3*a^2 + 2*a*b + b^2)*sinh(f*x + e)^6 + 8*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 - 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 - 30*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 + 9*a^2 - 6*a*b - 3*b^2)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^5 - 10*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 + 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^6 - 15*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 + 9*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 - 3*a^2 + 2*a*b + b^2)*sinh(f*x + e)^2 + 3*a^2 - 2*a*b - b^2 + 8*((3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^7 - 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^5 + 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 - (3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((3*a^2 - a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - a*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - a*b)*sinh(f*x + e)^6 - (11*a^2 - a*b)*cosh(f*x + e)^4 + (15*(3*a^2 - a*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*s

```
inh(f*x + e)^4 + 4*(5*(3*a^2 - a*b)*cosh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f
*x + e))*sinh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f*x + e)^2 + (15*(3*a^2 - a*
b)*cosh(f*x + e)^4 - 6*(11*a^2 - a*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*sinh(
f*x + e)^2 + 3*a^2 - a*b + 2*(3*(3*a^2 - a*b)*cosh(f*x + e)^5 - 2*(11*a^2 -
 a*b)*cosh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((
b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^8 + 8*a^2*
f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*f*sinh(f*x + e)^8 - 4*a^2*f*cosh(f*x
+ e)^6 + 6*a^2*f*cosh(f*x + e)^4 + 4*(7*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh
(f*x + e)^6 + 8*(7*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*sinh(f*x
+ e)^5 - 4*a^2*f*cosh(f*x + e)^2 + 2*(35*a^2*f*cosh(f*x + e)^4 - 30*a^2*f*c
osh(f*x + e)^2 + 3*a^2*f)*sinh(f*x + e)^4 + 8*(7*a^2*f*cosh(f*x + e)^5 - 10
*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*f + 4
*(7*a^2*f*cosh(f*x + e)^6 - 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f*x + e
)^2 - a^2*f)*sinh(f*x + e)^2 + 8*(a^2*f*cosh(f*x + e)^7 - 3*a^2*f*cosh(f*x
+ e)^5 + 3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.18, size = 381, normalized size = 2.65

$$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\,\left(6\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\,\left(\sinh^2\left(fx+e\right)\right)a^{\frac{5}{2}} -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(6*((a+b*sinh(f*x+e)^2)*cosh
(f*x+e)^2)^(1/2)*sinh(f*x+e)^2*a^(5/2)-3*a^3*ln(((a+b)*cosh(f*x+e)^2+2*a^(1
/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*
x+e)^4+2*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*
x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4*a^2+ln(((a+b)*cosh(f*x+e)^2
+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*
b^2*sinh(f*x+e)^4*a-2*b*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+
e)^2*a^(3/2)-4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*a^(5/2))/sinh(f*x+
e)^4/a^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a}\ \operatorname{csch}\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\sinh\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^5,x)
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.71 $\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=300

$$-\frac{(2a^2+3ab-8b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f}+\frac{(2a^2+3ab-8b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\big(}{15b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx}{a}}}$$

```
[Out] 1/15*(a-4*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+1/5*cosh
(f*x+e)*sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/15*(2*a^2+3*a*b-8*b^2)*
(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/15*(a-4*b)*(1/(1+sinh(
f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+
e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(
f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/15*(2*a^2+3*a*b-8*b^2)*(a+b*sinh(f*
x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Rubi [A] time = 0.33, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3188, 478, 582, 531, 418, 492, 411}

$$-\frac{(2a^2+3ab-8b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f}+\frac{(2a^2+3ab-8b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\big(}{15b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f
) + (Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) + ((2
*a^2 + 3*a*b - 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*
x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) - ((a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Se
ch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a +
 b*Sinh[e + f*x]^2))/a]) - ((2*a^2 + 3*a*b - 8*b^2)*Sqrt[a + b*Sinh[e + f*x
]^2]*Tanh[e + f*x])/(15*b^2*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{1+x^2}}dx, x, \sinh(e+f$$

$$= \frac{\cosh(e+fx)\sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{5f} - \frac{\left(\sqrt{\cosh^2(e+f$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+$$

Mathematica [C] time = 1.38, size = 210, normalized size = 0.70

$$\frac{\sqrt{2}b\sinh(2(e+fx))\left(8a^2+4b(4a-7b)\cosh(2(e+fx))-48ab+3b^2\cosh(4(e+fx))+25b^2\right)-32ia\left(a^2+ab-\right.}{240b^2f\sqrt{2a+b\cosh(2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((16*I)*a*(2*a^2 + 3*a*b - 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticE[I*(e + f*x), b/a] - (32*I)*a*(a^2 + a*b - 2*b^2)*Sqrt[(2*a - b + b
*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 - 48*
a*b + 25*b^2 + 4*(4*a - 7*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x)])
*Sinh[2*(e + f*x)])/(240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 3.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\sinh\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [A] time = 0.14, size = 512, normalized size = 1.71

$$\frac{3\sqrt{-\frac{b}{a}}\, b^2 \left(\sinh^7\left(fx+e\right)\right) + 4\sqrt{-\frac{b}{a}}\, ab \left(\sinh^5\left(fx+e\right)\right) - \sqrt{-\frac{b}{a}}\, b^2 \left(\sinh^5\left(fx+e\right)\right) + \sqrt{-\frac{b}{a}}\, a^2 \left(\sinh^3\left(fx+e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^7+4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5
-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-4*(-1/a*
b)^(1/2)*b^2*sinh(f*x+e)^3+a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+7*a*((a+b*sinh(f*
x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*b-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-2*((a+b*sinh(f*x+e)^2)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))*a^2-3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh
(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+(-
1/a*b)^(1/2)*a^2*sinh(f*x+e)-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/b/(-1/a*b)^(
1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a}\, \sinh\left(fx+e\right)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh\left(e+f\,x\right)^4\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)

[Out] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

3.72 $\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=177

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i(a-2b)\sqrt{a+b\sin\cdots}}{3bf\sqrt{\frac{b\cdots}{}}}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*I*(a-2*b)*(cos(
I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a
+b*sinh(f*x+e)^2)^(1/2)/b/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+1/3*I*a*(a-b)*(cos(
I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1
+b*sinh(f*x+e)^2/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3170, 3172, 3178, 3177, 3183, 3182}

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i(a-2b)\sqrt{a+b\sin\cdots}}{3bf\sqrt{\frac{b\cdots}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((I/3)*(a
 - 2*b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[
1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*
Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
```

```
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{1}{3}\int \frac{a-(a-\cdots}{\sqrt{a+\cdots}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{(a-2b)\int\sqrt{a\cdots}}{\cdots}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left((a-2b)\sqrt{a\cdots}\right.}{\cdots}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{i(a-2b)E\left(ie\cdots\right.}{3b\cdots}$$

Mathematica [A] time = 0.84, size = 170, normalized size = 0.96

$$\frac{b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)+2i\sqrt{2}\,a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right)-2i\sqrt{2}\,a(a-\cdots}{6bf\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 4.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\sinh\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.12, size = 341, normalized size = 1.93

$$\frac{\sqrt{-\frac{b}{a}}\, b \sinh\left(fx+e\right)\left(\cosh^4\left(fx+e\right)\right)+\left(\sqrt{-\frac{b}{a}}\, a-\sqrt{-\frac{b}{a}}\, b\right)\left(\cosh^2\left(fx+e\right)\right)\sinh\left(fx+e\right)-2a\sqrt{\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a}}\; +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^
(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)-2*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a-2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2
)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^2+a}\, \sinh\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{sinh}\left(e+f\,x\right)}^2\,\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh^{2}{\left(e + f x \right)}} \sinh^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sinh(e + f*x)**2, x)
```

3.73 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(
1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\\ &= -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 69, normalized size = 1.15

$$-\frac{ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,E\left(i(e+fx)\middle|\frac{b}{a}\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a]
)/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.10, size = 140, normalized size = 2.33

$$\frac{\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\left(a\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{b\,\mathrm{sinh}\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh^2\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

3.74 $\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=199

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\Big(}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\big(a+b\sin}{a}}}$$

```
[Out] -coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sin
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/
2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)
^2)/a)^(1/2)+b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*Elliptic
F(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(
f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+b*sinh(f
*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.18, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3188, 475, 21, 422, 418, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\Big(}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\big(a+b\sin}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (EllipticE[ArcTan[Sinh[e
 + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (b*EllipticF[ArcTan[Sinh[e + f*x
]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e +
 f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e
+ f*x])/f
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{1+x^2}}\,dx, x, \sinh(e+\ldots\right.}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e\ldots\right.}{\ldots}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e\ldots\right.}{\ldots}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e\ldots\right.}{\ldots}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{bF\left(\tan^{-1}(\sinh(e+fx))|1\ldots\right.}{af\sqrt{\frac{\operatorname{se}\ldots}{\ldots}}}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{E\left(\tan^{-1}(\sinh(e+fx))|1\ldots\right.}{f\sqrt{\frac{\operatorname{sech}\ldots}{\ldots}}}$$

Mathematica [C] time = 0.61, size = 151, normalized size = 0.76

$$\frac{\sqrt{2}\coth(e+fx)(-2a-b\cosh(2(e+fx))+b)+2i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)-2ia\sqrt{\frac{2a+b\cosh(2(e-}{a}}}{2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (2*I)*(a - b)*
Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(2*f*S
qrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 2.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\ \text{csch}\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.12, size = 161, normalized size = 0.81

$$-\frac{-b\sqrt{\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\ \sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \sinh\left(fx+e\right)\text{EllipticE}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right)+\sqrt{-\frac{b}{a}}\ b\left(\cosh^4\right.}{\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}}\ \cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -(-b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)*El
lipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+(-1/a*b)^(1/2)*b*cosh(f*x+e
)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*cosh(f*x+e)^2)/sinh(f*x+e)/(-1/a*b)
^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh\left(fx+e\right)^2+a}\ \text{csch}\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\sinh\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^2,x)
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^2\left(e+fx\right)}\,\operatorname{csch}^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*csch(e + f*x)**2, x)
```

3.75 $\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=276

$$-\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}+\frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth(e+fx)\operatorname{csch}^2(e}{}$$

```
[Out] 1/3*(2*a-b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-1/3*coth(f*x+e)*csch(
f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/3*(2*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2
)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-
b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*s
inh(f*x+e)^2)/a)^(1/2)-1/3*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^
(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x
+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/
2)-1/3*(2*a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/f
```

Rubi [A] time = 0.28, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3188, 475, 583, 531, 418, 492, 411}

$$-\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}+\frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth(e+fx)\operatorname{csch}^2(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e + f
*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + ((2*a - b)*Ellipti
cE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF
[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((2*a - b)*Sqr
t[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
```

```
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{1+x^2}}\,dx, x, \sinh(e+fx\right.}{f}$$

$$= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cosh^2(e+f\right.}{}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\operatorname{csch}^2(e+}{}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\operatorname{csch}^2(e+}{}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\operatorname{csch}^2(e+}{}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\operatorname{csch}^2(e+}{}$$

Mathematica [C] time = 2.99, size = 208, normalized size = 0.75

$$\frac{\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\left(4\left(2a^2-4ab+b^2\right)\cosh(2(e+fx))-(2a-b)(8a-b\cosh(4(e+fx))-3b)\right)}{2\sqrt{2}} - 4ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\right|}{6af\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (((4*(2*a^2 - 4*a*b + b^2)*Cosh[2*(e + f*x)] - (2*a - b)*(8*a - 3*b - b*Cos
h[4*(e + f*x)]))*Coth[e + f*x]*Csch[e + f*x]^2)/(2*Sqrt[2]) + (2*I)*a*(2*a
- b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] -
(4*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*
x), b/a])/(6*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 2.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\operatorname{csch}\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.16, size = 436, normalized size = 1.58

$$\frac{2\sqrt{-\frac{b}{a}}\,ab\left(\sinh^6\left(fx+e\right)\right)-\sqrt{-\frac{b}{a}}\,b^2\left(\sinh^6\left(fx+e\right)\right)+b\sqrt{\frac{a+b\left(\sinh^2\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+b*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*s
inh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+((a+b*sinh(f*x+
e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(
a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-(-1/a*b)^(
1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2-2*(-1/a*b)^(1/2)*a*
b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2)/a/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x
+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\ \mathrm{csch}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\sinh\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/sinh(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.76 $\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=177

$$-\frac{(a-b)^2(a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{16b^{3/2}f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6bf}-\frac{(a+5b)\cosh(e+fx)\left(\right.}{}$$

```
[Out] -1/16*(a-b)^2*(a+5*b)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/
2))/b^(3/2)/f-1/24*(a+5*b)*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(3/2)/b/f+1/6*
cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(5/2)/b/f-1/16*(a-b)*(a+5*b)*cosh(f*x+e)*
(a-b+b*cosh(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3186, 388, 195, 217, 206}

$$-\frac{(a-b)^2(a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{16b^{3/2}f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6bf}-\frac{(a+5b)\cosh(e+fx)\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a - b)^2*(a + 5*b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e
 + f*x]^2]])/(16*b^(3/2)*f) - ((a - b)*(a + 5*b)*Cosh[e + f*x]*Sqrt[a - b +
 b*Cosh[e + f*x]^2])/(16*b*f) - ((a + 5*b)*Cosh[e + f*x]*(a - b + b*Cosh[e
+ f*x]^2)^(3/2))/(24*b*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(5/2
))/(6*b*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a-b+bx^2\right)^{3/2}dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{5/2}}{6bf} - \frac{(a+5b)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a+5b)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{24bf} + \frac{\cosh(e}{}$$

$$= -\frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf} - \frac{(a+}{}$$

$$= -\frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf} - \frac{(a+}{}$$

$$= -\frac{(a-b)^2(a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{(a-b)(a+5b)}{}$$

Mathematica [A] time = 0.59, size = 151, normalized size = 0.85

$$\frac{\sqrt{2}\sqrt{b}\sqrt{2a+b\cosh(2(e+fx))-b}\left(\left(6a^2-51ab+37b^2\right)\cosh(e+fx)+b((7a-8b)\cosh(3(e+fx))+b\cosh\right.}{192b^{3/2}f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*Sqrt[b]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*((6*a^2 - 51*a*b + 37*
b^2)*Cosh[e + f*x] + b*((7*a - 8*b)*Cosh[3*(e + f*x)] + b*Cosh[5*(e + f*x)]
)) - 12*(a - b)^2*(a + 5*b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a -
b + b*Cosh[2*(e + f*x)]]])/(192*b^(3/2)*f)
```

fricas [B] time = 2.05, size = 4608, normalized size = 26.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(6*((a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^6 + 6*(a^3 + 3*a
^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 + 3*a^2*b -
 9*a*b^2 + 5*b^3)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 + 3*a^2*b - 9*a
*b^2 + 5*b^3)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 + 3*a^2*b - 9*a*b^2
 + 5*b^3)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh
(f*x + e)^6)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sin
```

h(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*((a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^6 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh(f*x + e)^6)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2)*(b^3*cosh(f*x + e)^10 + 10*b^3*cosh(f*x + e)*sinh(f*x + e)^9 + b^3*sinh(f*x + e)^10 + (7*a*b^2 - 8*b^3)*cosh(f*x + e)^8 + (45*b^3*cosh(f*x + e)^2 + 7*a*b^2 - 8*b^3)*sinh(f*x + e)^8 + 8*(15*b^3*cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^6 + (210*b^3*cosh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b^3 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^3 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^4 + (210*b^3*cosh(f*x + e)^6 + 70*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b^3 + 15*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(30*b^3*cosh(f*x + e)^7 + 14*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^5 + 5*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^3 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e)^2 + (45*b^3*cosh(f*x + e)^8 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^4 + 7*a*b^2 - 8*b^3 + 6*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^7 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^5 + 2*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 15*b^2*f*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*f*sinh(f*x + e)^6), 1/384*(12*((a^3 + 3*a^2*b - 9*a*b

```
^2 + 5*b^3)*cosh(f*x + e)^6 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x
+ e)^5*sinh(f*x + e) + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^4
*sinh(f*x + e)^2 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^3*sin
h(f*x + e)^3 + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^2*sinh(f*
x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)*sinh(f*x + e)^
5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh(f*x + e)^6)*sqrt(-b)*arctan(sqrt
(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^
2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cos
h(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(
f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3
*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*
x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*si
nh(f*x + e))) + 12*((a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^6 + 6*(
a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 +
3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 + 3*a^
2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 + 3*a^2*b
- 9*a*b^2 + 5*b^3)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a
*b^2 + 5*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*sinh(f*x + e)^6)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
 e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
 e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*
x + e))*sinh(f*x + e) + b)) + sqrt(2)*(b^3*cosh(f*x + e)^10 + 10*b^3*cosh(f
*x + e)*sinh(f*x + e)^9 + b^3*sinh(f*x + e)^10 + (7*a*b^2 - 8*b^3)*cosh(f*x
 + e)^8 + (45*b^3*cosh(f*x + e)^2 + 7*a*b^2 - 8*b^3)*sinh(f*x + e)^8 + 8*(1
5*b^3*cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 +
(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^6 + (210*b^3*cosh(f*x + e)^4 +
6*a^2*b - 51*a*b^2 + 37*b^3 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^
3 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (6*a^2
*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^4 + (210*b^3*cosh(f*x + e)^6 + 70*(7*
a*b^2 - 8*b^3)*cosh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b^3 + 15*(6*a^2*b
- 51*a*b^2 + 37*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(30*b^3*cosh(f*x
+ e)^7 + 14*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^5 + 5*(6*a^2*b - 51*a*b^2 + 37*
b^3)*cosh(f*x + e)^3 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e))*sinh(f*
x + e)^3 + b^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e)^2 + (45*b^3*cosh(f*x + e)^
8 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b - 51*a*b^2 + 37*b^3)
*cosh(f*x + e)^4 + 7*a*b^2 - 8*b^3 + 6*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f
*x + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*(7*a*b^2 - 8*b^3)
*cosh(f*x + e)^7 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^5 + 2*(6*a
^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e)
)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(co
sh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*c
osh(f*x + e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 15*b^2*f*cosh(f*x
+ e)^4*sinh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*b^2*
f*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*sinh(f*x + e)^5 +
 b^2*f*sinh(f*x + e)^6)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.15, size = 483, normalized size = 2.73

$$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\,\left(16b^{\frac{7}{2}}\sqrt{b\left(\cosh^4\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^2\left(fx+e\right)\right)}\,\left(\cosh^4\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/96*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(16*b^(7/2)*(b*cosh(f*x+e)^4
+(a-b)*cosh(f*x+e)^2)^(1/2)*cosh(f*x+e)^4+4*b^(5/2)*(b*cosh(f*x+e)^4+(a-b)*
cosh(f*x+e)^2)^(1/2)*(-13*b+7*a)*cosh(f*x+e)^2+66*b^(7/2)*(b*cosh(f*x+e)^4+
(a-b)*cosh(f*x+e)^2)^(1/2)-72*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)
*b^(5/2)+6*a^2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(3/2)-3*ln(1/2
*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a
-b)/b^(1/2))*a^3*b-9*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*a^2*b^2+27*b^3*a*ln(1/2*(2*b*cosh(f
*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))
-15*b^4*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(
1/2)*b^(1/2)+a-b)/b^(1/2)))/b^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e+f\,x\right)^3\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.77 $\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=121

$$\frac{3(a-b)\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{8f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4f}+\frac{3(a-b)^2\tanh^{-1}}{8}$$

```
[Out] 1/4*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(3/2)/f+3/8*(a-b)^2*arctanh(cosh(f*x+
e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/b^(1/2)+3/8*(a-b)*cosh(f*x+e)*(a-
b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 195, 217, 206}

$$\frac{3(a-b)\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{8f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4f}+\frac{3(a-b)^2\tanh^{-1}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*(a - b)^2*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2
]])/(8*Sqrt[b]*f) + (3*(a - b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2
])/(8*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a-b+bx^2\right)^{3/2}\,dx, x, \cosh(e+fx)\right)}{f}$$
$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a-b))\,\text{Subst}\left(\int \sqrt{\cdot}\right.}{}$$
$$= \frac{3(a-b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8f} + \frac{\cosh(e+fx)\left(a\right.}{}$$
$$= \frac{3(a-b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8f} + \frac{\cosh(e+fx)\left(a\right.}{}$$
$$= \frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3(a-b)\cosh(e+fx)\sqrt{a-}}{8f}$$

Mathematica [A] time = 0.32, size = 111, normalized size = 0.92

$$\frac{\frac{1}{2}\cosh(e+fx)(5a+b\cosh(2(e+fx))-4b)\sqrt{4a+2b\cosh(2(e+fx))-2b} + \frac{3(a-b)^2\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\right.}{\sqrt{b}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((Cosh[e + f*x]*(5*a - 4*b + b*Cosh[2*(e + f*x)])*Sqrt[4*a - 2*b + 2*b*Cosh
[2*(e + f*x)]])/2 + (3*(a - b)^2*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2
*a - b + b*Cosh[2*(e + f*x)]]])/Sqrt[b])/(8*f)
```

fricas [B] time = 4.83, size = 2977, normalized size = 24.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(6*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(
f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x +
 e)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b
+ b^2)*sinh(f*x + e)^4)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f
*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x
+ e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14
*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (
9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^
2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4
*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b -
4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cos
h(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)
^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x
+ e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5
 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 +
 a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sin
h(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18
*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f
```

```
*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*
cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^
3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*
x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x +
e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e
)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*((a^2 - 2*a*b
 + b^2)*cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x +
e) + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b
 + b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4
)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*s
inh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b
)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(
2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*
x + e)^6 + (10*a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*
a*b - 7*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (10*a*b - 7*b^2)*
cosh(f*x + e))*sinh(f*x + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e)^2 + (15*b^2
*cosh(f*x + e)^4 + 6*(10*a*b - 7*b^2)*cosh(f*x + e)^2 + 10*a*b - 7*b^2)*sin
h(f*x + e)^2 + b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 7*b^2)*cosh(f*x
 + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^
3*sinh(f*x + e) + 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x +
e)*sinh(f*x + e)^3 + b*f*sinh(f*x + e)^4), -1/64*(12*((a^2 - 2*a*b + b^2)*c
osh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b + b^2)*c
osh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4)*sqrt(-b
)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*s
inh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*
x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b -
 b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh
(f*x + e))*sinh(f*x + e))) + 12*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 4*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 2*a*b + b^2)*cos
h(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x
 + e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cos
h(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b
*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x +
 e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
 e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*(b^2*cosh(f*
x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (10*
a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*a*b - 7*b^2)*si
nh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e))*
sinh(f*x + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x + e)^
4 + 6*(10*a*b - 7*b^2)*cosh(f*x + e)^2 + 10*a*b - 7*b^2)*sinh(f*x + e)^2 +
b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 7*b^2)*cosh(f*x + e)^3 + (10*a
*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^3*sinh(f*x + e)
 + 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x + e)*sinh(f*x + e
)^3 + b*f*sinh(f*x + e)^4)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [**B**] time = 0.10, size = 336, normalized size = 2.78

$$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(4\sqrt{b\left(\cosh^4\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^2\left(fx+e\right)\right)}\,b^{\frac{3}{2}}\left(\cosh^2\left(fx+e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(4*(b*cosh(f*x+e)^4+(a-b)*co
sh(f*x+e)^2)^(1/2)*b^(3/2)*cosh(f*x+e)^2-10*b^(3/2)*(b*cosh(f*x+e)^4+(a-b)*
cosh(f*x+e)^2)^(1/2)+3*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*c
osh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*a^2-6*b*a*ln(1/2*(2*b*cosh(f*x+e)
^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))+3*b^
2*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b
^(1/2)+a-b)/b^(1/2))+10*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/
2))/b^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e+f\,x\right)\,{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.78 $\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}+\frac{b\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}+\frac{\sqrt{b}\,(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2f}$$

```
[Out] -a^(3/2)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f+1/2*(3*
a-b)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))*b^(1/2)/f+1/2
*b*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 416, 523, 217, 206, 377}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}+\frac{b\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}+\frac{\sqrt{b}\,(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Co
sh[e + f*x]^2]])/(2*f) + (b*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/
(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{1-x^2}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{b\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} + \frac{\operatorname{Subst}\left(\int \frac{-(a-b)(2a-b)-(3}{\left(1-x^2\right)\sqrt{a-b-}}\right.}{}$$

$$= \frac{b\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} - \frac{a^2\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a-b-}}\right.}{}$$

$$= \frac{b\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} - \frac{a^2\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x\right.}{f}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + \frac{(3a-b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(}{\sqrt{a-b+b\cosh}}\right.}{2f}$$

Mathematica [A] time = 0.56, size = 136, normalized size = 1.07

$$\frac{-4a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)+b\cosh(e+fx)\sqrt{4a+2b\cosh(2(e+fx))-2b}-2\sqrt{b}(b-3a)\log\left(\sqrt{2a+b}\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-4*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]] + b*Cosh[e + f*x]*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]] - 2
*Sqrt[b]*(-3*a + b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Co
sh[2*(e + f*x)]]])/(4*f)
```

fricas [B] time = 3.12, size = 5565, normalized size = 43.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e)
 + (3*a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*
```

cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cos
h(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 +
 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)
^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4
+ 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)
^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^
2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^
3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*
x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x
+ e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x +
e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e
))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^
4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*
cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh
(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*
a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^
2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(c
osh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(
f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f
*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) - 4*(a*cosh
(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(a)*
log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (
a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*
x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*
x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
+ e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*co
sh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + (
(3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e) + (3*a
- b)*sinh(f*x + e)^2)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cos
h(f*x + e)^2 + a - b)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*
sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e)
 + b*sinh(f*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))
/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x + e)^2),
 1/8*(8*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x +
 e)^2)*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(
f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b
)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*
x + e) + b)) - ((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(
f*x + e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 +
8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^
2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x
+ e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(
f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x
 + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(
f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3

```
 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*
b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)
*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e
)^2)*sinh(f*x + e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*s
inh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cos
h(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh
(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f
*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2
*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x +
e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
 + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b
- 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x
+ e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)
^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^
2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) - (
(3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e) + (3*a
- b)*sinh(f*x + e)^2)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cos
h(f*x + e)^2 + a - b)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*
sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e)
 + b*sinh(f*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))
/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x + e)^2),
 -1/8*(2*((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x +
e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2
+ 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh
(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x +
e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a
*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((3*a
 - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - b)
*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^
3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^
2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x +
e))*sinh(f*x + e) + b)) - 4*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x
 + e) + a*sinh(f*x + e)^2)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b
)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cos
h(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*
sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
+ 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*
cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x
 + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x
 + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cos
h(f*x + e))*sinh(f*x + e) + 1)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x
 + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) +
f*sinh(f*x + e)^2), 1/8*(8*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x
+ e) + a*sinh(f*x + e)^2)*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh
```

```
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f
*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f
*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cos
h(f*x + e))*sinh(f*x + e) + b)) - 2*((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b
)*cosh(f*x + e)*sinh(f*x + e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(-b)*arctan(
sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x +
 e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*c
osh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4
+ (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sin
h(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e)
)*sinh(f*x + e))) - 2*((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e
)*sinh(f*x + e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(
f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*(b*cosh(f*x +
e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 + b)*sqrt((b*cos
h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
 e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x +
e)*sinh(f*x + e) + f*sinh(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.17, size = 268, normalized size = 2.11

$$-\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(b^{\frac{3}{2}}\ln\left(\frac{2b\left(\cosh^2\left(fx+e\right)\right)+2\sqrt{b\left(\cosh^4\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^2\left(fx+e\right)\right)}\,\sqrt{b}+a-b}{2\sqrt{b}}\right)+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(b^(3/2)*ln(1/2*(2*b*cosh(f*
x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))+
2*a^(3/2)*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x
+e)^2)^(1/2)+a-b)/(cosh(f*x+e)^2-1))-3*b^(1/2)*a*ln(1/2*(2*b*cosh(f*x+e)^2+
2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-2*b*(b*
cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b \sinh \left(e+f x\right)^2+a\right)^{3/2}}{\sinh \left(e+f x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.79 $\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}+\frac{\sqrt{a}\,(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2f}-\frac{a\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}$$

```
[Out] b^(3/2)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f+1/2*(a-3
*b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))*a^(1/2)/f-1/2*
a*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.17, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3186, 413, 523, 217, 206, 377}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}+\frac{\sqrt{a}\,(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2f}-\frac{a\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e +
f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*C
osh[e + f*x]^2]])/f - (a*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch
[e + f*x])/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
```

```
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{\left(1-x^2\right)^2}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{a\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2f} - \frac{\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{a\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2f} + \frac{(a(a-3}{}$$

$$= -\frac{a\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\operatorname{csch}(e+fx)}{2f} + \frac{(a(a-3}{}$$

$$= \frac{\sqrt{a}\,(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2f} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e}{\sqrt{a-b+b\cosh}}\right.}{f}$$

Mathematica [A] time = 0.71, size = 143, normalized size = 1.10

$$\frac{4b^{3/2}\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)+2\sqrt{a}\,(a-3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)-a}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b +
 b*Cosh[2*(e + f*x)]]] - a*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]]*Coth[e +
 f*x]*Csch[e + f*x] + 4*b^(3/2)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*
a - b + b*Cosh[2*(e + f*x)]]])/(4*f)
```

fricas [B] time = 4.08, size = 6622, normalized size = 50.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
 e)^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 +
 4*(b*cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log((a^
```

```
2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*
x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 +
a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b
)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^
4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*
cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 +
a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x
+ e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e
)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b
^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 +
 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x
 + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f
*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x +
 e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sin
h(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*
a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(
f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*co
sh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x +
 e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)
^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 +
 sinh(f*x + e)^6)) - ((a - 3*b)*cosh(f*x + e)^4 + 4*(a - 3*b)*cosh(f*x + e)
*sinh(f*x + e)^3 + (a - 3*b)*sinh(f*x + e)^4 - 2*(a - 3*b)*cosh(f*x + e)^2
+ 2*(3*(a - 3*b)*cosh(f*x + e)^2 - a + 3*b)*sinh(f*x + e)^2 + 4*((a - 3*b)*
cosh(f*x + e)^3 - (a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt(a)
*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 +
(a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f
*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)
^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f
*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
 + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*c
osh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) +
(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
- 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*
cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log(-(b*cosh(
f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a -
 b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqr
t(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1
)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x
+ e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - 2*sqrt(2)*(a*cosh(f*x + e)^2
 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x + e)*s
inh(f*x + e)^3 + f*sinh(f*x + e)^4 - 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x
+ e)^2 - f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 - f*cosh(f*x + e))*sinh(
f*x + e) + f), -1/4*(2*((a - 3*b)*cosh(f*x + e)^4 + 4*(a - 3*b)*cosh(f*x +
e)*sinh(f*x + e)^3 + (a - 3*b)*sinh(f*x + e)^4 - 2*(a - 3*b)*cosh(f*x + e)^
2 + 2*(3*(a - 3*b)*cosh(f*x + e)^2 - a + 3*b)*sinh(f*x + e)^2 + 4*((a - 3*b
)*cosh(f*x + e)^3 - (a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt(
-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(
b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x +
```

```
e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))
 - (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)
^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*
(b*cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log((a^2*b
*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x +
 e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3
 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*c
osh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 +
 (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cos
h(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2
*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e
)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6
 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2
+ b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*
a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x +
e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x
+ e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)
^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f
*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b
 - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x
 + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(
f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e)
 + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3
+ 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + si
nh(f*x + e)^6)) - (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
b*sinh(f*x + e)^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(
f*x + e)^2 + 4*(b*cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqr
t(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sin
h(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + s
inh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2
*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)
) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(
f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 2*sqrt(2)*
(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 +
a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^4 + 4
*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 - 2*f*cosh(f*x + e)^2
+ 2*(3*f*cosh(f*x + e)^2 - f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 - f*co
sh(f*x + e))*sinh(f*x + e) + f), -1/4*(2*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x
+ e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cos
h(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 - b*cosh(f*x + e))
*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f
*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*si
nh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*
b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x
+ e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*(b*cosh(f*x + e)^
4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 - 2*b*cosh(f*x +
e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 -
 b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)
^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x
+ e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*
(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2
```

```
*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + ((a - 3*b)*cosh(f*x + e)^4 + 4
*(a - 3*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 3*b)*sinh(f*x + e)^4 - 2*(a
 - 3*b)*cosh(f*x + e)^2 + 2*(3*(a - 3*b)*cosh(f*x + e)^2 - a + 3*b)*sinh(f*
x + e)^2 + 4*((a - 3*b)*cosh(f*x + e)^3 - (a - 3*b)*cosh(f*x + e))*sinh(f*x
 + e) + a - 3*b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x
 + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)
^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(c
osh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(
a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x +
 e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 +
4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 -
1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e)
)*sinh(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*si
nh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh
(f*x + e)^4 - 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x + e)^2 - f)*sinh(f*x +
e)^2 + 4*(f*cosh(f*x + e)^3 - f*cosh(f*x + e))*sinh(f*x + e) + f), -1/2*(((
a - 3*b)*cosh(f*x + e)^4 + 4*(a - 3*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a -
 3*b)*sinh(f*x + e)^4 - 2*(a - 3*b)*cosh(f*x + e)^2 + 2*(3*(a - 3*b)*cosh(f
*x + e)^2 - a + 3*b)*sinh(f*x + e)^2 + 4*((a - 3*b)*cosh(f*x + e)^3 - (a -
3*b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt(-a)*arctan(sqrt(2)*(cosh(
f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + (b*cosh(f*x + e)^4 + 4
*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 - 2*b*cosh(f*x + e)^2
+ 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 - b*co
sh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2
+ 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh
(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x +
e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a
*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + (b*cosh
(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 - 2*b*c
osh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*
x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(cos
h(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b
*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x +
 e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
 e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*(a*cosh(f*x
+ e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x
+ e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 - 2*f*cosh(f*x + e)^2 + 2*(3*f*cos
h(f*x + e)^2 - f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 - f*cosh(f*x + e))
*sinh(f*x + e) + f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [B] time = 0.19, size = 297, normalized size = 2.28

$$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(a^{\frac{3}{2}}\ln\left(\frac{(a+b)\left(\cosh^{2}\left(fx+e\right)\right)+2\sqrt{a}\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+(a-b)\left(\cosh^{2}\left(fx+e\right)\right)}+a-b}{\sinh\left(fx+e\right)^{2}}\right)\left(\sinh\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(a^(3/2)*ln(((a+b)*cosh(f*x+e
)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^
2)*sinh(f*x+e)^2+2*b^(3/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-
b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*sinh(f*x+e)^2-3*a^(1/2)*b*ln(
((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+
a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-2*a*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(
1/2))/sinh(f*x+e)^2/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.80 $\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=135

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8\sqrt{a}f}-\frac{\coth(e+fx)\operatorname{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4f}+\frac{3(a-b)\coth}{}$$

[Out] -1/4*(a-b+b*cosh(f*x+e)^2)^(3/2)*coth(f*x+e)*csch(f*x+e)^3/f-3/8*(a-b)^2*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/a^(1/2)+3/8*(a-b)*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/f

Rubi [A] time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3186, 378, 377, 206}

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8\sqrt{a}f}-\frac{\coth(e+fx)\operatorname{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4f}+\frac{3(a-b)\coth}{}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (-3*(a - b)^2*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(8*Sqrt[a]*f) + (3*(a - b)*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x])/(8*f) - ((a - b + b*Cosh[e + f*x]^2)^(3/2)*Coth[e + f*x]*Csch[e + f*x]^3)/(4*f)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx=-\frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{\left(1-x^2\right)^3}dx,x,\cosh(e+fx)\right)}{f}$$

$$=-\frac{\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\operatorname{csch}^3(e+fx)}{4f}-\frac{(3(a-$$

$$=\frac{3(a-b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{8f}-\frac{(a$$

$$=\frac{3(a-b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{8f}-\frac{(a$$

$$=-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8\sqrt{a}f}+\frac{3(a-b)\sqrt{a-b+b\cosh$$

Mathematica [A] time = 0.64, size = 123, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{a}\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}\left(-2a\operatorname{csch}^2(e+fx)+3a-5b\right)-6(a-b)^2\tanh^{-1}\left(\right.}{16\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-6*(a - b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh
[2*(e + f*x)]]] + Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[
e + f*x]*Csch[e + f*x]*(3*a - 5*b - 2*a*Csch[e + f*x]^2))/(16*Sqrt[a]*f)
```

fricas [B] time = 1.24, size = 3133, normalized size = 23.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*cosh(
f*x + e)*sinh(f*x + e)^7 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 - 4*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^
2 + 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3
 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(a^2 - 2*a*b +
b^2)*cosh(f*x + e)^4 + 2*(35*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 30*(a^2
- 2*a*b + b^2)*cosh(f*x + e)^2 + 3*a^2 - 6*a*b + 3*b^2)*sinh(f*x + e)^4 + 8
*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 10*(a^2 - 2*a*b + b^2)*cosh(f*x +
 e)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^2 + 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 - 15*(
a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 9*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2
- a^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + a^2 - 2*a*b + b^2 + 8*((a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^7 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 3*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^3 - (a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f
*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a
```

```
 + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)
^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a
 - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x +
 e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x
+ e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x +
 e) + 1)) + 2*sqrt(2)*((3*a^2 - 5*a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - 5*a*b)*
cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b)*sinh(f*x + e)^6 - (11*a^2 -
 5*a*b)*cosh(f*x + e)^4 + (15*(3*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*
a*b)*sinh(f*x + e)^4 + 4*(5*(3*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a
*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e)^2 + (15
*(3*a^2 - 5*a*b)*cosh(f*x + e)^4 - 6*(11*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*
a^2 + 5*a*b)*sinh(f*x + e)^2 + 3*a^2 - 5*a*b + 2*(3*(3*a^2 - 5*a*b)*cosh(f*
x + e)^5 - 2*(11*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x +
 e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*
cosh(f*x + e)^8 + 8*a*f*cosh(f*x + e)*sinh(f*x + e)^7 + a*f*sinh(f*x + e)^8
 - 4*a*f*cosh(f*x + e)^6 + 4*(7*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^6
+ 6*a*f*cosh(f*x + e)^4 + 8*(7*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*s
inh(f*x + e)^5 + 2*(35*a*f*cosh(f*x + e)^4 - 30*a*f*cosh(f*x + e)^2 + 3*a*f
)*sinh(f*x + e)^4 - 4*a*f*cosh(f*x + e)^2 + 8*(7*a*f*cosh(f*x + e)^5 - 10*a
*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a*f*cosh(f
*x + e)^6 - 15*a*f*cosh(f*x + e)^4 + 9*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x
+ e)^2 + a*f + 8*(a*f*cosh(f*x + e)^7 - 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(
f*x + e)^3 - a*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(3*((a^2 - 2*a*b + b^2)
*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (a
^2 - 2*a*b + b^2)*sinh(f*x + e)^8 - 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 +
 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e
)^6 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e)^5 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 2*(35*
(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 30*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^
2 + 3*a^2 - 6*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(
f*x + e)^5 - 10*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)
*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 4
*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 - 15*(a^2 - 2*a*b + b^2)*cosh(f*x +
 e)^4 + 9*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x
 + e)^2 + a^2 - 2*a*b + b^2 + 8*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^7 - 3*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 -
 (a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*arctan(sqrt(2)*
(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqr
t(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 +
 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f
*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f
*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((3*a^2
- 5*a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - 5*a*b)*cosh(f*x + e)*sinh(f*x + e)^5
+ (3*a^2 - 5*a*b)*sinh(f*x + e)^6 - (11*a^2 - 5*a*b)*cosh(f*x + e)^4 + (15*
(3*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*a*b)*sinh(f*x + e)^4 + 4*(5*(3
*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e))*sinh(f*x +
e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e)^2 + (15*(3*a^2 - 5*a*b)*cosh(f*x + e)
^4 - 6*(11*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*a*b)*sinh(f*x + e)^2 +
 3*a^2 - 5*a*b + 2*(3*(3*a^2 - 5*a*b)*cosh(f*x + e)^5 - 2*(11*a^2 - 5*a*b)*
cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*co
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^8 + 8*a*f*cosh(f
*x + e)*sinh(f*x + e)^7 + a*f*sinh(f*x + e)^8 - 4*a*f*cosh(f*x + e)^6 + 4*(
7*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^6 + 6*a*f*cosh(f*x + e)^4 + 8*(7
*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a*f*cos
```

```
h(f*x + e)^4 - 30*a*f*cosh(f*x + e)^2 + 3*a*f)*sinh(f*x + e)^4 - 4*a*f*cosh
(f*x + e)^2 + 8*(7*a*f*cosh(f*x + e)^5 - 10*a*f*cosh(f*x + e)^3 + 3*a*f*cos
h(f*x + e))*sinh(f*x + e)^3 + 4*(7*a*f*cosh(f*x + e)^6 - 15*a*f*cosh(f*x +
e)^4 + 9*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 8*(a*f*cosh(f*x
 + e)^7 - 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e)
)*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.19, size = 379, normalized size = 2.81

$$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(-3a^2\ln\left(\frac{(a+b)\left(\cosh^2\left(fx+e\right)\right)+2\sqrt{a}\sqrt{b\left(\cosh^4\left(fx+e\right)\right)+(a-b)\left(\cosh^2\left(fx+e\right)\right)}+a-b}{\sinh\left(fx+e\right)^2}\right)\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-3*a^2*ln(((a+b)*cosh(f*x+e
)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^
2)*sinh(f*x+e)^4+6*a*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(
a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4-3*b^2*ln(((a+b)
*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/s
inh(f*x+e)^2)*sinh(f*x+e)^4+6*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(
1/2)*sinh(f*x+e)^2-10*b*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*a^(1/2)*s
inh(f*x+e)^2-4*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/a^(1/2)/s
inh(f*x+e)^4/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^5,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.81 $\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=199

$$\frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{16a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}^5(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6af}+\frac{(5a+b)\,\mathrm{cc}}{}$$

```
[Out] 1/16*(a-b)^2*(5*a+b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2
))/a^(3/2)/f+1/24*(5*a+b)*(a-b+b*cosh(f*x+e)^2)^(3/2)*coth(f*x+e)*csch(f*x+
e)^3/a/f-1/6*(a-b+b*cosh(f*x+e)^2)^(5/2)*coth(f*x+e)*csch(f*x+e)^5/a/f-1/16
*(a-b)*(5*a+b)*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.20, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 382, 378, 377, 206}

$$\frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{16a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}^5(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6af}+\frac{(5a+b)\,\mathrm{cc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e
+ f*x]^2]])/(16*a^(3/2)*f) - ((a - b)*(5*a + b)*Sqrt[a - b + b*Cosh[e + f*x
]^2]*Coth[e + f*x]*Csch[e + f*x])/(16*a*f) + ((5*a + b)*(a - b + b*Cosh[e +
 f*x]^2)^(3/2)*Coth[e + f*x]*Csch[e + f*x]^3)/(24*a*f) - ((a - b + b*Cosh[e
 + f*x]^2)^(5/2)*Coth[e + f*x]*Csch[e + f*x]^5)/(6*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
```

```
[q, -1]) && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{\left(1-x^2\right)^4}dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\left(a-b+b\cosh^2(e+fx)\right)^{5/2}\coth(e+fx)\operatorname{csch}^5(e+fx)}{6af} + \frac{(5a}{}$$

$$= \frac{(5a+b)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\operatorname{csch}^3(e+fx}{24af}$$

$$= -\frac{(a-b)(5a+b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e}{16af}$$

$$= -\frac{(a-b)(5a+b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e}{16af}$$

$$= \frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16a^{3/2}f} - \frac{(a-b)(5a+b)\sqrt{}}{}$$

Mathematica [A] time = 1.03, size = 174, normalized size = 0.87

$$\frac{\frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{a^{3/2}} - \frac{\coth(e+fx)\operatorname{csch}^5(e+fx)\sqrt{a+\frac{1}{2}b\cosh(2(e+fx))-\frac{b}{2}}\left(-4\left(25a^2-36ab+3b^2\right)\cosh(2(e+fx))+\left(15a^2-22\right.\right.}{24a}}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b
+ b*Cosh[2*(e + f*x)]]])/a^(3/2) - (Sqrt[a - b/2 + (b*Cosh[2*(e + f*x)])/2]
*(149*a^2 - 122*a*b + 9*b^2 - 4*(25*a^2 - 36*a*b + 3*b^2)*Cosh[2*(e + f*x)]
 + (15*a^2 - 22*a*b + 3*b^2)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]
^5)/(24*a))/(16*f)
```

fricas [B] time = 1.35, size = 7369, normalized size = 37.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*((5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^12 + 12*(5*a^3 -
9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (5*a^3 - 9*a^2*b
+ 3*a*b^2 + b^3)*sinh(f*x + e)^12 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cos
h(f*x + e)^10 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 11*(5*a^3 - 9*a^2*b +
3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 9*a^2*b
+ 3*a*b^2 + b^3)*cosh(f*x + e)^3 - 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh
(f*x + e))*sinh(f*x + e)^9 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x
+ e)^8 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 -
 9*a^2*b + 3*a*b^2 + b^3 - 18*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x +
e)^2)*sinh(f*x + e)^8 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x +
 e)^5 - 30*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 + 5*(5*a^3 - 9
*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^7 - 20*(5*a^3 - 9*a^2*
b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 + 4*(231*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^
3)*cosh(f*x + e)^6 - 315*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4
- 25*a^3 + 45*a^2*b - 15*a*b^2 - 5*b^3 + 105*(5*a^3 - 9*a^2*b + 3*a*b^2 + b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 +
b^3)*cosh(f*x + e)^7 - 63*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5
 + 35*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 - 5*(5*a^3 - 9*a^2*
b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 9*a^2*b + 3
*a*b^2 + b^3)*cosh(f*x + e)^4 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*co
sh(f*x + e)^8 - 84*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 + 70*(
5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 - 9*a^2*b + 3*a*b^
2 + b^3 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x +
e)^4 + 20*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^9 - 36*(5*a^3
 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 + 42*(5*a^3 - 9*a^2*b + 3*a*b^2
 + b^3)*cosh(f*x + e)^5 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e
)^3 + 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 +
5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(
f*x + e)^2 + 6*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^10 - 45*
(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 + 70*(5*a^3 - 9*a^2*b + 3
*a*b^2 + b^3)*cosh(f*x + e)^6 - 50*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f
*x + e)^4 - 5*a^3 + 9*a^2*b - 3*a*b^2 - b^3 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2
 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 9*a^2*b + 3*a*b^2 +
 b^3)*cosh(f*x + e)^11 - 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^
9 + 10*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 - 10*(5*a^3 - 9*a^
2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*
cosh(f*x + e)^3 - (5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh
(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*
(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x +
e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3
*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x
 + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*
x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x
 + e) + 1)) - 2*sqrt(2)*((15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^10 + 1
0*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^9 + (15*a^3 - 2
2*a^2*b + 3*a*b^2)*sinh(f*x + e)^10 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f
*x + e)^8 - (85*a^3 - 122*a^2*b + 9*a*b^2 - 45*(15*a^3 - 22*a^2*b + 3*a*b^2
)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*co
sh(f*x + e)^3 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e))*sinh(f*x + e)
^7 + 2*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^6 + 2*(105*(15*a^3 - 22*
a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 - 14*(85*a^3
 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(15*a^3 -
22*a^2*b + 3*a*b^2)*cosh(f*x + e)^5 - 14*(85*a^3 - 122*a^2*b + 9*a*b^2)*cos
h(f*x + e)^3 + 3*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e)
^5 + 2*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + 2*(105*(15*a^3 - 22*
a^2*b + 3*a*b^2)*cosh(f*x + e)^6 - 35*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f
```

```
*x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 + 15*(99*a^3 - 50*a^2*b + 3*a*b^2)*
cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh
(f*x + e)^7 - 7*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^5 + 5*(99*a^3
- 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^3 + (99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(
f*x + e))*sinh(f*x + e)^3 + 15*a^3 - 22*a^2*b + 3*a*b^2 - (85*a^3 - 122*a^2
*b + 9*a*b^2)*cosh(f*x + e)^2 + (45*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x
+ e)^8 - 28*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^6 + 30*(99*a^3 - 5
0*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 - 85*a^3 + 122*a^2*b - 9*a*b^2 + 12*(99*
a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(15*a^3 -
 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^9 - 4*(85*a^3 - 122*a^2*b + 9*a*b^2)*cos
h(f*x + e)^7 + 6*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^5 + 4*(99*a^3
- 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^3 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))
/(a^2*f*cosh(f*x + e)^12 + 12*a^2*f*cosh(f*x + e)*sinh(f*x + e)^11 + a^2*f*
sinh(f*x + e)^12 - 6*a^2*f*cosh(f*x + e)^10 + 15*a^2*f*cosh(f*x + e)^8 + 6*
(11*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^10 + 20*(11*a^2*f*cosh(f*x
 + e)^3 - 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 - 20*a^2*f*cosh(f*x + e)^6
 + 15*(33*a^2*f*cosh(f*x + e)^4 - 18*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*
x + e)^8 + 24*(33*a^2*f*cosh(f*x + e)^5 - 30*a^2*f*cosh(f*x + e)^3 + 5*a^2*
f*cosh(f*x + e))*sinh(f*x + e)^7 + 15*a^2*f*cosh(f*x + e)^4 + 4*(231*a^2*f*
cosh(f*x + e)^6 - 315*a^2*f*cosh(f*x + e)^4 + 105*a^2*f*cosh(f*x + e)^2 - 5
*a^2*f)*sinh(f*x + e)^6 + 24*(33*a^2*f*cosh(f*x + e)^7 - 63*a^2*f*cosh(f*x
+ e)^5 + 35*a^2*f*cosh(f*x + e)^3 - 5*a^2*f*cosh(f*x + e))*sinh(f*x + e)^5
- 6*a^2*f*cosh(f*x + e)^2 + 15*(33*a^2*f*cosh(f*x + e)^8 - 84*a^2*f*cosh(f*
x + e)^6 + 70*a^2*f*cosh(f*x + e)^4 - 20*a^2*f*cosh(f*x + e)^2 + a^2*f)*sin
h(f*x + e)^4 + 20*(11*a^2*f*cosh(f*x + e)^9 - 36*a^2*f*cosh(f*x + e)^7 + 42
*a^2*f*cosh(f*x + e)^5 - 20*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*
sinh(f*x + e)^3 + a^2*f + 6*(11*a^2*f*cosh(f*x + e)^10 - 45*a^2*f*cosh(f*x
+ e)^8 + 70*a^2*f*cosh(f*x + e)^6 - 50*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cos
h(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 12*(a^2*f*cosh(f*x + e)^11 - 5*a^2*
f*cosh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^7 - 10*a^2*f*cosh(f*x + e)^5 + 5
*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)), -1/48*(3*((5*
a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^12 + 12*(5*a^3 - 9*a^2*b + 3*a
*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (5*a^3 - 9*a^2*b + 3*a*b^2 + b
^3)*sinh(f*x + e)^12 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^10
 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3
)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b
^3)*cosh(f*x + e)^3 - 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*si
nh(f*x + e)^9 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 + 15*(
33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 - 9*a^2*b + 3*
a*b^2 + b^3 - 18*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^8 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 - 30*(5
*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 + 5*(5*a^3 - 9*a^2*b + 3*a*
b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^7 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 +
 b^3)*cosh(f*x + e)^6 + 4*(231*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x +
 e)^6 - 315*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 - 25*a^3 + 45
*a^2*b - 15*a*b^2 - 5*b^3 + 105*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^6 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x
 + e)^7 - 63*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 + 35*(5*a^3
- 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 - 5*(5*a^3 - 9*a^2*b + 3*a*b^2 +
 b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)
*cosh(f*x + e)^4 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8
 - 84*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 + 70*(5*a^3 - 9*a^2
*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 20*
(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 20*(11
*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^9 - 36*(5*a^3 - 9*a^2*b +
3*a*b^2 + b^3)*cosh(f*x + e)^7 + 42*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(
f*x + e)^5 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(5*a^
```

```
3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 5*a^3 - 9*a^2
*b + 3*a*b^2 + b^3 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2 +
6*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^10 - 45*(5*a^3 - 9*a^
2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 + 70*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)
*cosh(f*x + e)^6 - 50*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 - 5
*a^3 + 9*a^2*b - 3*a*b^2 - b^3 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(
f*x + e)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*
x + e)^11 - 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^9 + 10*(5*a^3
 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 - 10*(5*a^3 - 9*a^2*b + 3*a*b^2
 + b^3)*cosh(f*x + e)^5 + 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)
^3 - (5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-
a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b
*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x +
e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))
+ sqrt(2)*((15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^10 + 10*(15*a^3 - 22
*a^2*b + 3*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^9 + (15*a^3 - 22*a^2*b + 3*a*
b^2)*sinh(f*x + e)^10 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^8 - (8
5*a^3 - 122*a^2*b + 9*a*b^2 - 45*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e
)^2)*sinh(f*x + e)^8 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^3
- (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(99*a^3
 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^6 + 2*(105*(15*a^3 - 22*a^2*b + 3*a*b^
2)*cosh(f*x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 - 14*(85*a^3 - 122*a^2*b +
 9*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(15*a^3 - 22*a^2*b + 3*a
*b^2)*cosh(f*x + e)^5 - 14*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^3 +
 3*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(99*a^3
 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + 2*(105*(15*a^3 - 22*a^2*b + 3*a*b^
2)*cosh(f*x + e)^6 - 35*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^4 + 99
*a^3 - 50*a^2*b + 3*a*b^2 + 15*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^
2)*sinh(f*x + e)^4 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^7 -
7*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^5 + 5*(99*a^3 - 50*a^2*b + 3
*a*b^2)*cosh(f*x + e)^3 + (99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh
(f*x + e)^3 + 15*a^3 - 22*a^2*b + 3*a*b^2 - (85*a^3 - 122*a^2*b + 9*a*b^2)*
cosh(f*x + e)^2 + (45*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^8 - 28*(8
5*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^6 + 30*(99*a^3 - 50*a^2*b + 3*a*
b^2)*cosh(f*x + e)^4 - 85*a^3 + 122*a^2*b - 9*a*b^2 + 12*(99*a^3 - 50*a^2*b
 + 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(15*a^3 - 22*a^2*b + 3*
a*b^2)*cosh(f*x + e)^9 - 4*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^7 +
 6*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^5 + 4*(99*a^3 - 50*a^2*b + 3
*a*b^2)*cosh(f*x + e)^3 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f
*x + e)^12 + 12*a^2*f*cosh(f*x + e)*sinh(f*x + e)^11 + a^2*f*sinh(f*x + e)^
12 - 6*a^2*f*cosh(f*x + e)^10 + 15*a^2*f*cosh(f*x + e)^8 + 6*(11*a^2*f*cosh
(f*x + e)^2 - a^2*f)*sinh(f*x + e)^10 + 20*(11*a^2*f*cosh(f*x + e)^3 - 3*a^
2*f*cosh(f*x + e))*sinh(f*x + e)^9 - 20*a^2*f*cosh(f*x + e)^6 + 15*(33*a^2*
f*cosh(f*x + e)^4 - 18*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)^8 + 24*
(33*a^2*f*cosh(f*x + e)^5 - 30*a^2*f*cosh(f*x + e)^3 + 5*a^2*f*cosh(f*x + e
))*sinh(f*x + e)^7 + 15*a^2*f*cosh(f*x + e)^4 + 4*(231*a^2*f*cosh(f*x + e)^
6 - 315*a^2*f*cosh(f*x + e)^4 + 105*a^2*f*cosh(f*x + e)^2 - 5*a^2*f)*sinh(f
*x + e)^6 + 24*(33*a^2*f*cosh(f*x + e)^7 - 63*a^2*f*cosh(f*x + e)^5 + 35*a^
2*f*cosh(f*x + e)^3 - 5*a^2*f*cosh(f*x + e))*sinh(f*x + e)^5 - 6*a^2*f*cosh
(f*x + e)^2 + 15*(33*a^2*f*cosh(f*x + e)^8 - 84*a^2*f*cosh(f*x + e)^6 + 70*
a^2*f*cosh(f*x + e)^4 - 20*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)^4 +
 20*(11*a^2*f*cosh(f*x + e)^9 - 36*a^2*f*cosh(f*x + e)^7 + 42*a^2*f*cosh(f*
x + e)^5 - 20*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^
3 + a^2*f + 6*(11*a^2*f*cosh(f*x + e)^10 - 45*a^2*f*cosh(f*x + e)^8 + 70*a^
```

```
2*f*cosh(f*x + e)^6 - 50*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 -
 a^2*f)*sinh(f*x + e)^2 + 12*(a^2*f*cosh(f*x + e)^11 - 5*a^2*f*cosh(f*x + e
)^9 + 10*a^2*f*cosh(f*x + e)^7 - 10*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*
x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.21, size = 569, normalized size = 2.86

$$-\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(30a^{\frac{7}{2}}\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(\sinh^4\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/96*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(30*a^(7/2)*((a+b*sinh(f*x+
e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^4-44*b*((a+b*sinh(f*x+e)^2)*cosh(f*x
+e)^2)^(1/2)*sinh(f*x+e)^4*a^(5/2)-15*a^4*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)
*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e
)^6+27*a^3*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(
f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^6-9*b^2*ln(((a+b)*cosh(f*x+
e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)
^2)*sinh(f*x+e)^6*a^2-3*b^3*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e
)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^6*a-20*a^(7/
2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^2+6*b^2*((a+b*sinh
(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^4*a^(3/2)+28*b*((a+b*sinh(f*x+e
)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^2*a^(5/2)+16*a^(7/2)*((a+b*sinh(f*x+e
)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^6/a^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+
e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^7,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**7*(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.82 $\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=367

$$-\frac{2(a-2b)\left(a^2+4ab-4b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35b^2f}+\frac{\left(a^2-11ab+8b^2\right)\sinh(e+fx)\cosh(e+fx)}{35bf}$$

```
[Out] 1/35*(a^2-11*a*b+8*b^2)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b
/f+2/35*(4*a-3*b)*cosh(f*x+e)*sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/7
*b*cosh(f*x+e)*sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2)/f+2/35*(a-2*b)*(a^2+
4*a*b-4*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/35*(a^2-1
1*a*b+8*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(
sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/35*(a-2*b)*
(a^2+4*a*b-4*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Rubi [A] time = 0.47, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3188, 477, 582, 531, 418, 492, 411}

$$-\frac{2(a-2b)\left(a^2+4ab-4b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35b^2f}+\frac{\left(a^2-11ab+8b^2\right)\sinh(e+fx)\cosh(e+fx)}{35bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a^2 - 11*a*b + 8*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x
]^2])/(35*b*f) + (2*(4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Si
nh[e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]*Sinh[e + f*x]^5*Sqrt[a + b*Sinh[e
 + f*x]^2])/(7*f) + (2*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 11*a*b + 8*b^2
)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2
*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])
/(35*b^2*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\sqrt{1+x^2}}\,dx, x, \sinh\right.}{f}$$

$$= \frac{b\cosh(e+fx)\sinh^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{7f} + \frac{\left(\sqrt{\cosh\right.}{}$$

$$= \frac{2(4a-3b)\cosh(e+fx)\sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35f} +$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{35bf}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{35bf}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{35bf}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{35bf}$$

Mathematica [C] time = 2.82, size = 262, normalized size = 0.71

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(32a^3+b\left(144a^2-480ab+299b^2\right)\cosh(2(e+fx))-496a^2b+2b^2(26a-27b)\cosh(4(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((128*I)*a*(a^3 + 2*a^2*b - 12*a*b^2 + 8*b^3)*Sqrt[(2*a - b + b*Cosh[2*(e +
 f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (64*I)*a*(2*a^3 + 3*a^2*b - 13*a*b
^2 + 8*b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x),
b/a] + Sqrt[2]*b*(32*a^3 - 496*a^2*b + 684*a*b^2 - 250*b^3 + b*(144*a^2 - 4
80*a*b + 299*b^2)*Cosh[2*(e + f*x)] + 2*(26*a - 27*b)*b^2*Cosh[4*(e + f*x)]
 + 5*b^3*Cosh[6*(e + f*x)])*Sinh[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a - b + b
*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^6+a\sinh\left(fx+e\right)^4\right)\sqrt{b\sinh\left(fx+e\right)^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^6 + a*sinh(f*x + e)^4)*sqrt(b*sinh(f*x + e)^2 + a
), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.14, size = 743, normalized size = 2.02

$$\frac{5\sqrt{-\frac{b}{a}}\, b^3 \sinh\left(fx+e\right)\left(\cosh^8\left(fx+e\right)\right)+\left(13\sqrt{-\frac{b}{a}}\, a\, b^2-21\sqrt{-\frac{b}{a}}\, b^3\right)\left(\cosh^6\left(fx+e\right)\right)\sinh\left(fx+e\right)+\left(9\sqrt{-\frac{b}{a}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/35*(5*(-1/a*b)^(1/2)*b^3*sinh(f*x+e)*cosh(f*x+e)^8+(13*(-1/a*b)^(1/2)*a*b
^2-21*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-1/a*b)^(1/2)*a^2*b
-43*(-1/a*b)^(1/2)*a*b^2+35*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^4*sinh(f*x+e)+(
(-1/a*b)^(1/2)*a^3-20*(-1/a*b)^(1/2)*a^2*b+38*(-1/a*b)^(1/2)*a*b^2-19*(-1/a
*b)^(1/2)*b^3)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3
+15*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-32*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a*b^2+16*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(
sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)
)*a^3-4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+24*(b/a*cosh(f*x+e)^2+(a-b)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b^2-16*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/b/(-1/a*b)^(1/2)/cosh(f*x+
e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh\left(e+f\,x\right)^4\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.83 $\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=236

$$-\frac{i\left(3a^2-13ab+8b^2\right)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}+\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+$$

```
[Out] 1/5*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+1/15*(3*a-4*b)*cosh
(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/15*I*(3*a^2-13*a*b+8*b^2)
*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/
2))*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+1/15*I*a*(3*a
-4*b)*(a-b)*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x
),(b/a)^(1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3170, 3172, 3178, 3177, 3183, 3182}

$$-\frac{i\left(3a^2-13ab+8b^2\right)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}+\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f
) + (Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*f) - ((I
/15)*(3*a^2 - 13*a*b + 8*b^2)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/15)*a*(3*a - 4*b)*(
a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sq
rt[a + b*Sinh[e + f*x]^2])
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f} - \frac{1}{5}\int \left(a-\right.$$

$$= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{\mathrm{c}}{}$$

$$= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{\mathrm{c}}{}$$

$$= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{\mathrm{c}}{}$$

$$= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{\mathrm{c}}{}$$

Mathematica [A] time = 1.37, size = 213, normalized size = 0.90

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(48a^2+4b(9a-7b)\cosh(2(e+fx))-68ab+3b^2\cosh(4(e+fx))+25b^2\right)+16ia\left(3a^2-\right.}{240bf\sqrt{2a+b\,\mathrm{co}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-16*I)*a*(3*a^2 - 13*a*b + 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticE[I*(e + f*x), b/a] + (16*I)*a*(3*a^2 - 7*a*b + 4*b^2)*Sqrt[(2*a -
 b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(48*a^
2 - 68*a*b + 25*b^2 + 4*(9*a - 7*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e +
 f*x)])*Sinh[2*(e + f*x)])/(240*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^4+a\sinh\left(fx+e\right)^2\right)\sqrt{b\sinh\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^4 + a*sinh(f*x + e)^2)*sqrt(b*sinh(f*x + e)^2 + a
), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.12, size = 535, normalized size = 2.27

$$-\frac{-3\sqrt{-\frac{b}{a}}\,b^2\sinh\left(fx+e\right)\left(\cosh^6\left(fx+e\right)\right)+\left(-9\sqrt{-\frac{b}{a}}\,ab+10\sqrt{-\frac{b}{a}}\,b^2\right)\left(\cosh^4\left(fx+e\right)\right)\sinh\left(fx+e\right)+\left(-6\sqrt{\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15*(-3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^6+(-9*(-1/a*b)^(1/2)*a
*b+10*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(-6*(-1/a*b)^(1/2)*a^2+
13*(-1/a*b)^(1/2)*a*b-7*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+9*a^2
*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x
+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-17*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+8*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+13*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/(-1/a
*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\sinh\left(e+f\,x\right)^2\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.84 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\, dx$

Optimal. Leaf size=174

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\,\mathrm{s}\cdots}}{3f\sqrt{\cdots}}$$

```
[Out] 1/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2/3*I*(2*a-b)*(co
s(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*
(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+1/3*I*a*(a-b)*(cos(
I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1
+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\,\mathrm{s}\cdots}}{3f\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a-b)+2(2a-}{\sqrt{a+b\sinh}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}(a(a-b))\int \frac{}{\sqrt{a+b\sinh}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{\left(2(2a-b)\sqrt{a+b\sinh^2}\right.}{3\sqrt{1}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right.}{3f\sqrt{1+\frac{b}{}}}$$

Mathematica [A] time = 0.78, size = 169, normalized size = 0.97

$$\frac{b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)+2i\sqrt{2}a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)-4i\sqrt{2}a(2a}{6f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.11, size = 416, normalized size = 2.39

$$\frac{\sqrt{-\frac{b}{a}}\, b^2 \sinh\left(fx+e\right)\left(\cosh^4\left(fx+e\right)\right)+\left(\sqrt{-\frac{b}{a}}\, ab-\sqrt{-\frac{b}{a}}\, b^2\right)\left(\cosh^2\left(fx+e\right)\right)\sinh\left(fx+e\right)+3a^2\sqrt{\frac{b\left(\cosh^2\left(fx+\right.\right.}{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a*b-(-1/a
*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^
2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.85 $\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=204

$$\frac{(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{2b\operatorname{sech}(e+fx)\sqrt{a+b\sinh}}{f\sqrt{\frac{\operatorname{sech}}{}}}$$

```
[Out] -a*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2
)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-
b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sin
h(f*x+e)^2)/a)^(1/2)+2*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2
)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*
(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+
b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.21, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3188, 474, 531, 418, 492, 411}

$$\frac{(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{2b\operatorname{sech}(e+fx)\sqrt{a+b\sinh}}{f\sqrt{\frac{\operatorname{sech}}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - ((a + b)*EllipticE[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*b*EllipticF[ArcTan[
Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*Sqrt[a + b*Sinh[e
 + f*x]^2]*Tanh[e + f*x])/f
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
```

```
1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\sqrt{1+x^2}}\,dx, x, \sinh(e+\right.}{f}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e\right.}{}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(2ab\sqrt{\cosh^2(e+fx)}\operatorname{sec}\right.}{}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{2bF\left(\tan^{-1}(\sinh(e+fx)\right.}{f\sqrt{\cdots}}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{(a+b)E\left(\tan^{-1}(\sinh(e+\right.}{f\sqrt{\cdots}}$$

Mathematica [C] time = 0.96, size = 155, normalized size = 0.76

$$-\frac{a\left(\sqrt{2}\coth(e+fx)(2a+b\cosh(2(e+fx))-b)-2i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)+2i(a+b)\sqrt{\frac{2a+\cdots}{}}\right.}{2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(a*(Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Coth[e + f*x] + (2*I)*(a +
 b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (
2*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x),
 b/a]))/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \operatorname{csch}\left(fx+e\right)^{2} \sinh\left(fx+e\right)^{2} + a \operatorname{csch}\left(fx+e\right)^{2}\right)\sqrt{b \sinh\left(fx+e\right)^{2} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*csch(f*x + e)^2*sinh(f*x + e)^2 + a*csch(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.17, size = 244, normalized size = 1.20

$$-\frac{-\sinh\left(fx+e\right)\sqrt{\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\, b\left(a \operatorname{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) - b \operatorname{Ellipti}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(-sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(a
*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))+EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))*a+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))+(-1/a*b)^(1/2)*a*
b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a^2-(-1/a*b)^(1/2)*a*b)*cosh(f*x+e)^2)/sinh
(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sinh\left(fx+e\right)^{2} + a\right)^{\frac{3}{2}} \operatorname{csch}\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b \sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.86 $\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=267

$$-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{a\coth(e+fx)c}{}$$

```
[Out] 2/3*(a-2*b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*a*coth(f*x+e)*csch(
f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/f+2/3*(a-2*b)*(1/(1+sinh(f*x+e)^2))^(1/2
)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-
b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sin
h(f*x+e)^2)/a)^(1/2)-1/3*(a-3*b)*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*se
ch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/
a)^(1/2)-2/3*(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.31, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3188, 474, 583, 531, 418, 492, 411}

$$-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{a\coth(e+fx)c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (a*Coth[e +
 f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (2*(a - 2*b)*Ell
ipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a - 3*b)
*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2
*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^4\sqrt{1+x^2}}dx,x,\sinh(\right.}{f}$$

$$= -\frac{a\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cosh\right.}{}$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{a\coth(e+fx)c}{}$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{a\coth(e+fx)c}{}$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{a\coth(e+fx)c}{}$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{a\coth(e+fx)c}{}$$

Mathematica [C] time = 4.04, size = 213, normalized size = 0.80

$$\frac{-2i\left(2a^2-5ab+3b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)+\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\left(2\left(2a^2-7ab+4b^2\right)\cosh(2(e+fx))-8a^2+b(\right.}{\sqrt{2}}}{6f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (((-8*a^2 + 13*a*b - 6*b^2 + 2*(2*a^2 - 7*a*b + 4*b^2)*Cosh[2*(e + f*x)] +
(a - 2*b)*b*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + (4*
I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x
), b/a] - (2*I)*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)]
)/a]*EllipticF[I*(e + f*x), b/a])/(6*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\operatorname{csch}\left(fx+e\right)^4\sinh\left(fx+e\right)^2+a\operatorname{csch}\left(fx+e\right)^4\right)\sqrt{b\sinh\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*csch(f*x + e)^4*sinh(f*x + e)^2 + a*csch(f*x + e)^4)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.15, size = 454, normalized size = 1.70

$$2\sqrt{-\frac{b}{a}}\, ab\left(\sinh^6\left(fx+e\right)\right)-4\sqrt{-\frac{b}{a}}\, b^2\left(\sinh^6\left(fx+e\right)\right)+b\sqrt{\frac{a+b\left(\sinh^2\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+
b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+4*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-3*(-1/
a*b)^(1/2)*a*b*sinh(f*x+e)^4-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^(1
/2)*a^2*sinh(f*x+e)^2-5*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2
)/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}{\sinh\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/sinh(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.87 $\int \left(a+b\sinh^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=232

$$-\frac{i\left(23a^2-23ab+8b^2\right)\sqrt{a+b\sinh^2(c+dx)}\,E\left(ic+idx\middle|\frac{b}{a}\right)}{15d\sqrt{\frac{b\sinh^2(c+dx)}{a}+1}}+\frac{b\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx\right)}{5d}$$

```
[Out] 1/5*b*cosh(d*x+c)*sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^(3/2)/d+4/15*(2*a-b)*b*co
sh(d*x+c)*sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^(1/2)/d-1/15*I*(23*a^2-23*a*b+8*b
^2)*(cos(I*c+I*d*x)^2)^(1/2)/cos(I*c+I*d*x)*EllipticE(sin(I*c+I*d*x),(b/a)^
(1/2))*(a+b*sinh(d*x+c)^2)^(1/2)/d/(1+b*sinh(d*x+c)^2/a)^(1/2)+4/15*I*a*(a-
b)*(2*a-b)*(cos(I*c+I*d*x)^2)^(1/2)/cos(I*c+I*d*x)*EllipticF(sin(I*c+I*d*x)
,(b/a)^(1/2))*(1+b*sinh(d*x+c)^2/a)^(1/2)/d/(a+b*sinh(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3180, 3170, 3172, 3178, 3177, 3183, 3182}

$$-\frac{i\left(23a^2-23ab+8b^2\right)\sqrt{a+b\sinh^2(c+dx)}\,E\left(ic+idx\middle|\frac{b}{a}\right)}{15d\sqrt{\frac{b\sinh^2(c+dx)}{a}+1}}+\frac{b\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(5/2),x]
```

```
[Out] (4*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x]*Sqrt[a + b*Sinh[c + d*x]^2])/(15
*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^(3/2))/(5*d) -
 ((I/15)*(23*a^2 - 23*a*b + 8*b^2)*EllipticE[I*c + I*d*x, b/a]*Sqrt[a + b*S
inh[c + d*x]^2])/(d*Sqrt[1 + (b*Sinh[c + d*x]^2)/a]) + (((4*I)/15)*a*(a - b
)*(2*a - b)*EllipticF[I*c + I*d*x, b/a]*Sqrt[1 + (b*Sinh[c + d*x]^2)/a])/(d
*Sqrt[a + b*Sinh[c + d*x]^2])
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^{5/2} dx = \frac{b\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^{3/2}}{5d} + \frac{1}{5}\int \sqrt{a+b\sinh^2(c+dx}$$

$$= \frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d} + \frac{b\cosh(c+dx)\sin}{}$$

$$= \frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d} + \frac{b\cosh(c+dx)\sin}{}$$

$$= \frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d} + \frac{b\cosh(c+dx)\sin}{}$$

$$= \frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d} + \frac{b\cosh(c+dx)\sin}{}$$

Mathematica [A] time = 1.41, size = 208, normalized size = 0.90

$$\frac{\sqrt{2}b\sinh(2(c+dx))\left(88a^2+28b(2a-b)\cosh(2(c+dx))-88ab+3b^2\cosh(4(c+dx))+25b^2\right)+64ia\left(2a^2-3a\right.}{240d\sqrt{2a+b\cosh(2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(5/2),x]
```

```
[Out] ((-16*I)*a*(23*a^2 - 23*a*b + 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(c + d*x)])/a
]*EllipticE[I*(c + d*x), b/a] + (64*I)*a*(2*a^2 - 3*a*b + b^2)*Sqrt[(2*a -
b + b*Cosh[2*(c + d*x)])/a]*EllipticF[I*(c + d*x), b/a] + Sqrt[2]*b*(88*a^2
 - 88*a*b + 25*b^2 + 28*(2*a - b)*b*Cosh[2*(c + d*x)] + 3*b^2*Cosh[4*(c + d
*x)])*Sinh[2*(c + d*x)])/(240*d*Sqrt[2*a - b + b*Cosh[2*(c + d*x)]])
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sinh\left(dx+c\right)^4+2\,ab\sinh\left(dx+c\right)^2+a^2\right)\sqrt{b\sinh\left(dx+c\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sinh(d*x + c)^4 + 2*a*b*sinh(d*x + c)^2 + a^2)*sqrt(b*sinh(d*
x + c)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.12, size = 609, normalized size = 2.62

$$\frac{3\sqrt{-\frac{b}{a}}\, b^3\sinh\left(dx+c\right)\left(\cosh^6\left(dx+c\right)\right)+\left(14\sqrt{-\frac{b}{a}}\, a\, b^2-10\sqrt{-\frac{b}{a}}\, b^3\right)\left(\cosh^4\left(dx+c\right)\right)\sinh\left(dx+c\right)+\left(11\sqrt{-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^3*sinh(d*x+c)*cosh(d*x+c)^6+(14*(-1/a*b)^(1/2)*a*b
^2-10*(-1/a*b)^(1/2)*b^3)*cosh(d*x+c)^4*sinh(d*x+c)+(11*(-1/a*b)^(1/2)*a^2*
b-18*(-1/a*b)^(1/2)*a*b^2+7*(-1/a*b)^(1/2)*b^3)*cosh(d*x+c)^2*sinh(d*x+c)+1
5*a^3*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sin
h(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))-34*a^2*b*(b/a*cosh(d*x+c)^2+(a-b)/a)^(
1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2)
)+27*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sinh
(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-8*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/
2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))*
b^3+23*a^2*b*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*Ellipt
icE(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))-23*(b/a*cosh(d*x+c)^2+(a-b)/a)^
(1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticE(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2
))*a*b^2+8*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*Elliptic
E(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/(-1/a*b)^(1/2)/cosh(d*x+c)/(
a+b*sinh(d*x+c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\sinh\left(c+d\,x\right)^2+a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^(5/2),x)
[Out] int((a + b*sinh(c + d*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**(5/2),x)
[Out] Timed out
```

3.88 $\int \sqrt{1+\sinh^2(x)}\, dx$

Optimal. Leaf size=11

$$\sqrt{\cosh^2(x)}\ \tanh(x)$$

[Out] (cosh(x)^2)^(1/2)*tanh(x)

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3176, 3207, 2637}

$$\sqrt{\cosh^2(x)}\ \tanh(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sinh[x]^2],x]

[Out] Sqrt[Cosh[x]^2]*Tanh[x]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned} \int \sqrt{1+\sinh^2(x)}\, dx &= \int \sqrt{\cosh^2(x)}\, dx \\ &= \left(\sqrt{\cosh^2(x)}\,\text{sech}(x)\right) \int \cosh(x)\, dx \\ &= \sqrt{\cosh^2(x)}\ \tanh(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\sqrt{\cosh^2(x)}\ \tanh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Sinh[x]^2],x]

[Out] Sqrt[Cosh[x]^2]*Tanh[x]

fricas [A] time = 3.47, size = 2, normalized size = 0.18

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="fricas")

[Out] sinh(x)

giac [A] time = 0.11, size = 11, normalized size = 1.00

$$-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*e^(-x) + 1/2*e^x

maple [A] time = 0.08, size = 14, normalized size = 1.27

$$\frac{\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\,\sinh(x)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+sinh(x)^2)^(1/2),x)

[Out] (cosh(x)^2)^(1/2)*sinh(x)/cosh(x)

maxima [A] time = 0.41, size = 11, normalized size = 1.00

$$-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*e^(-x) + 1/2*e^x

mupad [B] time = 0.07, size = 2, normalized size = 0.18

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sinh(x)^2 + 1)^(1/2),x)

[Out] sinh(x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sinh^{2}(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sinh(x)**2)**(1/2),x)

[Out] Integral(sqrt(sinh(x)**2 + 1), x)

3.89 $\int \sqrt{-1-\sinh^2(x)}\,dx$

Optimal. Leaf size=13

$$\sqrt{-\cosh^2(x)}\,\tanh(x)$$

```
[Out] (-cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3176, 3207, 2637}

$$\sqrt{-\cosh^2(x)}\,\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] Sqrt[-Cosh[x]^2]*Tanh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1-\sinh^2(x)}\,dx &= \int \sqrt{-\cosh^2(x)}\,dx\\ &= \left(\sqrt{-\cosh^2(x)}\,\text{sech}(x)\right)\int \cosh(x)\,dx\\ &= \sqrt{-\cosh^2(x)}\,\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\sqrt{-\cosh^2(x)}\,\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] Sqrt[-Cosh[x]^2]*Tanh[x]
```

fricas [C] time = 0.73, size = 14, normalized size = 1.08

$$\frac{1}{2}\left(i\,e^{(2\,x)}-i\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(I*e^(2*x) - I)*e^(-x)
```

giac [C] time = 0.12, size = 11, normalized size = 0.85

$$-\frac{1}{2}i\,e^{(-x)}+\frac{1}{2}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*I*e^(-x) + 1/2*I*e^x
```

maple [A] time = 0.06, size = 15, normalized size = 1.15

$$-\frac{\cosh(x)\sinh(x)}{\sqrt{-\left(\cosh^{2}(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-sinh(x)^2)^(1/2),x)
[Out] -cosh(x)*sinh(x)/(-cosh(x)^2)^(1/2)
```

maxima [B] time = 0.42, size = 25, normalized size = 1.92

$$-\frac{e^{(-2\,x)}}{2\,\sqrt{-e^{(-2\,x)}}}+\frac{1}{2\,\sqrt{-e^{(-2\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*e^(-2*x)/sqrt(-e^(-2*x)) + 1/2/sqrt(-e^(-2*x))
```

mupad [B] time = 0.17, size = 5, normalized size = 0.38

$$\sinh(x)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- sinh(x)^2 - 1)^(1/2),x)
[Out] sinh(x)*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\sinh^{2}(x)-1}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)**2)**(1/2),x)
[Out] Integral(sqrt(-sinh(x)**2 - 1), x)
```

3.90 $\int \sqrt{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=11

$$-iE(ix|-1)$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticE(I*sinh(x),I)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3177}

$$-iE(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sinh[x]^2],x]
```

```
[Out] (-I)*EllipticE[I*x, -1]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{1-\sinh^2(x)}\,dx = -iE(ix|-1)$$

Mathematica [A] time = 0.02, size = 11, normalized size = 1.00

$$-iE(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sinh[x]^2],x]
```

```
[Out] (-I)*EllipticE[I*x, -1]
```

fricas [F] time = 1.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-\sinh(x)^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-sinh(x)^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sinh(x)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-sinh(x)^2 + 1), x)
```

maple [B] time = 0.12, size = 51, normalized size = 4.64

$$\frac{\sqrt{-\left(-1+\sinh^{2}(x)\right)\left(\cosh^{2}(x)\right)}\,\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\,\left(2\,\mathrm{EllipticF}\left(\sinh(x),i\right)-\mathrm{EllipticE}\left(\sinh(x),i\right)\right)}{\sqrt{1-\left(\sinh^{4}(x)\right)}\,\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sinh(x)^2)^(1/2),x)
```

```
[Out] (-(-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(2*EllipticF(sinh(x),I)
-EllipticE(sinh(x),I))/(1-sinh(x)^4)^(1/2)/cosh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sinh(x)^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-sinh(x)^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \sqrt{1-\sinh(x)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sinh(x)^2)^(1/2),x)
```

```
[Out] int((1 - sinh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-\sinh^{2}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sinh(x)**2), x)
```

3.91 $\int \sqrt{-1+\sinh^2(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{i\sqrt{\sinh^2(x)-1}\,E(ix|-1)}{\sqrt{1-\sinh^2(x)}}$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticE(I*sinh(x),I)*(-1+sinh(x)^2)^(1/2)/(1
-sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3178, 3177}

$$-\frac{i\sqrt{\sinh^2(x)-1}\,E(ix|-1)}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*x, -1]*Sqrt[-1 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1+\sinh^2(x)}\,dx &= \frac{\sqrt{-1+\sinh^2(x)}\int \sqrt{1-\sinh^2(x)}\,dx}{\sqrt{1-\sinh^2(x)}}\\ &= -\frac{iE(ix|-1)\sqrt{-1+\sinh^2(x)}}{\sqrt{1-\sinh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.00

$$\frac{i\sqrt{3-\cosh(2x)}\,E(ix|-1)}{\sqrt{\cosh(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] (I*Sqrt[3 - Cosh[2*x]]*EllipticE[I*x, -1])/Sqrt[-3 + Cosh[2*x]]
```

fricas [F] time = 3.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sinh(x)^2-1}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sinh(x)^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sinh(x)^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(x)^2 - 1), x)
```

maple [A] time = 0.10, size = 61, normalized size = 1.85

$$\frac{i\sqrt{\left(-1+\sinh^2(x)\right)\left(\cosh^2(x)\right)}\,\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\,\sqrt{1-\left(\sinh^2(x)\right)}\ \text{EllipticE}\left(i\sinh(x),i\right)}{\sqrt{\sinh^4(x)-1}\ \cosh(x)\sqrt{-1+\sinh^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sinh(x)^2)^(1/2),x)
```

```
[Out] I*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(1-sinh(x)^2)^(1/2)*El
lipticE(I*sinh(x),I)/(sinh(x)^4-1)^(1/2)/cosh(x)/(-1+sinh(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sinh(x)^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(x)^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{{\sinh(x)}^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)^2 - 1)^(1/2),x)
```

```
[Out] int((sinh(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sinh^2{(x)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(x)**2 - 1), x)
```

3.92 $\int \sqrt{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=42

$$-\frac{i\sqrt{a+b\sinh^2(x)}\,E\left(ix\left|\frac{b}{a}\right.\right)}{\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticE(I*sinh(x),(b/a)^(1/2))*(a+b*sinh(x)^
2)^(1/2)/(1+b*sinh(x)^2/a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3178, 3177}

$$-\frac{i\sqrt{a+b\sinh^2(x)}\,E\left(ix\left|\frac{b}{a}\right.\right)}{\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sinh^2(x)}\,dx &= \frac{\sqrt{a+b\sinh^2(x)}\int \sqrt{1+\frac{b\sinh^2(x)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(x)}{a}}}\\ &= -\frac{iE\left(ix\left|\frac{b}{a}\right.\right)\sqrt{a+b\sinh^2(x)}}{\sqrt{1+\frac{b\sinh^2(x)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 1.29

$$-\frac{ia\sqrt{\frac{2a+b\cosh(2x)-b}{a}}\,E\left(ix\left|\frac{b}{a}\right.\right)}{\sqrt{2a+b\cosh(2x)-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticE[I*x, b/a])/Sqrt[2*a - b +
 b*Cosh[2*x]]
```

fricas [F] time = 4.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh(x)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(x)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^2 + a), x)
```

maple [B] time = 0.10, size = 109, normalized size = 2.60

$$\frac{\sqrt{\frac{a+b\left(\sinh^2(x)\right)}{a}}\,\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\,\left(a\,\mathrm{EllipticF}\left(\sinh(x)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\,\mathrm{EllipticF}\left(\sinh(x)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+b\,\mathrm{EllipticE}\left(\mathrm{si}\right.\right.}{\sqrt{-\frac{b}{a}}\,\cosh(x)\sqrt{a+b\left(\sinh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x)^2)^(1/2),x)
```

```
[Out] ((a+b*sinh(x)^2)/a)^(1/2)*(cosh(x)^2)^(1/2)*(a*EllipticF(sinh(x)*(-1/a*b)^(
1/2),(a/b)^(1/2))-b*EllipticF(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))+b*Ellipti
cE(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/cosh(x)/(a+b*sinh(x)
^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{b\,{\mathrm{sinh}\left(x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x)^2)^(1/2),x)
```

```
[Out] int((a + b*sinh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh^2(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x)**2), x)
```

3.93 $\int \left(1+\sinh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=29

$$\frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)+\frac{2}{3}\sqrt{\cosh^2(x)}\tanh(x)$$

```
[Out] 1/3*(cosh(x)^2)^(3/2)*tanh(x)+2/3*(cosh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3176, 3203, 3207, 2637}

$$\frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)+\frac{2}{3}\sqrt{\cosh^2(x)}\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] (2*Sqrt[Cosh[x]^2]*Tanh[x])/3 + ((Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(1+\sinh^2(x)\right)^{3/2} dx &= \int \cosh^2(x)^{3/2} dx\\ &= \frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)+\frac{2}{3}\int \sqrt{\cosh^2(x)}\, dx\\ &= \frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)+\frac{1}{3}\left(2\sqrt{\cosh^2(x)}\operatorname{sech}(x)\right)\int \cosh(x)\, dx\\ &= \frac{2}{3}\sqrt{\cosh^2(x)}\tanh(x)+\frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 0.79

$$\frac{1}{12}(9\sinh(x)+\sinh(3x))\sqrt{\cosh^2(x)}\,\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)^(3/2),x]
[Out] (Sqrt[Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))/12
```

fricas [A] time = 2.91, size = 17, normalized size = 0.59

$$\frac{1}{12}\sinh(x)^3+\frac{1}{4}\left(\cosh(x)^2+3\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/12*sinh(x)^3 + 1/4*(cosh(x)^2 + 3)*sinh(x)
```

giac [A] time = 0.14, size = 25, normalized size = 0.86

$$-\frac{1}{24}\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}+\frac{1}{24}\,e^{(3\,x)}+\frac{3}{8}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/24*(9*e^(2*x) + 1)*e^(-3*x) + 1/24*e^(3*x) + 3/8*e^x
```

maple [A] time = 0.07, size = 21, normalized size = 0.72

$$\frac{\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\,\sinh(x)\left(\sinh^2(x)+3\right)}{3\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sinh(x)^2)^(3/2),x)
[Out] 1/3*(cosh(x)^2)^(1/2)*sinh(x)*(sinh(x)^2+3)/cosh(x)
```

maxima [A] time = 0.42, size = 23, normalized size = 0.79

$$\frac{1}{24}\,e^{(3\,x)}-\frac{3}{8}\,e^{(-x)}-\frac{1}{24}\,e^{(-3\,x)}+\frac{3}{8}\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(3/2),x, algorithm="maxima")
[Out] 1/24*e^(3*x) - 3/8*e^(-x) - 1/24*e^(-3*x) + 3/8*e^x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(\sinh(x)^2+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)^2 + 1)^(3/2),x)
[Out] int((sinh(x)^2 + 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sinh^2(x) + 1\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((sinh(x)**2 + 1)**(3/2), x)
```

3.94 $\int \left(-1-\sinh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=33

$$\frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)-\frac{2}{3}\sqrt{-\cosh^2(x)}\,\tanh(x)$$

[Out] 1/3*(-cosh(x)^2)^(3/2)*tanh(x)-2/3*(-cosh(x)^2)^(1/2)*tanh(x)

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3176, 3203, 3207, 2637}

$$\frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)-\frac{2}{3}\sqrt{-\cosh^2(x)}\,\tanh(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 - Sinh[x]^2)^(3/2),x]

[Out] (-2*Sqrt[-Cosh[x]^2]*Tanh[x])/3 + ((-Cosh[x]^2)^(3/2)*Tanh[x])/3

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(-1-\sinh^2(x)\right)^{3/2} dx &= \int \left(-\cosh^2(x)\right)^{3/2} dx\\
&= \frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)-\frac{2}{3}\int \sqrt{-\cosh^2(x)}\,dx\\
&= \frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)-\frac{1}{3}\left(2\sqrt{-\cosh^2(x)}\,\text{sech}(x)\right)\int \cosh(x)\,dx\\
&= -\frac{2}{3}\sqrt{-\cosh^2(x)}\,\tanh(x)+\frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.76

$$-\frac{1}{12}(9\sinh(x)+\sinh(3x))\sqrt{-\cosh^2(x)}\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Sinh[x]^2)^(3/2),x]
[Out] -1/12*(Sqrt[-Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))
```

fricas [C] time = 0.60, size = 26, normalized size = 0.79

$$\frac{1}{24}\left(-i\,e^{(6\,x)}-9i\,e^{(4\,x)}+9i\,e^{(2\,x)}+i\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/24*(-I*e^(6*x) - 9*I*e^(4*x) + 9*I*e^(2*x) + I)*e^(-3*x)
```

giac [C] time = 0.12, size = 25, normalized size = 0.76

$$\frac{1}{24}i\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}-\frac{1}{24}i\,e^{(3\,x)}-\frac{3}{8}i\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="giac")
[Out] 1/24*I*(9*e^(2*x) + 1)*e^(-3*x) - 1/24*I*e^(3*x) - 3/8*I*e^x
```

maple [A] time = 0.06, size = 21, normalized size = 0.64

$$\frac{\cosh(x)\sinh(x)\left(\cosh^2(x)+2\right)}{3\sqrt{-\left(\cosh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-sinh(x)^2)^(3/2),x)
[Out] 1/3*cosh(x)*sinh(x)*(cosh(x)^2+2)/(-cosh(x)^2)^(1/2)
```

maxima [B] time = 0.58, size = 53, normalized size = 1.61

$$\frac{3\,e^{(-2\,x)}}{8\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}}-\frac{3\,e^{(-4\,x)}}{8\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}}-\frac{e^{(-6\,x)}}{24\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}}+\frac{1}{24\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="maxima")
[Out] 3/8*e^(-2*x)/(-e^(-2*x))^(3/2) - 3/8*e^(-4*x)/(-e^(-2*x))^(3/2) - 1/24*e^(-
6*x)/(-e^(-2*x))^(3/2) + 1/24/(-e^(-2*x))^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\left(-\sinh(x)^2-1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((- sinh(x)^2 - 1)^(3/2),x)
```

```
[Out] int((- sinh(x)^2 - 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sinh^2(x) - 1\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((-sinh(x)**2 - 1)**(3/2), x)
```

3.95 $\int \left(1-\sinh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=45

$$\frac{2}{3}iF(ix|-1)-2iE(ix|-1)-\frac{1}{3}\sinh(x)\sqrt{1-\sinh^2(x)}\cosh(x)$$

```
[Out] -2*I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticE(I*sinh(x),I)+2/3*I*(cosh(x)^2)^(1/
2)/cosh(x)*EllipticF(I*sinh(x),I)-1/3*cosh(x)*sinh(x)*(1-sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3180, 3172, 3177, 3182}

$$\frac{2}{3}iF(ix|-1)-2iE(ix|-1)-\frac{1}{3}\sinh(x)\sqrt{1-\sinh^2(x)}\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] (-2*I)*EllipticE[I*x, -1] + ((2*I)/3)*EllipticF[I*x, -1] - (Cosh[x]*Sinh[x]
*Sqrt[1 - Sinh[x]^2])/3
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(1-\sinh^2(x)\right)^{3/2} dx &= -\frac{1}{3}\cosh(x)\sinh(x)\sqrt{1-\sinh^2(x)} + \frac{1}{3}\int \frac{4-6\sinh^2(x)}{\sqrt{1-\sinh^2(x)}} dx \\ &= -\frac{1}{3}\cosh(x)\sinh(x)\sqrt{1-\sinh^2(x)} - \frac{2}{3}\int \frac{1}{\sqrt{1-\sinh^2(x)}} dx + 2\int \sqrt{1-\sinh^2(x)}\, d \\ &= -2iE(ix|-1) + \frac{2}{3}iF(ix|-1) - \frac{1}{3}\cosh(x)\sinh(x)\sqrt{1-\sinh^2(x)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 45, normalized size = 1.00

$$\frac{1}{12}\left(8iF(ix|-1) - 24iE(ix|-1) - \sinh(2x)\sqrt{6-2\cosh(2x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(3/2),x]
[Out] ((-24*I)*EllipticE[I*x, -1] + (8*I)*EllipticF[I*x, -1] - Sqrt[6 - 2*Cosh[2*
x]]*Sinh[2*x])/12
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-\sinh(x)^2+1\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(3/2),x, algorithm="fricas")
[Out] integral((-sinh(x)^2 + 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sinh(x)^2+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(3/2),x, algorithm="giac")
[Out] integrate((-sinh(x)^2 + 1)^(3/2), x)
```

maple [A] time = 0.13, size = 103, normalized size = 2.29

$$\frac{\sqrt{-\left(-1+\sinh^2(x)\right)\left(\cosh^2(x)\right)}\,\left(\sinh(x)\left(\cosh^4(x)\right)+10\sqrt{-\left(\cosh^2(x)\right)+2}\,\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh(x\right.\right.}{3\sqrt{1-\left(\sinh^4(x)\right)}\,\cosh(x)\sqrt{1-\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sinh(x)^2)^(3/2),x)
[Out] 1/3*(-(-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(sinh(x)*cosh(x)^4+10*(-cosh(x)^2+2)^
(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x),I)-6*(-cosh(x)^2+2)^(1/2)*(cosh(x
)^2)^(1/2)*EllipticE(sinh(x),I)-2*cosh(x)^2*sinh(x))/(1-sinh(x)^4)^(1/2)/co
sh(x)/(1-sinh(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\sinh(x)^2+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((-sinh(x)^2 + 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(1-\sinh(x)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - sinh(x)^2)^(3/2),x)
[Out] int((1 - sinh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1-\sinh^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)**2)**(3/2),x)
[Out] Integral((1 - sinh(x)**2)**(3/2), x)
```

3.96 $\int \left(-1+\sinh^2(x)\right)^{3/2} \, dx$

Optimal. Leaf size=87

$$\frac{1}{3}\sinh(x)\sqrt{\sinh^2(x)-1}\cosh(x)+\frac{2i\sqrt{1-\sinh^2(x)}\,F(ix|-1)}{3\sqrt{\sinh^2(x)-1}}+\frac{2i\sqrt{\sinh^2(x)-1}\,E(ix|-1)}{\sqrt{1-\sinh^2(x)}}$$

```
[Out] 2/3*I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticF(I*sinh(x),I)*(1-sinh(x)^2)^(1/2)/
(-1+sinh(x)^2)^(1/2)+1/3*cosh(x)*sinh(x)*(-1+sinh(x)^2)^(1/2)+2*I*(cosh(x)^
2)^(1/2)/cosh(x)*EllipticE(I*sinh(x),I)*(-1+sinh(x)^2)^(1/2)/(1-sinh(x)^2)^
(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{1}{3}\sinh(x)\sqrt{\sinh^2(x)-1}\cosh(x)+\frac{2i\sqrt{1-\sinh^2(x)}\,F(ix|-1)}{3\sqrt{\sinh^2(x)-1}}+\frac{2i\sqrt{\sinh^2(x)-1}\,E(ix|-1)}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] (((2*I)/3)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2] + (
Cosh[x]*Sinh[x]*Sqrt[-1 + Sinh[x]^2])/3 + ((2*I)*EllipticE[I*x, -1]*Sqrt[-1
 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
```

```
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(-1+\sinh^2(x)\right)^{3/2}\,dx &= \frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}+\frac{1}{3}\int \frac{4-6\sinh^2(x)}{\sqrt{-1+\sinh^2(x)}}\,dx\\ &= \frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}-\frac{2}{3}\int \frac{1}{\sqrt{-1+\sinh^2(x)}}\,dx-2\int \sqrt{-1+\sinh^2(x)}\\ &= \frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}-\frac{\left(2\sqrt{1-\sinh^2(x)}\right)\int \frac{1}{\sqrt{1-\sinh^2(x)}}\,dx}{3\sqrt{-1+\sinh^2(x)}}-\frac{\left(2\sqrt{-1+\right.}{}\\ &= \frac{2iF(ix|-1)\sqrt{1-\sinh^2(x)}}{3\sqrt{-1+\sinh^2(x)}}+\frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}+\frac{2iE(ix|-1)\sqrt{-1+}}{\sqrt{1-\sinh^2}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 78, normalized size = 0.90

$$\frac{\frac{\sinh(4x)-6\sinh(2x)}{\sqrt{2}}+8i\sqrt{3-\cosh(2x)}\,F(ix|-1)-24i\sqrt{3-\cosh(2x)}\,E(ix|-1)}{12\sqrt{\cosh(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] ((-24*I)*Sqrt[3 - Cosh[2*x]]*EllipticE[I*x, -1] + (8*I)*Sqrt[3 - Cosh[2*x]]
*EllipticF[I*x, -1] + (-6*Sinh[2*x] + Sinh[4*x])/Sqrt[2])/(12*Sqrt[-3 + Cos
h[2*x]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\sinh(x)^2-1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((sinh(x)^2 - 1)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\sinh(x)^2-1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sinh(x)^2 - 1)^(3/2), x)
```

maple [A] time = 0.13, size = 106, normalized size = 1.22

$$\frac{\sqrt{\left(-1+\sinh^{2}(x)\right)\left(\cosh^{2}(x)\right)}\ \left(\sinh(x)\left(\cosh^{4}(x)\right)+2i\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\ \sqrt{-\left(\cosh^{2}(x)\right)+2}\ \text{EllipticF}\left(i\sinh(x),\right.\right.}{3\sqrt{\sinh^{4}(x)-1}\ \cosh(x)\sqrt{-1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sinh(x)^2)^(3/2),x)
```

```
[Out] 1/3*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(sinh(x)*cosh(x)^4+2*I*(cosh(x)^2)^(1/
2)*(-cosh(x)^2+2)^(1/2)*EllipticF(I*sinh(x),I)-6*I*(cosh(x)^2)^(1/2)*(-cosh
(x)^2+2)^(1/2)*EllipticE(I*sinh(x),I)-2*cosh(x)^2*sinh(x))/(sinh(x)^4-1)^(1
/2)/cosh(x)/(-1+sinh(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\sinh(x)^{2}-1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sinh(x)^2 - 1)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(\sinh(x)^{2}-1\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(x)^2 - 1)^(3/2),x)
```

```
[Out] int((sinh(x)^2 - 1)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\sinh^{2}(x)-1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)**2)**(3/2),x)
```

```
[Out] Integral((sinh(x)**2 - 1)**(3/2), x)
```

3.97 $\int \left(a+b\sinh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=123

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\sinh^2(x)}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(x)}{a}+1}\,F\left(ix\middle|\frac{b}{a}\right)}{3\sqrt{a+b\sinh^2(x)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(x)}\,E\left(ix\middle|\frac{b}{a}\right)}{3\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

```
[Out] 1/3*b*cosh(x)*sinh(x)*(a+b*sinh(x)^2)^(1/2)-2/3*I*(2*a-b)*(cosh(x)^2)^(1/2)
/cosh(x)*EllipticE(I*sinh(x),(b/a)^(1/2))*(a+b*sinh(x)^2)^(1/2)/(1+b*sinh(x
)^2/a)^(1/2)+1/3*I*a*(a-b)*(cosh(x)^2)^(1/2)/cosh(x)*EllipticF(I*sinh(x),(b
/a)^(1/2))*(1+b*sinh(x)^2/a)^(1/2)/(a+b*sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\sinh^2(x)}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(x)}{a}+1}\,F\left(ix\middle|\frac{b}{a}\right)}{3\sqrt{a+b\sinh^2(x)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(x)}\,E\left(ix\middle|\frac{b}{a}\right)}{3\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2])/3 - (((2*I)/3)*(2*a - b)*Elliptic
E[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a] + ((I/3)*a*(a
- b)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(x)\right)^{3/2}\,dx = \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)} + \frac{1}{3}\int \frac{a(3a-b)+2(2a-b)b\sinh^2(x)}{\sqrt{a+b\sinh^2(x)}}\,dx$$

$$= \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)} - \frac{1}{3}(a(a-b))\int \frac{1}{\sqrt{a+b\sinh^2(x)}}\,dx + \frac{1}{3}(2(2$$

$$= \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)} + \frac{\left(2(2a-b)\sqrt{a+b\sinh^2(x)}\right)\int \sqrt{1+\frac{b\sinh^2(}{a}}}{3\sqrt{1+\frac{b\sinh^2(x)}{a}}}$$

$$= \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)} - \frac{2i(2a-b)E\left(ix\,\middle|\,\frac{b}{a}\right)\sqrt{a+b\sinh^2(x)}}{3\sqrt{1+\frac{b\sinh^2(x)}{a}}} + \frac{ia(a}{}$$

Mathematica [A] time = 0.39, size = 132, normalized size = 1.07

$$\frac{\sqrt{2}\,b\sinh(2x)(2a+b\cosh(2x)-b)+4ia(a-b)\sqrt{\frac{2a+b\cosh(2x)-b}{a}}\,F\left(ix\,\middle|\,\frac{b}{a}\right)-8ia(2a-b)\sqrt{\frac{2a+b\cosh(2x)-b}{a}}\,E\left(ix\,\middle|\,\frac{b}{a}\right.}{12\sqrt{2a+b\cosh(2x)-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^2)^(3/2),x]
```

```
[Out] ((-8*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticE[I*x, b/a] + (
4*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticF[I*x, b/a] + Sqrt[2
]*b*(2*a - b + b*Cosh[2*x])*Sinh[2*x])/(12*Sqrt[2*a - b + b*Cosh[2*x]])
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh(x)^2+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(x)^2 + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh(x)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x)^2 + a)^(3/2), x)
```

maple [B] time = 0.11, size = 329, normalized size = 2.67

$$\frac{\sqrt{-\frac{b}{a}}\, b^2 \sinh(x) \left(\cosh^4(x)\right) + \left(\sqrt{-\frac{b}{a}}\, ab - \sqrt{-\frac{b}{a}}\, b^2\right) \left(\cosh^2(x)\right) \sinh(x) + 3a^2 \sqrt{\frac{b\left(\cosh^2(x)\right)}{a} + \frac{a-b}{a}}\, \sqrt{\frac{\cosh(2x)}{2} + \frac{1}{2}}\ \mathrm{El}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(x)*cosh(x)^4+((-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2
)*b^2)*cosh(x)^2*sinh(x)+3*a^2*(b/a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1
/2)*EllipticF(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))-5*a*b*(b/a*cosh(x)^2+(a-b
)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))+
2*(b/a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b^2+4*a*b*(b/a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1
/2)*EllipticE(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*(b/a*cosh(x)^2+(a-b)/a)
^(1/2)*(cosh(x)^2)^(1/2)*EllipticE(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)
/(-1/a*b)^(1/2)/cosh(x)/(a+b*sinh(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sinh(x)^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b \sinh(x)^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(x)^2)^(3/2),x)
```

```
[Out] int((a + b*sinh(x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.98 $\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=83

$$\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2bf}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2b^{3/2}f}$$

```
[Out] -1/2*(a+b)*arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(3/2)
/f+1/2*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3186, 388, 217, 206}

$$\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2bf}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((a + b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])
/(2*b^(3/2)*f) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*b*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \cosh(e$$

$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cosh(e+f}{\sqrt{a-b+b\cosh^2}}\right.}{2bf}$$

$$= -\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf}$$

Mathematica [**A**] time = 0.28, size = 98, normalized size = 1.18

$$\frac{\cosh(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}}{2\sqrt{2}bf} - \frac{(a+b)\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/(2*Sqrt[2]*b*f) - ((a +
 b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
])/(2*b^(3/2)*f)
```

fricas [**B**] time = 2.92, size = 2116, normalized size = 25.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((a + b)*cosh(f*x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a
 + b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*
x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x +
 e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*
a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9
*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2
*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*
(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh
(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^
4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x +
 e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5
+ a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 +
a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh
(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*
a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*
x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*c
osh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3
)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x
```

```
 + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e
)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)
^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a + b)*cosh(f*
x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a + b)*sinh(f*x + e)^2)
*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*si
nh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)
*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(c
osh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2
)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2
+ b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)
^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2), 1/8*(2*(
(a + b)*cosh(f*x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a + b)*s
inh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x +
 e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f
*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*co
sh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)
^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((a + b)*cosh(f*x + e
)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a + b)*sinh(f*x + e)^2)*sqrt
(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/
(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
+ 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x
+ e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)
) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f
*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cos
h(f*x + e)^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2)
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.13, size = 204, normalized size = 2.46

$$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(2b^{\frac{3}{2}}\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^{2}\left(fx+e\right)\right)}-b^{2}\ln\left(\frac{2b(c}{}\right.\right.}{4b^{\frac{5}{2}}\cosh\left(fx+e\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*b^(3/2)*(b*cosh(f*x+e)^4+(
a-b)*cosh(f*x+e)^2)^(1/2)-b^2*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+
(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-b*a*ln(1/2*(2*b*cosh(f*x+e
```

```
)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/b^
(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^3}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sinh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(e+f\,x\right)^3}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.99 $\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{b}\,f}$$

```
[Out] arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3186, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[b]*f
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{b}\,f}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 49, normalized size = 1.20

$$\frac{\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/(S
qrt[b]*f)
```

fricas [B] time = 3.91, size = 1654, normalized size = 40.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x +
 e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2
*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x +
e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2
+ b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^
3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*
x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cos
h(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(
14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3
 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(
a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x +
e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e
)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*
a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)
^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*
cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 +
3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3
 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(
f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x
 + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x +
 e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + sqrt(b)*log(-(b*cosh(f*x + e)^4 +
 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x
 + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sq
rt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*c
osh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a
- b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2)))/(b*f), -1/2*(sqrt(-b)*arctan(sqrt(2)*(a*c
osh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*s
qrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)
^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b
^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2
 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))) + sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh
(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a -
```

```
b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f
*x + e) + b)))/(b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.09, size = 108, normalized size = 2.63

$$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\,\ln\left(\frac{2b\left(\cosh^{2}\left(fx+e\right)\right)+2\sqrt{b\left(\cosh^{4}\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^{2}\left(fx+e\right)\right)}\,\sqrt{b}+a-b}{2\sqrt{b}}\right)}{2\sqrt{b}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*(
b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))/b^(1/2)/co
sh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sinh\left(e+f\,x\right)}{\sqrt{b\,\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sinh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.100 $\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{a}\,f}$$

[Out] -arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/f/a^(1/2)

Rubi [A] time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 377, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[a]*f))

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{a}\,f}$$

Mathematica [A] time = 0.18, size = 49, normalized size = 1.17

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]]/(Sqrt[a]*f))
```

fricas [B] time = 0.62, size = 572, normalized size = 13.62

$$\left[\frac{\log\left(-\frac{(a+b)\cosh\left(fx+e\right)^4+4\,(a+b)\cosh\left(fx+e\right)\sinh\left(fx+e\right)^3+(a+b)\sinh\left(fx+e\right)^4+2\,(3\,a-b)\cosh\left(fx+e\right)^2+2\left(3\,(a+b)\cosh\left(fx+e\right)^2+3\,a-b\right)\sinh(\cdots}{\cosh\left(fx+e\right)^4+4\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)^3+\sinh(f\cdots}\right)}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^
3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*co
sh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*c
osh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*co
sh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh
(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 -
 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)
)/(sqrt(a)*f), sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
 b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))
*sinh(f*x + e) + b))/(a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.12, size = 113, normalized size = 2.69

$$-\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\,\ln\left(\frac{(a+b)\left(\cosh^{2}(fx+e)\right)+2\sqrt{a}\,\sqrt{b\left(\cosh^{4}(fx+e)\right)+(a-b)\left(\cosh^{2}(fx+e)\right)}+a-b}{\sinh(fx+e)^{2}}\right)}{2\sqrt{a}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/a^(1/2)*ln(((a+b)*cosh(f*x+e
)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^
2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sinh\left(e+f\,x\right)\,\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csch(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.101 $\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2af}$$

```
[Out] 1/2*(a+b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(3/2)/
f-1/2*coth(f*x+e)*csch(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3186, 382, 377, 206}

$$\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*a^(3/2)*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x
])/(2*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\text{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\text{csch}(e+fx)}{2af} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a}}\right.}{2}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\text{csch}(e+fx)}{2af} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx\right.}{2a}$$

$$= \frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\sqrt{a-b+b\cosh^2(e+fx)}\,\coth(e+fx)\text{csch}}{2af}$$

Mathematica [A] time = 0.34, size = 102, normalized size = 1.15

$$\frac{2(a+b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right) - \sqrt{2}\sqrt{a}\coth(e+fx)\text{csch}(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}}{4a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (2*(a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e +
 f*x]*Csch[e + f*x])/(4*a^(3/2)*f)
```

fricas [B] time = 1.09, size = 1285, normalized size = 14.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 +
(a + b)*sinh(f*x + e)^4 - 2*(a + b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x
 + e)^2 - a - b)*sinh(f*x + e)^2 + 4*((a + b)*cosh(f*x + e)^3 - (a + b)*cos
h(f*x + e))*sinh(f*x + e) + a + b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 +
4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a
- b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e
)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(
(a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(
cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*
cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)
^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*(a*cosh(f*x + e)^2 + 2*
a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e
)*sinh(f*x + e)^3 + a^2*f*sinh(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f
 + 2*(3*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x
+ e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((a + b)*cosh(f*x + e)^
```

```
4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 - 2*(
a + b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 - a - b)*sinh(f*x + e
)^2 + 4*((a + b)*cosh(f*x + e)^3 - (a + b)*cosh(f*x + e))*sinh(f*x + e) + a
 + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)
*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x
 + e) + b)) + sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e)
+ a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/
(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*f*sinh
(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2
- a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*
sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.16, size = 234, normalized size = 2.63

$$-\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(-\ln\left(\frac{(a+b)\left(\cosh^2\left(fx+e\right)\right)+2\sqrt{a}\sqrt{b\left(\cosh^4\left(fx+e\right)\right)+(a-b)\left(\cosh^2\left(fx+e\right)\right)}+a-b}{\sinh\left(fx+e\right)^2}\right)\right.}{4\sinh\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-ln(((a+b)*cosh(f*x+e)^2+2*
a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sin
h(f*x+e)^2*a^2-ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*b*sinh(f*x+e)^2*a+2*a^(3/2)*((a+b*sin
h(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2/a^(5/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(fx+e\right)^3}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(e+fx\right)^3\sqrt{b\sinh\left(e+fx\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(e + fx)}{\sqrt{a + b\sinh^2(e + fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csch(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.102 $\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=229

$$-\frac{2(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f}+\frac{2(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+f\right.}{3b^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+2/3*(a+b)*(1/(1+s
inh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(
f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b^2/f/
(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(1/(1+sinh(f*x+e)^2))^(1/2)
*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b
/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(f*x+e)^2*(a+b*si
nh(f*x+e)^2)/a)^(1/2)-2/3*(a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Rubi [A] time = 0.22, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3188, 479, 531, 418, 492, 411}

$$-\frac{2(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f}+\frac{2(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+f\right.}{3b^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a +
 b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
 (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*
(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
```

```
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\,\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\left(a\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+f\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{3bf\sqrt{\frac{\operatorname{sech}^2(e+}{}}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{2(a+b)E\left(\tan^{-1}(\sinh(e+fx\right.}{3b^2f\sqrt{\frac{\operatorname{se}}{}}}$$

Mathematica [C] time = 0.96, size = 168, normalized size = 0.73

$$\frac{b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)-2i\sqrt{2}\,a(2a+b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right)+4i\sqrt{2}\,a(a+}{6b^2f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((4*I)*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[
I*(e + f*x), b/a] - (2*I)*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e +
 f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*
Sinh[2*(e + f*x)])/(6*b^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 1.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.11, size = 344, normalized size = 1.50

$$\frac{\sqrt{-\frac{b}{a}}\, b\sinh\left(fx+e\right)\left(\cosh^4\left(fx+e\right)\right)+\left(\sqrt{-\frac{b}{a}}\, a-\sqrt{-\frac{b}{a}}\, b\right)\left(\cosh^2\left(fx+e\right)\right)\sinh\left(fx+e\right)+a\sqrt{\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^
(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*(b/a*
cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a-2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1
/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh \left(e+f x\right)^4}{\sqrt{b \sinh \left(e+f x\right)^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.103 $\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=128

$$\frac{ia\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(
1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+I*a*(cos(I*
e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1+b
*sinh(f*x+e)^2/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3172, 3178, 3177, 3183, 3182}

$$\frac{ia\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[1
+ (b*Sinh[e + f*x]^2)/a]) + (I*a*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Si
nh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{b\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} - \frac{\left(a\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\right)\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}}{b\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} + \frac{iaF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{bf\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A] time = 0.25, size = 89, normalized size = 0.70

$$-\frac{i\sqrt{2a+b\cosh(2(e+fx))-b}\left(E\left(i(e+fx)\middle|\frac{b}{a}\right)-F\left(i(e+fx)\middle|\frac{b}{a}\right)\right)}{bf\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*(EllipticE[I*(e + f*x), b/a] - El
lipticF[I*(e + f*x), b/a]))/(b*f*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sinh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.10, size = 113, normalized size = 0.88

$$-\frac{\sqrt{\frac{a+b\left(\sinh^2(fx+e)\right)}{a}}\,\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\left(\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-\mathrm{EllipticE}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\right.\right.}{\sqrt{-\frac{b}{a}}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(Elli
pticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2)))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(e+f\,x\right)^2}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sinh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.104 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(
1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sinh^2(e+fx)}}\\ &= -\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 1.13

$$-\frac{i\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/
(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 2.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.08, size = 86, normalized size = 1.43

$$\frac{\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\;\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\;\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\sinh\left(e+fx\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sinh^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.105 $\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=134

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b}{a}}}$$

```
[Out] -coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-(1/(1+sinh(f*x+e)^2))^(1/2)*(1+s
inh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(
1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*
x+e)^2)/a)^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/f
```

Rubi [A] time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3188, 480, 12, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/(a*f)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}\sqrt{a+bx^2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\right.}{af}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\right.}{af}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{af} - \frac{\left(\sqrt{\right.}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} - \frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e\right.}{a}}}$$

Mathematica [C] time = 0.53, size = 150, normalized size = 1.12

$$\frac{\sqrt{2}\coth(e+fx)(-2a-b\cosh(2(e+fx))+b)+2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}{2af\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (2*I)*a*Sqrt[(
2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(2*a*f*Sqrt[
2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

[Out] integral(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.14, size = 189, normalized size = 1.41

$$-\frac{\sinh\left(fx+e\right)\sqrt{\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,b\left(\operatorname{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-\operatorname{EllipticE}\left(\mathrm{s}\right.\right.}{a\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}}\,\cosh\left(fx+e\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -(sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(El
lipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2)))+(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-
1/a*b)^(1/2)*b)*cosh(f*x+e)^2)/a/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+
b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(e+f\,x\right)^2\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csch(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.106 $\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=267

$$-\frac{2(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}+\frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+}}{3a^2}$$

```
[Out] 2/3*(a+b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f-1/3*coth(f*x+e)*csch(
f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a/f+2/3*(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2
)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-
b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b
*sinh(f*x+e)^2)/a)^(1/2)-1/3*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f
*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)
^(1/2)-2/3*(a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^2/f
```

Rubi [A] time = 0.28, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3188, 480, 583, 531, 418, 492, 411}

$$-\frac{2(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}+\frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (2*(a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f) - (Coth[e +
 f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) + (2*(a + b)*Ell
ipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*Ell
ipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a
+ b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+f\right.}{}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\,s}}{3af}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\,s}}{3af}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\,s}}{3af}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\,s}}{3af}$$

Mathematica [C] time = 3.54, size = 201, normalized size = 0.75

$$\frac{\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\left(\left(4a^2-2ab-4b^2\right)\cosh(2(e+fx))-8a^2+b(a+b)\cosh(4(e+fx))+ab+3b^2\right)}{\sqrt{2}} - 2ia(2a+b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+\right.}{6a^2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (((-8*a^2 + a*b + 3*b^2 + (4*a^2 - 2*a*b - 4*b^2)*Cosh[2*(e + f*x)] + b*(a
+ b)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + (4*I)*a*(a
 + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] -
 (2*I)*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e +
 f*x), b/a])/(6*a^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 2.09, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\operatorname{csch}\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.16, size = 456, normalized size = 1.71

$$2\sqrt{-\frac{b}{a}}\, ab\left(\sinh^{6}\left(fx+e\right)\right)+2\sqrt{-\frac{b}{a}}\, b^{2}\left(\sinh^{6}\left(fx+e\right)\right)+b\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+
b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b
^2*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3-2*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+3*(-
1/a*b)^(1/2)*a*b*sinh(f*x+e)^4+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^
(1/2)*a^2*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2
)/a^2/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(e+fx\right)^{4}\sqrt{b\sinh\left(e+fx\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{4}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csch(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.107 $\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{3/2}f}-\frac{a\cosh(e+fx)}{bf(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

```
[Out] arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(3/2)/f-a*cosh(f
*x+e)/(a-b)/b/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3186, 385, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{3/2}f}-\frac{a\cosh(e+fx)}{bf(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(3/2)*f
) - (a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx\right.}{bf}$$

$$= -\frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+f}}\right.}{bf}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.41, size = 98, normalized size = 1.18

$$\frac{\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{b^{3/2}f} - \frac{\sqrt{2}a\cosh(e+fx)}{bf(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Sqrt[2]*a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]
])) + Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]]/(b^(3/2)*f)
```

fricas [B] time = 1.11, size = 3038, normalized size = 36.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e)^3 + (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)
^2 + 2*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^
2 + a*b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh
(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*
x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(
14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 +
 (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*
a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b
- 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*c
osh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x +
e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)
^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2
 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*s
inh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 +
```

```
18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh
(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*
b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 +
b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(
f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x
+ e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x +
 e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a*b - b^2)*
cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)
*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(a*b - b^
2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*(
(a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
+ b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) +
 b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) -
4*sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e)*sinh(f*x + e) + a*b*si
nh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a*
b^3 - b^4)*f*cosh(f*x + e)^4 + 4*(a*b^3 - b^4)*f*cosh(f*x + e)*sinh(f*x + e
)^3 + (a*b^3 - b^4)*f*sinh(f*x + e)^4 + 2*(2*a^2*b^2 - 3*a*b^3 + b^4)*f*cos
h(f*x + e)^2 + 2*(3*(a*b^3 - b^4)*f*cosh(f*x + e)^2 + (2*a^2*b^2 - 3*a*b^3
+ b^4)*f)*sinh(f*x + e)^2 + (a*b^3 - b^4)*f + 4*((a*b^3 - b^4)*f*cosh(f*x +
 e)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(
((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3
+ (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2
*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*
b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x +
 e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f
*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*si
nh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*
b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x
+ e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + ((a*b - b^2)*cosh(f
*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(
f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(a*b - b^2)*cos
h(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b -
 b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))
*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh
(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e)
 + b)) + 2*sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e)*sinh(f*x + e)
 + a*b*sinh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2)))/((a*b^3 - b^4)*f*cosh(f*x + e)^4 + 4*(a*b^3 - b^4)*f*cosh(f*x + e)*sin
h(f*x + e)^3 + (a*b^3 - b^4)*f*sinh(f*x + e)^4 + 2*(2*a^2*b^2 - 3*a*b^3 + b
^4)*f*cosh(f*x + e)^2 + 2*(3*(a*b^3 - b^4)*f*cosh(f*x + e)^2 + (2*a^2*b^2 -
 3*a*b^3 + b^4)*f)*sinh(f*x + e)^2 + (a*b^3 - b^4)*f + 4*((a*b^3 - b^4)*f*c
osh(f*x + e)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)
)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.18, size = 146, normalized size = 1.76

$$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(\frac{\ln\left(\frac{\frac{a}{2}+\frac{b}{2}+b\left(\sinh^{2}\left(fx+e\right)\right)}{\sqrt{b}}+\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\right)}{2b^{\frac{3}{2}}}-\frac{a(\cdots}{b(a-b)\sqrt{\left(a+b\left(\mathrm{si}\cdots\right.\right.}}\right.}{\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b^(3/2)*ln((1/2*a+1/2*b+b*si
nh(f*x+e)^2)/b^(1/2)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))-a/b*cosh(f*
x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(e+f\,x\right)^{3}}{\left(b\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.108 $\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\cosh(e+fx)}{f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

```
[Out] cosh(f*x+e)/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3186, 191}

$$\frac{\cosh(e+fx)}{f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Cosh[e + f*x]/((a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{f}\\ &= \frac{\cosh(e+fx)}{(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 43, normalized size = 1.19

$$\frac{\sqrt{2}\cosh(e+fx)}{f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*Cosh[e + f*x])/((a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [B] time = 0.98, size = 296, normalized size = 8.22

$$\frac{\sqrt{2}\left(\cosh\left(fx+e\right)\right.}{\left(ab-b^2\right)f\cosh\left(fx+e\right)^4+4\left(ab-b^2\right)f\cosh\left(fx+e\right)\sinh\left(fx+e\right)^3+\left(ab-b^2\right)f\sinh\left(fx+e\right)^4+2\left(2a^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
+ 1)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*f*cosh(f
*x + e)^4 + 4*(a*b - b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*f*s
inh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a*b - b^
2)*f*cosh(f*x + e)^2 + (2*a^2 - 3*a*b + b^2)*f)*sinh(f*x + e)^2 + (a*b - b^
2)*f + 4*((a*b - b^2)*f*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*f*cosh(f*x
+ e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.08, size = 32, normalized size = 0.89

$$\frac{\cosh\left(fx+e\right)}{(a-b)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] cosh(f*x+e)/(a-b)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.45, size = 236, normalized size = 6.56

$$\frac{b^2e^{(-6fx-6e)}+2ab-b^2+\left(8a^2-8ab+3b^2\right)e^{(-2fx-2e)}+3\left(2ab-b^2\right)e^{(-4fx-4e)}}{2\left(a^2-ab\right)\left(2(2a-b)e^{(-2fx-2e)}+be^{(-4fx-4e)}+b\right)^{\frac{3}{2}}f}+\frac{b^2+3\left(2ab-b^2\right)e^{(-2fx-2e)}}{2\left(a^2-ab\right)\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*e^(-6*f*x - 6*e) + 2*a*b - b^2 + (8*a^2 - 8*a*b + 3*b^2)*e^(-2*f*x
 - 2*e) + 3*(2*a*b - b^2)*e^(-4*f*x - 4*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(3/2)*f) + 1/2*(b^2 + 3*(2*a*b - b^2)
*e^(-2*f*x - 2*e) + (8*a^2 - 8*a*b + 3*b^2)*e^(-4*f*x - 4*e) + (2*a*b - b^2
)*e^(-6*f*x - 6*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*
x - 4*e) + b)^(3/2)*f)
```

mupad [B] time = 0.92, size = 191, normalized size = 5.31

$$-\frac{e^{e+f\,x}\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,\left(\frac{2\,e^{e+f\,x}\sinh\left(e+f\,x\right)\,\left(b\,\left(2\,a-b\right)-b\,\left(4\,a-2\,b\right)\right)}{f\,\left(a\,b^2-a^2\,b\right)}+\frac{2\,b^2\cosh\left(e+f\,x\right)\,e^{e+f\,x}}{f\,\left(a\,b^2-a^2\,b\right)}+\frac{b\,e^{2\,e+2\,f\,x}\,\left(4\,a-2\,b\right)}{f\,\left(a\,b^2-a^2\,b\right)}\right)}{4\,a\,e^{2\,e+2\,f\,x}-2\,b\,e^{2\,e+2\,f\,x}+2\,b\,e^{2\,e+2\,f\,x}\cosh\left(2\,e+2\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] -(exp(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)*((2*exp(e + f*x)*sinh(e + f*x)
*(b*(2*a - b) - b*(4*a - 2*b)))/(f*(a*b^2 - a^2*b)) + (2*b^2*cosh(e + f*x)*
exp(e + f*x))/(f*(a*b^2 - a^2*b)) + (b*exp(2*e + 2*f*x)*(4*a - 2*b))/(f*(a*
b^2 - a^2*b))))/(4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + 2*b*exp(2*e
+ 2*f*x)*cosh(2*e + 2*f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sinh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.109 $\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{3/2}f}-\frac{b\cosh(e+fx)}{af(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

```
[Out] -arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(3/2)/f-b*cosh(
f*x+e)/a/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 382, 377, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{3/2}f}-\frac{b\cosh(e+fx)}{af(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(a^(3/2)
*f)) - (b*Cosh[e + f*x])/(a*(a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+\right.}{af}$$

$$= -\frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{af}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.38, size = 98, normalized size = 1.17

$$\frac{-\frac{\sqrt{2}\sqrt{a}b\cosh(e+fx)}{(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}} - \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]] - (Sqrt[2]*Sqrt[a]*b*Cosh[e + f*x])/((a - b)*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]]))/(a^(3/2)*f)
```

fricas [B] time = 1.95, size = 1641, normalized size = 19.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e)^3 + (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)
^2 + 2*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^
2 + a*b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)
*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh
(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*s
qrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 +
 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*c
osh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x
+ e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x
+ e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh
(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cos
h(f*x + e)*sinh(f*x + e) + a*b*sinh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)
^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2)))/((a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 4*(a^3*
```

```
b - a^2*b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^3*b - a^2*b^2)*f*sinh(f*x
 + e)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - a
^2*b^2)*f*cosh(f*x + e)^2 + (2*a^4 - 3*a^3*b + a^2*b^2)*f)*sinh(f*x + e)^2
+ (a^3*b - a^2*b^2)*f + 4*((a^3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^4 - 3
*a^3*b + a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), (((a*b - b^2)*cosh(f*x +
 e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x
+ e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(a*b - b^2)*cosh(f*
x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b - b^2
)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))*sqr
t(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sin
h(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x
 + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b
)) - sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e)*sinh(f*x + e) + a*b
*sinh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(
(a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 4*(a^3*b - a^2*b^2)*f*cosh(f*x + e)*s
inh(f*x + e)^3 + (a^3*b - a^2*b^2)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 3*a^3*b +
 a^2*b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (2
*a^4 - 3*a^3*b + a^2*b^2)*f)*sinh(f*x + e)^2 + (a^3*b - a^2*b^2)*f + 4*((a^
3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^4 - 3*a^3*b + a^2*b^2)*f*cosh(f*x +
 e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Error: Bad Argument Type
```

maple [B] time = 0.18, size = 154, normalized size = 1.83

$$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(-\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a(a-b)\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}}-\frac{\ln\left(\frac{2a+(a+b)\left(\sinh^{2}\left(fx+e\right)\right)+2\sqrt{a}\,\sqrt{(a\cdots}}{\sinh(fx\cdots}\right)}{2a^{\frac{3}{2}}}\cdots\right)}{\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/a*b*cosh(f*x+e)^2/(a-b)/((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)-1/2/a^(3/2)*ln((2*a+(a+b)*sinh(f*x+e)^
2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2))/cosh
(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(e+f\,x\right)\,\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csch(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.110 $\int \frac{\text{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{5/2}f}-\frac{b(a-3b)\cosh(e+fx)}{2a^2f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{\coth(e+fx)\text{csch}(e+fx)}{2af\sqrt{a+b\cosh^2(e+fx)-b}}$$

```
[Out] 1/2*(a+3*b)*arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(5/2
)/f-1/2*(a-3*b)*b*cosh(f*x+e)/a^2/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(1/2)-1/2*c
oth(f*x+e)*csch(f*x+e)/a/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3186, 414, 527, 12, 377, 206}

$$-\frac{b(a-3b)\cosh(e+fx)}{2a^2f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{5/2}f}-\frac{\coth(e+fx)\text{csch}(e+fx)}{2af\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/(2*a^(5/2)*f) - ((a - 3*b)*b*Cosh[e + f*x])/(2*a^2*(a - b)*f*Sqrt[a - b +
 b*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x])/(2*a*f*Sqrt[a - b + b*
Cosh[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{a+b+2bx^2}{\left(1-x^2\right)\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{2af}$$

$$= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\text{Sub}}{}$$

$$= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{(a+}{}$$

$$= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{(a+}{}$$

$$= \frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - $$

Mathematica [A] time = 0.72, size = 134, normalized size = 0.96

$$\frac{\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{a^{5/2}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)\left(2a^2+b(a-3b)\cosh(2(e+fx))-3ab+3b^2\right)}{a^2(a-b)\sqrt{4a+2b\cosh(2(e+fx))-2b}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (((a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]])/a^(5/2) - ((2*a^2 - 3*a*b + 3*b^2 + (a - 3*b)*b*Cosh[2*(e +
```

```
f*x)])*Coth[e + f*x]*Csch[e + f*x])/(a^2*(a - b)*Sqrt[4*a - 2*b + 2*b*Cosh[
2*(e + f*x)]]))/(2*f)
```

fricas [B] time = 3.00, size = 4441, normalized size = 31.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/4*(((a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 8*(a^2*b + 2*a*b^2 - 3*b
^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b + 2*a*b^2 - 3*b^3)*sinh(f*x + e)
^8 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 4*(a^3 + a^2*b - 5
*a*b^2 + 3*b^3 + 7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)
^6 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(a^3 + a^2*b - 5*a*
b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(4*a^3 + 5*a^2*b - 18*a*b^2
 + 9*b^3)*cosh(f*x + e)^4 + 2*(35*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4
 - 4*a^3 - 5*a^2*b + 18*a*b^2 - 9*b^3 + 30*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*
cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x
+ e)^5 + 10*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^3 - (4*a^3 + 5*a^
2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b + 2*a*b^2 -
3*b^3 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^2 + 4*(7*(a^2*b + 2
*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 15*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f
*x + e)^4 + a^3 + a^2*b - 5*a*b^2 + 3*b^3 - 3*(4*a^3 + 5*a^2*b - 18*a*b^2 +
 9*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b + 2*a*b^2 - 3*b^3)*cos
h(f*x + e)^7 + 3*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^5 - (4*a^3 +
 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^3 + (a^3 + a^2*b - 5*a*b^2 + 3*b
^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4
*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a -
 b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)
^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((
a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(c
osh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*c
osh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^
3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((a^2*b - 3*a*b^2)*cosh(
f*x + e)^6 + 6*(a^2*b - 3*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b - 3
*a*b^2)*sinh(f*x + e)^6 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + (4*
a^3 - 5*a^2*b + 3*a*b^2 + 15*(a^2*b - 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x +
e)^4 + 4*(5*(a^2*b - 3*a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)
*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b - 3*a*b^2 + (4*a^3 - 5*a^2*b + 3*a*
b^2)*cosh(f*x + e)^2 + (15*(a^2*b - 3*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 5*a^
2*b + 3*a*b^2 + 6*(4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e
)^2 + 2*(3*(a^2*b - 3*a*b^2)*cosh(f*x + e)^5 + 2*(4*a^3 - 5*a^2*b + 3*a*b^2
)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e
))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - a^3*b^2)*f*c
osh(f*x + e)^8 + 8*(a^4*b - a^3*b^2)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4
*b - a^3*b^2)*f*sinh(f*x + e)^8 + 4*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x +
e)^6 + 4*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)
*f)*sinh(f*x + e)^6 - 2*(4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^4 + 8
*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^3 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*cos
h(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^4 + 3
0*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^
2)*f)*sinh(f*x + e)^4 + 4*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^2 + 8*(
7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^5 + 10*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh
(f*x + e)^3 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^
3 + 4*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^6 + 15*(a^5 - 2*a^4*b + a^3*b^2)
*f*cosh(f*x + e)^4 - 3*(4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^2 + (a
```

```
^5 - 2*a^4*b + a^3*b^2)*f)*sinh(f*x + e)^2 + (a^4*b - a^3*b^2)*f + 8*((a^4*
b - a^3*b^2)*f*cosh(f*x + e)^7 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e
)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^3 + (a^5 - 2*a^4*b + a^
3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((a^2*b + 2*a*b^2 - 3*b^3)*co
sh(f*x + e)^8 + 8*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^7 +
 (a^2*b + 2*a*b^2 - 3*b^3)*sinh(f*x + e)^8 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b
^3)*cosh(f*x + e)^6 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3 + 7*(a^2*b + 2*a*b^2
 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)
*cosh(f*x + e)^3 + 3*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*
x + e)^5 - 2*(4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^4 + 2*(35*(
a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 - 4*a^3 - 5*a^2*b + 18*a*b^2 - 9*b
^3 + 30*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 10*(a^3 + a^2*b - 5*a*b^2
+ 3*b^3)*cosh(f*x + e)^3 - (4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x +
e))*sinh(f*x + e)^3 + a^2*b + 2*a*b^2 - 3*b^3 + 4*(a^3 + a^2*b - 5*a*b^2 +
3*b^3)*cosh(f*x + e)^2 + 4*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 1
5*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + a^3 + a^2*b - 5*a*b^2 +
 3*b^3 - 3*(4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^2)*sinh(f*x +
 e)^2 + 8*((a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 3*(a^3 + a^2*b - 5*a
*b^2 + 3*b^3)*cosh(f*x + e)^5 - (4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f
*x + e)^3 + (a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e))*s
qrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + s
inh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)
^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f
*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) +
 b)) + sqrt(2)*((a^2*b - 3*a*b^2)*cosh(f*x + e)^6 + 6*(a^2*b - 3*a*b^2)*cos
h(f*x + e)*sinh(f*x + e)^5 + (a^2*b - 3*a*b^2)*sinh(f*x + e)^6 + (4*a^3 - 5
*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + (4*a^3 - 5*a^2*b + 3*a*b^2 + 15*(a^2*b
- 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(a^2*b - 3*a*b^2)*cosh(f
*x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + a^
2*b - 3*a*b^2 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^2 + (15*(a^2*b -
3*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 5*a^2*b + 3*a*b^2 + 6*(4*a^3 - 5*a^2*b +
 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a^2*b - 3*a*b^2)*cosh(f*
x + e)^5 + 2*(4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b
 + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2)))/((a^4*b - a^3*b^2)*f*cosh(f*x + e)^8 + 8*(a^4*b - a^3*b^2)
*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4*b - a^3*b^2)*f*sinh(f*x + e)^8 + 4*
(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^6 + 4*(7*(a^4*b - a^3*b^2)*f*cosh
(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f)*sinh(f*x + e)^6 - 2*(4*a^5 - 7*a
^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^4 + 8*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e
)^3 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*
(a^4*b - a^3*b^2)*f*cosh(f*x + e)^4 + 30*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f
*x + e)^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f)*sinh(f*x + e)^4 + 4*(a^5 - 2*a
^4*b + a^3*b^2)*f*cosh(f*x + e)^2 + 8*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^
5 + 10*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^3 - (4*a^5 - 7*a^4*b + 3*a
^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^4*b - a^3*b^2)*f*cosh(f*
x + e)^6 + 15*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^4 - 3*(4*a^5 - 7*a^
4*b + 3*a^3*b^2)*f*cosh(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f)*sinh(f*x
+ e)^2 + (a^4*b - a^3*b^2)*f + 8*((a^4*b - a^3*b^2)*f*cosh(f*x + e)^7 + 3*(
a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*
f*cosh(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e
))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.47Error: Bad Argument Typ
e
```

maple [B] time = 0.21, size = 251, normalized size = 1.81

$$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(\frac{b^{2}\left(\cosh^{2}\left(fx+e\right)\right)}{a^{2}(a-b)\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}}-\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+\right.\right.}}{2a^{2}\sinh\left(fx+e\right)^{2}}\right.}{\cosh\left(fx+e\right)\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(b^2/a^2*cosh(f*x+e)^2/(a-b)/((a+
b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)-1/2/a^2/sinh(f*x+e)^2*((a+b*sinh(f*x+
e)^2)*cosh(f*x+e)^2)^(1/2)+1/4/a^(3/2)*ln((2*a+(a+b)*sinh(f*x+e)^2+2*a^(1/2
)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2)+3/4/a^(5/2)*b*l
n((2*a+(a+b)*sinh(f*x+e)^2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1
/2))/sinh(f*x+e)^2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(e+f\,x\right)^{3}\,\left(b\,\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{3}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csch(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.111 $\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=341

$$-\frac{\left(8a^2-3ab-2b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)}+\frac{\left(8a^2-3ab-2b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\right.}{3b^3f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e\right.}{a}}}$$

```
[Out] -a*cosh(f*x+e)*sinh(f*x+e)^3/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(4*a-b
)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/b^2/f+1/3*(8*a^2-
3*a*b-2*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/(a-b)/b^3/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(
4*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2
)^(1/2)/(a-b)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(8*a^2-
3*a*b-2*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/b^3/f
```

Rubi [A] time = 0.34, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3188, 470, 582, 531, 418, 492, 411}

$$-\frac{\left(8a^2-3ab-2b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)}+\frac{\left(8a^2-3ab-2b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\right.}{3b^3f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x]^3)/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(
3*(a - b)*b^2*f) + ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]]
, 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*b^3*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((4*a - b)*EllipticF[ArcTa
n[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a
 - b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 -
3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*b^3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
```

```
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^6}{\sqrt{1+x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{}}\right.}{(a-b)bf}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\,\mathrm{si}}}{3(a-b)b^2f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\,\mathrm{si}}}{3(a-b)b^2f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\,\mathrm{si}}}{3(a-b)b^2f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\,\mathrm{si}}}{3(a-b)b^2f}$$

Mathematica [**C**] time = 1.24, size = 211, normalized size = 0.62

$$\frac{-b\sinh(2(e+fx))\left(-8a^2+b(b-a)\cosh(2(e+fx))+3ab-b^2\right)-2i\sqrt{2}\,a\left(8a^2-7ab-b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F}{6b^3f(a-b)\sqrt{4a+2b\cosh(2(e+fx))-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[2]*a*(8*a^2 - 3*a*b - 2*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)
])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*a*(8*a^2 - 7*a*b - b^2)*S
qrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] - b*(-8*
a^2 + 3*a*b - b^2 + b*(-a + b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*(a
- b)*b^3*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [**F**] time = 2.93, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\sinh\left(fx+e\right)^6}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^6/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.16, size = 500, normalized size = 1.47

$$\frac{\sqrt{-\frac{b}{a}}\, ab\left(\sinh^5\left(fx+e\right)\right) - \sqrt{-\frac{b}{a}}\, b^2\left(\sinh^5\left(fx+e\right)\right) + 4\sqrt{-\frac{b}{a}}\, a^2\left(\sinh^3\left(fx+e\right)\right) - \sqrt{-\frac{b}{a}}\, b^2\left(\sinh^3\left(fx+e\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+4*(-
1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+4*a^2*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))-2*a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*((a+b*sinh(f*x+e)^
2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b
)^(1/2))*b^2-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+3*((a+b*sinh(f*x+e)^2)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a*b+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)-(-1/a
*b)^(1/2)*a*b*sinh(f*x+e))/b^2/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f
*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(e+f\,x\right)^6}{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.112 $\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=256

$$\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b^2f(a-b)}-\frac{(2a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+fx))|1\right.}{b^2f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -a*cosh(f*x+e)*sinh(f*x+e)/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-(2*a-b)*(1/(
1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+si
nh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-
b)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))^
(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2)
,(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/b/f/(sech(f*x+e
)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(2*a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x
+e)/(a-b)/b^2/f
```

Rubi [A] time = 0.24, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3188, 470, 531, 418, 492, 411}

$$\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b^2f(a-b)}-\frac{(2a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+fx))|1\right.}{b^2f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/((a - b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
 + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*
Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(
(a - b)*b^2*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
```

```
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int\right.}{(a-b)b}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(a\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\right.}{(a-b)b}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathrm{sech}(e+f}{(a-b)bf\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\,\mathrm{s}\right.}{a}}}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(2a-b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathrm{sec}}{(a-b)b^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)(}{}}}$$

Mathematica [C] time = 1.05, size = 156, normalized size = 0.61

$$\frac{a\left(4i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)-2i(2a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)-\sqrt{2}\,b\sinh(2\right.}{2b^2f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*((-2*I)*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] + (4*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellip
ticF[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]))/(2*(a - b)*b^2*f*Sqr
t[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 1.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\sinh\left(fx+e\right)^{4}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.13, size = 313, normalized size = 1.22

$$-\frac{\sqrt{-\frac{b}{a}}\,a\sinh\left(fx+e\right)\left(\cosh^{2}\left(fx+e\right)\right)+a\sqrt{\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/b*((-1/a*b)^(1/2)*a*sinh(f*x+e)*cosh(f*x+e)^2+a*(b/a*cosh(f*x+e)^2+(a-b)
/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+
(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sin
h(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(e+f\,x\right)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.113 $\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}+\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] cosh(f*x+e)*sinh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)+I*(cos(I*e+I*f*x)
^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*sinh(f*
x+e)^2)^(1/2)/(a-b)/b/f/(1+b*sinh(f*x+e)^2/a)^(1/2)-I*(cos(I*e+I*f*x)^2)^(1
/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1+b*sinh(f*x+e)^2
/a)^(1/2)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}+\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (I*
EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[
1 + (b*Sinh[e + f*x]^2)/a]) - (I*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Si
nh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
```

```
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{a+a\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{b} - \frac{\int \sqrt{a+b\sinh^2(e+}}{(a-b)b}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,d}{(a-b)b\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} - \frac{iF}{}$$

Mathematica [A] time = 0.49, size = 151, normalized size = 0.87

$$\frac{-i\sqrt{2}(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)+i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)+b\sinh(2(e+fx}{bf(a-b)\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x),
 b/a] - I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF
[I*(e + f*x), b/a] + b*Sinh[2*(e + f*x)])/((a - b)*b*f*Sqrt[4*a - 2*b + 2*b
*Cosh[2*(e + f*x)]])
```

fricas [F] time = 2.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\sinh\left(fx+e\right)^2}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.13, size = 127, normalized size = 0.73

$$-\frac{-\sqrt{-\frac{b}{a}}\,\sinh\left(fx+e\right)\left(\cosh^{2}\left(fx+e\right)\right)+\sqrt{\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\mathrm{EllipticE}\left(\sinh\left(fx+e\right)\sqrt{-\right.}{\left(a-b\right)\sqrt{-\frac{b}{a}}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(-(-1/a*b)^(1/2)*sinh(f*x+e)*cosh(f*x+e)^2+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))
/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(e+f\,x\right)^{2}}{\left(b\,\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.114 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] -b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-I*(cos(I*e+I
*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*si
nh(f*x+e)^2)^(1/2)/a/(a-b)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3184, 21, 3178, 3177}

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{-a-b\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,d}{a(a-b)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.18, size = 100, normalized size = 0.87

$$\frac{-\sqrt{2}b\sinh(2(e+fx)) - 2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,E\left(i(e+fx)\middle|\frac{b}{a}\right)}{2af(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x
+ e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.13, size = 253, normalized size = 2.20

$$-\frac{\sqrt{-\frac{b}{a}}\,b\sinh\left(fx+e\right)\left(\cosh^2\left(fx+e\right)\right) - a\sqrt{\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -((-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^2-a*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/a/(a-
b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b \sinh\left(e+fx\right)^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-3/2), x)
```

3.115 $\int \frac{\text{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=290

$$\frac{(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f(a-b)}-\frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f(a-b)}-\frac{b\text{sech}(e+fx)\sqrt{a+b\text{s}}}{a^2f(a-}$$

```
[Out] -b*coth(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-(a-2*b)*coth(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)/f-(a-2*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+si
nh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1
/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)/f/(sech(f*x+e)^2*(a+b*
sinh(f*x+e)^2)/a)^(1/2)-b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/
2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)
*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a
)^(1/2)+(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^2/(a-b)/f
```

Rubi [A] time = 0.31, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3188, 472, 583, 531, 418, 492, 411}

$$\frac{(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f(a-b)}-\frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f(a-b)}-\frac{b\text{sech}(e+fx)\sqrt{a+b\text{s}}}{a^2f(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b*Coth[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - ((a - 2*b)
*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)*f) - ((a - 2*b)*El
lipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
 (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + ((a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2*(a - b)*f
)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
```

```
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{}{x^2}\right.}{a(a-b)f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f}$$

Mathematica [C] time = 1.29, size = 185, normalized size = 0.64

$$\frac{-\left(\coth(e+fx)\left(2a^2+b(a-2b)\cosh(2(e+fx))-3ab+2b^2\right)\right)+i\sqrt{2}\,a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right.}{a^2f(a-b)\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-((2*a^2 - 3*a*b + 2*b^2 + (a - 2*b)*b*Cosh[2*(e + f*x)])*Coth[e + f*x]) -
 I*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*
(e + f*x), b/a] + I*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/
a]*EllipticF[I*(e + f*x), b/a])/(a^2*(a - b)*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*
(e + f*x)]])
```

fricas [F] time = 3.19, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\operatorname{csch}\left(fx+e\right)^2}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.17, size = 284, normalized size = 0.98

$$-\frac{\sinh\left(fx+e\right)\sqrt{\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,b\left(2a\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-2b\,\mathrm{Ellipti}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(2*
a*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*b*EllipticF(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*a+2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))+((-1/a*b)^(1
/2)*a*b-2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a^2-2*(-1/a*b)^
(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2)/a^2/(a-b)/(-1/a*b)^(1/2)/sin
h(f*x+e)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(e+f\,x\right)^2\,{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^{2}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csch(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.116 $\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{5/2}f}-\frac{a(3a-5b)\cosh(e+fx)}{3b^2f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{a\sinh^2(e+fx)\cosh(e+fx)}{3bf(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

```
[Out] arctanh(cosh(f*x+e)*b^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/b^(5/2)/f-1/3*a*co
sh(f*x+e)*sinh(f*x+e)^2/(a-b)/b/f/(a-b+b*cosh(f*x+e)^2)^(3/2)-1/3*a*(3*a-5*
b)*cosh(f*x+e)/(a-b)^2/b^2/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3186, 413, 385, 217, 206}

$$-\frac{a(3a-5b)\cosh(e+fx)}{3b^2f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{5/2}f}-\frac{a\sinh^2(e+fx)\cosh(e+fx)}{3bf(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(5/2)*f
) - (a*(3*a - 5*b)*Cosh[e + f*x])/(3*(a - b)^2*b^2*f*Sqrt[a - b + b*Cosh[e
+ f*x]^2]) - (a*Cosh[e + f*x]*Sinh[e + f*x]^2)/(3*(a - b)*b*f*(a - b + b*Co
sh[e + f*x]^2)^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+bx^2\right)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)bf\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-a+3b+3(a-b)x^2}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh\right.}{3(a-b)bf}$$

$$= -\frac{a(3a-5b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{a\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)bf\left(a-b+b\cosh^2(e+f\right.}$$

$$= -\frac{a(3a-5b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{a\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)bf\left(a-b+b\cosh^2(e+f\right.}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a(3a-5b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{}{3(a-}$$

Mathematica [A] time = 0.89, size = 130, normalized size = 0.91

$$\frac{\frac{\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{b^{5/2}} - \frac{2\sqrt{2}a\cosh(e+fx)\left(3a^2+b(2a-3b)\cosh(2(e+fx))-7ab+3b^2\right)}{3b^2(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-2*Sqrt[2]*a*Cosh[e + f*x]*(3*a^2 - 7*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*
(e + f*x)]))/(3*(a - b)^2*b^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2)) + Log[
Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/b^(5/2
))/f
```

fricas [B] time = 5.99, size = 7938, normalized size = 55.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3
+ b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x +
 e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*
b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2
```

```
)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a
^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4
 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b
^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*
b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sin
h(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cos
h(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (
8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x +
 e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2
*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
 b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24
*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 +
8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a
*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3
*b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)
)*sinh(f*x + e))*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)
*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6
+ 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*
cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b
 - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4
*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^
2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2
 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x +
 e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*
a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2
+ sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*
sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*s
inh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x +
 e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*co
sh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)
^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sq
rt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*
x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh
(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^
6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 +
20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6
*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 3*((a^2*b^2 - 2*a*b^3
+ b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x
 + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4
 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2
*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14
*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x
+ e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5
*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 - 2*a*
b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^5 + 10*(2*a^3*b -
5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2
 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*
x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^4 + 2*a^3
*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^
3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*
cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^5 +
 (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^3 + (2*a^
3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log(
-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e
```

)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*
cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^
2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - 8*sqrt(2)*((2*a^2*b
^2 - 3*a*b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)*sinh(
f*x + e)^5 + (2*a^2*b^2 - 3*a*b^3)*sinh(f*x + e)^6 + 3*(2*a^3*b - 4*a^2*b^2
+ a*b^3)*cosh(f*x + e)^4 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3 + 5*(2*a^2*b^2 -
3*a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 2*a^2*b^2 - 3*a*b^3 + 4*(5*(2*
a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^3 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f
*x + e))*sinh(f*x + e)^3 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^2
+ 3*(5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^4 + 2*a^3*b - 4*a^2*b^2 + a*b^3
+ 6*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((2*
a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^5 + 2*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f
*x + e)^3 + (2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqr
t((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*co
sh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2*b^5 - 2*a*b^6 + b^7)*f
*cosh(f*x + e)^8 + 8*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)*sinh(f*x + e
)^7 + (a^2*b^5 - 2*a*b^6 + b^7)*f*sinh(f*x + e)^8 + 4*(2*a^3*b^4 - 5*a^2*b^
5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cos
h(f*x + e)^2 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f)*sinh(f*x + e)^6 +
2*(8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e)
^4 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^3 + 3*(2*a^3*b^4 - 5*a^
2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b^5 -
2*a*b^6 + b^7)*f*cosh(f*x + e)^4 + 30*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^
7)*f*cosh(f*x + e)^2 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*
b^7)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(
f*x + e)^2 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^5 + 10*(2*a^3*b
^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^3 + (8*a^4*b^3 - 24*a^3*b^4
+ 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(
a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^6 + 15*(2*a^3*b^4 - 5*a^2*b^5 + 4*
a*b^6 - b^7)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 1
4*a*b^6 + 3*b^7)*f*cosh(f*x + e)^2 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7
)*f)*sinh(f*x + e)^2 + (a^2*b^5 - 2*a*b^6 + b^7)*f + 8*((a^2*b^5 - 2*a*b^6
+ b^7)*f*cosh(f*x + e)^7 + 3*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh
(f*x + e)^5 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*co
sh(f*x + e)^3 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e))*si
nh(f*x + e)), -1/6*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b
^2 - 2*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^
4)*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^
6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*co
sh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x +
e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)
^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 +
2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^
2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*
x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b
^3 + b^4)*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f
*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e
))*sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^
2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 +
3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(
f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*
a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 -
14*a*b^3 + 3*b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*
cosh(f*x + e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 +
2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x

```
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f
*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)
^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b
*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 3*((a^2*b
^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x
+ e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x + e)^8 + 4*(2*a^3
*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b - 5*a^2*b^2 +
4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^
6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2
 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27
*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b
^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30
*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a
^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^5 + 1
0*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b
 + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3
*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 +
 b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x +
 e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*
b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*
a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh
(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x +
 e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e))
*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh
(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e)
 + b)) + 4*sqrt(2)*((2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b^2 -
3*a*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b^2 - 3*a*b^3)*sinh(f*x + e
)^6 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 3*(2*a^3*b - 4*a^2*
b^2 + a*b^3 + 5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 2*
a^2*b^2 - 3*a*b^3 + 4*(5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^3 + 3*(2*a^3*b
 - 4*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a^3*b - 4*a^2*b
^2 + a*b^3)*cosh(f*x + e)^2 + 3*(5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^4 +
2*a^3*b - 4*a^2*b^2 + a*b^3 + 6*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)
^2)*sinh(f*x + e)^2 + 6*((2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^5 + 2*(2*a^3*b
 - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/
((a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^8 + 8*(a^2*b^5 - 2*a*b^6 + b^7)*
f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^5 - 2*a*b^6 + b^7)*f*sinh(f*x + e)
^8 + 4*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^6 + 4*(7*(a^
2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^2 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6
 - b^7)*f)*sinh(f*x + e)^6 + 2*(8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*
b^6 + 3*b^7)*f*cosh(f*x + e)^4 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x
+ e)^3 + 3*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e))*sinh(f*
x + e)^5 + 2*(35*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^4 + 30*(2*a^3*b^
4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^2 + (8*a^4*b^3 - 24*a^3*b^4
+ 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b^4 - 5*a^2*
b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^2 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*c
osh(f*x + e)^5 + 10*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)
^3 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x +
e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^6 + 15
*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 -
 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e)^2 + (2*a^3*b^4
 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f)*sinh(f*x + e)^2 + (a^2*b^5 - 2*a*b^6 + b^7
)*f + 8*((a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^7 + 3*(2*a^3*b^4 - 5*a^2
```

```
*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^5 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*
b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e)^3 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^
6 - b^7)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.45Error: Bad Argument Typ
e
```

maple [A] time = 0.33, size = 230, normalized size = 1.61

$$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(\frac{\ln\left(\frac{\frac{a}{2}+\frac{b}{2}+b\left(\sinh^2\left(fx+e\right)\right)}{\sqrt{b}}+\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\right)}{2b^{\frac{5}{2}}}-\frac{2a}{b^2\left(a-b\right)\sqrt{\left(a+b\left(s\right.\right.}}\right.}{\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^2\left(fx\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b^(5/2)*ln((1/2*a+1/2*b+b*si
nh(f*x+e)^2)/b^(1/2)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))-2*a/b^2*cos
h(f*x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)+1/3*a^2/b^2*(2*b
*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)^2/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/
2)/(a+b*sinh(f*x+e)^2)/(a^2-2*a*b+b^2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/
2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(e+f\,x\right)^5}{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sinh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.117 $\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sinh^2(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}-\frac{2\cosh(e+fx)}{3f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)^2/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(3/2)-2/3*cosh(
f*x+e)/(a-b)^2/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3186, 378, 191}

$$\frac{\sinh^2(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}-\frac{2\cosh(e+fx)}{3f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*Cosh[e + f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2]) + (Cosh[
e + f*x]*Sinh[e + f*x]^2)/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+bx^2\right)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+f\right.}{3(a-b)f}$$

$$= -\frac{2\cosh(e+fx)}{3(a-b)^2 f\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.33, size = 67, normalized size = 0.77

$$\frac{\sqrt{2}\cosh(e+fx)((a-3b)\cosh(2(e+fx))-5a+3b)}{3f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[2]*Cosh[e + f*x]*(-5*a + 3*b + (a - 3*b)*Cosh[2*(e + f*x)]))/(3*(a -
b)^2*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

fricas [B] time = 3.78, size = 1214, normalized size = 13.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*((a - 3*b)*cosh(f*x + e)^6 + 6*(a - 3*b)*cosh(f*x + e)*sinh(f*x
 + e)^5 + (a - 3*b)*sinh(f*x + e)^6 - 3*(3*a - b)*cosh(f*x + e)^4 + 3*(5*(a
 - 3*b)*cosh(f*x + e)^2 - 3*a + b)*sinh(f*x + e)^4 + 4*(5*(a - 3*b)*cosh(f*
x + e)^3 - 3*(3*a - b)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(3*a - b)*cosh(f*
x + e)^2 + 3*(5*(a - 3*b)*cosh(f*x + e)^4 - 6*(3*a - b)*cosh(f*x + e)^2 - 3
*a + b)*sinh(f*x + e)^2 + 6*((a - 3*b)*cosh(f*x + e)^5 - 2*(3*a - b)*cosh(f
*x + e)^3 - (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt((b*cosh(
f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2))/((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x +
 e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*
b^2 - 2*a*b^3 + b^4)*f*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
 b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^2
+ (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 24*
a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^2*b^2 -
2*a*b^3 + b^4)*f*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*
f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f
*x + e)^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + (8
*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f)*sinh(f*x + e)^4 + 4*(2*
a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + 8*(7*(a^2*b^2 - 2*a*
b^3 + b^4)*f*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*c
osh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(
f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^
6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^4 + 3*(8*a^4 -
 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^2 + (2*a^3*b - 5
*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*f
```

```
+ 8*((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 +
 4*a*b^3 - b^4)*f*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b
^3 + 3*b^4)*f*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cos
h(f*x + e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Evaluation time: 0.45Error: Bad Argument Typ
e
```

maple [A] time = 0.12, size = 64, normalized size = 0.74

$$\frac{\left(a\left(\sinh^{2}\left(fx+e\right)\right)-3b\left(\sinh^{2}\left(fx+e\right)\right)-2a\right)\cosh\left(fx+e\right)}{3\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(a^{2}-2ab+b^{2}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(a*sinh(f*x+e)^2-3*b*sinh(f*x+e)^2-2*a)*cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(3/2)/(a^2-2*a*b+b^2)/f
```

maxima [B] time = 0.50, size = 927, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b^4*e^(-10*f*x - 10*e) - 4*a^3*b + 6*a^2*b^2 - b^4 - (16*a^4 - 32*a^
3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-2*f*x - 2*e) + 10*(2*a^3*b - 3*a^2*
b^2 + 3*a*b^3 - b^4)*e^(-4*f*x - 4*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-
6*f*x - 6*e) + 5*(2*a*b^3 - b^4)*e^(-8*f*x - 8*e))/((a^4 - 2*a^3*b + a^2*b^
2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) - 1/4*(
2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-2*f
*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-4*f*x
 - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) + 5*(4*
a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x - 10
*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x
 - 4*e) + b)^(5/2)*f) - 1/4*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*
e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x - 4*e
) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x - 6*e)
+ 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^2 - 2
*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e
^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) - 1/12*(b^4 + 5*(2*a*b^3
 - b^4)*e^(-2*f*x - 2*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-4*f*x - 4*e)
+ 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^(-6*f*x - 6*e) - (16*a^4 - 32*
a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-8*f*x - 8*e) - (4*a^3*b - 6*a^2*b
^2 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

mupad [B] time = 1.62, size = 148, normalized size = 1.70

$$\frac{2\,e^{e+f\,x}\,\left(e^{2\,e+2\,f\,x}+1\right)\,\sqrt{a+b\,{\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)}^2}\,\left(a-3\,b-10\,a\,e^{2\,e+2\,f\,x}+a\,e^{4\,e+4\,f\,x}+6\,b\,e^{2\,e+2\,f\,x}-3\,b\,e^{4\,e+4\,f}\right.}{3\,f\,{\left(a-b\right)}^2\,{\left(b+4\,a\,e^{2\,e+2\,f\,x}-2\,b\,e^{2\,e+2\,f\,x}+b\,e^{4\,e+4\,f\,x}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] (2*exp(e + f*x)*(exp(2*e + 2*f*x) + 1)*(a + b*(exp(e + f*x)/2 - exp(- e - f
*x)/2)^2)^(1/2)*(a - 3*b - 10*a*exp(2*e + 2*f*x) + a*exp(4*e + 4*f*x) + 6*b
*exp(2*e + 2*f*x) - 3*b*exp(4*e + 4*f*x)))/(3*f*(a - b)^2*(b + 4*a*exp(2*e
+ 2*f*x) - 2*b*exp(2*e + 2*f*x) + b*exp(4*e + 4*f*x))^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.118 $\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2\cosh(e+fx)}{3f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{\cosh(e+fx)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

```
[Out] 1/3*cosh(f*x+e)/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(3/2)+2/3*cosh(f*x+e)/(a-b)^2
/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 192, 191}

$$\frac{2\cosh(e+fx)}{3f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{\cosh(e+fx)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Cosh[e + f*x]/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2)) + (2*Cosh[e +
 f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{5/2}}\,dx,x,\cosh(e+fx)\right)}{f}\\ &= \frac{\cosh(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{3/2}}\,dx,x,\cosh(e\right.}{3(a-b)f}\\ &= \frac{\cosh(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}+\frac{2\cosh(e+fx)}{3(a-b)^2f\sqrt{a-b+b\cosh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.18, size = 63, normalized size = 0.80

$$\frac{2\sqrt{2}\cosh(e+fx)(3a+b\cosh(2(e+fx))-2b)}{3f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*Sqrt[2]*Cosh[e + f*x]*(3*a - 2*b + b*Cosh[2*(e + f*x)]))/(3*(a - b)^2*f*
(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

fricas [B] time = 3.21, size = 1186, normalized size = 15.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(2)*(b*cosh(f*x + e)^6 + 6*b*cosh(f*x + e)*sinh(f*x + e)^5 + b*sinh
(f*x + e)^6 + 3*(2*a - b)*cosh(f*x + e)^4 + 3*(5*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^4 + 4*(5*b*cosh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e))*si
nh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e)^2 + 3*(5*b*cosh(f*x + e)^4 + 6*(2
*a - b)*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 6*(b*cosh(f*x + e)^5 +
 2*(2*a - b)*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a^2*b^2 - 2*a*b^3 + b^4)
*f*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)*sinh(f*x +
 e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*f*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cos
h(f*x + e)^2 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^6 + 2
*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^4 + 8*(
7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*
a*b^3 - b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b^2 - 2*a*b^3 +
b^4)*f*cosh(f*x + e)^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*
x + e)^2 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f)*sinh(f*x +
 e)^4 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + 8*(7*(a
^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b
^3 - b^4)*f*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3
*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*c
osh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^4
 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^2 +
 (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^2 + (a^2*b^2 - 2*a*
b^3 + b^4)*f + 8*((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^7 + 3*(2*a^3*b
- 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2
*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3
 - b^4)*f*cosh(f*x + e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.10, size = 57, normalized size = 0.72

$$\frac{\left(2b\left(\sinh^2\left(fx+e\right)\right)+3a-b\right)\cosh\left(fx+e\right)}{3\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(a^2-2ab+b^2\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(2*b*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2)/(a^2-2*
a*b+b^2)/f
```

maxima [B] time = 0.45, size = 485, normalized size = 6.14

$$\frac{2\,a^2b^2-2\,ab^3+b^4+5\left(4\,a^3b-6\,a^2b^2+4\,ab^3-b^4\right)e^{\left(-2\,fx-2\,e\right)}+2\left(24\,a^4-48\,a^3b+49\,a^2b^2-25\,ab^3+5\,b^4\right)e^{\left(-\right.}}{3\left(a^4-2\,a^3b+a^2b^2\right)\left(2\,(2\,a-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^
(-2*f*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-
4*f*x - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) +
5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x
 - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-
4*f*x - 4*e) + b)^(5/2)*f) + 1/3*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 +
b^4)*e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x
- 4*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x -
6*e) + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^
2 - 2*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a -
 b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

mupad [B] time = 1.29, size = 133, normalized size = 1.68

$$\frac{4\,e^{e+f\,x}\,\left(e^{2\,e+2\,f\,x}+1\right)\,\sqrt{a+b\,{\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)}^2}\,\left(b+6\,a\,e^{2\,e+2\,f\,x}-4\,b\,e^{2\,e+2\,f\,x}+b\,e^{4\,e+4\,f\,x}\right)}{3\,f\,{\left(a-b\right)}^2\,{\left(b+4\,a\,e^{2\,e+2\,f\,x}-2\,b\,e^{2\,e+2\,f\,x}+b\,e^{4\,e+4\,f\,x}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] (4*exp(e + f*x)*(exp(2*e + 2*f*x) + 1)*(a + b*(exp(e + f*x)/2 - exp(- e - f
*x)/2)^2)^(1/2)*(b + 6*a*exp(2*e + 2*f*x) - 4*b*exp(2*e + 2*f*x) + b*exp(4*
e + 4*f*x)))/(3*f*(a - b)^2*(b + 4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x
) + b*exp(4*e + 4*f*x))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.119 $\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{5/2}f}-\frac{b(5a-3b)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{b\cosh(e+fx)}{3af(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

```
[Out] -arctanh(cosh(f*x+e)*a^(1/2)/(a-b+b*cosh(f*x+e)^2)^(1/2))/a^(5/2)/f-1/3*b*c
osh(f*x+e)/a/(a-b)/f/(a-b+b*cosh(f*x+e)^2)^(3/2)-1/3*(5*a-3*b)*b*cosh(f*x+e
)/a^2/(a-b)^2/f/(a-b+b*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 12, 377, 206}

$$-\frac{b(5a-3b)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{5/2}f}-\frac{b\cosh(e+fx)}{3af(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(a^(5/2)
*f)) - (b*Cosh[e + f*x])/(3*a*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
- ((5*a - 3*b)*b*Cosh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a - b + b*Cosh[e +
f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+bx^2\right)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{-3a+b+2bx^2}{(1-x^2)\left(a-b+bx^2\right)^{3/2}}\,dx, x, \mathrm{c}\right.}{3a(a-b)f}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\cosh(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\cosh^2(e+}}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\cosh(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\cosh^2(e+}}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\cosh(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\cosh^2(e+}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{}{3a^2}$$

Mathematica [A] time = 0.79, size = 130, normalized size = 0.96

$$\frac{\frac{\sqrt{2}\,b\cosh(e+fx)\left(-12a^2+b(3b-5a)\cosh(2(e+fx))+13ab-3b^2\right)}{3a^2(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{a^{5/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x
)]]]/a^(5/2)) + (Sqrt[2]*b*Cosh[e + f*x]*(-12*a^2 + 13*a*b - 3*b^2 + b*(-5*
```

```
a + 3*b)*Cosh[2*(e + f*x)]))/(3*a^2*(a - b)^2*(2*a - b + b*Cosh[2*(e + f*x)
])^(3/2)))/f
```

fricas [B] time = 5.09, size = 5342, normalized size = 39.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 +
 b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x +
e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b
 - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)
*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^
3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4
- 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^
2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b
^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh
(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh
(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8
*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x +
e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*
b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*
a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8
*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*
b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*
b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))
*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x
+ e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^
2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(co
sh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x +
e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4
*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1
)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))
*sinh(f*x + e) + 1)) - 2*sqrt(2)*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^6 + 6
*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (5*a^2*b^2 - 3*a*b^3
)*sinh(f*x + e)^6 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 3*(8*
a^3*b - 7*a^2*b^2 + a*b^3 + 5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f
*x + e)^4 + 5*a^2*b^2 - 3*a*b^3 + 4*(5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^
3 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(8*a
^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^2 + 3*(5*(5*a^2*b^2 - 3*a*b^3)*cosh
(f*x + e)^4 + 8*a^3*b - 7*a^2*b^2 + a*b^3 + 6*(8*a^3*b - 7*a^2*b^2 + a*b^3)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^
5 + 2*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (8*a^3*b - 7*a^2*b^2
+ a*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
 + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^8 + 8*(a^5*b
^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 2*a^
4*b^3 + a^3*b^4)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a
^3*b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x
 + e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^6 +
2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^
4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^3 + 3*(2*a^6*b - 5
*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a
^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 5*a^5*b^2 +
 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 -
```

```
14*a^4*b^3 + 3*a^3*b^4)*f)*sinh(f*x + e)^4 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4
*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*
cosh(f*x + e)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x
 + e)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f
*x + e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x
+ e)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^4 +
 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)
^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^2 + (a^5*
b^2 - 2*a^4*b^3 + a^3*b^4)*f + 8*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*
x + e)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^5
+ (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^
3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x +
 e)), 1/3*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*
b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f
*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*
a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x +
e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*
(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8
*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a
^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 1
4*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)
*sinh(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)
*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3
 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f
*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*
(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b
^3 - b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4
- 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^
2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 +
 4*a*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3
 + 3*b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x
+ e))*sinh(f*x + e))*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2
+ b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x
 + e))*sinh(f*x + e) + b)) - sqrt(2)*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^6
 + 6*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (5*a^2*b^2 - 3*a
*b^3)*sinh(f*x + e)^6 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 3
*(8*a^3*b - 7*a^2*b^2 + a*b^3 + 5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*si
nh(f*x + e)^4 + 5*a^2*b^2 - 3*a*b^3 + 4*(5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x +
 e)^3 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*
(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^2 + 3*(5*(5*a^2*b^2 - 3*a*b^3)*
cosh(f*x + e)^4 + 8*a^3*b - 7*a^2*b^2 + a*b^3 + 6*(8*a^3*b - 7*a^2*b^2 + a*
b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x +
 e)^5 + 2*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (8*a^3*b - 7*a^2*
b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh
(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + s
inh(f*x + e)^2)))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^8 + 8*(a
^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 -
2*a^4*b^3 + a^3*b^4)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3
 - a^3*b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh
(f*x + e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^
6 + 2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x +
 e)^4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^3 + 3*(2*a^6*b
 - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(3
5*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 5*a^5*b
^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + (8*a^7 - 24*a^6*b + 27*a^5*b^
```

```
2 - 14*a^4*b^3 + 3*a^3*b^4)*f)*sinh(f*x + e)^4 + 4*(2*a^6*b - 5*a^5*b^2 + 4
*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4
)*f*cosh(f*x + e)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh
(f*x + e)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*co
sh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(
f*x + e)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)
^4 + 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x
+ e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^2 + (
a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f + 8*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cos
h(f*x + e)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e
)^5 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x +
 e)^3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f
*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.5Error: Bad Argument Type
```

maple [A] time = 0.25, size = 236, normalized size = 1.74

$$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(-\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a^2(a-b)\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}}-\frac{b\left(2b\left(\sinh^2\left(fx+e\right)\right)+\cdots}{3a\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+\cdots\right.\right.}}\cdots\right.}{\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^2\left(fx+e\cdots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/a^2*b*cosh(f*x+e)^2/(a-b)/((a
+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)-1/3/a*b*(2*b*sinh(f*x+e)^2+3*a-b)*co
sh(f*x+e)^2/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(a+b*sinh(f*x+e)^2)/(
a^2-2*a*b+b^2)-1/2/a^(5/2)*ln((2*a+(a+b)*sinh(f*x+e)^2+2*a^(1/2)*((a+b*sinh
(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2))/cosh(f*x+e)/(a+b*sinh(f*x+
e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh\left(e+fx\right)\left(b\sinh\left(e+fx\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)),x)
[Out] int(1/(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

3.120 $\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=344

$$\frac{\left(8a^2-13ab+3b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)^2}-\frac{\left(8a^2-13ab+3b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E}{3b^3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e\right.}{a}}}$$

```
[Out] -1/3*a*cosh(f*x+e)*sinh(f*x+e)^3/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*a*
(2*a-3*b)*cosh(f*x+e)*sinh(f*x+e)/(a-b)^2/b^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1
/3*(8*a^2-13*a*b+3*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)
*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(
a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/b^3/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a
)^(1/2)+2/3*(2*a-3*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+
b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^
(1/2)+1/3*(8*a^2-13*a*b+3*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)^
2/b^3/f
```

Rubi [A] time = 0.37, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3188, 470, 578, 531, 418, 492, 411}

$$\frac{\left(8a^2-13ab+3b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)^2}-\frac{\left(8a^2-13ab+3b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E}{3b^3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(a*Cosh[e + f*x]*Sinh[e + f*x]^3)/(3*(a - b)*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*a*(2*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^2*b^2*f*Sq
rt[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)
^2*b^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*(2*a - 3*b
)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*(a - b)^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
))/a]) + ((8*a^2 - 13*a*b + 3*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x
])/(3*(a - b)^2*b^3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^6}{\sqrt{1+x^2}\left(a+bx^2\right)^{5/2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\right.}{3(a-b}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

Mathematica [C] time = 2.09, size = 207, normalized size = 0.60

$$\frac{a\left(\sqrt{2}\,b\sinh(2(e+fx))\left(-8a^2+b(7b-5a)\cosh(2(e+fx))+17ab-7b^2\right)+2ia\left(8a^2-17ab+9b^2\right)\left(\frac{2a+b\cosh(2(e+\right.}{a}\right.}{6b^3f(a-b)^2(2a+b\cosh(2(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (a*((-2*I)*a*(8*a^2 - 13*a*b + 3*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(
3/2)*EllipticE[I*(e + f*x), b/a] + (2*I)*a*(8*a^2 - 17*a*b + 9*b^2)*((2*a -
 b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*
(-8*a^2 + 17*a*b - 7*b^2 + b*(-5*a + 7*b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*
x)]))/(6*(a - b)^2*b^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\sinh\left(fx+e\right)^6}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^6/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.49Error: Bad Argument Typ
e
```

maple [B] time = 0.25, size = 868, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((5*(-1/a*b)^(1/2)*a^2*b-7*(-1/a*b)^(1/2)*a*b^2)*sinh(f*x+e)*cosh(f*x+
e)^4+(4*(-1/a*b)^(1/2)*a^3-11*(-1/a*b)^(1/2)*a^2*b+7*(-1/a*b)^(1/2)*a*b^2)*
cosh(f*x+e)^2*sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)
^(1/2)*b*(4*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-7*Ellipti
cF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+3*EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*b^2-8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a^2+13*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-3*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2+4*(b/a*cosh(f
*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)
^(1/2),(a/b)^(1/2))*a^3-11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+10*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a*b^2-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-8*(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*a^3+21*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-16*(b/a
*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/(-1/
a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/b^2/cosh(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^{6}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(e+f\,x\right)^{6}}{\left(b\,\sinh\left(e+f\,x\right)^{2}+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sinh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.121 $\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=244

$$\frac{2\sqrt{a}\,(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3b^{3/2}f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{a\sinh(e+fx)\cosh(e+fx)}{3bf(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(a-3b)\text{sech}(e$$

```
[Out] -1/3*a*cosh(f*x+e)*sinh(f*x+e)/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*(a-2
*b)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)
*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^
(1/2))*a^(1/2)/(a-b)^2/b^(3/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2
)/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(a-3*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh
(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2
))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^2/b/f/(sech(f*x+e)^2*(a+b*
sinh(f*x+e)^2)/a)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3188, 470, 525, 418, 411}

$$\frac{2\sqrt{a}\,(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3b^{3/2}f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{a\sinh(e+fx)\cosh(e+fx)}{3bf(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(a-3b)\text{sech}(e$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(a*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)*b*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) + (2*Sqrt[a]*(a - 2*b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e +
 f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^2*b^(3/2)*f*Sqrt[(a*Cosh[e + f*x]^2)/
(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - ((a - 3*b)*Elliptic
F[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]
)/(3*a*(a - b)^2*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\Big(\ldots}{3(a-b\ldots}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left((a-3b)\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\ldots}{3(a-\ldots}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\sqrt{a}\,(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\,\mathrm{s}\ldots}{\ldots}\right.\right.}{3(a-b)^2b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\,\mathrm{s}\ldots}}$$

Mathematica [C] time = 1.67, size = 198, normalized size = 0.81

$$\frac{-\sqrt{2}\,b\sinh(2(e+fx))\left(-a^2-b(a-2b)\cosh(2(e+fx))+4ab-2b^2\right)-ia\left(2a^2-5ab+3b^2\right)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3\ldots}}{3b^2f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a - 2*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*
(e + f*x), b/a] - I*a*(2*a^2 - 5*a*b + 3*b^2)*((2*a - b + b*Cosh[2*(e + f*x
)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*(-a^2 + 4*a*b - 2*b^2
- (a - 2*b)*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*(a - b)^2*b^2*f*(2*a
 - b + b*Cosh[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\sinh\left(fx+e\right)^{4}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.44Error: Bad Argument Typ
e
```

maple [B] time = 0.17, size = 659, normalized size = 2.70

$$2\sqrt{-\frac{b}{a}}\,ab\left(\sinh^{5}\left(fx+e\right)\right)-4\sqrt{-\frac{b}{a}}\,b^{2}\left(\sinh^{5}\left(fx+e\right)\right)+\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
*sinh(f*x+e)^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^2+4*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-(-1/a*b)
^(1/2)*a*b*sinh(f*x+e)^3-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+a^2*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))-a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*((a+b*sinh(f*x+e)^2)/a)^(1
/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))
*a^2+4*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)-3*(-1/
a*b)^(1/2)*a*b*sinh(f*x+e))/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^
2/b/cosh(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(e+f\,x\right)^4}{{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(sinh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

3.122 $\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=241

$$\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{3af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3bf(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{i(a}{}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(a+b)*cos
h(f*x+e)*sinh(f*x+e)/a/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*I*(a+b)*(cos
(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(
a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^2/b/f/(1+b*sinh(f*x+e)^2/a)^(1/2)-1/3*I*(c
os(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))
*(1+b*sinh(f*x+e)^2/a)^(1/2)/(a-b)/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{3af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3bf(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{i(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) +
 ((a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*Sqrt[a + b*Sinh[e +
 f*x]^2]) + ((I/3)*(a + b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e +
f*x]^2])/(a*(a - b)^2*b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) - ((I/3)*Ellipti
cF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/((a - b)*b*f*Sqrt[a +
 b*Sinh[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{a-a\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{2a^2+\cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \frac{\cdots}{\sqrt{a+\cdots}}}{3\cdots}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left((a+\cdots\right.}{\cdots}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{i(a+\cdots}{\cdots}$$

Mathematica [A] time = 1.44, size = 187, normalized size = 0.78

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(4a^2+b(a+b)\cosh(2(e+fx))-ab-b^2\right)-2ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}F\left(i(e+fx)\right|\cdots}{6abf(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e
 + f*x), b/a] - (2*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)
*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(4*a^2 - a*b - b^2 + b*(a + b)*Cos
h[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*a*(a - b)^2*b*f*(2*a - b + b*Cosh[2*(
e + f*x)])^(3/2))
```

fricas [F] time = 1.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\sinh\left(fx+e\right)^2}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.47Error: Bad Argument Typ
e
```

maple [B] time = 0.16, size = 598, normalized size = 2.48

$$-\frac{\left(-\sqrt{-\frac{b}{a}}\,ab-\sqrt{-\frac{b}{a}}\,b^2\right)\sinh\left(fx+e\right)\left(\cosh^4\left(fx+e\right)\right)+\left(-2\sqrt{-\frac{b}{a}}\,a^2+\sqrt{-\frac{b}{a}}\,ab+\sqrt{-\frac{b}{a}}\,b^2\right)\left(\cosh^2\left(fx+e\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((-(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+(-
2*(-1/a*b)^(1/2)*a^2+(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*s
inh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*Ell
ipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))+EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a
+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2+a^2*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))-2*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x
+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b^2+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^
(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-(b/a*cosh(f*x+e
)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2)/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/a/cos
h(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(e+f\,x\right)^2}{{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(sinh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

3.123 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\mathrm{co}}{3af(a-b)\left(a+b\sin\right.}$$

```
[Out] -1/3*b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(2*a
-b)*b*cosh(f*x+e)*sinh(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-2/3*I
*(2*a-b)*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(
b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)^2/f/(1+b*sinh(f*x+e)^2/a)^(
1/2)+1/3*I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x)
,(b/a)^(1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/
2)
```

Rubi [A] time = 0.28, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\mathrm{co}}{3af(a-b)\left(a+b\sin\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] -(b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*S
qrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a
*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a+2b+b\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \ldots$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \ldots$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \ldots$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \ldots$$

Mathematica [A] time = 1.38, size = 190, normalized size = 0.76

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(-5a^2+b(b-2a)\cosh(2(e+fx))+5ab-b^2\right)+ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}F\left(i(e+f\ldots\right.}{3a^2f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*
EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b
)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cos
h[2*(e + f*x)])^(3/2))
```

fricas [F] time = 2.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*
x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.35, size = 406, normalized size = 1.62

$$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(-\frac{\sinh\left(fx+e\right)\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}}{3ab(a-b)\left(\sinh^{2}\left(fx+e\right)+\frac{a}{b}\right)^{2}}-\frac{2b\left(\cosh^{2}\left(fx+e\right)\right)\sinh}{3a^{2}(a-b)^{2}\sqrt{\left(a+b\left(\sinh^{2}\left(fx+\right.\right.\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a/b/(a-b)*sinh(f*x+e)*((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+a/b)^2-2/3*b*cosh(f*x+e
)^2/a^2/(a-b)^2*sinh(f*x+e)*(2*a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/
2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2/3*b*(2*a-b)/a^2/(a-b)^2/(-1/a
*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*
x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))))/cosh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-5/2), x)
```

3.124 $\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=385

$$-\frac{2b(3a-2b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{3a^3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2b(3a-2b)\coth(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+}}$$

```
[Out] -1/3*b*coth(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(3*a-2*b)*b*coth
(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(3*a^2-13*a*b+8*b^2)*co
th(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/(a-b)^2/f-1/3*(3*a^2-13*a*b+8*b^2)*
(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/a^3/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/3*(3*a-2*b)*b*
(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/a^3/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a^2-13*a*
b+8*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^3/(a-b)^2/f
```

Rubi [A] time = 0.46, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3188, 472, 579, 583, 531, 418, 492, 411}

$$\frac{\left(3a^2-13ab+8b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)^2}-\frac{\left(3a^2-13ab+8b^2\right)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*Coth[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*(3*a
 - 2*b)*b*Coth[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (
(3*a^2 - 13*a*b + 8*b^2)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*
(a - b)^2*f) - ((3*a^2 - 13*a*b + 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1
 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)^2*f*Sqrt[
(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(3*a - 2*b)*b*EllipticF[
ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/
(3*a^3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3
*a^2 - 13*a*b + 8*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^3*(a
 - b)^2*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx\right.}{f}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subs}}{3a(}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

Mathematica [C] time = 2.40, size = 234, normalized size = 0.61

$$\frac{i\left(4a^2\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\left(\left(3a^2-7ab+4b^2\right)F\left(i(e+fx)\middle|\frac{b}{a}\right)+\left(-3a^2+13ab-8b^2\right)E\left(i(e+fx)\middle|\frac{b}{a}\right)\right)+2i\sqrt{}\right.}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((I/12)*(4*a^2*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*((-3*a^2 + 13*a*b
- 8*b^2)*EllipticE[I*(e + f*x), b/a] + (3*a^2 - 7*a*b + 4*b^2)*EllipticF[I*
(e + f*x), b/a]) + (2*I)*Sqrt[2]*(3*(a - b)^2*(2*a - b + b*Cosh[2*(e + f*x)
])^2*Coth[e + f*x] - 2*a*(a - b)*b^2*Sinh[2*(e + f*x)] - (7*a - 5*b)*b^2*(2
*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])))/(a^3*(a - b)^2*f*(2*a -
b + b*Cosh[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\operatorname{csch}\left(fx+e\right)^2}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.57Error: Bad Argument Typ
e
```

maple [A] time = 0.21, size = 747, normalized size = 1.94

$$-\frac{\sqrt{\frac{\cosh(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{b\left(\cosh^2(fx+e)\right)}{a}+\frac{a-b}{a}}\,b^2\left(9\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)a^2-17\,\mathrm{EllipticF}\left(\sinh\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b^2*(9*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-17*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))*b^2-3*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+13*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8*EllipticE(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))*b^2)*sinh(f*x+e)*cosh(f*x+e)^2+(cosh(f*x+e)^2)^(1
/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(9*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))*a^3-26*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a^2*b+25*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-8*Elliptic
F(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-3*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a^3+16*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a^2*b-21*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+8*Ell
ipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)*sinh(f*x+e)+(3*(-1/a*b)
^(1/2)*a^2*b^2-13*(-1/a*b)^(1/2)*a*b^3+8*(-1/a*b)^(1/2)*b^4)*cosh(f*x+e)^6+
(6*(-1/a*b)^(1/2)*a^3*b-26*(-1/a*b)^(1/2)*a^2*b^2+38*(-1/a*b)^(1/2)*a*b^3-1
6*(-1/a*b)^(1/2)*b^4)*cosh(f*x+e)^4+(3*(-1/a*b)^(1/2)*a^4-12*(-1/a*b)^(1/2)
*a^3*b+26*(-1/a*b)^(1/2)*a^2*b^2-25*(-1/a*b)^(1/2)*a*b^3+8*(-1/a*b)^(1/2)*b
^4)*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(3/2)/sinh(f*x+e)/(-1/a*b)^(1/2)/(a-
b)^2/a^3/cosh(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(e+f\,x\right)^2\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.125 $\int \frac{1}{\sqrt{1+\sinh^2(x)}} \, dx$

Optimal. Leaf size=14

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}$$

```
[Out] arctan(sinh(x))*cosh(x)/(cosh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3176, 3207, 3770}

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Sinh[x]^2],x]
```

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[Cosh[x]^2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1+\sinh^2(x)}} \, dx &= \int \frac{1}{\sqrt{\cosh^2(x)}} \, dx \\
&= \frac{\cosh(x) \int \operatorname{sech}(x) \, dx}{\sqrt{\cosh^2(x)}} \\
&= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{\sqrt{\cosh^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.36

$$\frac{2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sinh[x]^2],x]
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[Cosh[x]^2]
```

fricas [A] time = 0.96, size = 8, normalized size = 0.57

$$2\arctan\left(\cosh(x)+\sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(cosh(x) + sinh(x))
```

giac [A] time = 0.14, size = 5, normalized size = 0.36

$$2\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="giac")
[Out] 2*arctan(e^x)
```

maple [A] time = 0.08, size = 15, normalized size = 1.07

$$\frac{\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\,\arctan\left(\sinh(x)\right)}{\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2)^(1/2),x)
[Out] (cosh(x)^2)^(1/2)*arctan(sinh(x))/cosh(x)
```

maxima [A] time = 0.49, size = 5, normalized size = 0.36

$$2\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="maxima")
[Out] 2*arctan(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{\sqrt{\sinh(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 + 1)^(1/2),x)
[Out] int(1/(sinh(x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sinh^2(x) + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sinh(x)**2 + 1), x)
```

3.126 $\int \frac{1}{\sqrt{1-\sinh^2(x)}}\,dx$

Optimal. Leaf size=11

$$-iF(ix|-1)$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticF(I*sinh(x),I)
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3182}

$$-iF(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Sinh[x]^2],x]
```

```
[Out] (-I)*EllipticF[I*x, -1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\sinh^2(x)}}\,dx = -iF(ix|-1)$$

Mathematica [A] time = 0.04, size = 11, normalized size = 1.00

$$-iF(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Sinh[x]^2],x]
```

```
[Out] (-I)*EllipticF[I*x, -1]
```

fricas [F] time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-\sinh(x)^2+1}}{\sinh(x)^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-sinh(x)^2 + 1)/(sinh(x)^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sinh(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-sinh(x)^2 + 1), x)
```

maple [A] time = 0.11, size = 41, normalized size = 3.73

$$\frac{\sqrt{-\left(-1+\sinh^{2}(x)\right)\left(\cosh^{2}(x)\right)}\,\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(x),i\right)}{\sqrt{1-\left(\sinh^{4}(x)\right)}\,\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2)^(1/2),x)
```

```
[Out] (-(-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)/(1-sinh(x)^4)^(1/2)*Ell
ipticF(sinh(x),I)/cosh(x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sinh(x)^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-sinh(x)^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{1}{\sqrt{1-\sinh(x)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - sinh(x)^2)^(1/2),x)
```

```
[Out] int(1/(1 - sinh(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{1-\sinh^{2}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - sinh(x)**2), x)
```

3.127 $\int \frac{1}{\sqrt{-1+\sinh^2(x)}}\,dx$

Optimal. Leaf size=33

$$-\frac{i\sqrt{1-\sinh^2(x)}\,F(ix|-1)}{\sqrt{\sinh^2(x)-1}}$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticF(I*sinh(x),I)*(1-sinh(x)^2)^(1/2)/(-1
+sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3183, 3182}

$$-\frac{i\sqrt{1-\sinh^2(x)}\,F(ix|-1)}{\sqrt{\sinh^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+\sinh^2(x)}}\,dx &= \frac{\sqrt{1-\sinh^2(x)}\int \frac{1}{\sqrt{1-\sinh^2(x)}}\,dx}{\sqrt{-1+\sinh^2(x)}}\\ &= -\frac{iF(ix|-1)\sqrt{1-\sinh^2(x)}}{\sqrt{-1+\sinh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 1.00

$$-\frac{i\sqrt{3-\cosh(2x)}\,F(ix|-1)}{\sqrt{\cosh(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[3 - Cosh[2*x]]*EllipticF[I*x, -1])/Sqrt[-3 + Cosh[2*x]]
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{\sinh(x)^2-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(sinh(x)^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sinh(x)^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(sinh(x)^2 - 1), x)
```

maple [A] time = 0.11, size = 61, normalized size = 1.85

$$-\frac{i\sqrt{\left(-1+\sinh^2(x)\right)\left(\cosh^2(x)\right)}\,\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\,\sqrt{1-\left(\sinh^2(x)\right)}\ \text{EllipticF}\left(i\sinh(x),i\right)}{\sqrt{\sinh^4(x)-1}\ \cosh(x)\sqrt{-1+\sinh^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+sinh(x)^2)^(1/2),x)
[Out] -I*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(1-sinh(x)^2)^(1/2)/(
sinh(x)^4-1)^(1/2)*EllipticF(I*sinh(x),I)/cosh(x)/(-1+sinh(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sinh(x)^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(sinh(x)^2 - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\sinh(x)^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^2 - 1)^(1/2),x)
[Out] int(1/(sinh(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sinh^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sinh(x)**2 - 1), x)
```

3.128 $\int \frac{1}{\sqrt{-1-\sinh^2(x)}} \, dx$

Optimal. Leaf size=16

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{-\cosh^2(x)}}$$

```
[Out] arctan(sinh(x))*cosh(x)/(-cosh(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3176, 3207, 3770}

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{-\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[-Cosh[x]^2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-\sinh^2(x)}} \, dx &= \int \frac{1}{\sqrt{-\cosh^2(x)}} \, dx\\ &= \frac{\cosh(x) \int \operatorname{sech}(x) \, dx}{\sqrt{-\cosh^2(x)}}\\ &= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{\sqrt{-\cosh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.31

$$\frac{2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Sinh[x]^2],x]
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[-Cosh[x]^2]
```

fricas [C] time = 1.03, size = 13, normalized size = 0.81

$$\log\left(e^x + i\right) - \log\left(e^x - i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="fricas")
[Out] log(e^x + I) - log(e^x - I)
```

giac [C] time = 0.14, size = 5, normalized size = 0.31

$$-2i\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="giac")
[Out] -2*I*arctan(e^x)
```

maple [B] time = 0.07, size = 34, normalized size = 2.12

$$-\frac{\cosh(x)\sqrt{-\left(\sinh^2(x)\right)}\ \operatorname{arctanh}\left(\frac{1}{\sqrt{-\left(\sinh^2(x)\right)}}\right)}{\sinh(x)\sqrt{-\left(\cosh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-sinh(x)^2)^(1/2),x)
[Out] -cosh(x)*(-sinh(x)^2)^(1/2)*arctanh(1/(-sinh(x)^2)^(1/2))/sinh(x)/(-cosh(x)
^2)^(1/2)
```

maxima [C] time = 0.59, size = 5, normalized size = 0.31

$$-2i\arctan\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="maxima")
[Out] -2*I*arctan(e^x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{-\sinh(x)^2 - 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- sinh(x)^2 - 1)^(1/2),x)
```

```
[Out] int(1/(- sinh(x)^2 - 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sinh^2(x) - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-sinh(x)**2 - 1), x)
```

3.129 $\int \frac{1}{\sqrt{a+b\sinh^2(x)}}\,dx$

Optimal. Leaf size=42

$$-\frac{i\sqrt{\frac{b\sinh^2(x)}{a}+1}\,F\left(ix\middle|\frac{b}{a}\right)}{\sqrt{a+b\sinh^2(x)}}$$

```
[Out] -I*(cosh(x)^2)^(1/2)/cosh(x)*EllipticF(I*sinh(x),(b/a)^(1/2))*(1+b*sinh(x)^
2/a)^(1/2)/(a+b*sinh(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b\sinh^2(x)}{a}+1}\,F\left(ix\middle|\frac{b}{a}\right)}{\sqrt{a+b\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sinh^2(x)}}\,dx &= \frac{\sqrt{1+\frac{b\sinh^2(x)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(x)}{a}}}\,dx}{\sqrt{a+b\sinh^2(x)}}\\ &= -\frac{iF\left(ix\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(x)}{a}}}{\sqrt{a+b\sinh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 1.26

$$-\frac{i\sqrt{\frac{2a+b\cosh(2x)-b}{a}}\,F\left(ix\middle|\frac{b}{a}\right)}{\sqrt{2a+b\cosh(2x)-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticF[I*x, b/a])/Sqrt[2*a - b + b
*Cosh[2*x]]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\sinh(x)^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sinh(x)^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sinh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(x)^2 + a), x)
```

maple [A] time = 0.09, size = 63, normalized size = 1.50

$$\frac{\sqrt{\frac{a+b\left(\sinh^2(x)\right)}{a}}\,\sqrt{\frac{\cosh(2x)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh(x)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\cosh(x)\sqrt{a+b\left(\sinh^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^2)^(1/2),x)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(x)^2)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh
(x)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(x)/(a+b*sinh(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sinh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(x)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\,{\sinh(x)}^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b*sinh(x)^2)^(1/2), x)
```

sympy [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sinh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(x)**2), x)
```

3.130 $\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=128

$$\frac{d\cosh(e+fx)\left(-\sinh^2(e+fx)\right)^{\frac{1-m}{2}}(d\sinh(e+fx))^{m-1}\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2}\right.}{f}$$

```
[Out] d*AppellF1(1/2,1/2-1/2*m,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh(
f*x+e)*(a-b+b*cosh(f*x+e)^2)^p*(d*sinh(f*x+e))^(-1+m)*(-sinh(f*x+e)^2)^(1/2
-1/2*m)/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3189, 430, 429}

$$\frac{d\cosh(e+fx)\left(-\sinh^2(e+fx)\right)^{\frac{1-m}{2}}(d\sinh(e+fx))^{m-1}\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)
/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*(d*Sinh[e + f*x])^(-
1 + m)*(-Sinh[e + f*x]^2)^((1 - m)/2))/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))
^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p\, dx = \frac{\left(d(d\sinh(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)} \left(-\sinh^2(e+fx)\right)^{\frac{1}{2}-\frac{m}{2}}\right) \text{Subst}\left(\int (\right.}{f}$$

$$= \frac{\left(d\left(a-b+b\cosh^2(e+fx)\right)^p \left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p} (d\sinh(e+\right.}{}$$

$$= \frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx}{}$$

Mathematica [F] time = 9.65, size = 0, normalized size = 0.00

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p \left(d\sinh\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p \left(d\sinh\left(fx+e\right)\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int \left(d\sinh\left(fx+e\right)\right)^m \left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p \left(d\sinh\left(fx+e\right)\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d \sinh\left(e + f x\right)\right)^{m} \left(b \sinh\left(e + f x\right)^{2} + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sinh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int((d*sinh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))**m*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.131 $\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=226

$$\frac{\left(3a^2+4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p\right.}{b^2f(2p+3)(2p+5)}$$

```
[Out] -(3*a+2*b*(2+p))*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1+p)/b^2/f/(4*p^2+16*p+
15)+(3*a^2+4*a*b*(1+p)+4*b^2*(p^2+3*p+2))*cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)
^p*hypergeom([1/2, -p],[3/2],-b*cosh(f*x+e)^2/(a-b))/b^2/f/(4*p^2+16*p+15)/
((1+b*cosh(f*x+e)^2/(a-b))^p)+cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1+p)*sinh(
f*x+e)^2/b/f/(5+2*p)
```

Rubi [A] time = 0.26, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3186, 416, 388, 246, 245}

$$\frac{\left(3a^2+4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p\right.}{b^2f(2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a + 2*b*(2 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(
b^2*f*(3 + 2*p)*(5 + 2*p))) + ((3*a^2 + 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^
2))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Co
sh[e + f*x]^2)/(a - b))^p) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1
+ p)*Sinh[e + f*x]^2)/(b*f*(5 + 2*p))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
```

```
1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a-b+bx^2\right)^p\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}\sinh^2(e+fx)}{bf(5+2p)} + \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{(3a+2b(2+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\cos}{}$$

$$= -\frac{(3a+2b(2+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\cos}{}$$

$$= -\frac{(3a+2b(2+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{(3a}{}$$

Mathematica [F] time = 11.80, size = 0, normalized size = 0.00

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 1.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \left(\sinh^{5}\left(fx+e\right)\right)\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sinh\left(e+f\,x\right)^{5}\left(b\sinh\left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p,x)
[Out] int(sinh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

3.132 $\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=137

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{p+1}}{bf(2p+3)}-\frac{(a+2b(p+1))\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}\right.}{bf(2p+3)}$$

```
[Out] cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^(1+p)/b/f/(3+2*p)-(a+2*b*(1+p))*cosh(f*x+
e)*(a-b+b*cosh(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],-b*cosh(f*x+e)^2/(a-b)
)/b/f/(3+2*p)/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3186, 388, 246, 245}

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{p+1}}{bf(2p+3)}-\frac{(a+2b(p+1))\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}\right.}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a +
 2*b*(1 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1
[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(b*f*(3 + 2*p)*(1 + (b*Cosh
[e + f*x]^2)/(a - b))^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = -\frac{\text{Subst}\left(\int (1-x^2)\left(a-b+bx^2\right)^p dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a+2b(1+p))\,\text{Sub}}{}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{\left((a+2b(1+p))\left(a\right.\right.}{}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a+2b(1+p))\cos}{}$$

Mathematica [F] time = 15.96, size = 0, normalized size = 0.00

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \left(\sinh^3\left(fx+e\right)\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e+f\,x\right)^3\,{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(sinh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.133 $\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] cosh(f*x+e)*(a-b+b*cosh(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],-b*cosh(f*x+e
)^2/(a-b))/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 246, 245}

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(a-b+bx^2\right)^p\,dx,x,\cosh(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{b}{a}\right.\right.}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}{}_2F_1\left(\right.}{f}$$

Mathematica [A] time = 0.22, size = 78, normalized size = 1.00

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int\sinh\left(fx+e\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sinh\left(e+f\,x\right)\,{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.134 $\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] -AppellF1(1/2,1,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh(f*x+e)*(a
-b+b*cosh(f*x+e)^2)^p/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 430, 429}

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b)
)]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)
/(a - b))^p))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^p}{1-x^2}\,dx,x,\cosh(e+fx)\right)}{f}$$

$$= -\frac{\left(\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a-}\right.}{1-x}\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b\right.}{f}$$

Mathematica [F] time = 4.47, size = 0, normalized size = 0.00

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(fx+e\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b \sinh\left(e+f\,x\right)^2+a\right)^p}{\sinh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.135 $\int \text{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=87

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] AppellF1(1/2,2,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh(f*x+e)*(a-
b+b*cosh(f*x+e)^2)^p/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 430, 429}

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))]
*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)/(
a - b))^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^p}{\left(1-x^2\right)^2}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right)^p}{\left(1-x^2\right)^2}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+\right.}{f}$$

Mathematica [F] time = 131.29, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(fx+e\right)^3\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^p}{\sinh\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^3,x)
[Out] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

3.136 $\int \text{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] -AppellF1(1/2,3,-p,3/2,cosh(f*x+e)^2,-b*cosh(f*x+e)^2/(a-b))*cosh(f*x+e)*(a
-b+b*cosh(f*x+e)^2)^p/f/((1+b*cosh(f*x+e)^2/(a-b))^p)
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 430, 429}

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 3, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b)
)]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)
/(a - b))^p))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=-\frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^p}{(1-x^2)^3}\,dx,x,\cosh(e+fx)\right)}{f}$$

$$=-\frac{\left(\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(1+\right.}{(1-}\right.}{f}$$

$$=-\frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-\right.}{f}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] $Aborted
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(fx+e\right)^5\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^p}{\sinh\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^5,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^5, x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.137 $\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{\sinh^4(e+fx)\sqrt{\cosh^2(e+fx)}\,\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};-\sinh^2(\right.}{5f}$$

```
[Out] 1/5*AppellF1(5/2,1/2,-p,7/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*sinh(f*x+e)^
4*(a+b*sinh(f*x+e)^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+
e)^2/a)^p)
```

Rubi [A] time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3188, 511, 510}

$$\frac{\sinh^4(e+fx)\sqrt{\cosh^2(e+fx)}\,\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};-\sinh^2(\right.}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x]
)/(5*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^p}{\sqrt{1+x^2}}\,dx, x, \sinh(e+\right.}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+}{a}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,\sinh}{5}$$

Mathematica [F] time = 11.00, size = 0, normalized size = 0.00

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \left(\sinh^4\left(fx+e\right)\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e+f\,x\right)^4\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(sinh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.138 $\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{\tanh^3(e+fx)\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(1-\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{-p}F_1\left(\frac{3}{2};p+2,-p;\frac{5}{2};\tanh^2(e+fx),\frac{(a-b}{}\right.}{3f}$$

[Out] 1/3*AppellF1(3/2,2+p,-p,5/2,tanh(f*x+e)^2,(a-b)*tanh(f*x+e)^2/a)*(sech(f*x+e)^2)^p*(a+b*sinh(f*x+e)^2)^p*tanh(f*x+e)^3/f/((1-(a-b)*tanh(f*x+e)^2/a)^p)

Rubi [A] time = 0.19, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3174, 511, 510}

$$\frac{\tanh^3(e+fx)\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(1-\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{-p}F_1\left(\frac{3}{2};p+2,-p;\frac{5}{2};\tanh^2(e+fx),\frac{(a-b}{}\right.}{3f}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (AppellF1[3/2, 2 + p, -p, 5/2, Tanh[e + f*x]^2, ((a - b)*Tanh[e + f*x]^2)/a]*(Sech[e + f*x]^2)^p*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x]^3)/(3*f*(1 - ((a - b)*Tanh[e + f*x]^2)/a)^p)

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3174

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[(ff*(a + b*Sin[e + f*x]^2)^p*(Sec[e + f*x]^2)^p)/(f*(a + (a + b)*Tan[e
+ f*x]^2)^p), Subst[Int[((a + (a + b)*ff^2*x^2)^p*(A + (A + B)*ff^2*x^2))/(
1 + ff^2*x^2)^(p + 2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, A
, B}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \frac{\left(\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(a-(a-b)\tanh^2(e+fx\right.\right.}{}$$

$$= \frac{\left(\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(a-(a-b)\tanh^2(e+fx\right.\right.}{}$$

$$= \frac{F_1\left(\frac{3}{2};2+p,-p;\frac{5}{2};\tanh^2(e+fx),\frac{(a-b)\tanh^2(e+fx)}{a}\right)\operatorname{sech}^2(e+fx)}{3f}$$

Mathematica [B] time = 0.78, size = 250, normalized size = 2.48

$$\frac{2^{-p-2}\operatorname{csch}(2(e+fx))\sqrt{-\frac{b\sinh^2(e+fx)}{a}}\sqrt{\frac{b\cosh^2(e+fx)}{b-a}}(2a+b\cosh(2(e+fx))-b)^{p+1}\left((p+1)(2a+b\cosh(2(e+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (2^(-2 - p)*Sqrt[(b*Cosh[e + f*x]^2)/(-a + b)]*(2*a - b + b*Cosh[2*(e + f*x
)])^(1 + p)*(-2*a*(2 + p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a - b + b*Cos
h[2*(e + f*x)])/(2*a), (2*a - b + b*Cosh[2*(e + f*x)])/(2*(a - b))] + (1 +
p)*AppellF1[2 + p, 1/2, 1/2, 3 + p, (2*a - b + b*Cosh[2*(e + f*x)])/(2*a),
(2*a - b + b*Cosh[2*(e + f*x)])/(2*(a - b))]*(2*a - b + b*Cosh[2*(e + f*x)]
))*Csch[2*(e + f*x)]*Sqrt[-((b*Sinh[e + f*x]^2)/a)])/(b^2*f*(1 + p)*(2 + p)
)
```

fricas [F] time = 2.13, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int\left(\sinh^2\left(fx+e\right)\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sinh\left(fx+e\right)^{2}+a\right)^{p} \sinh\left(fx+e\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sinh\left(e+f\,x\right)^{2}\,\left(b\,\sinh\left(e+f\,x\right)^{2}+a\right)^{p} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p,x)
[Out] int(sinh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

3.139 $\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};-\sinh\right.}{f}$$

```
[Out] -AppellF1(-1/2,1/2,-p,1/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*csch(f*x+e)*se
ch(f*x+e)*(a+b*sinh(f*x+e)^2)^p*(cosh(f*x+e)^2)^(1/2)/f/((1+b*sinh(f*x+e)^2
/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3188, 511, 510}

$$-\frac{\sqrt{\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};-\sinh\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1/2, -p, 1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]
*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^
p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^2\sqrt{1+x^2}}\,dx, x, \sinh(e+f$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+}{a}\right.\right.}{f}$$

$$= -\frac{F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,\text{c}}{}$$

Mathematica [F] time = 5.81, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(fx+e\right)^2\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^p}{\sinh\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^2,x)
[Out] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

3.140 $\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{\cosh^2(e+fx)}\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};-\sinh^2\right.}{3f}$$

```
[Out] -1/3*AppellF1(-3/2,1/2,-p,-1/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*csch(f*x+
e)^3*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p*(cosh(f*x+e)^2)^(1/2)/f/((1+b*sinh(f
*x+e)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3188, 511, 510}

$$-\frac{\sqrt{\cosh^2(e+fx)}\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};-\sinh^2\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, 1/2, -p, -1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]
*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2
)^p)/(3*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^4\sqrt{1+x^2}}\,dx, x, \sinh(e\right.}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh}{}\right.\right.}{f}$$

$$= -\frac{F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+}}{}$$

Mathematica [F] time = 10.86, size = 0, normalized size = 0.00

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \operatorname{csch}\left(fx+e\right)^4\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^p}{\sinh\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/sinh(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.141 $\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=106

$$\frac{a\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3a\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3ax}{8}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}+\frac{b\cosh}{}$$

```
[Out] 3/8*a*x-b*cosh(d*x+c)/d+b*cosh(d*x+c)^3/d-3/5*b*cosh(d*x+c)^5/d+1/7*b*cosh(
d*x+c)^7/d-3/8*a*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)*sinh(d*x+c)^3/
d
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3220, 2635, 8, 2633}

$$\frac{a\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3a\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3ax}{8}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}+\frac{b\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*a*x)/8 - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/d - (3*b*Cosh[c + d*x
]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d) - (3*a*Cosh[c + d*x]*Sinh[c + d*x])/
(8*d) + (a*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$
\begin{aligned}
\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)dx &= \int \left(a\sinh^4(c+dx)+b\sinh^7(c+dx)\right)dx \\
&= a\int \sinh^4(c+dx)\,dx + b\int \sinh^7(c+dx)\,dx \\
&= \frac{a\cosh(c+dx)\sinh^3(c+dx)}{4d} - \frac{1}{4}(3a)\int \sinh^2(c+dx)\,dx - \frac{b\,\text{Subst}}{} \\
&= -\frac{b\cosh(c+dx)}{d} + \frac{b\cosh^3(c+dx)}{d} - \frac{3b\cosh^5(c+dx)}{5d} + \frac{b\cosh^7(c}{7d} \\
&= \frac{3ax}{8} - \frac{b\cosh(c+dx)}{d} + \frac{b\cosh^3(c+dx)}{d} - \frac{3b\cosh^5(c+dx)}{5d} + \frac{b\cos}{}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 81, normalized size = 0.76

$$
\frac{-560a\sinh(2(c+dx)) + 70a\sinh(4(c+dx)) + 840ac + 840adx - 1225b\cosh(c+dx) + 245b\cosh(3(c+dx)) -}{2240d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (840*a*c + 840*a*d*x - 1225*b*Cosh[c + d*x] + 245*b*Cosh[3*(c + d*x)] - 49*
b*Cosh[5*(c + d*x)] + 5*b*Cosh[7*(c + d*x)] - 560*a*Sinh[2*(c + d*x)] + 70*
a*Sinh[4*(c + d*x)])/(2240*d)
```

fricas [A] time = 1.54, size = 188, normalized size = 1.77

$$
\frac{5\,b\cosh(dx+c)^7 + 35\,b\cosh(dx+c)\sinh(dx+c)^6 - 49\,b\cosh(dx+c)^5 + 280\,a\cosh(dx+c)\sinh(dx+c)^3 +}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b*cosh(d*x + c)^7 + 35*b*cosh(d*x + c)*sinh(d*x + c)^6 - 49*b*cos
h(d*x + c)^5 + 280*a*cosh(d*x + c)*sinh(d*x + c)^3 + 35*(5*b*cosh(d*x + c)^
3 - 7*b*cosh(d*x + c))*sinh(d*x + c)^4 + 245*b*cosh(d*x + c)^3 + 840*a*d*x
+ 35*(3*b*cosh(d*x + c)^5 - 14*b*cosh(d*x + c)^3 + 21*b*cosh(d*x + c))*sinh
(d*x + c)^2 - 1225*b*cosh(d*x + c) + 280*(a*cosh(d*x + c)^3 - 4*a*cosh(d*x
+ c))*sinh(d*x + c))/d
```

giac [A] time = 0.16, size = 182, normalized size = 1.72

$$
\frac{3}{8}ax + \frac{be^{(7dx+7c)}}{896d} - \frac{7be^{(5dx+5c)}}{640d} + \frac{ae^{(4dx+4c)}}{64d} + \frac{7be^{(3dx+3c)}}{128d} - \frac{ae^{(2dx+2c)}}{8d} - \frac{35be^{(dx+c)}}{128d} - \frac{35be^{(-dx-c)}}{128d} + \frac{ae^{(-2dx-2c)}}{8d} + \frac{7be^{(-3}}{12}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 3/8*a*x + 1/896*b*e^(7*d*x + 7*c)/d - 7/640*b*e^(5*d*x + 5*c)/d + 1/64*a*e^
(4*d*x + 4*c)/d + 7/128*b*e^(3*d*x + 3*c)/d - 1/8*a*e^(2*d*x + 2*c)/d - 35/
128*b*e^(d*x + c)/d - 35/128*b*e^(-d*x - c)/d + 1/8*a*e^(-2*d*x - 2*c)/d +
7/128*b*e^(-3*d*x - 3*c)/d - 1/64*a*e^(-4*d*x - 4*c)/d - 7/640*b*e^(-5*d*x
- 5*c)/d + 1/896*b*e^(-7*d*x - 7*c)/d
```

maple [A] time = 0.04, size = 82, normalized size = 0.77

$$
\frac{b\left(-\frac{16}{35} + \frac{(\sinh^6(dx+c))}{7} - \frac{6(\sinh^4(dx+c))}{35} + \frac{8(\sinh^2(dx+c))}{35}\right)\cosh(dx+c) + a\left(\left(\frac{(\sinh^3(dx+c))}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c) +\right.}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(b*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cos
h(d*x+c)+a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.32, size = 164, normalized size = 1.55

$$\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{4480}b\left(\frac{\left(49e^{(-2dx-2c)}-245e^{(-4dx-4c)}+1225\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/4480*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x -
 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) -
 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d)
```

mupad [B] time = 0.26, size = 85, normalized size = 0.80

$$\frac{280\,b\cosh\left(c+d\,x\right)^3-280\,b\cosh\left(c+d\,x\right)-168\,b\cosh\left(c+d\,x\right)^5+40\,b\cosh\left(c+d\,x\right)^7-175\,a\cosh\left(c+d\,x\right)}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^3),x)
```

```
[Out] (280*b*cosh(c + d*x)^3 - 280*b*cosh(c + d*x) - 168*b*cosh(c + d*x)^5 + 40*b
*cosh(c + d*x)^7 - 175*a*cosh(c + d*x)*sinh(c + d*x) + 105*a*d*x + 70*a*cos
h(c + d*x)^3*sinh(c + d*x))/(280*d)
```

sympy [A] time = 5.12, size = 192, normalized size = 1.81

$$\begin{cases}\frac{3ax\sinh^4(c+dx)}{8}-\frac{3ax\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3ax\cosh^4(c+dx)}{8}+\frac{5a\sinh^3(c+dx)\cosh(c+dx)}{8d}-\frac{3a\sinh(c+dx)\cosh^3(c+dx)}{8d}+\frac{b\,\mathrm{si}}{} \\ x\left(a+b\sinh^3(c)\right)\sinh^4(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b*sinh(c + d*x)**6*cosh(c + d*
x)/d - 2*b*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b*sinh(c + d*x)**2*cosh(
c + d*x)**5/(5*d) - 16*b*cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh
(c)**3)*sinh(c)**4, True))
```

3.142 $\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)dx$

Optimal. Leaf size=99

$$\frac{a\cosh^3(c+dx)}{3d}-\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b\sinh(c+dx)}{16d}$$

```
[Out] -5/16*b*x-a*cosh(d*x+c)/d+1/3*a*cosh(d*x+c)^3/d+5/16*b*cosh(d*x+c)*sinh(d*x
+c)/d-5/24*b*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3220, 2633, 2635, 8}

$$\frac{a\cosh^3(c+dx)}{3d}-\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b\sinh(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-5*b*x)/16 - (a*Cosh[c + d*x])/d + (a*Cosh[c + d*x]^3)/(3*d) + (5*b*Cosh[c
 + d*x]*Sinh[c + d*x])/(16*d) - (5*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d)
+ (b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)dx = i\int\left(-ia\sinh^3(c+dx)-ib\sinh^6(c+dx)\right)dx$$
$$= a\int\sinh^3(c+dx)\,dx+b\int\sinh^6(c+dx)\,dx$$
$$= \frac{b\cosh(c+dx)\sinh^5(c+dx)}{6d}-\frac{1}{6}(5b)\int\sinh^4(c+dx)\,dx-\frac{a\,\mathrm{Sub}}{}$$
$$= -\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{5b\cosh(c+dx)\sinh^3(c+}{24d}$$
$$= -\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{5b\cosh(c+dx)\sinh(c+d}{16d}$$
$$= -\frac{5bx}{16}-\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}+\frac{5b\cosh(c+dx)\sinh}{16d}$$

Mathematica [A] time = 0.13, size = 66, normalized size = 0.67

$$\frac{-144a\cosh(c+dx)+16a\cosh(3(c+dx))+b(45\sinh(2(c+dx))-9\sinh(4(c+dx))+\sinh(6(c+dx))-60c}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-144*a*Cosh[c + d*x] + 16*a*Cosh[3*(c + d*x)] + b*(-60*c - 60*d*x + 45*Sin
h[2*(c + d*x)] - 9*Sinh[4*(c + d*x)] + Sinh[6*(c + d*x)]))/(192*d)
```

fricas [A] time = 0.87, size = 135, normalized size = 1.36

$$\frac{3\,b\cosh(dx+c)\sinh(dx+c)^5+8\,a\cosh(dx+c)^3+24\,a\cosh(dx+c)\sinh(dx+c)^2+2\left(5\,b\cosh(dx+c)^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 8*a*cosh(d*x + c)^3 + 24*a*cosh(d
*x + c)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^3 - 9*b*cosh(d*x + c))*sinh(
d*x + c)^3 - 30*b*d*x - 72*a*cosh(d*x + c) + 3*(b*cosh(d*x + c)^5 - 6*b*cos
h(d*x + c)^3 + 15*b*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.14, size = 152, normalized size = 1.54

$$-\frac{5}{16}bx+\frac{be^{(6\,dx+6\,c)}}{384\,d}-\frac{3\,be^{(4\,dx+4\,c)}}{128\,d}+\frac{ae^{(3\,dx+3\,c)}}{24\,d}+\frac{15\,be^{(2\,dx+2\,c)}}{128\,d}-\frac{3\,ae^{(dx+c)}}{8\,d}-\frac{3\,ae^{(-dx-c)}}{8\,d}-\frac{15\,be^{(-2\,dx-2\,c)}}{128\,d}+\frac{ae^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -5/16*b*x + 1/384*b*e^(6*d*x + 6*c)/d - 3/128*b*e^(4*d*x + 4*c)/d + 1/24*a*
e^(3*d*x + 3*c)/d + 15/128*b*e^(2*d*x + 2*c)/d - 3/8*a*e^(d*x + c)/d - 3/8*
a*e^(-d*x - c)/d - 15/128*b*e^(-2*d*x - 2*c)/d + 1/24*a*e^(-3*d*x - 3*c)/d
+ 3/128*b*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x - 6*c)/d
```

maple [A] time = 0.03, size = 72, normalized size = 0.73

$$\frac{b\left(\left(\frac{\left(\sinh^5(dx+c)\right)}{6}-\frac{5\left(\sinh^3(dx+c)\right)}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)+a\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)
-5/16*d*x-5/16*c)+a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [A] time = 0.31, size = 143, normalized size = 1.44

$$-\frac{1}{384}b\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}+\frac{120(dx+c)}{d}+\frac{45e^{(-2dx-2c)}-9e^{(-4dx-4c)}+e^{(-6dx-6c)}}{d}\right)+\frac{1}{24}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/384*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d
+ 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x -
 6*c))/d) + 1/24*a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d
+ e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 0.45, size = 67, normalized size = 0.68

$$\frac{\frac{a\cosh(3c+3dx)}{12}-\frac{3a\cosh(c+dx)}{4}+\frac{15b\sinh(2c+2dx)}{64}-\frac{3b\sinh(4c+4dx)}{64}+\frac{b\sinh(6c+6dx)}{192}}{d}-\frac{5bx}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^3),x)
```

```
[Out] ((a*cosh(3*c + 3*d*x))/12 - (3*a*cosh(c + d*x))/4 + (15*b*sinh(2*c + 2*d*x)
)/64 - (3*b*sinh(4*c + 4*d*x))/64 + (b*sinh(6*c + 6*d*x))/192)/d - (5*b*x)/
16
```

sympy [A] time = 2.94, size = 194, normalized size = 1.96

$$\begin{cases} \frac{a\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{2a\cosh^3(c+dx)}{3d}+\frac{5bx\sinh^6(c+dx)}{16}-\frac{15bx\sinh^4(c+dx)\cosh^2(c+dx)}{16}+\frac{15bx\sinh^2(c+dx)\cosh^4(c+dx)}{16}-\frac{5bx\,c}{} \\ x\left(a+b\sinh^3(c)\right)\sinh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
+ 5*b*x*sinh(c + d*x)**6/16 - 15*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 +
 15*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*x*cosh(c + d*x)**6/16 +
11*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b*sinh(c + d*x)**3*cosh(c +
d*x)**3/(6*d) + 5*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a
 + b*sinh(c)**3)*sinh(c)**3, True))
```

3.143 $\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=70

$$\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{ax}{2}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh(c+dx)}{d}$$

```
[Out] -1/2*a*x+b*cosh(d*x+c)/d-2/3*b*cosh(d*x+c)^3/d+1/5*b*cosh(d*x+c)^5/d+1/2*a*
cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3220, 2635, 8, 2633}

$$\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{ax}{2}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -(a*x)/2 + (b*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c +
d*x]^5)/(5*d) + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= -\int \left(-a\sinh^2(c+dx)-b\sinh^5(c+dx)\right)\,dx\\ &= a\int \sinh^2(c+dx)\,dx+b\int \sinh^5(c+dx)\,dx\\ &= \frac{a\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{1}{2}a\int 1\,dx+\frac{b\,\text{Subst}\left(\int \left(1-2x^2+\right.\right.}{}\\ &= -\frac{ax}{2}+\frac{b\cosh(c+dx)}{d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}+\frac{a}{}\end{aligned}$$

Mathematica [A] time = 0.10, size = 79, normalized size = 1.13

$$\frac{a(-c-dx)}{2d}+\frac{a\sinh(2(c+dx))}{4d}+\frac{5b\cosh(c+dx)}{8d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (a*(-c - d*x))/(2*d) + (5*b*Cosh[c + d*x])/(8*d) - (5*b*Cosh[3*(c + d*x)])/
(48*d) + (b*Cosh[5*(c + d*x)])/(80*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

fricas [A] time = 1.47, size = 105, normalized size = 1.50

$$\frac{3\,b\cosh\left(dx+c\right)^{5}+15\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{4}-25\,b\cosh\left(dx+c\right)^{3}-120\,adx+120\,a\cosh\left(dx+c\right)\sinh\left(\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 - 25*b*cosh
(d*x + c)^3 - 120*a*d*x + 120*a*cosh(d*x + c)*sinh(d*x + c) + 15*(2*b*cosh(
d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c)^2 + 150*b*cosh(d*x + c))/d
```

giac [A] time = 0.15, size = 122, normalized size = 1.74

$$-\frac{1}{2}\,ax+\frac{be^{(5\,dx+5\,c)}}{160\,d}-\frac{5\,be^{(3\,dx+3\,c)}}{96\,d}+\frac{ae^{(2\,dx+2\,c)}}{8\,d}+\frac{5\,be^{(dx+c)}}{16\,d}+\frac{5\,be^{(-dx-c)}}{16\,d}-\frac{ae^{(-2\,dx-2\,c)}}{8\,d}-\frac{5\,be^{(-3\,dx-3\,c)}}{96\,d}+\frac{be^{(-5\,dx-5\,c)}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/2*a*x + 1/160*b*e^(5*d*x + 5*c)/d - 5/96*b*e^(3*d*x + 3*c)/d + 1/8*a*e^(
2*d*x + 2*c)/d + 5/16*b*e^(d*x + c)/d + 5/16*b*e^(-d*x - c)/d - 1/8*a*e^(-2
*d*x - 2*c)/d - 5/96*b*e^(-3*d*x - 3*c)/d + 1/160*b*e^(-5*d*x - 5*c)/d
```

maple [A] time = 0.03, size = 60, normalized size = 0.86

$$\frac{b\left(\frac{8}{15}+\frac{\left(\sinh^{4}(dx+c)\right)}{5}-\frac{4\left(\sinh^{2}(dx+c)\right)}{15}\right)\cosh\left(dx+c\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a*(1/2*cosh(
d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [A] time = 0.31, size = 120, normalized size = 1.71

$$-\frac{1}{8}\,a\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+\frac{1}{480}\,b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b*(3*e^(5*d*x
 + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d -
 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

mupad [**B**] time = 0.12, size = 55, normalized size = 0.79

$$\frac{b\cosh\left(c+d\,x\right)-\frac{2\,b\cosh(c+d\,x)^3}{3}+\frac{b\cosh(c+d\,x)^5}{5}+\frac{a\cosh(c+d\,x)\sinh(c+d\,x)}{2}}{d}-\frac{a\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3),x)
```

```
[Out] (b*cosh(c + d*x) - (2*b*cosh(c + d*x)^3)/3 + (b*cosh(c + d*x)^5)/5 + (a*cos
h(c + d*x)*sinh(c + d*x))/2)/d - (a*x)/2
```

sympy [**A**] time = 1.59, size = 117, normalized size = 1.67

$$\begin{cases}\frac{ax\sinh^2(c+dx)}{2}-\frac{ax\cosh^2(c+dx)}{2}+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{b\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{4b\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{8b\cosh^5(c}{15d}\\ x\left(a+b\sinh^3(c)\right)\sinh^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x
)*cosh(c + d*x)/(2*d) + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c + d
*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*
(a + b*sinh(c)**3)*sinh(c)**2, True))
```

3.144 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)dx$

Optimal. Leaf size=60

$$\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3bx}{8}$$

```
[Out] 3/8*b*x+a*cosh(d*x+c)/d-3/8*b*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)*s
inh(d*x+c)^3/d
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3220, 2638, 2635, 8}

$$\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*b*x)/8 + (a*Cosh[c + d*x])/d - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) +
 (b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)dx &= -\left(i\int\left(ia\sinh(c+dx)+ib\sinh^4(c+dx)\right)dx\right)\\
&= a\int \sinh(c+dx)\,dx+b\int \sinh^4(c+dx)\,dx\\
&= \frac{a\cosh(c+dx)}{d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}(3b)\int \sinh^2(c+\\
&= \frac{a\cosh(c+dx)}{d}-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh}{4d}\\
&= \frac{3bx}{8}+\frac{a\cosh(c+dx)}{d}-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+d}{}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 45, normalized size = 0.75

$$\frac{32a\cosh(c+dx)+b(12(c+dx)-8\sinh(2(c+dx))+\sinh(4(c+dx)))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (32*a*Cosh[c + d*x] + b*(12*(c + d*x) - 8*Sinh[2*(c + d*x)] + Sinh[4*(c + d
*x)]))/(32*d)
```

fricas [A] time = 1.82, size = 63, normalized size = 1.05

$$\frac{b\cosh(dx+c)\sinh(dx+c)^3+3\,bdx+8\,a\cosh(dx+c)+\left(b\cosh(dx+c)^3-4\,b\cosh(dx+c)\right)\sinh(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b*d*x + 8*a*cosh(d*x + c) + (b*cos
h(d*x + c)^3 - 4*b*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.15, size = 92, normalized size = 1.53

$$\frac{3}{8}\,bx+\frac{be^{(4\,dx+4\,c)}}{64\,d}-\frac{be^{(2\,dx+2\,c)}}{8\,d}+\frac{ae^{(dx+c)}}{2\,d}+\frac{ae^{(-dx-c)}}{2\,d}+\frac{be^{(-2\,dx-2\,c)}}{8\,d}-\frac{be^{(-4\,dx-4\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/64*b*e^(4*d*x + 4*c)/d - 1/8*b*e^(2*d*x + 2*c)/d + 1/2*a*e^(d*x
 + c)/d + 1/2*a*e^(-d*x - c)/d + 1/8*b*e^(-2*d*x - 2*c)/d - 1/64*b*e^(-4*d*
x - 4*c)/d
```

maple [A] time = 0.04, size = 50, normalized size = 0.83

$$\frac{b\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+a\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a*co
sh(d*x+c))
```

maxima [A] time = 0.30, size = 74, normalized size = 1.23

$$\frac{1}{64}\,b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{a\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + a*cosh(d*x + c)/d
```

mupad [B] time = 0.20, size = 42, normalized size = 0.70

$$\frac{3\,b\,x}{8}+\frac{a\cosh\left(c+d\,x\right)-\frac{b\,\mathrm{sinh}\left(2\,c+2\,d\,x\right)}{4}+\frac{b\,\mathrm{sinh}\left(4\,c+4\,d\,x\right)}{32}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^3),x)
```

```
[Out] (3*b*x)/8 + (a*cosh(c + d*x) - (b*sinh(2*c + 2*d*x))/4 + (b*sinh(4*c + 4*d*
x))/32)/d
```

sympy [A] time = 0.85, size = 121, normalized size = 2.02

$$\begin{cases} \frac{a\cosh\left(c+dx\right)}{d}+\frac{3bx\sinh^{4}\left(c+dx\right)}{8}-\frac{3bx\sinh^{2}\left(c+dx\right)\cosh^{2}\left(c+dx\right)}{4}+\frac{3bx\cosh^{4}\left(c+dx\right)}{8}+\frac{5b\sinh^{3}\left(c+dx\right)\cosh\left(c+dx\right)}{8d}-\frac{3b\sinh\left(c+dx\right)\cosh^{3}\left(\right.}{8d} \\ x\left(a+b\sinh^{3}\left(c\right)\right)\sinh\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((a*cosh(c + d*x)/d + 3*b*x*sinh(c + d*x)**4/8 - 3*b*x*sinh(c + d*
x)**2*cosh(c + d*x)**2/4 + 3*b*x*cosh(c + d*x)**4/8 + 5*b*sinh(c + d*x)**3*
cosh(c + d*x)/(8*d) - 3*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)),
(x*(a + b*sinh(c)**3)*sinh(c), True))
```

3.145 $\int \left(a+b\sinh^3(c+dx)\right) dx$

Optimal. Leaf size=32

$$ax+\frac{b\cosh^3(c+dx)}{3d}-\frac{b\cosh(c+dx)}{d}$$

```
[Out] a*x-b*cosh(d*x+c)/d+1/3*b*cosh(d*x+c)^3/d
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2633}

$$ax+\frac{b\cosh^3(c+dx)}{3d}-\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x]^3,x]
[Out] a*x - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^3(c+dx)\right) dx &= ax+b\int \sinh^3(c+dx)\,dx\\ &= ax-\frac{b\,\text{Subst}\left(\int \left(1-x^2\right) dx,x,\cosh(c+dx)\right)}{d}\\ &= ax-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.06

$$ax-\frac{3b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x]^3,x]
[Out] a*x - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])/(12*d)
```

fricas [A] time = 1.21, size = 47, normalized size = 1.47

$$\frac{b\cosh\left(dx+c\right)^3+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+12\,adx-9\,b\cosh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="fricas")
[Out] 1/12*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + 12*a*d*x - 9*
b*cosh(d*x + c))/d
```

giac [A] time = 0.12, size = 59, normalized size = 1.84

$$ax + \frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] a*x + 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-
3*d*x - 3*c)/d)
```

maple [A] time = 0.02, size = 28, normalized size = 0.88

$$ax + \frac{b\left(-\frac{2}{3} + \frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c)^3,x)
```

```
[Out] a*x+b/d*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)
```

maxima [A] time = 0.31, size = 59, normalized size = 1.84

$$ax + \frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] a*x + 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-
3*d*x - 3*c)/d)
```

mupad [B] time = 0.62, size = 29, normalized size = 0.91

$$a\,x - \frac{b\cosh\left(c + d\,x\right) - \frac{b\cosh(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sinh(c + d*x)^3,x)
```

```
[Out] a*x - (b*cosh(c + d*x) - (b*cosh(c + d*x)^3)/3)/d
```

sympy [A] time = 0.39, size = 41, normalized size = 1.28

$$ax + b\left(\begin{cases} \frac{\sinh^2\left(c+dx\right)\cosh\left(c+dx\right)}{d} - \frac{2\cosh^3\left(c+dx\right)}{3d} & \text{for}\ d \neq 0 \\ x\sinh^3\left(c\right) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)**3,x)
```

```
[Out] a*x + b*Piecewise((sinh(c + d*x)**2*cosh(c + d*x)/d - 2*cosh(c + d*x)**3/(3
*d), Ne(d, 0)), (x*sinh(c)**3, True))
```

3.146 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

```
[Out] -1/2*b*x-a*arctanh(cosh(d*x+c))/d+1/2*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3220, 3770, 2635, 8}

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -(b*x)/2 - (a*ArcTanh[Cosh[c + d*x]])/d + (b*Cosh[c + d*x]*Sinh[c + d*x])/(
2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= i\int\left(-ia\operatorname{csch}(c+dx)-ib\sinh^2(c+dx)\right)\,dx\\ &= a\int \operatorname{csch}(c+dx)\,dx+b\int \sinh^2(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{1}{2}b\int 1\\ &= -\frac{bx}{2}-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 1.80

$$\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b(-c-dx)}{2d}+\frac{b\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (b*(-c - d*x))/(2*d) - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[Sinh[c/2 + (
d*x)/2]])/d + (b*Sinh[2*(c + d*x)])/(4*d)
```

fricas [B] time = 2.18, size = 258, normalized size = 6.45

$$-\frac{4\,bdx\cosh\left(dx+c\right)^{2}-b\cosh\left(dx+c\right)^{4}-4\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}-b\sinh\left(dx+c\right)^{4}+2\left(2\,bdx-3\,b\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*b*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d
*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 8*(a*cosh(d*x + c)^2 +
2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*log(cosh(d*x + c) + si
nh(d*x + c) - 1) + 4*(2*b*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*x +
 c) + b)/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x
+ c)^2)
```

giac [A] time = 0.13, size = 62, normalized size = 1.55

$$-\frac{4\,(dx+c)b-be^{(2\,dx+2\,c)}+be^{(-2\,dx-2\,c)}+8\,a\log\left(e^{(dx+c)}+1\right)-8\,a\log\left(\left|e^{(dx+c)}-1\right|\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*b - b*e^(2*d*x + 2*c) + b*e^(-2*d*x - 2*c) + 8*a*log(e^(d
*x + c) + 1) - 8*a*log(abs(e^(d*x + c) - 1)))/d
```

maple [A] time = 0.09, size = 40, normalized size = 1.00

$$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)
)
```

maxima [A] time = 0.32, size = 50, normalized size = 1.25

$$-\frac{1}{8}\,b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*log(tanh(1/2*d*x
+ 1/2*c))/d
```

mupad [B] time = 0.69, size = 73, normalized size = 1.82

$$\frac{b\,e^{2\,c+2\,d\,x}}{8\,d}-\frac{b\,e^{-2\,c-2\,d\,x}}{8\,d}-\frac{b\,x}{2}-\frac{2\,\mathrm{atan}\left(\frac{a\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)/sinh(c + d*x),x)
```

```
[Out] (b*exp(2*c + 2*d*x))/(8*d) - (b*exp(- 2*c - 2*d*x))/(8*d) - (b*x)/2 - (2*at
an((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1
/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^{3}\left(c + dx\right)\right)\operatorname{csch}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**3)*csch(c + d*x), x)
```

3.147 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=24

$$\frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

```
[Out] b*cosh(d*x+c)/d-a*coth(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3220, 3767, 8, 2638}

$$\frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (b*Cosh[c + d*x])/d - (a*Coth[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)dx &= -\int \left(-a\operatorname{csch}^2(c+dx)-b\sinh(c+dx)\right)dx\\ &= a\int \operatorname{csch}^2(c+dx)\,dx+b\int \sinh(c+dx)\,dx\\ &= \frac{b\cosh(c+dx)}{d}-\frac{(ia)\operatorname{Subst}\left(\int 1\,dx,x,-i\coth(c+dx)\right)}{d}\\ &= \frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 1.46

$$-\frac{a\coth(c+dx)}{d}+\frac{b\sinh(c)\sinh(dx)}{d}+\frac{b\cosh(c)\cosh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Coth[c + d*x])/d + (b*Sinh[c]*Sinh[d*x])/d
```

fricas [A] time = 0.93, size = 40, normalized size = 1.67

$$-\frac{a\cosh\left(dx+c\right)-\left(b\cosh\left(dx+c\right)+a\right)\sinh\left(dx+c\right)}{d\sinh\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
[Out] -(a*cosh(d*x + c) - (b*cosh(d*x + c) + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

giac [B] time = 0.14, size = 59, normalized size = 2.46

$$\frac{be^{(dx+c)}+\frac{be^{(2\,dx+2\,c)}-4\,ae^{(dx+c)}-b}{e^{(3\,dx+3\,c)}-e^{(dx+c)}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
[Out] 1/2*(b*e^(d*x + c) + (b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c) - b)/(e^(3*d*x +
3*c) - e^(d*x + c)))/d
```

maple [A] time = 0.08, size = 23, normalized size = 0.96

$$\frac{-\coth\left(dx+c\right)a+b\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x)
[Out] 1/d*(-coth(d*x+c)*a+b*cosh(d*x+c))
```

maxima [A] time = 0.32, size = 47, normalized size = 1.96

$$\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{2\,a}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.09, size = 47, normalized size = 1.96

$$\frac{b\,\mathrm{e}^{-c-d\,x}}{2\,d}-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{b\,\mathrm{e}^{c+d\,x}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)/sinh(c + d*x)^2,x)
[Out] (b*exp(- c - d*x))/(2*d) - (2*a)/(d*(exp(2*c + 2*d*x) - 1)) + (b*exp(c + d*
x))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sinh^3(c + dx)\right) \operatorname{csch}^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3),x)
[Out] Integral((a + b*sinh(c + d*x)**3)*csch(c + d*x)**2, x)
```

3.148 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=39

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+bx$$

```
[Out] b*x+1/2*a*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3220, 3768, 3770}

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] b*x + (a*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/(2
*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= -\left(i\int \left(ib+ia\operatorname{csch}^3(c+dx)\right)\,dx\right)\\
&= bx+a\int \operatorname{csch}^3(c+dx)\,dx\\
&= bx-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{1}{2}a\int \operatorname{csch}(c+dx)\,dx\\
&= bx+\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 1.62

$$-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] b*x - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (a
*Sech[(c + d*x)/2]^2)/(8*d)
```

fricas [B] time = 0.62, size = 521, normalized size = 13.36

$$\frac{2\,bdx\cosh\left(dx+c\right)^{4}+2\,bdx\sinh\left(dx+c\right)^{4}-4\,bdx\cosh\left(dx+c\right)^{2}-2\,a\cosh\left(dx+c\right)^{3}+2\left(4\,bdx\cosh\left(dx+c\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x*cosh(d*x + c)^4 + 2*b*d*x*sinh(d*x + c)^4 - 4*b*d*x*cosh(d*x +
 c)^2 - 2*a*cosh(d*x + c)^3 + 2*(4*b*d*x*cosh(d*x + c) - a)*sinh(d*x + c)^3
 + 2*b*d*x + 2*(6*b*d*x*cosh(d*x + c)^2 - 2*b*d*x - 3*a*cosh(d*x + c))*sinh
(d*x + c)^2 - 2*a*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*si
nh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x +
 c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d
*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d*x + c)^4 +
4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2
 + 2*(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*c
osh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2
*(4*b*d*x*cosh(d*x + c)^3 - 4*b*d*x*cosh(d*x + c) - 3*a*cosh(d*x + c)^2 - a
)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d
*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d
*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.15, size = 73, normalized size = 1.87

$$\frac{2\left(dx+c\right)b+a\log\left(e^{\left(dx+c\right)}+1\right)-a\log\left(\left|e^{\left(dx+c\right)}-1\right|\right)-\frac{2\left(ae^{\left(3\,dx+3\,c\right)}+ae^{\left(dx+c\right)}\right)}{\left(e^{\left(2\,dx+2\,c\right)}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*b + a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x + c) - 1)) -
 2*(a*e^(3*d*x + 3*c) + a*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2)/d
```

maple [A] time = 0.10, size = 37, normalized size = 0.95

$$\frac{a\left(-\frac{\operatorname{csch}\left(dx+c\right)\coth\left(dx+c\right)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+\left(dx+c\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+(d*x+c)*b)
```

maxima [B] time = 0.32, size = 91, normalized size = 2.33

$$bx+\frac{1}{2}\,a\left(\frac{\log\left(e^{\left(-dx-c\right)}+1\right)}{d}-\frac{\log\left(e^{\left(-dx-c\right)}-1\right)}{d}+\frac{2\left(e^{\left(-dx-c\right)}+e^{\left(-3\,dx-3\,c\right)}\right)}{d\left(2\,e^{\left(-2\,dx-2\,c\right)}-e^{\left(-4\,dx-4\,c\right)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] b*x + 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x
 - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

mupad [B] time = 0.11, size = 102, normalized size = 2.62

$$bx+\frac{\operatorname{atan}\left(\frac{a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}-\frac{a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)/sinh(c + d*x)^3,x)
```

```
[Out] b*x + (atan((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/
(-d^2)^(1/2) - (a*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a*exp(c + d
*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.149 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)dx$

Optimal. Leaf size=41

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}$$

```
[Out] -b*arctanh(cosh(d*x+c))/d+a*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3220, 3770, 3767}

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -((b*ArcTanh[Cosh[c + d*x]])/d) + (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)
/(3*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)dx &= \int \left(b\operatorname{csch}(c+dx)+a\operatorname{csch}^4(c+dx)\right)dx\\ &= a\int \operatorname{csch}^4(c+dx)\,dx+b\int \operatorname{csch}(c+dx)\,dx\\ &= -\frac{b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{(ia)\operatorname{Subst}\left(\int \left(1+x^2\right)dx,x,-i\coth(c+\right.}{d}\\ &= -\frac{b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 1.85

$$\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+\frac{b\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b*Lo
g[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d
```

fricas [B] time = 2.93, size = 652, normalized size = 15.90

$$-\frac{12\,a\cosh\left(dx+c\right)^{2}+24\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+12\,a\sinh\left(dx+c\right)^{2}+3\left(b\cosh\left(dx+c\right)^{6}+6\,b\cosh\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*cosh(d*x + c)^2 + 24*a*cosh(d*x + c)*sinh(d*x + c) + 12*a*sinh(d
*x + c)^2 + 3*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*si
nh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*
cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*
x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*si
nh(d*x + c) - b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(b*cosh(d*x + c
)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x
+ c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)
^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh
(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)
^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*log(cosh(d*x
 + c) + sinh(d*x + c) - 1) - 4*a)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*si
nh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x +
 c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x
 + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 +
 d*cosh(d*x + c))*sinh(d*x + c) - d)
```

giac [A] time = 0.14, size = 62, normalized size = 1.51

$$-\frac{3\,b\log\left(e^{(dx+c)}+1\right)-3\,b\log\left(\left|e^{(dx+c)}-1\right|\right)+\frac{4\left(3\,ae^{(2\,dx+2\,c)}-a\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/3*(3*b*log(e^(d*x + c) + 1) - 3*b*log(abs(e^(d*x + c) - 1)) + 4*(3*a*e^(
2*d*x + 2*c) - a)/(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [A] time = 0.10, size = 36, normalized size = 0.88

$$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth\left(dx+c\right)-2b\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*b*arctanh(exp(d*x+c)))
```

maxima [B] time = 0.32, size = 131, normalized size = 3.20

$$-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)+\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{}{d\left(3\,e^{(-2\,dx-2\,c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a*(3*e^(-2*d*x
 - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)
) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 0.69, size = 110, normalized size = 2.68

$$-\frac{4\,a}{d\,\left({\mathrm{e}}^{4\,c+4\,d\,x}-2\,{\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}-\frac{8\,a}{3\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1\right)}-\frac{2\,\mathrm{atan}\left(\frac{b\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)/sinh(c + d*x)^4,x)
```

```
[Out] - (4*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a)/(3*d*(3*exp
(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (2*atan((b*ex
p(d*x)*exp(c)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(-d^2)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.150 $\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=192

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a^2\cosh(c+dx)}{d}+\frac{ab\sinh^5(c+dx)\cosh(c+dx)}{3d}-\frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{12d}+\frac{5ab\sinh}{}$$

```
[Out] -5/8*a*b*x-a^2*cosh(d*x+c)/d+b^2*cosh(d*x+c)/d+1/3*a^2*cosh(d*x+c)^3/d-4/3*
b^2*cosh(d*x+c)^3/d+6/5*b^2*cosh(d*x+c)^5/d-4/7*b^2*cosh(d*x+c)^7/d+1/9*b^2
*cosh(d*x+c)^9/d+5/8*a*b*cosh(d*x+c)*sinh(d*x+c)/d-5/12*a*b*cosh(d*x+c)*sin
h(d*x+c)^3/d+1/3*a*b*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Rubi [A] time = 0.18, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 2633, 2635, 8}

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a^2\cosh(c+dx)}{d}+\frac{ab\sinh^5(c+dx)\cosh(c+dx)}{3d}-\frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{12d}+\frac{5ab\sinh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-5*a*b*x)/8 - (a^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x])/d + (a^2*Cosh[c
+ d*x]^3)/(3*d) - (4*b^2*Cosh[c + d*x]^3)/(3*d) + (6*b^2*Cosh[c + d*x]^5)/(
5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[c + d*x]^9)/(9*d) + (5*a*b
*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) - (5*a*b*Cosh[c + d*x]*Sinh[c + d*x]^3)
/(12*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = i\int \left(-ia^2\sinh^3(c+dx)-2iab\sinh^6(c+dx)-ib^2\sinh^9(c+dx)\right) d$$
$$= a^2\int \sinh^3(c+dx)\,dx + (2ab)\int \sinh^6(c+dx)\,dx + b^2\int \sinh^9(c+$$
$$= \frac{ab\cosh(c+dx)\sinh^5(c+dx)}{3d} - \frac{1}{3}(5ab)\int \sinh^4(c+dx)\,dx - \frac{a^2\,\mathrm{Su}}{}$$
$$= -\frac{a^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d} + \frac{a^2\cosh^3(c+dx)}{3d} - \frac{4b^2\cosh}{3}$$
$$= -\frac{a^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d} + \frac{a^2\cosh^3(c+dx)}{3d} - \frac{4b^2\cosh}{3}$$
$$= -\frac{5}{8}abx - \frac{a^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d} + \frac{a^2\cosh^3(c+dx)}{3d} - $$

Mathematica [A] time = 0.68, size = 125, normalized size = 0.65

$$\frac{-1890\left(32a^2-21b^2\right)\cosh(c+dx)+420\left(16a^2-21b^2\right)\cosh(3(c+dx))+b(-840a(-45\sinh(2(c+dx))+9\sinh(}{80}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-1890*(32*a^2 - 21*b^2)*Cosh[c + d*x] + 420*(16*a^2 - 21*b^2)*Cosh[3*(c +
d*x)] + b*(2268*b*Cosh[5*(c + d*x)] - 405*b*Cosh[7*(c + d*x)] + 35*b*Cosh[9
*(c + d*x)] - 840*a*(60*c + 60*d*x - 45*Sinh[2*(c + d*x)] + 9*Sinh[4*(c + d
*x)] - Sinh[6*(c + d*x)])))/(80640*d)
```

fricas [B] time = 0.70, size = 355, normalized size = 1.85

$$\frac{35\,b^2\cosh(dx+c)^9+315\,b^2\cosh(dx+c)\sinh(dx+c)^8-405\,b^2\cosh(dx+c)^7+5040\,ab\cosh(dx+c)\sinh(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^2*cosh(d*x + c)^9 + 315*b^2*cosh(d*x + c)*sinh(d*x + c)^8 - 4
05*b^2*cosh(d*x + c)^7 + 5040*a*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2268*b^2*
cosh(d*x + c)^5 + 105*(28*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c))*sinh(
d*x + c)^6 + 315*(14*b^2*cosh(d*x + c)^5 - 45*b^2*cosh(d*x + c)^3 + 36*b^2*
cosh(d*x + c))*sinh(d*x + c)^4 - 50400*a*b*d*x + 420*(16*a^2 - 21*b^2)*cosh
(d*x + c)^3 + 3360*(5*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*sinh(d*x +
 c)^3 + 315*(4*b^2*cosh(d*x + c)^7 - 27*b^2*cosh(d*x + c)^5 + 72*b^2*cosh(d
*x + c)^3 + 4*(16*a^2 - 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 1890*(32*a
^2 - 21*b^2)*cosh(d*x + c) + 5040*(a*b*cosh(d*x + c)^5 - 6*a*b*cosh(d*x + c
)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.19, size = 301, normalized size = 1.57

$$-\frac{5}{8}abx+\frac{b^2e^{(9dx+9c)}}{4608d}-\frac{9b^2e^{(7dx+7c)}}{3584d}+\frac{abe^{(6dx+6c)}}{192d}+\frac{9b^2e^{(5dx+5c)}}{640d}-\frac{3abe^{(4dx+4c)}}{64d}+\frac{15abe^{(2dx+2c)}}{64d}-\frac{15abe^{(-2dx-2c)}}{64d}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -5/8*a*b*x + 1/4608*b^2*e^(9*d*x + 9*c)/d - 9/3584*b^2*e^(7*d*x + 7*c)/d +
1/192*a*b*e^(6*d*x + 6*c)/d + 9/640*b^2*e^(5*d*x + 5*c)/d - 3/64*a*b*e^(4*d
```

```
*x + 4*c)/d + 15/64*a*b*e^(2*d*x + 2*c)/d - 15/64*a*b*e^(-2*d*x - 2*c)/d +
3/64*a*b*e^(-4*d*x - 4*c)/d + 9/640*b^2*e^(-5*d*x - 5*c)/d - 1/192*a*b*e^(-
6*d*x - 6*c)/d - 9/3584*b^2*e^(-7*d*x - 7*c)/d + 1/4608*b^2*e^(-9*d*x - 9*c
)/d + 1/384*(16*a^2 - 21*b^2)*e^(3*d*x + 3*c)/d - 3/256*(32*a^2 - 21*b^2)*e
^(d*x + c)/d - 3/256*(32*a^2 - 21*b^2)*e^(-d*x - c)/d + 1/384*(16*a^2 - 21*
b^2)*e^(-3*d*x - 3*c)/d
```

maple [A] time = 0.04, size = 128, normalized size = 0.67

$$\frac{b^2\left(\frac{128}{315}+\frac{\left(\sinh^8(dx+c)\right)}{9}-\frac{8\left(\sinh^6(dx+c)\right)}{63}+\frac{16\left(\sinh^4(dx+c)\right)}{105}-\frac{64\left(\sinh^2(dx+c)\right)}{315}\right)\cosh(dx+c)+2ab\left(\left(\frac{\left(\sinh^5(dx+c)\right)}{6}-\frac{5\left(\sinh\right.}{}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(b^2*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4
-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+
c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a^2*(-2/3+1/3*sinh(d*x+
c)^2)*cosh(d*x+c))
```

maxima [A] time = 0.32, size = 272, normalized size = 1.42

$$-\frac{1}{161280}\,b^2\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\frac{39690\,e^{(-d}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/161280*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x
 - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*
e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 0.93, size = 149, normalized size = 0.78

$$\frac{\frac{a^2\cosh(c+d\,x)^3}{3}-a^2\cosh(c+d\,x)+\frac{\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)^5}{3}-\frac{13\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)^3}{12}+\frac{11\sinh(c+d\,x)\,a\,b\cosh(c+d\,x)}{8}-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^3)^2,x)
```

```
[Out] (b^2*cosh(c + d*x) - a^2*cosh(c + d*x) + (a^2*cosh(c + d*x)^3)/3 - (4*b^2*c
osh(c + d*x)^3)/3 + (6*b^2*cosh(c + d*x)^5)/5 - (4*b^2*cosh(c + d*x)^7)/7 +
 (b^2*cosh(c + d*x)^9)/9 - (13*a*b*cosh(c + d*x)^3*sinh(c + d*x))/12 + (a*b
*cosh(c + d*x)^5*sinh(c + d*x))/3 + (11*a*b*cosh(c + d*x)*sinh(c + d*x))/8
- (5*a*b*d*x)/8)/d
```

sympy [A] time = 13.68, size = 325, normalized size = 1.69

$$\begin{cases}\frac{a^2\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{2a^2\cosh^3(c+dx)}{3d}+\frac{5abx\sinh^6(c+dx)}{8}-\frac{15abx\sinh^4(c+dx)\cosh^2(c+dx)}{8}+\frac{15abx\sinh^2(c+dx)\cosh^4(c+dx)}{8} \\ x\left(a+b\sinh^3(c)\right)^2\sinh^3(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/
(3*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d*x
)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c + d*
x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c + d*x)
**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + b
**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**2*sinh(c + d*x)**6*cosh(c + d*x
)**3/(3*d) + 16*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b**2*sinh
(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**2*cosh(c + d*x)**9/(315*d), N
e(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c)**3, True))
```

3.151 $\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=180

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^2x}{2}+\frac{2ab\cosh^5(c+dx)}{5d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^7(c+d}{8}$$

```
[Out] -1/2*a^2*x+35/128*b^2*x+2*a*b*cosh(d*x+c)/d-4/3*a*b*cosh(d*x+c)^3/d+2/5*a*b
*cosh(d*x+c)^5/d+1/2*a^2*cosh(d*x+c)*sinh(d*x+c)/d-35/128*b^2*cosh(d*x+c)*s
inh(d*x+c)/d+35/192*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d-7/48*b^2*cosh(d*x+c)*si
nh(d*x+c)^5/d+1/8*b^2*cosh(d*x+c)*sinh(d*x+c)^7/d
```

Rubi [A] time = 0.15, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = $\{3220, 2635, 8, 2633\}$

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^2x}{2}+\frac{2ab\cosh^5(c+dx)}{5d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^7(c+d}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] -(a^2*x)/2 + (35*b^2*x)/128 + (2*a*b*Cosh[c + d*x])/d - (4*a*b*Cosh[c + d*x
]^3)/(3*d) + (2*a*b*Cosh[c + d*x]^5)/(5*d) + (a^2*Cosh[c + d*x]*Sinh[c + d*
x])/(2*d) - (35*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*b^2*Cosh[c +
 d*x]*Sinh[c + d*x]^3)/(192*d) - (7*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*
d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = -\int \left(-a^2\sinh^2(c+dx)-2ab\sinh^5(c+dx)-b^2\sinh^8(c+dx)\right)dx$$
$$= a^2\int \sinh^2(c+dx)\,dx+(2ab)\int \sinh^5(c+dx)\,dx+b^2\int \sinh^8(c+$$
$$= \frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^2\cosh(c+dx)\sinh^7(c+dx)}{8d}-\frac{1}{2}a$$
$$= -\frac{a^2x}{2}+\frac{2ab\cosh(c+dx)}{d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh^5(c+dx)}{5d}$$
$$= -\frac{a^2x}{2}+\frac{2ab\cosh(c+dx)}{d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh^5(c+dx)}{5d}$$
$$= -\frac{a^2x}{2}+\frac{2ab\cosh(c+dx)}{d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh^5(c+dx)}{5d}$$
$$= -\frac{a^2x}{2}+\frac{35b^2x}{128}+\frac{2ab\cosh(c+dx)}{d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh}{5}$$

Mathematica [A] time = 0.13, size = 133, normalized size = 0.74

$$\frac{3840a^2\sinh(2(c+dx))-7680a^2c-7680a^2dx+19200ab\cosh(c+dx)-3200ab\cosh(3(c+dx))+384ab\cosh(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-7680*a^2*c + 4200*b^2*c - 7680*a^2*d*x + 4200*b^2*d*x + 19200*a*b*Cosh[c
+ d*x] - 3200*a*b*Cosh[3*(c + d*x)] + 384*a*b*Cosh[5*(c + d*x)] + 3840*a^2*
Sinh[2*(c + d*x)] - 3360*b^2*Sinh[2*(c + d*x)] + 840*b^2*Sinh[4*(c + d*x)]
- 160*b^2*Sinh[6*(c + d*x)] + 15*b^2*Sinh[8*(c + d*x)])/(15360*d)
```

fricas [A] time = 1.53, size = 274, normalized size = 1.52

$$\frac{15\,b^2\cosh(dx+c)\sinh(dx+c)^7+48\,ab\cosh(dx+c)^5+240\,ab\cosh(dx+c)\sinh(dx+c)^4+15\left(7\,b^2\cosh(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/1920*(15*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 48*a*b*cosh(d*x + c)^5 + 240
*a*b*cosh(d*x + c)*sinh(d*x + c)^4 + 15*(7*b^2*cosh(d*x + c)^3 - 8*b^2*cosh
(d*x + c))*sinh(d*x + c)^5 - 400*a*b*cosh(d*x + c)^3 + 5*(21*b^2*cosh(d*x +
 c)^5 - 80*b^2*cosh(d*x + c)^3 + 84*b^2*cosh(d*x + c))*sinh(d*x + c)^3 - 15
*(64*a^2 - 35*b^2)*d*x + 2400*a*b*cosh(d*x + c) + 240*(2*a*b*cosh(d*x + c)^
3 - 5*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + 15*(b^2*cosh(d*x + c)^7 - 8*b^2*
cosh(d*x + c)^5 + 28*b^2*cosh(d*x + c)^3 + 8*(8*a^2 - 7*b^2)*cosh(d*x + c))
*sinh(d*x + c))/d
```

giac [A] time = 0.17, size = 260, normalized size = 1.44

$$-\frac{1}{128}\left(64\,a^2-35\,b^2\right)x+\frac{b^2e^{(8\,dx+8\,c)}}{2048\,d}-\frac{b^2e^{(6\,dx+6\,c)}}{192\,d}+\frac{abe^{(5\,dx+5\,c)}}{80\,d}+\frac{7\,b^2e^{(4\,dx+4\,c)}}{256\,d}-\frac{5\,abe^{(3\,dx+3\,c)}}{48\,d}+\frac{5\,abe^{(dx+c)}}{8\,d}+\frac{5\,abe^{(-dx-}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/128*(64*a^2 - 35*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d - 1/192*b^2*e^(6*
d*x + 6*c)/d + 1/80*a*b*e^(5*d*x + 5*c)/d + 7/256*b^2*e^(4*d*x + 4*c)/d - 5
```

```
/48*a*b*e^(3*d*x + 3*c)/d + 5/8*a*b*e^(d*x + c)/d + 5/8*a*b*e^(-d*x - c)/d
- 5/48*a*b*e^(-3*d*x - 3*c)/d - 7/256*b^2*e^(-4*d*x - 4*c)/d + 1/80*a*b*e^(
-5*d*x - 5*c)/d + 1/192*b^2*e^(-6*d*x - 6*c)/d - 1/2048*b^2*e^(-8*d*x - 8*c
)/d + 1/64*(8*a^2 - 7*b^2)*e^(2*d*x + 2*c)/d - 1/64*(8*a^2 - 7*b^2)*e^(-2*d
*x - 2*c)/d
```

maple [A] time = 0.05, size = 122, normalized size = 0.68

$$\frac{b^2\left(\left(\frac{\left(\sinh^7(dx+c)\right)}{8}-\frac{7\left(\sinh^5(dx+c)\right)}{48}+\frac{35\left(\sinh^3(dx+c)\right)}{192}-\frac{35\sinh(dx+c)}{128}\right)\cosh(dx+c)+\frac{35dx}{128}+\frac{35c}{128}\right)+2ab\left(\frac{8}{15}+\frac{\left(\sinh^4(dx\right.}{5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128
*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+2*a*b*(8/15+1/5*sinh(d*x+c)^
4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-
1/2*c))
```

maxima [A] time = 0.37, size = 237, normalized size = 1.32

$$-\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{6144}b^2\left(\frac{\left(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c)}-3\right)e^{(8dx+8c)}}{d}-\frac{16}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/6144*b^2*((32*e
^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x
 + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c)
 + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5
*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*
e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

mupad [B] time = 1.63, size = 126, normalized size = 0.70

$$\frac{480\,a^2\sinh(2\,c+2\,d\,x)-420\,b^2\sinh(2\,c+2\,d\,x)+105\,b^2\sinh(4\,c+4\,d\,x)-20\,b^2\sinh(6\,c+6\,d\,x)+\frac{15\,b^2\sin}{}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3)^2,x)
```

```
[Out] (480*a^2*sinh(2*c + 2*d*x) - 420*b^2*sinh(2*c + 2*d*x) + 105*b^2*sinh(4*c +
 4*d*x) - 20*b^2*sinh(6*c + 6*d*x) + (15*b^2*sinh(8*c + 8*d*x))/8 + 2400*a*
b*cosh(c + d*x) - 400*a*b*cosh(3*c + 3*d*x) + 48*a*b*cosh(5*c + 5*d*x) - 96
0*a^2*d*x + 525*b^2*d*x)/(1920*d)
```

sympy [A] time = 8.65, size = 340, normalized size = 1.89

$$\begin{cases}\frac{a^2x\sinh^2(c+dx)}{2}-\frac{a^2x\cosh^2(c+dx)}{2}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{2ab\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{8ab\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{16a}{} \\ x\left(a+b\sinh^3(c)\right)^2\sinh^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 8
*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15*
d) + 35*b**2*x*sinh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d
*x)**2/32 + 105*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sin
h(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**
2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c
 + d*x)**3/(384*d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 3
5*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)
**3)**2*sinh(c)**2, True))
```

3.152 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx$

Optimal. Leaf size=130

$$\frac{a^2\cosh(c+dx)}{d}+\frac{ab\sinh^3(c+dx)\cosh(c+dx)}{2d}-\frac{3ab\sinh(c+dx)\cosh(c+dx)}{4d}+\frac{3abx}{4}+\frac{b^2\cosh^7(c+dx)}{7d}-\frac{3b}{}$$

```
[Out] 3/4*a*b*x+a^2*cosh(d*x+c)/d-b^2*cosh(d*x+c)/d+b^2*cosh(d*x+c)^3/d-3/5*b^2*c
osh(d*x+c)^5/d+1/7*b^2*cosh(d*x+c)^7/d-3/4*a*b*cosh(d*x+c)*sinh(d*x+c)/d+1/
2*a*b*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Rubi [A] time = 0.12, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3220, 2638, 2635, 8, 2633}

$$\frac{a^2\cosh(c+dx)}{d}+\frac{ab\sinh^3(c+dx)\cosh(c+dx)}{2d}-\frac{3ab\sinh(c+dx)\cosh(c+dx)}{4d}+\frac{3abx}{4}+\frac{b^2\cosh^7(c+dx)}{7d}-\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (3*a*b*x)/4 + (a^2*Cosh[c + d*x])/d - (b^2*Cosh[c + d*x])/d + (b^2*Cosh[c +
 d*x]^3)/d - (3*b^2*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d) -
(3*a*b*Cosh[c + d*x]*Sinh[c + d*x])/(4*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x
]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = -\left(i\int\left(ia^2\sinh(c+dx)+2iab\sinh^4(c+dx)+ib^2\sinh^7(c+dx)\right)dx\right.$$
$$= a^2\int\sinh(c+dx)\,dx+(2ab)\int\sinh^4(c+dx)\,dx+b^2\int\sinh^7(c+$$
$$= \frac{a^2\cosh(c+dx)}{d}+\frac{ab\cosh(c+dx)\sinh^3(c+dx)}{2d}-\frac{1}{2}(3ab)\int\sinh^2($$
$$= \frac{a^2\cosh(c+dx)}{d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{d}-\frac{3b^2\cosh^5(}{5d}$$
$$= \frac{3abx}{4}+\frac{a^2\cosh(c+dx)}{d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{d}-\frac{3b^2}{}$$

Mathematica [A] time = 0.42, size = 92, normalized size = 0.71

$$\frac{35\left(64a^2-35b^2\right)\cosh(c+dx)+b(140a(12(c+dx)-8\sinh(2(c+dx))+\sinh(4(c+dx)))+245b\cosh(3(c+dx)}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (35*(64*a^2 - 35*b^2)*Cosh[c + d*x] + b*(245*b*Cosh[3*(c + d*x)] - 49*b*Cos
h[5*(c + d*x)] + 5*b*Cosh[7*(c + d*x)] + 140*a*(12*(c + d*x) - 8*Sinh[2*(c
+ d*x)] + Sinh[4*(c + d*x)])))/(2240*d)
```

fricas [A] time = 0.47, size = 220, normalized size = 1.69

$$5\,b^2\cosh(dx+c)^7+35\,b^2\cosh(dx+c)\sinh(dx+c)^6-49\,b^2\cosh(dx+c)^5+560\,ab\cosh(dx+c)\sinh(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b^2*cosh(d*x + c)^7 + 35*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 49*b
^2*cosh(d*x + c)^5 + 560*a*b*cosh(d*x + c)*sinh(d*x + c)^3 + 245*b^2*cosh(d
*x + c)^3 + 35*(5*b^2*cosh(d*x + c)^3 - 7*b^2*cosh(d*x + c))*sinh(d*x + c)^
4 + 1680*a*b*d*x + 35*(3*b^2*cosh(d*x + c)^5 - 14*b^2*cosh(d*x + c)^3 + 21*
b^2*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(64*a^2 - 35*b^2)*cosh(d*x + c) + 5
60*(a*b*cosh(d*x + c)^3 - 4*a*b*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.20, size = 219, normalized size = 1.68

$$\frac{3}{4}abx+\frac{b^2e^{(7dx+7c)}}{896d}-\frac{7b^2e^{(5dx+5c)}}{640d}+\frac{abe^{(4dx+4c)}}{32d}+\frac{7b^2e^{(3dx+3c)}}{128d}-\frac{abe^{(2dx+2c)}}{4d}+\frac{abe^{(-2dx-2c)}}{4d}+\frac{7b^2e^{(-3dx-3c)}}{128d}-\frac{abe^{(-4dx}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 3/4*a*b*x + 1/896*b^2*e^(7*d*x + 7*c)/d - 7/640*b^2*e^(5*d*x + 5*c)/d + 1/3
2*a*b*e^(4*d*x + 4*c)/d + 7/128*b^2*e^(3*d*x + 3*c)/d - 1/4*a*b*e^(2*d*x +
2*c)/d + 1/4*a*b*e^(-2*d*x - 2*c)/d + 7/128*b^2*e^(-3*d*x - 3*c)/d - 1/32*a
*b*e^(-4*d*x - 4*c)/d - 7/640*b^2*e^(-5*d*x - 5*c)/d + 1/896*b^2*e^(-7*d*x
- 7*c)/d + 1/128*(64*a^2 - 35*b^2)*e^(d*x + c)/d + 1/128*(64*a^2 - 35*b^2)*
e^(-d*x - c)/d
```

maple [A] time = 0.04, size = 96, normalized size = 0.74

$$\frac{b^2\left(-\frac{16}{35}+\frac{\left(\sinh^6(dx+c)\right)}{7}-\frac{6\left(\sinh^4(dx+c)\right)}{35}+\frac{8\left(\sinh^2(dx+c)\right)}{35}\right)\cosh(dx+c)+2ab\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*c
osh(d*x+c)+2*a*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3
/8*c)+a^2*cosh(d*x+c))
```

maxima [A] time = 0.33, size = 180, normalized size = 1.38

$$\frac{1}{32}ab\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) - \frac{1}{4480}b^2\left(\frac{\left(49e^{(-2dx-2c)} - 245e^{(-4dx-4c)} + 12\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d
*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x -
c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) +
a^2*cosh(d*x + c)/d
```

mupad [B] time = 0.25, size = 104, normalized size = 0.80

$$\frac{a^2\cosh(c+dx) + \frac{\sinh(c+dx)\,a\,b\cosh(c+dx)^3}{2} - \frac{5\sinh(c+dx)\,a\,b\cosh(c+dx)}{4} + \frac{3\,d\,x\,a\,b}{4} + \frac{b^2\cosh(c+dx)^7}{7} - \frac{3\,b^2\cosh(c+dx)^5}{5} + b^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^3)^2,x)
```

```
[Out] (a^2*cosh(c + d*x) - b^2*cosh(c + d*x) + b^2*cosh(c + d*x)^3 - (3*b^2*cosh(
c + d*x)^5)/5 + (b^2*cosh(c + d*x)^7)/7 + (a*b*cosh(c + d*x)^3*sinh(c + d*x
))/2 - (5*a*b*cosh(c + d*x)*sinh(c + d*x))/4 + (3*a*b*d*x)/4)/d
```

sympy [A] time = 5.16, size = 219, normalized size = 1.68

$$\begin{cases} \frac{a^2\cosh(c+dx)}{d} + \frac{3abx\sinh^4(c+dx)}{4} - \frac{3abx\sinh^2(c+dx)\cosh^2(c+dx)}{2} + \frac{3abx\cosh^4(c+dx)}{4} + \frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{4d} - \frac{3ab\sinh(c+ \\ x\left(a+b\sinh^3(c)\right)^2\sinh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*cosh(c + d*x)/d + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh
(c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c
 + d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d)
 + b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**2*sinh(c + d*x)**4*cosh(c +
 d*x)**3/d + 8*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**2*cosh(
c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c), True))
```

3.153 $\int \left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=114

$$a^2x+\frac{2ab\cosh^3(c+dx)}{3d}-\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b^2\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b^2}{}$$

```
[Out] a^2*x-5/16*b^2*x-2*a*b*cosh(d*x+c)/d+2/3*a*b*cosh(d*x+c)^3/d+5/16*b^2*cosh(
d*x+c)*sinh(d*x+c)/d-5/24*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b^2*cosh(d*x+
c)*sinh(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3213, 2633, 2635, 8}

$$a^2x+\frac{2ab\cosh^3(c+dx)}{3d}-\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b^2\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] a^2*x - (5*b^2*x)/16 - (2*a*b*Cosh[c + d*x])/d + (2*a*b*Cosh[c + d*x]^3)/(3
*d) + (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b^2*Cosh[c + d*x]*Sin
h[c + d*x]^3)/(24*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \left(a+b\sinh^3(c+dx)\right)^2\,dx = \int \left(a^2+2ab\sinh^3(c+dx)+b^2\sinh^6(c+dx)\right)\,dx$$
$$= a^2x+(2ab)\int \sinh^3(c+dx)\,dx+b^2\int \sinh^6(c+dx)\,dx$$
$$= a^2x+\frac{b^2\cosh(c+dx)\sinh^5(c+dx)}{6d}-\frac{1}{6}\left(5b^2\right)\int \sinh^4(c+dx)\,dx-\frac{(2ab)\,\text{Sub}}{}$$
$$= a^2x-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d}-\frac{5b^2\cosh(c+dx)\sinh^3(c+dx)}{24d}$$
$$= a^2x-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d}+\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{16d}$$
$$= a^2x-\frac{5b^2x}{16}-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d}+\frac{5b^2\cosh(c+dx)\sinh(}{16d}$$

Mathematica [A] time = 0.15, size = 94, normalized size = 0.82

$$\frac{192a^2c+192a^2dx-288ab\cosh(c+dx)+32ab\cosh(3(c+dx))+45b^2\sinh(2(c+dx))-9b^2\sinh(4(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (192*a^2*c - 60*b^2*c + 192*a^2*d*x - 60*b^2*d*x - 288*a*b*Cosh[c + d*x] +
32*a*b*Cosh[3*(c + d*x)] + 45*b^2*Sinh[2*(c + d*x)] - 9*b^2*Sinh[4*(c + d*x
)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.47, size = 160, normalized size = 1.40

$$\frac{3\,b^2\cosh(dx+c)\sinh(dx+c)^5+16\,ab\cosh(dx+c)^3+48\,ab\cosh(dx+c)\sinh(dx+c)^2+2\left(5\,b^2\cosh(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 16*a*b*cosh(d*x + c)^3 + 48*a*b
*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(5*b^2*cosh(d*x + c)^3 - 9*b^2*cosh(d*x
+ c))*sinh(d*x + c)^3 + 6*(16*a^2 - 5*b^2)*d*x - 144*a*b*cosh(d*x + c) + 3*
(b^2*cosh(d*x + c)^5 - 6*b^2*cosh(d*x + c)^3 + 15*b^2*cosh(d*x + c))*sinh(d
*x + c))/d
```

giac [A] time = 0.13, size = 178, normalized size = 1.56

$$\frac{1}{16}\left(16\,a^2-5\,b^2\right)x+\frac{b^2e^{(6\,dx+6\,c)}}{384\,d}-\frac{3\,b^2e^{(4\,dx+4\,c)}}{128\,d}+\frac{abe^{(3\,dx+3\,c)}}{12\,d}+\frac{15\,b^2e^{(2\,dx+2\,c)}}{128\,d}-\frac{3\,abe^{(dx+c)}}{4\,d}-\frac{3\,abe^{(-dx-c)}}{4\,d}-\frac{15\,b^2e^{(-2}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/16*(16*a^2 - 5*b^2)*x + 1/384*b^2*e^(6*d*x + 6*c)/d - 3/128*b^2*e^(4*d*x
+ 4*c)/d + 1/12*a*b*e^(3*d*x + 3*c)/d + 15/128*b^2*e^(2*d*x + 2*c)/d - 3/4*
a*b*e^(d*x + c)/d - 3/4*a*b*e^(-d*x - c)/d - 15/128*b^2*e^(-2*d*x - 2*c)/d
+ 1/12*a*b*e^(-3*d*x - 3*c)/d + 3/128*b^2*e^(-4*d*x - 4*c)/d - 1/384*b^2*e^
(-6*d*x - 6*c)/d
```

maple [A] time = 0.04, size = 85, normalized size = 0.75

$$\frac{b^2\left(\left(\frac{\left(\sinh^5(dx+c)\right)}{6}-\frac{5\left(\sinh^3(dx+c)\right)}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)+2ab\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+
c)-5/16*d*x-5/16*c)+2*a*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*(d*x+c))
```

maxima [A] time = 0.31, size = 151, normalized size = 1.32

$$a^2x - \frac{1}{384}b^2\left(\frac{\left(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1\right)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e^{(-6dx-6c)}}{d}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 1/384*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x
+ 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c))/d) + 1/12*a*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-
d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 0.46, size = 85, normalized size = 0.75

$$\frac{\frac{45b^2\sinh(2c+2dx)}{4} - \frac{9b^2\sinh(4c+4dx)}{4} + \frac{b^2\sinh(6c+6dx)}{4} - 72ab\cosh(c+dx) + 8ab\cosh(3c+3dx) + 48a^2dx - 15}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2,x)
```

```
[Out] ((45*b^2*sinh(2*c + 2*d*x))/4 - (9*b^2*sinh(4*c + 4*d*x))/4 + (b^2*sinh(6*c
 + 6*d*x))/4 - 72*a*b*cosh(c + d*x) + 8*a*b*cosh(3*c + 3*d*x) + 48*a^2*d*x
- 15*b^2*d*x)/(48*d)
```

sympy [A] time = 3.04, size = 212, normalized size = 1.86

$$\begin{cases} a^2x + \frac{2ab\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{4ab\cosh^3(c+dx)}{3d} + \frac{5b^2x\sinh^6(c+dx)}{16} - \frac{15b^2x\sinh^4(c+dx)\cosh^2(c+dx)}{16} + \frac{15b^2x\sinh^2(c+dx)\cosh^4(}{16} \\ x\left(a+b\sinh^3(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 4*a*b*cosh(c +
 d*x)**3/(3*d) + 5*b**2*x*sinh(c + d*x)**6/16 - 15*b**2*x*sinh(c + d*x)**4*
cosh(c + d*x)**2/16 + 15*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*
*2*x*cosh(c + d*x)**6/16 + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) -
5*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b**2*sinh(c + d*x)*cosh(
c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2, True))
```

3.154 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=88

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{ab\sinh(c+dx)\cosh(c+dx)}{d}-abx+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{2b^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh$$

```
[Out] -a*b*x-a^2*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-2/3*b^2*cosh(d*x+c)^3/d
+1/5*b^2*cosh(d*x+c)^5/d+a*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3220, 3770, 2635, 8, 2633}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{ab\sinh(c+dx)\cosh(c+dx)}{d}-abx+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{2b^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] -(a*b*x) - (a^2*ArcTanh[Cosh[c + d*x]])/d + (b^2*Cosh[c + d*x])/d - (2*b^2*
Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) + (a*b*Cosh[c + d*x]*S
inh[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = i\int\left(-ia^2\operatorname{csch}(c+dx)-2iab\sinh^2(c+dx)-ib^2\sinh^5(c+dx)\right)dx$$
$$= a^2\int \operatorname{csch}(c+dx)\,dx + (2ab)\int \sinh^2(c+dx)\,dx + b^2\int \sinh^5(c+d$$
$$= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d} + \frac{ab\cosh(c+dx)\sinh(c+dx)}{d} - (ab)\int$$
$$= -abx - \frac{a^2\tanh^{-1}(\cosh(c+dx))}{d} + \frac{b^2\cosh(c+dx)}{d} - \frac{2b^2\cosh^3(c+}{3d}$$

Mathematica [A] time = 0.31, size = 96, normalized size = 1.09

$$\frac{120a\left(b\sinh(2(c+dx))-2\left(-a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+bc+bdx\right)\right)+150b^2\cosh(c+}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (150*b^2*Cosh[c + d*x] - 25*b^2*Cosh[3*(c + d*x)] + 3*b^2*Cosh[5*(c + d*x)]
 + 120*a*(-2*(b*c + b*d*x + a*Log[Cosh[(c + d*x)/2]] - a*Log[Sinh[(c + d*x)
/2]]) + b*Sinh[2*(c + d*x)]))/(240*d)
```

fricas [B] time = 0.49, size = 1052, normalized size = 11.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^2*cosh(d*x + c)^10 + 30*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^
2*sinh(d*x + c)^10 - 25*b^2*cosh(d*x + c)^8 - 480*a*b*d*x*cosh(d*x + c)^5 +
 120*a*b*cosh(d*x + c)^7 + 5*(27*b^2*cosh(d*x + c)^2 - 5*b^2)*sinh(d*x + c)
^8 + 150*b^2*cosh(d*x + c)^6 + 40*(9*b^2*cosh(d*x + c)^3 - 5*b^2*cosh(d*x +
 c) + 3*a*b)*sinh(d*x + c)^7 + 10*(63*b^2*cosh(d*x + c)^4 - 70*b^2*cosh(d*x
 + c)^2 + 84*a*b*cosh(d*x + c) + 15*b^2)*sinh(d*x + c)^6 + 150*b^2*cosh(d*x
 + c)^4 + 4*(189*b^2*cosh(d*x + c)^5 - 350*b^2*cosh(d*x + c)^3 - 120*a*b*d*
x + 630*a*b*cosh(d*x + c)^2 + 225*b^2*cosh(d*x + c))*sinh(d*x + c)^5 - 120*
a*b*cosh(d*x + c)^3 + 10*(63*b^2*cosh(d*x + c)^6 - 175*b^2*cosh(d*x + c)^4
- 240*a*b*d*x*cosh(d*x + c) + 420*a*b*cosh(d*x + c)^3 + 225*b^2*cosh(d*x +
c)^2 + 15*b^2)*sinh(d*x + c)^4 - 25*b^2*cosh(d*x + c)^2 + 40*(9*b^2*cosh(d*
x + c)^7 - 35*b^2*cosh(d*x + c)^5 - 120*a*b*d*x*cosh(d*x + c)^2 + 105*a*b*c
osh(d*x + c)^4 + 75*b^2*cosh(d*x + c)^3 + 15*b^2*cosh(d*x + c) - 3*a*b)*sin
h(d*x + c)^3 + 5*(27*b^2*cosh(d*x + c)^8 - 140*b^2*cosh(d*x + c)^6 - 960*a*
b*d*x*cosh(d*x + c)^3 + 504*a*b*cosh(d*x + c)^5 + 450*b^2*cosh(d*x + c)^4 +
 180*b^2*cosh(d*x + c)^2 - 72*a*b*cosh(d*x + c) - 5*b^2)*sinh(d*x + c)^2 +
3*b^2 - 480*(a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^4*sinh(d*x + c) + 10
*a^2*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^2*cosh(d*x + c)^2*sinh(d*x + c)
^3 + 5*a^2*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*sinh(d*x + c)^5)*log(cosh(d*
x + c) + sinh(d*x + c) + 1) + 480*(a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c
)^4*sinh(d*x + c) + 10*a^2*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^2*cosh(d*
x + c)^2*sinh(d*x + c)^3 + 5*a^2*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*sinh(d
*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(3*b^2*cosh(d*x + c)
^9 - 20*b^2*cosh(d*x + c)^7 - 240*a*b*d*x*cosh(d*x + c)^4 + 84*a*b*cosh(d*x
 + c)^6 + 90*b^2*cosh(d*x + c)^5 + 60*b^2*cosh(d*x + c)^3 - 36*a*b*cosh(d*x
 + c)^2 - 5*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh
(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*d*cos
```

```
h(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d
*x + c)^5)
```

giac [A] time = 0.21, size = 154, normalized size = 1.75

$$-\frac{480\,(dx+c)ab - 3\,b^2 e^{(5\,dx+5\,c)} + 25\,b^2 e^{(3\,dx+3\,c)} - 120\,abe^{(2\,dx+2\,c)} - 150\,b^2 e^{(dx+c)} + 480\,a^2 \log\left(e^{(dx+c)}+1\right) - 48}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/480*(480*(d*x + c)*a*b - 3*b^2*e^(5*d*x + 5*c) + 25*b^2*e^(3*d*x + 3*c)
- 120*a*b*e^(2*d*x + 2*c) - 150*b^2*e^(d*x + c) + 480*a^2*log(e^(d*x + c) +
 1) - 480*a^2*log(abs(e^(d*x + c) - 1)) - (150*b^2*e^(4*d*x + 4*c) - 120*a*
b*e^(3*d*x + 3*c) - 25*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-5*d*x - 5*c))/d
```

maple [A] time = 0.12, size = 76, normalized size = 0.86

$$\frac{-2a^2\operatorname{arctanh}\left(e^{dx+c}\right) + 2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + b^2\left(\frac{8}{15} + \frac{\left(\sinh^4(dx+c)\right)}{5} - \frac{4\left(\sinh^2(dx+c)\right)}{15}\right)\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-
1/2*c)+b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [A] time = 0.31, size = 140, normalized size = 1.59

$$-\frac{1}{4}\,ab\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + \frac{1}{480}\,b^2\left(\frac{3\,e^{(5\,dx+5\,c)}}{d} - \frac{25\,e^{(3\,dx+3\,c)}}{d} + \frac{150\,e^{(dx+c)}}{d} + \frac{150\,e^{(-dx-c)}}{d} - \frac{25\,e^{(-3\,dx-}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b^2*(3*e^(5
*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)
/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^2*log(tanh(1/2*d*x +
 1/2*c))/d
```

mupad [B] time = 0.21, size = 177, normalized size = 2.01

$$\frac{5\,b^2\,e^{c+d\,x}}{16\,d} - \frac{2\,\mathrm{atan}\left(\frac{a^2\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}} - a\,b\,x + \frac{5\,b^2\,e^{-c-d\,x}}{16\,d} - \frac{5\,b^2\,e^{-3\,c-3\,d\,x}}{96\,d} - \frac{5\,b^2\,e^{3\,c+3\,d\,x}}{96\,d} + \frac{b^2\,e^{-5\,c-5\,d\,x}}{160\,d} + \frac{b^2\,e^{5\,c+5}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x),x)
```

```
[Out] (5*b^2*exp(c + d*x))/(16*d) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d
*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^2)^(1/2) - a*b*x + (5*b^2*exp(- c - d*x))/(
16*d) - (5*b^2*exp(- 3*c - 3*d*x))/(96*d) - (5*b^2*exp(3*c + 3*d*x))/(96*d)
 + (b^2*exp(- 5*c - 5*d*x))/(160*d) + (b^2*exp(5*c + 5*d*x))/(160*d) - (a*b
*exp(- 2*c - 2*d*x))/(4*d) + (a*b*exp(2*c + 2*d*x))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.155 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=82

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3b^2x}{8}$$

```
[Out] 3/8*b^2*x+2*a*b*cosh(d*x+c)/d-a^2*coth(d*x+c)/d-3/8*b^2*cosh(d*x+c)*sinh(d*
x+c)/d+1/4*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 3767, 8, 2638, 2635}

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3b^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (3*b^2*x)/8 + (2*a*b*Cosh[c + d*x])/d - (a^2*Coth[c + d*x])/d - (3*b^2*Cosh
[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = -\int \left(-a^2\operatorname{csch}^2(c+dx)-2ab\sinh(c+dx)-b^2\sinh^4(c+dx)\right)dx$$
$$= a^2\int \operatorname{csch}^2(c+dx)\,dx+(2ab)\int \sinh(c+dx)\,dx+b^2\int \sinh^4(c+$$
$$= \frac{2ab\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}\left(3b^2\right)\int \sin$$
$$= \frac{2ab\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)}{d}-\frac{3b^2\cosh(c+dx)\sinh(c+dx}{8d}$$
$$= \frac{3b^2x}{8}+\frac{2ab\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)}{d}-\frac{3b^2\cosh(c+dx)\sin}{8d}$$

Mathematica [A] time = 0.28, size = 92, normalized size = 1.12

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\sinh(c)\sinh(dx)}{d}+\frac{2ab\cosh(c)\cosh(dx)}{d}+\frac{3b^2(c+dx)}{8d}-\frac{b^2\sinh(2(c+dx))}{4d}+\frac{b^2\sinh(4(c+}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (3*b^2*(c + d*x))/(8*d) + (2*a*b*Cosh[c]*Cosh[d*x])/d - (a^2*Coth[c + d*x])
/d + (2*a*b*Sinh[c]*Sinh[d*x])/d - (b^2*Sinh[2*(c + d*x)])/(4*d) + (b^2*Sin
h[4*(c + d*x)])/(32*d)
```

fricas [A] time = 0.45, size = 142, normalized size = 1.73

$$\frac{b^2\cosh(dx+c)^5+5\,b^2\cosh(dx+c)\sinh(dx+c)^4-9\,b^2\cosh(dx+c)^3+\left(10\,b^2\cosh(dx+c)^3-27\,b^2\cosh(d\right.}{64\,d\sinh(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(b^2*cosh(d*x + c)^5 + 5*b^2*cosh(d*x + c)*sinh(d*x + c)^4 - 9*b^2*cos
h(d*x + c)^3 + (10*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c))*sinh(d*x + c
)^2 - 8*(8*a^2 - b^2)*cosh(d*x + c) + 8*(3*b^2*d*x + 16*a*b*cosh(d*x + c) +
 8*a^2)*sinh(d*x + c))/(d*sinh(d*x + c))
```

giac [A] time = 0.20, size = 149, normalized size = 1.82

$$\frac{24\,(dx+c)b^2+b^2e^{(4\,dx+4\,c)}-8\,b^2e^{(2\,dx+2\,c)}+64\,abe^{(dx+c)}+\frac{\left(64\,abe^{(5\,dx+5\,c)}-64\,abe^{(3\,dx+3\,c)}-9\,b^2e^{(2\,dx+2\,c)}+b^2-8\left(16\,a^2-b^2\right)e^{(4\,dx+4\,c)}\right.}{\left(e^{(dx+c)}+1\right)\left(e^{(dx+c)}-1\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*b^2 + b^2*e^(4*d*x + 4*c) - 8*b^2*e^(2*d*x + 2*c) + 64*a
*b*e^(d*x + c) + (64*a*b*e^(5*d*x + 5*c) - 64*a*b*e^(3*d*x + 3*c) - 9*b^2*e
^(2*d*x + 2*c) + b^2 - 8*(16*a^2 - b^2)*e^(4*d*x + 4*c))*e^(-4*d*x - 4*c)/(
(e^(d*x + c) + 1)*(e^(d*x + c) - 1)))/d
```

maple [A] time = 0.10, size = 65, normalized size = 0.79

$$\frac{-a^2\coth(dx+c)+2ab\cosh(dx+c)+b^2\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*cosh(d*x+c)+b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*
x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.33, size = 113, normalized size = 1.38

$$\frac{1}{64}\,b^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+ab\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + a*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a^2/(
d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.74, size = 123, normalized size = 1.50

$$\frac{3\,b^2\,x}{8}-\frac{2\,a^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}-\frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}-\frac{b^2\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d}+\frac{b^2\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d}+\frac{a\,b\,\mathrm{e}^{c+d\,x}}{d}+\frac{a\,b\,\mathrm{e}^{-c-d\,x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^2,x)
```

```
[Out] (3*b^2*x)/8 - (2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) + (b^2*exp(- 2*c - 2*d*x))
/(8*d) - (b^2*exp(2*c + 2*d*x))/(8*d) - (b^2*exp(- 4*c - 4*d*x))/(64*d) + (
b^2*exp(4*c + 4*d*x))/(64*d) + (a*b*exp(c + d*x))/d + (a*b*exp(- c - d*x))/
d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.156 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=77

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+2abx+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{b^2\cosh(c+dx)}{d}$$

```
[Out] 2*a*b*x+1/2*a^2*arctanh(cosh(d*x+c))/d-b^2*cosh(d*x+c)/d+1/3*b^2*cosh(d*x+c
)^3/d-1/2*a^2*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 3768, 3770, 2633}

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+2abx+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] 2*a*b*x + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^2*Cosh[c + d*x])/d + (b^2
*Cosh[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx &= -\left(i\int \left(2iab+ia^2\operatorname{csch}^3(c+dx)+ib^2\sinh^3(c+dx)\right)dx\right)\\
&= 2abx+a^2\int \operatorname{csch}^3(c+dx)\,dx+b^2\int \sinh^3(c+dx)\,dx\\
&= 2abx-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{1}{2}a^2\int \operatorname{csch}(c+dx)\,dx-\frac{b^2 S}{}\\
&= 2abx+\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+}{3d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 105, normalized size = 1.36

$$-\frac{a^2\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^2\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+2abx-\frac{3b^2\cosh(c+dx)}{4d}+\frac{b^2\cosh(3(c \ldots}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] 2*a*b*x - (3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c + d*x)])/(12*d) - (a
^2*Csch[(c + d*x)/2]^2)/(8*d) - (a^2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*S
ech[(c + d*x)/2]^2)/(8*d)
```

fricas [B] time = 0.68, size = 1616, normalized size = 20.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^10 + 10*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + b^2*sin
h(d*x + c)^10 + 48*a*b*d*x*cosh(d*x + c)^7 - 11*b^2*cosh(d*x + c)^8 - 96*a*
b*d*x*cosh(d*x + c)^5 + (45*b^2*cosh(d*x + c)^2 - 11*b^2)*sinh(d*x + c)^8 +
 8*(15*b^2*cosh(d*x + c)^3 + 6*a*b*d*x - 11*b^2*cosh(d*x + c))*sinh(d*x + c
)^7 + 48*a*b*d*x*cosh(d*x + c)^3 - 2*(12*a^2 - 5*b^2)*cosh(d*x + c)^6 + 2*(
105*b^2*cosh(d*x + c)^4 + 168*a*b*d*x*cosh(d*x + c) - 154*b^2*cosh(d*x + c)
^2 - 12*a^2 + 5*b^2)*sinh(d*x + c)^6 + 4*(63*b^2*cosh(d*x + c)^5 + 252*a*b*
d*x*cosh(d*x + c)^2 - 154*b^2*cosh(d*x + c)^3 - 24*a*b*d*x - 3*(12*a^2 - 5*
b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(12*a^2 - 5*b^2)*cosh(d*x + c)^4 +
2*(105*b^2*cosh(d*x + c)^6 + 840*a*b*d*x*cosh(d*x + c)^3 - 385*b^2*cosh(d*x
 + c)^4 - 240*a*b*d*x*cosh(d*x + c) - 15*(12*a^2 - 5*b^2)*cosh(d*x + c)^2 -
 12*a^2 + 5*b^2)*sinh(d*x + c)^4 - 11*b^2*cosh(d*x + c)^2 + 8*(15*b^2*cosh(
d*x + c)^7 + 210*a*b*d*x*cosh(d*x + c)^4 - 77*b^2*cosh(d*x + c)^5 - 120*a*b
*d*x*cosh(d*x + c)^2 + 6*a*b*d*x - 5*(12*a^2 - 5*b^2)*cosh(d*x + c)^3 - (12
*a^2 - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (45*b^2*cosh(d*x + c)^8 + 10
08*a*b*d*x*cosh(d*x + c)^5 - 308*b^2*cosh(d*x + c)^6 - 960*a*b*d*x*cosh(d*x
 + c)^3 + 144*a*b*d*x*cosh(d*x + c) - 30*(12*a^2 - 5*b^2)*cosh(d*x + c)^4 -
 12*(12*a^2 - 5*b^2)*cosh(d*x + c)^2 - 11*b^2)*sinh(d*x + c)^2 + b^2 + 12*(
a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6 + a^2*sinh(d*x +
c)^7 - 2*a^2*cosh(d*x + c)^5 + (21*a^2*cosh(d*x + c)^2 - 2*a^2)*sinh(d*x +
c)^5 + a^2*cosh(d*x + c)^3 + 5*(7*a^2*cosh(d*x + c)^3 - 2*a^2*cosh(d*x + c)
)*sinh(d*x + c)^4 + (35*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(d*x + c)^2 + a^2)
*sinh(d*x + c)^3 + (21*a^2*cosh(d*x + c)^5 - 20*a^2*cosh(d*x + c)^3 + 3*a^2
*cosh(d*x + c))*sinh(d*x + c)^2 + (7*a^2*cosh(d*x + c)^6 - 10*a^2*cosh(d*x
+ c)^4 + 3*a^2*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) + 1) - 12*(a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6
+ a^2*sinh(d*x + c)^7 - 2*a^2*cosh(d*x + c)^5 + (21*a^2*cosh(d*x + c)^2 - 2
*a^2)*sinh(d*x + c)^5 + a^2*cosh(d*x + c)^3 + 5*(7*a^2*cosh(d*x + c)^3 - 2*
a^2*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(
d*x + c)^2 + a^2)*sinh(d*x + c)^3 + (21*a^2*cosh(d*x + c)^5 - 20*a^2*cosh(d
*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^2 + (7*a^2*cosh(d*x + c)^6 -
 10*a^2*cosh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*
x + c) + sinh(d*x + c) - 1) + 2*(5*b^2*cosh(d*x + c)^9 + 168*a*b*d*x*cosh(d
*x + c)^6 - 44*b^2*cosh(d*x + c)^7 - 240*a*b*d*x*cosh(d*x + c)^4 + 72*a*b*d
*x*cosh(d*x + c)^2 - 6*(12*a^2 - 5*b^2)*cosh(d*x + c)^5 - 4*(12*a^2 - 5*b^2
)*cosh(d*x + c)^3 - 11*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7
 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 - 2*d*cosh(d*x + c
)^5 + (21*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3
 - 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x
+ c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5
```

```
- 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d*cosh(d*x
 + c)^6 - 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

giac [B] time = 0.20, size = 162, normalized size = 2.10

$$\frac{48\,(dx+c)ab + b^2e^{(3\,dx+3\,c)} - 9\,b^2e^{(dx+c)} + 12\,a^2\log\left(e^{(dx+c)}+1\right) - 12\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{\left(11\,b^2e^{(2\,dx+2\,c)}-b^2+3\left(8\,a^2+\right.\right.}{\left(e^{\cdot}\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/24*(48*(d*x + c)*a*b + b^2*e^(3*d*x + 3*c) - 9*b^2*e^(d*x + c) + 12*a^2*l
og(e^(d*x + c) + 1) - 12*a^2*log(abs(e^(d*x + c) - 1)) - (11*b^2*e^(2*d*x +
 2*c) - b^2 + 3*(8*a^2 + 3*b^2)*e^(6*d*x + 6*c) + (24*a^2 - 19*b^2)*e^(4*d*
x + 4*c))*e^(-3*d*x - 3*c)/((e^(d*x + c) + 1)^2*(e^(d*x + c) - 1)^2))/d
```

maple [A] time = 0.13, size = 63, normalized size = 0.82

$$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\,(dx+c)+b^2\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh\,(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(d*x+c)+b
^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.33, size = 152, normalized size = 1.97

$$2\,abx+\frac{1}{24}\,b^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(\right.}{d\left(2\,e^{(-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*x + 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d
+ e^(-3*d*x - 3*c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c)
 - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-
4*d*x - 4*c) - 1)))
```

mupad [B] time = 0.71, size = 175, normalized size = 2.27

$$\frac{\operatorname{atan}\left(\frac{a^2\,e^{d\,x}\,e^c\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}-\frac{3\,b^2\,e^{c+d\,x}}{8\,d}+2\,a\,b\,x-\frac{3\,b^2\,e^{-c-d\,x}}{8\,d}+\frac{b^2\,e^{-3\,c-3\,d\,x}}{24\,d}+\frac{b^2\,e^{3\,c+3\,d\,x}}{24\,d}-\frac{a^2\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2}{d\,\left(e^{4\,c+4\,d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^3,x)
```

```
[Out] (atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^
2)^(1/2) - (3*b^2*exp(c + d*x))/(8*d) + 2*a*b*x - (3*b^2*exp(- c - d*x))/(8
*d) + (b^2*exp(- 3*c - 3*d*x))/(24*d) + (b^2*exp(3*c + 3*d*x))/(24*d) - (a^
2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^2*exp(c + d*x))/(d*(exp(4
*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.157 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=76

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

```
[Out] -1/2*b^2*x-2*a*b*arctanh(cosh(d*x+c))/d+a^2*coth(d*x+c)/d-1/3*a^2*coth(d*x+
c)^3/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 3770, 3767, 2635, 8}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] -(b^2*x)/2 - (2*a*b*ArcTanh[Cosh[c + d*x]])/d + (a^2*Coth[c + d*x])/d - (a^
2*Coth[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = \int \left(2ab\operatorname{csch}(c+dx)+a^2\operatorname{csch}^4(c+dx)+b^2\sinh^2(c+dx)\right) dx$$
$$= a^2\int \operatorname{csch}^4(c+dx)\,dx+(2ab)\int \operatorname{csch}(c+dx)\,dx+b^2\int \sinh^2(c$$
$$= -\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{1}{2}$$
$$= -\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3}{3}$$

Mathematica [A] time = 0.41, size = 81, normalized size = 1.07

$$\frac{3b\left(b\sinh(2(c+dx))-2\left(-4a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+4a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+bc+bdx\right)\right)-4a^2\coth(c}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-4*a^2*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + 3*b*(-2*(b*c + b*d*x + 4*a*L
og[Cosh[(c + d*x)/2]] - 4*a*Log[Sinh[(c + d*x)/2]]) + b*Sinh[2*(c + d*x)]))
/(12*d)
```

fricas [B] time = 0.57, size = 1748, normalized size = 23.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^2*cosh(d*x + c)^10 + 30*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^2
*sinh(d*x + c)^10 - 3*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^8 - 3*(4*b^2*d*x -
45*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^8 + 24*(15*b^2*cosh(d*x + c)^
3 - (4*b^2*d*x + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 6*(6*b^2*d*x + b^2
)*cosh(d*x + c)^6 + 6*(105*b^2*cosh(d*x + c)^4 + 6*b^2*d*x - 14*(4*b^2*d*x
+ 3*b^2)*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 12*(63*b^2*cosh(d*x + c)^
5 - 14*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^3 + 3*(6*b^2*d*x + b^2)*cosh(d*x +
 c))*sinh(d*x + c)^5 - 6*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c)^4 + 6*(10
5*b^2*cosh(d*x + c)^6 - 35*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^4 - 6*b^2*d*x
+ 15*(6*b^2*d*x + b^2)*cosh(d*x + c)^2 - 16*a^2 + b^2)*sinh(d*x + c)^4 + 24
*(15*b^2*cosh(d*x + c)^7 - 7*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^5 + 5*(6*b^2
*d*x + b^2)*cosh(d*x + c)^3 - (6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c))*sin
h(d*x + c)^3 + (12*b^2*d*x + 32*a^2 - 9*b^2)*cosh(d*x + c)^2 + (135*b^2*cos
h(d*x + c)^8 - 84*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^6 + 90*(6*b^2*d*x + b^2
)*cosh(d*x + c)^4 + 12*b^2*d*x - 36*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c
)^2 + 32*a^2 - 9*b^2)*sinh(d*x + c)^2 + 3*b^2 - 48*(a*b*cosh(d*x + c)^8 + 8
*a*b*cosh(d*x + c)*sinh(d*x + c)^7 + a*b*sinh(d*x + c)^8 - 3*a*b*cosh(d*x +
 c)^6 + (28*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^6 + 3*a*b*cosh(d*x +
 c)^4 + 2*(28*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*sinh(d*x + c)^5 +
(70*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2 + 3*a*b)*sinh(d*x + c)^4 -
 a*b*cosh(d*x + c)^2 + 4*(14*a*b*cosh(d*x + c)^5 - 15*a*b*cosh(d*x + c)^3 +
 3*a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (28*a*b*cosh(d*x + c)^6 - 45*a*b*co
sh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^2 + 2*(4*a*b*co
sh(d*x + c)^7 - 9*a*b*cosh(d*x + c)^5 + 6*a*b*cosh(d*x + c)^3 - a*b*cosh(d*
x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 48*(a*b*cos
h(d*x + c)^8 + 8*a*b*cosh(d*x + c)*sinh(d*x + c)^7 + a*b*sinh(d*x + c)^8 -
3*a*b*cosh(d*x + c)^6 + (28*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^6 +
```

```
3*a*b*cosh(d*x + c)^4 + 2*(28*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*si
nh(d*x + c)^5 + (70*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2 + 3*a*b)*s
inh(d*x + c)^4 - a*b*cosh(d*x + c)^2 + 4*(14*a*b*cosh(d*x + c)^5 - 15*a*b*c
osh(d*x + c)^3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (28*a*b*cosh(d*x +
c)^6 - 45*a*b*cosh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)
^2 + 2*(4*a*b*cosh(d*x + c)^7 - 9*a*b*cosh(d*x + c)^5 + 6*a*b*cosh(d*x + c)
^3 - a*b*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) -
1) + 2*(15*b^2*cosh(d*x + c)^9 - 12*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^7 + 1
8*(6*b^2*d*x + b^2)*cosh(d*x + c)^5 - 12*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*
x + c)^3 + (12*b^2*d*x + 32*a^2 - 9*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*c
osh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 3*
d*cosh(d*x + c)^6 + (28*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(28*d*
cosh(d*x + c)^3 - 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 3*d*cosh(d*x + c)^4
+ (70*d*cosh(d*x + c)^4 - 45*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 4*(
14*d*cosh(d*x + c)^5 - 15*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
 c)^3 - d*cosh(d*x + c)^2 + (28*d*cosh(d*x + c)^6 - 45*d*cosh(d*x + c)^4 +
18*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 2*(4*d*cosh(d*x + c)^7 - 9*d*co
sh(d*x + c)^5 + 6*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 0.20, size = 151, normalized size = 1.99

$$-\frac{12\,(dx+c)b^2 - 3\,b^2e^{(2\,dx+2\,c)} + 48\,ab\log\left(e^{(dx+c)}+1\right) - 48\,ab\log\left(\left|e^{(dx+c)}-1\right|\right) + \frac{\left(3\,b^2e^{(6\,dx+6\,c)} - 3\,b^2 + 3\left(32\,a^2-3\,b^2\right)e^{(4\,dx+\cdots}}{\left(e^{(dx+c)}+1\right)^3\left(e\cdots\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(d*x + c)*b^2 - 3*b^2*e^(2*d*x + 2*c) + 48*a*b*log(e^(d*x + c) +
1) - 48*a*b*log(abs(e^(d*x + c) - 1)) + (3*b^2*e^(6*d*x + 6*c) - 3*b^2 + 3*
(32*a^2 - 3*b^2)*e^(4*d*x + 4*c) - (32*a^2 - 9*b^2)*e^(2*d*x + 2*c))*e^(-2*
d*x - 2*c)/((e^(d*x + c) + 1)^3*(e^(d*x + c) - 1)^3))/d
```

maple [A] time = 0.14, size = 65, normalized size = 0.86

$$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c) - 4ab\operatorname{arctanh}\left(e^{dx+c}\right) + b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-4*a*b*arctanh(exp(d*x+c))+b^2*
(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [B] time = 0.33, size = 170, normalized size = 2.24

$$-\frac{1}{8}\,b^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)+\frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-\cdots}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 2*a*b*(log(e^(-d*
x - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*
e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 0.13, size = 163, normalized size = 2.14

$$\frac{b^2\,e^{2\,c+2\,d\,x}}{8\,d}-\frac{4\,a^2}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{4\,\mathrm{atan}\left(\frac{a\,b\,e^{d\,x}\,e^c\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}-\frac{b^2\,e^{-2\,c-2\,d\,x}}{8\,d}-\frac{b^2\,x}{2}-\frac{}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^4,x)
```

```
[Out] (b^2*exp(2*c + 2*d*x))/(8*d) - (4*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2
*d*x) + 1)) - (4*atan((a*b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)
))*(a^2*b^2)^(1/2))/(-d^2)^(1/2) - (b^2*exp(- 2*c - 2*d*x))/(8*d) - (b^2*x)
/2 - (8*a^2)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*
x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.158 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=90

$$-\frac{3a^2\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{2ab\coth(c+dx)}{d}+\frac{b^2}{}$$

```
[Out] -3/8*a^2*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-2*a*b*coth(d*x+c)/d+3/8*a
^2*coth(d*x+c)*csch(d*x+c)/d-1/4*a^2*coth(d*x+c)*csch(d*x+c)^3/d
```

Rubi [A] time = 0.13, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3767, 8, 3768, 3770, 2638}

$$-\frac{3a^2\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{2ab\coth(c+dx)}{d}+\frac{b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-3*a^2*ArcTanh[Cosh[c + d*x]])/(8*d) + (b^2*Cosh[c + d*x])/d - (2*a*b*Coth
[c + d*x])/d + (3*a^2*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (a^2*Coth[c + d*
x]*Csch[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = i\int \left(-2iab\operatorname{csch}^2(c+dx)-ia^2\operatorname{csch}^5(c+dx)-ib^2\sinh(c+dx)\right)$$

$$= a^2\int \operatorname{csch}^5(c+dx)\,dx+(2ab)\int \operatorname{csch}^2(c+dx)\,dx+b^2\int \sinh($$

$$= \frac{b^2\cosh(c+dx)}{d}-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}-\frac{1}{4}\left(3a^2\right)\int \operatorname{cs}$$

$$= \frac{b^2\cosh(c+dx)}{d}-\frac{2ab\coth(c+dx)}{d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+}{8d}$$

$$= -\frac{3a^2\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{b^2\cosh(c+dx)}{d}-\frac{2ab\coth(c+d}{d}$$

Mathematica [A] time = 0.04, size = 149, normalized size = 1.66

$$-\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{3a^2\log\left(\tanh\left(\frac{1}{2}(\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (2*a*b*Coth[c + d*x])/d + (3*a^2*Csch[(c + d*x)
/2]^2)/(32*d) - (a^2*Csch[(c + d*x)/2]^4)/(64*d) + (3*a^2*Log[Tanh[(c + d*x
)/2]])/(8*d) + (3*a^2*Sech[(c + d*x)/2]^2)/(32*d) + (a^2*Sech[(c + d*x)/2]^
4)/(64*d) + (b^2*Sinh[c]*Sinh[d*x])/d
```

fricas [B] time = 0.53, size = 2119, normalized size = 23.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^2*cosh(d*x + c)^10 + 40*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 4*b^2*
sinh(d*x + c)^10 - 32*a*b*cosh(d*x + c)^7 + 6*(a^2 - 2*b^2)*cosh(d*x + c)^8
 + 6*(30*b^2*cosh(d*x + c)^2 + a^2 - 2*b^2)*sinh(d*x + c)^8 + 16*(30*b^2*co
sh(d*x + c)^3 - 2*a*b + 3*(a^2 - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 96
*a*b*cosh(d*x + c)^5 - 2*(11*a^2 - 4*b^2)*cosh(d*x + c)^6 + 2*(420*b^2*cosh
(d*x + c)^4 - 112*a*b*cosh(d*x + c) + 84*(a^2 - 2*b^2)*cosh(d*x + c)^2 - 11
*a^2 + 4*b^2)*sinh(d*x + c)^6 + 12*(84*b^2*cosh(d*x + c)^5 - 56*a*b*cosh(d*
x + c)^2 + 28*(a^2 - 2*b^2)*cosh(d*x + c)^3 + 8*a*b - (11*a^2 - 4*b^2)*cosh
(d*x + c))*sinh(d*x + c)^5 - 96*a*b*cosh(d*x + c)^3 - 2*(11*a^2 - 4*b^2)*co
sh(d*x + c)^4 + 2*(420*b^2*cosh(d*x + c)^6 - 560*a*b*cosh(d*x + c)^3 + 210*
(a^2 - 2*b^2)*cosh(d*x + c)^4 + 240*a*b*cosh(d*x + c) - 15*(11*a^2 - 4*b^2)
*cosh(d*x + c)^2 - 11*a^2 + 4*b^2)*sinh(d*x + c)^4 + 8*(60*b^2*cosh(d*x + c
)^7 - 140*a*b*cosh(d*x + c)^4 + 42*(a^2 - 2*b^2)*cosh(d*x + c)^5 + 120*a*b*
cosh(d*x + c)^2 - 5*(11*a^2 - 4*b^2)*cosh(d*x + c)^3 - 12*a*b - (11*a^2 - 4
*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 32*a*b*cosh(d*x + c) + 6*(a^2 - 2*b^
2)*cosh(d*x + c)^2 + 6*(30*b^2*cosh(d*x + c)^8 - 112*a*b*cosh(d*x + c)^5 +
28*(a^2 - 2*b^2)*cosh(d*x + c)^6 + 160*a*b*cosh(d*x + c)^3 - 5*(11*a^2 - 4*
b^2)*cosh(d*x + c)^4 - 48*a*b*cosh(d*x + c) - 2*(11*a^2 - 4*b^2)*cosh(d*x +
 c)^2 + a^2 - 2*b^2)*sinh(d*x + c)^2 + 4*b^2 - 3*(a^2*cosh(d*x + c)^9 + 9*a
^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 - 4*a^2*cosh(d*x + c
)^7 + 4*(9*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^7 + 6*a^2*cosh(d*x + c)
^5 + 28*(3*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(21
*a^2*cosh(d*x + c)^4 - 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^5 - 4*a^
2*cosh(d*x + c)^3 + 2*(63*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x + c)^3 + 15
```

```
*a^2*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^2*cosh(d*x + c)^6 - 35*a^2*co
sh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^3 + a^2*cosh(d*
x + c) + 12*(3*a^2*cosh(d*x + c)^7 - 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x
 + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^2*cosh(d*x + c)^8 - 28*
a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 - 12*a^2*cosh(d*x + c)^2 + a^2
)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*(a^2*cosh(d*x +
 c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 - 4*a^2*c
osh(d*x + c)^7 + 4*(9*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^7 + 6*a^2*co
sh(d*x + c)^5 + 28*(3*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c
)^6 + 6*(21*a^2*cosh(d*x + c)^4 - 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x +
c)^5 - 4*a^2*cosh(d*x + c)^3 + 2*(63*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x
+ c)^3 + 15*a^2*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^2*cosh(d*x + c)^6
- 35*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^3 +
a^2*cosh(d*x + c) + 12*(3*a^2*cosh(d*x + c)^7 - 7*a^2*cosh(d*x + c)^5 + 5*a
^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^2*cosh(d*x +
 c)^8 - 28*a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 - 12*a^2*cosh(d*x +
 c)^2 + a^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(10*
b^2*cosh(d*x + c)^9 - 56*a*b*cosh(d*x + c)^6 + 12*(a^2 - 2*b^2)*cosh(d*x +
c)^7 + 120*a*b*cosh(d*x + c)^4 - 3*(11*a^2 - 4*b^2)*cosh(d*x + c)^5 - 72*a*
b*cosh(d*x + c)^2 - 2*(11*a^2 - 4*b^2)*cosh(d*x + c)^3 + 8*a*b + 3*(a^2 - 2
*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*
sinh(d*x + c)^8 + d*sinh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 4*(9*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*s
inh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cosh(d*x + c)^4 - 14*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x + c)^5 - 70*d*cosh(d*x +
 c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 - 4*d*cosh(d*x + c)^3 + 4*(21*d
*cosh(d*x + c)^6 - 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*
x + c)^3 + 12*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c
)^3 - d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (9*d*cosh(d*x +
c)^8 - 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 +
 d)*sinh(d*x + c))
```

giac [B] time = 0.23, size = 172, normalized size = 1.91

$$\frac{4\,b^2e^{(dx+c)} + 4\,b^2e^{(-dx-c)} - 3\,a^2\log\left(e^{(dx+c)}+1\right) + 3\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right) + \frac{2\left(3\,a^2e^{(7\,dx+7\,c)} - 16\,abe^{(6\,dx+6\,c)} - 11\,a^2e^{(5\,dx+5\,c)} + 48\,a \cdots\right.}{\left(e^{\cdots}\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*e^(d*x + c) + 4*b^2*e^(-d*x - c) - 3*a^2*log(e^(d*x + c) + 1) +
3*a^2*log(abs(e^(d*x + c) - 1)) + 2*(3*a^2*e^(7*d*x + 7*c) - 16*a*b*e^(6*d*
x + 6*c) - 11*a^2*e^(5*d*x + 5*c) + 48*a*b*e^(4*d*x + 4*c) - 11*a^2*e^(3*d*
x + 3*c) - 48*a*b*e^(2*d*x + 2*c) + 3*a^2*e^(d*x + c) + 16*a*b)/(e^(2*d*x +
 2*c) - 1)^4)/d
```

maple [A] time = 0.18, size = 66, normalized size = 0.73

$$\frac{a^2\left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-2ab\coth(dx+c)+b^2\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(a^2*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(
d*x+c)))-2*a*b*coth(d*x+c)+b^2*cosh(d*x+c))
```

maxima [B] time = 0.33, size = 188, normalized size = 2.09

$$\frac{1}{2}\,b^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)-\frac{1}{8}\,a^2\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}-11\,e^{(-3\,dx-3\,c)}-11\,e^{(-}}{d\left(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) - 1/8*a^2*(3*log(e^(-d*x - c) + 1)
/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) -
11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 4*a*b/(d*(e^(-2
*d*x - 2*c) - 1))
```

mupad [B] time = 0.14, size = 355, normalized size = 3.94

$$\frac{\frac{3\,a^2\,{\mathrm{e}}^{c+d\,x}}{4\,d}-\frac{2\,a\,b}{d}}{{\mathrm{e}}^{2\,c+2\,d\,x}-1}-\frac{\frac{4\,a^2\,{\mathrm{e}}^{3\,c+3\,d\,x}}{d}-\frac{a\,b}{d}+\frac{3\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{d}-\frac{3\,a\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{d}+\frac{a\,b\,{\mathrm{e}}^{6\,c+6\,d\,x}}{d}}{6\,{\mathrm{e}}^{4\,c+4\,d\,x}-4\,{\mathrm{e}}^{2\,c+2\,d\,x}-4\,{\mathrm{e}}^{6\,c+6\,d\,x}+{\mathrm{e}}^{8\,c+8\,d\,x}+1}-\frac{\frac{2\,a^2\,{\mathrm{e}}^{c+d\,x}}{d}+\frac{a\,b}{d}-\frac{2\,a\,b\,{\mathrm{e}}^{2\,c+2\,d\,x}}{d}+\frac{a\,b\,{\mathrm{e}}^{4\,c+4\,d\,x}}{d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^5,x)
```

```
[Out] ((3*a^2*exp(c + d*x))/(4*d) - (2*a*b)/d)/(exp(2*c + 2*d*x) - 1) - ((4*a^2*e
xp(3*c + 3*d*x))/d - (a*b)/d + (3*a*b*exp(2*c + 2*d*x))/d - (3*a*b*exp(4*c
+ 4*d*x))/d + (a*b*exp(6*c + 6*d*x))/d)/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2
*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*a^2*exp(c + d*x))/
d + (a*b)/d - (2*a*b*exp(2*c + 2*d*x))/d + (a*b*exp(4*c + 4*d*x))/d)/(3*exp
(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (b^2*exp(c + d
*x))/(2*d) - (3*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a
^4)^(1/2))/(4*(-d^2)^(1/2)) + (b^2*exp(- c - d*x))/(2*d) - (a^2*exp(c + d*x
))/(2*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.159 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=88

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{ab\coth(c+dx)\operatorname{csch}(c+dx)}{d}$$

```
[Out] b^2*x+a*b*arctanh(cosh(d*x+c))/d-a^2*coth(d*x+c)/d+2/3*a^2*coth(d*x+c)^3/d-
1/5*a^2*coth(d*x+c)^5/d-a*b*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 3768, 3770, 3767}

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{ab\coth(c+dx)\operatorname{csch}(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] b^2*x + (a*b*ArcTanh[Cosh[c + d*x]])/d - (a^2*Coth[c + d*x])/d + (2*a^2*Cot
h[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]^5)/(5*d) - (a*b*Coth[c + d*x]*Csch
[c + d*x])/d
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = -\int \left(-b^2-2ab\operatorname{csch}^3(c+dx)-a^2\operatorname{csch}^6(c+dx)\right)dx$$
$$= b^2x + a^2\int \operatorname{csch}^6(c+dx)\,dx + (2ab)\int \operatorname{csch}^3(c+dx)\,dx$$
$$= b^2x - \frac{ab\coth(c+dx)\operatorname{csch}(c+dx)}{d} - (ab)\int \operatorname{csch}(c+dx)\,dx - \frac{(}{}$$
$$= b^2x + \frac{ab\tanh^{-1}(\cosh(c+dx))}{d} - \frac{a^2\coth(c+dx)}{d} + \frac{2a^2\coth^3(c}{3d}$$

Mathematica [B] time = 0.94, size = 197, normalized size = 2.24

$$\frac{16\left(-16a^2\tanh\left(\frac{1}{2}(c+dx)\right)-12a^2\sinh^6\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^5(c+dx)-19a^2\sinh^4\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^3(c+dx)-1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-256*a^2*Coth[(c + d*x)/2] - 240*a*b*Csch[(c + d*x)/2]^2 + 19*a^2*Csch[(c
+ d*x)/2]^4*Sinh[c + d*x] - 3*a^2*Csch[(c + d*x)/2]^6*Sinh[c + d*x] + 16*(6
0*b^2*c + 60*b^2*d*x - 60*a*b*Log[Tanh[(c + d*x)/2]] - 15*a*b*Sech[(c + d*x
)/2]^2 - 19*a^2*Csch[c + d*x]^3*Sinh[(c + d*x)/2]^4 - 12*a^2*Csch[c + d*x]^
5*Sinh[(c + d*x)/2]^6 - 16*a^2*Tanh[(c + d*x)/2]))/(960*d)
```

fricas [B] time = 0.66, size = 2310, normalized size = 26.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(15*b^2*d*x*cosh(d*x + c)^10 + 15*b^2*d*x*sinh(d*x + c)^10 - 75*b^2*d*
x*cosh(d*x + c)^8 - 30*a*b*cosh(d*x + c)^9 + 150*b^2*d*x*cosh(d*x + c)^6 +
30*(5*b^2*d*x*cosh(d*x + c) - a*b)*sinh(d*x + c)^9 + 60*a*b*cosh(d*x + c)^7
 + 15*(45*b^2*d*x*cosh(d*x + c)^2 - 5*b^2*d*x - 18*a*b*cosh(d*x + c))*sinh(
d*x + c)^8 + 60*(30*b^2*d*x*cosh(d*x + c)^3 - 10*b^2*d*x*cosh(d*x + c) - 18
*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^7 + 30*(105*b^2*d*x*cosh(d*x + c)
^4 - 70*b^2*d*x*cosh(d*x + c)^2 - 84*a*b*cosh(d*x + c)^3 + 5*b^2*d*x + 14*a
*b*cosh(d*x + c))*sinh(d*x + c)^6 + 60*(63*b^2*d*x*cosh(d*x + c)^5 - 70*b^2
*d*x*cosh(d*x + c)^3 - 63*a*b*cosh(d*x + c)^4 + 15*b^2*d*x*cosh(d*x + c) +
21*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^5 - 60*a*b*cosh(d*x + c)^3 - 10*(15*b
^2*d*x + 16*a^2)*cosh(d*x + c)^4 + 10*(315*b^2*d*x*cosh(d*x + c)^6 - 525*b^
2*d*x*cosh(d*x + c)^4 - 378*a*b*cosh(d*x + c)^5 + 225*b^2*d*x*cosh(d*x + c)
^2 + 210*a*b*cosh(d*x + c)^3 - 15*b^2*d*x - 16*a^2)*sinh(d*x + c)^4 - 15*b^
2*d*x + 20*(90*b^2*d*x*cosh(d*x + c)^7 - 210*b^2*d*x*cosh(d*x + c)^5 - 126*
a*b*cosh(d*x + c)^6 + 150*b^2*d*x*cosh(d*x + c)^3 + 105*a*b*cosh(d*x + c)^4
 - 3*a*b - 2*(15*b^2*d*x + 16*a^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*a*b*
cosh(d*x + c) + 5*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)^2 + 5*(135*b^2*d*x*co
sh(d*x + c)^8 - 420*b^2*d*x*cosh(d*x + c)^6 - 216*a*b*cosh(d*x + c)^7 + 450
*b^2*d*x*cosh(d*x + c)^4 + 252*a*b*cosh(d*x + c)^5 + 15*b^2*d*x - 36*a*b*co
sh(d*x + c) - 12*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)^2 + 16*a^2)*sinh(d*x +
 c)^2 - 16*a^2 + 15*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x +
 c)^9 + a*b*sinh(d*x + c)^10 - 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x +
c)^2 - a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x +
 c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 - 1
4*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^6 - 10*a*b*cosh(d*x + c)^4 + 4*(
```

```
63*a*b*cosh(d*x + c)^5 - 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sin
h(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 - 35*a*b*cosh(d*x + c)^4 + 15*a*b
*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b
*cosh(d*x + c)^7 - 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 - a*b*cosh
(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 - 28*a*b*cosh(d*x + c
)^6 + 30*a*b*cosh(d*x + c)^4 - 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^
2 - a*b + 10*(a*b*cosh(d*x + c)^9 - 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x
+ c)^5 - 4*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c))*log(cosh
(d*x + c) + sinh(d*x + c) + 1) - 15*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x
 + c)*sinh(d*x + c)^9 + a*b*sinh(d*x + c)^10 - 5*a*b*cosh(d*x + c)^8 + 5*(9
*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(
3*a*b*cosh(d*x + c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cos
h(d*x + c)^4 - 14*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^6 - 10*a*b*cosh(
d*x + c)^4 + 4*(63*a*b*cosh(d*x + c)^5 - 70*a*b*cosh(d*x + c)^3 + 15*a*b*co
sh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 - 35*a*b*cosh(d*x
 + c)^4 + 15*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x +
c)^2 + 40*(3*a*b*cosh(d*x + c)^7 - 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x +
 c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 - 28*
a*b*cosh(d*x + c)^6 + 30*a*b*cosh(d*x + c)^4 - 12*a*b*cosh(d*x + c)^2 + a*b
)*sinh(d*x + c)^2 - a*b + 10*(a*b*cosh(d*x + c)^9 - 4*a*b*cosh(d*x + c)^7 +
 6*a*b*cosh(d*x + c)^5 - 4*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(15*b^2*d*x*cosh(d*x +
c)^9 - 60*b^2*d*x*cosh(d*x + c)^7 - 27*a*b*cosh(d*x + c)^8 + 90*b^2*d*x*cos
h(d*x + c)^5 + 42*a*b*cosh(d*x + c)^6 - 18*a*b*cosh(d*x + c)^2 - 4*(15*b^2*
d*x + 16*a^2)*cosh(d*x + c)^3 + 3*a*b + (15*b^2*d*x + 16*a^2)*cosh(d*x + c)
)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 +
 d*sinh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 - d)*sin
h(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^7 +
 10*d*cosh(d*x + c)^6 + 10*(21*d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*c
osh(d*x + c))*sinh(d*x + c)^5 - 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x +
c)^6 - 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4
0*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh
(d*x + c))*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 -
 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + d)*si
nh(d*x + c)^2 + 10*(d*cosh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x
+ c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

giac [A] time = 0.21, size = 141, normalized size = 1.60

$$\frac{15\,(dx+c)b^2 + 15\,ab\log\left(e^{(dx+c)}+1\right) - 15\,ab\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\left(15\,abe^{(9\,dx+9\,c)} - 30\,abe^{(7\,dx+7\,c)} + 80\,a^2e^{(4\,dx+4\,c)} + 30\,abe^{(3\,dx+3\,c}}{\left(e^{(2\,dx+2\,c)}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*b^2 + 15*a*b*log(e^(d*x + c) + 1) - 15*a*b*log(abs(e^(d*
x + c) - 1)) - 2*(15*a*b*e^(9*d*x + 9*c) - 30*a*b*e^(7*d*x + 7*c) + 80*a^2*
e^(4*d*x + 4*c) + 30*a*b*e^(3*d*x + 3*c) - 40*a^2*e^(2*d*x + 2*c) - 15*a*b*
e^(d*x + c) + 8*a^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

maple [A] time = 0.20, size = 73, normalized size = 0.83

$$\frac{a^2\left(-\frac{8}{15} - \frac{\operatorname{csch}(dx+c)^4}{5} + \frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c) + 2ab\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2} + \operatorname{arctanh}\left(e^{dx+c}\right)\right) + b^2(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(a^2*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(-1
/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b^2*(d*x+c))
```

maxima [B] time = 0.33, size = 303, normalized size = 3.44

$$b^2x+ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)-\frac{16}{15}\,a^2\left(\frac{}{d\left(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] b^2*x + a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x
 - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
- 16/15*a^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1
0*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*
d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*
d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) - 1)))
```

mupad [B] time = 0.65, size = 351, normalized size = 3.99

$$b^2\,x-\frac{\frac{32\,a^2\,e^{4\,c+4\,d\,x}}{5\,d}-\frac{8\,a\,b\,e^{c+d\,x}}{5\,d}+\frac{24\,a\,b\,e^{3\,c+3\,d\,x}}{5\,d}-\frac{24\,a\,b\,e^{5\,c+5\,d\,x}}{5\,d}+\frac{8\,a\,b\,e^{7\,c+7\,d\,x}}{5\,d}}{5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1}+\frac{2\,\mathrm{atan}\left(\frac{a\,b\,e^{d\,x}\,e^c\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}-\frac{}{15\,d\,\left(3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^6,x)
```

```
[Out] b^2*x - ((32*a^2*exp(4*c + 4*d*x))/(5*d) - (8*a*b*exp(c + d*x))/(5*d) + (24
*a*b*exp(3*c + 3*d*x))/(5*d) - (24*a*b*exp(5*c + 5*d*x))/(5*d) + (8*a*b*exp
(7*c + 7*d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*
c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) + (2*atan((a*b*ex
p(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(-d^2)^(1
/2) - (64*a^2)/(15*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6
*d*x) - 1)) - (16*a^2)/(5*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (2*a*b*exp(c + d*x))/(d*(exp(2*c
+ 2*d*x) - 1)) - (12*a*b*exp(c + d*x))/(5*d*(exp(4*c + 4*d*x) - 2*exp(2*c +
 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.160 $\int \text{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=133

$$\frac{5a^2\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^2\coth(c+dx)\text{csch}^5(c+dx)}{6d}+\frac{5a^2\coth(c+dx)\text{csch}^3(c+dx)}{24d}-\frac{5a^2\coth(c+dx)\text{csch}(}{16d}$$

```
[Out] 5/16*a^2*arctanh(cosh(d*x+c))/d-b^2*arctanh(cosh(d*x+c))/d+2*a*b*coth(d*x+c
)/d-2/3*a*b*coth(d*x+c)^3/d-5/16*a^2*coth(d*x+c)*csch(d*x+c)/d+5/24*a^2*cot
h(d*x+c)*csch(d*x+c)^3/d-1/6*a^2*coth(d*x+c)*csch(d*x+c)^5/d
```

Rubi [A] time = 0.17, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 3770, 3767, 3768}

$$\frac{5a^2\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^2\coth(c+dx)\text{csch}^5(c+dx)}{6d}+\frac{5a^2\coth(c+dx)\text{csch}^3(c+dx)}{24d}-\frac{5a^2\coth(c+dx)\text{csch}(}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (5*a^2*ArcTanh[Cosh[c + d*x]])/(16*d) - (b^2*ArcTanh[Cosh[c + d*x]])/d + (2
*a*b*Coth[c + d*x])/d - (2*a*b*Coth[c + d*x]^3)/(3*d) - (5*a^2*Coth[c + d*x
]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a
^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = -\left(i\int \left(ib^2\operatorname{csch}(c+dx)+2iab\operatorname{csch}^4(c+dx)+ia^2\operatorname{csch}^7(c+dx)\right)\right.$$
$$= a^2\int \operatorname{csch}^7(c+dx)\,dx+(2ab)\int \operatorname{csch}^4(c+dx)\,dx+b^2\int \operatorname{csch}($$
$$= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}-\frac{1}{6}($$
$$= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{2ab\coth(c+dx)}{d}-\frac{2ab\coth^3(c+}{3d}$$
$$= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{2ab\coth(c+dx)}{d}-\frac{2ab\coth^3(c+}{3d}$$
$$= \frac{5a^2\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{2ab\coth}{}$$

Mathematica [A] time = 0.05, size = 235, normalized size = 1.77

$$-\frac{a^2\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{5a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a^2\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (4*a*b*Coth[c + d*x])/(3*d) - (5*a^2*Csch[(c + d*x)/2]^2)/(64*d) + (a^2*Csc
h[(c + d*x)/2]^4)/(64*d) - (a^2*Csch[(c + d*x)/2]^6)/(384*d) - (2*a*b*Coth[
c + d*x]*Csch[c + d*x]^2)/(3*d) - (b^2*Log[Cosh[c/2 + (d*x)/2]])/d + (b^2*L
og[Sinh[c/2 + (d*x)/2]])/d - (5*a^2*Log[Tanh[(c + d*x)/2]])/(16*d) - (5*a^2
*Sech[(c + d*x)/2]^2)/(64*d) - (a^2*Sech[(c + d*x)/2]^4)/(64*d) - (a^2*Sech
[(c + d*x)/2]^6)/(384*d)
```

fricas [B] time = 0.52, size = 3607, normalized size = 27.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(30*a^2*cosh(d*x + c)^11 + 330*a^2*cosh(d*x + c)*sinh(d*x + c)^10 + 3
0*a^2*sinh(d*x + c)^11 - 170*a^2*cosh(d*x + c)^9 + 384*a*b*cosh(d*x + c)^8
+ 10*(165*a^2*cosh(d*x + c)^2 - 17*a^2)*sinh(d*x + c)^9 + 396*a^2*cosh(d*x
+ c)^7 + 6*(825*a^2*cosh(d*x + c)^3 - 255*a^2*cosh(d*x + c) + 64*a*b)*sinh(
d*x + c)^8 - 1280*a*b*cosh(d*x + c)^6 + 12*(825*a^2*cosh(d*x + c)^4 - 510*a
^2*cosh(d*x + c)^2 + 256*a*b*cosh(d*x + c) + 33*a^2)*sinh(d*x + c)^7 + 396*
a^2*cosh(d*x + c)^5 + 4*(3465*a^2*cosh(d*x + c)^5 - 3570*a^2*cosh(d*x + c)^
3 + 2688*a*b*cosh(d*x + c)^2 + 693*a^2*cosh(d*x + c) - 320*a*b)*sinh(d*x +
c)^6 + 1536*a*b*cosh(d*x + c)^4 + 12*(1155*a^2*cosh(d*x + c)^6 - 1785*a^2*c
osh(d*x + c)^4 + 1792*a*b*cosh(d*x + c)^3 + 693*a^2*cosh(d*x + c)^2 - 640*a
*b*cosh(d*x + c) + 33*a^2)*sinh(d*x + c)^5 - 170*a^2*cosh(d*x + c)^3 + 12*(
825*a^2*cosh(d*x + c)^7 - 1785*a^2*cosh(d*x + c)^5 + 2240*a*b*cosh(d*x + c)
^4 + 1155*a^2*cosh(d*x + c)^3 - 1600*a*b*cosh(d*x + c)^2 + 165*a^2*cosh(d*x
 + c) + 128*a*b)*sinh(d*x + c)^4 - 768*a*b*cosh(d*x + c)^2 + 2*(2475*a^2*co
sh(d*x + c)^8 - 7140*a^2*cosh(d*x + c)^6 + 10752*a*b*cosh(d*x + c)^5 + 6930
*a^2*cosh(d*x + c)^4 - 12800*a*b*cosh(d*x + c)^3 + 1980*a^2*cosh(d*x + c)^2
 + 3072*a*b*cosh(d*x + c) - 85*a^2)*sinh(d*x + c)^3 + 30*a^2*cosh(d*x + c)
+ 6*(275*a^2*cosh(d*x + c)^9 - 1020*a^2*cosh(d*x + c)^7 + 1792*a*b*cosh(d*x
 + c)^6 + 1386*a^2*cosh(d*x + c)^5 - 3200*a*b*cosh(d*x + c)^4 + 660*a^2*cos
```

h(d*x + c)^3 + 1536*a*b*cosh(d*x + c)^2 - 85*a^2*cosh(d*x + c) - 128*a*b)*s
inh(d*x + c)^2 + 128*a*b - 3*((5*a^2 - 16*b^2)*cosh(d*x + c)^12 + 12*(5*a^2
 - 16*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^2 - 16*b^2)*sinh(d*x + c)^
12 - 6*(5*a^2 - 16*b^2)*cosh(d*x + c)^10 + 6*(11*(5*a^2 - 16*b^2)*cosh(d*x
+ c)^2 - 5*a^2 + 16*b^2)*sinh(d*x + c)^10 + 20*(11*(5*a^2 - 16*b^2)*cosh(d*
x + c)^3 - 3*(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a^2 -
16*b^2)*cosh(d*x + c)^8 + 15*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 - 18*(5*a
^2 - 16*b^2)*cosh(d*x + c)^2 + 5*a^2 - 16*b^2)*sinh(d*x + c)^8 + 24*(33*(5*
a^2 - 16*b^2)*cosh(d*x + c)^5 - 30*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 + 5*(5*
a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(5*a^2 - 16*b^2)*cosh(d*x
 + c)^6 + 4*(231*(5*a^2 - 16*b^2)*cosh(d*x + c)^6 - 315*(5*a^2 - 16*b^2)*co
sh(d*x + c)^4 + 105*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 - 25*a^2 + 80*b^2)*sin
h(d*x + c)^6 + 24*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 - 63*(5*a^2 - 16*b^2
)*cosh(d*x + c)^5 + 35*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - 5*(5*a^2 - 16*b^2
)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 + 15
(33(5*a^2 - 16*b^2)*cosh(d*x + c)^8 - 84*(5*a^2 - 16*b^2)*cosh(d*x + c)^6
 + 70*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 - 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^
2 + 5*a^2 - 16*b^2)*sinh(d*x + c)^4 + 20*(11*(5*a^2 - 16*b^2)*cosh(d*x + c)
^9 - 36*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 + 42*(5*a^2 - 16*b^2)*cosh(d*x + c
)^5 - 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 + 3*(5*a^2 - 16*b^2)*cosh(d*x + c
))*sinh(d*x + c)^3 - 6*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 + 6*(11*(5*a^2 - 16
*b^2)*cosh(d*x + c)^10 - 45*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 + 70*(5*a^2 -
16*b^2)*cosh(d*x + c)^6 - 50*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 + 15*(5*a^2 -
 16*b^2)*cosh(d*x + c)^2 - 5*a^2 + 16*b^2)*sinh(d*x + c)^2 + 5*a^2 - 16*b^2
 + 12*((5*a^2 - 16*b^2)*cosh(d*x + c)^11 - 5*(5*a^2 - 16*b^2)*cosh(d*x + c)
^9 + 10*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 - 10*(5*a^2 - 16*b^2)*cosh(d*x + c
)^5 + 5*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - (5*a^2 - 16*b^2)*cosh(d*x + c))*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*((5*a^2 - 16*b^2)
*cosh(d*x + c)^12 + 12*(5*a^2 - 16*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5
*a^2 - 16*b^2)*sinh(d*x + c)^12 - 6*(5*a^2 - 16*b^2)*cosh(d*x + c)^10 + 6*(
11*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 - 5*a^2 + 16*b^2)*sinh(d*x + c)^10 + 20
(11(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 - 16*b^2)*cosh(d*x + c))*s
inh(d*x + c)^9 + 15*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 + 15*(33*(5*a^2 - 16*b
^2)*cosh(d*x + c)^4 - 18*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 + 5*a^2 - 16*b^2)
*sinh(d*x + c)^8 + 24*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^5 - 30*(5*a^2 - 16
*b^2)*cosh(d*x + c)^3 + 5*(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 -
 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^6 + 4*(231*(5*a^2 - 16*b^2)*cosh(d*x + c
)^6 - 315*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 + 105*(5*a^2 - 16*b^2)*cosh(d*x
+ c)^2 - 25*a^2 + 80*b^2)*sinh(d*x + c)^6 + 24*(33*(5*a^2 - 16*b^2)*cosh(d*
x + c)^7 - 63*(5*a^2 - 16*b^2)*cosh(d*x + c)^5 + 35*(5*a^2 - 16*b^2)*cosh(d
x + c)^3 - 5(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(5*a^2 -
 16*b^2)*cosh(d*x + c)^4 + 15*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 - 84*(5*
a^2 - 16*b^2)*cosh(d*x + c)^6 + 70*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 - 20*(5
*a^2 - 16*b^2)*cosh(d*x + c)^2 + 5*a^2 - 16*b^2)*sinh(d*x + c)^4 + 20*(11*(
5*a^2 - 16*b^2)*cosh(d*x + c)^9 - 36*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 + 42*
(5*a^2 - 16*b^2)*cosh(d*x + c)^5 - 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 + 3*
(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(5*a^2 - 16*b^2)*cosh(d
x + c)^2 + 6(11*(5*a^2 - 16*b^2)*cosh(d*x + c)^10 - 45*(5*a^2 - 16*b^2)*c
osh(d*x + c)^8 + 70*(5*a^2 - 16*b^2)*cosh(d*x + c)^6 - 50*(5*a^2 - 16*b^2)*
cosh(d*x + c)^4 + 15*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 - 5*a^2 + 16*b^2)*sin
h(d*x + c)^2 + 5*a^2 - 16*b^2 + 12*((5*a^2 - 16*b^2)*cosh(d*x + c)^11 - 5*(
5*a^2 - 16*b^2)*cosh(d*x + c)^9 + 10*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 - 10*
(5*a^2 - 16*b^2)*cosh(d*x + c)^5 + 5*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - (5*
a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x +
c) - 1) + 6*(55*a^2*cosh(d*x + c)^10 - 255*a^2*cosh(d*x + c)^8 + 512*a*b*co
sh(d*x + c)^7 + 462*a^2*cosh(d*x + c)^6 - 1280*a*b*cosh(d*x + c)^5 + 330*a^
2*cosh(d*x + c)^4 + 1024*a*b*cosh(d*x + c)^3 - 85*a^2*cosh(d*x + c)^2 - 256
*a*b*cosh(d*x + c) + 5*a^2)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(
d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 + 6*(

```
11*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 - 3*d
*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x
+ c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)
^5 - 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*cosh(
d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*cosh(
d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cosh(d*
x + c)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d
*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*c
osh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(
d*x + c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x +
c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*c
osh(d*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d
*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)
^11 - 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5
*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [A] time = 0.25, size = 204, normalized size = 1.53

$$\frac{3\left(5a^2-16b^2\right)\log\left(e^{(dx+c)}+1\right)-3\left(5a^2-16b^2\right)\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{2\left(15a^2e^{(11dx+11c)}-85a^2e^{(9dx+9c)}+192abe^{(8dx+8c)}+\cdots\right.}{\cdots}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/48*(3*(5*a^2 - 16*b^2)*log(e^(d*x + c) + 1) - 3*(5*a^2 - 16*b^2)*log(abs(
e^(d*x + c) - 1)) - 2*(15*a^2*e^(11*d*x + 11*c) - 85*a^2*e^(9*d*x + 9*c) +
192*a*b*e^(8*d*x + 8*c) + 198*a^2*e^(7*d*x + 7*c) - 640*a*b*e^(6*d*x + 6*c)
 + 198*a^2*e^(5*d*x + 5*c) + 768*a*b*e^(4*d*x + 4*c) - 85*a^2*e^(3*d*x + 3*
c) - 384*a*b*e^(2*d*x + 2*c) + 15*a^2*e^(d*x + c) + 64*a*b)/(e^(2*d*x + 2*c
) - 1)^6)/d
```

maple [A] time = 0.18, size = 90, normalized size = 0.68

$$\frac{a^2\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6}+\frac{5\operatorname{csch}(dx+c)^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+2ab\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(a^2*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x
+c)+5/8*arctanh(exp(d*x+c)))+2*a*b*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*b^
2*arctanh(exp(d*x+c)))
```

maxima [B] time = 0.33, size = 316, normalized size = 2.38

$$\frac{1}{48}a^2\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15e^{(-dx-c)}-85e^{(-3dx-3c)}+198e^{(-5dx-5c)}+198e^{(-7dx\cdots}\right.}{d\left(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/48*a^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e
^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x -
7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c)
- 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-1
0*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) - b^2*(log(e^(-d*x - c) + 1)/d -
```

```
log(e^(-d*x - c) - 1)/d) + 8/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*
c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c)
 - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 0.16, size = 434, normalized size = 3.26

$$\frac{\frac{5\,a^2\,\mathrm{e}^{c+d\,x}}{12\,d}-\frac{8\,a\,b}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1}-\frac{\frac{a^2\,\mathrm{e}^{c+d\,x}}{3\,d}+\frac{16\,a\,b}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(5\,a^2\,\sqrt{-d^2}-16\,b^2\,\sqrt{-d^2}\right)}{d\,\sqrt{25\,a^4-160\,a^2\,b^2+256\,b^4}}\right)\,\sqrt{25\,a^4-160\,a^2}}{8\,\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^2/sinh(c + d*x)^7,x)
```

```
[Out] ((5*a^2*exp(c + d*x))/(12*d) - (8*a*b)/d)/(exp(4*c + 4*d*x) - 2*exp(2*c + 2
*d*x) + 1) - ((a^2*exp(c + d*x))/(3*d) + (16*a*b)/(3*d))/(3*exp(2*c + 2*d*x
) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (atan((exp(d*x)*exp(c)*(5*
a^2*(-d^2)^(1/2) - 16*b^2*(-d^2)^(1/2)))/(d*(25*a^4 + 256*b^4 - 160*a^2*b^2
)^(1/2)))*(25*a^4 + 256*b^4 - 160*a^2*b^2)^(1/2))/(8*(-d^2)^(1/2)) - (18*a^
2*exp(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6
*d*x) + exp(8*c + 8*d*x) + 1)) - (80*a^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*
d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp
(10*c + 10*d*x) - 1)) - (32*a^2*exp(c + d*x))/(3*d*(15*exp(4*c + 4*d*x) - 6
*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c
+ 10*d*x) + exp(12*c + 12*d*x) + 1)) - (5*a^2*exp(c + d*x))/(8*d*(exp(2*c +
 2*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.161 $\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=291

$$\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^3x}{2}+\frac{3a^2b\cosh^5(c+dx)}{5d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)}{d}+\frac{3ab^2\sinh^7(}{}$$

```
[Out] -1/2*a^3*x+105/128*a*b^2*x+3*a^2*b*cosh(d*x+c)/d-b^3*cosh(d*x+c)/d-2*a^2*b*
cosh(d*x+c)^3/d+5/3*b^3*cosh(d*x+c)^3/d+3/5*a^2*b*cosh(d*x+c)^5/d-2*b^3*cos
h(d*x+c)^5/d+10/7*b^3*cosh(d*x+c)^7/d-5/9*b^3*cosh(d*x+c)^9/d+1/11*b^3*cosh
(d*x+c)^11/d+1/2*a^3*cosh(d*x+c)*sinh(d*x+c)/d-105/128*a*b^2*cosh(d*x+c)*si
nh(d*x+c)/d+35/64*a*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d-7/16*a*b^2*cosh(d*x+c)*
sinh(d*x+c)^5/d+3/8*a*b^2*cosh(d*x+c)*sinh(d*x+c)^7/d
```

Rubi [A] time = 0.21, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 2635, 8, 2633}

$$\frac{3a^2b\cosh^5(c+dx)}{5d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)}{d}+\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^3x}{2}+\frac{3ab^2\sinh^7(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] -(a^3*x)/2 + (105*a*b^2*x)/128 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c +
d*x])/d - (2*a^2*b*Cosh[c + d*x]^3)/d + (5*b^3*Cosh[c + d*x]^3)/(3*d) + (3*
a^2*b*Cosh[c + d*x]^5)/(5*d) - (2*b^3*Cosh[c + d*x]^5)/d + (10*b^3*Cosh[c +
 d*x]^7)/(7*d) - (5*b^3*Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^11)/(11
*d) + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (105*a*b^2*Cosh[c + d*x]*Si
nh[c + d*x])/(128*d) + (35*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(64*d) - (7
*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(16*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[
c + d*x]^7)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx = -\int \left(-a^3\sinh^2(c+dx) - 3a^2b\sinh^5(c+dx) - 3ab^2\sinh^8(c+dx)\right.$$
$$= a^3\int \sinh^2(c+dx)\,dx + \left(3a^2b\right)\int \sinh^5(c+dx)\,dx + \left(3ab^2\right)\int \sin$$
$$= \frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{3ab^2\cosh(c+dx)\sinh^7(c+dx)}{8d} -$$
$$= -\frac{a^3x}{2} + \frac{3a^2b\cosh(c+dx)}{d} - \frac{b^3\cosh(c+dx)}{d} - \frac{2a^2b\cosh^3(c+dx)}{d}$$
$$= -\frac{a^3x}{2} + \frac{3a^2b\cosh(c+dx)}{d} - \frac{b^3\cosh(c+dx)}{d} - \frac{2a^2b\cosh^3(c+dx)}{d}$$
$$= -\frac{a^3x}{2} + \frac{3a^2b\cosh(c+dx)}{d} - \frac{b^3\cosh(c+dx)}{d} - \frac{2a^2b\cosh^3(c+dx)}{d}$$
$$= -\frac{a^3x}{2} + \frac{105}{128}ab^2x + \frac{3a^2b\cosh(c+dx)}{d} - \frac{b^3\cosh(c+dx)}{d} - \frac{2a^2b\,\mathrm{co}}{}$$

Mathematica [A] time = 0.47, size = 194, normalized size = 0.67

$$\frac{-27720a\left(64a^2-105b^2\right)(c+dx)+110880a\left(8a^2-21b^2\right)\sinh(2(c+dx))-20790b\left(77b^2-320a^2\right)\cosh(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-27720*a*(64*a^2 - 105*b^2)*(c + d*x) - 20790*b*(-320*a^2 + 77*b^2)*Cosh[c
 + d*x] + 34650*b*(-32*a^2 + 11*b^2)*Cosh[3*(c + d*x)] - 2079*b*(-64*a^2 +
55*b^2)*Cosh[5*(c + d*x)] + 27225*b^3*Cosh[7*(c + d*x)] - 4235*b^3*Cosh[9*(
c + d*x)] + 315*b^3*Cosh[11*(c + d*x)] + 110880*a*(8*a^2 - 21*b^2)*Sinh[2*(
c + d*x)] + 582120*a*b^2*Sinh[4*(c + d*x)] - 110880*a*b^2*Sinh[6*(c + d*x)]
 + 10395*a*b^2*Sinh[8*(c + d*x)])/(3548160*d)
```

fricas [B] time = 0.52, size = 568, normalized size = 1.95

$$\frac{315\,b^3\cosh(dx+c)^{11} + 3465\,b^3\cosh(dx+c)\sinh(dx+c)^{10} - 4235\,b^3\cosh(dx+c)^9 + 83160\,ab^2\cosh(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/3548160*(315*b^3*cosh(d*x + c)^11 + 3465*b^3*cosh(d*x + c)*sinh(d*x + c)^
10 - 4235*b^3*cosh(d*x + c)^9 + 83160*a*b^2*cosh(d*x + c)*sinh(d*x + c)^7 +
 27225*b^3*cosh(d*x + c)^7 + 3465*(15*b^3*cosh(d*x + c)^3 - 11*b^3*cosh(d*x
 + c))*sinh(d*x + c)^8 + 1155*(126*b^3*cosh(d*x + c)^5 - 308*b^3*cosh(d*x +
 c)^3 + 165*b^3*cosh(d*x + c))*sinh(d*x + c)^6 + 2079*(64*a^2*b - 55*b^3)*c
osh(d*x + c)^5 + 83160*(7*a*b^2*cosh(d*x + c)^3 - 8*a*b^2*cosh(d*x + c))*si
nh(d*x + c)^5 + 3465*(30*b^3*cosh(d*x + c)^7 - 154*b^3*cosh(d*x + c)^5 + 27
5*b^3*cosh(d*x + c)^3 + 3*(64*a^2*b - 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^
4 - 34650*(32*a^2*b - 11*b^3)*cosh(d*x + c)^3 + 27720*(21*a*b^2*cosh(d*x +
c)^5 - 80*a*b^2*cosh(d*x + c)^3 + 84*a*b^2*cosh(d*x + c))*sinh(d*x + c)^3 -
 27720*(64*a^3 - 105*a*b^2)*d*x + 3465*(5*b^3*cosh(d*x + c)^9 - 44*b^3*cosh
(d*x + c)^7 + 165*b^3*cosh(d*x + c)^5 + 6*(64*a^2*b - 55*b^3)*cosh(d*x + c)
^3 - 30*(32*a^2*b - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 20790*(320*a^2
*b - 77*b^3)*cosh(d*x + c) + 27720*(3*a*b^2*cosh(d*x + c)^7 - 24*a*b^2*cosh
(d*x + c)^5 + 84*a*b^2*cosh(d*x + c)^3 + 8*(8*a^3 - 21*a*b^2)*cosh(d*x + c)
)*sinh(d*x + c))/d
```

giac [A] time = 0.26, size = 431, normalized size = 1.48

$$\frac{b^3 e^{(11\,dx+11\,c)}}{22528\,d} - \frac{11\,b^3 e^{(9\,dx+9\,c)}}{18432\,d} + \frac{3\,ab^2 e^{(8\,dx+8\,c)}}{2048\,d} + \frac{55\,b^3 e^{(7\,dx+7\,c)}}{14336\,d} - \frac{ab^2 e^{(6\,dx+6\,c)}}{64\,d} + \frac{21\,ab^2 e^{(4\,dx+4\,c)}}{256\,d} - \frac{21\,ab^2 e^{(-4\,dx-4\,c)}}{256\,d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/22528*b^3*e^(11*d*x + 11*c)/d - 11/18432*b^3*e^(9*d*x + 9*c)/d + 3/2048*a
*b^2*e^(8*d*x + 8*c)/d + 55/14336*b^3*e^(7*d*x + 7*c)/d - 1/64*a*b^2*e^(6*d
*x + 6*c)/d + 21/256*a*b^2*e^(4*d*x + 4*c)/d - 21/256*a*b^2*e^(-4*d*x - 4*c
)/d + 1/64*a*b^2*e^(-6*d*x - 6*c)/d + 55/14336*b^3*e^(-7*d*x - 7*c)/d - 3/2
048*a*b^2*e^(-8*d*x - 8*c)/d - 11/18432*b^3*e^(-9*d*x - 9*c)/d + 1/22528*b^
3*e^(-11*d*x - 11*c)/d - 1/128*(64*a^3 - 105*a*b^2)*x + 3/10240*(64*a^2*b -
 55*b^3)*e^(5*d*x + 5*c)/d - 5/1024*(32*a^2*b - 11*b^3)*e^(3*d*x + 3*c)/d +
 1/64*(8*a^3 - 21*a*b^2)*e^(2*d*x + 2*c)/d + 3/1024*(320*a^2*b - 77*b^3)*e^
(d*x + c)/d + 3/1024*(320*a^2*b - 77*b^3)*e^(-d*x - c)/d - 1/64*(8*a^3 - 21
*a*b^2)*e^(-2*d*x - 2*c)/d - 5/1024*(32*a^2*b - 11*b^3)*e^(-3*d*x - 3*c)/d
+ 3/10240*(64*a^2*b - 55*b^3)*e^(-5*d*x - 5*c)/d
```

maple [A] time = 0.14, size = 188, normalized size = 0.65

$$b^3\left(-\frac{256}{693}+\frac{\left(\sinh^{10}(dx+c)\right)}{11}-\frac{10\left(\sinh^{8}(dx+c)\right)}{99}+\frac{80\left(\sinh^{6}(dx+c)\right)}{693}-\frac{32\left(\sinh^{4}(dx+c)\right)}{231}+\frac{128\left(\sinh^{2}(dx+c)\right)}{693}\right)\cosh(dx+c)+3a\,b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(b^3*(-256/693+1/11*sinh(d*x+c)^10-10/99*sinh(d*x+c)^8+80/693*sinh(d*x+
c)^6-32/231*sinh(d*x+c)^4+128/693*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*((1/8*
sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d*x+c))*c
osh(d*x+c)+35/128*d*x+35/128*c)+3*a^2*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d
*x+c)^2)*cosh(d*x+c)+a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [A] time = 0.33, size = 387, normalized size = 1.33

$$-\frac{1}{8}\,a^3\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{1419264}\,b^3\left(\frac{\left(847\,e^{(-2\,dx-2\,c)}-5445\,e^{(-4\,dx-4\,c)}+22869\,e^{(-6\,dx-6\,c)}-76230\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/1419264*b^3*((8
47*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6*d*x - 6*c) - 7623
0*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(11*d*x + 11*c)/d +
(320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(-5*d*x - 5*c) - 54
45*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*x - 11*c))/d) - 1/
2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6
*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168
*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/160*a^
2*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e
^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

mupad [B] time = 1.09, size = 231, normalized size = 0.79

$$\frac{\sinh(c+d\,x)\,a^3\cosh(c+d\,x)}{2}-\frac{d\,x\,a^3}{2}+\frac{3\,a^2\,b\cosh(c+d\,x)^5}{5}-2\,a^2\,b\cosh\left(c+d\,x\right)^3+3\,a^2\,b\cosh\left(c+d\,x\right)+\frac{3\sinh(c+d\,x)\,a\,b^2\cos}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3)^3,x)
```

```
[Out] ((5*b^3*cosh(c + d*x)^3)/3 - b^3*cosh(c + d*x) - 2*b^3*cosh(c + d*x)^5 + (1
0*b^3*cosh(c + d*x)^7)/7 - (5*b^3*cosh(c + d*x)^9)/9 + (b^3*cosh(c + d*x)^1
1)/11 - 2*a^2*b*cosh(c + d*x)^3 + (3*a^2*b*cosh(c + d*x)^5)/5 + 3*a^2*b*cos
h(c + d*x) + (a^3*cosh(c + d*x)*sinh(c + d*x))/2 - (a^3*d*x)/2 - (279*a*b^2
*cosh(c + d*x)*sinh(c + d*x))/128 + (105*a*b^2*d*x)/128 + (163*a*b^2*cosh(c
 + d*x)^3*sinh(c + d*x))/64 - (25*a*b^2*cosh(c + d*x)^5*sinh(c + d*x))/16 +
 (3*a*b^2*cosh(c + d*x)^7*sinh(c + d*x))/8)/d
```

sympy [A] time = 33.47, size = 498, normalized size = 1.71

$$\begin{cases} \frac{a^3 x \sinh^2(c+dx)}{2} - \frac{a^3 x \cosh^2(c+dx)}{2} + \frac{a^3 \sinh(c+dx)\cosh(c+dx)}{2d} + \frac{3a^2 b \sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4a^2 b \sinh^2(c+dx)\cosh^3(c+dx)}{d} + \frac{8a^2 b c}{} \\ x\left(a + b\sinh^3(c)\right)^3 \sinh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d
- 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5/
(5*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*c
osh(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 10
5*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x
)**8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*s
inh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh
(c + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) +
b**3*sinh(c + d*x)**10*cosh(c + d*x)/d - 10*b**3*sinh(c + d*x)**8*cosh(c +
d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**6*cosh(c + d*x)**5/(3*d) - 32*b**3*s
inh(c + d*x)**4*cosh(c + d*x)**7/(7*d) + 128*b**3*sinh(c + d*x)**2*cosh(c +
 d*x)**9/(63*d) - 256*b**3*cosh(c + d*x)**11/(693*d), Ne(d, 0)), (x*(a + b*
sinh(c)**3)**3*sinh(c)**2, True))
```

3.162 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx$

Optimal. Leaf size=267

$$\frac{a^3\cosh(c+dx)}{d}+\frac{3a^2b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9a^2b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}a^2bx+\frac{3ab^2\cosh^7(c+d$$

```
[Out] 9/8*a^2*b*x-63/256*b^3*x+a^3*cosh(d*x+c)/d-3*a*b^2*cosh(d*x+c)/d+3*a*b^2*co
sh(d*x+c)^3/d-9/5*a*b^2*cosh(d*x+c)^5/d+3/7*a*b^2*cosh(d*x+c)^7/d-9/8*a^2*b
*cosh(d*x+c)*sinh(d*x+c)/d+63/256*b^3*cosh(d*x+c)*sinh(d*x+c)/d+3/4*a^2*b*c
osh(d*x+c)*sinh(d*x+c)^3/d-21/128*b^3*cosh(d*x+c)*sinh(d*x+c)^3/d+21/160*b^
3*cosh(d*x+c)*sinh(d*x+c)^5/d-9/80*b^3*cosh(d*x+c)*sinh(d*x+c)^7/d+1/10*b^3
*cosh(d*x+c)*sinh(d*x+c)^9/d
```

Rubi [A] time = 0.22, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3220, 2638, 2635, 8, 2633}

$$\frac{3a^2b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9a^2b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh^7(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (9*a^2*b*x)/8 - (63*b^3*x)/256 + (a^3*Cosh[c + d*x])/d - (3*a*b^2*Cosh[c +
d*x])/d + (3*a*b^2*Cosh[c + d*x]^3)/d - (9*a*b^2*Cosh[c + d*x]^5)/(5*d) + (
3*a*b^2*Cosh[c + d*x]^7)/(7*d) - (9*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d
) + (63*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + (3*a^2*b*Cosh[c + d*x]*S
inh[c + d*x]^3)/(4*d) - (21*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(128*d) + (2
1*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(160*d) - (9*b^3*Cosh[c + d*x]*Sinh[c
+ d*x]^7)/(80*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^9)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
```

```
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\left(i\int\left(ia^3\sinh(c+dx)+3ia^2b\sinh^4(c+dx)+3iab^2\sinh^7(c+dx)\right.\right.$$
$$= a^3\int\sinh(c+dx)\,dx+\left(3a^2b\right)\int\sinh^4(c+dx)\,dx+\left(3ab^2\right)\int\sinh^7$$
$$= \frac{a^3\cosh(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)\sinh^3(c+dx)}{4d}+\frac{b^3\cosh(c+d}{1}$$
$$= \frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx)}{d}-\frac{9ab^2c}{}$$
$$= \frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx)}{d}$$
$$= \frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx)}{d}$$
$$= \frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx)}{d}$$
$$= \frac{9}{8}a^2bx-\frac{63b^3x}{256}+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh}{d}$$

Mathematica [A] time = 0.45, size = 184, normalized size = 0.69

$$\frac{1120a\left(64a^2-105b^2\right)\cosh(c+dx)+b\left(-53760a^2\sinh(2(c+dx))+6720a^2\sinh(4(c+dx))+80640a^2c+8064\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (1120*a*(64*a^2 - 105*b^2)*Cosh[c + d*x] + b*(80640*a^2*c - 17640*b^2*c + 8
0640*a^2*d*x - 17640*b^2*d*x + 23520*a*b*Cosh[3*(c + d*x)] - 4704*a*b*Cosh[
5*(c + d*x)] + 480*a*b*Cosh[7*(c + d*x)] - 53760*a^2*Sinh[2*(c + d*x)] + 14
700*b^2*Sinh[2*(c + d*x)] + 6720*a^2*Sinh[4*(c + d*x)] - 4200*b^2*Sinh[4*(c
 + d*x)] + 1050*b^2*Sinh[6*(c + d*x)] - 175*b^2*Sinh[8*(c + d*x)] + 14*b^2*
Sinh[10*(c + d*x)]))/(71680*d)
```

fricas [A] time = 0.56, size = 453, normalized size = 1.70

$$\frac{35\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^9+120\,ab^2\cosh\left(dx+c\right)^7+840\,ab^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6-1176\,ab^2\cosh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/17920*(35*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 120*a*b^2*cosh(d*x + c)^7 +
 840*a*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 1176*a*b^2*cosh(d*x + c)^5 + 70*
(6*b^3*cosh(d*x + c)^3 - 5*b^3*cosh(d*x + c))*sinh(d*x + c)^7 + 5880*a*b^2*
cosh(d*x + c)^3 + 7*(126*b^3*cosh(d*x + c)^5 - 350*b^3*cosh(d*x + c)^3 + 22
5*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 840*(5*a*b^2*cosh(d*x + c)^3 - 7*a*b
^2*cosh(d*x + c))*sinh(d*x + c)^4 + 70*(6*b^3*cosh(d*x + c)^7 - 35*b^3*cosh
(d*x + c)^5 + 75*b^3*cosh(d*x + c)^3 + 12*(8*a^2*b - 5*b^3)*cosh(d*x + c))*
sinh(d*x + c)^3 + 630*(32*a^2*b - 7*b^3)*d*x + 840*(3*a*b^2*cosh(d*x + c)^5
 - 14*a*b^2*cosh(d*x + c)^3 + 21*a*b^2*cosh(d*x + c))*sinh(d*x + c)^2 + 280
*(64*a^3 - 105*a*b^2)*cosh(d*x + c) + 35*(b^3*cosh(d*x + c)^9 - 10*b^3*cosh
```

```
(d*x + c)^7 + 45*b^3*cosh(d*x + c)^5 + 24*(8*a^2*b - 5*b^3)*cosh(d*x + c)^3
 - 6*(128*a^2*b - 35*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.25, size = 379, normalized size = 1.42

$$\frac{b^3e^{(10\,dx+10\,c)}}{10240\,d}-\frac{5\,b^3e^{(8\,dx+8\,c)}}{4096\,d}+\frac{3\,ab^2e^{(7\,dx+7\,c)}}{896\,d}+\frac{15\,b^3e^{(6\,dx+6\,c)}}{2048\,d}-\frac{21\,ab^2e^{(5\,dx+5\,c)}}{640\,d}+\frac{21\,ab^2e^{(3\,dx+3\,c)}}{128\,d}+\frac{21\,ab^2e^{(-3\,dx-3\,c)}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/10240*b^3*e^(10*d*x + 10*c)/d - 5/4096*b^3*e^(8*d*x + 8*c)/d + 3/896*a*b^
2*e^(7*d*x + 7*c)/d + 15/2048*b^3*e^(6*d*x + 6*c)/d - 21/640*a*b^2*e^(5*d*x
 + 5*c)/d + 21/128*a*b^2*e^(3*d*x + 3*c)/d + 21/128*a*b^2*e^(-3*d*x - 3*c)/
d - 21/640*a*b^2*e^(-5*d*x - 5*c)/d - 15/2048*b^3*e^(-6*d*x - 6*c)/d + 3/89
6*a*b^2*e^(-7*d*x - 7*c)/d + 5/4096*b^3*e^(-8*d*x - 8*c)/d - 1/10240*b^3*e^
(-10*d*x - 10*c)/d + 9/256*(32*a^2*b - 7*b^3)*x + 3/512*(8*a^2*b - 5*b^3)*e
^(4*d*x + 4*c)/d - 3/1024*(128*a^2*b - 35*b^3)*e^(2*d*x + 2*c)/d + 1/128*(6
4*a^3 - 105*a*b^2)*e^(d*x + c)/d + 1/128*(64*a^3 - 105*a*b^2)*e^(-d*x - c)/
d + 3/1024*(128*a^2*b - 35*b^3)*e^(-2*d*x - 2*c)/d - 3/512*(8*a^2*b - 5*b^3
)*e^(-4*d*x - 4*c)/d
```

maple [A] time = 0.05, size = 168, normalized size = 0.63

$$\frac{b^3\left(\left(\frac{\left(\sinh^9(dx+c)\right)}{10}-\frac{9\left(\sinh^7(dx+c)\right)}{80}+\frac{21\left(\sinh^5(dx+c)\right)}{160}-\frac{21\left(\sinh^3(dx+c)\right)}{128}+\frac{63\sinh(dx+c)}{256}\right)\cosh\left(dx+c\right)-\frac{63dx}{256}-\frac{63c}{256}\right)+3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(b^3*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/12
8*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+3*a*b^
2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x
+c)+3*a^2*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)
+a^3*cosh(d*x+c))
```

maxima [A] time = 0.33, size = 318, normalized size = 1.19

$$\frac{3}{64}\,a^2b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{20480}\,b^3\left(\frac{\left(25\,e^{(-2\,dx-2\,c)}-150\,e^{(-4\,dx-4\,c)}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((25*e^(-2*d*x - 2*c) - 150*e^(-
4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x
+ 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c
) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) -
 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x
 - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c)
+ 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + a^3*cosh(d*x + c)/d
```

mupad [B] time = 2.95, size = 189, normalized size = 0.71

$$\frac{8960\,a^3\cosh\left(c+d\,x\right)+\frac{3675\,b^3\sinh(2\,c+2\,d\,x)}{2}-525\,b^3\sinh\left(4\,c+4\,d\,x\right)+\frac{525\,b^3\sinh(6\,c+6\,d\,x)}{4}-\frac{175\,b^3\sinh(8\,c+8\,d\,x)}{8}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^3)^3,x)
```

```
[Out] (8960*a^3*cosh(c + d*x) + (3675*b^3*sinh(2*c + 2*d*x))/2 - 525*b^3*sinh(4*c
 + 4*d*x) + (525*b^3*sinh(6*c + 6*d*x))/4 - (175*b^3*sinh(8*c + 8*d*x))/8 +
 (7*b^3*sinh(10*c + 10*d*x))/4 + 2940*a*b^2*cosh(3*c + 3*d*x) - 588*a*b^2*c
osh(5*c + 5*d*x) + 60*a*b^2*cosh(7*c + 7*d*x) - 6720*a^2*b*sinh(2*c + 2*d*x
) + 840*a^2*b*sinh(4*c + 4*d*x) - 14700*a*b^2*cosh(c + d*x) - 2205*b^3*d*x
+ 10080*a^2*b*d*x)/(8960*d)
```

sympy [A] time = 21.92, size = 496, normalized size = 1.86

$$\begin{cases} \frac{a^3\cosh(c+dx)}{d} + \frac{9a^2bx\sinh^4(c+dx)}{8} - \frac{9a^2bx\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{9a^2bx\cosh^4(c+dx)}{8} + \frac{15a^2b\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{9a^2b\sinh(\\ x\left(a+b\sinh^3(c)\right)^3\sinh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*
x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*
a**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c
 + d*x)**3/(8*d) + 3*a*b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 6*a*b**2*sin
h(c + d*x)**4*cosh(c + d*x)**3/d + 24*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)
**5/(5*d) - 48*a*b**2*cosh(c + d*x)**7/(35*d) + 63*b**3*x*sinh(c + d*x)**10
/256 - 315*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 315*b**3*x*sinh(c
 + d*x)**6*cosh(c + d*x)**4/128 - 315*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)
**6/128 + 315*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 63*b**3*x*cosh
(c + d*x)**10/256 + 193*b**3*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) - 237*b
**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 21*b**3*sinh(c + d*x)**5*co
sh(c + d*x)**5/(10*d) - 147*b**3*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d)
+ 63*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh
(c)**3)**3*sinh(c), True))
```

3.163 $\int \left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=204

$$a^3x+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{3a^2b\cosh(c+dx)}{d}+\frac{ab^2\sinh^5(c+dx)\cosh(c+dx)}{2d}-\frac{5ab^2\sinh^3(c+dx)\cosh(c+dx)}{8d}$$

```
[Out] a^3*x-15/16*a*b^2*x-3*a^2*b*cosh(d*x+c)/d+b^3*cosh(d*x+c)/d+a^2*b*cosh(d*x+
c)^3/d-4/3*b^3*cosh(d*x+c)^3/d+6/5*b^3*cosh(d*x+c)^5/d-4/7*b^3*cosh(d*x+c)^
7/d+1/9*b^3*cosh(d*x+c)^9/d+15/16*a*b^2*cosh(d*x+c)*sinh(d*x+c)/d-5/8*a*b^2
*cosh(d*x+c)*sinh(d*x+c)^3/d+1/2*a*b^2*cosh(d*x+c)*sinh(d*x+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3213, 2633, 2635, 8}

$$\frac{a^2b\cosh^3(c+dx)}{d}-\frac{3a^2b\cosh(c+dx)}{d}+a^3x+\frac{ab^2\sinh^5(c+dx)\cosh(c+dx)}{2d}-\frac{5ab^2\sinh^3(c+dx)\cosh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] a^3*x - (15*a*b^2*x)/16 - (3*a^2*b*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x])/d
 + (a^2*b*Cosh[c + d*x]^3)/d - (4*b^3*Cosh[c + d*x]^3)/(3*d) + (6*b^3*Cosh[
c + d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(
9*d) + (15*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*a*b^2*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(8*d) + (a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \left(a+b\sinh^3(c+dx)\right)^3 dx = \int \left(a^3+3a^2b\sinh^3(c+dx)+3ab^2\sinh^6(c+dx)+b^3\sinh^9(c+dx)\right) dx$$
$$= a^3x+\left(3a^2b\right)\int \sinh^3(c+dx)\,dx+\left(3ab^2\right)\int \sinh^6(c+dx)\,dx+b^3\int \sinh^9(c+$$
$$= a^3x+\frac{ab^2\cosh(c+dx)\sinh^5(c+dx)}{2d}-\frac{1}{2}\left(5ab^2\right)\int \sinh^4(c+dx)\,dx-\frac{\left(3a^2b\right)\mathrm{Su}}{}$$
$$= a^3x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{4b^3\cosh^3(c+}{3d}$$
$$= a^3x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{4b^3\cosh^3(c+}{3d}$$
$$= a^3x-\frac{15}{16}ab^2x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{4b^3}{}$$

Mathematica [A] time = 0.26, size = 159, normalized size = 0.78

$$\frac{80640a^3c+80640a^3dx+1260\left(16a^2b-7b^3\right)\cosh(3(c+dx))+5670b\left(7b^2-32a^2\right)\cosh(c+dx)+56700ab^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (80640*a^3*c - 75600*a*b^2*c + 80640*a^3*d*x - 75600*a*b^2*d*x + 5670*b*(-3
2*a^2 + 7*b^2)*Cosh[c + d*x] + 1260*(16*a^2*b - 7*b^3)*Cosh[3*(c + d*x)] +
2268*b^3*Cosh[5*(c + d*x)] - 405*b^3*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(c +
 d*x)] + 56700*a*b^2*Sinh[2*(c + d*x)] - 11340*a*b^2*Sinh[4*(c + d*x)] + 12
60*a*b^2*Sinh[6*(c + d*x)])/(80640*d)
```

fricas [B] time = 0.45, size = 380, normalized size = 1.86

$$\frac{35\,b^3\cosh\left(dx+c\right)^9+315\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8-405\,b^3\cosh\left(dx+c\right)^7+7560\,ab^2\cosh\left(dx+c\right)\sinh\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^3*cosh(d*x + c)^9 + 315*b^3*cosh(d*x + c)*sinh(d*x + c)^8 - 4
05*b^3*cosh(d*x + c)^7 + 7560*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2268*b^
3*cosh(d*x + c)^5 + 105*(28*b^3*cosh(d*x + c)^3 - 27*b^3*cosh(d*x + c))*sin
h(d*x + c)^6 + 315*(14*b^3*cosh(d*x + c)^5 - 45*b^3*cosh(d*x + c)^3 + 36*b^
3*cosh(d*x + c))*sinh(d*x + c)^4 + 1260*(16*a^2*b - 7*b^3)*cosh(d*x + c)^3
+ 5040*(5*a*b^2*cosh(d*x + c)^3 - 9*a*b^2*cosh(d*x + c))*sinh(d*x + c)^3 +
5040*(16*a^3 - 15*a*b^2)*d*x + 315*(4*b^3*cosh(d*x + c)^7 - 27*b^3*cosh(d*x
 + c)^5 + 72*b^3*cosh(d*x + c)^3 + 12*(16*a^2*b - 7*b^3)*cosh(d*x + c))*sin
h(d*x + c)^2 - 5670*(32*a^2*b - 7*b^3)*cosh(d*x + c) + 7560*(a*b^2*cosh(d*x
 + c)^5 - 6*a*b^2*cosh(d*x + c)^3 + 15*a*b^2*cosh(d*x + c))*sinh(d*x + c))/
d
```

giac [A] time = 0.19, size = 327, normalized size = 1.60

$$\frac{b^3e^{(9\,dx+9\,c)}}{4608\,d}-\frac{9\,b^3e^{(7\,dx+7\,c)}}{3584\,d}+\frac{ab^2e^{(6\,dx+6\,c)}}{128\,d}+\frac{9\,b^3e^{(5\,dx+5\,c)}}{640\,d}-\frac{9\,ab^2e^{(4\,dx+4\,c)}}{128\,d}+\frac{45\,ab^2e^{(2\,dx+2\,c)}}{128\,d}-\frac{45\,ab^2e^{(-2\,dx-2\,c)}}{128\,d}+\frac{9\,ab^2e^{(}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/4608*b^3*e^(9*d*x + 9*c)/d - 9/3584*b^3*e^(7*d*x + 7*c)/d + 1/128*a*b^2*e
^(6*d*x + 6*c)/d + 9/640*b^3*e^(5*d*x + 5*c)/d - 9/128*a*b^2*e^(4*d*x + 4*c
)/d + 45/128*a*b^2*e^(2*d*x + 2*c)/d - 45/128*a*b^2*e^(-2*d*x - 2*c)/d + 9/
128*a*b^2*e^(-4*d*x - 4*c)/d + 9/640*b^3*e^(-5*d*x - 5*c)/d - 1/128*a*b^2*e
^(-6*d*x - 6*c)/d - 9/3584*b^3*e^(-7*d*x - 7*c)/d + 1/4608*b^3*e^(-9*d*x -
9*c)/d + 1/16*(16*a^3 - 15*a*b^2)*x + 1/128*(16*a^2*b - 7*b^3)*e^(3*d*x + 3
*c)/d - 9/256*(32*a^2*b - 7*b^3)*e^(d*x + c)/d - 9/256*(32*a^2*b - 7*b^3)*e
^(-d*x - c)/d + 1/128*(16*a^2*b - 7*b^3)*e^(-3*d*x - 3*c)/d
```

maple [A] time = 0.05, size = 141, normalized size = 0.69

$$\frac{b^3\left(\frac{128}{315}+\frac{\left(\sinh^8(dx+c)\right)}{9}-\frac{8\left(\sinh^6(dx+c)\right)}{63}+\frac{16\left(\sinh^4(dx+c)\right)}{105}-\frac{64\left(\sinh^2(dx+c)\right)}{315}\right)\cosh(dx+c)+3ab^2\left(\left(\left(\frac{\left(\sinh^5(dx+c)\right)}{6}-\frac{5(\sin}{}\right.\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(b^3*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4
-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*
x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+3*a^2*b*(-2/3+1/3*sin
h(d*x+c)^2)*cosh(d*x+c)+a^3*(d*x+c))
```

maxima [A] time = 0.33, size = 280, normalized size = 1.37

$$a^3x-\frac{1}{161280}b^3\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\frac{39690\,e}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*
e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*
e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*
x - 7*c) + 35*e^(-9*d*x - 9*c))/d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*
e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x -
 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/8*a^2*b*(e^(3*d*x + 3
*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 0.93, size = 164, normalized size = 0.80

$$\frac{d\,x\,a^3+a^2\,b\,\cosh\left(c+d\,x\right)^3-3\,a^2\,b\,\cosh\left(c+d\,x\right)+\frac{\sinh(c+d\,x)\,a\,b^2\,\cosh(c+d\,x)^5}{2}-\frac{13\,\sinh(c+d\,x)\,a\,b^2\,\cosh(c+d\,x)^3}{8}+\frac{33\,\sinh}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3,x)
```

```
[Out] (b^3*cosh(c + d*x) - (4*b^3*cosh(c + d*x)^3)/3 + (6*b^3*cosh(c + d*x)^5)/5
- (4*b^3*cosh(c + d*x)^7)/7 + (b^3*cosh(c + d*x)^9)/9 + a^2*b*cosh(c + d*x)
^3 - 3*a^2*b*cosh(c + d*x) + a^3*d*x + (33*a*b^2*cosh(c + d*x)*sinh(c + d*x
))/16 - (15*a*b^2*d*x)/16 - (13*a*b^2*cosh(c + d*x)^3*sinh(c + d*x))/8 + (a
*b^2*cosh(c + d*x)^5*sinh(c + d*x))/2)/d
```

sympy [A] time = 13.92, size = 340, normalized size = 1.67

$$\begin{cases} a^3x+\frac{3a^2b\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{15ab^2x\sinh^6(c+dx)}{16}-\frac{45ab^2x\sinh^4(c+dx)\cosh^2(c+dx)}{16}+\frac{45ab^2x\sinh^2(c+}{16} \\ x\left(a+b\sinh^3(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*b*co
sh(c + d*x)**3/d + 15*a*b**2*x*sinh(c + d*x)**6/16 - 45*a*b**2*x*sinh(c + d
*x)**4*cosh(c + d*x)**2/16 + 45*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/
16 - 15*a*b**2*x*cosh(c + d*x)**6/16 + 33*a*b**2*sinh(c + d*x)**5*cosh(c +
d*x)/(16*d) - 5*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a*b**2*
sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + b**3*sinh(c + d*x)**8*cosh(c + d*x)
/d - 8*b**3*sinh(c + d*x)**6*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)
**4*cosh(c + d*x)**5/(5*d) - 64*b**3*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*
d) + 128*b**3*cosh(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**3
, True))
```

3.164 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=201

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}a^2bx+\frac{3ab^2\cosh^5(c+dx)}{5d}-\frac{2ab^2\cosh^3(c+dx)}{d}$$

```
[Out] -3/2*a^2*b*x+35/128*b^3*x-a^3*arctanh(cosh(d*x+c))/d+3*a*b^2*cosh(d*x+c)/d-
2*a*b^2*cosh(d*x+c)^3/d+3/5*a*b^2*cosh(d*x+c)^5/d+3/2*a^2*b*cosh(d*x+c)*sin
h(d*x+c)/d-35/128*b^3*cosh(d*x+c)*sinh(d*x+c)/d+35/192*b^3*cosh(d*x+c)*sinh
(d*x+c)^3/d-7/48*b^3*cosh(d*x+c)*sinh(d*x+c)^5/d+1/8*b^3*cosh(d*x+c)*sinh(d
*x+c)^7/d
```

Rubi [A] time = 0.19, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3220, 3770, 2635, 8, 2633}

$$\frac{3a^2b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh^5(c+dx)}{5d}-\frac{2ab^2\cosh^3(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-3*a^2*b*x)/2 + (35*b^3*x)/128 - (a^3*ArcTanh[Cosh[c + d*x]])/d + (3*a*b^2
*Cosh[c + d*x])/d - (2*a*b^2*Cosh[c + d*x]^3)/d + (3*a*b^2*Cosh[c + d*x]^5)
/(5*d) + (3*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (35*b^3*Cosh[c + d*x
]*Sinh[c + d*x])/(128*d) + (35*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) -
 (7*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*d) + (b^3*Cosh[c + d*x]*Sinh[c +
 d*x]^7)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \text{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = i\int\left(-ia^3\text{csch}(c+dx)-3ia^2b\sinh^2(c+dx)-3iab^2\sinh^5(c+dx)-\right.$$
$$= a^3\int \text{csch}(c+dx)\,dx+\left(3a^2b\right)\int \sinh^2(c+dx)\,dx+\left(3ab^2\right)\int \sinh^5$$
$$= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^3\,\text{co}}{}$$
$$= -\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab^2\cos}{}$$
$$= -\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab^2\cos}{}$$
$$= -\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab^2\cos}{}$$
$$= -\frac{3}{2}a^2bx+\frac{35b^3x}{128}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-$$

Mathematica [A] time = 0.22, size = 158, normalized size = 0.79

$$\frac{15360a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+11520a^2b\sinh(2(c+dx))-23040a^2bc-23040a^2bdx+28800ab^2\cosh(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-23040*a^2*b*c + 4200*b^3*c - 23040*a^2*b*d*x + 4200*b^3*d*x + 28800*a*b^2
*Cosh[c + d*x] - 4800*a*b^2*Cosh[3*(c + d*x)] + 576*a*b^2*Cosh[5*(c + d*x)]
 + 15360*a^3*Log[Tanh[(c + d*x)/2]] + 11520*a^2*b*Sinh[2*(c + d*x)] - 3360*
b^3*Sinh[2*(c + d*x)] + 840*b^3*Sinh[4*(c + d*x)] - 160*b^3*Sinh[6*(c + d*x
)] + 15*b^3*Sinh[8*(c + d*x)])/(15360*d)
```

fricas [B] time = 0.60, size = 2609, normalized size = 12.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/30720*(15*b^3*cosh(d*x + c)^16 + 240*b^3*cosh(d*x + c)*sinh(d*x + c)^15 +
 15*b^3*sinh(d*x + c)^16 - 160*b^3*cosh(d*x + c)^14 + 576*a*b^2*cosh(d*x +
c)^13 + 840*b^3*cosh(d*x + c)^12 + 40*(45*b^3*cosh(d*x + c)^2 - 4*b^3)*sinh
(d*x + c)^14 - 4800*a*b^2*cosh(d*x + c)^11 + 16*(525*b^3*cosh(d*x + c)^3 -
140*b^3*cosh(d*x + c) + 36*a*b^2)*sinh(d*x + c)^13 + 4*(6825*b^3*cosh(d*x +
 c)^4 - 3640*b^3*cosh(d*x + c)^2 + 1872*a*b^2*cosh(d*x + c) + 210*b^3)*sinh
(d*x + c)^12 + 28800*a*b^2*cosh(d*x + c)^9 + 16*(4095*b^3*cosh(d*x + c)^5 -
 3640*b^3*cosh(d*x + c)^3 + 2808*a*b^2*cosh(d*x + c)^2 + 630*b^3*cosh(d*x +
 c) - 300*a*b^2)*sinh(d*x + c)^11 - 240*(192*a^2*b - 35*b^3)*d*x*cosh(d*x +
 c)^8 + 480*(24*a^2*b - 7*b^3)*cosh(d*x + c)^10 + 8*(15015*b^3*cosh(d*x + c
)^6 - 20020*b^3*cosh(d*x + c)^4 + 20592*a*b^2*cosh(d*x + c)^3 + 6930*b^3*co
sh(d*x + c)^2 - 6600*a*b^2*cosh(d*x + c) + 1440*a^2*b - 420*b^3)*sinh(d*x +
 c)^10 + 28800*a*b^2*cosh(d*x + c)^7 + 80*(2145*b^3*cosh(d*x + c)^7 - 4004*
b^3*cosh(d*x + c)^5 + 5148*a*b^2*cosh(d*x + c)^4 + 2310*b^3*cosh(d*x + c)^3
 - 3300*a*b^2*cosh(d*x + c)^2 + 360*a*b^2 + 60*(24*a^2*b - 7*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^9 + 6*(32175*b^3*cosh(d*x + c)^8 - 80080*b^3*cosh(d*x +
 c)^6 + 123552*a*b^2*cosh(d*x + c)^5 + 69300*b^3*cosh(d*x + c)^4 - 132000*a
```

```
*b^2*cosh(d*x + c)^3 + 43200*a*b^2*cosh(d*x + c) - 40*(192*a^2*b - 35*b^3)*
d*x + 3600*(24*a^2*b - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 - 4800*a*b^2
*cosh(d*x + c)^5 + 48*(3575*b^3*cosh(d*x + c)^9 - 11440*b^3*cosh(d*x + c)^7
 + 20592*a*b^2*cosh(d*x + c)^6 + 13860*b^3*cosh(d*x + c)^5 - 33000*a*b^2*co
sh(d*x + c)^4 + 21600*a*b^2*cosh(d*x + c)^2 - 40*(192*a^2*b - 35*b^3)*d*x*c
osh(d*x + c) + 1200*(24*a^2*b - 7*b^3)*cosh(d*x + c)^3 + 600*a*b^2)*sinh(d*
x + c)^7 - 840*b^3*cosh(d*x + c)^4 - 480*(24*a^2*b - 7*b^3)*cosh(d*x + c)^6
 + 24*(5005*b^3*cosh(d*x + c)^10 - 20020*b^3*cosh(d*x + c)^8 + 41184*a*b^2*
cosh(d*x + c)^7 + 32340*b^3*cosh(d*x + c)^6 - 92400*a*b^2*cosh(d*x + c)^5 +
 100800*a*b^2*cosh(d*x + c)^3 - 280*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^
2 + 4200*(24*a^2*b - 7*b^3)*cosh(d*x + c)^4 + 8400*a*b^2*cosh(d*x + c) - 48
0*a^2*b + 140*b^3)*sinh(d*x + c)^6 + 576*a*b^2*cosh(d*x + c)^3 + 16*(4095*b
^3*cosh(d*x + c)^11 - 20020*b^3*cosh(d*x + c)^9 + 46332*a*b^2*cosh(d*x + c)
^8 + 41580*b^3*cosh(d*x + c)^7 - 138600*a*b^2*cosh(d*x + c)^6 + 226800*a*b^
2*cosh(d*x + c)^4 - 840*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^3 + 7560*(24
*a^2*b - 7*b^3)*cosh(d*x + c)^5 + 37800*a*b^2*cosh(d*x + c)^2 - 300*a*b^2 -
 180*(24*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 160*b^3*cosh(d*x +
 c)^2 + 20*(1365*b^3*cosh(d*x + c)^12 - 8008*b^3*cosh(d*x + c)^10 + 20592*a
*b^2*cosh(d*x + c)^9 + 20790*b^3*cosh(d*x + c)^8 - 79200*a*b^2*cosh(d*x + c
)^7 + 181440*a*b^2*cosh(d*x + c)^5 - 840*(192*a^2*b - 35*b^3)*d*x*cosh(d*x
+ c)^4 + 5040*(24*a^2*b - 7*b^3)*cosh(d*x + c)^6 + 50400*a*b^2*cosh(d*x + c
)^3 - 1200*a*b^2*cosh(d*x + c) - 42*b^3 - 360*(24*a^2*b - 7*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 + 16*(525*b^3*cosh(d*x + c)^13 - 3640*b^3*cosh(d*x +
 c)^11 + 10296*a*b^2*cosh(d*x + c)^10 + 11550*b^3*cosh(d*x + c)^9 - 49500*a
*b^2*cosh(d*x + c)^8 + 151200*a*b^2*cosh(d*x + c)^6 - 840*(192*a^2*b - 35*b
^3)*d*x*cosh(d*x + c)^5 + 3600*(24*a^2*b - 7*b^3)*cosh(d*x + c)^7 + 63000*a
*b^2*cosh(d*x + c)^4 - 3000*a*b^2*cosh(d*x + c)^2 - 210*b^3*cosh(d*x + c) -
 600*(24*a^2*b - 7*b^3)*cosh(d*x + c)^3 + 36*a*b^2)*sinh(d*x + c)^3 - 15*b^
3 + 8*(225*b^3*cosh(d*x + c)^14 - 1820*b^3*cosh(d*x + c)^12 + 5616*a*b^2*co
sh(d*x + c)^11 + 6930*b^3*cosh(d*x + c)^10 - 33000*a*b^2*cosh(d*x + c)^9 +
129600*a*b^2*cosh(d*x + c)^7 - 840*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^6
 + 2700*(24*a^2*b - 7*b^3)*cosh(d*x + c)^8 + 75600*a*b^2*cosh(d*x + c)^5 -
6000*a*b^2*cosh(d*x + c)^3 - 630*b^3*cosh(d*x + c)^2 - 900*(24*a^2*b - 7*b^
3)*cosh(d*x + c)^4 + 216*a*b^2*cosh(d*x + c) + 20*b^3)*sinh(d*x + c)^2 - 30
720*(a^3*cosh(d*x + c)^8 + 8*a^3*cosh(d*x + c)^7*sinh(d*x + c) + 28*a^3*cos
h(d*x + c)^6*sinh(d*x + c)^2 + 56*a^3*cosh(d*x + c)^5*sinh(d*x + c)^3 + 70*
a^3*cosh(d*x + c)^4*sinh(d*x + c)^4 + 56*a^3*cosh(d*x + c)^3*sinh(d*x + c)^
5 + 28*a^3*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*a^3*cosh(d*x + c)*sinh(d*x +
 c)^7 + a^3*sinh(d*x + c)^8)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 30720
*(a^3*cosh(d*x + c)^8 + 8*a^3*cosh(d*x + c)^7*sinh(d*x + c) + 28*a^3*cosh(d
*x + c)^6*sinh(d*x + c)^2 + 56*a^3*cosh(d*x + c)^5*sinh(d*x + c)^3 + 70*a^3
*cosh(d*x + c)^4*sinh(d*x + c)^4 + 56*a^3*cosh(d*x + c)^3*sinh(d*x + c)^5 +
 28*a^3*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*a^3*cosh(d*x + c)*sinh(d*x + c)
^7 + a^3*sinh(d*x + c)^8)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 16*(15*b
^3*cosh(d*x + c)^15 - 140*b^3*cosh(d*x + c)^13 + 468*a*b^2*cosh(d*x + c)^12
 + 630*b^3*cosh(d*x + c)^11 - 3300*a*b^2*cosh(d*x + c)^10 + 16200*a*b^2*cos
h(d*x + c)^8 - 120*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^7 + 300*(24*a^2*b
 - 7*b^3)*cosh(d*x + c)^9 + 12600*a*b^2*cosh(d*x + c)^6 - 1500*a*b^2*cosh(d
*x + c)^4 - 210*b^3*cosh(d*x + c)^3 - 180*(24*a^2*b - 7*b^3)*cosh(d*x + c)^
5 + 108*a*b^2*cosh(d*x + c)^2 + 20*b^3*cosh(d*x + c))*sinh(d*x + c))/(d*cos
h(d*x + c)^8 + 8*d*cosh(d*x + c)^7*sinh(d*x + c) + 28*d*cosh(d*x + c)^6*sin
h(d*x + c)^2 + 56*d*cosh(d*x + c)^5*sinh(d*x + c)^3 + 70*d*cosh(d*x + c)^4*
sinh(d*x + c)^4 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)
^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8)
```

giac [A] time = 0.28, size = 279, normalized size = 1.39

$$15\,b^3e^{(8\,dx+8\,c)} - 160\,b^3e^{(6\,dx+6\,c)} + 576\,ab^2e^{(5\,dx+5\,c)} + 840\,b^3e^{(4\,dx+4\,c)} - 4800\,ab^2e^{(3\,dx+3\,c)} + 11520\,a^2be^{(2\,dx+2\,c)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/30720*(15*b^3*e^(8*d*x + 8*c) - 160*b^3*e^(6*d*x + 6*c) + 576*a*b^2*e^(5*
d*x + 5*c) + 840*b^3*e^(4*d*x + 4*c) - 4800*a*b^2*e^(3*d*x + 3*c) + 11520*a
^2*b*e^(2*d*x + 2*c) - 3360*b^3*e^(2*d*x + 2*c) + 28800*a*b^2*e^(d*x + c) -
 30720*a^3*log(e^(d*x + c) + 1) + 30720*a^3*log(abs(e^(d*x + c) - 1)) - 240
*(192*a^2*b - 35*b^3)*(d*x + c) + (28800*a*b^2*e^(7*d*x + 7*c) - 4800*a*b^2
*e^(5*d*x + 5*c) - 840*b^3*e^(4*d*x + 4*c) + 576*a*b^2*e^(3*d*x + 3*c) + 16
0*b^3*e^(2*d*x + 2*c) - 15*b^3 - 480*(24*a^2*b - 7*b^3)*e^(6*d*x + 6*c))*e^
(-8*d*x - 8*c))/d
```

maple [A] time = 0.15, size = 138, normalized size = 0.69

$$\frac{-2a^3\operatorname{arctanh}\left(e^{dx+c}\right)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a\,b^2\left(\frac{8}{15}+\frac{\left(\sinh^4(dx+c)\right)}{5}-\frac{4\left(\sinh^2(dx+c)\right)}{15}\right)\cosh(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*
x-1/2*c)+3*a*b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+b^
3*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d
*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c))
```

maxima [A] time = 0.33, size = 257, normalized size = 1.28

$$-\frac{3}{8}a^2b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{6144}b^3\left(\frac{\left(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c)}-3\right)e^{(8dx+8c)}}{d}-\frac{168}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/6144*b^3*((32
*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d
*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*
c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/160*a*b^2*(3*e^(5*d*x
 + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d -
 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^3*log(tanh(1/2*d*x + 1/2
*c))/d
```

mupad [B] time = 0.50, size = 315, normalized size = 1.57

$$\frac{7\,b^3\,e^{4\,c+4\,d\,x}}{256\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\sqrt{a^6}}{\sqrt{-d^2}}-\frac{7\,b^3\,e^{-4\,c-4\,d\,x}}{256\,d}-x\left(\frac{3\,a^2\,b}{2}-\frac{35\,b^3}{128}\right)+\frac{b^3\,e^{-6\,c-6\,d\,x}}{192\,d}-\frac{b^3\,e^{6\,c+6\,d\,x}}{192\,d}-\frac{b^3\,e^{-8\,c-8}}{2048\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x),x)
```

```
[Out] (7*b^3*exp(4*c + 4*d*x))/(256*d) - (2*atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2
))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2) - (7*b^3*exp(- 4*c - 4*d*x))/
(256*d) - x*((3*a^2*b)/2 - (35*b^3)/128) + (b^3*exp(- 6*c - 6*d*x))/(192*d)
 - (b^3*exp(6*c + 6*d*x))/(192*d) - (b^3*exp(- 8*c - 8*d*x))/(2048*d) + (b^
3*exp(8*c + 8*d*x))/(2048*d) - (exp(- 2*c - 2*d*x)*(24*a^2*b - 7*b^3))/(64*
d) + (exp(2*c + 2*d*x)*(24*a^2*b - 7*b^3))/(64*d) + (15*a*b^2*exp(- c - d*x
```

```
))/(16*d) - (5*a*b^2*exp(- 3*c - 3*d*x))/(32*d) - (5*a*b^2*exp(3*c + 3*d*x)
)/(32*d) + (3*a*b^2*exp(- 5*c - 5*d*x))/(160*d) + (3*a*b^2*exp(5*c + 5*d*x)
)/(160*d) + (15*a*b^2*exp(c + d*x))/(16*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

3.165 $\int \text{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx$

Optimal. Leaf size=152

$$-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)}{d}+\frac{3ab^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9ab^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}ab^2x$$

```
[Out] 9/8*a*b^2*x+3*a^2*b*cosh(d*x+c)/d-b^3*cosh(d*x+c)/d+b^3*cosh(d*x+c)^3/d-3/5
*b^3*cosh(d*x+c)^5/d+1/7*b^3*cosh(d*x+c)^7/d-a^3*coth(d*x+c)/d-9/8*a*b^2*co
sh(d*x+c)*sinh(d*x+c)/d+3/4*a*b^2*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Rubi [A] time = 0.14, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3220, 3767, 8, 2638, 2635, 2633}

$$\frac{3a^2b\cosh(c+dx)}{d}-\frac{a^3\coth(c+dx)}{d}+\frac{3ab^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9ab^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}ab^2x+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (9*a*b^2*x)/8 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c + d*x])/d + (b^3*Co
sh[c + d*x]^3)/d - (3*b^3*Cosh[c + d*x]^5)/(5*d) + (b^3*Cosh[c + d*x]^7)/(7
*d) - (a^3*Coth[c + d*x])/d - (9*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) +
 (3*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx = -\int \left(-a^3\operatorname{csch}^2(c+dx)-3a^2b\sinh(c+dx)-3ab^2\sinh^4(c+dx\right.$$
$$= a^3\int \operatorname{csch}^2(c+dx)\,dx+\left(3a^2b\right)\int \sinh(c+dx)\,dx+\left(3ab^2\right)\int \mathrm{s}$$
$$= \frac{3a^2b\cosh(c+dx)}{d}+\frac{3ab^2\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}\left(9ab^2\right.$$
$$= \frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{d}-\frac{3b^3\,\mathrm{c}}{}$$
$$= \frac{9}{8}ab^2x+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{d}$$

Mathematica [A] time = 1.13, size = 140, normalized size = 0.92

$$\frac{-1120a^3\tanh\left(\frac{1}{2}(c+dx)\right)-1120a^3\coth\left(\frac{1}{2}(c+dx)\right)+35b\left(192a^2-35b^2\right)\cosh(c+dx)-1680ab^2\sinh(2(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (2520*a*b^2*c + 2520*a*b^2*d*x + 35*b*(192*a^2 - 35*b^2)*Cosh[c + d*x] + 24
5*b^3*Cosh[3*(c + d*x)] - 49*b^3*Cosh[5*(c + d*x)] + 5*b^3*Cosh[7*(c + d*x)
] - 1120*a^3*Coth[(c + d*x)/2] - 1680*a*b^2*Sinh[2*(c + d*x)] + 210*a*b^2*S
inh[4*(c + d*x)] - 1120*a^3*Tanh[(c + d*x)/2])/(2240*d)
```

fricas [B] time = 0.44, size = 302, normalized size = 1.99

$$\frac{20\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+105\,ab^2\cosh\left(dx+c\right)^5+525\,ab^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-945\,ab^2\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(20*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 105*a*b^2*cosh(d*x + c)^5 +
525*a*b^2*cosh(d*x + c)*sinh(d*x + c)^4 - 945*a*b^2*cosh(d*x + c)^3 + 2*(70
*b^3*cosh(d*x + c)^3 - 81*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(35*b^3*co
sh(d*x + c)^5 - 135*b^3*cosh(d*x + c)^3 + 147*b^3*cosh(d*x + c))*sinh(d*x +
 c)^3 + 105*(10*a*b^2*cosh(d*x + c)^3 - 27*a*b^2*cosh(d*x + c))*sinh(d*x +
c)^2 - 280*(8*a^3 - 3*a*b^2)*cosh(d*x + c) + 2*(10*b^3*cosh(d*x + c)^7 - 81
*b^3*cosh(d*x + c)^5 + 294*b^3*cosh(d*x + c)^3 + 1260*a*b^2*d*x + 1120*a^3
+ 105*(32*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*sinh(d*x + c))
```

giac [A] time = 0.30, size = 276, normalized size = 1.82

$$\frac{5040\left(dx+c\right)ab^2+5\,b^3e^{(7\,dx+7\,c)}-49\,b^3e^{(5\,dx+5\,c)}+210\,ab^2e^{(4\,dx+4\,c)}+245\,b^3e^{(3\,dx+3\,c)}-1680\,ab^2e^{(2\,dx+2\,c)}+67}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5040*(d*x + c)*a*b^2 + 5*b^3*e^(7*d*x + 7*c) - 49*b^3*e^(5*d*x + 5*
c) + 210*a*b^2*e^(4*d*x + 4*c) + 245*b^3*e^(3*d*x + 3*c) - 1680*a*b^2*e^(2*
d*x + 2*c) + 6720*a^2*b*e^(d*x + c) - 1225*b^3*e^(d*x + c) - (1890*a*b^2*e^
(5*d*x + 5*c) + 294*b^3*e^(4*d*x + 4*c) - 210*a*b^2*e^(3*d*x + 3*c) - 54*b^
3*e^(2*d*x + 2*c) + 5*b^3 - 35*(192*a^2*b - 35*b^3)*e^(8*d*x + 8*c) + 560*(
16*a^3 - 3*a*b^2)*e^(7*d*x + 7*c) + 210*(32*a^2*b - 7*b^3)*e^(6*d*x + 6*c))
*e^(-7*d*x - 7*c)/((e^(d*x + c) + 1)*(e^(d*x + c) - 1)))/d
```

maple [A] time = 0.14, size = 111, normalized size = 0.73

$$\frac{-a^3 \coth(dx+c) + 3a^2 b \cosh(dx+c) + 3ab^2\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8}\right) + b^3\left(-\frac{16}{35} + \right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*cosh(d*x+c)+3*a*b^2*((1/4*sinh(d*x+c)^3-3/8*s
inh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b^3*(-16/35+1/7*sinh(d*x+c)^6-6/35*s
inh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [A] time = 0.32, size = 220, normalized size = 1.45

$$\frac{3}{64}\,ab^2\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) - \frac{1}{4480}\,b^3\left(\frac{\left(49\,e^{(-2\,dx-2\,c)} - 245\,e^{(-4\,dx-4\,c)} + 1225\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4
*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x
- c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d)
+ 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a^3/(d*(e^(-2*d*x - 2*c) -
 1))
```

mupad [B] time = 0.40, size = 252, normalized size = 1.66

$$\frac{e^{c+d\,x}\,\left(192\,a^2\,b - 35\,b^3\right)}{128\,d} - \frac{2\,a^3}{d\,\left(e^{2\,c+2\,d\,x} - 1\right)} + \frac{7\,b^3\,e^{-3\,c-3\,d\,x}}{128\,d} + \frac{7\,b^3\,e^{3\,c+3\,d\,x}}{128\,d} - \frac{7\,b^3\,e^{-5\,c-5\,d\,x}}{640\,d} - \frac{7\,b^3\,e^{5\,c+5\,d\,x}}{640\,d} + \frac{b^3\,e^{-7\,c-7\,d}}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^2,x)
```

```
[Out] (exp(c + d*x)*(192*a^2*b - 35*b^3))/(128*d) - (2*a^3)/(d*(exp(2*c + 2*d*x)
- 1)) + (7*b^3*exp(- 3*c - 3*d*x))/(128*d) + (7*b^3*exp(3*c + 3*d*x))/(128*
d) - (7*b^3*exp(- 5*c - 5*d*x))/(640*d) - (7*b^3*exp(5*c + 5*d*x))/(640*d)
+ (b^3*exp(- 7*c - 7*d*x))/(896*d) + (b^3*exp(7*c + 7*d*x))/(896*d) + (exp(
- c - d*x)*(192*a^2*b - 35*b^3))/(128*d) + (9*a*b^2*x)/8 + (3*a*b^2*exp(- 2
*c - 2*d*x))/(8*d) - (3*a*b^2*exp(2*c + 2*d*x))/(8*d) - (3*a*b^2*exp(- 4*c
- 4*d*x))/(64*d) + (3*a*b^2*exp(4*c + 4*d*x))/(64*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

3.166 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=156

$$\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+3a^2bx+\frac{ab^2\cosh^3(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{b^3\sin}{}$$

```
[Out] 3*a^2*b*x-5/16*b^3*x+1/2*a^3*arctanh(cosh(d*x+c))/d-3*a*b^2*cosh(d*x+c)/d+a
*b^2*cosh(d*x+c)^3/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d+5/16*b^3*cosh(d*x+c)
*sinh(d*x+c)/d-5/24*b^3*cosh(d*x+c)*sinh(d*x+c)^3/d+1/6*b^3*cosh(d*x+c)*sin
h(d*x+c)^5/d
```

Rubi [A] time = 0.17, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3768, 3770, 2633, 2635, 8}

$$3a^2bx+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{ab^2\cosh^3(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{b^3\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] 3*a^2*b*x - (5*b^3*x)/16 + (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) - (3*a*b^2*Co
sh[c + d*x])/d + (a*b^2*Cosh[c + d*x]^3)/d - (a^3*Coth[c + d*x]*Csch[c + d*
x])/(2*d) + (5*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b^3*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(24*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\left(i\int\left(3ia^2b+ia^3\operatorname{csch}^3(c+dx)+3iab^2\sinh^3(c+dx)+ib^3\sinh^6(\right.\right.$$
$$= 3a^2bx+a^3\int\operatorname{csch}^3(c+dx)\,dx+\left(3ab^2\right)\int\sinh^3(c+dx)\,dx+b^3\int$$
$$= 3a^2bx-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b^3\cosh(c+dx)\sinh^5(c+d}{6d}$$
$$= 3a^2bx+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{ab^2\cosh^3}{d}$$
$$= 3a^2bx+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{ab^2\cosh^3}{d}$$
$$= 3a^2bx-\frac{5b^3x}{16}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{ab}{}$$

Mathematica [A] time = 3.44, size = 150, normalized size = 0.96

$$\frac{-24a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-24a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)-96a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+576a^2bc+576a^2bdx-432ab^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (576*a^2*b*c - 60*b^3*c + 576*a^2*b*d*x - 60*b^3*d*x - 432*a*b^2*Cosh[c + d
*x] + 48*a*b^2*Cosh[3*(c + d*x)] - 24*a^3*Csch[(c + d*x)/2]^2 - 96*a^3*Log[
Tanh[(c + d*x)/2]] - 24*a^3*Sech[(c + d*x)/2]^2 + 45*b^3*Sinh[2*(c + d*x)]
- 9*b^3*Sinh[4*(c + d*x)] + b^3*Sinh[6*(c + d*x)])/(192*d)
```

fricas [B] time = 0.63, size = 3627, normalized size = 23.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(b^3*cosh(d*x + c)^16 + 16*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + b^3*s
inh(d*x + c)^16 - 11*b^3*cosh(d*x + c)^14 + 48*a*b^2*cosh(d*x + c)^13 + 64*
b^3*cosh(d*x + c)^12 + (120*b^3*cosh(d*x + c)^2 - 11*b^3)*sinh(d*x + c)^14
- 528*a*b^2*cosh(d*x + c)^11 + 2*(280*b^3*cosh(d*x + c)^3 - 77*b^3*cosh(d*x
 + c) + 24*a*b^2)*sinh(d*x + c)^13 + (1820*b^3*cosh(d*x + c)^4 - 1001*b^3*c
osh(d*x + c)^2 + 624*a*b^2*cosh(d*x + c) + 64*b^3)*sinh(d*x + c)^12 + 4*(10
92*b^3*cosh(d*x + c)^5 - 1001*b^3*cosh(d*x + c)^3 + 936*a*b^2*cosh(d*x + c)
^2 + 192*b^3*cosh(d*x + c) - 132*a*b^2)*sinh(d*x + c)^11 - 48*(48*a^2*b - 5
*b^3)*d*x*cosh(d*x + c)^8 - 3*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x
+ c)^10 + (8008*b^3*cosh(d*x + c)^6 - 11011*b^3*cosh(d*x + c)^4 + 13728*a*b
^2*cosh(d*x + c)^3 + 4224*b^3*cosh(d*x + c)^2 - 5808*a*b^2*cosh(d*x + c) -
99*b^3 + 24*(48*a^2*b - 5*b^3)*d*x)*sinh(d*x + c)^10 - 96*(4*a^3 - 5*a*b^2)
*cosh(d*x + c)^9 + 2*(5720*b^3*cosh(d*x + c)^7 - 11011*b^3*cosh(d*x + c)^5
+ 17160*a*b^2*cosh(d*x + c)^4 + 7040*b^3*cosh(d*x + c)^3 - 14520*a*b^2*cosh
(d*x + c)^2 - 192*a^3 + 240*a*b^2 - 15*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*
cosh(d*x + c))*sinh(d*x + c)^9 + 3*(4290*b^3*cosh(d*x + c)^8 - 11011*b^3*co
```

```
sh(d*x + c)^6 + 20592*a*b^2*cosh(d*x + c)^5 + 10560*b^3*cosh(d*x + c)^4 - 2
9040*a*b^2*cosh(d*x + c)^3 - 16*(48*a^2*b - 5*b^3)*d*x - 45*(33*b^3 - 8*(48
*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^2 - 288*(4*a^3 - 5*a*b^2)*cosh(d*x + c))
*sinh(d*x + c)^8 - 528*a*b^2*cosh(d*x + c)^5 - 96*(4*a^3 - 5*a*b^2)*cosh(d*
x + c)^7 + 8*(1430*b^3*cosh(d*x + c)^9 - 4719*b^3*cosh(d*x + c)^7 + 10296*a
*b^2*cosh(d*x + c)^6 + 6336*b^3*cosh(d*x + c)^5 - 21780*a*b^2*cosh(d*x + c)
^4 - 48*(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c) - 45*(33*b^3 - 8*(48*a^2*b - 5
*b^3)*d*x)*cosh(d*x + c)^3 - 48*a^3 + 60*a*b^2 - 432*(4*a^3 - 5*a*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^7 - 64*b^3*cosh(d*x + c)^4 + 3*(33*b^3 + 8*(48*a
^2*b - 5*b^3)*d*x)*cosh(d*x + c)^6 + (8008*b^3*cosh(d*x + c)^10 - 33033*b^3
*cosh(d*x + c)^8 + 82368*a*b^2*cosh(d*x + c)^7 + 59136*b^3*cosh(d*x + c)^6
- 243936*a*b^2*cosh(d*x + c)^5 - 1344*(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c)^
2 - 630*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^4 - 8064*(4*a^3 -
 5*a*b^2)*cosh(d*x + c)^3 + 99*b^3 + 24*(48*a^2*b - 5*b^3)*d*x - 672*(4*a^3
 - 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 48*a*b^2*cosh(d*x + c)^3 + 2*(
2184*b^3*cosh(d*x + c)^11 - 11011*b^3*cosh(d*x + c)^9 + 30888*a*b^2*cosh(d*
x + c)^8 + 25344*b^3*cosh(d*x + c)^7 - 121968*a*b^2*cosh(d*x + c)^6 - 1344*
(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c)^3 - 378*(33*b^3 - 8*(48*a^2*b - 5*b^3)
*d*x)*cosh(d*x + c)^5 - 6048*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^4 - 264*a*b^2
- 1008*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^2 + 9*(33*b^3 + 8*(48*a^2*b - 5*b^3)
*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 11*b^3*cosh(d*x + c)^2 + (1820*b^3*c
osh(d*x + c)^12 - 11011*b^3*cosh(d*x + c)^10 + 34320*a*b^2*cosh(d*x + c)^9
+ 31680*b^3*cosh(d*x + c)^8 - 174240*a*b^2*cosh(d*x + c)^7 - 3360*(48*a^2*b
 - 5*b^3)*d*x*cosh(d*x + c)^4 - 630*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cos
h(d*x + c)^6 - 12096*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^5 - 2640*a*b^2*cosh(d*
x + c) - 3360*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^3 - 64*b^3 + 45*(33*b^3 + 8*(
48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(140*b^3*cosh(d
*x + c)^13 - 1001*b^3*cosh(d*x + c)^11 + 3432*a*b^2*cosh(d*x + c)^10 + 3520
*b^3*cosh(d*x + c)^9 - 21780*a*b^2*cosh(d*x + c)^8 - 672*(48*a^2*b - 5*b^3)
*d*x*cosh(d*x + c)^5 - 90*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)
^7 - 2016*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^6 - 1320*a*b^2*cosh(d*x + c)^2 -
840*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^4 - 64*b^3*cosh(d*x + c) + 15*(33*b^3 +
 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^3 + 12*a*b^2)*sinh(d*x + c)^3 - b^
3 + (120*b^3*cosh(d*x + c)^14 - 1001*b^3*cosh(d*x + c)^12 + 3744*a*b^2*cosh
(d*x + c)^11 + 4224*b^3*cosh(d*x + c)^10 - 29040*a*b^2*cosh(d*x + c)^9 - 13
44*(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c)^6 - 135*(33*b^3 - 8*(48*a^2*b - 5*b
^3)*d*x)*cosh(d*x + c)^8 - 3456*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^7 - 5280*a*
b^2*cosh(d*x + c)^3 - 2016*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^5 - 384*b^3*cosh
(d*x + c)^2 + 45*(33*b^3 + 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^4 + 144*
a*b^2*cosh(d*x + c) + 11*b^3)*sinh(d*x + c)^2 + 192*(a^3*cosh(d*x + c)^10 +
 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sinh(d*x + c)^10 - 2*a^3*cosh(d
*x + c)^8 + a^3*cosh(d*x + c)^6 + (45*a^3*cosh(d*x + c)^2 - 2*a^3)*sinh(d*x
 + c)^8 + 8*(15*a^3*cosh(d*x + c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x + c)^7
+ (210*a^3*cosh(d*x + c)^4 - 56*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6
+ 2*(126*a^3*cosh(d*x + c)^5 - 56*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c)
)*sinh(d*x + c)^5 + 5*(42*a^3*cosh(d*x + c)^6 - 28*a^3*cosh(d*x + c)^4 + 3*
a^3*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(30*a^3*cosh(d*x + c)^7 - 28*a^3*c
osh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3)*sinh(d*x + c)^3 + (45*a^3*cosh(d*x
+ c)^8 - 56*a^3*cosh(d*x + c)^6 + 15*a^3*cosh(d*x + c)^4)*sinh(d*x + c)^2 +
 2*(5*a^3*cosh(d*x + c)^9 - 8*a^3*cosh(d*x + c)^7 + 3*a^3*cosh(d*x + c)^5)*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 192*(a^3*cosh(d*x +
 c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sinh(d*x + c)^10 - 2*a^
3*cosh(d*x + c)^8 + a^3*cosh(d*x + c)^6 + (45*a^3*cosh(d*x + c)^2 - 2*a^3)*
sinh(d*x + c)^8 + 8*(15*a^3*cosh(d*x + c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x
 + c)^7 + (210*a^3*cosh(d*x + c)^4 - 56*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x
 + c)^6 + 2*(126*a^3*cosh(d*x + c)^5 - 56*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(
d*x + c))*sinh(d*x + c)^5 + 5*(42*a^3*cosh(d*x + c)^6 - 28*a^3*cosh(d*x + c
)^4 + 3*a^3*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(30*a^3*cosh(d*x + c)^7 -
28*a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3)*sinh(d*x + c)^3 + (45*a^3*c
```

```
osh(d*x + c)^8 - 56*a^3*cosh(d*x + c)^6 + 15*a^3*cosh(d*x + c)^4)*sinh(d*x
+ c)^2 + 2*(5*a^3*cosh(d*x + c)^9 - 8*a^3*cosh(d*x + c)^7 + 3*a^3*cosh(d*x
+ c)^5)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(8*b^3*co
sh(d*x + c)^15 - 77*b^3*cosh(d*x + c)^13 + 312*a*b^2*cosh(d*x + c)^12 + 384
*b^3*cosh(d*x + c)^11 - 2904*a*b^2*cosh(d*x + c)^10 - 192*(48*a^2*b - 5*b^3
)*d*x*cosh(d*x + c)^7 - 15*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c
)^9 - 432*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^8 - 1320*a*b^2*cosh(d*x + c)^4 -
336*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^6 - 128*b^3*cosh(d*x + c)^3 + 9*(33*b^3
 + 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^5 + 72*a*b^2*cosh(d*x + c)^2 + 1
1*b^3*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c
)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 - 2*d*cosh(d*x + c)^8 + (45*d*cosh(d
*x + c)^2 - 2*d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 - 2*d*cosh(d*x +
 c))*sinh(d*x + c)^7 + d*cosh(d*x + c)^6 + (210*d*cosh(d*x + c)^4 - 56*d*co
sh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d*x + c)^5 - 56*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 5*(42*d*cosh(d*x + c)^6 -
28*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(30*d*cosh(
d*x + c)^7 - 28*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3)*sinh(d*x + c)^3 +
(45*d*cosh(d*x + c)^8 - 56*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4)*sinh(d
*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 - 8*d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c
)^5)*sinh(d*x + c))
```

giac [B] time = 0.33, size = 289, normalized size = 1.85

$$\frac{b^3e^{(6\,dx+6\,c)} - 9\,b^3e^{(4\,dx+4\,c)} + 48\,ab^2e^{(3\,dx+3\,c)} + 45\,b^3e^{(2\,dx+2\,c)} - 432\,ab^2e^{(dx+c)} + 192\,a^3\log\left(e^{(dx+c)}+1\right) - 192\,a^3\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/384*(b^3*e^(6*d*x + 6*c) - 9*b^3*e^(4*d*x + 4*c) + 48*a*b^2*e^(3*d*x + 3*
c) + 45*b^3*e^(2*d*x + 2*c) - 432*a*b^2*e^(d*x + c) + 192*a^3*log(e^(d*x +
c) + 1) - 192*a^3*log(abs(e^(d*x + c) - 1)) + 24*(48*a^2*b - 5*b^3)*(d*x +
c) - (45*b^3*e^(8*d*x + 8*c) - 99*b^3*e^(6*d*x + 6*c) + 528*a*b^2*e^(5*d*x
+ 5*c) + 64*b^3*e^(4*d*x + 4*c) - 48*a*b^2*e^(3*d*x + 3*c) - 11*b^3*e^(2*d*
x + 2*c) + b^3 + 48*(8*a^3 + 9*a*b^2)*e^(9*d*x + 9*c) + 48*(8*a^3 - 19*a*b^
2)*e^(7*d*x + 7*c))*e^(-6*d*x - 6*c)/((e^(d*x + c) + 1)^2*(e^(d*x + c) - 1)
^2))/d
```

maple [A] time = 0.14, size = 115, normalized size = 0.74

$$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2} + \operatorname{arctanh}\left(e^{dx+c}\right)\right) + 3a^2b\,(dx+c) + 3a\,b^2\left(-\frac{2}{3} + \frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh\,(dx+c) + b^3\left(\left(\frac{\left(\sinh^5(\right.}{6}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(d*x+c)
+3*a*b^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*((1/6*sinh(d*x+c)^5-5/24*
sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c))
```

maxima [A] time = 0.33, size = 244, normalized size = 1.56

$$3\,a^2bx - \frac{1}{384}\,b^3\left(\frac{\left(9\,e^{(-2\,dx-2\,c)} - 45\,e^{(-4\,dx-4\,c)} - 1\right)e^{(6\,dx+6\,c)}}{d} + \frac{120\,(dx+c)}{d} + \frac{45\,e^{(-2\,dx-2\,c)} - 9\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] 3*a^2*b*x - 1/384*b^3*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/8*a*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))

mupad [B] time = 0.93, size = 290, normalized size = 1.86

$$x\left(3\,a^2\,b-\frac{5\,b^3}{16}\right)+\frac{\mathrm{atan}\left(\frac{a^3\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}-\frac{15\,b^3\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{128\,d}+\frac{15\,b^3\,{\mathrm{e}}^{2\,c+2\,d\,x}}{128\,d}+\frac{3\,b^3\,{\mathrm{e}}^{-4\,c-4\,d\,x}}{128\,d}-\frac{3\,b^3\,{\mathrm{e}}^{4\,c+4\,d\,x}}{128\,d}-\frac{b^3\,\mathrm{e}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^3,x)

[Out] x*(3*a^2*b - (5*b^3)/16) + (atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2) - (15*b^3*exp(- 2*c - 2*d*x))/(128*d) + (15*b^3*exp(2*c + 2*d*x))/(128*d) + (3*b^3*exp(- 4*c - 4*d*x))/(128*d) - (3*b^3*exp(4*c + 4*d*x))/(128*d) - (b^3*exp(- 6*c - 6*d*x))/(384*d) + (b^3*exp(6*c + 6*d*x))/(384*d) - (9*a*b^2*exp(- c - d*x))/(8*d) + (a*b^2*exp(- 3*c - 3*d*x))/(8*d) + (a*b^2*exp(3*c + 3*d*x))/(8*d) - (9*a*b^2*exp(c + d*x))/(8*d) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3)**3,x)

[Out] Timed out

3.167 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=129

$$-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}ab^2x+\frac{b^3\cos}{}$$

```
[Out] -3/2*a*b^2*x-3*a^2*b*arctanh(cosh(d*x+c))/d+b^3*cosh(d*x+c)/d-2/3*b^3*cosh(
d*x+c)^3/d+1/5*b^3*cosh(d*x+c)^5/d+a^3*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/
d+3/2*a*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3220, 3770, 3767, 2635, 8, 2633}

$$-\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}ab^2x+\frac{b^3\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cosh[c + d*x]])/d + (b^3*Cosh[c + d*x])/d
 - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) + (a^3*Coth[
c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = \int \left(3a^2b\operatorname{csch}(c+dx)+a^3\operatorname{csch}^4(c+dx)+3ab^2\sinh^2(c+dx)+\right.$$
$$= a^3\int \operatorname{csch}^4(c+dx)\,dx+\left(3a^2b\right)\int \operatorname{csch}(c+dx)\,dx+\left(3ab^2\right)\int \mathrm{s}$$
$$= -\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)\sinh(c+dx)}{2d}-$$
$$= -\frac{3}{2}ab^2x-\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^3\cosh(c+dx)}{d}-\frac{2b^3\,\mathrm{c}}{}$$

Mathematica [A] time = 0.44, size = 169, normalized size = 1.31

$$\frac{80a^3\tanh\left(\frac{1}{2}(c+dx)\right)+80a^3\coth\left(\frac{1}{2}(c+dx)\right)-5a^3\sinh(c+dx)\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)+80a^3\sinh^4\left(\frac{1}{2}(c+dx)\right)\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-360*a*b^2*c - 360*a*b^2*d*x + 150*b^3*Cosh[c + d*x] - 25*b^3*Cosh[3*(c +
d*x)] + 3*b^3*Cosh[5*(c + d*x)] + 80*a^3*Coth[(c + d*x)/2] + 720*a^2*b*Log[
Tanh[(c + d*x)/2]] + 80*a^3*Csch[c + d*x]^3*Sinh[(c + d*x)/2]^4 - 5*a^3*Csc
h[(c + d*x)/2]^4*Sinh[c + d*x] + 180*a*b^2*Sinh[2*(c + d*x)] + 80*a^3*Tanh[
(c + d*x)/2])/(240*d)
```

fricas [B] time = 0.58, size = 3801, normalized size = 29.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^16 + 48*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + 3*b
^3*sinh(d*x + c)^16 - 34*b^3*cosh(d*x + c)^14 + 180*a*b^2*cosh(d*x + c)^13
+ 234*b^3*cosh(d*x + c)^12 + 2*(180*b^3*cosh(d*x + c)^2 - 17*b^3)*sinh(d*x
+ c)^14 + 4*(420*b^3*cosh(d*x + c)^3 - 119*b^3*cosh(d*x + c) + 45*a*b^2)*si
nh(d*x + c)^13 - 378*b^3*cosh(d*x + c)^10 + 26*(210*b^3*cosh(d*x + c)^4 - 1
19*b^3*cosh(d*x + c)^2 + 90*a*b^2*cosh(d*x + c) + 9*b^3)*sinh(d*x + c)^12 -
 180*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^11 + 4*(3276*b^3*cosh(d*x + c)^5
 - 3094*b^3*cosh(d*x + c)^3 - 180*a*b^2*d*x + 3510*a*b^2*cosh(d*x + c)^2 +
702*b^3*cosh(d*x + c) - 135*a*b^2)*sinh(d*x + c)^11 + 2*(12012*b^3*cosh(d*x
 + c)^6 - 17017*b^3*cosh(d*x + c)^4 + 25740*a*b^2*cosh(d*x + c)^3 + 7722*b^
3*cosh(d*x + c)^2 - 189*b^3 - 990*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c))*si
nh(d*x + c)^10 + 360*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^9 + 4*(8580*b^3*co
sh(d*x + c)^7 - 17017*b^3*cosh(d*x + c)^5 + 32175*a*b^2*cosh(d*x + c)^4 + 1
2870*b^3*cosh(d*x + c)^3 + 540*a*b^2*d*x - 945*b^3*cosh(d*x + c) + 90*a*b^2
 - 2475*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 378*b^3*
cosh(d*x + c)^6 + 6*(6435*b^3*cosh(d*x + c)^8 - 17017*b^3*cosh(d*x + c)^6 +
 38610*a*b^2*cosh(d*x + c)^5 + 19305*b^3*cosh(d*x + c)^4 - 2835*b^3*cosh(d*
x + c)^2 - 4950*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^3 + 540*(6*a*b^2*d*x
+ a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 - 120*(18*a*b^2*d*x + 16*a^3 - 3*a*
b^2)*cosh(d*x + c)^7 + 24*(1430*b^3*cosh(d*x + c)^9 - 4862*b^3*cosh(d*x + c
```

)^7 + 12870*a*b^2*cosh(d*x + c)^6 + 7722*b^3*cosh(d*x + c)^5 - 1890*b^3*cos
h(d*x + c)^3 - 90*a*b^2*d*x - 2475*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^4
- 80*a^3 + 15*a*b^2 + 540*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^7 - 234*b^3*cosh(d*x + c)^4 + 6*(4004*b^3*cosh(d*x + c)^10 - 17017*b^3*
cosh(d*x + c)^8 + 51480*a*b^2*cosh(d*x + c)^7 + 36036*b^3*cosh(d*x + c)^6 -
13230*b^3*cosh(d*x + c)^4 - 13860*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^5
+ 5040*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^3 + 63*b^3 - 140*(18*a*b^2*d*x +
16*a^3 - 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 180*a*b^2*cosh(d*x + c)
^3 + 20*(36*a*b^2*d*x + 32*a^3 - 27*a*b^2)*cosh(d*x + c)^5 + 4*(3276*b^3*co
sh(d*x + c)^11 - 17017*b^3*cosh(d*x + c)^9 + 57915*a*b^2*cosh(d*x + c)^8 +
46332*b^3*cosh(d*x + c)^7 - 23814*b^3*cosh(d*x + c)^5 - 20790*(4*a*b^2*d*x
+ 3*a*b^2)*cosh(d*x + c)^6 + 180*a*b^2*d*x + 11340*(6*a*b^2*d*x + a*b^2)*co
sh(d*x + c)^4 + 567*b^3*cosh(d*x + c) + 160*a^3 - 135*a*b^2 - 630*(18*a*b^2
*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 34*b^3*cosh(d*x
+ c)^2 + 2*(2730*b^3*cosh(d*x + c)^12 - 17017*b^3*cosh(d*x + c)^10 + 64350
*a*b^2*cosh(d*x + c)^9 + 57915*b^3*cosh(d*x + c)^8 - 39690*b^3*cosh(d*x + c
)^6 - 29700*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^7 + 22680*(6*a*b^2*d*x +
a*b^2)*cosh(d*x + c)^5 + 2835*b^3*cosh(d*x + c)^2 - 2100*(18*a*b^2*d*x + 16
*a^3 - 3*a*b^2)*cosh(d*x + c)^3 - 117*b^3 + 50*(36*a*b^2*d*x + 32*a^3 - 27*
a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(420*b^3*cosh(d*x + c)^13 - 3094*
b^3*cosh(d*x + c)^11 + 12870*a*b^2*cosh(d*x + c)^10 + 12870*b^3*cosh(d*x +
c)^9 - 11340*b^3*cosh(d*x + c)^7 - 7425*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x +
c)^8 + 7560*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^6 + 1890*b^3*cosh(d*x + c)^
3 - 1050*(18*a*b^2*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^4 - 234*b^3*cosh(d
*x + c) + 45*a*b^2 + 50*(36*a*b^2*d*x + 32*a^3 - 27*a*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 - 3*b^3 + 2*(180*b^3*cosh(d*x + c)^14 - 1547*b^3*cosh(d*x
+ c)^12 + 7020*a*b^2*cosh(d*x + c)^11 + 7722*b^3*cosh(d*x + c)^10 - 8505*b^
3*cosh(d*x + c)^8 - 4950*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^9 + 6480*(6*
a*b^2*d*x + a*b^2)*cosh(d*x + c)^7 + 2835*b^3*cosh(d*x + c)^4 - 1260*(18*a*
b^2*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^5 - 702*b^3*cosh(d*x + c)^2 + 270
*a*b^2*cosh(d*x + c) + 100*(36*a*b^2*d*x + 32*a^3 - 27*a*b^2)*cosh(d*x + c)
^3 + 17*b^3)*sinh(d*x + c)^2 - 1440*(a^2*b*cosh(d*x + c)^11 + 11*a^2*b*cosh
(d*x + c)*sinh(d*x + c)^10 + a^2*b*sinh(d*x + c)^11 - 3*a^2*b*cosh(d*x + c)
^9 + 3*a^2*b*cosh(d*x + c)^7 + (55*a^2*b*cosh(d*x + c)^2 - 3*a^2*b)*sinh(d*
x + c)^9 + 3*(55*a^2*b*cosh(d*x + c)^3 - 9*a^2*b*cosh(d*x + c))*sinh(d*x +
c)^8 - a^2*b*cosh(d*x + c)^5 + 3*(110*a^2*b*cosh(d*x + c)^4 - 36*a^2*b*cosh
(d*x + c)^2 + a^2*b)*sinh(d*x + c)^7 + 21*(22*a^2*b*cosh(d*x + c)^5 - 12*a^
2*b*cosh(d*x + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c)^6 + (462*a^2*b*cos
h(d*x + c)^6 - 378*a^2*b*cosh(d*x + c)^4 + 63*a^2*b*cosh(d*x + c)^2 - a^2*b
)*sinh(d*x + c)^5 + (330*a^2*b*cosh(d*x + c)^7 - 378*a^2*b*cosh(d*x + c)^5
+ 105*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^4 + (165
*a^2*b*cosh(d*x + c)^8 - 252*a^2*b*cosh(d*x + c)^6 + 105*a^2*b*cosh(d*x + c
)^4 - 10*a^2*b*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*a^2*b*cosh(d*x + c)^9
- 108*a^2*b*cosh(d*x + c)^7 + 63*a^2*b*cosh(d*x + c)^5 - 10*a^2*b*cosh(d*x
+ c)^3)*sinh(d*x + c)^2 + (11*a^2*b*cosh(d*x + c)^10 - 27*a^2*b*cosh(d*x +
c)^8 + 21*a^2*b*cosh(d*x + c)^6 - 5*a^2*b*cosh(d*x + c)^4)*sinh(d*x + c))*
log(cosh(d*x + c) + sinh(d*x + c) + 1) + 1440*(a^2*b*cosh(d*x + c)^11 + 11*
a^2*b*cosh(d*x + c)*sinh(d*x + c)^10 + a^2*b*sinh(d*x + c)^11 - 3*a^2*b*cos
h(d*x + c)^9 + 3*a^2*b*cosh(d*x + c)^7 + (55*a^2*b*cosh(d*x + c)^2 - 3*a^2*
b)*sinh(d*x + c)^9 + 3*(55*a^2*b*cosh(d*x + c)^3 - 9*a^2*b*cosh(d*x + c))*s
inh(d*x + c)^8 - a^2*b*cosh(d*x + c)^5 + 3*(110*a^2*b*cosh(d*x + c)^4 - 36*
a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^7 + 21*(22*a^2*b*cosh(d*x + c)
^5 - 12*a^2*b*cosh(d*x + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c)^6 + (462
*a^2*b*cosh(d*x + c)^6 - 378*a^2*b*cosh(d*x + c)^4 + 63*a^2*b*cosh(d*x + c)
^2 - a^2*b)*sinh(d*x + c)^5 + (330*a^2*b*cosh(d*x + c)^7 - 378*a^2*b*cosh(d
*x + c)^5 + 105*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c
)^4 + (165*a^2*b*cosh(d*x + c)^8 - 252*a^2*b*cosh(d*x + c)^6 + 105*a^2*b*co
sh(d*x + c)^4 - 10*a^2*b*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*a^2*b*cosh(
d*x + c)^9 - 108*a^2*b*cosh(d*x + c)^7 + 63*a^2*b*cosh(d*x + c)^5 - 10*a^2*

```
b*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (11*a^2*b*cosh(d*x + c)^10 - 27*a^2*b*
cosh(d*x + c)^8 + 21*a^2*b*cosh(d*x + c)^6 - 5*a^2*b*cosh(d*x + c)^4)*sinh(
d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(12*b^3*cosh(d*x + c)^
15 - 119*b^3*cosh(d*x + c)^13 + 585*a*b^2*cosh(d*x + c)^12 + 702*b^3*cosh(d
*x + c)^11 - 945*b^3*cosh(d*x + c)^9 - 495*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x
 + c)^10 + 810*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^8 + 567*b^3*cosh(d*x + c
)^5 - 210*(18*a*b^2*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^6 - 234*b^3*cosh(
d*x + c)^3 + 135*a*b^2*cosh(d*x + c)^2 + 25*(36*a*b^2*d*x + 32*a^3 - 27*a*b
^2)*cosh(d*x + c)^4 + 17*b^3*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)
^11 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^11 - 3*d*cosh(d
*x + c)^9 + (55*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x
 + c)^3 - 9*d*cosh(d*x + c))*sinh(d*x + c)^8 + 3*d*cosh(d*x + c)^7 + 3*(110
*d*cosh(d*x + c)^4 - 36*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 21*(22*d*c
osh(d*x + c)^5 - 12*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^6 -
d*cosh(d*x + c)^5 + (462*d*cosh(d*x + c)^6 - 378*d*cosh(d*x + c)^4 + 63*d*c
osh(d*x + c)^2 - d)*sinh(d*x + c)^5 + (330*d*cosh(d*x + c)^7 - 378*d*cosh(d
*x + c)^5 + 105*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^4 + (1
65*d*cosh(d*x + c)^8 - 252*d*cosh(d*x + c)^6 + 105*d*cosh(d*x + c)^4 - 10*d
*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*d*cosh(d*x + c)^9 - 108*d*cosh(d*x
+ c)^7 + 63*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (11
*d*cosh(d*x + c)^10 - 27*d*cosh(d*x + c)^8 + 21*d*cosh(d*x + c)^6 - 5*d*cos
h(d*x + c)^4)*sinh(d*x + c))
```

giac [B] time = 0.34, size = 285, normalized size = 2.21

$$-\frac{720\,(dx+c)ab^2 - 3\,b^3e^{(5\,dx+5\,c)} + 25\,b^3e^{(3\,dx+3\,c)} - 180\,ab^2e^{(2\,dx+2\,c)} - 150\,b^3e^{(dx+c)} + 1440\,a^2b\log\left(e^{(dx+c)}+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/480*(720*(d*x + c)*a*b^2 - 3*b^3*e^(5*d*x + 5*c) + 25*b^3*e^(3*d*x + 3*c
) - 180*a*b^2*e^(2*d*x + 2*c) - 150*b^3*e^(d*x + c) + 1440*a^2*b*log(e^(d*x
 + c) + 1) - 1440*a^2*b*log(abs(e^(d*x + c) - 1)) - (150*b^3*e^(10*d*x + 10
*c) - 180*a*b^2*e^(9*d*x + 9*c) - 475*b^3*e^(8*d*x + 8*c) + 528*b^3*e^(6*d*
x + 6*c) - 234*b^3*e^(4*d*x + 4*c) + 180*a*b^2*e^(3*d*x + 3*c) + 34*b^3*e^(
2*d*x + 2*c) - 3*b^3 - 60*(32*a^3 - 9*a*b^2)*e^(7*d*x + 7*c) + 20*(32*a^3 -
 27*a*b^2)*e^(5*d*x + 5*c))*e^(-5*d*x - 5*c)/((e^(d*x + c) + 1)^3*(e^(d*x +
 c) - 1)^3))/d
```

maple [A] time = 0.17, size = 101, normalized size = 0.78

$$\frac{a^3\left(\frac{2}{3}-\frac{\mathrm{csch}(dx+c)^2}{3}\right)\coth\left(dx+c\right) - 6a^2b\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right) + 3a\,b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right) + b^3\left(\frac{8}{15}+\frac{(\sinh^4}{5}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-6*a^2*b*arctanh(exp(d*x+c))+3*
a*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^3*(8/15+1/5*sinh(d*x+c)
^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.33, size = 260, normalized size = 2.02

$$-\frac{3}{8}\,ab^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+\frac{1}{480}\,b^3\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,d}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b^3*(3*e^
(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x -
c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) - 3*a^2*b*(log(e^(-d*x
 - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3
*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e
^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 0.88, size = 267, normalized size = 2.07

$$\frac{5\,b^3\,e^{c+d\,x}}{16\,d}-\frac{4\,a^3}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}+\frac{5\,b^3\,e^{-c-d\,x}}{16\,d}-\frac{5\,b^3\,e^{-3\,c-3\,d\,x}}{96\,d}-\frac{5\,b^3\,e^{3\,c+3\,d\,x}}{96\,d}+\frac{b^3\,e^{-5\,c-5\,d\,x}}{160\,d}+\frac{b^3\,e^{5\,c+5\,d\,x}}{160\,d}-\frac{}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^4,x)
```

```
[Out] (5*b^3*exp(c + d*x))/(16*d) - (4*a^3)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*
d*x) + 1)) + (5*b^3*exp(- c - d*x))/(16*d) - (5*b^3*exp(- 3*c - 3*d*x))/(96
*d) - (5*b^3*exp(3*c + 3*d*x))/(96*d) + (b^3*exp(- 5*c - 5*d*x))/(160*d) +
(b^3*exp(5*c + 5*d*x))/(160*d) - (8*a^3)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4
*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (6*atan((a^2*b*exp(d*x)*exp(c)*(-d^2
)^(1/2))/(d*(a^4*b^2)^(1/2)))*(a^4*b^2)^(1/2))/(-d^2)^(1/2) - (3*a*b^2*x)/2
 - (3*a*b^2*exp(- 2*c - 2*d*x))/(8*d) + (3*a*b^2*exp(2*c + 2*d*x))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

3.168 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=148

$$-\frac{3a^3\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{3a^2b\coth(c+dx)}{d}$$

```
[Out] 3/8*b^3*x-3/8*a^3*arctanh(cosh(d*x+c))/d+3*a*b^2*cosh(d*x+c)/d-3*a^2*b*coth
(d*x+c)/d+3/8*a^3*coth(d*x+c)*csch(d*x+c)/d-1/4*a^3*coth(d*x+c)*csch(d*x+c)
^3/d-3/8*b^3*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b^3*cosh(d*x+c)*sinh(d*x+c)^3/d
```

Rubi [A] time = 0.17, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 3767, 8, 3768, 3770, 2638, 2635}

$$-\frac{3a^2b\coth(c+dx)}{d}-\frac{3a^3\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (3*b^3*x)/8 - (3*a^3*ArcTanh[Cosh[c + d*x]])/(8*d) + (3*a*b^2*Cosh[c + d*x]
)/d - (3*a^2*b*Coth[c + d*x])/d + (3*a^3*Coth[c + d*x]*Csch[c + d*x])/(8*d)
 - (a^3*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d) - (3*b^3*Cosh[c + d*x]*Sinh[c
+ d*x])/(8*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx &= i\int \left(-3ia^2b\operatorname{csch}^2(c+dx)-ia^3\operatorname{csch}^5(c+dx)-3iab^2\sinh(c+dx)-\right.\\ &= a^3\int \operatorname{csch}^5(c+dx)\,dx+\left(3a^2b\right)\int \operatorname{csch}^2(c+dx)\,dx+\left(3ab^2\right)\int \sin\\ &= \frac{3ab^2\cosh(c+dx)}{d}-\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{b^3\cosh(c+d}{}\\ &= \frac{3ab^2\cosh(c+dx)}{d}-\frac{3a^2b\coth(c+dx)}{d}+\frac{3a^3\coth(c+dx)\operatorname{csch}(c+}{8d}\\ &= \frac{3b^3x}{8}-\frac{3a^3\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{3a^2b\coth}{d}\end{aligned}$$

Mathematica [A] time = 6.14, size = 218, normalized size = 1.47

$$-\frac{a^3\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a^3\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{3a^3\log\left(\tanh\left(\frac{1}{2}(c+\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (3*b^3*(c + d*x))/(8*d) + (3*a*b^2*Cosh[c + d*x])/d - (3*a^2*b*Coth[(c + d*
x)/2])/(2*d) + (3*a^3*Csch[(c + d*x)/2]^2)/(32*d) - (a^3*Csch[(c + d*x)/2]^
4)/(64*d) + (3*a^3*Log[Tanh[(c + d*x)/2]])/(8*d) + (3*a^3*Sech[(c + d*x)/2]
^2)/(32*d) + (a^3*Sech[(c + d*x)/2]^4)/(64*d) - (b^3*Sinh[2*(c + d*x)])/(4*
d) + (b^3*Sinh[4*(c + d*x)])/(32*d) - (3*a^2*b*Tanh[(c + d*x)/2])/(2*d)
```

fricas [B] time = 0.62, size = 4541, normalized size = 30.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(b^3*cosh(d*x + c)^16 + 16*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + b^3*si
nh(d*x + c)^16 - 12*b^3*cosh(d*x + c)^14 + 96*a*b^2*cosh(d*x + c)^13 + 12*(
10*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^14 + 8*(70*b^3*cosh(d*x + c)^3
- 21*b^3*cosh(d*x + c) + 12*a*b^2)*sinh(d*x + c)^13 + 2*(12*b^3*d*x + 19*b^
3)*cosh(d*x + c)^12 + 2*(910*b^3*cosh(d*x + c)^4 + 12*b^3*d*x - 546*b^3*cos
h(d*x + c)^2 + 624*a*b^2*cosh(d*x + c) + 19*b^3)*sinh(d*x + c)^12 + 48*(a^3
 - 6*a*b^2)*cosh(d*x + c)^11 + 24*(182*b^3*cosh(d*x + c)^5 - 182*b^3*cosh(d
*x + c)^3 + 312*a*b^2*cosh(d*x + c)^2 + 2*a^3 - 12*a*b^2 + (12*b^3*d*x + 19
*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 4*(24*b^3*d*x + 96*a^2*b + 11*b^3)*
cosh(d*x + c)^10 + 4*(2002*b^3*cosh(d*x + c)^6 - 3003*b^3*cosh(d*x + c)^4 +
 6864*a*b^2*cosh(d*x + c)^3 - 24*b^3*d*x - 96*a^2*b - 11*b^3 + 33*(12*b^3*d
*x + 19*b^3)*cosh(d*x + c)^2 + 132*(a^3 - 6*a*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^10 - 16*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^9 + 8*(1430*b^3*cosh(d*x + c
```

```
)^7 - 3003*b^3*cosh(d*x + c)^5 + 8580*a*b^2*cosh(d*x + c)^4 + 55*(12*b^3*d*
x + 19*b^3)*cosh(d*x + c)^3 - 22*a^3 + 24*a*b^2 + 330*(a^3 - 6*a*b^2)*cosh(
d*x + c)^2 - 5*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c))*sinh(d*x + c
)^9 + 144*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^8 + 18*(715*b^3*cosh(d*x + c)^8
 - 2002*b^3*cosh(d*x + c)^6 + 6864*a*b^2*cosh(d*x + c)^5 + 8*b^3*d*x + 55*(
12*b^3*d*x + 19*b^3)*cosh(d*x + c)^4 + 440*(a^3 - 6*a*b^2)*cosh(d*x + c)^3
+ 64*a^2*b - 10*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^2 - 8*(11*a^
3 - 12*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 - 16*(11*a^3 - 12*a*b^2)*cosh(
d*x + c)^7 + 16*(715*b^3*cosh(d*x + c)^9 - 2574*b^3*cosh(d*x + c)^7 + 10296
*a*b^2*cosh(d*x + c)^6 + 99*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^5 + 990*(a^
3 - 6*a*b^2)*cosh(d*x + c)^4 - 30*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x
 + c)^3 - 11*a^3 + 12*a*b^2 - 36*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^2 + 72*(
b^3*d*x + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(24*b^3*d*x + 288*a^2
*b - 11*b^3)*cosh(d*x + c)^6 + 4*(2002*b^3*cosh(d*x + c)^10 - 9009*b^3*cosh
(d*x + c)^8 + 41184*a*b^2*cosh(d*x + c)^7 + 462*(12*b^3*d*x + 19*b^3)*cosh(
d*x + c)^6 + 5544*(a^3 - 6*a*b^2)*cosh(d*x + c)^5 - 24*b^3*d*x - 210*(24*b^
3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^4 - 336*(11*a^3 - 12*a*b^2)*cosh(d
*x + c)^3 - 288*a^2*b + 11*b^3 + 1008*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^2 -
 28*(11*a^3 - 12*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 96*a*b^2*cosh(d*x
+ c)^3 + 48*(a^3 - 6*a*b^2)*cosh(d*x + c)^5 + 24*(182*b^3*cosh(d*x + c)^11
- 1001*b^3*cosh(d*x + c)^9 + 5148*a*b^2*cosh(d*x + c)^8 + 66*(12*b^3*d*x +
19*b^3)*cosh(d*x + c)^7 + 924*(a^3 - 6*a*b^2)*cosh(d*x + c)^6 - 42*(24*b^3*
d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^5 - 84*(11*a^3 - 12*a*b^2)*cosh(d*x
+ c)^4 + 336*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^3 + 2*a^3 - 12*a*b^2 - 14*(1
1*a^3 - 12*a*b^2)*cosh(d*x + c)^2 - (24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(
d*x + c))*sinh(d*x + c)^5 + 12*b^3*cosh(d*x + c)^2 + 2*(12*b^3*d*x + 192*a^
2*b - 19*b^3)*cosh(d*x + c)^4 + 2*(910*b^3*cosh(d*x + c)^12 - 6006*b^3*cosh
(d*x + c)^10 + 34320*a*b^2*cosh(d*x + c)^9 + 495*(12*b^3*d*x + 19*b^3)*cosh
(d*x + c)^8 + 7920*(a^3 - 6*a*b^2)*cosh(d*x + c)^7 - 420*(24*b^3*d*x + 96*a
^2*b + 11*b^3)*cosh(d*x + c)^6 - 1008*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^5 +
 12*b^3*d*x + 5040*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^4 - 280*(11*a^3 - 12*a
*b^2)*cosh(d*x + c)^3 + 192*a^2*b - 19*b^3 - 30*(24*b^3*d*x + 288*a^2*b - 1
1*b^3)*cosh(d*x + c)^2 + 120*(a^3 - 6*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4
 + 8*(70*b^3*cosh(d*x + c)^13 - 546*b^3*cosh(d*x + c)^11 + 3432*a*b^2*cosh(
d*x + c)^10 + 55*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^9 + 990*(a^3 - 6*a*b^2
)*cosh(d*x + c)^8 - 60*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^7 - 1
68*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^6 + 1008*(b^3*d*x + 8*a^2*b)*cosh(d*x
+ c)^5 - 70*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^4 - 10*(24*b^3*d*x + 288*a^2*
b - 11*b^3)*cosh(d*x + c)^3 + 12*a*b^2 + 60*(a^3 - 6*a*b^2)*cosh(d*x + c)^2
 + (12*b^3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 +
 12*(10*b^3*cosh(d*x + c)^14 - 91*b^3*cosh(d*x + c)^12 + 624*a*b^2*cosh(d*x
 + c)^11 + 11*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^10 + 220*(a^3 - 6*a*b^2)*
cosh(d*x + c)^9 - 15*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^8 - 48*
(11*a^3 - 12*a*b^2)*cosh(d*x + c)^7 + 336*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)
^6 - 28*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^5 - 5*(24*b^3*d*x + 288*a^2*b - 1
1*b^3)*cosh(d*x + c)^4 + 24*a*b^2*cosh(d*x + c) + 40*(a^3 - 6*a*b^2)*cosh(d
*x + c)^3 + b^3 + (12*b^3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 - 24*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^1
1 + a^3*sinh(d*x + c)^12 - 4*a^3*cosh(d*x + c)^10 + 6*a^3*cosh(d*x + c)^8 +
 2*(33*a^3*cosh(d*x + c)^2 - 2*a^3)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x
+ c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x + c)^9 - 4*a^3*cosh(d*x + c)^6 + 3*(
165*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2 + 2*a^3)*sinh(d*x + c)^8 +
 24*(33*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c)^3 + 2*a^3*cosh(d*x + c))
*sinh(d*x + c)^7 + a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c)^6 - 210*a
^3*cosh(d*x + c)^4 + 42*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^6 + 24*(33
*a^3*cosh(d*x + c)^7 - 42*a^3*cosh(d*x + c)^5 + 14*a^3*cosh(d*x + c)^3 - a^
3*cosh(d*x + c))*sinh(d*x + c)^5 + (495*a^3*cosh(d*x + c)^8 - 840*a^3*cosh(
d*x + c)^6 + 420*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2 + a^3)*sinh(d
*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 - 120*a^3*cosh(d*x + c)^7 + 84*a^3*co
```

```
sh(d*x + c)^5 - 20*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c)^3
 + 6*(11*a^3*cosh(d*x + c)^10 - 30*a^3*cosh(d*x + c)^8 + 28*a^3*cosh(d*x +
c)^6 - 10*a^3*cosh(d*x + c)^4 + a^3*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3
*a^3*cosh(d*x + c)^11 - 10*a^3*cosh(d*x + c)^9 + 12*a^3*cosh(d*x + c)^7 - 6
*a^3*cosh(d*x + c)^5 + a^3*cosh(d*x + c)^3)*sinh(d*x + c))*log(cosh(d*x + c
) + sinh(d*x + c) + 1) + 24*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*si
nh(d*x + c)^11 + a^3*sinh(d*x + c)^12 - 4*a^3*cosh(d*x + c)^10 + 6*a^3*cosh
(d*x + c)^8 + 2*(33*a^3*cosh(d*x + c)^2 - 2*a^3)*sinh(d*x + c)^10 + 20*(11*
a^3*cosh(d*x + c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x + c)^9 - 4*a^3*cosh(d*x
 + c)^6 + 3*(165*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2 + 2*a^3)*sinh
(d*x + c)^8 + 24*(33*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c)^3 + 2*a^3*c
osh(d*x + c))*sinh(d*x + c)^7 + a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x +
 c)^6 - 210*a^3*cosh(d*x + c)^4 + 42*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x +
c)^6 + 24*(33*a^3*cosh(d*x + c)^7 - 42*a^3*cosh(d*x + c)^5 + 14*a^3*cosh(d*
x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c)^5 + (495*a^3*cosh(d*x + c)^8 -
840*a^3*cosh(d*x + c)^6 + 420*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2
+ a^3)*sinh(d*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 - 120*a^3*cosh(d*x + c)^
7 + 84*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*si
nh(d*x + c)^3 + 6*(11*a^3*cosh(d*x + c)^10 - 30*a^3*cosh(d*x + c)^8 + 28*a^
3*cosh(d*x + c)^6 - 10*a^3*cosh(d*x + c)^4 + a^3*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 + 4*(3*a^3*cosh(d*x + c)^11 - 10*a^3*cosh(d*x + c)^9 + 12*a^3*cosh(d
*x + c)^7 - 6*a^3*cosh(d*x + c)^5 + a^3*cosh(d*x + c)^3)*sinh(d*x + c))*log
(cosh(d*x + c) + sinh(d*x + c) - 1) + 8*(2*b^3*cosh(d*x + c)^15 - 21*b^3*co
sh(d*x + c)^13 + 156*a*b^2*cosh(d*x + c)^12 + 3*(12*b^3*d*x + 19*b^3)*cosh(
d*x + c)^11 + 66*(a^3 - 6*a*b^2)*cosh(d*x + c)^10 - 5*(24*b^3*d*x + 96*a^2*
b + 11*b^3)*cosh(d*x + c)^9 - 18*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^8 + 144*
(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^7 - 14*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^
6 - 3*(24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c)^5 + 36*a*b^2*cosh(d*x
 + c)^2 + 30*(a^3 - 6*a*b^2)*cosh(d*x + c)^4 + 3*b^3*cosh(d*x + c) + (12*b^
3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))/(d*cosh(d*x + c
)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 4*d*cosh(
d*x + c)^10 + 2*(33*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^10 + 20*(11*d*co
sh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(d*x + c)^9 + 6*d*cosh(d*x + c)^8 +
3*(165*d*cosh(d*x + c)^4 - 60*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 24
*(33*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x
 + c)^7 - 4*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 210*d*cosh(d*x +
 c)^4 + 42*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^
7 - 42*d*cosh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x
 + c)^5 + d*cosh(d*x + c)^4 + (495*d*cosh(d*x + c)^8 - 840*d*cosh(d*x + c)^
6 + 420*d*cosh(d*x + c)^4 - 60*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(
55*d*cosh(d*x + c)^9 - 120*d*cosh(d*x + c)^7 + 84*d*cosh(d*x + c)^5 - 20*d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(11*d*cosh(d*x + c)^
10 - 30*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 - 10*d*cosh(d*x + c)^4 + d
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*d*cosh(d*x + c)^11 - 10*d*cosh(d*x
 + c)^9 + 12*d*cosh(d*x + c)^7 - 6*d*cosh(d*x + c)^5 + d*cosh(d*x + c)^3)*s
inh(d*x + c))
```

giac [B] time = 0.39, size = 329, normalized size = 2.22

$$24\,(dx+c)b^3 + b^3e^{(4\,dx+4\,c)} - 8\,b^3e^{(2\,dx+2\,c)} + 96\,ab^2e^{(dx+c)} - 24\,a^3\log\left(e^{(dx+c)}+1\right) + 24\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right) + \frac{(96\,a\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*b^3 + b^3*e^(4*d*x + 4*c) - 8*b^3*e^(2*d*x + 2*c) + 96*a
*b^2*e^(d*x + c) - 24*a^3*log(e^(d*x + c) + 1) + 24*a^3*log(abs(e^(d*x + c)
 - 1)) + (96*a*b^2*e^(3*d*x + 3*c) + 12*b^3*e^(2*d*x + 2*c) - b^3 + 48*(a^3
```

```
 + 2*a*b^2)*e^(11*d*x + 11*c) - 8*(48*a^2*b - b^3)*e^(10*d*x + 10*c) - 16*(
11*a^3 + 24*a*b^2)*e^(9*d*x + 9*c) + 3*(384*a^2*b - 11*b^3)*e^(8*d*x + 8*c)
 - 16*(11*a^3 - 36*a*b^2)*e^(7*d*x + 7*c) - 4*(288*a^2*b - 13*b^3)*e^(6*d*x
 + 6*c) + 48*(a^3 - 8*a*b^2)*e^(5*d*x + 5*c) + 2*(192*a^2*b - 19*b^3)*e^(4*
d*x + 4*c))*e^(-4*d*x - 4*c)/((e^(d*x + c) + 1)^4*(e^(d*x + c) - 1)^4))/d
```

maple [A] time = 0.19, size = 108, normalized size = 0.73

$$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-3a^2b\coth(dx+c)+3ab^2\cosh(dx+c)+b^3\Bigl(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(a^3*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(
d*x+c)))-3*a^2*b*coth(d*x+c)+3*a*b^2*cosh(d*x+c)+b^3*((1/4*sinh(d*x+c)^3-3/
8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.34, size = 255, normalized size = 1.72

$$\frac{1}{64}b^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{3}{2}ab^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)-\frac{1}{8}a^3\Bigl(\frac{3\log\left(e^{(-dx-}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/2*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) - 1
/8*a^3*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*
x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d
*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x
- 8*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.87, size = 451, normalized size = 3.05

$$\frac{3b^3x}{8}-\frac{\frac{3a^2b}{2d}+\frac{2a^3e^{c+dx}}{d}-\frac{3a^2be^{2c+2dx}}{d}+\frac{3a^2be^{4c+4dx}}{2d}}{3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1}-\frac{\frac{4a^3e^{3c+3dx}}{d}-\frac{3a^2b}{2d}+\frac{9a^2be^{2c+2dx}}{2d}-\frac{9a^2be^{4c+4dx}}{2d}+\frac{3a^2be^{6c+6}}{2d}}{6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^5,x)
```

```
[Out] (3*b^3*x)/8 - ((3*a^2*b)/(2*d) + (2*a^3*exp(c + d*x))/d - (3*a^2*b*exp(2*c
+ 2*d*x))/d + (3*a^2*b*exp(4*c + 4*d*x))/(2*d))/(3*exp(2*c + 2*d*x) - 3*exp
(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((4*a^3*exp(3*c + 3*d*x))/d - (3*a^
2*b)/(2*d) + (9*a^2*b*exp(2*c + 2*d*x))/(2*d) - (9*a^2*b*exp(4*c + 4*d*x))/
(2*d) + (3*a^2*b*exp(6*c + 6*d*x))/(2*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c +
 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((3*a^2*b)/d - (3*a^
3*exp(c + d*x))/(4*d))/(exp(2*c + 2*d*x) - 1) - (3*atan((a^3*exp(d*x)*exp(c
)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(4*(-d^2)^(1/2)) + (b^3*exp(-
 2*c - 2*d*x))/(8*d) - (b^3*exp(2*c + 2*d*x))/(8*d) - (b^3*exp(- 4*c - 4*d*
x))/(64*d) + (b^3*exp(4*c + 4*d*x))/(64*d) + (3*a*b^2*exp(- c - d*x))/(2*d)
 + (3*a*b^2*exp(c + d*x))/(2*d) - (a^3*exp(c + d*x))/(2*d*(exp(4*c + 4*d*x)
 - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

3.169 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=131

$$-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3a^2b\coth(c+dx)\operatorname{csch}(c$$

```
[Out] 3*a*b^2*x+3/2*a^2*b*arctanh(cosh(d*x+c))/d-b^3*cosh(d*x+c)/d+1/3*b^3*cosh(d
*x+c)^3/d-a^3*coth(d*x+c)/d+2/3*a^3*coth(d*x+c)^3/d-1/5*a^3*coth(d*x+c)^5/d
-3/2*a^2*b*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 3768, 3770, 3767, 2633}

$$\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3a^2b\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}-\frac{a^3\coth(c-}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] 3*a*b^2*x + (3*a^2*b*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^3*Cosh[c + d*x])/d
+ (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x])/d + (2*a^3*Coth[c + d*x
]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) - (3*a^2*b*Coth[c + d*x]*Csch[c +
d*x])/(2*d)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\int \left(-3ab^2-3a^2b\operatorname{csch}^3(c+dx)-a^3\operatorname{csch}^6(c+dx)-b^3\sinh^3(c+\right.$$

$$= 3ab^2x + a^3\int \operatorname{csch}^6(c+dx)\,dx + \left(3a^2b\right)\int \operatorname{csch}^3(c+dx)\,dx + b^3\int$$

$$= 3ab^2x - \frac{3a^2b\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{1}{2}\left(3a^2b\right)\int \operatorname{csch}(c+dx)$$

$$= 3ab^2x + \frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{b^3\cosh(c+dx)}{d} + \frac{b^3\cosh^3(}{3d}$$

Mathematica [A] time = 1.87, size = 225, normalized size = 1.72

$$\frac{1}{2}a\left(8\left(-32a^2\tanh\left(\frac{1}{2}(c+dx)\right)-24a^2\sinh^6\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^5(c+dx)-38a^2\sinh^4\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^3(c+dx)-4\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-360*b^3*Cosh[c + d*x] + 40*b^3*Cosh[3*(c + d*x)] + (a*(-256*a^2*Coth[(c +
 d*x)/2] - 360*a*b*Csch[(c + d*x)/2]^2 + 19*a^2*Csch[(c + d*x)/2]^4*Sinh[c
+ d*x] - 3*a^2*Csch[(c + d*x)/2]^6*Sinh[c + d*x] + 8*(180*b*(2*b*(c + d*x)
- a*Log[Tanh[(c + d*x)/2]]) - 45*a*b*Sech[(c + d*x)/2]^2 - 38*a^2*Csch[c +
d*x]^3*Sinh[(c + d*x)/2]^4 - 24*a^2*Csch[c + d*x]^5*Sinh[(c + d*x)/2]^6 - 3
2*a^2*Tanh[(c + d*x)/2])))/2)/(480*d)
```

fricas [B] time = 0.80, size = 4629, normalized size = 35.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(5*b^3*cosh(d*x + c)^16 + 80*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + 5*b
^3*sinh(d*x + c)^16 + 360*a*b^2*d*x*cosh(d*x + c)^13 - 70*b^3*cosh(d*x + c)
^14 - 1800*a*b^2*d*x*cosh(d*x + c)^11 + 10*(60*b^3*cosh(d*x + c)^2 - 7*b^3)
*sinh(d*x + c)^14 + 3600*a*b^2*d*x*cosh(d*x + c)^9 + 20*(140*b^3*cosh(d*x +
 c)^3 + 18*a*b^2*d*x - 49*b^3*cosh(d*x + c))*sinh(d*x + c)^13 - 10*(36*a^2*
b - 23*b^3)*cosh(d*x + c)^12 + 10*(910*b^3*cosh(d*x + c)^4 + 468*a*b^2*d*x*
cosh(d*x + c) - 637*b^3*cosh(d*x + c)^2 - 36*a^2*b + 23*b^3)*sinh(d*x + c)^
12 + 40*(546*b^3*cosh(d*x + c)^5 + 702*a*b^2*d*x*cosh(d*x + c)^2 - 637*b^3*
cosh(d*x + c)^3 - 45*a*b^2*d*x - 3*(36*a^2*b - 23*b^3)*cosh(d*x + c))*sinh(
d*x + c)^11 + 90*(8*a^2*b - 3*b^3)*cosh(d*x + c)^10 + 10*(4004*b^3*cosh(d*x
 + c)^6 + 10296*a*b^2*d*x*cosh(d*x + c)^3 - 7007*b^3*cosh(d*x + c)^4 - 1980
*a*b^2*d*x*cosh(d*x + c) + 72*a^2*b - 27*b^3 - 66*(36*a^2*b - 23*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^10 + 20*(2860*b^3*cosh(d*x + c)^7 + 12870*a*b^2*d
*x*cosh(d*x + c)^4 - 7007*b^3*cosh(d*x + c)^5 - 4950*a*b^2*d*x*cosh(d*x + c
)^2 + 180*a*b^2*d*x - 110*(36*a^2*b - 23*b^3)*cosh(d*x + c)^3 + 45*(8*a^2*b
 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 30*(2145*b^3*cosh(d*x + c)^8 + 1
5444*a*b^2*d*x*cosh(d*x + c)^5 - 7007*b^3*cosh(d*x + c)^6 - 9900*a*b^2*d*x*
cosh(d*x + c)^3 + 1080*a*b^2*d*x*cosh(d*x + c) - 165*(36*a^2*b - 23*b^3)*co
sh(d*x + c)^4 + 135*(8*a^2*b - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 - 80
*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^7 + 80*(715*b^3*cosh(d*x + c)^9 + 77
22*a*b^2*d*x*cosh(d*x + c)^6 - 3003*b^3*cosh(d*x + c)^7 - 7425*a*b^2*d*x*co
sh(d*x + c)^4 + 1620*a*b^2*d*x*cosh(d*x + c)^2 - 99*(36*a^2*b - 23*b^3)*cos
h(d*x + c)^5 - 45*a*b^2*d*x + 135*(8*a^2*b - 3*b^3)*cosh(d*x + c)^3 - 16*a^
3)*sinh(d*x + c)^7 - 90*(8*a^2*b - 3*b^3)*cosh(d*x + c)^6 + 10*(4004*b^3*co
```

```
sh(d*x + c)^10 + 61776*a*b^2*d*x*cosh(d*x + c)^7 - 21021*b^3*cosh(d*x + c)^
8 - 83160*a*b^2*d*x*cosh(d*x + c)^5 + 30240*a*b^2*d*x*cosh(d*x + c)^3 - 924
*(36*a^2*b - 23*b^3)*cosh(d*x + c)^6 + 1890*(8*a^2*b - 3*b^3)*cosh(d*x + c)
^4 - 72*a^2*b + 27*b^3 - 56*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c))*sinh(d*x
 + c)^6 + 40*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^5 + 20*(1092*b^3*cosh(d*
x + c)^11 + 23166*a*b^2*d*x*cosh(d*x + c)^8 - 7007*b^3*cosh(d*x + c)^9 - 41
580*a*b^2*d*x*cosh(d*x + c)^6 + 22680*a*b^2*d*x*cosh(d*x + c)^4 - 396*(36*a
^2*b - 23*b^3)*cosh(d*x + c)^7 + 1134*(8*a^2*b - 3*b^3)*cosh(d*x + c)^5 + 9
0*a*b^2*d*x + 32*a^3 - 84*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^2 - 27*(8*a
^2*b - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 70*b^3*cosh(d*x + c)^2 + 10*
(36*a^2*b - 23*b^3)*cosh(d*x + c)^4 + 10*(910*b^3*cosh(d*x + c)^12 + 25740*
a*b^2*d*x*cosh(d*x + c)^9 - 7007*b^3*cosh(d*x + c)^10 - 59400*a*b^2*d*x*cos
h(d*x + c)^7 + 45360*a*b^2*d*x*cosh(d*x + c)^5 - 495*(36*a^2*b - 23*b^3)*co
sh(d*x + c)^8 + 1890*(8*a^2*b - 3*b^3)*cosh(d*x + c)^6 - 280*(45*a*b^2*d*x
+ 16*a^3)*cosh(d*x + c)^3 + 36*a^2*b - 23*b^3 - 135*(8*a^2*b - 3*b^3)*cosh(
d*x + c)^2 + 20*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 8*
(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^3 + 8*(350*b^3*cosh(d*x + c)^13 + 128
70*a*b^2*d*x*cosh(d*x + c)^10 - 3185*b^3*cosh(d*x + c)^11 - 37125*a*b^2*d*x
*cosh(d*x + c)^8 + 37800*a*b^2*d*x*cosh(d*x + c)^6 - 275*(36*a^2*b - 23*b^3
)*cosh(d*x + c)^9 + 1350*(8*a^2*b - 3*b^3)*cosh(d*x + c)^7 - 45*a*b^2*d*x -
 350*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^4 - 225*(8*a^2*b - 3*b^3)*cosh(d
*x + c)^3 - 16*a^3 + 50*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^2 + 5*(36*a^2
*b - 23*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*b^3 + 2*(300*b^3*cosh(d*x +
 c)^14 + 14040*a*b^2*d*x*cosh(d*x + c)^11 - 3185*b^3*cosh(d*x + c)^12 - 495
00*a*b^2*d*x*cosh(d*x + c)^9 + 64800*a*b^2*d*x*cosh(d*x + c)^7 - 330*(36*a^
2*b - 23*b^3)*cosh(d*x + c)^10 + 2025*(8*a^2*b - 3*b^3)*cosh(d*x + c)^8 - 8
40*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^5 - 675*(8*a^2*b - 3*b^3)*cosh(d*x
 + c)^4 + 200*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^3 + 35*b^3 + 30*(36*a^2
*b - 23*b^3)*cosh(d*x + c)^2 - 12*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c))*si
nh(d*x + c)^2 + 180*(a^2*b*cosh(d*x + c)^13 + 13*a^2*b*cosh(d*x + c)*sinh(d
*x + c)^12 + a^2*b*sinh(d*x + c)^13 - 5*a^2*b*cosh(d*x + c)^11 + 10*a^2*b*c
osh(d*x + c)^9 + (78*a^2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^11 + 11
*(26*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^10 - 10*a
^2*b*cosh(d*x + c)^7 + 5*(143*a^2*b*cosh(d*x + c)^4 - 55*a^2*b*cosh(d*x + c
)^2 + 2*a^2*b)*sinh(d*x + c)^9 + 3*(429*a^2*b*cosh(d*x + c)^5 - 275*a^2*b*c
osh(d*x + c)^3 + 30*a^2*b*cosh(d*x + c))*sinh(d*x + c)^8 + 5*a^2*b*cosh(d*x
 + c)^5 + 2*(858*a^2*b*cosh(d*x + c)^6 - 825*a^2*b*cosh(d*x + c)^4 + 180*a^
2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^7 + 2*(858*a^2*b*cosh(d*x + c)
^7 - 1155*a^2*b*cosh(d*x + c)^5 + 420*a^2*b*cosh(d*x + c)^3 - 35*a^2*b*cosh
(d*x + c))*sinh(d*x + c)^6 - a^2*b*cosh(d*x + c)^3 + (1287*a^2*b*cosh(d*x +
 c)^8 - 2310*a^2*b*cosh(d*x + c)^6 + 1260*a^2*b*cosh(d*x + c)^4 - 210*a^2*b
*cosh(d*x + c)^2 + 5*a^2*b)*sinh(d*x + c)^5 + 5*(143*a^2*b*cosh(d*x + c)^9
- 330*a^2*b*cosh(d*x + c)^7 + 252*a^2*b*cosh(d*x + c)^5 - 70*a^2*b*cosh(d*x
 + c)^3 + 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^4 + (286*a^2*b*cosh(d*x + c)
^10 - 825*a^2*b*cosh(d*x + c)^8 + 840*a^2*b*cosh(d*x + c)^6 - 350*a^2*b*cos
h(d*x + c)^4 + 50*a^2*b*cosh(d*x + c)^2 - a^2*b)*sinh(d*x + c)^3 + (78*a^2*
b*cosh(d*x + c)^11 - 275*a^2*b*cosh(d*x + c)^9 + 360*a^2*b*cosh(d*x + c)^7
- 210*a^2*b*cosh(d*x + c)^5 + 50*a^2*b*cosh(d*x + c)^3 - 3*a^2*b*cosh(d*x +
 c))*sinh(d*x + c)^2 + (13*a^2*b*cosh(d*x + c)^12 - 55*a^2*b*cosh(d*x + c)^
10 + 90*a^2*b*cosh(d*x + c)^8 - 70*a^2*b*cosh(d*x + c)^6 + 25*a^2*b*cosh(d*
x + c)^4 - 3*a^2*b*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh
(d*x + c) + 1) - 180*(a^2*b*cosh(d*x + c)^13 + 13*a^2*b*cosh(d*x + c)*sinh(
d*x + c)^12 + a^2*b*sinh(d*x + c)^13 - 5*a^2*b*cosh(d*x + c)^11 + 10*a^2*b*
cosh(d*x + c)^9 + (78*a^2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^11 + 1
1*(26*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^10 - 10*
a^2*b*cosh(d*x + c)^7 + 5*(143*a^2*b*cosh(d*x + c)^4 - 55*a^2*b*cosh(d*x +
c)^2 + 2*a^2*b)*sinh(d*x + c)^9 + 3*(429*a^2*b*cosh(d*x + c)^5 - 275*a^2*b*
cosh(d*x + c)^3 + 30*a^2*b*cosh(d*x + c))*sinh(d*x + c)^8 + 5*a^2*b*cosh(d*
x + c)^5 + 2*(858*a^2*b*cosh(d*x + c)^6 - 825*a^2*b*cosh(d*x + c)^4 + 180*a
```

```
^2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^7 + 2*(858*a^2*b*cosh(d*x + c
)^7 - 1155*a^2*b*cosh(d*x + c)^5 + 420*a^2*b*cosh(d*x + c)^3 - 35*a^2*b*cos
h(d*x + c))*sinh(d*x + c)^6 - a^2*b*cosh(d*x + c)^3 + (1287*a^2*b*cosh(d*x
+ c)^8 - 2310*a^2*b*cosh(d*x + c)^6 + 1260*a^2*b*cosh(d*x + c)^4 - 210*a^2*
b*cosh(d*x + c)^2 + 5*a^2*b)*sinh(d*x + c)^5 + 5*(143*a^2*b*cosh(d*x + c)^9
 - 330*a^2*b*cosh(d*x + c)^7 + 252*a^2*b*cosh(d*x + c)^5 - 70*a^2*b*cosh(d*
x + c)^3 + 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^4 + (286*a^2*b*cosh(d*x + c
)^10 - 825*a^2*b*cosh(d*x + c)^8 + 840*a^2*b*cosh(d*x + c)^6 - 350*a^2*b*co
sh(d*x + c)^4 + 50*a^2*b*cosh(d*x + c)^2 - a^2*b)*sinh(d*x + c)^3 + (78*a^2
*b*cosh(d*x + c)^11 - 275*a^2*b*cosh(d*x + c)^9 + 360*a^2*b*cosh(d*x + c)^7
 - 210*a^2*b*cosh(d*x + c)^5 + 50*a^2*b*cosh(d*x + c)^3 - 3*a^2*b*cosh(d*x
+ c))*sinh(d*x + c)^2 + (13*a^2*b*cosh(d*x + c)^12 - 55*a^2*b*cosh(d*x + c)
^10 + 90*a^2*b*cosh(d*x + c)^8 - 70*a^2*b*cosh(d*x + c)^6 + 25*a^2*b*cosh(d
*x + c)^4 - 3*a^2*b*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sin
h(d*x + c) - 1) + 4*(20*b^3*cosh(d*x + c)^15 + 1170*a*b^2*d*x*cosh(d*x + c)
^12 - 245*b^3*cosh(d*x + c)^13 - 4950*a*b^2*d*x*cosh(d*x + c)^10 + 8100*a*b
^2*d*x*cosh(d*x + c)^8 - 30*(36*a^2*b - 23*b^3)*cosh(d*x + c)^11 + 225*(8*a
^2*b - 3*b^3)*cosh(d*x + c)^9 - 140*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^6
 - 135*(8*a^2*b - 3*b^3)*cosh(d*x + c)^5 + 50*(45*a*b^2*d*x + 16*a^3)*cosh(
d*x + c)^4 + 35*b^3*cosh(d*x + c) + 10*(36*a^2*b - 23*b^3)*cosh(d*x + c)^3
- 6*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c
)^13 + 13*d*cosh(d*x + c)*sinh(d*x + c)^12 + d*sinh(d*x + c)^13 - 5*d*cosh(
d*x + c)^11 + (78*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^11 + 11*(26*d*cosh
(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^10 + 10*d*cosh(d*x + c)^9 +
5*(143*d*cosh(d*x + c)^4 - 55*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^9 + 3*
(429*d*cosh(d*x + c)^5 - 275*d*cosh(d*x + c)^3 + 30*d*cosh(d*x + c))*sinh(d
*x + c)^8 - 10*d*cosh(d*x + c)^7 + 2*(858*d*cosh(d*x + c)^6 - 825*d*cosh(d*
x + c)^4 + 180*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^7 + 2*(858*d*cosh(d*x
 + c)^7 - 1155*d*cosh(d*x + c)^5 + 420*d*cosh(d*x + c)^3 - 35*d*cosh(d*x +
c))*sinh(d*x + c)^6 + 5*d*cosh(d*x + c)^5 + (1287*d*cosh(d*x + c)^8 - 2310*
d*cosh(d*x + c)^6 + 1260*d*cosh(d*x + c)^4 - 210*d*cosh(d*x + c)^2 + 5*d)*s
inh(d*x + c)^5 + 5*(143*d*cosh(d*x + c)^9 - 330*d*cosh(d*x + c)^7 + 252*d*c
osh(d*x + c)^5 - 70*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^4
- d*cosh(d*x + c)^3 + (286*d*cosh(d*x + c)^10 - 825*d*cosh(d*x + c)^8 + 840
*d*cosh(d*x + c)^6 - 350*d*cosh(d*x + c)^4 + 50*d*cosh(d*x + c)^2 - d)*sinh
(d*x + c)^3 + (78*d*cosh(d*x + c)^11 - 275*d*cosh(d*x + c)^9 + 360*d*cosh(d
*x + c)^7 - 210*d*cosh(d*x + c)^5 + 50*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c
))*sinh(d*x + c)^2 + (13*d*cosh(d*x + c)^12 - 55*d*cosh(d*x + c)^10 + 90*d*
cosh(d*x + c)^8 - 70*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 - 3*d*cosh(d*
x + c)^2)*sinh(d*x + c))
```

giac [B] time = 0.34, size = 270, normalized size = 2.06

$$360\,(dx+c)ab^2 + 5\,b^3e^{(3\,dx+3\,c)} - 45\,b^3e^{(dx+c)} + 180\,a^2b\log\left(e^{(dx+c)}+1\right) - 180\,a^2b\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{(475\,b^3e^{(8\,dx+8\,c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/120*(360*(d*x + c)*a*b^2 + 5*b^3*e^(3*d*x + 3*c) - 45*b^3*e^(d*x + c) + 1
80*a^2*b*log(e^(d*x + c) + 1) - 180*a^2*b*log(abs(e^(d*x + c) - 1)) - (475*
b^3*e^(8*d*x + 8*c) + 1280*a^3*e^(7*d*x + 7*c) - 640*a^3*e^(5*d*x + 5*c) +
128*a^3*e^(3*d*x + 3*c) - 70*b^3*e^(2*d*x + 2*c) + 5*b^3 + 45*(8*a^2*b + b^
3)*e^(12*d*x + 12*c) - 10*(72*a^2*b + 23*b^3)*e^(10*d*x + 10*c) + 20*(36*a^
2*b - 25*b^3)*e^(6*d*x + 6*c) - 5*(72*a^2*b - 55*b^3)*e^(4*d*x + 4*c))*e^(-
3*d*x - 3*c)/((e^(d*x + c) + 1)^5*(e^(d*x + c) - 1)^5))/d
```

maple [A] time = 0.18, size = 99, normalized size = 0.76

$$\frac{a^3\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)+3a^2b\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a\,b^2\,(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(a^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(
-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a*b^2*(d*x+c)+b^3*(-2/3
+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.72, size = 365, normalized size = 2.79

$$3\,ab^2x+\frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3}{2}\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3*a*b^2*x + 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/
d + e^(-3*d*x - 3*c)/d) + 3/2*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x
- c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c) - 1))) - 16/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2
*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-
4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c
) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6
*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)))
```

mupad [B] time = 0.81, size = 432, normalized size = 3.30

$$\frac{b^3\,e^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b^3\,e^{c+d\,x}}{8\,d}-\frac{3\,b^3\,e^{-c-d\,x}}{8\,d}-\frac{\frac{32\,a^3\,e^{4\,c+4\,d\,x}}{5\,d}+\frac{36\,a^2\,b\,e^{3\,c+3\,d\,x}}{5\,d}-\frac{36\,a^2\,b\,e^{5\,c+5\,d\,x}}{5\,d}+\frac{12\,a^2\,b\,e^{7\,c+7\,d\,x}}{5\,d}-\frac{12\,a^2\,b\,e^{c+d\,x}}{5\,d}}{5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1}+\frac{b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^6,x)
```

```
[Out] (b^3*exp(- 3*c - 3*d*x))/(24*d) - (3*b^3*exp(c + d*x))/(8*d) - (3*b^3*exp(-
 c - d*x))/(8*d) - ((32*a^3*exp(4*c + 4*d*x))/(5*d) + (36*a^2*b*exp(3*c + 3
*d*x))/(5*d) - (36*a^2*b*exp(5*c + 5*d*x))/(5*d) + (12*a^2*b*exp(7*c + 7*d*
x))/(5*d) - (12*a^2*b*exp(c + d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c
 + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) -
 1) + (b^3*exp(3*c + 3*d*x))/(24*d) - (64*a^3)/(15*d*(3*exp(2*c + 2*d*x) -
3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (16*a^3)/(5*d*(6*exp(4*c + 4*
d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (
3*atan((a^2*b*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4*b^2)^(1/2)))*(a^4*b^2)^
(1/2))/(-d^2)^(1/2) + 3*a*b^2*x - (3*a^2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*
x) - 1)) - (18*a^2*b*exp(c + d*x))/(5*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d
*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

3.170 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=166

$$\frac{5a^3\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}-\frac{5a^3\coth(c+dx)\text{cs}}{16d}$$

```
[Out] -1/2*b^3*x+5/16*a^3*arctanh(cosh(d*x+c))/d-3*a*b^2*arctanh(cosh(d*x+c))/d+3
*a^2*b*coth(d*x+c)/d-a^2*b*coth(d*x+c)^3/d-5/16*a^3*coth(d*x+c)*csch(d*x+c)
/d+5/24*a^3*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a^3*coth(d*x+c)*csch(d*x+c)^5/d
+1/2*b^3*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3770, 3767, 3768, 2635, 8}

$$-\frac{a^2b\coth^3(c+dx)}{d}+\frac{3a^2b\coth(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^3\coth(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] -(b^3*x)/2 + (5*a^3*ArcTanh[Cosh[c + d*x]])/(16*d) - (3*a*b^2*ArcTanh[Cosh[
c + d*x]])/d + (3*a^2*b*Coth[c + d*x])/d - (a^2*b*Coth[c + d*x]^3)/d - (5*a
^3*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^3*Coth[c + d*x]*Csch[c + d*x]
^3)/(24*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d) + (b^3*Cosh[c + d*x]
*Sinh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\left(i\int\left(3iab^2\operatorname{csch}(c+dx)+3ia^2b\operatorname{csch}^4(c+dx)+ia^3\operatorname{csch}^7(c+dx)\right.\right.$$
$$= a^3\int \operatorname{csch}^7(c+dx)\,dx+\left(3a^2b\right)\int \operatorname{csch}^4(c+dx)\,dx+\left(3ab^2\right)\int \operatorname{csc}$$
$$= -\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{b^3 c}{}$$
$$= -\frac{b^3x}{2}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\coth(c+dx)}{d}-\frac{a^2b\coth}{}$$
$$= -\frac{b^3x}{2}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\coth(c+dx)}{d}-\frac{a^2b\coth}{}$$
$$= -\frac{b^3x}{2}+\frac{5a^3\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}+$$

Mathematica [A] time = 1.55, size = 236, normalized size = 1.42

$$-\frac{a^3\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)+30a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+a^3\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)+6a^3\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)+30a^3\operatorname{sech}^2\left(\frac{1}{2}(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] -1/384*(192*b^3*c + 192*b^3*d*x - 384*a^2*b*Coth[(c + d*x)/2] + 30*a^3*Csch
[(c + d*x)/2]^2 + a^3*Csch[(c + d*x)/2]^6 + 120*a^3*Log[Tanh[(c + d*x)/2]]
- 1152*a*b^2*Log[Tanh[(c + d*x)/2]] + 30*a^3*Sech[(c + d*x)/2]^2 + 6*a^3*Se
ch[(c + d*x)/2]^4 + a^3*Sech[(c + d*x)/2]^6 - 384*a^2*b*Csch[c + d*x]^3*Sin
h[(c + d*x)/2]^4 - 6*a^2*Csch[(c + d*x)/2]^4*(a - 4*b*Sinh[c + d*x]) - 96*b
^3*Sinh[2*(c + d*x)] - 384*a^2*b*Tanh[(c + d*x)/2])/d
```

fricas [B] time = 0.67, size = 6210, normalized size = 37.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^3*cosh(d*x + c)^16 + 96*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + 6*b^
3*sinh(d*x + c)^16 - 30*a^3*cosh(d*x + c)^13 - 12*(2*b^3*d*x + 3*b^3)*cosh(
d*x + c)^14 - 12*(2*b^3*d*x - 60*b^3*cosh(d*x + c)^2 + 3*b^3)*sinh(d*x + c)
^14 + 170*a^3*cosh(d*x + c)^11 + 6*(560*b^3*cosh(d*x + c)^3 - 5*a^3 - 28*(2
*b^3*d*x + 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 + 12*(12*b^3*d*x + 7*b^3)
*cosh(d*x + c)^12 + 6*(1820*b^3*cosh(d*x + c)^4 + 24*b^3*d*x - 65*a^3*cosh(
d*x + c) + 14*b^3 - 182*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
12 - 396*a^3*cosh(d*x + c)^9 + 2*(13104*b^3*cosh(d*x + c)^5 - 1170*a^3*cosh
(d*x + c)^2 - 2184*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^3 + 85*a^3 + 72*(12*b^
3*d*x + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 12*(30*b^3*d*x + 48*a^2*b
+ 7*b^3)*cosh(d*x + c)^10 + 2*(24024*b^3*cosh(d*x + c)^6 - 4290*a^3*cosh(d*
x + c)^3 - 180*b^3*d*x - 6006*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^4 + 935*a^3
*cosh(d*x + c) - 288*a^2*b - 42*b^3 + 396*(12*b^3*d*x + 7*b^3)*cosh(d*x + c
```

)^2)*sinh(d*x + c)^10 - 396*a^3*cosh(d*x + c)^7 + 2*(34320*b^3*cosh(d*x + c
)^7 - 10725*a^3*cosh(d*x + c)^4 - 12012*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^5
 + 4675*a^3*cosh(d*x + c)^2 + 1320*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^3 - 1
98*a^3 - 60*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^9
+ 480*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^8 + 6*(12870*b^3*cosh(d*x + c)^8 -
6435*a^3*cosh(d*x + c)^5 - 6006*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^6 + 4675*
a^3*cosh(d*x + c)^3 + 80*b^3*d*x + 990*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^4
 - 594*a^3*cosh(d*x + c) + 320*a^2*b - 90*(30*b^3*d*x + 48*a^2*b + 7*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^8 + 170*a^3*cosh(d*x + c)^5 + 12*(5720*b^3*co
sh(d*x + c)^9 - 4290*a^3*cosh(d*x + c)^6 - 3432*(2*b^3*d*x + 3*b^3)*cosh(d*
x + c)^7 + 4675*a^3*cosh(d*x + c)^4 + 792*(12*b^3*d*x + 7*b^3)*cosh(d*x + c
)^5 - 1188*a^3*cosh(d*x + c)^2 - 120*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d
*x + c)^3 - 33*a^3 + 320*(b^3*d*x + 4*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7
 - 12*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x + c)^6 + 12*(4004*b^3*cosh(
d*x + c)^10 - 4290*a^3*cosh(d*x + c)^7 - 3003*(2*b^3*d*x + 3*b^3)*cosh(d*x
+ c)^8 + 6545*a^3*cosh(d*x + c)^5 + 924*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^
6 - 2772*a^3*cosh(d*x + c)^3 - 30*b^3*d*x - 210*(30*b^3*d*x + 48*a^2*b + 7*
b^3)*cosh(d*x + c)^4 - 231*a^3*cosh(d*x + c) - 192*a^2*b + 7*b^3 + 1120*(b^
3*d*x + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 30*a^3*cosh(d*x + c)^3
+ 2*(13104*b^3*cosh(d*x + c)^11 - 19305*a^3*cosh(d*x + c)^8 - 12012*(2*b^3*
d*x + 3*b^3)*cosh(d*x + c)^9 + 39270*a^3*cosh(d*x + c)^6 + 4752*(12*b^3*d*x
 + 7*b^3)*cosh(d*x + c)^7 - 24948*a^3*cosh(d*x + c)^4 - 1512*(30*b^3*d*x +
48*a^2*b + 7*b^3)*cosh(d*x + c)^5 - 4158*a^3*cosh(d*x + c)^2 + 13440*(b^3*d
*x + 4*a^2*b)*cosh(d*x + c)^3 + 85*a^3 - 36*(30*b^3*d*x + 192*a^2*b - 7*b^3
)*cosh(d*x + c))*sinh(d*x + c)^5 + 12*(12*b^3*d*x + 96*a^2*b - 7*b^3)*cosh(
d*x + c)^4 + 2*(5460*b^3*cosh(d*x + c)^12 - 10725*a^3*cosh(d*x + c)^9 - 600
6*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^10 + 28050*a^3*cosh(d*x + c)^7 + 2970*(
12*b^3*d*x + 7*b^3)*cosh(d*x + c)^8 - 24948*a^3*cosh(d*x + c)^5 - 1260*(30*
b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c)^6 - 6930*a^3*cosh(d*x + c)^3 + 72
*b^3*d*x + 16800*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^4 + 425*a^3*cosh(d*x + c
) + 576*a^2*b - 42*b^3 - 90*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^4 + 2*(1680*b^3*cosh(d*x + c)^13 - 4290*a^3*cosh(d*x + c)^
10 - 2184*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^11 + 14025*a^3*cosh(d*x + c)^8
+ 1320*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^9 - 16632*a^3*cosh(d*x + c)^6 - 7
20*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c)^7 - 6930*a^3*cosh(d*x + c)
^4 + 13440*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^5 + 850*a^3*cosh(d*x + c)^2 -
120*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x + c)^3 - 15*a^3 + 24*(12*b^3*
d*x + 96*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*b^3 - 12*(2*b^3*
d*x + 16*a^2*b - 3*b^3)*cosh(d*x + c)^2 + 2*(360*b^3*cosh(d*x + c)^14 - 117
0*a^3*cosh(d*x + c)^11 - 546*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^12 + 4675*a^
3*cosh(d*x + c)^9 + 396*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^10 - 7128*a^3*co
sh(d*x + c)^7 - 270*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c)^8 - 4158*
a^3*cosh(d*x + c)^5 + 6720*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^6 + 850*a^3*co
sh(d*x + c)^3 - 12*b^3*d*x - 90*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x +
 c)^4 - 45*a^3*cosh(d*x + c) - 96*a^2*b + 18*b^3 + 36*(12*b^3*d*x + 96*a^2*
b - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((5*a^3 - 48*a*b^2)*cosh(d*
x + c)^14 + 14*(5*a^3 - 48*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^13 + (5*a^3 -
 48*a*b^2)*sinh(d*x + c)^14 - 6*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^12 - (30*a
^3 - 288*a*b^2 - 91*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^12 +
4*(91*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 - 18*(5*a^3 - 48*a*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^11 + 15*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^10 + (1001*(5*a
^3 - 48*a*b^2)*cosh(d*x + c)^4 + 75*a^3 - 720*a*b^2 - 396*(5*a^3 - 48*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(5*a^3 - 48*a*b^2)*cosh(d*x +
 c)^5 - 660*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 + 75*(5*a^3 - 48*a*b^2)*cosh
(d*x + c))*sinh(d*x + c)^9 - 20*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 + (3003*
(5*a^3 - 48*a*b^2)*cosh(d*x + c)^6 - 2970*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^
4 - 100*a^3 + 960*a*b^2 + 675*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x
+ c)^8 + 8*(429*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 - 594*(5*a^3 - 48*a*b^2)
*cosh(d*x + c)^5 + 225*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 - 20*(5*a^3 - 48*

```
a*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 15*(5*a^3 - 48*a*b^2)*cosh(d*x + c)
^6 + (3003*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 - 5544*(5*a^3 - 48*a*b^2)*cos
h(d*x + c)^6 + 3150*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 + 75*a^3 - 720*a*b^2
 - 560*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(5*a^3
 - 48*a*b^2)*cosh(d*x + c)^9 - 2376*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 + 18
90*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^5 - 560*(5*a^3 - 48*a*b^2)*cosh(d*x + c
)^3 + 45*(5*a^3 - 48*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 6*(5*a^3 - 48*
a*b^2)*cosh(d*x + c)^4 + (1001*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^10 - 2970*(
5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 + 3150*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^6
 - 1400*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 - 30*a^3 + 288*a*b^2 + 225*(5*a^
3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(91*(5*a^3 - 48*a*b^2)*c
osh(d*x + c)^11 - 330*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^9 + 450*(5*a^3 - 48*
a*b^2)*cosh(d*x + c)^7 - 280*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^5 + 75*(5*a^3
 - 48*a*b^2)*cosh(d*x + c)^3 - 6*(5*a^3 - 48*a*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^3 + (5*a^3 - 48*a*b^2)*cosh(d*x + c)^2 + (91*(5*a^3 - 48*a*b^2)*cosh(
d*x + c)^12 - 396*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^10 + 675*(5*a^3 - 48*a*b
^2)*cosh(d*x + c)^8 - 560*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^6 + 225*(5*a^3 -
 48*a*b^2)*cosh(d*x + c)^4 + 5*a^3 - 48*a*b^2 - 36*(5*a^3 - 48*a*b^2)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 2*(7*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^13 - 36
*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^11 + 75*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^
9 - 80*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 + 45*(5*a^3 - 48*a*b^2)*cosh(d*x
+ c)^5 - 12*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 + (5*a^3 - 48*a*b^2)*cosh(d*
x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*((5*a^3 -
 48*a*b^2)*cosh(d*x + c)^14 + 14*(5*a^3 - 48*a*b^2)*cosh(d*x + c)*sinh(d*x
+ c)^13 + (5*a^3 - 48*a*b^2)*sinh(d*x + c)^14 - 6*(5*a^3 - 48*a*b^2)*cosh(d
*x + c)^12 - (30*a^3 - 288*a*b^2 - 91*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*s
inh(d*x + c)^12 + 4*(91*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 - 18*(5*a^3 - 48
*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 + 15*(5*a^3 - 48*a*b^2)*cosh(d*x +
c)^10 + (1001*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 + 75*a^3 - 720*a*b^2 - 396
*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(5*a^3 - 48
*a*b^2)*cosh(d*x + c)^5 - 660*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 + 75*(5*a^
3 - 48*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 20*(5*a^3 - 48*a*b^2)*cosh(d
*x + c)^8 + (3003*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^6 - 2970*(5*a^3 - 48*a*b
^2)*cosh(d*x + c)^4 - 100*a^3 + 960*a*b^2 + 675*(5*a^3 - 48*a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^8 + 8*(429*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 - 594*
(5*a^3 - 48*a*b^2)*cosh(d*x + c)^5 + 225*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3
 - 20*(5*a^3 - 48*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 15*(5*a^3 - 48*a*
b^2)*cosh(d*x + c)^6 + (3003*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 - 5544*(5*a
^3 - 48*a*b^2)*cosh(d*x + c)^6 + 3150*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 +
75*a^3 - 720*a*b^2 - 560*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 2*(1001*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^9 - 2376*(5*a^3 - 48*a*b^2)*co
sh(d*x + c)^7 + 1890*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^5 - 560*(5*a^3 - 48*a
*b^2)*cosh(d*x + c)^3 + 45*(5*a^3 - 48*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^
5 - 6*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 + (1001*(5*a^3 - 48*a*b^2)*cosh(d*
x + c)^10 - 2970*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 + 3150*(5*a^3 - 48*a*b^
2)*cosh(d*x + c)^6 - 1400*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 - 30*a^3 + 288
*a*b^2 + 225*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(91*(5
*a^3 - 48*a*b^2)*cosh(d*x + c)^11 - 330*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^9
+ 450*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 - 280*(5*a^3 - 48*a*b^2)*cosh(d*x
+ c)^5 + 75*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 - 6*(5*a^3 - 48*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^3 + (5*a^3 - 48*a*b^2)*cosh(d*x + c)^2 + (91*(5*a^3
 - 48*a*b^2)*cosh(d*x + c)^12 - 396*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^10 + 6
75*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 - 560*(5*a^3 - 48*a*b^2)*cosh(d*x + c
)^6 + 225*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 + 5*a^3 - 48*a*b^2 - 36*(5*a^3
 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(7*(5*a^3 - 48*a*b^2)*cos
h(d*x + c)^13 - 36*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^11 + 75*(5*a^3 - 48*a*b
^2)*cosh(d*x + c)^9 - 80*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 + 45*(5*a^3 - 4
8*a*b^2)*cosh(d*x + c)^5 - 12*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 + (5*a^3 -
 48*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
```

```
- 1) + 2*(48*b^3*cosh(d*x + c)^15 - 195*a^3*cosh(d*x + c)^12 - 84*(2*b^3*d*
x + 3*b^3)*cosh(d*x + c)^13 + 935*a^3*cosh(d*x + c)^10 + 72*(12*b^3*d*x + 7
*b^3)*cosh(d*x + c)^11 - 1782*a^3*cosh(d*x + c)^8 - 60*(30*b^3*d*x + 48*a^2
*b + 7*b^3)*cosh(d*x + c)^9 - 1386*a^3*cosh(d*x + c)^6 + 1920*(b^3*d*x + 4*
a^2*b)*cosh(d*x + c)^7 + 425*a^3*cosh(d*x + c)^4 - 36*(30*b^3*d*x + 192*a^2
*b - 7*b^3)*cosh(d*x + c)^5 - 45*a^3*cosh(d*x + c)^2 + 24*(12*b^3*d*x + 96*
a^2*b - 7*b^3)*cosh(d*x + c)^3 - 12*(2*b^3*d*x + 16*a^2*b - 3*b^3)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^14 + 14*d*cosh(d*x + c)*sinh(d*x + c
)^13 + d*sinh(d*x + c)^14 - 6*d*cosh(d*x + c)^12 + (91*d*cosh(d*x + c)^2 -
6*d)*sinh(d*x + c)^12 + 4*(91*d*cosh(d*x + c)^3 - 18*d*cosh(d*x + c))*sinh(
d*x + c)^11 + 15*d*cosh(d*x + c)^10 + (1001*d*cosh(d*x + c)^4 - 396*d*cosh(
d*x + c)^2 + 15*d)*sinh(d*x + c)^10 + 2*(1001*d*cosh(d*x + c)^5 - 660*d*cos
h(d*x + c)^3 + 75*d*cosh(d*x + c))*sinh(d*x + c)^9 - 20*d*cosh(d*x + c)^8 +
 (3003*d*cosh(d*x + c)^6 - 2970*d*cosh(d*x + c)^4 + 675*d*cosh(d*x + c)^2 -
 20*d)*sinh(d*x + c)^8 + 8*(429*d*cosh(d*x + c)^7 - 594*d*cosh(d*x + c)^5 +
 225*d*cosh(d*x + c)^3 - 20*d*cosh(d*x + c))*sinh(d*x + c)^7 + 15*d*cosh(d*
x + c)^6 + (3003*d*cosh(d*x + c)^8 - 5544*d*cosh(d*x + c)^6 + 3150*d*cosh(d
*x + c)^4 - 560*d*cosh(d*x + c)^2 + 15*d)*sinh(d*x + c)^6 + 2*(1001*d*cosh(
d*x + c)^9 - 2376*d*cosh(d*x + c)^7 + 1890*d*cosh(d*x + c)^5 - 560*d*cosh(d
*x + c)^3 + 45*d*cosh(d*x + c))*sinh(d*x + c)^5 - 6*d*cosh(d*x + c)^4 + (10
01*d*cosh(d*x + c)^10 - 2970*d*cosh(d*x + c)^8 + 3150*d*cosh(d*x + c)^6 - 1
400*d*cosh(d*x + c)^4 + 225*d*cosh(d*x + c)^2 - 6*d)*sinh(d*x + c)^4 + 4*(9
1*d*cosh(d*x + c)^11 - 330*d*cosh(d*x + c)^9 + 450*d*cosh(d*x + c)^7 - 280*
d*cosh(d*x + c)^5 + 75*d*cosh(d*x + c)^3 - 6*d*cosh(d*x + c))*sinh(d*x + c)
^3 + d*cosh(d*x + c)^2 + (91*d*cosh(d*x + c)^12 - 396*d*cosh(d*x + c)^10 +
675*d*cosh(d*x + c)^8 - 560*d*cosh(d*x + c)^6 + 225*d*cosh(d*x + c)^4 - 36*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(7*d*cosh(d*x + c)^13 - 36*d*cos
h(d*x + c)^11 + 75*d*cosh(d*x + c)^9 - 80*d*cosh(d*x + c)^7 + 45*d*cosh(d*x
 + c)^5 - 12*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 0.38, size = 327, normalized size = 1.97

$$-\frac{24\,(dx+c)b^3 - 6\,b^3 e^{(2\,dx+2\,c)} - 3\left(5\,a^3 - 48\,ab^2\right)\log\left(e^{(dx+c)}+1\right) + 3\left(5\,a^3 - 48\,ab^2\right)\log\left(\left|e^{(dx+c)}-1\right|\right) + \frac{2\,(15\,a^3}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/48*(24*(d*x + c)*b^3 - 6*b^3*e^(2*d*x + 2*c) - 3*(5*a^3 - 48*a*b^2)*log(
e^(d*x + c) + 1) + 3*(5*a^3 - 48*a*b^2)*log(abs(e^(d*x + c) - 1)) + 2*(15*a
^3*e^(13*d*x + 13*c) + 3*b^3*e^(12*d*x + 12*c) - 85*a^3*e^(11*d*x + 11*c) +
 198*a^3*e^(9*d*x + 9*c) + 198*a^3*e^(7*d*x + 7*c) - 85*a^3*e^(5*d*x + 5*c)
 + 15*a^3*e^(3*d*x + 3*c) + 3*b^3 + 18*(16*a^2*b - b^3)*e^(10*d*x + 10*c) -
 15*(64*a^2*b - 3*b^3)*e^(8*d*x + 8*c) + 12*(96*a^2*b - 5*b^3)*e^(6*d*x + 6
*c) - 9*(64*a^2*b - 5*b^3)*e^(4*d*x + 4*c) + 6*(16*a^2*b - 3*b^3)*e^(2*d*x
+ 2*c))*e^(-2*d*x - 2*c)/((e^(d*x + c) + 1)^6*(e^(d*x + c) - 1)^6))/d
```

maple [A] time = 0.19, size = 119, normalized size = 0.72

$$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6} + \frac{5\operatorname{csch}(dx+c)^3}{24} - \frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c) + \frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right) + 3a^2b\left(\frac{2}{3} - \frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(a^3*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x
+c)+5/8*arctanh(exp(d*x+c)))+3*a^2*b*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-6*
a*b^2*arctanh(exp(d*x+c))+b^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [B] time = 0.43, size = 355, normalized size = 2.14

$$-\frac{1}{8}b^3\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + \frac{1}{48}a^3\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{15\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(15e^{(-dx-c)} - 85e^{(-3d}\right.}{d\left(6e^{(-2dx-2c)} - 15e^{(-4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/48*a^3*(15*log(
e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*
e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d
*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x -
4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e
^(-12*d*x - 12*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x -
 c) - 1)/d) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 0.29, size = 486, normalized size = 2.93

$$\frac{\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\left(5a^3\sqrt{-d^2}-48ab^2\sqrt{-d^2}\right)}{d\sqrt{25a^6-480a^4b^2+2304a^2b^4}}\right)\sqrt{25a^6-480a^4b^2+2304a^2b^4}}{8\sqrt{-d^2}} - \frac{b^3x}{2} - \frac{\frac{12a^2b}{d} - \frac{5a^3e^{c+dx}}{12d}}{e^{4c+4dx}-2e^{2c+2dx}+1} - \frac{\frac{8a}{c}}{3e^{2c+2dx}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^3)^3/sinh(c + d*x)^7,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*a^3*(-d^2)^(1/2) - 48*a*b^2*(-d^2)^(1/2)))/(d*(25
*a^6 + 2304*a^2*b^4 - 480*a^4*b^2)^(1/2)))*(25*a^6 + 2304*a^2*b^4 - 480*a^4
*b^2)^(1/2))/(8*(-d^2)^(1/2)) - (b^3*x)/2 - ((12*a^2*b)/d - (5*a^3*exp(c +
d*x))/(12*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((8*a^2*b)/d +
(a^3*exp(c + d*x))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) - 1) - (b^3*exp(- 2*c - 2*d*x))/(8*d) + (b^3*exp(2*c + 2*d*x))/(
8*d) - (18*a^3*exp(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) -
4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (80*a^3*exp(c + d*x))/(3*d*(5
*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c +
 8*d*x) + exp(10*c + 10*d*x) - 1)) - (32*a^3*exp(c + d*x))/(3*d*(15*exp(4*c
 + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x)
- 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (5*a^3*exp(c + d*x))/(8
*d*(exp(2*c + 2*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

3.171 $\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=328

$$-\frac{2a^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\ldots\right)}{3b^2d\sqrt{\ldots}}$$

```
[Out] -a*x/b^2-cosh(d*x+c)/b/d+1/3*cosh(d*x+c)^3/b/d-2/3*(-1)^(2/3)*a^(4/3)*arcta
n((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)
*a^(2/3)-b^(2/3))^(1/2))/b^2/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)-2/3*a^(4/
3)*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/b
^2/d/(a^(2/3)+b^(2/3))^(1/2)-2/3*(-1)^(2/3)*a^(4/3)*arctan((-1)^(1/6)*((-1)
^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3
)*b^(2/3))^(1/2))/b^2/d/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 2633, 3213, 2660, 618, 204}

$$-\frac{2a^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\ldots\right)}{3b^2d\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -((a*x)/b^2) - (2*(-1)^(2/3)*a^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3)
 + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/
3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^2*d) - (2*(-1)^(2/
3)*a^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x
)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^
(2/3)]*b^2*d) - (2*a^(4/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sq
rt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^2*d) - Cosh[c + d*x]/(
b*d) + Cosh[c + d*x]^3/(3*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\int \left(\frac{a}{b^2}-\frac{\sinh^3(c+dx)}{b}-\frac{a^2}{b^2\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$= -\frac{ax}{b^2}+\frac{a^2\int \frac{1}{a+b\sinh^3(c+dx)}\,dx}{b^2}+\frac{\int \sinh^3(c+dx)\,dx}{b}$$

$$= -\frac{ax}{b^2}+\frac{a^2\int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{\cdots}\right.}\right.}{b^2}$$

$$= -\frac{ax}{b^2}-\frac{\cosh(c+dx)}{bd}+\frac{\cosh^3(c+dx)}{3bd}+\frac{\left(\sqrt[6]{-1}a^{4/3}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^2}+\frac{\left(\sqrt[\cdots]{-}\right.}{}$$

$$= -\frac{ax}{b^2}-\frac{\cosh(c+dx)}{bd}+\frac{\cosh^3(c+dx)}{3bd}-\frac{\left(2(-1)^{2/3}a^{4/3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}\cdots}\right.}{3b^2d}$$

$$= -\frac{ax}{b^2}-\frac{\cosh(c+dx)}{bd}+\frac{\cosh^3(c+dx)}{3bd}+\frac{\left(4(-1)^{2/3}a^{4/3}\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,d\cdots\right.}{3b^2\cdots}$$

$$= -\frac{ax}{b^2}+\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^2d}-\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-\cdots}{\sqrt{\sqrt[3]{-1}\cdots}}\right.}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2\cdots}}}$$

Mathematica [C] time = 0.37, size = 168, normalized size = 0.51

$$\frac{8a^2\mathrm{RootSum}\left[\#1^6b-3\#1^4b+8\#1^3a+3\#1^2b-b\&,\frac{2\#1\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\cdots\right.}{\#1^4b-2\#1^2b+4\#1a+b}}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-12*a*c - 12*a*d*x - 9*b*Cosh[c + d*x] + b*Cosh[3*(c + d*x)] + 8*a^2*RootS
um[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[
```

```
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(12*b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)^6}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
[Out] integrate(sinh(d*x + c)^6/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.12, size = 259, normalized size = 0.79

$$-\frac{1}{3db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,b^2}+-\frac{}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x)
[Out] -1/3/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/
b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/3/d/b/(tanh
(1/2*d*x+1/2*c)+1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/b/(tanh(1/2*d*
x+1/2*c)+1)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/3/d*a^2/b^2*sum((_R^4-2*_
R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf
(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$8\,a^2\int\frac{e^{(3\,dx+3\,c)}}{b^3e^{(6\,dx+6\,c)}-3\,b^3e^{(4\,dx+4\,c)}+8\,ab^2e^{(3\,dx+3\,c)}+3\,b^3e^{(2\,dx+2\,c)}-b^3}\,dx-\frac{\left(24\,adxe^{(3\,dx+3\,c)}-be^{(6\,dx+6\,c)}+9\,be^{(}\right.}{24\,l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
[Out] 8*a^2*integrate(e^(3*d*x + 3*c)/(b^3*e^(6*d*x + 6*c) - 3*b^3*e^(4*d*x + 4*c
) + 8*a*b^2*e^(3*d*x + 3*c) + 3*b^3*e^(2*d*x + 2*c) - b^3), x) - 1/24*(24*a
*d*x*e^(3*d*x + 3*c) - b*e^(6*d*x + 6*c) + 9*b*e^(4*d*x + 4*c) + 9*b*e^(2*d
*x + 2*c) - b)*e^(-3*d*x - 3*c)/(b^2*d)
```

mupad [B] time = 10.80, size = 1579, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^6/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log((294912*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b
^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^2*a^7*b^5*d^2 - 98304*a^10*exp
(d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^
4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) + 1327104*root(729*a^2*b^12*d^6*z^6 +
729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^3*
a^6*b^7*d^3 + 2654208*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^
4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^4*a^5*b^9*d^4 + 1990656*roo
t(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^
4*d^2*z^2 - a^8, z, k)^5*a^4*b^11*d^5 + 24576*root(729*a^2*b^12*d^6*z^6 + 7
29*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)*a^8
*b^3*d + 589824*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*
d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^2*a^8*b^4*d^2*exp(d*x)*exp(root(7
29*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d
^2*z^2 - a^8, z, k)) + 5308416*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6
 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^3*a^7*b^6*d^3*exp(
d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4
 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) - 663552*root(729*a^2*b^12*d^6*z^6 + 72
9*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^4*a^
4*b^10*d^4*exp(d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*
a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) + 2654208*root(729*a^2*b
^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 -
 a^8, z, k)^4*a^6*b^8*d^4*exp(d*x)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14
*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)) - 9953280
*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^
6*b^4*d^2*z^2 - a^8, z, k)^5*a^3*b^12*d^5*exp(d*x)*exp(root(729*a^2*b^12*d^
6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8,
z, k)) - 7962624*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8
*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)^5*a^5*b^10*d^5*exp(d*x)*exp(root
(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4
*d^2*z^2 - a^8, z, k)) - 491520*root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^
6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k)*a^9*b^2*d*exp(d*x
)*exp(root(729*a^2*b^12*d^6*z^6 + 729*b^14*d^6*z^6 - 243*a^4*b^8*d^4*z^4 +
27*a^6*b^4*d^2*z^2 - a^8, z, k)))/b^15)*root(729*a^2*b^12*d^6*z^6 + 729*b^1
4*d^6*z^6 - 243*a^4*b^8*d^4*z^4 + 27*a^6*b^4*d^2*z^2 - a^8, z, k), k, 1, 6)
 - (3*exp(c + d*x))/(8*b*d) - (3*exp(- c - d*x))/(8*b*d) + exp(- 3*c - 3*d*
x)/(24*b*d) + exp(3*c + 3*d*x)/(24*b*d) - (a*x)/b^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)**6/(a + b*sinh(c + d*x)**3), x)
```

3.172 $\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=295

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2a\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2a\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}$$

```
[Out] -1/2*x/b+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d+2/3*a*arctan((-1)^(1/6)*((-1)^(5/6
)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)
)/b^(5/3)/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)+2/3*a*arctan((-1)^(5/6)*((-1
)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)-b^(2/3)
)^(1/2))/b^(5/3)/d/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2)+2/3*a*arctanh((b^(1/
3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/b^(5/3)/d/(a^(2/3)
+b^(2/3))^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 2635, 8, 2660, 618, 206, 204}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2a\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2a\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -x/(2*b) + (2*a*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c
+ d*x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^
(2/3)) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3)
+ I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqr
t[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) - a^(1/3
)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b
^(5/3)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\left(i\int\left(\frac{i\sinh^2(c+dx)}{b}-\frac{ia\sinh^2(c+dx)}{b\left(a+b\sinh^3(c+dx)\right)}\right)dx\right)$$

$$=\frac{\int\sinh^2(c+dx)\,dx}{b}-\frac{a\int\frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{b}$$

$$=\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}-\frac{\int 1\,dx}{2b}+\frac{a\int\left(\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{i}{3b^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{\ldots}\right.}\right.}{b}$$

$$=-\frac{x}{2b}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}+\frac{(ia)\int\frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{5/3}}+\frac{(ia)\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\ldots}}{3b^{5/3}}$$

$$=-\frac{x}{2b}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}+\frac{(2a)\operatorname{Subst}\left(\int\frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}x-i\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+\ldots\right.\right.}{3b^{5/3}d}$$

$$=-\frac{x}{2b}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}-\frac{(4a)\operatorname{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}+2\ldots\right.}{3b^{5/3}d}$$

$$=-\frac{x}{2b}-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,b^{5/3}d}-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^{5/3}d}+\ldots$$

Mathematica [C] time = 0.33, size = 299, normalized size = 1.01

$$-2a\operatorname{RootSum}\left[\#1^6b-3\#1^4b+8\#1^3a+3\#1^2b-b\&,\frac{2\#1^4\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx\right.\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-6*(c + d*x) - 2*a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &
, (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*
x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1
^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cos
h[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3
 + b*#1^5) & ] + 3*Sinh[2*(c + d*x)])/(12*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)^5}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^5/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.11, size = 207, normalized size = 0.70

$$\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2db}-\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{2db(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/2/d/b*l
n(tanh(1/2*d*x+1/2*c)-1)-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(tanh(1/
2*d*x+1/2*c)+1)-1/2/d/b*ln(tanh(1/2*d*x+1/2*c)+1)+4/3/d*a/b*sum(_R^2/(_R^5*
a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^
4*a-8*_Z^3*b+3*_Z^2*a-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(4\,dxe^{(2\,dx+2\,c)}-e^{(4\,dx+4\,c)}+1\right)e^{(-2\,dx-2\,c)}}{8\,bd}-\frac{1}{32}\int\frac{64\left(ae^{(5\,dx+5\,c)}-2\,ae^{(3\,dx+3\,c)}+ae^{(dx+c)}\right)}{b^2e^{(6\,dx+6\,c)}-3\,b^2e^{(4\,dx+4\,c)}+8\,abe^{(3\,dx+3\,c)}+3\,b^2e^{(2\,dx+2\,c)}-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*(4*d*x*e^(2*d*x + 2*c) - e^(4*d*x + 4*c) + 1)*e^(-2*d*x - 2*c)/(b*d) -
 1/32*integrate(64*(a*e^(5*d*x + 5*c) - 2*a*e^(3*d*x + 3*c) + a*e^(d*x + c)
```

```
)/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c) + 8*a*b*e^(3*d*x + 3*c) + 3*
b^2*e^(2*d*x + 2*c) - b^2), x)
```

mupad [B] time = 11.48, size = 1114, normalized size = 3.78

$$\left(\sum_{k=1}^{6}\ln\left(-\mathrm{root}\left(729\,a^2\,b^{10}\,d^6\,z^6+729\,b^{12}\,d^6\,z^6-243\,a^2\,b^8\,d^4\,z^4+27\,a^4\,b^4\,d^2\,z^2-a^6,z,k\right)\left(\mathrm{root}\left(729\,a^2\,b^{10}\,d^6\,z^6+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log(- root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4
*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)*(root(729*a^2*b^10*d^6*z^6 + 729*b^1
2*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)*(root(729
*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2
*z^2 - a^6, z, k)*(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b
^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k)*((663552*(8*a^6*d^4 + 4*a^4*b^
2*d^4 - 5*a^5*b*d^4*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z
^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^7 + (199065
6*root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a
^4*b^4*d^2*z^2 - a^6, z, k)*(4*a^5*d^5*exp(d*x)*exp(root(729*a^2*b^10*d^6*z
^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z,
k)) - a^4*b*d^5 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 72
9*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b
^5) + (442368*(4*a^6*b*d^3 + 8*a^7*d^3*exp(d*x)*exp(root(729*a^2*b^10*d^6*z
^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z,
k)) - 5*a^5*b^2*d^3*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z
^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^9) - (29491
2*a^6*d^2*(2*b - 5*a*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*
z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^10) - (245
76*a^7*d*(8*a - 5*b*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z
^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^12) - (3276
8*a^8*(b - 4*a*exp(d*x)*exp(root(729*a^2*b^10*d^6*z^6 + 729*b^12*d^6*z^6 -
243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - a^6, z, k))))/b^14)*root(729*a^2
*b^10*d^6*z^6 + 729*b^12*d^6*z^6 - 243*a^2*b^8*d^4*z^4 + 27*a^4*b^4*d^2*z^2
 - a^6, z, k), k, 1, 6) - x/(2*b) - exp(- 2*c - 2*d*x)/(8*b*d) + exp(2*c +
2*d*x)/(8*b*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)**5/(a + b*sinh(c + d*x)**3), x)
```

3.173 $\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=303

$$-\frac{2a^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}a^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\ldots\right.}{\sqrt{\ldots}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2}\ldots}}$$

```
[Out] cosh(d*x+c)/b/d+2/3*(-1)^(1/3)*a^(2/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3
)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/b^(4/3
)/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)-2/3*a^(2/3)*arctanh((b^(1/3)-a^(1/3)
*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)+b^(2/3))^
(1/2)-2/3*a^(2/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*
d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b^(4/3)/d/((-1)^
(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 2638, 2660, 618, 204}

$$-\frac{2a^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}a^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\ldots\right.}{\sqrt{\ldots}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2}\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*a^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*
a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(4/3)*d) + (2*(-1)^(1/3)*a^(2/3)*ArcTan[((-
1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3
)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(4/3)*d) - (
2*a^(2/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2
/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(4/3)*d) + Cosh[c + d*x]/(b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \left(\frac{\sinh(c+dx)}{b} - \frac{a\sinh(c+dx)}{b\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \sinh(c+dx)\,dx}{b} - \frac{a\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{b}$$

$$= \frac{\cosh(c+dx)}{bd} + \frac{(ia)\int \left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+d\right.}\right.}{b}$$

$$= \frac{\cosh(c+dx)}{bd} - \frac{\left(ia^{2/3}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{4/3}} + \frac{\left(\sqrt[6]{-1}\,a^{2/3}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}}}{3b^{4/3}}$$

$$= \frac{\cosh(c+dx)}{bd} - \frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{4/3}d}$$

$$= \frac{\cosh(c+dx)}{bd} + \frac{\left(4a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{-1}\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\right.}{3b^{4/3}d}$$

$$= -\frac{2\sqrt[3]{-1}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,b^{4/3}d} - \frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+\right.}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}\right.}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d}$$

Mathematica [C] time = 0.36, size = 214, normalized size = 0.71

$$\frac{3\cosh(c+dx) - a\text{RootSum}\left[\#1^6b - 3\#1^4b + 8\#1^3a + 3\#1^2b - b\&, \frac{2\#1^2\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+\right.\right.}{}\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*Cosh[c + d*x] - a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &
 , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x
)/2]*#1 - Sinh[(c + d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2
)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(3*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)^4}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^4/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.10, size = 128, normalized size = 0.42

$$\frac{1}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3db}-\frac{1}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d/b/(tanh(1/2*d*x+1/2*c)+1)-2/3/d*a/b*sum((_R^3-_R)/(_R^5*a-2*_R^3*a-4*_R
^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*
_Z^2*a-a))-1/d/b/(tanh(1/2*d*x+1/2*c)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2\,dx+2\,c)}+1\right)e^{(-dx-c)}}{2\,bd}-\frac{1}{16}\int\frac{64\left(ae^{(4\,dx+4\,c)}-ae^{(2\,dx+2\,c)}\right)}{b^2e^{(6\,dx+6\,c)}-3\,b^2e^{(4\,dx+4\,c)}+8\,abe^{(3\,dx+3\,c)}+3\,b^2e^{(2\,dx+2\,c)}-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/16*integrate(64*(a*e^(4*d*
x + 4*c) - a*e^(2*d*x + 2*c))/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c)
+ 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(2*d*x + 2*c) - b^2), x)
```

mupad [B] time = 23.50, size = 906, normalized size = 2.99

$$\left(\sum_{k=1}^{6}\ln\left(-\mathrm{root}\left(729\,a^2\,b^8\,d^6\,z^6+729\,b^{10}\,d^6\,z^6+243\,a^2\,b^6\,d^4\,z^4-a^4,z,k\right)\left(\mathrm{root}\left(729\,a^2\,b^8\,d^6\,z^6+729\,b^{10}\,d^6\,z^6\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log((8192*a^6*(8*a - b*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b
^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^12 - root(729*a^2*b^8*d
^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(root(729*a^2*
b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(root(729
*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(roo
t(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)
*((663552*(4*a^5*b*d^4 + 16*a^6*d^4*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 +
 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)) + 11*a^4*b^2*d^4*exp(
d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4
```

```
- a^4, z, k))))/b^7 + (1990656*root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6
+ 243*a^2*b^6*d^4*z^4 - a^4, z, k)*(4*a^5*d^5*exp(d*x)*exp(root(729*a^2*b^8
*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k)) - a^4*b*d^5
 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 +
 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^5) - (221184*(8*a^6*d^3 + a^4*b^2*d^
3 - 25*a^5*b*d^3*exp(d*x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 +
 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^8) - (294912*a^5*d^2*(b - 7*a*exp(d*
x)*exp(root(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 -
a^4, z, k))))/b^9) - (196608*a^6*d*(b - 2*a*exp(d*x)*exp(root(729*a^2*b^8*d
^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k))))/b^11))*root
(729*a^2*b^8*d^6*z^6 + 729*b^10*d^6*z^6 + 243*a^2*b^6*d^4*z^4 - a^4, z, k),
 k, 1, 6) + exp(c + d*x)/(2*b*d) + exp(- c - d*x)/(2*b*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(c + dx)}{a + b\sinh^3(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)**4/(a + b*sinh(c + d*x)**3), x)
```

3.174 $\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{2\sqrt[3]{a}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+b^{2/3}}}+\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\cdots\right)}{3bd\sqrt{\cdots}}$$

```
[Out] x/b+2/3*(-1)^(2/3)*a^(1/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*
tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/b/d/((-1)^(1/3)*a^
(2/3)-b^(2/3))^(1/2)+2/3*a^(1/3)*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*
c))/(a^(2/3)+b^(2/3))^(1/2))/b/d/(a^(2/3)+b^(2/3))^(1/2)+2/3*(-1)^(2/3)*a^(
1/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/(
(-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b/d/((-1)^(1/3)*a^(2/3)-(-1)^
(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 3213, 2660, 618, 204}

$$\frac{2\sqrt[3]{a}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+b^{2/3}}}+\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\cdots\right)}{3bd\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] x/b + (2*(-1)^(2/3)*a^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1
/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*
Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d) + (2*(-1)^(2/3)*a^(1/3)*
ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt
[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b*d)
 + (2*a^(1/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) +
b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = i\int\left(-\frac{i}{b}+\frac{ia}{b\left(a+b\sinh^3(c+dx)\right)}\right)dx$$
$$= \frac{x}{b}-\frac{a\int\frac{1}{a+b\sinh^3(c+dx)}\,dx}{b}$$
$$= \frac{x}{b}-\frac{a\int\left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+(-\right.}\right.}{b}$$
$$= \frac{x}{b}-\frac{\left(\sqrt[6]{-1}\sqrt[3]{a}\right)\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b}-\frac{\left(\sqrt[6]{-1}\sqrt[3]{a}\right)\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b}$$
$$= \frac{x}{b}+\frac{\left(2(-1)^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3bd}+\frac{\left(2(-1\right.}{}$$
$$= \frac{x}{b}-\frac{\left(4(-1)^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+i\right.\right.}{3bd}$$
$$= \frac{x}{b}-\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,bd}+\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}}{\sqrt{\sqrt[3]{-1}a^{2/3}}}\right.}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b}}$$

Mathematica [C] time = 0.23, size = 145, normalized size = 0.49

$$\frac{-2a\text{RootSum}\left[\#1^6b-3\#1^4b+8\#1^3a+3\#1^2b-b\&,\frac{2\#1\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right.\right.}{\#1^4b-2\#1^2b+4\#1a+b}\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*c + 3*d*x - 2*a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & ,
 (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ]
)/(3*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)^3}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
[Out] integrate(sinh(d*x + c)^3/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.09, size = 129, normalized size = 0.44

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{db}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{db}+\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}\right.\right.}{\_R^5a-2\_R^3a-4\_R^2b-}\right.}{3db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x)
[Out] -1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)+1/3/d*a/b*
sum((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-
_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-8\,a\int \frac{e^{(3\,dx+3\,c)}}{b^2e^{(6\,dx+6\,c)}-3\,b^2e^{(4\,dx+4\,c)}+8\,abe^{(3\,dx+3\,c)}+3\,b^2e^{(2\,dx+2\,c)}-b^2}\,dx+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
[Out] -8*a*integrate(e^(3*d*x + 3*c)/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c)
 + 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(2*d*x + 2*c) - b^2), x) + x/b
```

mupad [B] time = 10.76, size = 1498, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a + b*sinh(c + d*x)^3),x)
[Out] symsum(log(-(24576*a^3*(405*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 24
3*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^5*b^7*d^5*exp(root(729*
a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^
2 - a^2, z, k) + d*x) - root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^
2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)*a*b^2*d - 27*root(729*a^2*b
^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a
^2, z, k)^4*b^6*d^4*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^
2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x) - 4*a^2*exp(root(729
*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z
^2 - a^2, z, k) + d*x) + 12*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 24
3*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^2*a*b^3*d^2 - 54*root(7
29*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2
*z^2 - a^2, z, k)^3*a*b^4*d^3 + 108*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*
```

```
z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^4*a*b^5*d^4 - 8
1*root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2
*b^2*d^2*z^2 - a^2, z, k)^5*a*b^6*d^5 + 20*root(729*a^2*b^6*d^6*z^6 + 729*b
^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)*a^2*b*d*
exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a
^2*b^2*d^2*z^2 - a^2, z, k) + d*x) + 24*root(729*a^2*b^6*d^6*z^6 + 729*b^8*
d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^2*a^2*b^2*d
^2*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 2
7*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x) - 216*root(729*a^2*b^6*d^6*z^6 + 729*
b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^3*a^2*b
^3*d^3*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4
 + 27*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x) + 108*root(729*a^2*b^6*d^6*z^6 +
729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)^4*a
^2*b^4*d^4*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4
*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x) + 324*root(729*a^2*b^6*d^6*z^
6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k)
^5*a^2*b^5*d^5*exp(root(729*a^2*b^6*d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4
*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2, z, k) + d*x)))/b^10)*root(729*a^2*b^6*
d^6*z^6 + 729*b^8*d^6*z^6 - 243*a^2*b^4*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - a^2,
 z, k), k, 1, 6) + x/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**3),x)
[Out] Integral(sinh(c + d*x)**3/(a + b*sinh(c + d*x)**3), x)
```

3.175 $\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=262

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}$$

```
[Out] -2/3*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/(
(-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/b^(2/3)/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(
1/2)-2/3*arctan((-1)^(5/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c
))/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2))/b^(2/3)/d/(-(-1)^(2/3)*a^(2/3)-b^(2
/3))^(1/2)-2/3*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/
3))^(1/2))/b^(2/3)/d/(a^(2/3)+b^(2/3))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 2660, 618, 206, 204}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/
Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(
2/3)]*b^(2/3)*d) - (2*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Ta
nh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a
^(2/3) - b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)
/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(2/3)*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\int\left(\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{i}{3b^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{}{3b^{2}}\right.$$

$$= -\frac{i\int\frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{2/3}}-\frac{i\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{2/3}}-\frac{i\int\frac{1}{(-1)^{5/6}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}}{3b^{2/3}}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int\frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}x-i\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{2/3}d}-\frac{2\,\mathrm{Subst}\left(\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{b}x+\sqrt[6]{}}\right.}{3}$$

$$= \frac{4\,\mathrm{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{2/3}d}+\frac{4\,\mathrm{Subst}}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,b^{2/3}d}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^{2/3}d}-\frac{2\tanh^{-1}\left(\right.}{3\sqrt{a^{2}}}$$

Mathematica [C] time = 0.18, size = 275, normalized size = 1.05

$$\mathrm{RootSum}\left[\#1^6b-3\#1^4b+8\#1^3a+3\#1^2b-b\&,\frac{2\#1^4\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)+}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c + d*x + 2*Log[-
Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*
x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1
^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) & ]/(6*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)^2}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.08, size = 78, normalized size = 0.30

$$-\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] -4/3/d*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),
_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)^2}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

mupad [B] time = 11.13, size = 932, normalized size = 3.56

$$\sum_{k=1}^{6}\ln\left(\mathrm{root}\left(729\,a^2\,b^4\,d^6\,z^6+729\,b^6\,d^6\,z^6-243\,b^4\,d^4\,z^4+27\,b^2\,d^2\,z^2-1,z,k\right)\left(\mathrm{root}\left(729\,a^2\,b^4\,d^6\,z^6+729\,b^6\,d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 2
7*b^2*d^2*z^2 - 1, z, k)*(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*
b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)*(root(729*a^2*b^4*d^6*z^6 + 729*b^6
*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)*(root(729*a^2*b^4*d^
6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)*((663
552*(8*a^5*d^4 + 4*a^3*b^2*d^4 - 5*a^4*b*d^4*exp(d*x)*exp(root(729*a^2*b^4*
d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k))))/
b^6 - (1990656*root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4
 + 27*b^2*d^2*z^2 - 1, z, k)*(4*a^5*d^5*exp(d*x)*exp(root(729*a^2*b^4*d^6*z
^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)) - a^4*b
*d^5 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^
6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k))))/b^5) - (442368*(4*a^4*b*
d^3 + 8*a^5*d^3*exp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 2
43*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)) - 5*a^3*b^2*d^3*exp(d*x)*exp(ro
ot(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2
```

```
 - 1, z, k))))/b^7) - (294912*a^3*d^2*(2*b - 5*a*exp(d*x)*exp(root(729*a^2*
b^4*d^6*z^6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k)
)))/b^7) + (24576*a^3*d*(8*a - 5*b*exp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 +
729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k))))/b^8) + (32
768*a^3*(b - 4*a*exp(d*x)*exp(root(729*a^2*b^4*d^6*z^6 + 729*b^6*d^6*z^6 -
243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k))))/b^9)*root(729*a^2*b^4*d^6*z^
6 + 729*b^6*d^6*z^6 - 243*b^4*d^4*z^4 + 27*b^2*d^2*z^2 - 1, z, k), k, 1, 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(sinh(c + d*x)**2/(a + b*sinh(c + d*x)**3), x)
```

3.176 $\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=290

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] -2/3*(-1)^(1/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*
x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^(1/3)/b^(1/3)/d/((-1)^(1/3)
*a^(2/3)-b^(2/3))^(1/2)+2/3*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(
a^(2/3)+b^(2/3))^(1/2))/a^(1/3)/b^(1/3)/d/(a^(2/3)+b^(2/3))^(1/2)+2/3*arcta
n((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)
*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(1/3)/b^(1/3)/d/((-1)^(1/3)*a^(2/3)-(
-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3220, 2660, 618, 204}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/S
qrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a
^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(1/3)*d) - (2*(-1)^(1/3)*ArcTan[((-1)^(1/6)*
((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3)
 - b^(2/3)]])/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2
*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3
*a^(1/3)*Sqrt[a^(2/3) + b^(2/3)]*b^(1/3)*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\left(i\int\left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}-\frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh\right.}\right.\right.$$

$$=\frac{i\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\sqrt[6]{-1}\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{(-1)^{5/6}\int\frac{\cdots}{\cdots}}{\cdots}$$

$$=\frac{2\,\mathrm{Subst}\left(\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{b}d}-\frac{\left(2\sqrt[3]{-1}\right)\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$=-\frac{4\,\mathrm{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2}\,dx,x,2\sqrt[3]{-1}\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{b}d}-\cdots$$

$$=\frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\sqrt[3]{b}d}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\sqrt[3]{b}d}+\cdots$$

Mathematica [C] time = 0.25, size = 199, normalized size = 0.69

$$\frac{\mathrm{RootSum}\left[\#1^6b-3\#1^4b+8\#1^3a+3\#1^2b-b\&,\frac{2\#1^2\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\cdots}{\#1^4\cdots}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (-c - d*x - 2*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/
2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2)/(b + 4*a*#1 - 2*b*#
1^2 + b*#1^4) & ]/(3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(dx+c)}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.09, size = 82, normalized size = 0.28

$$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\,\_Z^6-3a\,\_Z^4-8b\,\_Z^3+3a\,\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 2/3/d*sum((_R^3-_R)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-
_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh\left(dx+c\right)}{b\sinh\left(dx+c\right)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

mupad [B] time = 21.84, size = 857, normalized size = 2.96

$$\sum_{k=1}^{6}\ln\left(\mathrm{root}\left(729\,a^4\,b^2\,d^6\,z^6+729\,a^2\,b^4\,d^6\,z^6+243\,a^2\,b^2\,d^4\,z^4-1,z,k\right)\left(\mathrm{root}\left(729\,a^4\,b^2\,d^6\,z^6+729\,a^2\,b^4\,d^6\,z^6\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4
*z^4 - 1, z, k)*(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b
^2*d^4*z^4 - 1, z, k)*(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243
*a^2*b^2*d^4*z^4 - 1, z, k)*(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6
 + 243*a^2*b^2*d^4*z^4 - 1, z, k)*((663552*(4*a^4*b*d^4 + 16*a^5*d^4*exp(d*
x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4
 - 1, z, k)) + 11*a^3*b^2*d^4*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a
^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/b^6 - (1990656*root(729*
a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k)*(4*a
^5*d^5*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^
2*b^2*d^4*z^4 - 1, z, k)) - a^4*b*d^5 + 5*a^3*b^2*d^5*exp(d*x)*exp(root(729
*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/
b^5) + (221184*(8*a^4*d^3 + a^2*b^2*d^3 - 25*a^3*b*d^3*exp(d*x)*exp(root(72
9*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))
/b^6) - (294912*a^2*d^2*(b - 7*a*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 72
9*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/b^6) + (196608*a^2*d*
(b - 2*a*exp(d*x)*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*
a^2*b^2*d^4*z^4 - 1, z, k))))/b^7) - (8192*a*(8*a - b*exp(d*x)*exp(root(729
*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 - 1, z, k))))/
b^7)*root(729*a^4*b^2*d^6*z^6 + 729*a^2*b^4*d^6*z^6 + 243*a^2*b^2*d^4*z^4 -
 1, z, k), k, 1, 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**3),x)
[Out] Integral(sinh(c + d*x)/(a + b*sinh(c + d*x)**3), x)
```

3.177 $\int \frac{1}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=280

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}\cdots\right.}{\sqrt{\sqrt[3]{-}\cdots}}\right.}{3a^{2/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}\cdots}}$$

```
[Out] -2/3*(-1)^(2/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*
x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^(2/3)/d/((-1)^(1/3)*a^(2/3)
-b^(2/3))^(1/2)-2/3*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+
b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)+b^(2/3))^(1/2)-2/3*(-1)^(2/3)*arctan((-1
)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2
/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/d/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(
2/3))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3213, 2660, 618, 204}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}\cdots\right.}{\sqrt{\sqrt[3]{-}\cdots}}\right.}{3a^{2/3}d\sqrt{\sqrt[3]{-1}\,a^{2/3}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^3)^(-1),x]
```

```
[Out] (-2*(-1)^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c +
 d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[
(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - (2*(-1)^(2/3)*ArcTan[((-1)^(1
/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(
2/3) - b^(2/3)]])/(3*a^(2/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - (2*Arc
Tanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*a^(
2/3)*Sqrt[a^(2/3) + b^(2/3)]*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\sinh^3(c+dx)}dx = \int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)}\right.$$

$$=\frac{\sqrt[6]{-1}\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}dx}{3a^{2/3}}+\frac{\sqrt[6]{-1}\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)}dx}{3a^{2/3}}+\frac{\sqrt[6]{-1}\int\frac{}{\sqrt[6]{-1}}}{}$$

$$=-\frac{\left(2(-1)^{2/3}\right)\text{Subst}\left(\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3a^{2/3}d}-\frac{\left(2(-1)^{2/3}\right)\text{Su}}{}$$

$$=\frac{\left(4(-1)^{2/3}\right)\text{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}dx,x,-2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3a^{2/3}d}+$$

$$=\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d}-\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right.}{3a^{2/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}$$

Mathematica [C] time = 0.17, size = 131, normalized size = 0.47

$$\frac{2\text{RootSum}\left[\#1^6b-3\#1^4b+8\#1^3a+3\#1^2b-b\&,\frac{2\#1\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)+}{\#1^4b-2\#1^2b+4\#1a+b}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^3)^(-1),x]
```

```
[Out] (2*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x*#1
+ 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sin
h[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.09, size = 87, normalized size = 0.31

$$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\_Z^{4}-8b\_Z^{3}+3a\_Z^{2}-a\right)}\frac{\left(-\_R^{4}+2\_R^{2}-1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a-2\_R^{3}a-4\_R^{2}b+\_Ra}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/3/d*sum((-_R^4+2*_R^2-1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+
1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sinh\left(dx+c\right)^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(d*x + c)^3 + a), x)
```

mupad [B] time = 9.49, size = 1261, normalized size = 4.50

$$\sum_{k=1}^{6}\ln\left(\frac{\left(-4\,e^{\mathrm{root}\left(729\,a^4\,b^2\,d^6\,z^6+729\,a^6\,d^6\,z^6-243\,a^4\,d^4\,z^4+27\,a^2\,d^2\,z^2-1,z,k\right)+d\,x}+\mathrm{root}\left(729\,a^4\,b^2\,d^6\,z^6+729\,a^6\,d^6\,z^6-243\,a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(c + d*x)^3),x)
```

```
[Out] symsum(log((24576*(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4
*z^4 + 27*a^2*d^2*z^2 - 1, z, k)*b*d - 4*exp(root(729*a^4*b^2*d^6*z^6 + 729
*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 12*root
(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 -
 1, z, k)^2*a*b*d^2 - 20*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a
^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)*a*d*exp(root(729*a^4*b^2*d^6*z^6 + 7
29*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 24*ro
ot(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2
 - 1, z, k)^2*a^2*d^2*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*
a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 216*root(729*a^4*b^2*d^6*z
^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)^3*a^3*d^
3*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2
*d^2*z^2 - 1, z, k) + d*x) + 108*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6
 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)^4*a^4*d^4*exp(root(729*a^4*b
^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)
+ d*x) - 324*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 +
 27*a^2*d^2*z^2 - 1, z, k)^5*a^5*d^5*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^6
*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) + 54*root(729
*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1,
z, k)^3*a^2*b*d^3 + 108*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^
4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)^4*a^3*b*d^4 + 81*root(729*a^4*b^2*d^6
*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k)^5*a^4*
b*d^5 - 27*root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 2
7*a^2*d^2*z^2 - 1, z, k)^4*a^2*b^2*d^4*exp(root(729*a^4*b^2*d^6*z^6 + 729*a
^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x) - 405*root(
729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 -
```

```
1, z, k)^5*a^3*b^2*d^5*exp(root(729*a^4*b^2*d^6*z^6 + 729*a^6*d^6*z^6 - 243
*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k) + d*x)))/b^5)*root(729*a^4*b^2*d^6
*z^6 + 729*a^6*d^6*z^6 - 243*a^4*d^4*z^4 + 27*a^2*d^2*z^2 - 1, z, k), k, 1,
 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sinh^3(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*sinh(c + d*x)**3), x)
```

3.178 $\int \frac{\mathrm{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=286

$$\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}+b^{2/3}}}+\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{\ldots}\right.}{\sqrt{\sqrt[3]{-1}\,a^{2}\ldots}}\right.}{3ad\sqrt{\sqrt[3]{-1}\,a^{2/3}-\ldots}}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d+2/3*b^(1/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)
+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a/d/((-
1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)+2/3*b^(1/3)*arctan((-1)^(5/6)*((-1)^(1/6)*b
^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2))/
a/d/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2)+2/3*b^(1/3)*arctanh((b^(1/3)-a^(1/3
)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/a/d/(a^(2/3)+b^(2/3))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3220, 3770, 2660, 618, 206, 204}

$$\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}+b^{2/3}}}+\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{\ldots}\right.}{\sqrt{\sqrt[3]{-1}\,a^{2}\ldots}}\right.}{3ad\sqrt{\sqrt[3]{-1}\,a^{2/3}-\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*b^(1/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x
)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*a*Sqrt[-((-1)^(2/3)*a^(2/
3)) - b^(2/3)]*d) + (2*b^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a
^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a*Sqrt[(
-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - ArcTanh[Cosh[c + d*x]]/(a*d) + (2*b^(1/3)
*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3
*a*Sqrt[a^(2/3) + b^(2/3)]*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx = i\int\left(-\frac{i\operatorname{csch}(c+dx)}{a}+\frac{ib\sinh^2(c+dx)}{a\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \operatorname{csch}(c+dx)\,dx}{a} - \frac{b\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{b\int\left(\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{i}{3b^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}\right.}{a}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\left(i\sqrt[3]{b}\right)\int\frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3a} + \frac{\left(i\sqrt[3]{b}\right)\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(}}{3a}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\left(2\sqrt[3]{b}\right)\operatorname{Subst}\left(\int\frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}x-i\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3ad}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{\left(4\sqrt[3]{b}\right)\operatorname{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{}\right.}{3ad}$$

$$= -\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,d} - \frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d} - \tan$$

Mathematica [C] time = 0.26, size = 295, normalized size = 1.03

$$6\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - b\operatorname{RootSum}\left[\#1^6b-3\#1^4b+8\#1^3a+3\#1^2b-b\&,\frac{2\#1^4\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (6*Log[Tanh[(c + d*x)/2]] - b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 +
 b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[
(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
```

```
/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2
- 2*b*#1^3 + b*#1^5) & ])/(6*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.15, size = 100, normalized size = 0.35

$$\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3da}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 4/3/d/a*b*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_
R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))+1/d/a*ln(tanh(1/2*d*x+1/
2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad}+\frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad}-2\int\frac{be^{(5\,dx+5\,c)}-2\,be^{(3\,dx+3\,c)}+be^{(dx+c)}}{abe^{(6\,dx+6\,c)}-3\,abe^{(4\,dx+4\,c)}+8\,a^2e^{(3\,dx+3\,c)}+3\,abe^{(2\,dx+2\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(5*d*x + 5*c) - 2*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a*
b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x + 3*c) + 3*a*b*e
^(2*d*x + 2*c) - a*b), x)
```

mupad [B] time = 55.94, size = 2970, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a + b*sinh(c + d*x)^3)),x)
```

```
[Out] symsum(log(-(2147483648*a*b*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6
- 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 1073741824
```

```
*b^2 - 86973087744*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2
*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^6*d^4 + 86973087744*root(729
*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z
^2 - b^2, z, k)^6*a^8*d^6 + 134217728*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^
6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*b^3*d + 32212
25472*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - b^2, z, k)*a^2*b*d + 18589155328*root(729*a^6*b^2*d^6*z^
6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)
^2*a^2*b^2*d^2 - 2818572288*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 24
3*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^3*d^3 - 8818104
7296*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*
a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^4*b^2*d^4 + 18119393280*root(729*a^6*b^2*d
^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2,
z, k)^5*a^4*b^3*d^5 + 70665633792*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^
6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^6*b^2*d^6 - 3
2614907904*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4
 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^6*b^3*d^7 - 57982058496*root(729*a^6
*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 -
 b^2, z, k)^3*a^5*d^3*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*
a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 333396836352*roo
t(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*
d^2*z^2 - b^2, z, k)^5*a^7*d^5*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z
^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) + 3913788
94848*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^9*d^7*exp(root(729*a^6*b^2*d^6*z^6 + 729*
a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x)
- 17716740096*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*
z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^4*b*d^3 + 30802968576*root(729*a^
6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2
- b^2, z, k)^5*a^6*b*d^5 - 40768634880*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d
^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^8*b*d^7
+ 268435456*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^
4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a*b^3*d^2*exp(root(729*a^6*b^2*d^6*z^
6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)
 + d*x) - 16642998272*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*
b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^2*a^3*b*d^2*exp(root(729*a^6*
b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 -
b^2, z, k) + d*x) + 36238786560*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6
- 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^4*a^5*b*d^4*exp(roo
t(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*
d^2*z^2 - b^2, z, k) + d*x) + 2717908992*root(729*a^6*b^2*d^6*z^6 + 729*a^8
*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^7*b*d^
6*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 5637144576*root(729*a^6*b^2*d^6*z^6
+ 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*a
*b^2*d*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4
 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) + 100763959296*root(729*a^6*b^2*d
^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2,
z, k)^3*a^3*b^2*d^3*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^
4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 4831838208*root(72
9*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*
z^2 - b^2, z, k)^4*a^3*b^3*d^4*exp(root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z
^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d*x) - 4946594
36544*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^5*b^2*d^5*exp(root(729*a^6*b^2*d^6*z^6 +
729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k) + d
*x) + 21743271936*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*
d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^5*b^3*d^6*exp(root(729*a^6*b^
2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^
```

```
2, z, k) + d*x) + 399532621824*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 -
 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^7*a^7*b^2*d^7*exp(ro
ot(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2
*d^2*z^2 - b^2, z, k) + d*x))/b^9)*root(729*a^6*b^2*d^6*z^6 + 729*a^8*d^6*z
^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k), k, 1, 6) + log(
exp(d*x + 1/(a*d)) - 1)/(a*d) - log(exp(d*x - 1/(a*d)) + 1)/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.179 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=304

$$-\frac{2b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}b^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}\cdots\right.}{\sqrt{\sqrt[3]{-1}\cdots}}\right.}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}\cdots}}$$

```
[Out] -coth(d*x+c)/a/d+2/3*(-1)^(1/3)*b^(2/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/
3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^(4/
3)/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)-2/3*b^(2/3)*arctanh((b^(1/3)-a^(1/3
)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)+b^(2/3))
^(1/2)-2/3*b^(2/3)*arctan((-1)^(1/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2
*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(4/3)/d/((-1)
^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3767, 8, 2660, 618, 204}

$$-\frac{2b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}b^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}\cdots\right.}{\sqrt{\sqrt[3]{-1}\cdots}}\right.}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*b^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[(-1
)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) + (2*(-1)^(1/3)*b^(2/3)*ArcTan[((-
1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3
)*a^(2/3) - b^(2/3)]])/(3*a^(4/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - (
2*b^(2/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2
/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) + b^(2/3)]*d) - Coth[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\int\left(-\frac{\operatorname{csch}^2(c+dx)}{a}+\frac{b\sinh(c+dx)}{a\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$=\frac{\int \operatorname{csch}^2(c+dx)\,dx}{a}-\frac{b\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{a}$$

$$=\frac{(ib)\int\left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}-\frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)}-\frac{}{3\sqrt[3]{a}\sqrt[3]{b}(\cdots}\right.}{a}$$

$$=-\frac{\coth(c+dx)}{ad}-\frac{\left(ib^{2/3}\right)\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3a^{4/3}}+\frac{\left(\sqrt[6]{-1}\,b^{2/3}\right)\int\frac{}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-}\cdots}}{3a^{4/3}}$$

$$=-\frac{\coth(c+dx)}{ad}-\frac{\left(2b^{2/3}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+id\cdots\right.\right.}{3a^{4/3}d}$$

$$=-\frac{\coth(c+dx)}{ad}+\frac{\left(4b^{2/3}\right)\operatorname{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2}\,dx,x,2\sqrt[3]{-1}\sqrt[3]{b}+2\sqrt[6]{-}\cdots\right.}{3a^{4/3}d}$$

$$=-\frac{2\sqrt[3]{-1}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d}-\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\cdots\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2}\cdots}}\right.}{3a^{4/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}\cdots}}$$

Mathematica [C] time = 0.42, size = 230, normalized size = 0.76

$$-\frac{2b\operatorname{RootSum}\left[\#1^6b-3\#1^4b+8\#1^3a+3\#1^2b-b\&,\frac{2\#1^2\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c\cdots\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -1/6*(3*Coth[(c + d*x)/2] + 2*b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4
 + b*#1^6 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Co
sh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cos
h[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/
```

```
2]*#1]*#1^2)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*Tanh[(c + d*x)/2])/(a
*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
[Out] integrate(csch(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.16, size = 123, normalized size = 0.40

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3da}-\frac{1}{2da\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x)
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-2/3/d/a*b*sum((_R^3-_R)/(_R^5*a-2*_R^3*a-4*_R^
2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_
Z^2*a-a))-1/2/d/a/tanh(1/2*d*x+1/2*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{ade^{(2\,dx+2\,c)}-ad}-4\int\frac{be^{(4\,dx+4\,c)}-be^{(2\,dx+2\,c)}}{abe^{(6\,dx+6\,c)}-3\,abe^{(4\,dx+4\,c)}+8\,a^2e^{(3\,dx+3\,c)}+3\,abe^{(2\,dx+2\,c)}-ab}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d*
x + 2*c))/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x + 3
*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x)
```

mupad [B] time = 23.06, size = 1293, normalized size = 4.25

$$\left(\sum_{k=1}^{6}\ln\left(-\frac{8192\,b^4\,\mathrm{e}^{\mathrm{root}\left(729\,a^8\,b^2\,d^6\,z^6+729\,a^{10}\,d^6\,z^6+243\,a^6\,b^2\,d^4\,z^4-b^4,z,k\right)+d\,x}-65536\,a\,b^3-\mathrm{root}\left(729\,a^8\,b^2\,d^6\,z^6+729\,a^{10}\,d\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^3)),x)
```

```
[Out] symsum(log(-(8192*b^4*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243
*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) - 65536*a*b^3 - 294912*root(729*a^8*b^
2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^2*a^3*b^3*d
^2 - 221184*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z
^4 - b^4, z, k)^3*a^4*b^3*d^3 - 196608*root(729*a^8*b^2*d^6*z^6 + 729*a^10*
d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)*a^2*b^3*d + 10616832*root(729*a^
8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^4*a^8*d
^4*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 -
b^4, z, k) + d*x) + 7962624*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 2
43*a^6*b^2*d^4*z^4 - b^4, z, k)^5*a^9*d^5*exp(root(729*a^8*b^2*d^6*z^6 + 72
9*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) - 1769472*root(729
*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^3*a^
6*b*d^3 + 2654208*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2
*d^4*z^4 - b^4, z, k)^4*a^7*b*d^4 - 1990656*root(729*a^8*b^2*d^6*z^6 + 729*
a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^5*a^8*b*d^5 + 2064384*root(
729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^2
*a^4*b^2*d^2*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*
d^4*z^4 - b^4, z, k) + d*x) + 5529600*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d
^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^3*a^5*b^2*d^3*exp(root(729*a^8*b^
2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) + 72
99072*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b
^4, z, k)^4*a^6*b^2*d^4*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 2
43*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x) + 9953280*root(729*a^8*b^2*d^6*z^6 +
 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)^5*a^7*b^2*d^5*exp(root
(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k)
+ d*x) + 393216*root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d
^4*z^4 - b^4, z, k)*a^3*b^2*d*exp(root(729*a^8*b^2*d^6*z^6 + 729*a^10*d^6*z
^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k) + d*x))/(a^4*b^5))*root(729*a^8*b^2*d
^6*z^6 + 729*a^10*d^6*z^6 + 243*a^6*b^2*d^4*z^4 - b^4, z, k), k, 1, 6) + 2/
(a*d - a*d*exp(2*c + 2*d*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.180 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=322

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+\cdots\right.}{\sqrt{\sqrt[3]{-1}\cdots}}\right.}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}\cdots}}$$

```
[Out] 1/2*arctanh(cosh(d*x+c))/a/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d+2/3*(-1)^(2/3)
*b*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-
1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^(5/3)/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/
2)+2/3*b*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3)+b^(2/3))^(1
/2))/a^(5/3)/d/(a^(2/3)+b^(2/3))^(1/2)+2/3*(-1)^(2/3)*b*arctan((-1)^(1/6)*(
(-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)-(-1)^
(2/3)*b^(2/3))^(1/2))/a^(5/3)/d/((-1)^(1/3)*a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/
2)
```

Rubi [A] time = 0.46, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 3768, 3770, 3213, 2660, 618, 204}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+\cdots\right.}{\sqrt{\sqrt[3]{-1}\cdots}}\right.}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*(-1)^(2/3)*b*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c
+ d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(5/3)*Sqrt
[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) + (2*(-1)^(2/3)*b*ArcTan[((-1)
^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*
a^(2/3) - b^(2/3)]])/(3*a^(5/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) + Arc
Tanh[Cosh[c + d*x]]/(2*a*d) + (2*b*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x
)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*a^(5/3)*Sqrt[a^(2/3) + b^(2/3)]*d) - (Co
th[c + d*x]*Csch[c + d*x])/(2*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\left(i\int\left(\frac{i\operatorname{csch}^3(c+dx)}{a}-\frac{ib}{a\left(a+b\sinh^3(c+dx)\right)}\right)dx\right)$$

$$=\frac{\int \operatorname{csch}^3(c+dx)\,dx}{a}-\frac{b\int\frac{1}{a+b\sinh^3(c+dx)}\,dx}{a}$$

$$=-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}-\frac{\int \operatorname{csch}(c+dx)\,dx}{2a}-\frac{b\int\left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\,\sqrt[3]{a}-i\sqrt[3]{b}\,\sinh(c+dx)\right)}\right.}{}$$

$$=\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}-\frac{\left(\sqrt[6]{-1}\,b\right)\int\frac{1}{\sqrt[6]{-1}\,\sqrt[3]{a}-i\sqrt[3]{b}\,\sinh(c+}}{3a^{5/3}}$$

$$=\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}+\frac{\left(2(-1)^{2/3}b\right)\operatorname{Subst}\left(\int\frac{}{\sqrt[6]{-1}\,\sqrt[3]{a}}\right.}{}$$

$$=\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}-\frac{\left(4(-1)^{2/3}b\right)\operatorname{Subst}\left(\int\frac{}{-4\left(\sqrt[3]{-1}\right.}\right.}{}$$

$$=-\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\,\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,d}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\,\mathrm{ta}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)}}\right.}{3a^{5/3}\sqrt{\sqrt[3]{-1}\,a^{2/3}-(-1)^{2/3}b^{2}}}$$

Mathematica [C] time = 0.51, size = 178, normalized size = 0.55

$$-\frac{16b\operatorname{RootSum}\left[\#1^6b-3\#1^4b+8\#1^3a+3\#1^2b-b\&,\frac{2\#1\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(\right.\right.}{\#1^4b-2\#1^2b+4\#1a+b}\right.}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -1/24*(16*b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1
+ d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]
*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*(C
sch[(c + d*x)/2]^2 + 4*Log[Tanh[(c + d*x)/2]] + Sech[(c + d*x)/2]^2))/(a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}\left(dx+c\right)^{3}}{b\sinh\left(dx+c\right)^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)^3/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.18, size = 146, normalized size = 0.45

$$\frac{\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{6}-3a\_Z^{4}-8b\_Z^{3}+3a\_Z^{2}-a\right)}\frac{\left(\_R^{4}-2\_R^{2}+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{5}a-2\_R^{3}a-4\_R^{2}b+\_Ra}\right)}{3da}-\frac{1}{8da\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-\ln\left(t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2+1/3/d/a*b*sum((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3*
a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^
3*b+3*_Z^2*a-a))-1/8/d/a/tanh(1/2*d*x+1/2*c)^2-1/2/d/a*ln(tanh(1/2*d*x+1/2*
c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-8\,b\int\frac{e^{(3\,dx+3\,c)}}{abe^{(6\,dx+6\,c)}-3\,abe^{(4\,dx+4\,c)}+8\,a^{2}e^{(3\,dx+3\,c)}+3\,abe^{(2\,dx+2\,c)}-ab}\,dx-\frac{e^{(3\,dx+3\,c)}+e^{(dx+c)}}{ade^{(4\,dx+4\,c)}-2\,ade^{(2\,dx+2\,c)}+ad}+\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -8*b*integrate(e^(3*d*x + 3*c)/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c)
 + 8*a^2*e^(3*d*x + 3*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x) - (e^(3*d*x + 3
*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/
2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) - 1/2*log((e^(d*x + c) - 1)*e^(-c))/(
a*d)
```

mupad [B] time = 89.25, size = 3605, normalized size = 11.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^3)),x)
```

```
[Out] symsum(log(-(16777216*b^7*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12
*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 5033164
8*a*b^6 + 33554432*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b
^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)*a*b^7*d + 671088640*a^2*b^5*ex
p(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z
^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) + 201326592*root(729*a^10*b^2*d^6*z^6
 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)
^2*a^3*b^6*d^2 - 1509949440*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 -
243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^2*a^5*b^4*d^2 - 27179
08992*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 +
27*a^4*b^4*d^2*z^2 - b^6, z, k)^3*a^5*b^5*d^3 + 2717908992*root(729*a^10*b^
2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b
^6, z, k)^3*a^7*b^3*d^3 + 6039797760*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d
^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^4*a^7*b^4*d^
4 - 4076863488*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d
^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^4*a^9*b^2*d^4 - 679477248*root(729
*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2
*z^2 - b^6, z, k)^5*a^9*b^3*d^5 + 16307453952*root(729*a^10*b^2*d^6*z^6 + 7
29*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^6*a
^11*b^2*d^6 - 32614907904*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 24
3*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^7*a^11*b^3*d^7 + 452984
832*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27
*a^4*b^4*d^2*z^2 - b^6, z, k)*a^3*b^5*d + 4076863488*root(729*a^10*b^2*d^6*
z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z,
 k)^5*a^11*b*d^5 - 40768634880*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6
 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^7*a^13*b*d^7 - 978
44723712*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4
 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^5*a^12*d^5*exp(d*x)*exp(root(729*a^10*b^
2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b
^6, z, k)) + 391378894848*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 24
3*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^7*a^14*d^7*exp(d*x)*exp
(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^
4*b^4*d^2*z^2 - b^6, z, k)) + 10871635968*root(729*a^10*b^2*d^6*z^6 + 729*a
^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^4*a^10*
b*d^4*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b
^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 55717134336*root(729*a^10*b
^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 -
b^6, z, k)^6*a^12*b*d^6*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d
^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 306184192
0*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a
^4*b^4*d^2*z^2 - b^6, z, k)^2*a^4*b^5*d^2*exp(d*x)*exp(root(729*a^10*b^2*d^
6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6,
z, k)) - 7247757312*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*
b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^2*a^6*b^3*d^2*exp(d*x)*exp(ro
ot(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b
^4*d^2*z^2 - b^6, z, k)) + 9688842240*root(729*a^10*b^2*d^6*z^6 + 729*a^12*
d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^3*a^6*b^4*d
^3*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*
d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) + 36238786560*root(729*a^10*b^2*
d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6
, z, k)^3*a^8*b^2*d^3*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6
*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 301989888*r
oot(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*
b^4*d^2*z^2 - b^6, z, k)^4*a^6*b^5*d^4*exp(d*x)*exp(root(729*a^10*b^2*d^6*z
^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z,
k)) + 48695869440*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^
```

```
2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^4*a^8*b^3*d^4*exp(d*x)*exp(root
(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4
*d^2*z^2 - b^6, z, k)) + 6341787648*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^
6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^5*a^8*b^4*d^5
*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^
4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 244838301696*root(729*a^10*b^2*d
^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6,
 z, k)^5*a^10*b^2*d^5*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6
*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)) - 74742497280
*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^
4*b^4*d^2*z^2 - b^6, z, k)^6*a^10*b^3*d^6*exp(d*x)*exp(root(729*a^10*b^2*d^
6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6,
z, k)) + 399532621824*root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^
8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)^7*a^12*b^2*d^7*exp(d*x)*exp
(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^
4*b^4*d^2*z^2 - b^6, z, k)) - 2818572288*root(729*a^10*b^2*d^6*z^6 + 729*a^
12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)*a^4*b^4*
d*exp(d*x)*exp(root(729*a^10*b^2*d^6*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d
^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z, k)))/(a^5*b^9))*root(729*a^10*b^2*d^6
*z^6 + 729*a^12*d^6*z^6 - 243*a^8*b^2*d^4*z^4 + 27*a^4*b^4*d^2*z^2 - b^6, z
, k), k, 1, 6) - (2*exp(c + d*x))/(a*d - 2*a*d*exp(2*c + 2*d*x) + a*d*exp(4
*c + 4*d*x)) + exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - log(191102976*a^
6*b - 16777216*b^7 + 113246208*a^2*b^5 - 63700992*a^4*b^3 + 16777216*b^7*ex
p(-1/(2*a*d))*exp(d*x) - 191102976*a^6*b*exp(-1/(2*a*d))*exp(d*x) - 1132462
08*a^2*b^5*exp(-1/(2*a*d))*exp(d*x) + 63700992*a^4*b^3*exp(-1/(2*a*d))*exp(
d*x))/(2*a*d) + log(191102976*a^6*b - 16777216*b^7 + 113246208*a^2*b^5 - 63
700992*a^4*b^3 - 16777216*b^7*exp(1/(2*a*d))*exp(d*x) + 191102976*a^6*b*exp
(1/(2*a*d))*exp(d*x) + 113246208*a^2*b^5*exp(1/(2*a*d))*exp(d*x) - 63700992
*a^4*b^3*exp(1/(2*a*d))*exp(d*x))/(2*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.181 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=317

$$\frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2b^{4/3}\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-2b^{4/3}\ldots$$

```
[Out] b*arctanh(cosh(d*x+c))/a^2/d+coth(d*x+c)/a/d-1/3*coth(d*x+c)^3/a/d-2/3*b^(4
/3)*arctan((-1)^(1/6)*((-1)^(5/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/2*c))/((
-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2))/a^2/d/((-1)^(1/3)*a^(2/3)-b^(2/3))^(1/2)-
2/3*b^(4/3)*arctan((-1)^(5/6)*((-1)^(1/6)*b^(1/3)+I*a^(1/3)*tanh(1/2*d*x+1/
2*c))/(-(-1)^(2/3)*a^(2/3)-b^(2/3))^(1/2))/a^2/d/(-(-1)^(2/3)*a^(2/3)-b^(2/
3))^(1/2)-2/3*b^(4/3)*arctanh((b^(1/3)-a^(1/3)*tanh(1/2*d*x+1/2*c))/(a^(2/3
)+b^(2/3))^(1/2))/a^2/d/(a^(2/3)+b^(2/3))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 3770, 3767, 2660, 618, 206, 204}

$$-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2b^{4/3}\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+\ldots\right)}{\sqrt{\sqrt[3]{-1}\ldots}}\right)}{3a^2d\sqrt{\sqrt[3]{-1}\,a^{2/3}\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*b^(4/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*a^2*Sqrt[-((-1)^(2/3)*a^
(2/3)) - b^(2/3)]*d) - (2*b^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) +
I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a^2*S
qrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) + (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) -
 (2*b^(4/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^
(2/3)]])/(3*a^2*Sqrt[a^(2/3) + b^(2/3)]*d) + Coth[c + d*x]/(a*d) - Coth[c +
 d*x]^3/(3*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\text{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \left(-\frac{b\,\text{csch}(c+dx)}{a^2} + \frac{\text{csch}^4(c+dx)}{a} - \frac{b^2\sinh^2(c+dx)}{a^2\left(-a-b\sinh^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \text{csch}^4(c+dx)\,dx}{a} - \frac{b\int \text{csch}(c+dx)\,dx}{a^2} - \frac{b^2\int \frac{\sinh^2(c+dx)}{-a-b\sinh^3(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{b^2\int\left(-\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)} - \frac{i}{3b^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx\right.}\right.}{a^2}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad} - \frac{\left(ib^{4/3}\right)\int \frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+d}}{3a^2}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad} - \frac{\left(2b^{4/3}\right)\text{Subst}\left(\int \frac{}{-i\sqrt[3]{a}-2\sqrt[3]{}}\right.}{}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\coth(c+dx)}{ad} - \frac{\coth^3(c+dx)}{3ad} + \frac{\left(4b^{4/3}\right)\text{Subst}\left(\int \frac{}{-4\left(\sqrt[3]{-1}\,a\right.}\right.}{}$$

$$= \frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,d} + \frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{\sqrt[3]{-1}\,a^{2/3}-b^{2/3}}\,d} + \frac{b\,\text{ta}}{}$$

Mathematica [C] time = 5.79, size = 370, normalized size = 1.17

$$4b^2\text{RootSum}\left[\#1^6 b - 3\#1^4 b + 8\#1^3 a + 3\#1^2 b - b\&, \frac{2\#1^4\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (8*a*Coth[(c + d*x)/2] - 24*b*Log[Tanh[(c + d*x)/2]] + 4*b^2*RootSum[-b + 3
*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*
c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + 8*a*Csch[c + d*x]
^3*Sinh[(c + d*x)/2]^4 - (a*Csch[(c + d*x)/2]^4*Sinh[c + d*x])/2 + 8*a*Tanh
[(c + d*x)/2])/(24*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(dx+c)^4}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)^4/(b*sinh(d*x + c)^3 + a), x)
```

maple [C] time = 0.18, size = 178, normalized size = 0.56

$$-\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}+\frac{3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{4b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\right)}{3d\,a^2}-\frac{}{24da\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)-4/3/d*b^2/a^2*s
um(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootO
f(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))-1/24/d/a/tanh(1/2*d*x+1/2*c)^3+3/8/
d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{4\left(3e^{(2dx+2c)}-1\right)}{3\left(ade^{(6dx+6c)}-3ade^{(4dx+4c)}+3ade^{(2dx+2c)}-ad\right)}+\frac{b\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{a^2d}-\frac{b\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{a^2d}+16\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*e^(6*d*x + 6*c) - 3*a*d*e^(4*d*x + 4*c) +
 3*a*d*e^(2*d*x + 2*c) - a*d) + b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - b
```

```
*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 16*integrate(1/8*(b^2*e^(5*d*x + 5
*c) - 2*b^2*e^(3*d*x + 3*c) + b^2*e^(d*x + c))/(a^2*b*e^(6*d*x + 6*c) - 3*a
^2*b*e^(4*d*x + 4*c) + 8*a^3*e^(3*d*x + 3*c) + 3*a^2*b*e^(2*d*x + 2*c) - a^
2*b), x)
```

mupad [B] time = 59.69, size = 3086, normalized size = 9.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^3)),x)
```

```
[Out] symsum(log((18589155328*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*
a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^2*a^4*b^7*d^2 - 134217728
*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^
4*b^6*d^2*z^2 - b^8, z, k)*a*b^9*d - 1073741824*b^9 + 2818572288*root(729*a
^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z
^2 - b^8, z, k)^3*a^5*b^7*d^3 + 17716740096*root(729*a^12*b^2*d^6*z^6 + 729
*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^3*a^7
*b^5*d^3 - 88181047296*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a
^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^4*a^8*b^5*d^4 - 8697308774
4*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a
^4*b^6*d^2*z^2 - b^8, z, k)^4*a^10*b^3*d^4 - 18119393280*root(729*a^12*b^2*
d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8
, z, k)^5*a^9*b^5*d^5 - 30802968576*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^
6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^5*a^11*b^3*d^
5 + 70665633792*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*
d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^6*a^12*b^3*d^6 + 32614907904*root
(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6
*d^2*z^2 - b^8, z, k)^7*a^13*b^3*d^7 - 3221225472*root(729*a^12*b^2*d^6*z^6
 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)
*a^3*b^7*d + 2147483648*a*b^8*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*
a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) + 869
73087744*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4
 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^6*a^14*b*d^6 + 40768634880*root(729*a^12
*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2
 - b^8, z, k)^7*a^15*b*d^7 - 391378894848*root(729*a^12*b^2*d^6*z^6 + 729*a^
14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^7*a^16*d
^7*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*
d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) + 268435456*root(729*a^12*b^2*d^
6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8,
z, k)^2*a^3*b^8*d^2*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z
^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) - 16642998272*r
oot(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*
b^6*d^2*z^2 - b^8, z, k)^2*a^5*b^6*d^2*exp(d*x)*exp(root(729*a^12*b^2*d^6*z
^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z,
k)) - 100763959296*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b
^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^3*a^6*b^6*d^3*exp(d*x)*exp(roo
t(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^
6*d^2*z^2 - b^8, z, k)) + 57982058496*root(729*a^12*b^2*d^6*z^6 + 729*a^14*
d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^3*a^8*b^4*d
^3*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*
d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) - 4831838208*root(729*a^12*b^2*d
^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8,
 z, k)^4*a^7*b^6*d^4*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*
z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) + 36238786560*
root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4
*b^6*d^2*z^2 - b^8, z, k)^4*a^9*b^4*d^4*exp(d*x)*exp(root(729*a^12*b^2*d^6*
z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z,
 k)) + 494659436544*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*
```

```
b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^5*a^10*b^4*d^5*exp(d*x)*exp(r
oot(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*
b^6*d^2*z^2 - b^8, z, k)) + 333396836352*root(729*a^12*b^2*d^6*z^6 + 729*a^
14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^5*a^12*b
^2*d^5*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*
b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) + 21743271936*root(729*a^12*
b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 -
 b^8, z, k)^6*a^11*b^4*d^6*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^1
4*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)) + 271790
8992*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 2
7*a^4*b^6*d^2*z^2 - b^8, z, k)^6*a^13*b^2*d^6*exp(d*x)*exp(root(729*a^12*b^
2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b
^8, z, k)) - 399532621824*root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 24
3*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)^7*a^14*b^2*d^7*exp(d*x)
*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 2
7*a^4*b^6*d^2*z^2 - b^8, z, k)) + 5637144576*root(729*a^12*b^2*d^6*z^6 + 72
9*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)*a^2*
b^8*d*exp(d*x)*exp(root(729*a^12*b^2*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b
^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^8, z, k)))/(a^7*b^9))*root(729*a^12*b^2
*d^6*z^6 + 729*a^14*d^6*z^6 - 243*a^8*b^4*d^4*z^4 + 27*a^4*b^6*d^2*z^2 - b^
8, z, k), k, 1, 6) + 8/(3*(a*d - 3*a*d*exp(2*c + 2*d*x) + 3*a*d*exp(4*c + 4
*d*x) - a*d*exp(6*c + 6*d*x))) - 4/(a*d - 2*a*d*exp(2*c + 2*d*x) + a*d*exp(
4*c + 4*d*x)) + (b*log(exp(d*x + b/(a^2*d)) + 1))/(a^2*d) - (b*log(exp(d*x
- b/(a^2*d)) - 1))/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.182 $\int \frac{1}{1+\sinh^3(x)}\,dx$

Optimal. Leaf size=139

$$-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}\sqrt[6]{-1}\log\left(-\sqrt[6]{-1}\tanh\left(\frac{x}{2}\right)+(-1)^{5/6}+1\right)+\frac{1}{3}\sqrt[6]{-1}\log\left(\sqrt[3]{-1}\tanh\left(\frac{x}{2}\right)+\sqrt[6]{-1}+\right.$$

```
[Out] -1/3*(-1)^(1/6)*ln(1+(-1)^(5/6)-(-1)^(1/6)*tanh(1/2*x))+1/3*(-1)^(1/6)*ln(1
+(-1)^(1/6)+(-1)^(1/3)*tanh(1/2*x))-1/3*arctanh(1/2*(1-tanh(1/2*x))*2^(1/2)
)*2^(1/2)-2/3*(-1)^(1/6)*arctan((I+(-1)^(1/6)*tanh(1/2*x))/(1-(-1)^(1/3))^(
1/2))/(1-(-1)^(1/3))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3213, 2660, 618, 204, 617, 206, 616, 31}

$$-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}\sqrt[6]{-1}\log\left(-\sqrt[6]{-1}\tanh\left(\frac{x}{2}\right)+(-1)^{5/6}+1\right)+\frac{1}{3}\sqrt[6]{-1}\log\left(\sqrt[3]{-1}\tanh\left(\frac{x}{2}\right)+\sqrt[6]{-1}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^3)^(-1),x]
```

```
[Out] (-2*(-1)^(1/6)*ArcTan[(I + (-1)^(1/6)*Tanh[x/2])/Sqrt[1 - (-1)^(1/3)]])/(3*
Sqrt[1 - (-1)^(1/3)]) - (Sqrt[2]*ArcTanh[(1 - Tanh[x/2])/Sqrt[2]])/3 - ((-1
)^(1/6)*Log[1 + (-1)^(5/6) - (-1)^(1/6)*Tanh[x/2]])/3 + ((-1)^(1/6)*Log[1 +
 (-1)^(1/6) + (-1)^(1/3)*Tanh[x/2]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sinh^3(x)}\,dx &= \int \left(\frac{\sqrt[6]{-1}}{3\left(\sqrt[6]{-1}-i\sinh(x)\right)}+\frac{\sqrt[6]{-1}}{3\left(\sqrt[6]{-1}+\sqrt[6]{-1}\sinh(x)\right)}+\frac{\sqrt[6]{-1}}{3\left(\sqrt[6]{-1}+(-1)^{5/6}\sinh(x)\right)}\right)dx\\
&= \frac{1}{3}\sqrt[6]{-1}\int \frac{1}{\sqrt[6]{-1}-i\sinh(x)}\,dx+\frac{1}{3}\sqrt[6]{-1}\int \frac{1}{\sqrt[6]{-1}+\sqrt[6]{-1}\sinh(x)}\,dx+\frac{1}{3}\sqrt[6]{-1}\int \frac{}{\sqrt[6]{-1}+(}\\
&= \frac{1}{3}\left(2\sqrt[6]{-1}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}-2ix-\sqrt[6]{-1}x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{3}\left(2\sqrt[6]{-1}\right)\text{Subst}\left(\int \frac{}{\sqrt[6]{-1}}\right.\\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,1-\tanh\left(\frac{x}{2}\right)\right)\right)-\frac{1}{3}\left(4\sqrt[6]{-1}\right)\text{Subst}\left(\int \frac{1}{-4\left(1-\sqrt[3]{-1}\right)-x^2}\right.\\
&= -\frac{2\sqrt[6]{-1}\tan^{-1}\left(\frac{i+\sqrt[6]{-1}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}}-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}\sqrt[6]{-1}\log\left(1+(-1)^{5/6}-\right.\end{aligned}$$

Mathematica [A] time = 1.50, size = 156, normalized size = 1.12

$$\frac{2\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)+i\sqrt{-1-i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{2+\left(1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2+2i\sqrt{3}}}\right)+\left(-1-i\sqrt{3}\right)\sqrt{-1+i\sqrt{3}}\tan^{-1}\left(\frac{2+(1+}{\sqrt{}}\right.}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^3)^(-1),x]
```

```
[Out] (I*Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[(2 + (1 - I*Sqrt[3])*Tanh[x/2]
)/Sqrt[-2 + (2*I)*Sqrt[3]]] + (-1 - I*Sqrt[3])*Sqrt[-1 + I*Sqrt[3]]*ArcTan[
(2 + (1 + I*Sqrt[3])*Tanh[x/2])/Sqrt[-2 - (2*I)*Sqrt[3]]] + 2*ArcTanh[(-1 +
 Tanh[x/2])/Sqrt[2]])/(3*Sqrt[2])
```

fricas [B] time = 0.90, size = 185, normalized size = 1.33

$$-\frac{1}{6}\sqrt{3}\log\left(-4\left(\sqrt{3}+1\right)e^{x}+4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{3}\log\left(4\left(\sqrt{3}-1\right)e^{x}-4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{2}\log\left(-\frac{2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*log(-4*(sqrt(3) + 1)*e^x + 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sq
rt(3)*log(4*(sqrt(3) - 1)*e^x - 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sqrt(2)*lo
g(-(2*(sqrt(2) - 1)*e^x + 2*sqrt(2) - e^(2*x) - 3)/(e^(2*x) + 2*e^x - 1)) +
 2/3*arctan(-(sqrt(3) + 1)*e^x + sqrt((sqrt(3) - 1)*e^x - sqrt(3) + e^(2*x)
 + 2)*(sqrt(3) + 1) - 1) - 2/3*arctan(-(sqrt(3) - 1)*e^x + 1/2*sqrt(-4*(sqr
t(3) + 1)*e^x + 4*sqrt(3) + 4*e^(2*x) + 8)*(sqrt(3) - 1) + 1)
```

giac [A] time = 0.13, size = 102, normalized size = 0.73

$$\frac{1}{6}\pi+\frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3}+e^{x}-1\right)^{2}+e^{(2x)}\right)-\frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3}-e^{x}+1\right)^{2}+e^{(2x)}\right)+\frac{1}{6}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2e^{x}+2\right|}{2\left(\sqrt{2}+e^{x}+1\right)}\right)+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^3),x, algorithm="giac")
```

```
[Out] 1/6*pi + 1/6*sqrt(3)*log((sqrt(3) + e^x - 1)^2 + e^(2*x)) - 1/6*sqrt(3)*log
((sqrt(3) - e^x + 1)^2 + e^(2*x)) + 1/6*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*
e^x + 2)/(sqrt(2) + e^x + 1)) + 1/3*arctan(-(sqrt(3) + 1)*e^x - 1) + 1/3*ar
ctan((sqrt(3) - 1)*e^x - 1)
```

maple [C] time = 0.06, size = 82, normalized size = 0.59

$$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+2\_Z^{3}+2\_Z^{2}-2\_Z+1\right)}\frac{\left(-\_R^{2}-\_R+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^{3}+3\_R^{2}+2\_R-1}\right)}{3}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^3),x)
```

```
[Out] 2/3*sum((-_R^2-_R+1)/(2*_R^3+3*_R^2+2*_R-1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z
^4+2*_Z^3+2*_Z^2-2*_Z+1))+1/3*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{x}-1}{\sqrt{2}+e^{x}+1}\right)-\int\frac{2\left(e^{(3x)}-4e^{(2x)}-e^{x}\right)}{3\left(e^{(4x)}-2e^{(3x)}+2e^{(2x)}+2e^{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) - integrate(2/3*(
e^(3*x) - 4*e^(2*x) - e^x)/(e^(4*x) - 2*e^(3*x) + 2*e^(2*x) + 2*e^x + 1), x
)
```

mupad [B] time = 1.77, size = 203, normalized size = 1.46

$$\frac{\operatorname{atan}\left(\frac{77824\,e^{x}-32768\sqrt{3}-45056\sqrt{3}\,e^{x}+57344}{77824\,e^{x}-45056\sqrt{3}\,e^{x}}\right)}{3}-\frac{\operatorname{atan}\left(\frac{77824\,e^{x}+45056\sqrt{3}\,e^{x}}{77824\,e^{x}+32768\sqrt{3}+45056\sqrt{3}\,e^{x}+57344}\right)}{3}-\frac{\sqrt{2}\,\ln\left(41984\sqrt{2}\,e^{x}-17408\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^3 + 1),x)
```

```
[Out] atan((77824*exp(x) - 32768*3^(1/2) - 45056*3^(1/2)*exp(x) + 57344)/(77824*e
xp(x) - 45056*3^(1/2)*exp(x)))/3 - atan((77824*exp(x) + 45056*3^(1/2)*exp(x
))/(77824*exp(x) + 32768*3^(1/2) + 45056*3^(1/2)*exp(x) + 57344))/3 - (2^(1
/2)*log(41984*2^(1/2)*exp(x) - 17408*2^(1/2) - 59392*exp(x) + 24576))/6 + (
2^(1/2)*log(17408*2^(1/2) - 59392*exp(x) - 41984*2^(1/2)*exp(x) + 24576))/6
 - (3^(1/2)*log((77824*exp(x) - 32768*3^(1/2) - 45056*3^(1/2)*exp(x) + 5734
4)^2 + (77824*exp(x) - 45056*3^(1/2)*exp(x))^2))/6 + (3^(1/2)*log((77824*ex
p(x) + 32768*3^(1/2) + 45056*3^(1/2)*exp(x) + 57344)^2 + (77824*exp(x) + 45
056*3^(1/2)*exp(x))^2))/6
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**3),x)
```

```
[Out] Timed out
```

3.183 $\int \frac{1}{1-\sinh^3(x)}\,dx$

Optimal. Leaf size=133

$$\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)-\frac{1}{3}(-1)^{5/6}\log\left((-1)^{2/3}\tanh\left(\frac{x}{2}\right)+(-1)^{5/6}+1\right)+\frac{1}{3}(-1)^{5/6}\log\left((-1)^{5/6}\tanh\left(\frac{x}{2}\right)+\sqrt[6]{-}\right.$$

```
[Out] -1/3*(-1)^(5/6)*ln(1+(-1)^(5/6)+(-1)^(2/3)*tanh(1/2*x))+1/3*(-1)^(5/6)*ln(1
+(-1)^(1/6)+(-1)^(5/6)*tanh(1/2*x))+1/3*arctanh(1/2*(1+tanh(1/2*x))*2^(1/2)
)*2^(1/2)+2/3*(-1)^(5/6)*arctan((I-(-1)^(5/6)*tanh(1/2*x))/(1+(-1)^(2/3))^(
1/2))/(1+(-1)^(2/3))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3213, 2660, 618, 204, 616, 31, 617, 206}

$$\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)-\frac{1}{3}(-1)^{5/6}\log\left((-1)^{2/3}\tanh\left(\frac{x}{2}\right)+(-1)^{5/6}+1\right)+\frac{1}{3}(-1)^{5/6}\log\left((-1)^{5/6}\tanh\left(\frac{x}{2}\right)+\sqrt[6]{-}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^3)^(-1),x]
```

```
[Out] (2*(-1)^(5/6)*ArcTan[(I - (-1)^(5/6)*Tanh[x/2])/Sqrt[1 + (-1)^(2/3)]])/(3*S
qrt[1 + (-1)^(2/3)]) + (Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[2]])/3 - ((-1)
^(5/6)*Log[1 + (-1)^(5/6) + (-1)^(2/3)*Tanh[x/2]])/3 + ((-1)^(5/6)*Log[1 +
(-1)^(1/6) + (-1)^(5/6)*Tanh[x/2]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{1-\sinh^3(x)}\,dx = \int \left(-\frac{(-1)^{5/6}}{3\left(-(-1)^{5/6}-i\sinh(x)\right)} - \frac{(-1)^{5/6}}{3\left(-(-1)^{5/6}+\sqrt[6]{-1}\,\sinh(x)\right)} - \frac{(-1)^{5/6}}{3\left(-(-1)^{5/6}+(-1)^{5/6}\sinh\right.}\right.$$

$$= -\left(\frac{1}{3}(-1)^{5/6}\int \frac{1}{-(-1)^{5/6}-i\sinh(x)}\,dx\right) - \frac{1}{3}(-1)^{5/6}\int \frac{1}{-(-1)^{5/6}+\sqrt[6]{-1}\,\sinh(x)}\,dx - \frac{1}{3}(-1$$

$$= -\left(\frac{1}{3}\left(2(-1)^{5/6}\right)\text{Subst}\left(\int \frac{1}{-(-1)^{5/6}-2ix+(-1)^{5/6}x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\right) - \frac{1}{3}\left(2(-1)^{5/6}\right)\text{Su}$$

$$= \frac{2}{3}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, 1+\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{3}(-1)^{2/3}\,\text{Subst}\left(\int \frac{1}{-1+\sqrt[6]{-1}+(-1)^{5/6}x}\,dx, x$$

$$= \frac{2(-1)^{5/6}\tan^{-1}\left(\frac{i-(-1)^{5/6}\tanh\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}} + \frac{1}{3}\sqrt{2}\,\tanh^{-1}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - \frac{1}{3}(-1)^{5/6}\log\left(1+(-1)^{5/}\right.$$

Mathematica [A] time = 1.28, size = 156, normalized size = 1.17

$$\frac{2\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right) + \sqrt{-1+i\sqrt{3}}\,\left(1+i\sqrt{3}\right)\tan^{-1}\left(\frac{2+\left(-1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2-2i\sqrt{3}}}\right) + \sqrt{-1-i\sqrt{3}}\,\left(1-i\sqrt{3}\right)\tan^{-1}\left(\frac{2+i\left(\sqrt{\right.}}{\sqrt{-}}\right.}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^3)^(-1),x]
```

```
[Out] (Sqrt[-1 + I*Sqrt[3]]*(1 + I*Sqrt[3])*ArcTan[(2 + (-1 - I*Sqrt[3])*Tanh[x/2
])/Sqrt[-2 - (2*I)*Sqrt[3]]] + Sqrt[-1 - I*Sqrt[3]]*(1 - I*Sqrt[3])*ArcTan[
(2 + I*(I + Sqrt[3])*Tanh[x/2])/Sqrt[-2 + (2*I)*Sqrt[3]]] + 2*ArcTanh[(1 +
Tanh[x/2])/Sqrt[2]])/(3*Sqrt[2])
```

fricas [B] time = 1.48, size = 180, normalized size = 1.35

$$-\frac{1}{6}\sqrt{3}\log\left(4\left(\sqrt{3}+1\right)e^{x}+4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{3}\log\left(-4\left(\sqrt{3}-1\right)e^{x}-4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{2}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*log(4*(sqrt(3) + 1)*e^x + 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sqr
t(3)*log(-4*(sqrt(3) - 1)*e^x - 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sqrt(2)*lo
g((2*(sqrt(2) - 1)*e^x - 2*sqrt(2) + e^(2*x) + 3)/(e^(2*x) - 2*e^x - 1)) -
2/3*arctan(-(sqrt(3) + 1)*e^x + 1/2*sqrt(-4*(sqrt(3) - 1)*e^x - 4*sqrt(3) +
 4*e^(2*x) + 8)*(sqrt(3) + 1) + 1) + 2/3*arctan(-(sqrt(3) - 1)*e^x + sqrt((
sqrt(3) + 1)*e^x + sqrt(3) + e^(2*x) + 2)*(sqrt(3) - 1) - 1)
```

giac [A] time = 0.16, size = 106, normalized size = 0.80

$$-\frac{1}{6}\pi-\frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3}+e^{x}+1\right)^{2}+e^{(2x)}\right)+\frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3}-e^{x}-1\right)^{2}+e^{(2x)}\right)-\frac{1}{6}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2e^{x}-2\right|}{\left|2\sqrt{2}+2e^{x}-2\right|}\right)-\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^3),x, algorithm="giac")
```

```
[Out] -1/6*pi - 1/6*sqrt(3)*log((sqrt(3) + e^x + 1)^2 + e^(2*x)) + 1/6*sqrt(3)*lo
g((sqrt(3) - e^x - 1)^2 + e^(2*x)) - 1/6*sqrt(2)*log(abs(-2*sqrt(2) + 2*e^x
 - 2)/abs(2*sqrt(2) + 2*e^x - 2)) - 1/3*arctan(-(sqrt(3) + 1)*e^x + 1) - 1/
3*arctan((sqrt(3) - 1)*e^x + 1)
```

maple [C] time = 0.05, size = 80, normalized size = 0.60

$$\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{3}+\frac{2\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}-2\_Z^{3}+2\_Z^{2}+2\_Z+1\right)}\frac{\left(-\_R^{2}+\_R+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^{3}-3\_R^{2}+2\_R+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^3),x)
```

```
[Out] 1/3*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))+2/3*sum((-_R^2+_R+1)/(2*
_R^3-3*_R^2+2*_R+1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^4-2*_Z^3+2*_Z^2+2*_Z+1)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{x}+1}{\sqrt{2}+e^{x}-1}\right)+\int\frac{2\left(e^{(3x)}+4e^{(2x)}-e^{x}\right)}{3\left(e^{(4x)}+2e^{(3x)}+2e^{(2x)}-2e^{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + integrate(2/3*
(e^(3*x) + 4*e^(2*x) - e^x)/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1),
x)
```

mupad [B] time = 2.07, size = 225, normalized size = 1.69

$$\frac{\operatorname{atan}\left(\frac{77824\,e^{x}+32768\sqrt{3}-45056\sqrt{3}\,e^{x}-57344}{77824\,e^{x}-45056\sqrt{3}\,e^{x}}\right)}{3}+\frac{\operatorname{atan}\left(\frac{77824\,e^{x}-32768\sqrt{3}+45056\sqrt{3}\,e^{x}-57344}{77824\,e^{x}+45056\sqrt{3}\,e^{x}}\right)}{3}+\frac{\pi\operatorname{sign}\left(77824\,e^{x}+32768\sqrt{3}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^3 - 1),x)
```

```
[Out] atan((77824*exp(x) + 32768*3^(1/2) - 45056*3^(1/2)*exp(x) - 57344)/(77824*e
xp(x) - 45056*3^(1/2)*exp(x)))/3 + atan((77824*exp(x) - 32768*3^(1/2) + 450
56*3^(1/2)*exp(x) - 57344)/(77824*exp(x) + 45056*3^(1/2)*exp(x)))/3 + (pi*s
ign(77824*exp(x) + 32768*3^(1/2) - 45056*3^(1/2)*exp(x) - 57344))/3 - (2^(1
/2)*log(59392*exp(x) - 17408*2^(1/2) - 41984*2^(1/2)*exp(x) + 24576))/6 + (
2^(1/2)*log(59392*exp(x) + 17408*2^(1/2) + 41984*2^(1/2)*exp(x) + 24576))/6
 + (3^(1/2)*log((77824*exp(x) - 32768*3^(1/2) + 45056*3^(1/2)*exp(x) - 5734
4)^2 + (77824*exp(x) + 45056*3^(1/2)*exp(x))^2))/6 - (3^(1/2)*log((77824*ex
p(x) + 32768*3^(1/2) - 45056*3^(1/2)*exp(x) - 57344)^2 + (77824*exp(x) - 45
056*3^(1/2)*exp(x))^2))/6
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**3),x)
```

```
[Out] Timed out
```

3.184 $\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=111

$$\frac{(48a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{(80a+93b)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x(48a+35b)+\frac{b\sinh(c+d\ldots}{}$$

[Out] 1/128*(48*a+35*b)*x-1/128*(80*a+93*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*(48*a+163*b)*cosh(d*x+c)^3*sinh(d*x+c)/d-25/48*b*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b*cosh(d*x+c)^7*sinh(d*x+c)/d

Rubi [A] time = 0.14, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3217, 1257, 1814, 1157, 385, 206}

$$\frac{(48a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{(80a+93b)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x(48a+35b)+\frac{b\sinh(c+d\ldots}{}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]

[Out] ((48*a + 35*b)*x)/128 - ((80*a + 93*b)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + ((48*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) - (25*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x])/(8*d)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)dx=\frac{\text{Subst}\left(\int \frac{x^4\left(a-2ax^2+(a+b)x^4\right)}{\left(1-x^2\right)^5}dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b\cosh^7(c+dx)\sinh(c+dx)}{8d}-\frac{\text{Subst}\left(\int \frac{b+8bx^2-8(a-b)x^4+8(a+b)x^6}{\left(1-x^2\right)^4}d\right.}{8d}$$

$$=-\frac{25b\cosh^5(c+dx)\sinh(c+dx)}{48d}+\frac{b\cosh^7(c+dx)\sinh(c+dx)}{8d}$$

$$=\frac{(48a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d}-\frac{25b\cosh^5(c+dx)\sin}{48d}$$

$$=-\frac{(80a+93b)\cosh(c+dx)\sinh(c+dx)}{128d}+\frac{(48a+163b)\cosh^3(c}{192d}$$

$$=\frac{1}{128}(48a+35b)x-\frac{(80a+93b)\cosh(c+dx)\sinh(c+dx)}{128d}+\frac{(48a}{}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 0.74

$$\frac{-96(8a+7b)\sinh(2(c+dx))+24(4a+7b)\sinh(4(c+dx))+1152ac+1152adx-32b\sinh(6(c+dx))+3b\,\text{s}}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (1152*a*c + 840*b*c + 1152*a*d*x + 840*b*d*x - 96*(8*a + 7*b)*Sinh[2*(c + d
*x)] + 24*(4*a + 7*b)*Sinh[4*(c + d*x)] - 32*b*Sinh[6*(c + d*x)] + 3*b*Sinh
[8*(c + d*x)])/(3072*d)
```

fricas [A] time = 1.07, size = 174, normalized size = 1.57

$$\frac{3b\cosh(dx+c)\sinh(dx+c)^7+3\left(7b\cosh(dx+c)^3-8b\cosh(dx+c)\right)\sinh(dx+c)^5+\left(21b\cosh(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/384*(3*b*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b*cosh(d*x + c)^3 - 8*b*cos
h(d*x + c))*sinh(d*x + c)^5 + (21*b*cosh(d*x + c)^5 - 80*b*cosh(d*x + c)^3
+ 12*(4*a + 7*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(48*a + 35*b)*d*x + 3*(
b*cosh(d*x + c)^7 - 8*b*cosh(d*x + c)^5 + 4*(4*a + 7*b)*cosh(d*x + c)^3 - 8
*(8*a + 7*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.17, size = 155, normalized size = 1.40

$$\frac{1}{128}\,(48\,a+35\,b)x+\frac{be^{(8\,dx+8\,c)}}{2048\,d}-\frac{be^{(6\,dx+6\,c)}}{192\,d}+\frac{(4\,a+7\,b)e^{(4\,dx+4\,c)}}{256\,d}-\frac{(8\,a+7\,b)e^{(2\,dx+2\,c)}}{64\,d}+\frac{(8\,a+7\,b)e^{(-2\,dx-2\,c)}}{64\,d}-\frac{(4\,a+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/128*(48*a + 35*b)*x + 1/2048*b*e^(8*d*x + 8*c)/d - 1/192*b*e^(6*d*x + 6*c
)/d + 1/256*(4*a + 7*b)*e^(4*d*x + 4*c)/d - 1/64*(8*a + 7*b)*e^(2*d*x + 2*c
)/d + 1/64*(8*a + 7*b)*e^(-2*d*x - 2*c)/d - 1/256*(4*a + 7*b)*e^(-4*d*x - 4
*c)/d + 1/192*b*e^(-6*d*x - 6*c)/d - 1/2048*b*e^(-8*d*x - 8*c)/d
```

maple [A] time = 0.04, size = 98, normalized size = 0.88

$$\frac{b\left(\left(\frac{\left(\sinh^{7}(dx+c)\right)}{8}-\frac{7\left(\sinh^{5}(dx+c)\right)}{48}+\frac{35\left(\sinh^{3}(dx+c)\right)}{192}-\frac{35\sinh(dx+c)}{128}\right)\cosh\left(dx+c\right)+\frac{35dx}{128}+\frac{35c}{128}\right)+a\left(\left(\frac{\left(\sinh^{3}(dx+c)\right)}{4}-\frac{3\sinh}{}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(b*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*s
inh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+a*((1/4*sinh(d*x+c)^3-3/8*sinh
(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.32, size = 175, normalized size = 1.58

$$\frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{6144}\,b\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/6144*b*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x -
 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (6
72*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*
d*x - 8*c))/d)
```

mupad [B] time = 0.92, size = 88, normalized size = 0.79

$$\frac{12\,a\sinh\left(4\,c+4\,d\,x\right)-96\,a\sinh\left(2\,c+2\,d\,x\right)-84\,b\sinh\left(2\,c+2\,d\,x\right)+21\,b\sinh\left(4\,c+4\,d\,x\right)-4\,b\sinh\left(6\,c+6\,d\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*sinh(c + d*x)^4),x)
```

```
[Out] (12*a*sinh(4*c + 4*d*x) - 96*a*sinh(2*c + 2*d*x) - 84*b*sinh(2*c + 2*d*x) +
 21*b*sinh(4*c + 4*d*x) - 4*b*sinh(6*c + 6*d*x) + (3*b*sinh(8*c + 8*d*x))/8
 + 144*a*d*x + 105*b*d*x)/(384*d)
```

sympy [A] time = 8.33, size = 306, normalized size = 2.76

$$\begin{cases} \frac{3ax\sinh^4(c+dx)}{8} - \frac{3ax\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3ax\cosh^4(c+dx)}{8} + \frac{5a\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{3a\sinh(c+dx)\cosh^3(c+dx)}{8d} + \frac{35b \\ x\left(a + b\sinh^4(c)\right)\sinh^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 35*b*x*sinh(c + d*x)**8/128 -
35*b*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b*x*sinh(c + d*x)**4*cosh
(c + d*x)**4/64 - 35*b*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b*x*cosh
(c + d*x)**8/128 + 93*b*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b*sinh
(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b*sinh(c + d*x)**3*cosh(c + d*x
)**5/(384*d) - 35*b*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(
a + b*sinh(c)**4)*sinh(c)**4, True))
```

3.185 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)dx$

Optimal. Leaf size=67

$$\frac{(a+3b)\cosh^3(c+dx)}{3d}-\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}$$

```
[Out] -(a+b)*cosh(d*x+c)/d+1/3*(a+3*b)*cosh(d*x+c)^3/d-3/5*b*cosh(d*x+c)^5/d+1/7*
b*cosh(d*x+c)^7/d
```

Rubi [A] time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3215, 1153}

$$\frac{(a+3b)\cosh^3(c+dx)}{3d}-\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -(((a + b)*Cosh[c + d*x])/d) + ((a + 3*b)*Cosh[c + d*x]^3)/(3*d) - (3*b*Cos
h[c + d*x]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d)
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)dx &= -\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+b-2bx^2+bx^4\right)dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int\left(a\left(1+\frac{b}{a}\right)-(a+3b)x^2+3bx^4-bx^6\right)dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{(a+b)\cosh(c+dx)}{d}+\frac{(a+3b)\cosh^3(c+dx)}{3d}-\frac{3b\cosh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 1.39

$$-\frac{3a\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}-\frac{35b\cosh(c+dx)}{64d}+\frac{7b\cosh(3(c+dx))}{64d}-\frac{7b\cosh(5(c+dx))}{320d}+\frac{b\cosh(7(c+}{448d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) - (35*b*Cosh[c + d*x])/(64*d) + (a*Cosh[3*(c + d
*x)])/(12*d) + (7*b*Cosh[3*(c + d*x)])/(64*d) - (7*b*Cosh[5*(c + d*x)])/(32
0*d) + (b*Cosh[7*(c + d*x)])/(448*d)
```

fricas [B] time = 1.81, size = 155, normalized size = 2.31

$$\frac{15\,b\cosh\left(dx+c\right)^{7}+105\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}-147\,b\cosh\left(dx+c\right)^{5}+105\left(5\,b\cosh\left(dx+c\right)^{3}-7\,b\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/6720*(15*b*cosh(d*x + c)^7 + 105*b*cosh(d*x + c)*sinh(d*x + c)^6 - 147*b*
cosh(d*x + c)^5 + 105*(5*b*cosh(d*x + c)^3 - 7*b*cosh(d*x + c))*sinh(d*x +
c)^4 + 35*(16*a + 21*b)*cosh(d*x + c)^3 + 105*(3*b*cosh(d*x + c)^5 - 14*b*c
osh(d*x + c)^3 + (16*a + 21*b)*cosh(d*x + c))*sinh(d*x + c)^2 - 105*(48*a +
 35*b)*cosh(d*x + c))/d
```

giac [B] time = 0.17, size = 142, normalized size = 2.12

$$\frac{be^{(7\,dx+7\,c)}}{896\,d}-\frac{7\,be^{(5\,dx+5\,c)}}{640\,d}+\frac{(16\,a+21\,b)e^{(3\,dx+3\,c)}}{384\,d}-\frac{(48\,a+35\,b)e^{(dx+c)}}{128\,d}-\frac{(48\,a+35\,b)e^{(-dx-c)}}{128\,d}+\frac{(16\,a+21\,b)e^{(-3\,dx-}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/896*b*e^(7*d*x + 7*c)/d - 7/640*b*e^(5*d*x + 5*c)/d + 1/384*(16*a + 21*b)
*e^(3*d*x + 3*c)/d - 1/128*(48*a + 35*b)*e^(d*x + c)/d - 1/128*(48*a + 35*b
)*e^(-d*x - c)/d + 1/384*(16*a + 21*b)*e^(-3*d*x - 3*c)/d - 7/640*b*e^(-5*d
*x - 5*c)/d + 1/896*b*e^(-7*d*x - 7*c)/d
```

maple [A] time = 0.03, size = 66, normalized size = 0.99

$$\frac{b\left(-\frac{16}{35}+\frac{\left(\sinh^{6}(dx+c)\right)}{7}-\frac{6\left(\sinh^{4}(dx+c)\right)}{35}+\frac{8\left(\sinh^{2}(dx+c)\right)}{35}\right)\cosh\left(dx+c\right)+a\left(-\frac{2}{3}+\frac{\left(\sinh^{2}(dx+c)\right)}{3}\right)\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(b*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cos
h(d*x+c)+a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.31, size = 157, normalized size = 2.34

$$-\frac{1}{4480}\,b\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/4480*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6
*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49
*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a*(e^(3*d*x + 3*c)/d - 9*
e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 0.79, size = 66, normalized size = 0.99

$$-\frac{a\cosh\left(c+d\,x\right)+b\cosh\left(c+d\,x\right)-\frac{a\cosh(c+d\,x)^3}{3}-b\cosh\left(c+d\,x\right)^3+\frac{3\,b\cosh(c+d\,x)^5}{5}-\frac{b\cosh(c+d\,x)^7}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^4),x)
```

```
[Out] -(a*cosh(c + d*x) + b*cosh(c + d*x) - (a*cosh(c + d*x)^3)/3 - b*cosh(c + d*
x)^3 + (3*b*cosh(c + d*x)^5)/5 - (b*cosh(c + d*x)^7)/7)/d
```

sympy [A] time = 4.81, size = 128, normalized size = 1.91

$$\begin{cases} \frac{a\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a\cosh^3(c+dx)}{3d} + \frac{b\sinh^6(c+dx)\cosh(c+dx)}{d} - \frac{2b\sinh^4(c+dx)\cosh^3(c+dx)}{d} + \frac{8b\sinh^2(c+dx)\cosh^5(c+dx)}{5d} - \\ x\left(a + b\sinh^4(c)\right)\sinh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
+ b*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b*sinh(c + d*x)**4*cosh(c + d*x)**
3/d + 8*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b*cosh(c + d*x)**7/(
35*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**3, True))
```

3.186 $\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=83

$$\frac{(8a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}x(8a+5b)+\frac{b\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b\sinh(c+dx)\cosh^3(c+dx)}{24d}$$

```
[Out] -1/16*(8*a+5*b)*x+1/16*(8*a+11*b)*cosh(d*x+c)*sinh(d*x+c)/d-13/24*b*cosh(d*
x+c)^3*sinh(d*x+c)/d+1/6*b*cosh(d*x+c)^5*sinh(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3217, 1257, 1157, 385, 206}

$$\frac{(8a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}x(8a+5b)+\frac{b\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b\sinh(c+dx)\cosh^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -((8*a + 5*b)*x)/16 + ((8*a + 11*b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (
13*b*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*Cosh[c + d*x]^5*Sinh[c + d*
x])/(6*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-2ax^2+(a+b)x^4\right)}{\left(1-x^2\right)^4}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d} + \frac{\text{Subst}\left(\int \frac{-b+6(a-b)x^2-6(a+b)x^4}{\left(1-x^2\right)^3}dx, x, \text{ta}\right.}{6d}$$

$$= -\frac{13b\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d} - $$

$$= \frac{(8a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b\cosh^3(c+dx)\sinh(c+}{24d}$$

$$= -\frac{1}{16}(8a+5b)x + \frac{(8a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b\cosh^3(}{}$$

Mathematica [A] time = 0.08, size = 63, normalized size = 0.76

$$\frac{(48a+45b)\sinh(2(c+dx)) - 96ac - 96adx - 9b\sinh(4(c+dx)) + b\sinh(6(c+dx)) - 60bc - 60bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-96*a*c - 60*b*c - 96*a*d*x - 60*b*d*x + (48*a + 45*b)*Sinh[2*(c + d*x)] -
 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

fricas [A] time = 3.30, size = 109, normalized size = 1.31

$$\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5 + 2\left(5\,b\cosh\left(dx+c\right)^3 - 9\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^3 - 6\,(8\,a+5\,b)dx + 3\left(b\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 - 9*b*cosh
(d*x + c))*sinh(d*x + c)^3 - 6*(8*a + 5*b)*d*x + 3*(b*cosh(d*x + c)^5 - 6*b
*cosh(d*x + c)^3 + (16*a + 15*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.16, size = 113, normalized size = 1.36

$$-\frac{1}{16}\,(8\,a+5\,b)x + \frac{be^{(6\,dx+6\,c)}}{384\,d} - \frac{3\,be^{(4\,dx+4\,c)}}{128\,d} + \frac{(16\,a+15\,b)e^{(2\,dx+2\,c)}}{128\,d} - \frac{(16\,a+15\,b)e^{(-2\,dx-2\,c)}}{128\,d} + \frac{3\,be^{(-4\,dx-4\,c)}}{128\,d} - \frac{be^{(-6\,dx-6}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/16*(8*a + 5*b)*x + 1/384*b*e^(6*d*x + 6*c)/d - 3/128*b*e^(4*d*x + 4*c)/d
 + 1/128*(16*a + 15*b)*e^(2*d*x + 2*c)/d - 1/128*(16*a + 15*b)*e^(-2*d*x -
2*c)/d + 3/128*b*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x - 6*c)/d
```

maple [A] time = 0.04, size = 76, normalized size = 0.92

$$\frac{b\left(\left(\frac{\left(\sinh^{5}(dx+c)\right)}{6}-\frac{5\left(\sinh^{3}(dx+c)\right)}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)
-5/16*d*x-5/16*c)+a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [A] time = 0.32, size = 122, normalized size = 1.47

$$-\frac{1}{8}a\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{384}b\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}+\frac{120(dx+c)}{d}+\frac{45e^{(-2d}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b*((9*e^(-2*d
*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d +
(45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

mupad [B] time = 0.16, size = 64, normalized size = 0.77

$$\frac{12\,a\sinh\left(2\,c+2\,d\,x\right)+\frac{45\,b\sinh(2\,c+2\,d\,x)}{4}-\frac{9\,b\sinh(4\,c+4\,d\,x)}{4}+\frac{b\sinh(6\,c+6\,d\,x)}{4}-24\,a\,d\,x-15\,b\,d\,x}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^4),x)
```

```
[Out] (12*a*sinh(2*c + 2*d*x) + (45*b*sinh(2*c + 2*d*x))/4 - (9*b*sinh(4*c + 4*d*
x))/4 + (b*sinh(6*c + 6*d*x))/4 - 24*a*d*x - 15*b*d*x)/(48*d)
```

sympy [A] time = 2.95, size = 206, normalized size = 2.48

$$\begin{cases}\frac{ax\sinh^{2}(c+dx)}{2}-\frac{ax\cosh^{2}(c+dx)}{2}+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{5bx\sinh^{6}(c+dx)}{16}-\frac{15bx\sinh^{4}(c+dx)\cosh^{2}(c+dx)}{16}+\frac{15bx\sinh^{2}(c+dx)}{16} \\ x\left(a+b\sinh^{4}(c)\right)\sinh^{2}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x
)*cosh(c + d*x)/(2*d) + 5*b*x*sinh(c + d*x)**6/16 - 15*b*x*sinh(c + d*x)**4
*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*x*
cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b*sinh
(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c + d*x)*cosh(c + d*x)**5/(1
6*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**2, True))
```

3.187 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=46

$$\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}$$

```
[Out] (a+b)*cosh(d*x+c)/d-2/3*b*cosh(d*x+c)^3/d+1/5*b*cosh(d*x+c)^5/d
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3215}

$$\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] ((a + b)*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c + d*x]^
5)/(5*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(a+b-2bx^2+bx^4\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{(a+b)\cosh(c+dx)}{d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 1.50

$$\frac{a\sinh(c)\sinh(dx)}{d}+\frac{a\cosh(c)\cosh(dx)}{d}+\frac{5b\cosh(c+dx)}{8d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d + (5*b*Cosh[c + d*x])/(8*d) - (5*b*Cosh[3*(c + d*x)
])/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d) + (a*Sinh[c]*Sinh[d*x])/d
```

fricas [B] time = 0.80, size = 91, normalized size = 1.98

$$\frac{3\,b\cosh\left(dx+c\right)^5+15\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-25\,b\cosh\left(dx+c\right)^3+15\left(2\,b\cosh\left(dx+c\right)^3-5\,b\cosh\left(dx\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 - 25*b*cosh
(d*x + c)^3 + 15*(2*b*cosh(d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c)^2
+ 30*(8*a + 5*b)*cosh(d*x + c))/d
```

giac [B] time = 0.14, size = 100, normalized size = 2.17

$$\frac{be^{(5\,dx+5\,c)}}{160\,d}-\frac{5\,be^{(3\,dx+3\,c)}}{96\,d}+\frac{(8\,a+5\,b)e^{(dx+c)}}{16\,d}+\frac{(8\,a+5\,b)e^{(-dx-c)}}{16\,d}-\frac{5\,be^{(-3\,dx-3\,c)}}{96\,d}+\frac{be^{(-5\,dx-5\,c)}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/160*b*e^(5*d*x + 5*c)/d - 5/96*b*e^(3*d*x + 3*c)/d + 1/16*(8*a + 5*b)*e^(
d*x + c)/d + 1/16*(8*a + 5*b)*e^(-d*x - c)/d - 5/96*b*e^(-3*d*x - 3*c)/d +
1/160*b*e^(-5*d*x - 5*c)/d
```

maple [A] time = 0.03, size = 44, normalized size = 0.96

$$\frac{b\left(\frac{8}{15}+\frac{\left(\sinh^4(dx+c)\right)}{5}-\frac{4\left(\sinh^2(dx+c)\right)}{15}\right)\cosh\left(dx+c\right)+a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a*cosh(d*x+c
))
```

maxima [B] time = 0.31, size = 97, normalized size = 2.11

$$\frac{1}{480}\,b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/480*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 1
50*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a*cosh(
d*x + c)/d
```

mupad [B] time = 0.10, size = 46, normalized size = 1.00

$$\frac{15\,a\cosh\left(c+d\,x\right)+15\,b\cosh\left(c+d\,x\right)-10\,b\cosh\left(c+d\,x\right)^3+3\,b\cosh\left(c+d\,x\right)^5}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^4),x)
```

```
[Out] (15*a*cosh(c + d*x) + 15*b*cosh(c + d*x) - 10*b*cosh(c + d*x)^3 + 3*b*cosh(
c + d*x)^5)/(15*d)
```

sympy [A] time = 1.52, size = 80, normalized size = 1.74

$$\begin{cases}\frac{a\cosh\left(c+dx\right)}{d}+\frac{b\sinh^4\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{4b\sinh^2\left(c+dx\right)\cosh^3\left(c+dx\right)}{3d}+\frac{8b\cosh^5\left(c+dx\right)}{15d} & \text{for}\ d\neq 0\\ x\left(a+b\sinh^4\left(c\right)\right)\sinh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((a*cosh(c + d*x)/d + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sin
h(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0
)), (x*(a + b*sinh(c)**4)*sinh(c), True))
```

3.188 $\int \left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=52

$$ax+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3bx}{8}$$

```
[Out] a*x+3/8*b*x-3/8*b*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)*sinh(d*x+c)^3
/d
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2635, 8}

$$ax+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x]^4,x]
```

```
[Out] a*x + (3*b*x)/8 - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c + d*x
]*Sinh[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\sinh^4(c+dx)\right)\,dx &= ax+b\int \sinh^4(c+dx)\,dx\\
&= ax+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}(3b)\int \sinh^2(c+dx)\,dx\\
&= ax-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}+\frac{1}{8}(3b)\int 1\\
&= ax+\frac{3bx}{8}-\frac{3b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 0.94

$$ax+\frac{3b(c+dx)}{8d}-\frac{b\sinh(2(c+dx))}{4d}+\frac{b\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x]^4,x]
```

```
[Out] a*x + (3*b*(c + d*x))/(8*d) - (b*Sinh[2*(c + d*x)])/(4*d) + (b*Sinh[4*(c +
d*x)])/(32*d)
```

fricas [A] time = 0.65, size = 59, normalized size = 1.13

$$\frac{b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\left(8\,a+3\,b\right)dx+\left(b\cosh\left(dx+c\right)^{3}-4\,b\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + (8*a + 3*b)*d*x + (b*cosh(d*x + c)^3
 - 4*b*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.15, size = 66, normalized size = 1.27

$$\frac{1}{64}\,b\left(24\,x+\frac{e^{\left(4\,dx+4\,c\right)}}{d}-\frac{8\,e^{\left(2\,dx+2\,c\right)}}{d}+\frac{8\,e^{\left(-2\,dx-2\,c\right)}}{d}-\frac{e^{\left(-4\,dx-4\,c\right)}}{d}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + a*x
```

maple [A] time = 0.02, size = 44, normalized size = 0.85

$$ax+\frac{b\left(\left(\frac{\left(\sinh^{3}(dx+c)\right)}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh\left(dx+c\right)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c)^4,x)
```

```
[Out] a*x+b/d*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)
```

maxima [A] time = 0.32, size = 66, normalized size = 1.27

$$\frac{1}{64}\,b\left(24\,x+\frac{e^{\left(4\,dx+4\,c\right)}}{d}-\frac{8\,e^{\left(2\,dx+2\,c\right)}}{d}+\frac{8\,e^{\left(-2\,dx-2\,c\right)}}{d}-\frac{e^{\left(-4\,dx-4\,c\right)}}{d}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + a*x
```

mupad [B] time = 0.70, size = 38, normalized size = 0.73

$$a\,x+\frac{3\,b\,x}{8}-\frac{\frac{b\sinh(2\,c+2\,d\,x)}{4}-\frac{b\sinh(4\,c+4\,d\,x)}{32}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sinh(c + d*x)^4,x)
```

```
[Out] a*x + (3*b*x)/8 - ((b*sinh(2*c + 2*d*x))/4 - (b*sinh(4*c + 4*d*x))/32)/d
```

sympy [A] time = 0.81, size = 100, normalized size = 1.92

$$ax+b\left(\begin{cases}\frac{3x\sinh^{4}(c+dx)}{8}-\frac{3x\sinh^{2}(c+dx)\cosh^{2}(c+dx)}{4}+\frac{3x\cosh^{4}(c+dx)}{8}+\frac{5\sinh^{3}(c+dx)\cosh(c+dx)}{8d}-\frac{3\sinh(c+dx)\cosh^{3}(c+dx)}{8d} & \text{for} \\ x\sinh^{4}(c) & \text{oth}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)**4,x)
```

```
[Out] a*x + b*Piecewise((3*x*sinh(c + d*x)**4/8 - 3*x*sinh(c + d*x)**2*cosh(c + d
*x)**2/4 + 3*x*cosh(c + d*x)**4/8 + 5*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*sinh(c)**4, True))
```

3.189 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=42

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh^3(c+dx)}{3d}-\frac{b\cosh(c+dx)}{d}$$

```
[Out] -a*arctanh(cosh(d*x+c))/d-b*cosh(d*x+c)/d+1/3*b*cosh(d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3215, 1153, 206}

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh^3(c+dx)}{3d}-\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)
/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{1-x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(b-bx^2+\frac{a}{1-x^2}\right)\,dx,x,\cosh(c+dx)\right)}{d}\\
&= -\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cosh(c+\right.}{d}\\
&= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.67

$$\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{3b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])/(12*d) - (a*Log[Cosh[c/2
 + (d*x)/2]])/d + (a*Log[Sinh[c/2 + (d*x)/2]])/d
```

fricas [B] time = 3.14, size = 395, normalized size = 9.40

$$\frac{b\cosh\left(dx+c\right)^{6}+6\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+b\sinh\left(dx+c\right)^{6}-9\,b\cosh\left(dx+c\right)^{4}+3\left(5\,b\cosh\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/24*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x +
c)^6 - 9*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - 3*b)*sinh(d*x + c)^4
+ 4*(5*b*cosh(d*x + c)^3 - 9*b*cosh(d*x + c))*sinh(d*x + c)^3 - 9*b*cosh(d*
x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 18*b*cosh(d*x + c)^2 - 3*b)*sinh(d*x +
c)^2 - 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*cosh
(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*log(cosh(d*x + c) + sinh(d*x
 + c) + 1) + 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*
a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*log(cosh(d*x + c) + si
nh(d*x + c) - 1) + 6*(b*cosh(d*x + c)^5 - 6*b*cosh(d*x + c)^3 - 3*b*cosh(d*
x + c))*sinh(d*x + c) + b)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*
x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

giac [A] time = 0.17, size = 78, normalized size = 1.86

$$\frac{be^{(3\,dx+3\,c)}-9\,be^{(dx+c)}-\left(9\,be^{(2\,dx+2\,c)}-b\right)e^{(-3\,dx-3\,c)}-24\,a\log\left(e^{(dx+c)}+1\right)+24\,a\log\left(\left|e^{(dx+c)}-1\right|\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/24*(b*e^(3*d*x + 3*c) - 9*b*e^(d*x + c) - (9*b*e^(2*d*x + 2*c) - b)*e^(-3
*d*x - 3*c) - 24*a*log(e^(d*x + c) + 1) + 24*a*log(abs(e^(d*x + c) - 1)))/d
```

maple [A] time = 0.07, size = 36, normalized size = 0.86

$$\frac{-2a\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(-\frac{2}{3}+\frac{\left(\sinh^{2}\left(dx+c\right)\right)}{3}\right)\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [A] time = 0.31, size = 71, normalized size = 1.69

$$\frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
- 3*c)/d) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

mupad [B] time = 0.13, size = 96, normalized size = 2.29

$$\frac{b\,e^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b\,e^{-c-d\,x}}{8\,d}+\frac{b\,e^{3\,c+3\,d\,x}}{24\,d}-\frac{3\,b\,e^{c+d\,x}}{8\,d}-\frac{2\,\mathrm{atan}\left(\frac{a\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x),x)
```

```
[Out] (b*exp(- 3*c - 3*d*x))/(24*d) - (3*b*exp(- c - d*x))/(8*d) + (b*exp(3*c + 3
*d*x))/(24*d) - (3*b*exp(c + d*x))/(8*d) - (2*atan((a*exp(d*x)*exp(c)*(-d^2
)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^4\left(c+dx\right)\right)\operatorname{csch}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**4)*csch(c + d*x), x)
```

3.190 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=39

$$-\frac{a\coth(c+dx)}{d}+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

```
[Out] -1/2*b*x-a*coth(d*x+c)/d+1/2*b*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3217, 1259, 453, 206}

$$-\frac{a\coth(c+dx)}{d}+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -(b*x)/2 - (a*Coth[c + d*x])/d + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)dx=\frac{\operatorname{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^2(1-x^2)^2}dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{\operatorname{Subst}\left(\int \frac{-2a+(2a+b)x^2}{x^2(1-x^2)}dx,x,\tanh(c+dx)\right)}{2d}$$
$$=-\frac{a\coth(c+dx)}{d}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{b\operatorname{Subst}\left(\int \frac{1}{1-x^2}dx,x,\tanh(c+dx)\right)}{2d}$$
$$=-\frac{bx}{2}-\frac{a\coth(c+dx)}{d}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}$$

Mathematica [A] time = 0.12, size = 45, normalized size = 1.15

$$-\frac{a\coth(c+dx)}{d}+\frac{b(-c-dx)}{2d}+\frac{b\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
[Out] (b*(-c - d*x))/(2*d) - (a*Coth[c + d*x])/d + (b*Sinh[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.80, size = 70, normalized size = 1.79

$$\frac{b\cosh(dx+c)^3+3\,b\cosh(dx+c)\sinh(dx+c)^2-(8\,a+b)\cosh(dx+c)-4\,(bdx-2\,a)\sinh(dx+c)}{8\,d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
[Out] 1/8*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 - (8*a + b)*cosh
(d*x + c) - 4*(b*d*x - 2*a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

giac [B] time = 0.18, size = 88, normalized size = 2.26

$$-\frac{4\,(dx+c)b-be^{(2\,dx+2\,c)}-\frac{be^{(4\,dx+4\,c)}-16\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-e^{(2\,dx+2\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
[Out] -1/8*(4*(d*x + c)*b - b*e^(2*d*x + 2*c) - (b*e^(4*d*x + 4*c) - 16*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)/(e^(4*d*x + 4*c) - e^(2*d*x + 2*c)))/d
```

maple [A] time = 0.06, size = 39, normalized size = 1.00

$$\frac{-\coth(dx+c)\,a+b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x)
[Out] 1/d*(-coth(d*x+c)*a+b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [A] time = 0.32, size = 54, normalized size = 1.38

$$-\frac{1}{8}\,b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+\frac{2\,a}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a/(d*(e^(-2*d*x -
 2*c) - 1))
```

mupad [B] time = 0.72, size = 54, normalized size = 1.38

$$\frac{b\,e^{2\,c+2\,d\,x}}{8\,d}-\frac{2\,a}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{b\,e^{-2\,c-2\,d\,x}}{8\,d}-\frac{b\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^2,x)
```

```
[Out] (b*exp(2*c + 2*d*x))/(8*d) - (2*a)/(d*(exp(2*c + 2*d*x) - 1)) - (b*exp(- 2*
c - 2*d*x))/(8*d) - (b*x)/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.191 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=47

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\cosh(c+dx)}{d}$$

```
[Out] 1/2*a*arctanh(cosh(d*x+c))/d+b*cosh(d*x+c)/d-1/2*a*coth(d*x+c)*csch(d*x+c)/
d
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3215, 1157, 388, 206}

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (a*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*Cosh[c + d*x])/d - (a*Coth[c + d*x]*C
sch[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^2}dx,x,\cosh(c+dx)\right)}{d}$$
$$= -\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-a-2b+2bx^2}{1-x^2}dx,x,\cosh(c\right.}{2d}$$
$$= \frac{b\cosh(c+dx)}{d} - \frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}d\right.}{2}$$
$$= \frac{a\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b\cosh(c+dx)}{d} - \frac{a\coth(c+dx)\operatorname{csch}(c}{2d}$$

Mathematica [A] time = 0.03, size = 82, normalized size = 1.74

$$-\frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\sinh(c)\sinh(dx)}{d}+\frac{b\cosh(c)\cosh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c +
d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sinh[c]*Sinh[d*x])/d
```

fricas [B] time = 1.58, size = 690, normalized size = 14.68

$$\frac{b\cosh(dx+c)^6 + 6\,b\cosh(dx+c)\sinh(dx+c)^5 + b\sinh(dx+c)^6 - (2\,a+b)\cosh(dx+c)^4 + \left(15\,b\cosh(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c
)^6 - (2*a + b)*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x
 + c)^4 + 4*(5*b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c)^3
 - (2*a + b)*cosh(d*x + c)^2 + (15*b*cosh(d*x + c)^4 - 6*(2*a + b)*cosh(d*x
 + c)^2 - 2*a - b)*sinh(d*x + c)^2 + (a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)
*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + 2*(5*a*cosh(d*
x + c)^2 - a)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))
*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x +
c)^2 + a)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d
*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 - 2*a*cos
h(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*
x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh
(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*log(cosh(d*x + c) + s
inh(d*x + c) - 1) + 2*(3*b*cosh(d*x + c)^5 - 2*(2*a + b)*cosh(d*x + c)^3 -
(2*a + b)*cosh(d*x + c))*sinh(d*x + c) + b)/(d*cosh(d*x + c)^5 + 5*d*cosh(d
*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + 2*(5*d*
cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*
x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 - 6*d*cosh
(d*x + c)^2 + d)*sinh(d*x + c))
```

giac [B] time = 0.19, size = 107, normalized size = 2.28

$$\frac{2\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+a\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-a\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*(e^(d*x + c) + e^(-d*x - c)) + a*log(e^(d*x + c) + e^(-d*x - c) +
2) - a*log(e^(d*x + c) + e^(-d*x - c) - 2) - 4*a*(e^(d*x + c) + e^(-d*x - c
))/((e^(d*x + c) + e^(-d*x - c))^2 - 4))/d
```

maple [A] time = 0.09, size = 38, normalized size = 0.81

$$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*cosh(d*x+c))
```

maxima [B] time = 0.31, size = 115, normalized size = 2.45

$$\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(log(e^(-d*x - c) + 1)/d - l
og(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*
x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

mupad [B] time = 0.72, size = 126, normalized size = 2.68

$$\frac{b\,e^{-c-d\,x}}{2\,d}+\frac{b\,e^{c+d\,x}}{2\,d}+\frac{\operatorname{atan}\left(\frac{a\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}-\frac{a\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^3,x)
```

```
[Out] (b*exp(- c - d*x))/(2*d) + (b*exp(c + d*x))/(2*d) + (atan((a*exp(d*x)*exp(c
)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c + d*x
))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2
*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.192 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}+bx$$

```
[Out] b*x+a*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d
```

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3217, 1261, 207}

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] b*x + (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^4\left(1-x^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^4}-\frac{a}{x^2}-\frac{b}{-1+x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= bx+\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.29

$$\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\text{csch}^2(c+dx)}{3d}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
[Out] b*x + (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d)
```

fricas [B] time = 0.73, size = 129, normalized size = 4.16

$$\frac{2\,a\cosh\left(dx+c\right)^{3}+6\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+\left(3\,bdx-2\,a\right)\sinh\left(dx+c\right)^{3}-6\,a\cosh\left(dx+c\right)-3\left(3\,bdx-\right.}{3\left(d\sinh\left(dx+c\right)^{3}+3\left(d\cosh\left(dx+c\right)^{2}-d\right)\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
[Out] 1/3*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + (3*b*d*x - 2
*a)*sinh(d*x + c)^3 - 6*a*cosh(d*x + c) - 3*(3*b*d*x - (3*b*d*x - 2*a)*cosh
(d*x + c)^2 - 2*a)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2
 - d)*sinh(d*x + c))
```

giac [A] time = 0.16, size = 45, normalized size = 1.45

$$\frac{3\left(dx+c\right)b-\frac{4\left(3\,ae^{\left(2\,dx+2\,c\right)}-a\right)}{\left(e^{\left(2\,dx+2\,c\right)}-1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
[Out] 1/3*(3*(d*x + c)*b - 4*(3*a*e^(2*d*x + 2*c) - a)/(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [A] time = 0.08, size = 33, normalized size = 1.06

$$\frac{a\left(\frac{2}{3}-\frac{\text{csch}(dx+c)^{2}}{3}\right)\coth\left(dx+c\right)+\left(dx+c\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x)
[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+(d*x+c)*b)
```

maxima [B] time = 0.32, size = 97, normalized size = 3.13

$$bx+\frac{4}{3}\,a\left(\frac{3\,e^{\left(-2\,dx-2\,c\right)}}{d\left(3\,e^{\left(-2\,dx-2\,c\right)}-3\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}-1\right)}-\frac{1}{d\left(3\,e^{\left(-2\,dx-2\,c\right)}-3\,e^{\left(-4\,dx-4\,c\right)}+e^{\left(-6\,dx-6\,c\right)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
[Out] b*x + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 0.70, size = 81, normalized size = 2.61

$$\frac{4\,a-12\,a\,{\mathrm{e}}^{2\,c+2\,d\,x}-3\,b\,d\,x+9\,b\,d\,x\,{\mathrm{e}}^{2\,c+2\,d\,x}-9\,b\,d\,x\,{\mathrm{e}}^{4\,c+4\,d\,x}+3\,b\,d\,x\,{\mathrm{e}}^{6\,c+6\,d\,x}}{3\,d\,{\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^4,x)
```

```
[Out] (4*a - 12*a*exp(2*c + 2*d*x) - 3*b*d*x + 9*b*d*x*exp(2*c + 2*d*x) - 9*b*d*x
*exp(4*c + 4*d*x) + 3*b*d*x*exp(6*c + 6*d*x))/(3*d*(exp(2*c + 2*d*x) - 1)^3
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.193 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)dx$

Optimal. Leaf size=64

$$-\frac{(3a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a\coth(c+dx)\operatorname{csch}(c+dx)}{8d}$$

```
[Out] -1/8*(3*a+8*b)*arctanh(cosh(d*x+c))/d+3/8*a*coth(d*x+c)*csch(d*x+c)/d-1/4*a
*coth(d*x+c)*csch(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3215, 1157, 385, 206}

$$-\frac{(3a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a\coth(c+dx)\operatorname{csch}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -((3*a + 8*b)*ArcTanh[Cosh[c + d*x]])/(8*d) + (3*a*Coth[c + d*x]*Csch[c + d
*x])/(8*d) - (a*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)dx = -\frac{\operatorname{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^3}dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{a\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{-3a-4b+4bx^2}{(1-x^2)^2}dx, x, \cosh(\right.}{4d}$$
$$= \frac{3a\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} - \frac{(3a}{}$$
$$= -\frac{(3a+8b)\tanh^{-1}(\cosh(c+dx))}{8d} + \frac{3a\coth(c+dx)\operatorname{csch}(c+dx)}{8d}$$

Mathematica [B] time = 0.03, size = 139, normalized size = 2.17

$$-\frac{a\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a\log\left(\tanh\left(\frac{1}{2}(c+d\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (3*a*Csch[(c + d*x)/2]^2)/(32*d) - (a*Csch[(c + d*x)/2]^4)/(64*d) - (b*Log[
Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d + (3*a*Log[Tanh[(c
 + d*x)/2]])/(8*d) + (3*a*Sech[(c + d*x)/2]^2)/(32*d) + (a*Sech[(c + d*x)/2
]^4)/(64*d)
```

fricas [B] time = 0.58, size = 1476, normalized size = 23.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(6*a*cosh(d*x + c)^7 + 42*a*cosh(d*x + c)*sinh(d*x + c)^6 + 6*a*sinh(d*
x + c)^7 - 22*a*cosh(d*x + c)^5 + 2*(63*a*cosh(d*x + c)^2 - 11*a)*sinh(d*x
+ c)^5 + 10*(21*a*cosh(d*x + c)^3 - 11*a*cosh(d*x + c))*sinh(d*x + c)^4 - 2
2*a*cosh(d*x + c)^3 + 2*(105*a*cosh(d*x + c)^4 - 110*a*cosh(d*x + c)^2 - 11
*a)*sinh(d*x + c)^3 + 2*(63*a*cosh(d*x + c)^5 - 110*a*cosh(d*x + c)^3 - 33*
a*cosh(d*x + c))*sinh(d*x + c)^2 + 6*a*cosh(d*x + c) - ((3*a + 8*b)*cosh(d*
x + c)^8 + 8*(3*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + 8*b)*sinh(d
*x + c)^8 - 4*(3*a + 8*b)*cosh(d*x + c)^6 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^
2 - 3*a - 8*b)*sinh(d*x + c)^6 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^3 - 3*(3*a
+ 8*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a + 8*b)*cosh(d*x + c)^4 + 2*(
35*(3*a + 8*b)*cosh(d*x + c)^4 - 30*(3*a + 8*b)*cosh(d*x + c)^2 + 9*a + 24*
b)*sinh(d*x + c)^4 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^5 - 10*(3*a + 8*b)*cosh
(d*x + c)^3 + 3*(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a + 8*b)*
cosh(d*x + c)^2 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^6 - 15*(3*a + 8*b)*cosh(d*
x + c)^4 + 9*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^2 + 8*(
(3*a + 8*b)*cosh(d*x + c)^7 - 3*(3*a + 8*b)*cosh(d*x + c)^5 + 3*(3*a + 8*b)
*cosh(d*x + c)^3 - (3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 3*a + 8*b)*lo
g(cosh(d*x + c) + sinh(d*x + c) + 1) + ((3*a + 8*b)*cosh(d*x + c)^8 + 8*(3*
a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + 8*b)*sinh(d*x + c)^8 - 4*(3
*a + 8*b)*cosh(d*x + c)^6 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*s
inh(d*x + c)^6 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^3 - 3*(3*a + 8*b)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 6*(3*a + 8*b)*cosh(d*x + c)^4 + 2*(35*(3*a + 8*b)*c
```

```
osh(d*x + c)^4 - 30*(3*a + 8*b)*cosh(d*x + c)^2 + 9*a + 24*b)*sinh(d*x + c)
^4 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^5 - 10*(3*a + 8*b)*cosh(d*x + c)^3 + 3*
(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a + 8*b)*cosh(d*x + c)^2
+ 4*(7*(3*a + 8*b)*cosh(d*x + c)^6 - 15*(3*a + 8*b)*cosh(d*x + c)^4 + 9*(3*
a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^2 + 8*((3*a + 8*b)*cosh
(d*x + c)^7 - 3*(3*a + 8*b)*cosh(d*x + c)^5 + 3*(3*a + 8*b)*cosh(d*x + c)^3
 - (3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 3*a + 8*b)*log(cosh(d*x + c)
+ sinh(d*x + c) - 1) + 2*(21*a*cosh(d*x + c)^6 - 55*a*cosh(d*x + c)^4 - 33*
a*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x +
 c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh
(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*
cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh
(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*
(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh
(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c
)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.18, size = 124, normalized size = 1.94

$$-\frac{(3\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(3\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(3\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{3}-20\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}-4\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/16*((3*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (3*a + 8*b)*log(e^
(d*x + c) + e^(-d*x - c) - 2) - 4*(3*a*(e^(d*x + c) + e^(-d*x - c))^3 - 20*
a*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^2)/d
```

maple [A] time = 0.09, size = 54, normalized size = 0.84

$$\frac{a\left(\left(-\frac{\operatorname{csch}(dx+c)^{3}}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth\left(dx+c\right)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-2b\operatorname{arctanh}\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(a*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(d*
x+c)))-2*b*arctanh(exp(d*x+c)))
```

maxima [B] time = 0.34, size = 174, normalized size = 2.72

$$-\frac{1}{8}\,a\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}-11\,e^{(-3\,dx-3\,c)}-11\,e^{(-5\,dx-5\,c)}+3\,e^{(-7\,dx-7\,c)}\right)}{d\left(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1\right)}\right)-b\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*
x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d
*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x
- 8*c) - 1))) - b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

mupad [B] time = 0.74, size = 242, normalized size = 3.78

$$\frac{3\,a\,\mathrm{e}^{c+d\,x}}{4\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(3\,a\,\sqrt{-d^2}+8\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{9\,a^2+48\,a\,b+64\,b^2}}\right)\,\sqrt{9\,a^2+48\,a\,b+64\,b^2}}{4\,\sqrt{-d^2}}-\frac{a\,\mathrm{e}^{c+d\,x}}{2\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{}{d\,\left(3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^5,x)
```

```
[Out] (3*a*exp(c + d*x))/(4*d*(exp(2*c + 2*d*x) - 1)) - (atan((exp(d*x)*exp(c)*(3
*a*(-d^2)^(1/2) + 8*b*(-d^2)^(1/2)))/(d*(48*a*b + 9*a^2 + 64*b^2)^(1/2)))*(
48*a*b + 9*a^2 + 64*b^2)^(1/2))/(4*(-d^2)^(1/2)) - (a*exp(c + d*x))/(2*d*(e
xp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (6*a*exp(c + d*x))/(d*(3*exp(2
*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*a*exp(c + d*
x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(
8*c + 8*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.194 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=47

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^5(c+dx)}{5d}+\frac{2a\coth^3(c+dx)}{3d}$$

```
[Out] -(a+b)*coth(d*x+c)/d+2/3*a*coth(d*x+c)^3/d-1/5*a*coth(d*x+c)^5/d
```

Rubi [A] time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3217, 14}

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^5(c+dx)}{5d}+\frac{2a\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -(((a + b)*Coth[c + d*x])/d) + (2*a*Coth[c + d*x]^3)/(3*d) - (a*Coth[c + d*
x]^5)/(5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx&=\frac{\operatorname{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^6}\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^6}-\frac{2a}{x^4}+\frac{a+b}{x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}\\&=-\frac{(a+b)\coth(c+dx)}{d}+\frac{2a\coth^3(c+dx)}{3d}-\frac{a\coth^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.51

$$-\frac{8a\coth(c+dx)}{15d}-\frac{a\coth(c+dx)\operatorname{csch}^4(c+dx)}{5d}+\frac{4a\coth(c+dx)\operatorname{csch}^2(c+dx)}{15d}-\frac{b\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-8*a*Coth[c + d*x])/(15*d) - (b*Coth[c + d*x])/d + (4*a*Coth[c + d*x]*Csch
[c + d*x]^2)/(15*d) - (a*Coth[c + d*x]*Csch[c + d*x]^4)/(5*d)
```

fricas [B] time = 0.79, size = 333, normalized size = 7.09

$$-\frac{4\left((4a+15b)\cosh(dx+c)^4-16a\cosh(dx+c)\sin\right.}{15\left(d\cosh(dx+c)^6+6d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6-6d\cosh(dx+c)^4+3\left(5d\cosh(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -4/15*((4*a + 15*b)*cosh(d*x + c)^4 - 16*a*cosh(d*x + c)*sinh(d*x + c)^3 +
(4*a + 15*b)*sinh(d*x + c)^4 - 20*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(4*a + 1
5*b)*cosh(d*x + c)^2 - 10*a - 30*b)*sinh(d*x + c)^2 - 8*(2*a*cosh(d*x + c)^
3 - 5*a*cosh(d*x + c))*sinh(d*x + c) + 40*a + 45*b)/(d*cosh(d*x + c)^6 + 6*
d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 6*d*cosh(d*x + c)^4 +
 3*(5*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 4
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x
+ c)^4 - 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)
^5 - 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c) - 10*d)
```

giac [B] time = 0.19, size = 97, normalized size = 2.06

$$-\frac{2\left(15\,be^{(8\,dx+8\,c)}-60\,be^{(6\,dx+6\,c)}+80\,ae^{(4\,dx+4\,c)}+90\,be^{(4\,dx+4\,c)}-40\,ae^{(2\,dx+2\,c)}-60\,be^{(2\,dx+2\,c)}+8\,a+15\,b\right)}{15\,d\left(e^{(2\,dx+2\,c)}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -2/15*(15*b*e^(8*d*x + 8*c) - 60*b*e^(6*d*x + 6*c) + 80*a*e^(4*d*x + 4*c) +
 90*b*e^(4*d*x + 4*c) - 40*a*e^(2*d*x + 2*c) - 60*b*e^(2*d*x + 2*c) + 8*a +
 15*b)/(d*(e^(2*d*x + 2*c) - 1)^5)
```

maple [A] time = 0.09, size = 45, normalized size = 0.96

$$\frac{a\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth(dx+c)-b\coth(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(a*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-b*coth(d*x+
c))
```

maxima [B] time = 0.33, size = 228, normalized size = 4.85

$$-\frac{16}{15}a\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1\right)}-\frac{}{d\left(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,d}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -16/15*a*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e
^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.72, size = 337, normalized size = 7.17

$$\frac{\frac{2b}{5d}+\frac{6b\,e^{4c+4dx}}{5d}-\frac{2b\,e^{6c+6dx}}{5d}-\frac{2\,e^{2c+2dx}(8a+3b)}{5d}}{6\,e^{4c+4dx}-4\,e^{2c+2dx}-4\,e^{6c+6dx}+e^{8c+8dx}+1}-\frac{\frac{2(8a+3b)}{15d}-\frac{4b\,e^{2c+2dx}}{5d}+\frac{2b\,e^{4c+4dx}}{5d}}{3\,e^{2c+2dx}-3\,e^{4c+4dx}+e^{6c+6dx}-1}-\frac{\frac{2b}{5d}-\frac{8b\,e^{2c+2dx}}{5d}-\frac{8b\,e}{}}{5\,e^{2c+2dx}-10\,e^{4c+4dx}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^6,x)
```

```
[Out] ((2*b)/(5*d) + (6*b*exp(4*c + 4*d*x))/(5*d) - (2*b*exp(6*c + 6*d*x))/(5*d)
- (2*exp(2*c + 2*d*x)*(8*a + 3*b))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c +
 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(8*a + 3*b))/(15
*d) - (4*b*exp(2*c + 2*d*x))/(5*d) + (2*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2
*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*b)/(5*d) - (
8*b*exp(2*c + 2*d*x))/(5*d) - (8*b*exp(6*c + 6*d*x))/(5*d) + (2*b*exp(8*c +
 8*d*x))/(5*d) + (4*exp(4*c + 4*d*x)*(8*a + 3*b))/(5*d))/(5*exp(2*c + 2*d*x
) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10
*c + 10*d*x) - 1) - (4*b)/(5*d*(exp(2*c + 2*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.195 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=92

$$\frac{(5a+8b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{(5a+8b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}-\frac{a\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a\coth(c$$

```
[Out] 1/16*(5*a+8*b)*arctanh(cosh(d*x+c))/d-1/16*(5*a+8*b)*coth(d*x+c)*csch(d*x+c
)/d+5/24*a*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a*coth(d*x+c)*csch(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3215, 1157, 385, 199, 206}

$$\frac{(5a+8b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{(5a+8b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}-\frac{a\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a\coth(c$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] ((5*a + 8*b)*ArcTanh[Cosh[c + d*x]])/(16*d) - ((5*a + 8*b)*Coth[c + d*x]*Cs
ch[c + d*x])/(16*d) + (5*a*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a*Coth[
c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^4}dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d} - \frac{\text{Subst}\left(\int \frac{-5a-6b+6bx^2}{(1-x^2)^3}dx, x, \cosh(c+\right.}{6d}$$

$$= \frac{5a\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d} - \frac{a\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d} + \frac{(5a}{}$$

$$= -\frac{(5a+8b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a\coth(c+dx)\operatorname{csch}^3(c+dx}{24d}$$

$$= \frac{(5a+8b)\tanh^{-1}(\cosh(c+dx))}{16d} - \frac{(5a+8b)\coth(c+dx)\operatorname{csch}(c+dx}{16d}$$

Mathematica [B] time = 0.04, size = 199, normalized size = 2.16

$$-\frac{a\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{a\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{5a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{a\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{5a\operatorname{sec}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-5*a*Csch[(c + d*x)/2]^2)/(64*d) - (b*Csch[(c + d*x)/2]^2)/(8*d) + (a*Csch
[(c + d*x)/2]^4)/(64*d) - (a*Csch[(c + d*x)/2]^6)/(384*d) - (5*a*Log[Tanh[(
c + d*x)/2]])/(16*d) - (b*Log[Tanh[(c + d*x)/2]])/(2*d) - (5*a*Sech[(c + d*
x)/2]^2)/(64*d) - (b*Sech[(c + d*x)/2]^2)/(8*d) - (a*Sech[(c + d*x)/2]^4)/(
64*d) - (a*Sech[(c + d*x)/2]^6)/(384*d)
```

fricas [B] time = 2.01, size = 3115, normalized size = 33.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/48*(6*(5*a + 8*b)*cosh(d*x + c)^11 + 66*(5*a + 8*b)*cosh(d*x + c)*sinh(d
*x + c)^10 + 6*(5*a + 8*b)*sinh(d*x + c)^11 - 2*(85*a + 72*b)*cosh(d*x + c)
^9 + 2*(165*(5*a + 8*b)*cosh(d*x + c)^2 - 85*a - 72*b)*sinh(d*x + c)^9 + 18
*(55*(5*a + 8*b)*cosh(d*x + c)^3 - (85*a + 72*b)*cosh(d*x + c))*sinh(d*x +
c)^8 + 12*(33*a + 8*b)*cosh(d*x + c)^7 + 12*(165*(5*a + 8*b)*cosh(d*x + c)^
4 - 6*(85*a + 72*b)*cosh(d*x + c)^2 + 33*a + 8*b)*sinh(d*x + c)^7 + 84*(33*
(5*a + 8*b)*cosh(d*x + c)^5 - 2*(85*a + 72*b)*cosh(d*x + c)^3 + (33*a + 8*b
)*cosh(d*x + c))*sinh(d*x + c)^6 + 12*(33*a + 8*b)*cosh(d*x + c)^5 + 12*(23
1*(5*a + 8*b)*cosh(d*x + c)^6 - 21*(85*a + 72*b)*cosh(d*x + c)^4 + 21*(33*a
 + 8*b)*cosh(d*x + c)^2 + 33*a + 8*b)*sinh(d*x + c)^5 + 12*(165*(5*a + 8*b)
*cosh(d*x + c)^7 - 21*(85*a + 72*b)*cosh(d*x + c)^5 + 35*(33*a + 8*b)*cosh(
d*x + c)^3 + 5*(33*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(85*a + 72*b
```

)*cosh(d*x + c)^3 + 2*(495*(5*a + 8*b)*cosh(d*x + c)^8 - 84*(85*a + 72*b)*c
osh(d*x + c)^6 + 210*(33*a + 8*b)*cosh(d*x + c)^4 + 60*(33*a + 8*b)*cosh(d*
x + c)^2 - 85*a - 72*b)*sinh(d*x + c)^3 + 6*(55*(5*a + 8*b)*cosh(d*x + c)^9
 - 12*(85*a + 72*b)*cosh(d*x + c)^7 + 42*(33*a + 8*b)*cosh(d*x + c)^5 + 20*
(33*a + 8*b)*cosh(d*x + c)^3 - (85*a + 72*b)*cosh(d*x + c))*sinh(d*x + c)^2
 + 6*(5*a + 8*b)*cosh(d*x + c) - 3*((5*a + 8*b)*cosh(d*x + c)^12 + 12*(5*a
+ 8*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a + 8*b)*sinh(d*x + c)^12 - 6*(5
*a + 8*b)*cosh(d*x + c)^10 + 6*(11*(5*a + 8*b)*cosh(d*x + c)^2 - 5*a - 8*b)
*sinh(d*x + c)^10 + 20*(11*(5*a + 8*b)*cosh(d*x + c)^3 - 3*(5*a + 8*b)*cosh
(d*x + c))*sinh(d*x + c)^9 + 15*(5*a + 8*b)*cosh(d*x + c)^8 + 15*(33*(5*a +
 8*b)*cosh(d*x + c)^4 - 18*(5*a + 8*b)*cosh(d*x + c)^2 + 5*a + 8*b)*sinh(d*
x + c)^8 + 24*(33*(5*a + 8*b)*cosh(d*x + c)^5 - 30*(5*a + 8*b)*cosh(d*x + c
)^3 + 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(5*a + 8*b)*cosh(d*
x + c)^6 + 4*(231*(5*a + 8*b)*cosh(d*x + c)^6 - 315*(5*a + 8*b)*cosh(d*x +
c)^4 + 105*(5*a + 8*b)*cosh(d*x + c)^2 - 25*a - 40*b)*sinh(d*x + c)^6 + 24*
(33*(5*a + 8*b)*cosh(d*x + c)^7 - 63*(5*a + 8*b)*cosh(d*x + c)^5 + 35*(5*a
+ 8*b)*cosh(d*x + c)^3 - 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*
(5*a + 8*b)*cosh(d*x + c)^4 + 15*(33*(5*a + 8*b)*cosh(d*x + c)^8 - 84*(5*a
+ 8*b)*cosh(d*x + c)^6 + 70*(5*a + 8*b)*cosh(d*x + c)^4 - 20*(5*a + 8*b)*co
sh(d*x + c)^2 + 5*a + 8*b)*sinh(d*x + c)^4 + 20*(11*(5*a + 8*b)*cosh(d*x +
c)^9 - 36*(5*a + 8*b)*cosh(d*x + c)^7 + 42*(5*a + 8*b)*cosh(d*x + c)^5 - 20
*(5*a + 8*b)*cosh(d*x + c)^3 + 3*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3
 - 6*(5*a + 8*b)*cosh(d*x + c)^2 + 6*(11*(5*a + 8*b)*cosh(d*x + c)^10 - 45*
(5*a + 8*b)*cosh(d*x + c)^8 + 70*(5*a + 8*b)*cosh(d*x + c)^6 - 50*(5*a + 8*
b)*cosh(d*x + c)^4 + 15*(5*a + 8*b)*cosh(d*x + c)^2 - 5*a - 8*b)*sinh(d*x +
 c)^2 + 12*((5*a + 8*b)*cosh(d*x + c)^11 - 5*(5*a + 8*b)*cosh(d*x + c)^9 +
10*(5*a + 8*b)*cosh(d*x + c)^7 - 10*(5*a + 8*b)*cosh(d*x + c)^5 + 5*(5*a +
8*b)*cosh(d*x + c)^3 - (5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 5*a + 8*b
)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*((5*a + 8*b)*cosh(d*x + c)^12
+ 12*(5*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a + 8*b)*sinh(d*x + c)
^12 - 6*(5*a + 8*b)*cosh(d*x + c)^10 + 6*(11*(5*a + 8*b)*cosh(d*x + c)^2 -
5*a - 8*b)*sinh(d*x + c)^10 + 20*(11*(5*a + 8*b)*cosh(d*x + c)^3 - 3*(5*a +
 8*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a + 8*b)*cosh(d*x + c)^8 + 15*
(33*(5*a + 8*b)*cosh(d*x + c)^4 - 18*(5*a + 8*b)*cosh(d*x + c)^2 + 5*a + 8*
b)*sinh(d*x + c)^8 + 24*(33*(5*a + 8*b)*cosh(d*x + c)^5 - 30*(5*a + 8*b)*co
sh(d*x + c)^3 + 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(5*a + 8*
b)*cosh(d*x + c)^6 + 4*(231*(5*a + 8*b)*cosh(d*x + c)^6 - 315*(5*a + 8*b)*c
osh(d*x + c)^4 + 105*(5*a + 8*b)*cosh(d*x + c)^2 - 25*a - 40*b)*sinh(d*x +
c)^6 + 24*(33*(5*a + 8*b)*cosh(d*x + c)^7 - 63*(5*a + 8*b)*cosh(d*x + c)^5
+ 35*(5*a + 8*b)*cosh(d*x + c)^3 - 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x +
c)^5 + 15*(5*a + 8*b)*cosh(d*x + c)^4 + 15*(33*(5*a + 8*b)*cosh(d*x + c)^8
- 84*(5*a + 8*b)*cosh(d*x + c)^6 + 70*(5*a + 8*b)*cosh(d*x + c)^4 - 20*(5*a
 + 8*b)*cosh(d*x + c)^2 + 5*a + 8*b)*sinh(d*x + c)^4 + 20*(11*(5*a + 8*b)*c
osh(d*x + c)^9 - 36*(5*a + 8*b)*cosh(d*x + c)^7 + 42*(5*a + 8*b)*cosh(d*x +
 c)^5 - 20*(5*a + 8*b)*cosh(d*x + c)^3 + 3*(5*a + 8*b)*cosh(d*x + c))*sinh(
d*x + c)^3 - 6*(5*a + 8*b)*cosh(d*x + c)^2 + 6*(11*(5*a + 8*b)*cosh(d*x + c
)^10 - 45*(5*a + 8*b)*cosh(d*x + c)^8 + 70*(5*a + 8*b)*cosh(d*x + c)^6 - 50
*(5*a + 8*b)*cosh(d*x + c)^4 + 15*(5*a + 8*b)*cosh(d*x + c)^2 - 5*a - 8*b)*
sinh(d*x + c)^2 + 12*((5*a + 8*b)*cosh(d*x + c)^11 - 5*(5*a + 8*b)*cosh(d*x
 + c)^9 + 10*(5*a + 8*b)*cosh(d*x + c)^7 - 10*(5*a + 8*b)*cosh(d*x + c)^5 +
 5*(5*a + 8*b)*cosh(d*x + c)^3 - (5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) +
 5*a + 8*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(11*(5*a + 8*b)*cosh
(d*x + c)^10 - 3*(85*a + 72*b)*cosh(d*x + c)^8 + 14*(33*a + 8*b)*cosh(d*x +
 c)^6 + 10*(33*a + 8*b)*cosh(d*x + c)^4 - (85*a + 72*b)*cosh(d*x + c)^2 + 5
*a + 8*b)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x
+ c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)
^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*si
nh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 - 18*d*cosh
(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 - 30*d*cosh(d*x

```
 + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*cosh(d*x + c)^6 + 4*(23
1*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 - 5*d)*
sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cosh(d*x + c)^5 + 35*d*co
sh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 +
 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 - 2
0*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 - 36*d*
cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*
x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 -
45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d*x + c)^4 + 15*d*c
osh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 - 5*d*cosh(d*x
 + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3
 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.20, size = 207, normalized size = 2.25

$$\frac{3\,(5\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\,(5\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(15\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+24\,b\left(e^{(dx+c)}+e^{(\cdots}\right.\right.}{\cdots}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/96*(3*(5*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*(5*a + 8*b)*log
(e^(d*x + c) + e^(-d*x - c) - 2) - 4*(15*a*(e^(d*x + c) + e^(-d*x - c))^5 +
 24*b*(e^(d*x + c) + e^(-d*x - c))^5 - 160*a*(e^(d*x + c) + e^(-d*x - c))^3
 - 192*b*(e^(d*x + c) + e^(-d*x - c))^3 + 528*a*(e^(d*x + c) + e^(-d*x - c)
) + 384*b*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4
)^3)/d
```

maple [A] time = 0.09, size = 78, normalized size = 0.85

$$\frac{a\left(\left(-\frac{\mathrm{csch}(dx+c)^5}{6}+\frac{5\mathrm{csch}(dx+c)^3}{24}-\frac{5\,\mathrm{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\,\mathrm{arctanh}\left(e^{dx+c}\right)}{8}\right)+b\left(-\frac{\mathrm{csch}(dx+c)\coth(dx+c)}{2}+\mathrm{arctanh}\left(e^{d\cdots}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(a*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x+c
)+5/8*arctanh(exp(d*x+c)))+b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+
c))))
```

maxima [B] time = 0.32, size = 268, normalized size = 2.91

$$\frac{1}{48}\,a\left(\frac{15\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15\,e^{(-dx-c)}-85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5\,dx-5\,c)}+198\,e^{(-7\,dx-7\,c)}\cdots\right.}{d\left(6\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}-15\,e^{(-8\,dx-8\,c)}+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/48*a*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(
-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*
c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) -
15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*
d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 1/2*b*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

mupad [B] time = 0.77, size = 472, normalized size = 5.13

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\, e^{c}\left(5\,a\,\sqrt{-d^2}+8\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{25\,a^2+80\,a\,b+64\,b^2}}\right)\sqrt{25\,a^2+80\,a\,b+64\,b^2}}{8\,\sqrt{-d^2}}-\frac{\frac{2\,b\,e^{9\,c+9\,d\,x}}{3\,d}-\frac{8\,b\,e^{7\,c+7\,d\,x}}{3\,d}-\frac{8\,b\,e^{3\,c+3\,d\,x}}{3\,d}+\frac{4\,e^{5\,c+5\,d\,x}(}{3\,d}}{15\,e^{4\,c+4\,d\,x}-6\,e^{2\,c+2\,d\,x}-20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}-6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)/sinh(c + d*x)^7,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*a*(-d^2)^(1/2) + 8*b*(-d^2)^(1/2)))/(d*(80*a*b +
25*a^2 + 64*b^2)^(1/2)))*(80*a*b + 25*a^2 + 64*b^2)^(1/2))/(8*(-d^2)^(1/2))
 - ((2*b*exp(9*c + 9*d*x))/(3*d) - (8*b*exp(7*c + 7*d*x))/(3*d) - (8*b*exp(
3*c + 3*d*x))/(3*d) + (4*exp(5*c + 5*d*x)*(8*a + 3*b))/(3*d) + (2*b*exp(c +
 d*x))/(3*d))/(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*
x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) +
 (exp(c + d*x)*(5*a - 16*b))/(12*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) +
 1)) - (a*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp
(6*c + 6*d*x) - 1)) - (22*a*exp(c + d*x))/(3*d*(6*exp(4*c + 4*d*x) - 4*exp(
2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (exp(c + d*x)*
(5*a + 8*b))/(8*d*(exp(2*c + 2*d*x) - 1)) - (16*a*exp(c + d*x))/(3*d*(5*exp
(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d
*x) + exp(10*c + 10*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.196 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=120

$$\frac{2b(a+5b)\cosh^7(c+dx)}{7d}-\frac{2b(3a+5b)\cosh^5(c+dx)}{5d}+\frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d}-\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{b^2 c}{}$$

```
[Out] -(a+b)^2*cosh(d*x+c)/d+1/3*(a+b)*(a+5*b)*cosh(d*x+c)^3/d-2/5*b*(3*a+5*b)*co
sh(d*x+c)^5/d+2/7*b*(a+5*b)*cosh(d*x+c)^7/d-5/9*b^2*cosh(d*x+c)^9/d+1/11*b^
2*cosh(d*x+c)^11/d
```

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3215, 1153}

$$\frac{2b(a+5b)\cosh^7(c+dx)}{7d}-\frac{2b(3a+5b)\cosh^5(c+dx)}{5d}+\frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d}-\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{b^2 c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(((a + b)^2*Cosh[c + d*x])/d) + ((a + b)*(a + 5*b)*Cosh[c + d*x]^3)/(3*d)
- (2*b*(3*a + 5*b)*Cosh[c + d*x]^5)/(5*d) + (2*b*(a + 5*b)*Cosh[c + d*x]^7)
/(7*d) - (5*b^2*Cosh[c + d*x]^9)/(9*d) + (b^2*Cosh[c + d*x]^11)/(11*d)
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+b-2bx^2+bx^4\right)^2\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left((a+b)^2+(-a-5b)(a+b)x^2+2b(3a+5b)x^4-2b(a+\right.\right.}{d}\\ &= -\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d}-\frac{2b(3a+}{}\end{aligned}$$

Mathematica [A] time = 0.06, size = 207, normalized size = 1.72

$$-\frac{3a^2\cosh(c+dx)}{4d}+\frac{a^2\cosh(3(c+dx))}{12d}-\frac{35ab\cosh(c+dx)}{32d}+\frac{7ab\cosh(3(c+dx))}{32d}-\frac{7ab\cosh(5(c+dx))}{160d}+\frac{ab\cosh}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-3*a^2*Cosh[c + d*x])/(4*d) - (35*a*b*Cosh[c + d*x])/(32*d) - (231*b^2*Cos
h[c + d*x])/(512*d) + (a^2*Cosh[3*(c + d*x)])/(12*d) + (7*a*b*Cosh[3*(c + d
*x)])/(32*d) + (55*b^2*Cosh[3*(c + d*x)])/(512*d) - (7*a*b*Cosh[5*(c + d*x)
])/(160*d) - (33*b^2*Cosh[5*(c + d*x)])/(1024*d) + (a*b*Cosh[7*(c + d*x)])/
(224*d) + (55*b^2*Cosh[7*(c + d*x)])/(7168*d) - (11*b^2*Cosh[9*(c + d*x)])/
(9216*d) + (b^2*Cosh[11*(c + d*x)])/(11264*d)
```

fricas [B] time = 0.41, size = 404, normalized size = 3.37

$$\frac{315\,b^2\cosh\left(dx+c\right)^{11} + 3465\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{10} - 4235\,b^2\cosh\left(dx+c\right)^{9} + 3465\left(15\,b^2\cosh\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/3548160*(315*b^2*cosh(d*x + c)^11 + 3465*b^2*cosh(d*x + c)*sinh(d*x + c)^
10 - 4235*b^2*cosh(d*x + c)^9 + 3465*(15*b^2*cosh(d*x + c)^3 - 11*b^2*cosh(
d*x + c))*sinh(d*x + c)^8 + 495*(32*a*b + 55*b^2)*cosh(d*x + c)^7 + 1155*(1
26*b^2*cosh(d*x + c)^5 - 308*b^2*cosh(d*x + c)^3 + 3*(32*a*b + 55*b^2)*cosh
(d*x + c))*sinh(d*x + c)^6 - 693*(224*a*b + 165*b^2)*cosh(d*x + c)^5 + 3465
*(30*b^2*cosh(d*x + c)^7 - 154*b^2*cosh(d*x + c)^5 + 5*(32*a*b + 55*b^2)*co
sh(d*x + c)^3 - (224*a*b + 165*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 2310*(
128*a^2 + 336*a*b + 165*b^2)*cosh(d*x + c)^3 + 3465*(5*b^2*cosh(d*x + c)^9
- 44*b^2*cosh(d*x + c)^7 + 3*(32*a*b + 55*b^2)*cosh(d*x + c)^5 - 2*(224*a*b
 + 165*b^2)*cosh(d*x + c)^3 + 2*(128*a^2 + 336*a*b + 165*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^2 - 6930*(384*a^2 + 560*a*b + 231*b^2)*cosh(d*x + c))/d
```

giac [B] time = 0.21, size = 278, normalized size = 2.32

$$\frac{b^2e^{(11\,dx+11\,c)}}{22528\,d}-\frac{11\,b^2e^{(9\,dx+9\,c)}}{18432\,d}-\frac{11\,b^2e^{(-9\,dx-9\,c)}}{18432\,d}+\frac{b^2e^{(-11\,dx-11\,c)}}{22528\,d}+\frac{\left(32\,ab+55\,b^2\right)e^{(7\,dx+7\,c)}}{14336\,d}-\frac{\left(224\,ab+165\,b^2\right)e^{(5\,dx}}{10240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/22528*b^2*e^(11*d*x + 11*c)/d - 11/18432*b^2*e^(9*d*x + 9*c)/d - 11/18432
*b^2*e^(-9*d*x - 9*c)/d + 1/22528*b^2*e^(-11*d*x - 11*c)/d + 1/14336*(32*a*
b + 55*b^2)*e^(7*d*x + 7*c)/d - 1/10240*(224*a*b + 165*b^2)*e^(5*d*x + 5*c)
/d + 1/3072*(128*a^2 + 336*a*b + 165*b^2)*e^(3*d*x + 3*c)/d - 1/1024*(384*a
^2 + 560*a*b + 231*b^2)*e^(d*x + c)/d - 1/1024*(384*a^2 + 560*a*b + 231*b^2
)*e^(-d*x - c)/d + 1/3072*(128*a^2 + 336*a*b + 165*b^2)*e^(-3*d*x - 3*c)/d
- 1/10240*(224*a*b + 165*b^2)*e^(-5*d*x - 5*c)/d + 1/14336*(32*a*b + 55*b^2
)*e^(-7*d*x - 7*c)/d
```

maple [A] time = 0.04, size = 132, normalized size = 1.10

$$\frac{b^2\left(-\frac{256}{693}+\frac{\left(\sinh^{10}(dx+c)\right)}{11}-\frac{10\left(\sinh^{8}(dx+c)\right)}{99}+\frac{80\left(\sinh^{6}(dx+c)\right)}{693}-\frac{32\left(\sinh^{4}(dx+c)\right)}{231}+\frac{128\left(\sinh^{2}(dx+c)\right)}{693}\right)\cosh\left(dx+c\right)+2ab\left(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(b^2*(-256/693+1/11*sinh(d*x+c)^10-10/99*sinh(d*x+c)^8+80/693*sinh(d*x+
c)^6-32/231*sinh(d*x+c)^4+128/693*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(-16/35+
1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*(-
2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.33, size = 307, normalized size = 2.56

$$-\frac{1}{1419264}b^2\left(\frac{\left(847\,e^{(-2\,dx-2\,c)}-5445\,e^{(-4\,dx-4\,c)}+22869\,e^{(-6\,dx-6\,c)}-76230\,e^{(-8\,dx-8\,c)}+320166\,e^{(-10\,dx-10\,c)}-63\right)e}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/1419264*b^2*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6
*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(1
1*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(
-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*
x - 11*c))/d) - 1/2240*a*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1
225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-
3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a^2*(e^(
3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 0.36, size = 150, normalized size = 1.25

$$-\frac{-\frac{a^2\cosh(c+d\,x)^3}{3}+a^2\cosh\left(c+d\,x\right)-\frac{2\,a\,b\cosh(c+d\,x)^7}{7}+\frac{6\,a\,b\cosh(c+d\,x)^5}{5}-2\,a\,b\cosh\left(c+d\,x\right)^3+2\,a\,b\cosh\left(c+d\,x\right)-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^4)^2,x)
```

```
[Out] -(a^2*cosh(c + d*x) + b^2*cosh(c + d*x) - (a^2*cosh(c + d*x)^3)/3 - (5*b^2*
cosh(c + d*x)^3)/3 + 2*b^2*cosh(c + d*x)^5 - (10*b^2*cosh(c + d*x)^7)/7 + (
5*b^2*cosh(c + d*x)^9)/9 - (b^2*cosh(c + d*x)^11)/11 + 2*a*b*cosh(c + d*x)
- 2*a*b*cosh(c + d*x)^3 + (6*a*b*cosh(c + d*x)^5)/5 - (2*a*b*cosh(c + d*x)^
7)/7)/d
```

sympy [A] time = 32.00, size = 280, normalized size = 2.33

$$\begin{cases}\frac{a^2\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{2a^2\cosh^3(c+dx)}{3d}+\frac{2ab\sinh^6(c+dx)\cosh(c+dx)}{d}-\frac{4ab\sinh^4(c+dx)\cosh^3(c+dx)}{d}+\frac{16ab\sinh^2(c+dx)\cosh^5(c+}{5d} \\ x\left(a+b\sinh^4(c)\right)^2\sinh^3(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/
(3*d) + 2*a*b*sinh(c + d*x)**6*cosh(c + d*x)/d - 4*a*b*sinh(c + d*x)**4*cos
h(c + d*x)**3/d + 16*a*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 32*a*b*c
osh(c + d*x)**7/(35*d) + b**2*sinh(c + d*x)**10*cosh(c + d*x)/d - 10*b**2*s
inh(c + d*x)**8*cosh(c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**6*cosh(c +
d*x)**5/(3*d) - 32*b**2*sinh(c + d*x)**4*cosh(c + d*x)**7/(7*d) + 128*b**2*
sinh(c + d*x)**2*cosh(c + d*x)**9/(63*d) - 256*b**2*cosh(c + d*x)**11/(693*
d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c)**3, True))
```

3.197 $\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=161

$$\frac{\left(128a^2+352ab+193b^2\right)\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{1}{256}x\left(128a^2+160ab+63b^2\right)+\frac{b(160a+513b)\sinh(c+}{480d}$$

[Out] -1/256*(128*a^2+160*a*b+63*b^2)*x+1/256*(128*a^2+352*a*b+193*b^2)*cosh(d*x+c)*sinh(d*x+c)/d-1/384*b*(416*a+447*b)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/480*b*(160*a+513*b)*cosh(d*x+c)^5*sinh(d*x+c)/d-41/80*b^2*cosh(d*x+c)^7*sinh(d*x+c)/d+1/10*b^2*cosh(d*x+c)^9*sinh(d*x+c)/d

Rubi [A] time = 0.28, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3217, 1257, 1814, 1157, 385, 206}

$$\frac{\left(128a^2+352ab+193b^2\right)\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{1}{256}x\left(128a^2+160ab+63b^2\right)+\frac{b(160a+513b)\sinh(c+}{480d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((128*a^2 + 160*a*b + 63*b^2)*x)/256 + ((128*a^2 + 352*a*b + 193*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) - (b*(416*a + 447*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(384*d) + (b*(160*a + 513*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(480*d) - (41*b^2*Cosh[c + d*x]^7*Sinh[c + d*x])/(80*d) + (b^2*Cosh[c + d*x]^9*Sinh[c + d*x])/(10*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
```

```
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-2ax^2+(a+b)x^4\right)^2}{\left(1-x^2\right)^6} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh^9(c+dx)\sinh(c+dx)}{10d} + \frac{\text{Subst}\left(\int \frac{-b^2+10\left(a^2-b^2\right)x^2-10\left(3a^2+b^2\right)x}{\left(1-\right.}\right.}{}$$

$$= -\frac{41b^2\cosh^7(c+dx)\sinh(c+dx)}{80d} + \frac{b^2\cosh^9(c+dx)\sinh(c+dx)}{10d}$$

$$= \frac{b(160a+513b)\cosh^5(c+dx)\sinh(c+dx)}{480d} - \frac{41b^2\cosh^7(c+dx)\sin}{80d}$$

$$= -\frac{b(416a+447b)\cosh^3(c+dx)\sinh(c+dx)}{384d} + \frac{b(160a+513b)\cos}{4}$$

$$= \frac{\left(128a^2+352ab+193b^2\right)\cosh(c+dx)\sinh(c+dx)}{256d} - \frac{b(416a+44}{}$$

$$= -\frac{1}{256}\left(128a^2+160ab+63b^2\right)x + \frac{\left(128a^2+352ab+193b^2\right)\cosh(c}{256d}$$

Mathematica [A] time = 0.34, size = 139, normalized size = 0.86

$$-\frac{-60\left(128a^2+240ab+105b^2\right)\sinh(2(c+dx))+15360a^2c+15360a^2dx-320ab\sinh(6(c+dx))+360b(8a+5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/30720*(15360*a^2*c + 19200*a*b*c + 7560*b^2*c + 15360*a^2*d*x + 19200*a*
b*d*x + 7560*b^2*d*x - 60*(128*a^2 + 240*a*b + 105*b^2)*Sinh[2*(c + d*x)] +
```

```
360*b*(8*a + 5*b)*Sinh[4*(c + d*x)] - 320*a*b*Sinh[6*(c + d*x)] - 450*b^2*
Sinh[6*(c + d*x)] + 75*b^2*Sinh[8*(c + d*x)] - 6*b^2*Sinh[10*(c + d*x)])/d
```

fricas [B] time = 0.61, size = 305, normalized size = 1.89

$$\frac{15\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^9+30\left(6\,b^2\cosh\left(dx+c\right)^3-5\,b^2\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^7+3\left(126\,b^2\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/7680*(15*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 30*(6*b^2*cosh(d*x + c)^3 -
5*b^2*cosh(d*x + c))*sinh(d*x + c)^7 + 3*(126*b^2*cosh(d*x + c)^5 - 350*b^2
*cosh(d*x + c)^3 + 5*(32*a*b + 45*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*
(18*b^2*cosh(d*x + c)^7 - 105*b^2*cosh(d*x + c)^5 + 5*(32*a*b + 45*b^2)*cos
h(d*x + c)^3 - 36*(8*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 30*(128*
a^2 + 160*a*b + 63*b^2)*d*x + 15*(b^2*cosh(d*x + c)^9 - 10*b^2*cosh(d*x + c
)^7 + (32*a*b + 45*b^2)*cosh(d*x + c)^5 - 24*(8*a*b + 5*b^2)*cosh(d*x + c)^
3 + 2*(128*a^2 + 240*a*b + 105*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.21, size = 241, normalized size = 1.50

$$-\frac{1}{256}\left(128\,a^2+160\,ab+63\,b^2\right)x+\frac{b^2e^{(10\,dx+10\,c)}}{10240\,d}-\frac{5\,b^2e^{(8\,dx+8\,c)}}{4096\,d}+\frac{5\,b^2e^{(-8\,dx-8\,c)}}{4096\,d}-\frac{b^2e^{(-10\,dx-10\,c)}}{10240\,d}+\frac{\left(32\,ab+45\,b^2\right)e}{6144\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/256*(128*a^2 + 160*a*b + 63*b^2)*x + 1/10240*b^2*e^(10*d*x + 10*c)/d - 5
/4096*b^2*e^(8*d*x + 8*c)/d + 5/4096*b^2*e^(-8*d*x - 8*c)/d - 1/10240*b^2*e
^(-10*d*x - 10*c)/d + 1/6144*(32*a*b + 45*b^2)*e^(6*d*x + 6*c)/d - 3/512*(8
*a*b + 5*b^2)*e^(4*d*x + 4*c)/d + 1/1024*(128*a^2 + 240*a*b + 105*b^2)*e^(2
*d*x + 2*c)/d - 1/1024*(128*a^2 + 240*a*b + 105*b^2)*e^(-2*d*x - 2*c)/d + 3
/512*(8*a*b + 5*b^2)*e^(-4*d*x - 4*c)/d - 1/6144*(32*a*b + 45*b^2)*e^(-6*d*
x - 6*c)/d
```

maple [A] time = 0.05, size = 148, normalized size = 0.92

$$\frac{b^2\left(\left(\frac{\left(\sinh^9(dx+c)\right)}{10}-\frac{9\left(\sinh^7(dx+c)\right)}{80}+\frac{21\left(\sinh^5(dx+c)\right)}{160}-\frac{21\left(\sinh^3(dx+c)\right)}{128}+\frac{63\sinh(dx+c)}{256}\right)\cosh\left(dx+c\right)-\frac{63dx}{256}-\frac{63c}{256}\right)+2a}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(b^2*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/12
8*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+2*a*b*
((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d
*x-5/16*c)+a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [A] time = 0.35, size = 260, normalized size = 1.61

$$-\frac{1}{8}\,a^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{20480}\,b^2\left(\frac{\left(25\,e^{(-2\,dx-2\,c)}-150\,e^{(-4\,dx-4\,c)}+600\,e^{(-6\,dx-6\,c)}-2100\,e^{(-8\,dx-8\,c}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/20480*b^2*((25*
e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8
*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x -
 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) +
 2*e^(-10*d*x - 10*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x -
 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e
^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

mupad [B] time = 0.41, size = 149, normalized size = 0.93

$$960\,a^2\sinh\left(2\,c+2\,d\,x\right)+\frac{1575\,b^2\sinh(2\,c+2\,d\,x)}{2}-225\,b^2\sinh\left(4\,c+4\,d\,x\right)+\frac{225\,b^2\sinh(6\,c+6\,d\,x)}{4}-\frac{75\,b^2\sinh(8\,c+8\,d\,x)}{8}+\frac{3\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^4)^2,x)
```

```
[Out] (960*a^2*sinh(2*c + 2*d*x) + (1575*b^2*sinh(2*c + 2*d*x))/2 - 225*b^2*sinh(
4*c + 4*d*x) + (225*b^2*sinh(6*c + 6*d*x))/4 - (75*b^2*sinh(8*c + 8*d*x))/8
 + (3*b^2*sinh(10*c + 10*d*x))/4 + 1800*a*b*sinh(2*c + 2*d*x) - 360*a*b*sin
h(4*c + 4*d*x) + 40*a*b*sinh(6*c + 6*d*x) - 1920*a^2*d*x - 945*b^2*d*x - 24
00*a*b*d*x)/(3840*d)
```

sympy [A] time = 21.71, size = 484, normalized size = 3.01

$$\begin{cases} \frac{a^2x\sinh^2(c+dx)}{2}-\frac{a^2x\cosh^2(c+dx)}{2}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{5abx\sinh^6(c+dx)}{8}-\frac{15abx\sinh^4(c+dx)\cosh^2(c+dx)}{8}+\frac{15abx\sinh^2(c+ \\ x\left(a+b\sinh^4(c)\right)^2\sinh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*sinh
(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**
4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8
*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d*x)*c
osh(c + d*x)**5/(8*d) + 63*b**2*x*sinh(c + d*x)**10/256 - 315*b**2*x*sinh(c
 + d*x)**8*cosh(c + d*x)**2/256 + 315*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)
**4/128 - 315*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**6/128 + 315*b**2*x*sin
h(c + d*x)**2*cosh(c + d*x)**8/256 - 63*b**2*x*cosh(c + d*x)**10/256 + 193*
b**2*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) - 237*b**2*sinh(c + d*x)**7*cos
h(c + d*x)**3/(128*d) + 21*b**2*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) -
147*b**2*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 63*b**2*sinh(c + d*x)*
cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c)**2, T
rue))
```

3.198 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=92

$$\frac{2b(a+3b)\cosh^5(c+dx)}{5d}-\frac{4b(a+b)\cosh^3(c+dx)}{3d}+\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^9(c+dx)}{9d}-\frac{4b^2\cosh^7(c+a}{7d}$$

```
[Out] (a+b)^2*cosh(d*x+c)/d-4/3*b*(a+b)*cosh(d*x+c)^3/d+2/5*b*(a+3*b)*cosh(d*x+c)
^5/d-4/7*b^2*cosh(d*x+c)^7/d+1/9*b^2*cosh(d*x+c)^9/d
```

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3215, 1090}

$$\frac{2b(a+3b)\cosh^5(c+dx)}{5d}-\frac{4b(a+b)\cosh^3(c+dx)}{3d}+\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^9(c+dx)}{9d}-\frac{4b^2\cosh^7(c+a}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((a + b)^2*Cosh[c + d*x])/d - (4*b*(a + b)*Cosh[c + d*x]^3)/(3*d) + (2*b*(a
 + 3*b)*Cosh[c + d*x]^5)/(5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[
c + d*x]^9)/(9*d)
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int\left(a+b-2bx^2+bx^4\right)^2\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)-4ab\left(1+\frac{b}{a}\right)x^2+2ab\left(1+\frac{3b}{a}\right)x^4-4b\right.\right.}{d}\\ &= \frac{(a+b)^2\cosh(c+dx)}{d}-\frac{4b(a+b)\cosh^3(c+dx)}{3d}+\frac{2b(a+3b)\cos}{5d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 164, normalized size = 1.78

$$\frac{a^2\sinh(c)\sinh(dx)}{d}+\frac{a^2\cosh(c)\cosh(dx)}{d}+\frac{5ab\cosh(c+dx)}{4d}-\frac{5ab\cosh(3(c+dx))}{24d}+\frac{ab\cosh(5(c+dx))}{40d}+\frac{63b^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

[Out] (a^2*Cosh[c]*Cosh[d*x])/d + (5*a*b*Cosh[c + d*x])/(4*d) + (63*b^2*Cosh[c + d*x])/(128*d) - (5*a*b*Cosh[3*(c + d*x)])/(24*d) - (7*b^2*Cosh[3*(c + d*x)])/(64*d) + (a*b*Cosh[5*(c + d*x)])/(40*d) + (9*b^2*Cosh[5*(c + d*x)])/(320*d) - (9*b^2*Cosh[7*(c + d*x)])/(1792*d) + (b^2*Cosh[9*(c + d*x)])/(2304*d) + (a^2*Sinh[c]*Sinh[d*x])/d

fricas [B] time = 1.67, size = 279, normalized size = 3.03

$$\frac{35\,b^2\cosh\left(dx+c\right)^9+315\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8-405\,b^2\cosh\left(dx+c\right)^7+105\left(28\,b^2\cosh\left(dx+c\right)^3-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out] 1/80640*(35*b^2*cosh(d*x + c)^9 + 315*b^2*cosh(d*x + c)*sinh(d*x + c)^8 - 405*b^2*cosh(d*x + c)^7 + 105*(28*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c))*sinh(d*x + c)^6 + 252*(8*a*b + 9*b^2)*cosh(d*x + c)^5 + 315*(14*b^2*cosh(d*x + c)^5 - 45*b^2*cosh(d*x + c)^3 + 4*(8*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - 420*(40*a*b + 21*b^2)*cosh(d*x + c)^3 + 315*(4*b^2*cosh(d*x + c)^7 - 27*b^2*cosh(d*x + c)^5 + 8*(8*a*b + 9*b^2)*cosh(d*x + c)^3 - 4*(40*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 630*(128*a^2 + 160*a*b + 63*b^2)*cosh(d*x + c))/d

giac [B] time = 0.23, size = 220, normalized size = 2.39

$$\frac{b^2e^{(9\,dx+9\,c)}}{4608\,d}-\frac{9\,b^2e^{(7\,dx+7\,c)}}{3584\,d}-\frac{9\,b^2e^{(-7\,dx-7\,c)}}{3584\,d}+\frac{b^2e^{(-9\,dx-9\,c)}}{4608\,d}+\frac{\left(8\,ab+9\,b^2\right)e^{(5\,dx+5\,c)}}{640\,d}-\frac{\left(40\,ab+21\,b^2\right)e^{(3\,dx+3\,c)}}{384\,d}+\frac{\left(128\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out] 1/4608*b^2*e^(9*d*x + 9*c)/d - 9/3584*b^2*e^(7*d*x + 7*c)/d - 9/3584*b^2*e^(-7*d*x - 7*c)/d + 1/4608*b^2*e^(-9*d*x - 9*c)/d + 1/640*(8*a*b + 9*b^2)*e^(5*d*x + 5*c)/d - 1/384*(40*a*b + 21*b^2)*e^(3*d*x + 3*c)/d + 1/256*(128*a^2 + 160*a*b + 63*b^2)*e^(d*x + c)/d + 1/256*(128*a^2 + 160*a*b + 63*b^2)*e^(-d*x - c)/d - 1/384*(40*a*b + 21*b^2)*e^(-3*d*x - 3*c)/d + 1/640*(8*a*b + 9*b^2)*e^(-5*d*x - 5*c)/d

maple [A] time = 0.04, size = 100, normalized size = 1.09

$$\frac{b^2\left(\frac{128}{315}+\frac{\left(\sinh^8(dx+c)\right)}{9}-\frac{8\left(\sinh^6(dx+c)\right)}{63}+\frac{16\left(\sinh^4(dx+c)\right)}{105}-\frac{64\left(\sinh^2(dx+c)\right)}{315}\right)\cosh\left(dx+c\right)+2ab\left(\frac{8}{15}+\frac{\left(\sinh^4(dx+c)\right)}{5}-\frac{4\left(\sinh\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x)

[Out] 1/d*(b^2*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*cosh(d*x+c))

maxima [B] time = 0.33, size = 226, normalized size = 2.46

$$-\frac{1}{161280}\,b^2\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\frac{39690\,e^{(-dx-c)}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

```
[Out] -1/161280*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x
+ 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d +
 3*e^(-5*d*x - 5*c)/d) + a^2*cosh(d*x + c)/d
```

mupad [B] time = 0.88, size = 111, normalized size = 1.21

$$\frac{a^2\cosh(c+d\,x)+\frac{2\,a\,b\cosh(c+d\,x)^5}{5}-\frac{4\,a\,b\cosh(c+d\,x)^3}{3}+2\,a\,b\cosh(c+d\,x)+\frac{b^2\cosh(c+d\,x)^9}{9}-\frac{4\,b^2\cosh(c+d\,x)^7}{7}+\frac{6\,b^2\cos}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^4)^2,x)
```

```
[Out] (a^2*cosh(c + d*x) + b^2*cosh(c + d*x) - (4*b^2*cosh(c + d*x)^3)/3 + (6*b^2
*cosh(c + d*x)^5)/5 - (4*b^2*cosh(c + d*x)^7)/7 + (b^2*cosh(c + d*x)^9)/9 +
 2*a*b*cosh(c + d*x) - (4*a*b*cosh(c + d*x)^3)/3 + (2*a*b*cosh(c + d*x)^5)/
5)/d
```

sympy [A] time = 13.08, size = 204, normalized size = 2.22

$$\begin{cases} \frac{a^2\cosh(c+dx)}{d}+\frac{2ab\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{8ab\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{16ab\cosh^5(c+dx)}{15d}+\frac{b^2\sinh^8(c+dx)\cosh(c+dx)}{d}-\frac{8b^2}{} \\ x\left(a+b\sinh^4(c)\right)^2\sinh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*cosh(c + d*x)/d + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d -
8*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15
*d) + b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**2*sinh(c + d*x)**6*cosh(
c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b*
*2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**2*cosh(c + d*x)**9/(31
5*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c), True))
```

3.199 $\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=125

$$\frac{1}{128}x\left(128a^2+96ab+35b^2\right)+\frac{b(96a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{b(160a+93b)\sinh(c+dx)\cosh(c+dx)}{128d}$$

```
[Out] 1/128*(128*a^2+96*a*b+35*b^2)*x-1/128*b*(160*a+93*b)*cosh(d*x+c)*sinh(d*x+c
)/d+1/192*b*(96*a+163*b)*cosh(d*x+c)^3*sinh(d*x+c)/d-25/48*b^2*cosh(d*x+c)^
5*sinh(d*x+c)/d+1/8*b^2*cosh(d*x+c)^7*sinh(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3209, 1157, 1814, 385, 206}

$$\frac{1}{128}x\left(128a^2+96ab+35b^2\right)+\frac{b(96a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{b(160a+93b)\sinh(c+dx)\cosh(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((128*a^2 + 96*a*b + 35*b^2)*x)/128 - (b*(160*a + 93*b)*Cosh[c + d*x]*Sinh[
c + d*x])/(128*d) + (b*(96*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d
) - (25*b^2*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^2*Cosh[c + d*x]^7*Si
nh[c + d*x])/(8*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh^7(c+dx)\sinh(c+dx)}{8d} - \frac{\text{Subst}\left(\int \frac{-8a^2+b^2+8\left(3a^2+b^2\right)x^2-8(3a-b)(a+b)x^4+8(a+b}{\left(1-x^2\right)^4}\right.}{8d}$$

$$= -\frac{25b^2\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^2\cosh^7(c+dx)\sinh(c+dx)}{8d} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{b(96a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d} - \frac{25b^2\cosh^5(c+dx)\sinh(c+dx)}{48d}$$

$$= -\frac{b(160a+93b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b(96a+163b)\cosh^3(c+dx)\sinh}{192d}$$

$$= \frac{1}{128}\left(128a^2+96ab+35b^2\right)x - \frac{b(160a+93b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b(9}{}$$

Mathematica [A] time = 0.17, size = 92, normalized size = 0.74

$$\frac{24\left(128a^2+96ab+35b^2\right)(c+dx)-96b(16a+7b)\sinh(2(c+dx))+24b(8a+7b)\sinh(4(c+dx))-32b^2\sin}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (24*(128*a^2 + 96*a*b + 35*b^2)*(c + d*x) - 96*b*(16*a + 7*b)*Sinh[2*(c + d
*x)] + 24*b*(8*a + 7*b)*Sinh[4*(c + d*x)] - 32*b^2*Sinh[6*(c + d*x)] + 3*b^
2*Sinh[8*(c + d*x)])/(3072*d)
```

fricas [A] time = 0.99, size = 205, normalized size = 1.64

$$3\,b^2\cosh(dx+c)\sinh(dx+c)^7 + 3\left(7\,b^2\cosh(dx+c)^3 - 8\,b^2\cosh(dx+c)\right)\sinh(dx+c)^5 + \left(21\,b^2\cosh(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 - 8*b
^2*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 - 80*b^2*cosh(d
*x + c)^3 + 12*(8*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(128*a^2
+ 96*a*b + 35*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 - 8*b^2*cosh(d*x + c)^5 + 4
*(8*a*b + 7*b^2)*cosh(d*x + c)^3 - 8*(16*a*b + 7*b^2)*cosh(d*x + c))*sinh(d
*x + c))/d
```

giac [A] time = 0.14, size = 183, normalized size = 1.46

$$\frac{1}{128}\left(128\,a^2+96\,ab+35\,b^2\right)x+\frac{b^2e^{(8\,dx+8\,c)}}{2048\,d}-\frac{b^2e^{(6\,dx+6\,c)}}{192\,d}+\frac{b^2e^{(-6\,dx-6\,c)}}{192\,d}-\frac{b^2e^{(-8\,dx-8\,c)}}{2048\,d}+\frac{\left(8\,ab+7\,b^2\right)e^{(4\,dx+4\,c)}}{256\,d}-\frac{(16\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/128*(128*a^2 + 96*a*b + 35*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d - 1/192*
b^2*e^(6*d*x + 6*c)/d + 1/192*b^2*e^(-6*d*x - 6*c)/d - 1/2048*b^2*e^(-8*d*x
 - 8*c)/d + 1/256*(8*a*b + 7*b^2)*e^(4*d*x + 4*c)/d - 1/64*(16*a*b + 7*b^2)
*e^(2*d*x + 2*c)/d + 1/64*(16*a*b + 7*b^2)*e^(-2*d*x - 2*c)/d - 1/256*(8*a*
b + 7*b^2)*e^(-4*d*x - 4*c)/d
```

maple [A] time = 0.04, size = 111, normalized size = 0.89

$$\frac{b^2\left(\left(\frac{\left(\sinh^7(dx+c)\right)}{8}-\frac{7\left(\sinh^5(dx+c)\right)}{48}+\frac{35\left(\sinh^3(dx+c)\right)}{192}-\frac{35\sinh(dx+c)}{128}\right)\cosh\left(dx+c\right)+\frac{35dx}{128}+\frac{35c}{128}\right)+2ab\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4}-\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128
*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+2*a*b*((1/4*sinh(d*x+c)^3-3/
8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^2*(d*x+c))
```

maxima [A] time = 0.33, size = 183, normalized size = 1.46

$$\frac{1}{32}\,ab\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+a^2x-\frac{1}{6144}\,b^2\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + a^2*x - 1/6144*b^2*((32*e^(-2*d*x - 2*c) - 168
*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x
 + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c
) - 3*e^(-8*d*x - 8*c))/d)
```

mupad [B] time = 0.29, size = 108, normalized size = 0.86

$$\frac{21\,b^2\sinh\left(4\,c+4\,d\,x\right)-84\,b^2\sinh\left(2\,c+2\,d\,x\right)-4\,b^2\sinh\left(6\,c+6\,d\,x\right)+\frac{3\,b^2\sinh(8\,c+8\,d\,x)}{8}-192\,a\,b\sinh\left(2\,c+2\,d\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2,x)
```

```
[Out] (21*b^2*sinh(4*c + 4*d*x) - 84*b^2*sinh(2*c + 2*d*x) - 4*b^2*sinh(6*c + 6*d
*x) + (3*b^2*sinh(8*c + 8*d*x))/8 - 192*a*b*sinh(2*c + 2*d*x) + 24*a*b*sinh
(4*c + 4*d*x) + 384*a^2*d*x + 105*b^2*d*x + 288*a*b*d*x)/(384*d)
```

sympy [A] time = 8.56, size = 332, normalized size = 2.66

$$\begin{cases} a^2x+\frac{3abx\sinh^4(c+dx)}{4}-\frac{3abx\sinh^2(c+dx)\cosh^2(c+dx)}{2}+\frac{3abx\cosh^4(c+dx)}{4}+\frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3ab\sinh(c+dx)\cosh^3(c+d}{4d} \\ x\left(a+b\sinh^4(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*x + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh(c + d*x)**2*c
osh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c + d*x)**3*cos
h(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) + 35*b**2*x*s
inh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*
b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)**2*co
sh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c + d*x)*
*7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*
d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*sinh(c +
d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2, True))
```

3.200 $\int \operatorname{csch}(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx$

Optimal. Leaf size=92

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a+3b)\cosh^3(c+dx)}{3d}-\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b^2\cosh^7(c+dx)}{7d}-\frac{3b^2\cosh^5(c+}{5d}$$

```
[Out] -a^2*arctanh(cosh(d*x+c))/d-b*(2*a+b)*cosh(d*x+c)/d+1/3*b*(2*a+3*b)*cosh(d*
x+c)^3/d-3/5*b^2*cosh(d*x+c)^5/d+1/7*b^2*cosh(d*x+c)^7/d
```

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3215, 1153, 206}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a+3b)\cosh^3(c+dx)}{3d}-\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b^2\cosh^7(c+dx)}{7d}-\frac{3b^2\cosh^5(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) - (b*(2*a + b)*Cosh[c + d*x])/d + (b*(2*a
 + 3*b)*Cosh[c + d*x]^3)/(3*d) - (3*b^2*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[
c + d*x]^7)/(7*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{1-x^2}\,dx,x,\cosh(c+dx)\right)}{d}\\
&= -\frac{\operatorname{Subst}\left(\int \left(b(2a+b)-b(2a+3b)x^2+3b^2x^4-b^2x^6+\frac{a^2}{1-x^2}\right)dx,x,c\right.}{d}\\
&= -\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b(2a+3b)\cosh^3(c+dx)}{3d}-\frac{3b^2\cosh^5(c-}{5d}\\
&= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b(2a+3b)\text{co}}{3d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 146, normalized size = 1.59

$$\frac{a^2\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a^2\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{3ab\cosh(c+dx)}{2d}+\frac{ab\cosh(3(c+dx))}{6d}-\frac{35b^2\cosh(c+dx)}{64d}+$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-3*a*b*Cosh[c + d*x])/(2*d) - (35*b^2*Cosh[c + d*x])/(64*d) + (a*b*Cosh[3*
(c + d*x)])/(6*d) + (7*b^2*Cosh[3*(c + d*x)])/(64*d) - (7*b^2*Cosh[5*(c + d
*x)])/(320*d) + (b^2*Cosh[7*(c + d*x)])/(448*d) - (a^2*Log[Cosh[c/2 + (d*x)
/2]])/d + (a^2*Log[Sinh[c/2 + (d*x)/2]])/d
```

fricas [B] time = 0.85, size = 1575, normalized size = 17.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/13440*(15*b^2*cosh(d*x + c)^14 + 210*b^2*cosh(d*x + c)*sinh(d*x + c)^13 +
 15*b^2*sinh(d*x + c)^14 - 147*b^2*cosh(d*x + c)^12 + 21*(65*b^2*cosh(d*x +
 c)^2 - 7*b^2)*sinh(d*x + c)^12 + 84*(65*b^2*cosh(d*x + c)^3 - 21*b^2*cosh(
d*x + c))*sinh(d*x + c)^11 + 35*(32*a*b + 21*b^2)*cosh(d*x + c)^10 + 7*(214
5*b^2*cosh(d*x + c)^4 - 1386*b^2*cosh(d*x + c)^2 + 160*a*b + 105*b^2)*sinh(
d*x + c)^10 + 70*(429*b^2*cosh(d*x + c)^5 - 462*b^2*cosh(d*x + c)^3 + 5*(32
*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 105*(96*a*b + 35*b^2)*cosh(
d*x + c)^8 + 105*(429*b^2*cosh(d*x + c)^6 - 693*b^2*cosh(d*x + c)^4 + 15*(3
2*a*b + 21*b^2)*cosh(d*x + c)^2 - 96*a*b - 35*b^2)*sinh(d*x + c)^8 + 24*(21
45*b^2*cosh(d*x + c)^7 - 4851*b^2*cosh(d*x + c)^5 + 175*(32*a*b + 21*b^2)*c
osh(d*x + c)^3 - 35*(96*a*b + 35*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 105*
(96*a*b + 35*b^2)*cosh(d*x + c)^6 + 21*(2145*b^2*cosh(d*x + c)^8 - 6468*b^2
*cosh(d*x + c)^6 + 350*(32*a*b + 21*b^2)*cosh(d*x + c)^4 - 140*(96*a*b + 35
*b^2)*cosh(d*x + c)^2 - 480*a*b - 175*b^2)*sinh(d*x + c)^6 + 42*(715*b^2*co
sh(d*x + c)^9 - 2772*b^2*cosh(d*x + c)^7 + 210*(32*a*b + 21*b^2)*cosh(d*x +
 c)^5 - 140*(96*a*b + 35*b^2)*cosh(d*x + c)^3 - 15*(96*a*b + 35*b^2)*cosh(d
*x + c))*sinh(d*x + c)^5 + 35*(32*a*b + 21*b^2)*cosh(d*x + c)^4 + 35*(429*b
^2*cosh(d*x + c)^10 - 2079*b^2*cosh(d*x + c)^8 + 210*(32*a*b + 21*b^2)*cosh
(d*x + c)^6 - 210*(96*a*b + 35*b^2)*cosh(d*x + c)^4 - 45*(96*a*b + 35*b^2)*
cosh(d*x + c)^2 + 32*a*b + 21*b^2)*sinh(d*x + c)^4 - 147*b^2*cosh(d*x + c)^
2 + 140*(39*b^2*cosh(d*x + c)^11 - 231*b^2*cosh(d*x + c)^9 + 30*(32*a*b + 2
1*b^2)*cosh(d*x + c)^7 - 42*(96*a*b + 35*b^2)*cosh(d*x + c)^5 - 15*(96*a*b
 + 35*b^2)*cosh(d*x + c)^3 + (32*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^
3 + 21*(65*b^2*cosh(d*x + c)^12 - 462*b^2*cosh(d*x + c)^10 + 75*(32*a*b + 2
1*b^2)*cosh(d*x + c)^8 - 140*(96*a*b + 35*b^2)*cosh(d*x + c)^6 - 75*(96*a*b
 + 35*b^2)*cosh(d*x + c)^4 + 10*(32*a*b + 21*b^2)*cosh(d*x + c)^2 - 7*b^2)*
sinh(d*x + c)^2 + 15*b^2 - 13440*(a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)
^6*sinh(d*x + c) + 21*a^2*cosh(d*x + c)^5*sinh(d*x + c)^2 + 35*a^2*cosh(d*x
 + c)^4*sinh(d*x + c)^3 + 35*a^2*cosh(d*x + c)^3*sinh(d*x + c)^4 + 21*a^2*c
osh(d*x + c)^2*sinh(d*x + c)^5 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6 + a^2*
sinh(d*x + c)^7)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 13440*(a^2*cosh(d
*x + c)^7 + 7*a^2*cosh(d*x + c)^6*sinh(d*x + c) + 21*a^2*cosh(d*x + c)^5*si
nh(d*x + c)^2 + 35*a^2*cosh(d*x + c)^4*sinh(d*x + c)^3 + 35*a^2*cosh(d*x +
c)^3*sinh(d*x + c)^4 + 21*a^2*cosh(d*x + c)^2*sinh(d*x + c)^5 + 7*a^2*cosh(
d*x + c)*sinh(d*x + c)^6 + a^2*sinh(d*x + c)^7)*log(cosh(d*x + c) + sinh(d*
x + c) - 1) + 14*(15*b^2*cosh(d*x + c)^13 - 126*b^2*cosh(d*x + c)^11 + 25*(
32*a*b + 21*b^2)*cosh(d*x + c)^9 - 60*(96*a*b + 35*b^2)*cosh(d*x + c)^7 - 4
5*(96*a*b + 35*b^2)*cosh(d*x + c)^5 + 10*(32*a*b + 21*b^2)*cosh(d*x + c)^3
- 21*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x +
```

```
c)^6*sinh(d*x + c) + 21*d*cosh(d*x + c)^5*sinh(d*x + c)^2 + 35*d*cosh(d*x +
 c)^4*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)^3*sinh(d*x + c)^4 + 21*d*cosh(d*
x + c)^2*sinh(d*x + c)^5 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x +
 c)^7)
```

giac [B] time = 0.22, size = 196, normalized size = 2.13

$$\frac{15\,b^2 e^{(7\,dx+7\,c)} - 147\,b^2 e^{(5\,dx+5\,c)} + 1120\,abe^{(3\,dx+3\,c)} + 735\,b^2 e^{(3\,dx+3\,c)} - 10080\,abe^{(dx+c)} - 3675\,b^2 e^{(dx+c)} - 13440\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*b^2*e^(7*d*x + 7*c) - 147*b^2*e^(5*d*x + 5*c) + 1120*a*b*e^(3*d
*x + 3*c) + 735*b^2*e^(3*d*x + 3*c) - 10080*a*b*e^(d*x + c) - 3675*b^2*e^(d
*x + c) - 13440*a^2*log(e^(d*x + c) + 1) + 13440*a^2*log(abs(e^(d*x + c) -
1)) - (10080*a*b*e^(6*d*x + 6*c) + 3675*b^2*e^(6*d*x + 6*c) - 1120*a*b*e^(4
*d*x + 4*c) - 735*b^2*e^(4*d*x + 4*c) + 147*b^2*e^(2*d*x + 2*c) - 15*b^2)*e
^(-7*d*x - 7*c))/d
```

maple [A] time = 0.08, size = 82, normalized size = 0.89

$$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right) + 2ab\left(-\frac{2}{3} + \frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh(dx+c) + b^2\left(-\frac{16}{35} + \frac{\left(\sinh^6(dx+c)\right)}{7} - \frac{6\left(\sinh^4(dx+c)\right)}{35} + \frac{8\left(\sinh^2(dx+c\right.}{35}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+
b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d
*x+c))
```

maxima [B] time = 0.34, size = 177, normalized size = 1.92

$$-\frac{1}{4480}\,b^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)} - 245\,e^{(-4\,dx-4\,c)} + 1225\,e^{(-6\,dx-6\,c)} - 5\right)e^{(7\,dx+7\,c)}}{d} + \frac{1225\,e^{(-dx-c)} - 245\,e^{(-3\,dx-3\,c)} + 49\,e^{(-5}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/12*a*b*(e^(3*d*x + 3*c)/d
- 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a^2*log(tanh(1
/2*d*x + 1/2*c))/d
```

mupad [B] time = 0.34, size = 198, normalized size = 2.15

$$\frac{b^2\,e^{-7\,c-7\,d\,x}}{896\,d} - \frac{2\,\mathrm{atan}\left(\frac{a^2\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\sqrt{a^4}}{\sqrt{-d^2}} - \frac{e^{-c-d\,x}\,\left(35\,b^2 + 96\,a\,b\right)}{128\,d} - \frac{7\,b^2\,e^{-5\,c-5\,d\,x}}{640\,d} - \frac{7\,b^2\,e^{5\,c+5\,d\,x}}{640\,d} - \frac{e^{c+d\,x}\,\left(35\,b^2 + 96\right.}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x),x)
```

```
[Out] (b^2*exp(- 7*c - 7*d*x))/(896*d) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2
))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^2)^(1/2) - (exp(- c - d*x)*(96*a*b + 3
```

```
5*b^2))/(128*d) - (7*b^2*exp(- 5*c - 5*d*x))/(640*d) - (7*b^2*exp(5*c + 5*d
*x))/(640*d) - (exp(c + d*x)*(96*a*b + 35*b^2))/(128*d) + (b^2*exp(7*c + 7*
d*x))/(896*d) + (b*exp(- 3*c - 3*d*x)*(32*a + 21*b))/(384*d) + (b*exp(3*c +
 3*d*x)*(32*a + 21*b))/(384*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.201 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=103

$$-\frac{a^2\coth(c+dx)}{d}+\frac{b(16a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}bx(16a+5b)+\frac{b^2\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b^2}{}$$

```
[Out] -1/16*b*(16*a+5*b)*x-a^2*coth(d*x+c)/d+1/16*b*(16*a+11*b)*cosh(d*x+c)*sinh(
d*x+c)/d-13/24*b^2*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b^2*cosh(d*x+c)^5*sinh(d
*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3217, 1259, 1805, 453, 206}

$$-\frac{a^2\coth(c+dx)}{d}+\frac{b(16a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}bx(16a+5b)+\frac{b^2\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(b*(16*a + 5*b)*x)/16 - (a^2*Coth[c + d*x])/d + (b*(16*a + 11*b)*Cosh[c +
d*x]*Sinh[c + d*x])/(16*d) - (13*b^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d)
+ (b^2*Cosh[c + d*x]^5*Sinh[c + d*x])/(6*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a-2ax^2+(a+b)x^4\right)^2}{x^2\left(1-x^2\right)^4}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b^2\cosh^5(c+dx)\sinh(c+dx)}{6d}-\frac{\operatorname{Subst}\left(\int\frac{-6a^2+\left(18a^2+b^2\right)x^2-6(3a-b)}{x^2\left(1-x^2\right)}\right.}{}$$

$$=-\frac{13b^2\cosh^3(c+dx)\sinh(c+dx)}{24d}+\frac{b^2\cosh^5(c+dx)\sinh(c+d}{6d}$$

$$=\frac{b(16a+11b)\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{13b^2\cosh^3(c+dx)\sin}{24d}$$

$$=-\frac{a^2\coth(c+dx)}{d}+\frac{b(16a+11b)\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{13}{}$$

$$=-\frac{1}{16}b(16a+5b)x-\frac{a^2\coth(c+dx)}{d}+\frac{b(16a+11b)\cosh(c+dx)}{16d}$$

Mathematica [A] time = 0.31, size = 77, normalized size = 0.75

$$\frac{b((96a+45b)\sinh(2(c+dx))-192ac-192adx-9b\sinh(4(c+dx))+b\sinh(6(c+dx))-60bc-60bdx)-1}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-192*a^2*Coth[c + d*x] + b*(-192*a*c - 60*b*c - 192*a*d*x - 60*b*d*x + (96
*a + 45*b)*Sinh[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])
)/(192*d)
```

fricas [B] time = 0.99, size = 217, normalized size = 2.11

$$\frac{b^2\cosh(dx+c)^7+7b^2\cosh(dx+c)\sinh(dx+c)^6-10b^2\cosh(dx+c)^5+5\left(7b^2\cosh(dx+c)^3-10b^2\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(b^2*cosh(d*x + c)^7 + 7*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 10*b^2*c
osh(d*x + c)^5 + 5*(7*b^2*cosh(d*x + c)^3 - 10*b^2*cosh(d*x + c))*sinh(d*x
+ c)^4 + 6*(16*a*b + 9*b^2)*cosh(d*x + c)^3 + (21*b^2*cosh(d*x + c)^5 - 100
*b^2*cosh(d*x + c)^3 + 18*(16*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 -
```

```
 3*(128*a^2 + 32*a*b + 15*b^2)*cosh(d*x + c) - 24*((16*a*b + 5*b^2)*d*x - 1
6*a^2)*sinh(d*x + c))/(d*sinh(d*x + c))
```

giac [A] time = 0.24, size = 179, normalized size = 1.74

$$\frac{b^2e^{(6\,dx+6\,c)} - 9\,b^2e^{(4\,dx+4\,c)} + 96\,abe^{(2\,dx+2\,c)} + 45\,b^2e^{(2\,dx+2\,c)} - 24\left(16\,ab + 5\,b^2\right)(dx+c) + \left(352\,abe^{(6\,dx+6\,c)} + 110\,b^2\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/384*(b^2*e^(6*d*x + 6*c) - 9*b^2*e^(4*d*x + 4*c) + 96*a*b*e^(2*d*x + 2*c)
 + 45*b^2*e^(2*d*x + 2*c) - 24*(16*a*b + 5*b^2)*(d*x + c) + (352*a*b*e^(6*d
*x + 6*c) + 110*b^2*e^(6*d*x + 6*c) - 96*a*b*e^(4*d*x + 4*c) - 45*b^2*e^(4*
d*x + 4*c) + 9*b^2*e^(2*d*x + 2*c) - b^2)*e^(-6*d*x - 6*c) - 768*a^2/(e^(2*
d*x + 2*c) - 1))/d
```

maple [A] time = 0.08, size = 91, normalized size = 0.88

$$\frac{-a^2\coth\left(dx+c\right) + 2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + b^2\left(\left(\frac{\left(\sinh^5(dx+c)\right)}{6} - \frac{5\left(\sinh^3(dx+c)\right)}{24} + \frac{5\sinh(dx+c)}{16}\right)\cosh\left(dx+\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^2
*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*
d*x-5/16*c))
```

maxima [A] time = 0.36, size = 146, normalized size = 1.42

$$-\frac{1}{4}\,ab\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) - \frac{1}{384}\,b^2\left(\frac{\left(9\,e^{(-2\,dx-2\,c)} - 45\,e^{(-4\,dx-4\,c)} - 1\right)e^{(6\,dx+6\,c)}}{d} + \frac{120\,(dx+c)}{d} + \frac{45\,e^{(-2\,dx}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b^2*((9*e^(
-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/
d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 2*a^
2/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.23, size = 148, normalized size = 1.44

$$\frac{3\,b^2\,\mathrm{e}^{-4\,c-4\,d\,x}}{128\,d} - \frac{2\,a^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} - \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,\left(15\,b^2+32\,a\,b\right)}{128\,d} - x\,\left(\frac{5\,b^2}{16}+a\,b\right) - \frac{3\,b^2\,\mathrm{e}^{4\,c+4\,d\,x}}{128\,d} - \frac{b^2\,\mathrm{e}^{-6\,c-6\,d\,x}}{384\,d} + \frac{b^2\,\mathrm{e}^{6\,c+6\,d}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^2,x)
```

```
[Out] (3*b^2*exp(- 4*c - 4*d*x))/(128*d) - (2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) - (
exp(- 2*c - 2*d*x)*(32*a*b + 15*b^2))/(128*d) - x*(a*b + (5*b^2)/16) - (3*b
^2*exp(4*c + 4*d*x))/(128*d) - (b^2*exp(- 6*c - 6*d*x))/(384*d) + (b^2*exp(
6*c + 6*d*x))/(384*d) + (b*exp(2*c + 2*d*x)*(32*a + 15*b))/(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.202 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=92

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{2b^2\cosh^3(c+dx)}{3d}$$

```
[Out] 1/2*a^2*arctanh(cosh(d*x+c))/d+b*(2*a+b)*cosh(d*x+c)/d-2/3*b^2*cosh(d*x+c)^
3/d+1/5*b^2*cosh(d*x+c)^5/d-1/2*a^2*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3215, 1157, 1810, 206}

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{2b^2\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(2*a + b)*Cosh[c + d*x])/d - (2*b^2
*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) - (a^2*Coth[c + d*x]*
Csch[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{\left(1-x^2\right)^2}dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-a^2-4ab-2b^2+2b(2a+3b)x^2-}{1-x^2}\right.}{2}$$
$$= -\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int\left(-2b(2a+b)+4b^2x^2\right.\right.}{}$$
$$= \frac{b(2a+b)\cosh(c+dx)}{d} - \frac{2b^2\cosh^3(c+dx)}{3d} + \frac{b^2\cosh^5(c+dx)}{5d}$$
$$= \frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b(2a+b)\cosh(c+dx)}{d} - \frac{2b^2\cosh^3(}{3d}$$

Mathematica [A] time = 0.05, size = 144, normalized size = 1.57

$$-\frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{2ab\sinh(c)\sinh(dx)}{d} + \frac{2ab\cosh(c)\operatorname{co}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (2*a*b*Cosh[c]*Cosh[d*x])/d + (5*b^2*Cosh[c + d*x])/(8*d) - (5*b^2*Cosh[3*(
c + d*x)])/(48*d) + (b^2*Cosh[5*(c + d*x)])/(80*d) - (a^2*Csch[(c + d*x)/2]
^2)/(8*d) - (a^2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/
(8*d) + (2*a*b*Sinh[c]*Sinh[d*x])/d
```

fricas [B] time = 3.95, size = 2272, normalized size = 24.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^2*cosh(d*x + c)^14 + 42*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + 3*b
^2*sinh(d*x + c)^14 - 31*b^2*cosh(d*x + c)^12 + (273*b^2*cosh(d*x + c)^2 -
31*b^2)*sinh(d*x + c)^12 + 12*(91*b^2*cosh(d*x + c)^3 - 31*b^2*cosh(d*x + c
))*sinh(d*x + c)^11 + (480*a*b + 203*b^2)*cosh(d*x + c)^10 + (3003*b^2*cosh
(d*x + c)^4 - 2046*b^2*cosh(d*x + c)^2 + 480*a*b + 203*b^2)*sinh(d*x + c)^1
0 + 2*(3003*b^2*cosh(d*x + c)^5 - 3410*b^2*cosh(d*x + c)^3 + 5*(480*a*b + 2
03*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(
d*x + c)^8 + (9009*b^2*cosh(d*x + c)^6 - 15345*b^2*cosh(d*x + c)^4 + 45*(48
0*a*b + 203*b^2)*cosh(d*x + c)^2 - 480*a^2 - 480*a*b - 175*b^2)*sinh(d*x +
c)^8 + 8*(1287*b^2*cosh(d*x + c)^7 - 3069*b^2*cosh(d*x + c)^5 + 15*(480*a*b
 + 203*b^2)*cosh(d*x + c)^3 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c))*s
inh(d*x + c)^7 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^6 + (9009*b^2*c
osh(d*x + c)^8 - 28644*b^2*cosh(d*x + c)^6 + 210*(480*a*b + 203*b^2)*cosh(d
*x + c)^4 - 140*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^2 - 480*a^2 - 480*
a*b - 175*b^2)*sinh(d*x + c)^6 + 2*(3003*b^2*cosh(d*x + c)^9 - 12276*b^2*co
sh(d*x + c)^7 + 126*(480*a*b + 203*b^2)*cosh(d*x + c)^5 - 140*(96*a^2 + 96*
a*b + 35*b^2)*cosh(d*x + c)^3 - 15*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^5 + (480*a*b + 203*b^2)*cosh(d*x + c)^4 + (3003*b^2*cosh(d*
x + c)^10 - 15345*b^2*cosh(d*x + c)^8 + 210*(480*a*b + 203*b^2)*cosh(d*x +
c)^6 - 350*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^4 - 75*(96*a^2 + 96*a*b
```

```
 + 35*b^2)*cosh(d*x + c)^2 + 480*a*b + 203*b^2)*sinh(d*x + c)^4 - 31*b^2*co
sh(d*x + c)^2 + 4*(273*b^2*cosh(d*x + c)^11 - 1705*b^2*cosh(d*x + c)^9 + 30
*(480*a*b + 203*b^2)*cosh(d*x + c)^7 - 70*(96*a^2 + 96*a*b + 35*b^2)*cosh(d
*x + c)^5 - 25*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^3 + (480*a*b + 203*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (273*b^2*cosh(d*x + c)^12 - 2046*b^2*
cosh(d*x + c)^10 + 45*(480*a*b + 203*b^2)*cosh(d*x + c)^8 - 140*(96*a^2 + 9
6*a*b + 35*b^2)*cosh(d*x + c)^6 - 75*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x +
c)^4 + 6*(480*a*b + 203*b^2)*cosh(d*x + c)^2 - 31*b^2)*sinh(d*x + c)^2 + 3*
b^2 + 240*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*
sinh(d*x + c)^9 - 2*a^2*cosh(d*x + c)^7 + 2*(18*a^2*cosh(d*x + c)^2 - a^2)*
sinh(d*x + c)^7 + a^2*cosh(d*x + c)^5 + 14*(6*a^2*cosh(d*x + c)^3 - a^2*cos
h(d*x + c))*sinh(d*x + c)^6 + (126*a^2*cosh(d*x + c)^4 - 42*a^2*cosh(d*x +
c)^2 + a^2)*sinh(d*x + c)^5 + (126*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x +
c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(42*a^2*cosh(d*x + c)^6 - 3
5*a^2*cosh(d*x + c)^4 + 5*a^2*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*a^2*
cosh(d*x + c)^7 - 21*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3)*sinh(d*x
+ c)^2 + (9*a^2*cosh(d*x + c)^8 - 14*a^2*cosh(d*x + c)^6 + 5*a^2*cosh(d*x +
 c)^4)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 240*(a^2*cos
h(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 -
2*a^2*cosh(d*x + c)^7 + 2*(18*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^7 +
a^2*cosh(d*x + c)^5 + 14*(6*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d
*x + c)^6 + (126*a^2*cosh(d*x + c)^4 - 42*a^2*cosh(d*x + c)^2 + a^2)*sinh(d
*x + c)^5 + (126*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x + c)^3 + 5*a^2*cosh(
d*x + c))*sinh(d*x + c)^4 + 2*(42*a^2*cosh(d*x + c)^6 - 35*a^2*cosh(d*x + c
)^4 + 5*a^2*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*a^2*cosh(d*x + c)^7 -
21*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (9*a^2*co
sh(d*x + c)^8 - 14*a^2*cosh(d*x + c)^6 + 5*a^2*cosh(d*x + c)^4)*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(21*b^2*cosh(d*x + c)^13 - 1
86*b^2*cosh(d*x + c)^11 + 5*(480*a*b + 203*b^2)*cosh(d*x + c)^9 - 20*(96*a^
2 + 96*a*b + 35*b^2)*cosh(d*x + c)^7 - 15*(96*a^2 + 96*a*b + 35*b^2)*cosh(d
*x + c)^5 + 2*(480*a*b + 203*b^2)*cosh(d*x + c)^3 - 31*b^2*cosh(d*x + c))*s
inh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*si
nh(d*x + c)^9 - 2*d*cosh(d*x + c)^7 + 2*(18*d*cosh(d*x + c)^2 - d)*sinh(d*x
 + c)^7 + 14*(6*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^6 + d*co
sh(d*x + c)^5 + (126*d*cosh(d*x + c)^4 - 42*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^5 + (126*d*cosh(d*x + c)^5 - 70*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c)
)*sinh(d*x + c)^4 + 2*(42*d*cosh(d*x + c)^6 - 35*d*cosh(d*x + c)^4 + 5*d*co
sh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*d*cosh(d*x + c)^7 - 21*d*cosh(d*x +
c)^5 + 5*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (9*d*cosh(d*x + c)^8 - 14*d*c
osh(d*x + c)^6 + 5*d*cosh(d*x + c)^4)*sinh(d*x + c))
```

giac [B] time = 0.23, size = 182, normalized size = 1.98

$$\frac{3\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5-40\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+480\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)+240\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+120\,a^2}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^2*(e^(d*x + c) + e^(-d*x - c))^5 - 40*b^2*(e^(d*x + c) + e^(-d*x
 - c))^3 + 480*a*b*(e^(d*x + c) + e^(-d*x - c)) + 240*b^2*(e^(d*x + c) + e^
(-d*x - c)) + 120*a^2*log(e^(d*x + c) + e^(-d*x - c) + 2) - 120*a^2*log(e^(
d*x + c) + e^(-d*x - c) - 2) - 480*a^2*(e^(d*x + c) + e^(-d*x - c))/((e^(d*
x + c) + e^(-d*x - c))^2 - 4))/d
```

maple [A] time = 0.10, size = 74, normalized size = 0.80

$$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\cosh(dx+c)+b^2\left(\frac{8}{15}+\frac{\left(\sinh^4(dx+c)\right)}{5}-\frac{4\left(\sinh^2(dx+c)\right)}{15}\right)\cosh(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*cosh(d*x+
c)+b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.34, size = 204, normalized size = 2.22

$$\frac{1}{480}b^2\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)+ab\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-d}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/480*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a*b*(
e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^
(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2
*c) - e^(-4*d*x - 4*c) - 1)))
```

mupad [B] time = 0.83, size = 214, normalized size = 2.33

$$\frac{\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}-\frac{5\,b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{96\,d}-\frac{5\,b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{96\,d}+\frac{b^2\,\mathrm{e}^{-5\,c-5\,d\,x}}{160\,d}+\frac{b^2\,\mathrm{e}^{5\,c+5\,d\,x}}{160\,d}+\frac{b\,\mathrm{e}^{-c-d\,x}\,\left(16\,a+5\,b\right)}{16\,d}-\frac{a^2}{d\,\left(\mathrm{e}^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^3,x)
```

```
[Out] (atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a^4)^(1/2))/(-d^
2)^(1/2) - (5*b^2*exp(- 3*c - 3*d*x))/(96*d) - (5*b^2*exp(3*c + 3*d*x))/(96
*d) + (b^2*exp(- 5*c - 5*d*x))/(160*d) + (b^2*exp(5*c + 5*d*x))/(160*d) + (
b*exp(- c - d*x)*(16*a + 5*b))/(16*d) - (a^2*exp(c + d*x))/(d*(exp(2*c + 2*
d*x) - 1)) + (b*exp(c + d*x)*(16*a + 5*b))/(16*d) - (2*a^2*exp(c + d*x))/(d
*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.203 $\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right)^2 \, dx$

Optimal. Leaf size=91

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}+\frac{1}{8}bx(16a+3b)+\frac{b^2\sinh(c+dx)\cosh^3(c+dx)}{4d}-\frac{5b^2\sinh(c+dx)\cosh(c+dx)}{8d}$$

```
[Out] 1/8*b*(16*a+3*b)*x+a^2*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d-5/8*b^2*cosh(d
*x+c)*sinh(d*x+c)/d+1/4*b^2*cosh(d*x+c)^3*sinh(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3217, 1259, 1805, 1261, 207}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}+\frac{1}{8}bx(16a+3b)+\frac{b^2\sinh(c+dx)\cosh^3(c+dx)}{4d}-\frac{5b^2\sinh(c+dx)\cosh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (b*(16*a + 3*b)*x)/8 + (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
- (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]^3*Sinh[c +
 d*x])/(4*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{x^4\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{4a^2-12a^2x^2+\left(12a^2+8ab-b^2\right.}{x^4\left(1-x^2\right)^2}\right.}{4}$$

$$= -\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^2\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

$$= -\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^2\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

$$= \frac{a^2\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} - \frac{5b^2\cosh(c+dx)\sinh(c+}{8d}$$

$$= \frac{1}{8}b(16a+3b)x + \frac{a^2\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} - 5b^2\cosh$$

Mathematica [A] time = 0.33, size = 68, normalized size = 0.75

$$\frac{3b(64adx-8b\sinh(2(c+dx))+b\sinh(4(c+dx))+12bc+12bdx)-32a^2\coth(c+dx)\left(\operatorname{csch}^2(c+dx)-2\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-32*a^2*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + 3*b*(12*b*c + 64*a*d*x + 12
*b*d*x - 8*b*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]))/(96*d)
```

fricas [B] time = 2.06, size = 300, normalized size = 3.30

$$3\,b^2\cosh(dx+c)^7 + 21\,b^2\cosh(dx+c)\sinh(dx+c)^6 - 33\,b^2\cosh(dx+c)^5 + 15\left(7\,b^2\cosh(dx+c)^3 - 11\,b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^2*cosh(d*x + c)^7 + 21*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 33*b^
2*cosh(d*x + c)^5 + 15*(7*b^2*cosh(d*x + c)^3 - 11*b^2*cosh(d*x + c))*sinh(
d*x + c)^4 + (128*a^2 + 81*b^2)*cosh(d*x + c)^3 + 8*(3*(16*a*b + 3*b^2)*d*x
 - 16*a^2)*sinh(d*x + c)^3 + 3*(21*b^2*cosh(d*x + c)^5 - 110*b^2*cosh(d*x +
 c)^3 + (128*a^2 + 81*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(128*a^2 + 17
*b^2)*cosh(d*x + c) - 24*(3*(16*a*b + 3*b^2)*d*x - (3*(16*a*b + 3*b^2)*d*x
- 16*a^2)*cosh(d*x + c)^2 - 16*a^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(
d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

giac [A] time = 0.24, size = 142, normalized size = 1.56

$$\frac{3\,b^2e^{(4\,dx+4\,c)} - 24\,b^2e^{(2\,dx+2\,c)} + 24\left(16\,ab + 3\,b^2\right)(dx + c) - 3\left(96\,abe^{(4\,dx+4\,c)} + 18\,b^2e^{(4\,dx+4\,c)} - 8\,b^2e^{(2\,dx+2\,c)} + b^2\right)e}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*b^2*e^(4*d*x + 4*c) - 24*b^2*e^(2*d*x + 2*c) + 24*(16*a*b + 3*b^2)
*(d*x + c) - 3*(96*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) - 8*b^2*e^(
2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c) - 256*(3*a^2*e^(2*d*x + 2*c) - a^2)/(e
^(2*d*x + 2*c) - 1)^3)/d
```

maple [A] time = 0.09, size = 75, normalized size = 0.82

$$\frac{a^2\left(\frac{2}{3} - \frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth\left(dx + c\right) + 2ab\left(dx + c\right) + b^2\left(\left(\frac{\left(\sinh^3(dx+c)\right)}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh\left(dx + c\right) + \frac{3dx}{8} + \frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(d*x+c)+b^2*((1/4*sinh(d
*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.34, size = 165, normalized size = 1.81

$$\frac{1}{64}\,b^2\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) + 2\,abx + \frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)} - 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,d}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 2*a*b*x + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^
(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-
2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 0.17, size = 164, normalized size = 1.80

$$\frac{b\,x\,\left(16\,a + 3\,b\right)}{8} - \frac{4\,a^2}{3\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x} - 2\,\mathrm{e}^{2\,c+2\,d\,x} + 1\right)} + \frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d} - \frac{b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d} - \frac{b^2\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d} + \frac{b^2\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d} - \frac{}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^4,x)
```

```
[Out] (b*x*(16*a + 3*b))/8 - (4*a^2)/(3*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
+ 1)) + (b^2*exp(- 2*c - 2*d*x))/(8*d) - (b^2*exp(2*c + 2*d*x))/(8*d) - (b^
2*exp(- 4*c - 4*d*x))/(64*d) + (b^2*exp(4*c + 4*d*x))/(64*d) - (8*a^2*exp(2
*c + 2*d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*
x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.204 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=101

$$-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{a(3a+16b)\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{b^2\cosh^3(c}{3d}$$

```
[Out] -1/8*a*(3*a+16*b)*arctanh(cosh(d*x+c))/d-b^2*cosh(d*x+c)/d+1/3*b^2*cosh(d*x
+c)^3/d+3/8*a^2*coth(d*x+c)*csch(d*x+c)/d-1/4*a^2*coth(d*x+c)*csch(d*x+c)^3
/d
```

Rubi [A] time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 1153, 206}

$$-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{a(3a+16b)\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{b^2\cosh^3(c}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(a*(3*a + 16*b)*ArcTanh[Cosh[c + d*x]])/(8*d) - (b^2*Cosh[c + d*x])/d + (b
^2*Cosh[c + d*x]^3)/(3*d) + (3*a^2*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (a^
2*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{\left(1-x^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{-(a+2b)(3a+2b)+4b(2a+3b)x^2-}{\left(1-x^2\right)^2}\right.}{4c}$$

$$= \frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} - \frac{\text{Su}}{}$$

$$= \frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} - \frac{\text{Su}}{}$$

$$= -\frac{b^2\cosh(c+dx)}{d} + \frac{b^2\cosh^3(c+dx)}{3d} + \frac{3a^2\coth(c+dx)\operatorname{csch}(c+d}{8d}$$

$$= -\frac{a(3a+16b)\tanh^{-1}(\cosh(c+dx))}{8d} - \frac{b^2\cosh(c+dx)}{d} + \frac{b^2\cosh^3(c}{3d}$$

Mathematica [A] time = 0.04, size = 186, normalized size = 1.84

$$-\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a^2\log\left(\tanh\left(\frac{1}{2}(c+\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c + d*x)])/(12*d) + (3*a^2*Csch
[(c + d*x)/2]^2)/(32*d) - (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (2*a*b*Log[Cos
h[c/2 + (d*x)/2]])/d + (2*a*b*Log[Sinh[c/2 + (d*x)/2]])/d + (3*a^2*Log[Tanh
[(c + d*x)/2]])/(8*d) + (3*a^2*Sech[(c + d*x)/2]^2)/(32*d) + (a^2*Sech[(c +
 d*x)/2]^4)/(64*d)
```

fricas [B] time = 1.07, size = 3356, normalized size = 33.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^14 + 14*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + b^2*si
nh(d*x + c)^14 - 13*b^2*cosh(d*x + c)^12 + 13*(7*b^2*cosh(d*x + c)^2 - b^2)
*sinh(d*x + c)^12 + 52*(7*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d
*x + c)^11 + 3*(6*a^2 + 11*b^2)*cosh(d*x + c)^10 + (1001*b^2*cosh(d*x + c)^
4 - 858*b^2*cosh(d*x + c)^2 + 18*a^2 + 33*b^2)*sinh(d*x + c)^10 + 2*(1001*b
^2*cosh(d*x + c)^5 - 1430*b^2*cosh(d*x + c)^3 + 15*(6*a^2 + 11*b^2)*cosh(d*
x + c))*sinh(d*x + c)^9 - 3*(22*a^2 + 7*b^2)*cosh(d*x + c)^8 + 3*(1001*b^2*
```

cosh(d*x + c)^6 - 2145*b^2*cosh(d*x + c)^4 + 45*(6*a^2 + 11*b^2)*cosh(d*x + c)^2 - 22*a^2 - 7*b^2)*sinh(d*x + c)^8 + 24*(143*b^2*cosh(d*x + c)^7 - 429*b^2*cosh(d*x + c)^5 + 15*(6*a^2 + 11*b^2)*cosh(d*x + c)^3 - (22*a^2 + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 3*(22*a^2 + 7*b^2)*cosh(d*x + c)^6 + 3*(1001*b^2*cosh(d*x + c)^8 - 4004*b^2*cosh(d*x + c)^6 + 210*(6*a^2 + 11*b^2)*cosh(d*x + c)^4 - 28*(22*a^2 + 7*b^2)*cosh(d*x + c)^2 - 22*a^2 - 7*b^2)*sinh(d*x + c)^6 + 2*(1001*b^2*cosh(d*x + c)^9 - 5148*b^2*cosh(d*x + c)^7 + 378*(6*a^2 + 11*b^2)*cosh(d*x + c)^5 - 84*(22*a^2 + 7*b^2)*cosh(d*x + c)^3 - 9*(22*a^2 + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(6*a^2 + 11*b^2)*cosh(d*x + c)^4 + (1001*b^2*cosh(d*x + c)^10 - 6435*b^2*cosh(d*x + c)^8 + 630*(6*a^2 + 11*b^2)*cosh(d*x + c)^6 - 210*(22*a^2 + 7*b^2)*cosh(d*x + c)^4 - 45*(22*a^2 + 7*b^2)*cosh(d*x + c)^2 + 18*a^2 + 33*b^2)*sinh(d*x + c)^4 - 13*b^2*cosh(d*x + c)^2 + 4*(91*b^2*cosh(d*x + c)^11 - 715*b^2*cosh(d*x + c)^9 + 90*(6*a^2 + 11*b^2)*cosh(d*x + c)^7 - 42*(22*a^2 + 7*b^2)*cosh(d*x + c)^5 - 15*(22*a^2 + 7*b^2)*cosh(d*x + c)^3 + 3*(6*a^2 + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (91*b^2*cosh(d*x + c)^12 - 858*b^2*cosh(d*x + c)^10 + 135*(6*a^2 + 11*b^2)*cosh(d*x + c)^8 - 84*(22*a^2 + 7*b^2)*cosh(d*x + c)^6 - 45*(22*a^2 + 7*b^2)*cosh(d*x + c)^4 + 18*(6*a^2 + 11*b^2)*cosh(d*x + c)^2 - 13*b^2)*sinh(d*x + c)^2 + b^2 - 3*((3*a^2 + 16*a*b)*cosh(d*x + c)^11 + 11*(3*a^2 + 16*a*b)*cosh(d*x + c)*sinh(d*x + c)^10 + (3*a^2 + 16*a*b)*sinh(d*x + c)^11 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 + (55*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 - 12*a^2 - 64*a*b)*sinh(d*x + c)^9 + 3*(55*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 - 12*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^8 + 6*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 + 6*(55*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 24*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b)*sinh(d*x + c)^7 + 42*(11*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 - 8*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 + 2*(231*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 252*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 + 63*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 - 6*a^2 - 32*a*b)*sinh(d*x + c)^5 + 2*(165*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 - 252*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 + 105*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 - 10*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (165*(3*a^2 + 16*a*b)*cosh(d*x + c)^8 - 336*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 + 210*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b)*sinh(d*x + c)^3 + (55*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 - 144*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 + 126*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 - 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (11*(3*a^2 + 16*a*b)*cosh(d*x + c)^10 - 36*(3*a^2 + 16*a*b)*cosh(d*x + c)^8 + 42*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 20*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*((3*a^2 + 16*a*b)*cosh(d*x + c)^11 + 11*(3*a^2 + 16*a*b)*cosh(d*x + c)*sinh(d*x + c)^10 + (3*a^2 + 16*a*b)*sinh(d*x + c)^11 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 + (55*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 - 12*a^2 - 64*a*b)*sinh(d*x + c)^9 + 3*(55*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 - 12*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^8 + 6*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 + 6*(55*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 24*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b)*sinh(d*x + c)^7 + 42*(11*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 - 8*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 + 2*(231*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 252*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 + 63*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 - 6*a^2 - 32*a*b)*sinh(d*x + c)^5 + 2*(165*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 - 252*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 + 105*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 - 10*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (165*(3*a^2 + 16*a*b)*cosh(d*x + c)^8 - 336*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 + 210*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b)*sinh(d*x + c)^3 + (55*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 - 144*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 + 126*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 - 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c))*si

```
nh(d*x + c)^2 + (11*(3*a^2 + 16*a*b)*cosh(d*x + c)^10 - 36*(3*a^2 + 16*a*b)
*cosh(d*x + c)^8 + 42*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 20*(3*a^2 + 16*a*b
)*cosh(d*x + c)^4 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(
cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*b^2*cosh(d*x + c)^13 - 78*b^2*cos
h(d*x + c)^11 + 15*(6*a^2 + 11*b^2)*cosh(d*x + c)^9 - 12*(22*a^2 + 7*b^2)*c
osh(d*x + c)^7 - 9*(22*a^2 + 7*b^2)*cosh(d*x + c)^5 + 6*(6*a^2 + 11*b^2)*co
sh(d*x + c)^3 - 13*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^11 +
11*d*cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^11 - 4*d*cosh(d*x + c
)^9 + (55*d*cosh(d*x + c)^2 - 4*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^
3 - 12*d*cosh(d*x + c))*sinh(d*x + c)^8 + 6*d*cosh(d*x + c)^7 + 6*(55*d*cos
h(d*x + c)^4 - 24*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 42*(11*d*cosh(d*
x + c)^5 - 8*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^6 - 4*d*cos
h(d*x + c)^5 + 2*(231*d*cosh(d*x + c)^6 - 252*d*cosh(d*x + c)^4 + 63*d*cosh
(d*x + c)^2 - 2*d)*sinh(d*x + c)^5 + 2*(165*d*cosh(d*x + c)^7 - 252*d*cosh(
d*x + c)^5 + 105*d*cosh(d*x + c)^3 - 10*d*cosh(d*x + c))*sinh(d*x + c)^4 +
d*cosh(d*x + c)^3 + (165*d*cosh(d*x + c)^8 - 336*d*cosh(d*x + c)^6 + 210*d*
cosh(d*x + c)^4 - 40*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (55*d*cosh(d*
x + c)^9 - 144*d*cosh(d*x + c)^7 + 126*d*cosh(d*x + c)^5 - 40*d*cosh(d*x +
c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (11*d*cosh(d*x + c)^10 - 36*d*c
osh(d*x + c)^8 + 42*d*cosh(d*x + c)^6 - 20*d*cosh(d*x + c)^4 + 3*d*cosh(d*x
 + c)^2)*sinh(d*x + c))
```

giac [A] time = 0.27, size = 179, normalized size = 1.77

$$\frac{2\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-24\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-3\left(3\,a^2+16\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)+3\left(3\,a^2+16\,ab\right)\mathrm{lo}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 24*b^2*(e^(d*x + c) + e^(-d*x
- c)) - 3*(3*a^2 + 16*a*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 3*(3*a^2 +
 16*a*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) + 12*(3*a^2*(e^(d*x + c) + e^(
-d*x - c))^3 - 20*a^2*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x
 - c))^2 - 4)^2)/d
```

maple [A] time = 0.10, size = 79, normalized size = 0.78

$$\frac{a^2\left(\left(-\frac{\mathrm{csch}(dx+c)^3}{4}+\frac{3\,\mathrm{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)}{4}\right)-4ab\,\mathrm{arctanh}\left(\mathrm{e}^{dx+c}\right)+b^2\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)\cosh}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(
d*x+c)))-4*a*b*arctanh(exp(d*x+c))+b^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)
)
```

maxima [B] time = 0.35, size = 234, normalized size = 2.32

$$\frac{1}{24}\,b^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)-\frac{1}{8}\,a^2\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx}}{d\left(4\,e^{(-2\,dx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) - 1/8*a^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/
d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7
*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*
c) - e^(-8*d*x - 8*c) - 1))) - 2*a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x
 - c) - 1)/d)
```

mupad [B] time = 0.22, size = 328, normalized size = 3.25

$$\frac{b^2\,\mathrm{e}^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b^2\,\mathrm{e}^{-c-d\,x}}{8\,d}-\frac{3\,b^2\,\mathrm{e}^{c+d\,x}}{8\,d}+\frac{b^2\,\mathrm{e}^{3\,c+3\,d\,x}}{24\,d}-\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(3\,a^2\,\sqrt{-d^2}+16\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{9\,a^4+96\,a^3\,b+256\,a^2\,b^2}}\right)\,\sqrt{9\,a^4+96\,a^3\,b+256\,a^2\,b}}{4\,\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^5,x)
```

```
[Out] (b^2*exp(- 3*c - 3*d*x))/(24*d) - (3*b^2*exp(- c - d*x))/(8*d) - (3*b^2*exp
(c + d*x))/(8*d) + (b^2*exp(3*c + 3*d*x))/(24*d) - (atan((exp(d*x)*exp(c)*(
3*a^2*(-d^2)^(1/2) + 16*a*b*(-d^2)^(1/2)))/(d*(96*a^3*b + 9*a^4 + 256*a^2*b
^2)^(1/2)))*(96*a^3*b + 9*a^4 + 256*a^2*b^2)^(1/2))/(4*(-d^2)^(1/2)) - (6*a
^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*
d*x) - 1)) - (4*a^2*exp(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*
x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (3*a^2*exp(c + d*x))/(4*
d*(exp(2*c + 2*d*x) - 1)) - (a^2*exp(c + d*x))/(2*d*(exp(4*c + 4*d*x) - 2*e
xp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.205 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=84

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a(a+2b)\coth(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

```
[Out] -1/2*b^2*x-a*(a+2*b)*coth(d*x+c)/d+2/3*a^2*coth(d*x+c)^3/d-1/5*a^2*coth(d*x
+c)^5/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3217, 1259, 1802, 207}

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a(a+2b)\coth(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(b^2*x)/2 - (a*(a + 2*b)*Coth[c + d*x])/d + (2*a^2*Coth[c + d*x]^3)/(3*d)
- (a^2*Coth[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{x^6\left(1-x^2\right)^2}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-2a^2+6a^2x^2-2a(3a+2b)x^4+(2}{x^6(1-x^2)}\right.}{2}$$

$$= \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int\left(-\frac{2a^2}{x^6}+\frac{4a^2}{x^4}-\frac{2a(a+2b)}{x^2}\right.\right.}{2d}$$

$$= -\frac{a(a+2b)\coth(c+dx)}{d} + \frac{2a^2\coth^3(c+dx)}{3d} - \frac{a^2\coth^5(c+dx)}{5d}$$

$$= -\frac{b^2x}{2} - \frac{a(a+2b)\coth(c+dx)}{d} + \frac{2a^2\coth^3(c+dx)}{3d} - \frac{a^2\coth^5(c}{5d}$$

Mathematica [A] time = 0.83, size = 67, normalized size = 0.80

$$\frac{15b^2(\sinh(2(c+dx))-2(c+dx))-4a\coth(c+dx)\left(3a\operatorname{csch}^4(c+dx)-4a\operatorname{csch}^2(c+dx)+8a+30b\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-4*a*Coth[c + d*x]*(8*a + 30*b - 4*a*Csch[c + d*x]^2 + 3*a*Csch[c + d*x]^4
) + 15*b^2*(-2*(c + d*x) + Sinh[2*(c + d*x)]))/(60*d)
```

fricas [B] time = 1.31, size = 457, normalized size = 5.44

$$15\,b^2\cosh(dx+c)^7 + 105\,b^2\cosh(dx+c)\sinh(dx+c)^6 - \left(64\,a^2+240\,ab+75\,b^2\right)\cosh(dx+c)^5 - 4\left(15\,b^2 d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(15*b^2*cosh(d*x + c)^7 + 105*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - (64
*a^2 + 240*a*b + 75*b^2)*cosh(d*x + c)^5 - 4*(15*b^2*d*x - 16*a^2 - 60*a*b)
*sinh(d*x + c)^5 + 5*(105*b^2*cosh(d*x + c)^3 - (64*a^2 + 240*a*b + 75*b^2)
*cosh(d*x + c))*sinh(d*x + c)^4 + 5*(64*a^2 + 144*a*b + 27*b^2)*cosh(d*x +
c)^3 + 20*(15*b^2*d*x - 2*(15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^2 -
16*a^2 - 60*a*b)*sinh(d*x + c)^3 + 5*(63*b^2*cosh(d*x + c)^5 - 2*(64*a^2 +
240*a*b + 75*b^2)*cosh(d*x + c)^3 + 3*(64*a^2 + 144*a*b + 27*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^2 - 5*(128*a^2 + 96*a*b + 15*b^2)*cosh(d*x + c) - 20*((
15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^4 + 30*b^2*d*x - 3*(15*b^2*d*x
- 16*a^2 - 60*a*b)*cosh(d*x + c)^2 - 32*a^2 - 120*a*b)*sinh(d*x + c))/(d*si
nh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x
 + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

giac [B] time = 0.26, size = 166, normalized size = 1.98

$$-\frac{60\,(dx+c)b^2 - 15\,b^2e^{(2\,dx+2\,c)} - 15\left(2\,b^2e^{(2\,dx+2\,c)}-b^2\right)e^{(-2\,dx-2\,c)} + \frac{32\left(15\,abe^{(8\,dx+8\,c)}-60\,abe^{(6\,dx+6\,c)}+40\,a^2e^{(4\,dx+4\,c)}+90\,ab\right.}{\left(e^{(2\,dx+2\right.}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/120*(60*(d*x + c)*b^2 - 15*b^2*e^(2*d*x + 2*c) - 15*(2*b^2*e^(2*d*x + 2*
c) - b^2)*e^(-2*d*x - 2*c) + 32*(15*a*b*e^(8*d*x + 8*c) - 60*a*b*e^(6*d*x +
 6*c) + 40*a^2*e^(4*d*x + 4*c) + 90*a*b*e^(4*d*x + 4*c) - 20*a^2*e^(2*d*x +
 2*c) - 60*a*b*e^(2*d*x + 2*c) + 4*a^2 + 15*a*b)/(e^(2*d*x + 2*c) - 1)^5)/d
```

maple [A] time = 0.10, size = 74, normalized size = 0.88

$$\frac{a^2\left(-\frac{8}{15}-\frac{\operatorname{csch}(dx+c)^4}{5}+\frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth\left(dx+c\right)-2ab\coth\left(dx+c\right)+b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-2*a*b*cot
h(d*x+c)+b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [B] time = 0.34, size = 267, normalized size = 3.18

$$-\frac{1}{8}b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{16}{15}a^2\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 16/15*a^2*(5*e^(-
2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/
(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 0.75, size = 397, normalized size = 4.73

$$\frac{b^2 e^{2c+2dx}}{8d}-\frac{\frac{4e^{2c+2dx}\left(4a^2+3ba\right)}{5d}-\frac{4ab}{5d}-\frac{12abe^{4c+4dx}}{5d}+\frac{4abe^{6c+6dx}}{5d}}{6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1}-\frac{b^2x}{2}-\frac{\frac{4\left(4a^2+3ba\right)}{15d}-\frac{8abe^{2c+2dx}}{5d}+\frac{4abe^{4c+4dx}}{5d}}{3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1}-\frac{b^2 e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^6,x)
```

```
[Out] (b^2*exp(2*c + 2*d*x))/(8*d) - ((4*exp(2*c + 2*d*x)*(3*a*b + 4*a^2))/(5*d)
- (4*a*b)/(5*d) - (12*a*b*exp(4*c + 4*d*x))/(5*d) + (4*a*b*exp(6*c + 6*d*x)
)/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + ex
p(8*c + 8*d*x) + 1) - (b^2*x)/2 - ((4*(3*a*b + 4*a^2))/(15*d) - (8*a*b*exp(
2*c + 2*d*x))/(5*d) + (4*a*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d*x) -
 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (b^2*exp(- 2*c - 2*d*x))/(8*d
) - ((8*exp(4*c + 4*d*x)*(3*a*b + 4*a^2))/(5*d) + (4*a*b)/(5*d) - (16*a*b*e
xp(2*c + 2*d*x))/(5*d) - (16*a*b*exp(6*c + 6*d*x))/(5*d) + (4*a*b*exp(8*c +
 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*
d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - (8*a*b)/(5*d*(exp(2*c
 + 2*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.206 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=111

$$-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}+\frac{a(5a+16b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a(5a+16b)}{}$$

```
[Out] 1/16*a*(5*a+16*b)*arctanh(cosh(d*x+c))/d+b^2*cosh(d*x+c)/d-1/16*a*(5*a+16*b
)*coth(d*x+c)*csch(d*x+c)/d+5/24*a^2*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a^2*co
th(d*x+c)*csch(d*x+c)^5/d
```

Rubi [A] time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 388, 206}

$$-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}+\frac{a(5a+16b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a(5a+16b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (a*(5*a + 16*b)*ArcTanh[Cosh[c + d*x]])/(16*d) + (b^2*Cosh[c + d*x])/d - (a
*(5*a + 16*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Cs
ch[c + d*x]^3)/(24*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{\left(1-x^2\right)^4}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d} - \frac{\operatorname{Subst}\left(\int \frac{-5a^2-12ab-6b^2+6b(2a+3b}{\left(1-x^2\right)^3}\right.}{}$$

$$= \frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d} - \frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d} +$$

$$= -\frac{a(5a+16b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a^2\coth(c+dx)\operatorname{csch}^3}{24d}$$

$$= \frac{b^2\cosh(c+dx)}{d} - \frac{a(5a+16b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a^2 c}{}$$

$$= \frac{a(5a+16b)\tanh^{-1}(\cosh(c+dx))}{16d} + \frac{b^2\cosh(c+dx)}{d} - \frac{a(5a+1}{}$$

Mathematica [B] time = 0.04, size = 240, normalized size = 2.16

$$-\frac{a^2\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a^2\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (5*a^2*Csch[(c + d*x)/2]^2)/(64*d) - (a*b*Csch[
(c + d*x)/2]^2)/(4*d) + (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (a^2*Csch[(c + d
*x)/2]^6)/(384*d) - (5*a^2*Log[Tanh[(c + d*x)/2]])/(16*d) - (a*b*Log[Tanh[(
c + d*x)/2]])/d - (5*a^2*Sech[(c + d*x)/2]^2)/(64*d) - (a*b*Sech[(c + d*x)/
2]^2)/(4*d) - (a^2*Sech[(c + d*x)/2]^4)/(64*d) - (a^2*Sech[(c + d*x)/2]^6)/
(384*d) + (b^2*Sinh[c]*Sinh[d*x])/d
```

fricas [B] time = 1.07, size = 4500, normalized size = 40.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(24*b^2*cosh(d*x + c)^14 + 336*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + 24
*b^2*sinh(d*x + c)^14 - 6*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^12 + 6*(3
64*b^2*cosh(d*x + c)^2 - 5*a^2 - 16*a*b - 20*b^2)*sinh(d*x + c)^12 + 24*(36
4*b^2*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^11 + 2*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^10 + 2*(12012*b^2*c
```

osh(d*x + c)^4 - 198*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^2 + 85*a^2 + 1
44*a*b + 108*b^2)*sinh(d*x + c)^10 + 4*(12012*b^2*cosh(d*x + c)^5 - 330*(5*
a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^3 + 5*(85*a^2 + 144*a*b + 108*b^2)*cos
h(d*x + c))*sinh(d*x + c)^9 - 12*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^8
 + 6*(12012*b^2*cosh(d*x + c)^6 - 495*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x +
c)^4 + 15*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^2 - 66*a^2 - 32*a*b -
20*b^2)*sinh(d*x + c)^8 + 48*(1716*b^2*cosh(d*x + c)^7 - 99*(5*a^2 + 16*a*b
 + 20*b^2)*cosh(d*x + c)^5 + 5*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^3
 - 2*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 12*(33*a^2
 + 16*a*b + 10*b^2)*cosh(d*x + c)^6 + 12*(6006*b^2*cosh(d*x + c)^8 - 462*(5
*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^6 + 35*(85*a^2 + 144*a*b + 108*b^2)*c
osh(d*x + c)^4 - 28*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^2 - 33*a^2 - 1
6*a*b - 10*b^2)*sinh(d*x + c)^6 + 24*(2002*b^2*cosh(d*x + c)^9 - 198*(5*a^2
 + 16*a*b + 20*b^2)*cosh(d*x + c)^7 + 21*(85*a^2 + 144*a*b + 108*b^2)*cosh(
d*x + c)^5 - 28*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^3 - 3*(33*a^2 + 16
*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(85*a^2 + 144*a*b + 108*b
^2)*cosh(d*x + c)^4 + 2*(12012*b^2*cosh(d*x + c)^10 - 1485*(5*a^2 + 16*a*b
+ 20*b^2)*cosh(d*x + c)^8 + 210*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^
6 - 420*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^4 - 90*(33*a^2 + 16*a*b +
10*b^2)*cosh(d*x + c)^2 + 85*a^2 + 144*a*b + 108*b^2)*sinh(d*x + c)^4 + 8*(
1092*b^2*cosh(d*x + c)^11 - 165*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^9 +
 30*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^7 - 84*(33*a^2 + 16*a*b + 10
*b^2)*cosh(d*x + c)^5 - 30*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^3 + (85
*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(5*a^2 + 16*a*
b + 20*b^2)*cosh(d*x + c)^2 + 6*(364*b^2*cosh(d*x + c)^12 - 66*(5*a^2 + 16*
a*b + 20*b^2)*cosh(d*x + c)^10 + 15*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x +
 c)^8 - 56*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^6 - 30*(33*a^2 + 16*a*b
 + 10*b^2)*cosh(d*x + c)^4 + 2*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^2
 - 5*a^2 - 16*a*b - 20*b^2)*sinh(d*x + c)^2 + 24*b^2 + 3*((5*a^2 + 16*a*b)*
cosh(d*x + c)^13 + 13*(5*a^2 + 16*a*b)*cosh(d*x + c)*sinh(d*x + c)^12 + (5*
a^2 + 16*a*b)*sinh(d*x + c)^13 - 6*(5*a^2 + 16*a*b)*cosh(d*x + c)^11 + 6*(1
3*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 - 5*a^2 - 16*a*b)*sinh(d*x + c)^11 + 22*
(13*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b)*cosh(d*x + c))*si
nh(d*x + c)^10 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c)^9 + 5*(143*(5*a^2 + 16*a
*b)*cosh(d*x + c)^4 - 66*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 + 15*a^2 + 48*a*b
)*sinh(d*x + c)^9 + 9*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 - 110*(5*a^2 +
16*a*b)*cosh(d*x + c)^3 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^
8 - 20*(5*a^2 + 16*a*b)*cosh(d*x + c)^7 + 4*(429*(5*a^2 + 16*a*b)*cosh(d*x
+ c)^6 - 495*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 + 135*(5*a^2 + 16*a*b)*cosh(d
*x + c)^2 - 25*a^2 - 80*a*b)*sinh(d*x + c)^7 + 4*(429*(5*a^2 + 16*a*b)*cosh
(d*x + c)^7 - 693*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 + 315*(5*a^2 + 16*a*b)*c
osh(d*x + c)^3 - 35*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 15*(5
*a^2 + 16*a*b)*cosh(d*x + c)^5 + 3*(429*(5*a^2 + 16*a*b)*cosh(d*x + c)^8 -
924*(5*a^2 + 16*a*b)*cosh(d*x + c)^6 + 630*(5*a^2 + 16*a*b)*cosh(d*x + c)^4
 - 140*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 + 25*a^2 + 80*a*b)*sinh(d*x + c)^5
+ 5*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^9 - 396*(5*a^2 + 16*a*b)*cosh(d*x +
 c)^7 + 378*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 - 140*(5*a^2 + 16*a*b)*cosh(d*
x + c)^3 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^4 - 6*(5*a^2 +
16*a*b)*cosh(d*x + c)^3 + 2*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^10 - 495*(5
*a^2 + 16*a*b)*cosh(d*x + c)^8 + 630*(5*a^2 + 16*a*b)*cosh(d*x + c)^6 - 350
*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 + 75*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 - 1
5*a^2 - 48*a*b)*sinh(d*x + c)^3 + 6*(13*(5*a^2 + 16*a*b)*cosh(d*x + c)^11 -
 55*(5*a^2 + 16*a*b)*cosh(d*x + c)^9 + 90*(5*a^2 + 16*a*b)*cosh(d*x + c)^7
- 70*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 + 25*(5*a^2 + 16*a*b)*cosh(d*x + c)^3
 - 3*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^2 + 16*a*b)*cos
h(d*x + c) + (13*(5*a^2 + 16*a*b)*cosh(d*x + c)^12 - 66*(5*a^2 + 16*a*b)*co
sh(d*x + c)^10 + 135*(5*a^2 + 16*a*b)*cosh(d*x + c)^8 - 140*(5*a^2 + 16*a*b
)*cosh(d*x + c)^6 + 75*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 - 18*(5*a^2 + 16*a*
b)*cosh(d*x + c)^2 + 5*a^2 + 16*a*b)*sinh(d*x + c))*log(cosh(d*x + c) + sin

```
h(d*x + c) + 1) - 3*((5*a^2 + 16*a*b)*cosh(d*x + c)^13 + 13*(5*a^2 + 16*a*b
)*cosh(d*x + c)*sinh(d*x + c)^12 + (5*a^2 + 16*a*b)*sinh(d*x + c)^13 - 6*(5
*a^2 + 16*a*b)*cosh(d*x + c)^11 + 6*(13*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 -
5*a^2 - 16*a*b)*sinh(d*x + c)^11 + 22*(13*(5*a^2 + 16*a*b)*cosh(d*x + c)^3
- 3*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^10 + 15*(5*a^2 + 16*a*b)*
cosh(d*x + c)^9 + 5*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 - 66*(5*a^2 + 16*
a*b)*cosh(d*x + c)^2 + 15*a^2 + 48*a*b)*sinh(d*x + c)^9 + 9*(143*(5*a^2 + 1
6*a*b)*cosh(d*x + c)^5 - 110*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 + 15*(5*a^2 +
 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^8 - 20*(5*a^2 + 16*a*b)*cosh(d*x + c)
^7 + 4*(429*(5*a^2 + 16*a*b)*cosh(d*x + c)^6 - 495*(5*a^2 + 16*a*b)*cosh(d*
x + c)^4 + 135*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 - 25*a^2 - 80*a*b)*sinh(d*x
 + c)^7 + 4*(429*(5*a^2 + 16*a*b)*cosh(d*x + c)^7 - 693*(5*a^2 + 16*a*b)*co
sh(d*x + c)^5 + 315*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 - 35*(5*a^2 + 16*a*b)*
cosh(d*x + c))*sinh(d*x + c)^6 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 + 3*(4
29*(5*a^2 + 16*a*b)*cosh(d*x + c)^8 - 924*(5*a^2 + 16*a*b)*cosh(d*x + c)^6
+ 630*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 - 140*(5*a^2 + 16*a*b)*cosh(d*x + c)
^2 + 25*a^2 + 80*a*b)*sinh(d*x + c)^5 + 5*(143*(5*a^2 + 16*a*b)*cosh(d*x +
c)^9 - 396*(5*a^2 + 16*a*b)*cosh(d*x + c)^7 + 378*(5*a^2 + 16*a*b)*cosh(d*x
 + c)^5 - 140*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 + 15*(5*a^2 + 16*a*b)*cosh(d
*x + c))*sinh(d*x + c)^4 - 6*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 + 2*(143*(5*a
^2 + 16*a*b)*cosh(d*x + c)^10 - 495*(5*a^2 + 16*a*b)*cosh(d*x + c)^8 + 630*
(5*a^2 + 16*a*b)*cosh(d*x + c)^6 - 350*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 + 7
5*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 - 15*a^2 - 48*a*b)*sinh(d*x + c)^3 + 6*(
13*(5*a^2 + 16*a*b)*cosh(d*x + c)^11 - 55*(5*a^2 + 16*a*b)*cosh(d*x + c)^9
+ 90*(5*a^2 + 16*a*b)*cosh(d*x + c)^7 - 70*(5*a^2 + 16*a*b)*cosh(d*x + c)^5
 + 25*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b)*cosh(d*x + c))*
sinh(d*x + c)^2 + (5*a^2 + 16*a*b)*cosh(d*x + c) + (13*(5*a^2 + 16*a*b)*cos
h(d*x + c)^12 - 66*(5*a^2 + 16*a*b)*cosh(d*x + c)^10 + 135*(5*a^2 + 16*a*b)
*cosh(d*x + c)^8 - 140*(5*a^2 + 16*a*b)*cosh(d*x + c)^6 + 75*(5*a^2 + 16*a*
b)*cosh(d*x + c)^4 - 18*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 + 5*a^2 + 16*a*b)*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(84*b^2*cosh(d*x
+ c)^13 - 18*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^11 + 5*(85*a^2 + 144*a
*b + 108*b^2)*cosh(d*x + c)^9 - 24*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)
^7 - 18*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^5 + 2*(85*a^2 + 144*a*b +
108*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c))*sinh(
d*x + c))/(d*cosh(d*x + c)^13 + 13*d*cosh(d*x + c)*sinh(d*x + c)^12 + d*sin
h(d*x + c)^13 - 6*d*cosh(d*x + c)^11 + 6*(13*d*cosh(d*x + c)^2 - d)*sinh(d*
x + c)^11 + 22*(13*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^10
+ 15*d*cosh(d*x + c)^9 + 5*(143*d*cosh(d*x + c)^4 - 66*d*cosh(d*x + c)^2 +
3*d)*sinh(d*x + c)^9 + 9*(143*d*cosh(d*x + c)^5 - 110*d*cosh(d*x + c)^3 + 1
5*d*cosh(d*x + c))*sinh(d*x + c)^8 - 20*d*cosh(d*x + c)^7 + 4*(429*d*cosh(d
*x + c)^6 - 495*d*cosh(d*x + c)^4 + 135*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x +
 c)^7 + 4*(429*d*cosh(d*x + c)^7 - 693*d*cosh(d*x + c)^5 + 315*d*cosh(d*x +
 c)^3 - 35*d*cosh(d*x + c))*sinh(d*x + c)^6 + 15*d*cosh(d*x + c)^5 + 3*(429
*d*cosh(d*x + c)^8 - 924*d*cosh(d*x + c)^6 + 630*d*cosh(d*x + c)^4 - 140*d*
cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(143*d*cosh(d*x + c)^9 - 396*d*c
osh(d*x + c)^7 + 378*d*cosh(d*x + c)^5 - 140*d*cosh(d*x + c)^3 + 15*d*cosh(
d*x + c))*sinh(d*x + c)^4 - 6*d*cosh(d*x + c)^3 + 2*(143*d*cosh(d*x + c)^10
 - 495*d*cosh(d*x + c)^8 + 630*d*cosh(d*x + c)^6 - 350*d*cosh(d*x + c)^4 +
75*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^3 + 6*(13*d*cosh(d*x + c)^11 - 55
*d*cosh(d*x + c)^9 + 90*d*cosh(d*x + c)^7 - 70*d*cosh(d*x + c)^5 + 25*d*cos
h(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (13*d
*cosh(d*x + c)^12 - 66*d*cosh(d*x + c)^10 + 135*d*cosh(d*x + c)^8 - 140*d*c
osh(d*x + c)^6 + 75*d*cosh(d*x + c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c))
```

giac [B] time = 0.28, size = 243, normalized size = 2.19

$$\frac{48\, b^2\left(e^{(dx+c)} + e^{(-dx-c)}\right) + 3\left(5\, a^2 + 16\, ab\right)\log\left(e^{(dx+c)} + e^{(-dx-c)} + 2\right) - 3\left(5\, a^2 + 16\, ab\right)\log\left(e^{(dx+c)} + e^{(-dx-c)} - 2\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/96*(48*b^2*(e^(d*x + c) + e^(-d*x - c)) + 3*(5*a^2 + 16*a*b)*log(e^(d*x +
 c) + e^(-d*x - c) + 2) - 3*(5*a^2 + 16*a*b)*log(e^(d*x + c) + e^(-d*x - c)
 - 2) - 4*(15*a^2*(e^(d*x + c) + e^(-d*x - c))^5 + 48*a*b*(e^(d*x + c) + e^
(-d*x - c))^5 - 160*a^2*(e^(d*x + c) + e^(-d*x - c))^3 - 384*a*b*(e^(d*x +
c) + e^(-d*x - c))^3 + 528*a^2*(e^(d*x + c) + e^(-d*x - c)) + 768*a*b*(e^(d
*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^3)/d
```

maple [A] time = 0.10, size = 92, normalized size = 0.83

$$\frac{a^2\left(\left(-\frac{\operatorname{csch}(dx+c)^5}{6} + \frac{5\operatorname{csch}(dx+c)^3}{24} - \frac{5\operatorname{csch}(dx+c)}{16}\right)\coth\left(dx+c\right) + \frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right) + 2ab\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2} + \operatorname{arctanh}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x
+c)+5/8*arctanh(exp(d*x+c)))+2*a*b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(ex
p(d*x+c)))+b^2*cosh(d*x+c))
```

maxima [B] time = 0.35, size = 299, normalized size = 2.69

$$\frac{1}{2}\, b^2\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) + \frac{1}{48}\, a^2\left(\frac{15\,\log\left(e^{(-dx-c)} + 1\right)}{d} - \frac{15\,\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2\left(15\, e^{(-dx-c)} - 85\, e^{(-3\,dx-3\,c)} + 198\right.}{d\left(6\, e^{(-2\,dx-2\,c)} - 15\, e^{(-4\,dx-4\,c)} + 20\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/48*a^2*(15*log(e^(-d*x - c) +
1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c
) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*
e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6
*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*
c) - 1))) + a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
)))
```

mupad [B] time = 0.82, size = 535, normalized size = 4.82

$$\frac{b^2\,\mathrm{e}^{c+d\,x}}{2\,d} - \frac{\frac{8\,\mathrm{e}^{5\,c+5\,d\,x}\left(4\,a^2+3\,b\,a\right)}{3\,d} + \frac{4\,a\,b\,\mathrm{e}^{c+d\,x}}{3\,d} - \frac{16\,a\,b\,\mathrm{e}^{3\,c+3\,d\,x}}{3\,d} - \frac{16\,a\,b\,\mathrm{e}^{7\,c+7\,d\,x}}{3\,d} + \frac{4\,a\,b\,\mathrm{e}^{9\,c+9\,d\,x}}{3\,d}}{15\,\mathrm{e}^{4\,c+4\,d\,x} - 6\,\mathrm{e}^{2\,c+2\,d\,x} - 20\,\mathrm{e}^{6\,c+6\,d\,x} + 15\,\mathrm{e}^{8\,c+8\,d\,x} - 6\,\mathrm{e}^{10\,c+10\,d\,x} + \mathrm{e}^{12\,c+12\,d\,x} + 1} + \frac{b^2\,\mathrm{e}^{-c-d\,x}}{2\,d} + \frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^2/sinh(c + d*x)^7,x)
```

```
[Out] (b^2*exp(c + d*x))/(2*d) - ((8*exp(5*c + 5*d*x)*(3*a*b + 4*a^2))/(3*d) + (4
*a*b*exp(c + d*x))/(3*d) - (16*a*b*exp(3*c + 3*d*x))/(3*d) - (16*a*b*exp(7*
c + 7*d*x))/(3*d) + (4*a*b*exp(9*c + 9*d*x))/(3*d))/(15*exp(4*c + 4*d*x) -
6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c
 + 10*d*x) + exp(12*c + 12*d*x) + 1) + (b^2*exp(- c - d*x))/(2*d) + (atan((
exp(d*x)*exp(c)*(5*a^2*(-d^2)^(1/2) + 16*a*b*(-d^2)^(1/2)))/(d*(160*a^3*b +
 25*a^4 + 256*a^2*b^2)^(1/2)))*(160*a^3*b + 25*a^4 + 256*a^2*b^2)^(1/2))/(8
*(-d^2)^(1/2)) - (a^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) - 1)) - (22*a^2*exp(c + d*x))/(3*d*(6*exp(4*c + 4
*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) -
(16*a^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (exp(c +
d*x)*(16*a*b + 5*a^2))/(8*d*(exp(2*c + 2*d*x) - 1)) - (exp(c + d*x)*(32*a*b
 - 5*a^2))/(12*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.207 $\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=220

$$\frac{b\left(3a^2+45ab+70b^2\right)\cosh^9(c+dx)}{9d}-\frac{4b\left(3a^2+15ab+14b^2\right)\cosh^7(c+dx)}{7d}+\frac{(a+b)\left(a^2+17ab+28b^2\right)\cosh^5(c}{5d}$$

```
[Out] (a+b)^3*cosh(d*x+c)/d-2/3*(a+b)^2*(a+4*b)*cosh(d*x+c)^3/d+1/5*(a+b)*(a^2+17
*a*b+28*b^2)*cosh(d*x+c)^5/d-4/7*b*(3*a^2+15*a*b+14*b^2)*cosh(d*x+c)^7/d+1/
9*b*(3*a^2+45*a*b+70*b^2)*cosh(d*x+c)^9/d-2/11*b^2*(9*a+28*b)*cosh(d*x+c)^1
1/d+1/13*b^2*(3*a+28*b)*cosh(d*x+c)^13/d-8/15*b^3*cosh(d*x+c)^15/d+1/17*b^3
*cosh(d*x+c)^17/d
```

Rubi [A] time = 0.22, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3215, 1153}

$$\frac{b\left(3a^2+45ab+70b^2\right)\cosh^9(c+dx)}{9d}-\frac{4b\left(3a^2+15ab+14b^2\right)\cosh^7(c+dx)}{7d}+\frac{(a+b)\left(a^2+17ab+28b^2\right)\cosh^5(c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*Cosh[c + d*x])/d - (2*(a + b)^2*(a + 4*b)*Cosh[c + d*x]^3)/(3*d)
 + ((a + b)*(a^2 + 17*a*b + 28*b^2)*Cosh[c + d*x]^5)/(5*d) - (4*b*(3*a^2 +
15*a*b + 14*b^2)*Cosh[c + d*x]^7)/(7*d) + (b*(3*a^2 + 45*a*b + 70*b^2)*Cosh
[c + d*x]^9)/(9*d) - (2*b^2*(9*a + 28*b)*Cosh[c + d*x]^11)/(11*d) + (b^2*(3
*a + 28*b)*Cosh[c + d*x]^13)/(13*d) - (8*b^3*Cosh[c + d*x]^15)/(15*d) + (b^
3*Cosh[c + d*x]^17)/(17*d)
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a+b-2bx^2+bx^4\right)^3\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left((a+b)^3-2(a+b)^2(a+4b)x^2+(a+b)\left(a^2+17ab+28b\right.\right.\right.}{}\\ &= \frac{(a+b)^3\cosh(c+dx)}{d}-\frac{2(a+b)^2(a+4b)\cosh^3(c+dx)}{3d}+\frac{(a+b)\left(\right.}{}\end{aligned}$$

Mathematica [A] time = 2.28, size = 288, normalized size = 1.31

$$\frac{627314688a^3\cosh(5(c+dx))+4234374144a^2b\cosh(5(c+dx))-756138240a^2b\cosh(7(c+dx))+6534528\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (1531530*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*Cosh[c + d*x]
- 2042040*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*Cosh[3*(c + d*x)]
 + 627314688*a^3*Cosh[5*(c + d*x)] + 4234374144*a^2*b*Cosh[5*(c + d*x)] + 5
256210960*a*b^2*Cosh[5*(c + d*x)] + 1895421528*b^3*Cosh[5*(c + d*x)] - 7561
38240*a^2*b*Cosh[7*(c + d*x)] - 1501774560*a*b^2*Cosh[7*(c + d*x)] - 676936
260*b^3*Cosh[7*(c + d*x)] + 65345280*a^2*b*Cosh[9*(c + d*x)] + 318558240*a*
b^2*Cosh[9*(c + d*x)] + 202502300*b^3*Cosh[9*(c + d*x)] - 43439760*a*b^2*Co
sh[11*(c + d*x)] - 47338200*b^3*Cosh[11*(c + d*x)] + 2827440*a*b^2*Cosh[13*
(c + d*x)] + 8011080*b^3*Cosh[13*(c + d*x)] - 867867*b^3*Cosh[15*(c + d*x)]
 + 45045*b^3*Cosh[17*(c + d*x)])/(50185175040*d)
```

fricas [B] time = 0.85, size = 1030, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/50185175040*(45045*b^3*cosh(d*x + c)^17 + 765765*b^3*cosh(d*x + c)*sinh(d
*x + c)^16 - 867867*b^3*cosh(d*x + c)^15 + 765765*(40*b^3*cosh(d*x + c)^3 -
 17*b^3*cosh(d*x + c))*sinh(d*x + c)^14 + 471240*(6*a*b^2 + 17*b^3)*cosh(d*
x + c)^13 + 255255*(1092*b^3*cosh(d*x + c)^5 - 1547*b^3*cosh(d*x + c)^3 + 2
4*(6*a*b^2 + 17*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 - 556920*(78*a*b^2 + 8
5*b^3)*cosh(d*x + c)^11 + 153153*(5720*b^3*cosh(d*x + c)^7 - 17017*b^3*cosh
(d*x + c)^5 + 880*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^3 - 40*(78*a*b^2 + 85*b^
3)*cosh(d*x + c))*sinh(d*x + c)^10 + 340340*(192*a^2*b + 936*a*b^2 + 595*b^
3)*cosh(d*x + c)^9 + 765765*(1430*b^3*cosh(d*x + c)^9 - 7293*b^3*cosh(d*x +
 c)^7 + 792*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^5 - 120*(78*a*b^2 + 85*b^3)*co
sh(d*x + c)^3 + 4*(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^8 - 437580*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*cosh(d*x + c)^7 + 255
255*(2184*b^3*cosh(d*x + c)^11 - 17017*b^3*cosh(d*x + c)^9 + 3168*(6*a*b^2
+ 17*b^3)*cosh(d*x + c)^7 - 1008*(78*a*b^2 + 85*b^3)*cosh(d*x + c)^5 + 112*
(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c)^3 - 12*(1728*a^2*b + 3432*a
*b^2 + 1547*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 1225224*(512*a^3 + 3456*a
^2*b + 4290*a*b^2 + 1547*b^3)*cosh(d*x + c)^5 + 765765*(140*b^3*cosh(d*x +
c)^13 - 1547*b^3*cosh(d*x + c)^11 + 440*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^9
- 240*(78*a*b^2 + 85*b^3)*cosh(d*x + c)^7 + 56*(192*a^2*b + 936*a*b^2 + 595
*b^3)*cosh(d*x + c)^5 - 20*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*cosh(d*x +
c)^3 + 8*(512*a^3 + 3456*a^2*b + 4290*a*b^2 + 1547*b^3)*cosh(d*x + c))*sinh
(d*x + c)^4 - 2042040*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*cosh(
d*x + c)^3 + 765765*(8*b^3*cosh(d*x + c)^15 - 119*b^3*cosh(d*x + c)^13 + 48
*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^11 - 40*(78*a*b^2 + 85*b^3)*cosh(d*x + c)
^9 + 16*(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c)^7 - 12*(1728*a^2*b
+ 3432*a*b^2 + 1547*b^3)*cosh(d*x + c)^5 + 16*(512*a^3 + 3456*a^2*b + 4290*
a*b^2 + 1547*b^3)*cosh(d*x + c)^3 - 8*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 +
 2431*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 1531530*(20480*a^3 + 48384*a^2*
b + 41184*a*b^2 + 12155*b^3)*cosh(d*x + c))/d
```

giac [B] time = 0.39, size = 520, normalized size = 2.36

$$\frac{b^3e^{(17\,dx+17\,c)}}{2228224\,d}-\frac{17\,b^3e^{(15\,dx+15\,c)}}{1966080\,d}-\frac{17\,b^3e^{(-15\,dx-15\,c)}}{1966080\,d}+\frac{b^3e^{(-17\,dx-17\,c)}}{2228224\,d}+\frac{\left(6\,ab^2+17\,b^3\right)e^{(13\,dx+13\,c)}}{212992\,d}-\frac{\left(78\,ab^2+85\,b^3\right)e\ldots}{180224\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2228224*b^3*e^(17*d*x + 17*c)/d - 17/1966080*b^3*e^(15*d*x + 15*c)/d - 17
/1966080*b^3*e^(-15*d*x - 15*c)/d + 1/2228224*b^3*e^(-17*d*x - 17*c)/d + 1/
212992*(6*a*b^2 + 17*b^3)*e^(13*d*x + 13*c)/d - 1/180224*(78*a*b^2 + 85*b^3
)*e^(11*d*x + 11*c)/d + 1/294912*(192*a^2*b + 936*a*b^2 + 595*b^3)*e^(9*d*x
 + 9*c)/d - 1/229376*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*e^(7*d*x + 7*c)/d
 + 1/81920*(512*a^3 + 3456*a^2*b + 4290*a*b^2 + 1547*b^3)*e^(5*d*x + 5*c)/d
 - 1/49152*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*e^(3*d*x + 3*c)/
d + 1/65536*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*e^(d*x + c)
/d + 1/65536*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*e^(-d*x -
c)/d - 1/49152*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*e^(-3*d*x -
3*c)/d + 1/81920*(512*a^3 + 3456*a^2*b + 4290*a*b^2 + 1547*b^3)*e^(-5*d*x -
 5*c)/d - 1/229376*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*e^(-7*d*x - 7*c)/d
+ 1/294912*(192*a^2*b + 936*a*b^2 + 595*b^3)*e^(-9*d*x - 9*c)/d - 1/180224*
(78*a*b^2 + 85*b^3)*e^(-11*d*x - 11*c)/d + 1/212992*(6*a*b^2 + 17*b^3)*e^(-
13*d*x - 13*c)/d
```

maple [A] time = 0.17, size = 258, normalized size = 1.17

$$b^3\left(\frac{32768}{109395}+\frac{\left(\sinh^{16}(dx+c)\right)}{17}-\frac{16\left(\sinh^{14}(dx+c)\right)}{255}+\frac{224\left(\sinh^{12}(dx+c)\right)}{3315}-\frac{896\left(\sinh^{10}(dx+c)\right)}{12155}+\frac{1792\left(\sinh^{8}(dx+c)\right)}{21879}-\frac{2048\left(\sinh^{6}(dx+c)\right)}{21879}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(b^3*(32768/109395+1/17*sinh(d*x+c)^16-16/255*sinh(d*x+c)^14+224/3315*s
inh(d*x+c)^12-896/12155*sinh(d*x+c)^10+1792/21879*sinh(d*x+c)^8-2048/21879*
sinh(d*x+c)^6+4096/36465*sinh(d*x+c)^4-16384/109395*sinh(d*x+c)^2)*cosh(d*x
+c)+3*a*b^2*(1024/3003+1/13*sinh(d*x+c)^12-12/143*sinh(d*x+c)^10+40/429*sin
h(d*x+c)^8-320/3003*sinh(d*x+c)^6+128/1001*sinh(d*x+c)^4-512/3003*sinh(d*x+
c)^2)*cosh(d*x+c)+3*a^2*b*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/
105*sinh(d*x+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*(8/15+1/5*sinh(d*x+
c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.34, size = 600, normalized size = 2.73

$$-\frac{1}{14338621440}b^3\left(\frac{\left(123981\,e^{(-2\,dx-2\,c)}-1144440\,e^{(-4\,dx-4\,c)}+6762600\,e^{(-6\,dx-6\,c)}-28928900\,e^{(-8\,dx-8\,c)}+9670518\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/14338621440*b^3*((123981*e^(-2*d*x - 2*c) - 1144440*e^(-4*d*x - 4*c) + 6
762600*e^(-6*d*x - 6*c) - 28928900*e^(-8*d*x - 8*c) + 96705180*e^(-10*d*x -
 10*c) - 270774504*e^(-12*d*x - 12*c) + 709171320*e^(-14*d*x - 14*c) - 2659
392450*e^(-16*d*x - 16*c) - 6435)*e^(17*d*x + 17*c)/d - (2659392450*e^(-d*x
 - c) - 709171320*e^(-3*d*x - 3*c) + 270774504*e^(-5*d*x - 5*c) - 96705180*
e^(-7*d*x - 7*c) + 28928900*e^(-9*d*x - 9*c) - 6762600*e^(-11*d*x - 11*c) +
 1144440*e^(-13*d*x - 13*c) - 123981*e^(-15*d*x - 15*c) + 6435*e^(-17*d*x -
 17*c))/d) - 1/8200192*a*b^2*((3549*e^(-2*d*x - 2*c) - 26026*e^(-4*d*x - 4*
c) + 122694*e^(-6*d*x - 6*c) - 429429*e^(-8*d*x - 8*c) + 1288287*e^(-10*d*x
 - 10*c) - 5153148*e^(-12*d*x - 12*c) - 231)*e^(13*d*x + 13*c)/d - (5153148
*e^(-d*x - c) - 1288287*e^(-3*d*x - 3*c) + 429429*e^(-5*d*x - 5*c) - 122694
*e^(-7*d*x - 7*c) + 26026*e^(-9*d*x - 9*c) - 3549*e^(-11*d*x - 11*c) + 231*
e^(-13*d*x - 13*c))/d) - 1/53760*a^2*b*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*
```

```
d*x - 4*c) + 8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x
+ 9*c)/d - (39690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5
*c) - 405*e^(-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/480*a^3*(3*e^(5*d*
x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d
- 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

mupad [B] time = 1.76, size = 319, normalized size = 1.45

$$\frac{\frac{a^3\cosh(c+d\,x)^5}{5}-\frac{2\,a^3\cosh(c+d\,x)^3}{3}+a^3\cosh\left(c+d\,x\right)+\frac{a^2\,b\cosh(c+d\,x)^9}{3}-\frac{12\,a^2\,b\cosh(c+d\,x)^7}{7}+\frac{18\,a^2\,b\cosh(c+d\,x)^5}{5}-4\,a^2\,b\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5*(a + b*sinh(c + d*x)^4)^3,x)
```

```
[Out] (a^3*cosh(c + d*x) + b^3*cosh(c + d*x) - (2*a^3*cosh(c + d*x)^3)/3 + (a^3*c
osh(c + d*x)^5)/5 - (8*b^3*cosh(c + d*x)^3)/3 + (28*b^3*cosh(c + d*x)^5)/5
- 8*b^3*cosh(c + d*x)^7 + (70*b^3*cosh(c + d*x)^9)/9 - (56*b^3*cosh(c + d*x
)^11)/11 + (28*b^3*cosh(c + d*x)^13)/13 - (8*b^3*cosh(c + d*x)^15)/15 + (b^
3*cosh(c + d*x)^17)/17 - 6*a*b^2*cosh(c + d*x)^3 - 4*a^2*b*cosh(c + d*x)^3
+ 9*a*b^2*cosh(c + d*x)^5 + (18*a^2*b*cosh(c + d*x)^5)/5 - (60*a*b^2*cosh(c
 + d*x)^7)/7 - (12*a^2*b*cosh(c + d*x)^7)/7 + 5*a*b^2*cosh(c + d*x)^9 + (a^
2*b*cosh(c + d*x)^9)/3 - (18*a*b^2*cosh(c + d*x)^11)/11 + (3*a*b^2*cosh(c +
 d*x)^13)/13 + 3*a*b^2*cosh(c + d*x) + 3*a^2*b*cosh(c + d*x))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.208 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=183

$$\frac{b\left(3a^2+30ab+35b^2\right)\cosh^7(c+dx)}{7d}+\frac{3b^2(a+7b)\cosh^{11}(c+dx)}{11d}-\frac{5b^2(3a+7b)\cosh^9(c+dx)}{9d}-\frac{3b(a+b)(3a+7}{5}$$

```
[Out] -(a+b)^3*cosh(d*x+c)/d+1/3*(a+b)^2*(a+7*b)*cosh(d*x+c)^3/d-3/5*b*(a+b)*(3*a
+7*b)*cosh(d*x+c)^5/d+1/7*b*(3*a^2+30*a*b+35*b^2)*cosh(d*x+c)^7/d-5/9*b^2*(
3*a+7*b)*cosh(d*x+c)^9/d+3/11*b^2*(a+7*b)*cosh(d*x+c)^11/d-7/13*b^3*cosh(d*
x+c)^13/d+1/15*b^3*cosh(d*x+c)^15/d
```

Rubi [A] time = 0.18, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3215, 1153}

$$\frac{b\left(3a^2+30ab+35b^2\right)\cosh^7(c+dx)}{7d}+\frac{3b^2(a+7b)\cosh^{11}(c+dx)}{11d}-\frac{5b^2(3a+7b)\cosh^9(c+dx)}{9d}-\frac{3b(a+b)(3a+7}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(((a + b)^3*Cosh[c + d*x])/d) + ((a + b)^2*(a + 7*b)*Cosh[c + d*x]^3)/(3*d
) - (3*b*(a + b)*(3*a + 7*b)*Cosh[c + d*x]^5)/(5*d) + (b*(3*a^2 + 30*a*b +
35*b^2)*Cosh[c + d*x]^7)/(7*d) - (5*b^2*(3*a + 7*b)*Cosh[c + d*x]^9)/(9*d)
+ (3*b^2*(a + 7*b)*Cosh[c + d*x]^11)/(11*d) - (7*b^3*Cosh[c + d*x]^13)/(13*
d) + (b^3*Cosh[c + d*x]^15)/(15*d)
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= -\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+b-2bx^2+bx^4\right)^3\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int\left((a+b)^3-(a+b)^2(a+7b)x^2+3b(a+b)(3a+7b)x^4-\right.\right.}{}\\ &= -\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{(a+b)^2(a+7b)\cosh^3(c+dx)}{3d}-\frac{3b(a+}{}\end{aligned}$$

Mathematica [A] time = 2.46, size = 185, normalized size = 1.01

$$\frac{b\left(-27027\left(1792a^2+2640ab+1001b^2\right)\cosh(5(c+dx))+19305\left(256a^2+880ab+455b^2\right)\cosh(7(c+dx))-7b(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-135135*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*Cosh[c + d*x] + 15
015*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*Cosh[3*(c + d*x)] + b
*(-27027*(1792*a^2 + 2640*a*b + 1001*b^2)*Cosh[5*(c + d*x)] + 19305*(256*a^
2 + 880*a*b + 455*b^2)*Cosh[7*(c + d*x)] - 7*b*(715*(528*a + 455*b)*Cosh[9*
(c + d*x)] - 1755*(16*a + 35*b)*Cosh[11*(c + d*x)] + 7425*b*Cosh[13*(c + d*
x)] - 429*b*Cosh[15*(c + d*x)])))/(738017280*d)
```

fricas [B] time = 1.21, size = 795, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/738017280*(3003*b^3*cosh(d*x + c)^15 + 45045*b^3*cosh(d*x + c)*sinh(d*x +
 c)^14 - 51975*b^3*cosh(d*x + c)^13 + 15015*(91*b^3*cosh(d*x + c)^3 - 45*b^
3*cosh(d*x + c))*sinh(d*x + c)^12 + 12285*(16*a*b^2 + 35*b^3)*cosh(d*x + c)
^11 + 9009*(1001*b^3*cosh(d*x + c)^5 - 1650*b^3*cosh(d*x + c)^3 + 15*(16*a*
b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 - 5005*(528*a*b^2 + 455*b^3)*
cosh(d*x + c)^9 + 45045*(429*b^3*cosh(d*x + c)^7 - 1485*b^3*cosh(d*x + c)^5
 + 45*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^3 - (528*a*b^2 + 455*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^8 + 19305*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x +
c)^7 + 15015*(1001*b^3*cosh(d*x + c)^9 - 5940*b^3*cosh(d*x + c)^7 + 378*(16
*a*b^2 + 35*b^3)*cosh(d*x + c)^5 - 28*(528*a*b^2 + 455*b^3)*cosh(d*x + c)^3
 + 9*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 270
27*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(d*x + c)^5 + 45045*(91*b^3*cos
h(d*x + c)^11 - 825*b^3*cosh(d*x + c)^9 + 90*(16*a*b^2 + 35*b^3)*cosh(d*x +
 c)^7 - 14*(528*a*b^2 + 455*b^3)*cosh(d*x + c)^5 + 15*(256*a^2*b + 880*a*b^
2 + 455*b^3)*cosh(d*x + c)^3 - 3*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(
d*x + c))*sinh(d*x + c)^4 + 15015*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5
005*b^3)*cosh(d*x + c)^3 + 45045*(7*b^3*cosh(d*x + c)^13 - 90*b^3*cosh(d*x
+ c)^11 + 15*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^9 - 4*(528*a*b^2 + 455*b^3)*
cosh(d*x + c)^7 + 9*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x + c)^5 - 6*(
1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 + (4096*a^3 + 16128*a^2
*b + 15840*a*b^2 + 5005*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 135135*(4096*
a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*cosh(d*x + c))/d
```

giac [B] time = 0.40, size = 446, normalized size = 2.44

$$\frac{b^3e^{(15\,dx+15\,c)}}{491520\,d}-\frac{15\,b^3e^{(13\,dx+13\,c)}}{425984\,d}-\frac{15\,b^3e^{(-13\,dx-13\,c)}}{425984\,d}+\frac{b^3e^{(-15\,dx-15\,c)}}{491520\,d}+\frac{3\left(16\,ab^2+35\,b^3\right)e^{(11\,dx+11\,c)}}{360448\,d}-\frac{\left(528\,ab^2+45\right.}{2949}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/491520*b^3*e^(15*d*x + 15*c)/d - 15/425984*b^3*e^(13*d*x + 13*c)/d - 15/4
25984*b^3*e^(-13*d*x - 13*c)/d + 1/491520*b^3*e^(-15*d*x - 15*c)/d + 3/3604
48*(16*a*b^2 + 35*b^3)*e^(11*d*x + 11*c)/d - 1/294912*(528*a*b^2 + 455*b^3)
*e^(9*d*x + 9*c)/d + 3/229376*(256*a^2*b + 880*a*b^2 + 455*b^3)*e^(7*d*x +
7*c)/d - 3/163840*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*e^(5*d*x + 5*c)/d +
1/98304*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*e^(3*d*x + 3*c)/d
 - 3/32768*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*e^(d*x + c)/d -
3/32768*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*e^(-d*x - c)/d + 1/
98304*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*e^(-3*d*x - 3*c)/d
- 3/163840*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*e^(-5*d*x - 5*c)/d + 3/2293
76*(256*a^2*b + 880*a*b^2 + 455*b^3)*e^(-7*d*x - 7*c)/d - 1/294912*(528*a*b
```

```
^2 + 455*b^3)*e^(-9*d*x - 9*c)/d + 3/360448*(16*a*b^2 + 35*b^3)*e^(-11*d*x
- 11*c)/d
```

maple [A] time = 0.13, size = 218, normalized size = 1.19

$$b^3\left(-\frac{2048}{6435}+\frac{\left(\sinh^{14}(dx+c)\right)}{15}-\frac{14\left(\sinh^{12}(dx+c)\right)}{195}+\frac{56\left(\sinh^{10}(dx+c)\right)}{715}-\frac{112\left(\sinh^{8}(dx+c)\right)}{1287}+\frac{128\left(\sinh^{6}(dx+c)\right)}{1287}-\frac{256\left(\sinh^{4}(dx+c)\right)}{2145}+\frac{102}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(b^3*(-2048/6435+1/15*sinh(d*x+c)^14-14/195*sinh(d*x+c)^12+56/715*sinh(
d*x+c)^10-112/1287*sinh(d*x+c)^8+128/1287*sinh(d*x+c)^6-256/2145*sinh(d*x+c
)^4+1024/6435*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(-256/693+1/11*sinh(d*x+c)
^10-10/99*sinh(d*x+c)^8+80/693*sinh(d*x+c)^6-32/231*sinh(d*x+c)^4+128/693*s
inh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c
)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c
))
```

maxima [B] time = 0.34, size = 501, normalized size = 2.74

$$-\frac{1}{210862080}b^3\left(\frac{\left(7425\,e^{(-2\,dx-2\,c)}-61425\,e^{(-4\,dx-4\,c)}+325325\,e^{(-6\,dx-6\,c)}-1254825\,e^{(-8\,dx-8\,c)}+3864861\,e^{(-10\,dx-1}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/210862080*b^3*((7425*e^(-2*d*x - 2*c) - 61425*e^(-4*d*x - 4*c) + 325325*
e^(-6*d*x - 6*c) - 1254825*e^(-8*d*x - 8*c) + 3864861*e^(-10*d*x - 10*c) -
10735725*e^(-12*d*x - 12*c) + 41409225*e^(-14*d*x - 14*c) - 429)*e^(15*d*x
+ 15*c)/d + (41409225*e^(-d*x - c) - 10735725*e^(-3*d*x - 3*c) + 3864861*e^
(-5*d*x - 5*c) - 1254825*e^(-7*d*x - 7*c) + 325325*e^(-9*d*x - 9*c) - 61425
*e^(-11*d*x - 11*c) + 7425*e^(-13*d*x - 13*c) - 429*e^(-15*d*x - 15*c))/d)
- 1/473088*a*b^2*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(
-6*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^
(11*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e
^(-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*
d*x - 11*c))/d) - 3/4480*a^2*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c)
 + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*
e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a^3*
(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/
d)
```

mupad [B] time = 1.33, size = 266, normalized size = 1.45

$$-\frac{-\frac{a^3\cosh(c+d\,x)^3}{3}+a^3\cosh\left(c+d\,x\right)-\frac{3\,a^2\,b\cosh(c+d\,x)^7}{7}+\frac{9\,a^2\,b\cosh(c+d\,x)^5}{5}-3\,a^2\,b\cosh\left(c+d\,x\right)^3+3\,a^2\,b\cosh\left(c+d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*sinh(c + d*x)^4)^3,x)
```

```
[Out] -(a^3*cosh(c + d*x) + b^3*cosh(c + d*x) - (a^3*cosh(c + d*x)^3)/3 - (7*b^3*
cosh(c + d*x)^3)/3 + (21*b^3*cosh(c + d*x)^5)/5 - 5*b^3*cosh(c + d*x)^7 + (
35*b^3*cosh(c + d*x)^9)/9 - (21*b^3*cosh(c + d*x)^11)/11 + (7*b^3*cosh(c +
d*x)^13)/13 - (b^3*cosh(c + d*x)^15)/15 - 5*a*b^2*cosh(c + d*x)^3 - 3*a^2*b
*cosh(c + d*x)^3 + 6*a*b^2*cosh(c + d*x)^5 + (9*a^2*b*cosh(c + d*x)^5)/5 -
(30*a*b^2*cosh(c + d*x)^7)/7 - (3*a^2*b*cosh(c + d*x)^7)/7 + (5*a*b^2*cosh(
```

```
c + d*x)^9)/3 - (3*a*b^2*cosh(c + d*x)^11)/11 + 3*a*b^2*cosh(c + d*x) + 3*a
^2*b*cosh(c + d*x))/d
```

sympy [A] time = 139.98, size = 484, normalized size = 2.64

$$\begin{cases} \frac{a^3\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^3\cosh^3(c+dx)}{3d} + \frac{3a^2b\sinh^6(c+dx)\cosh(c+dx)}{d} - \frac{6a^2b\sinh^4(c+dx)\cosh^3(c+dx)}{d} + \frac{24a^2b\sinh^2(c+dx)\cosh^5(c+dx)}{5d} \\ x\left(a+b\sinh^4(c)\right)^3\sinh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**3*cosh(c + d*x)**3/
(3*d) + 3*a**2*b*sinh(c + d*x)**6*cosh(c + d*x)/d - 6*a**2*b*sinh(c + d*x)*
*4*cosh(c + d*x)**3/d + 24*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) -
 48*a**2*b*cosh(c + d*x)**7/(35*d) + 3*a*b**2*sinh(c + d*x)**10*cosh(c + d*
x)/d - 10*a*b**2*sinh(c + d*x)**8*cosh(c + d*x)**3/d + 16*a*b**2*sinh(c + d
*x)**6*cosh(c + d*x)**5/d - 96*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**7/(7*
d) + 128*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**9/(21*d) - 256*a*b**2*cosh(
c + d*x)**11/(231*d) + b**3*sinh(c + d*x)**14*cosh(c + d*x)/d - 14*b**3*sin
h(c + d*x)**12*cosh(c + d*x)**3/(3*d) + 56*b**3*sinh(c + d*x)**10*cosh(c +
d*x)**5/(5*d) - 16*b**3*sinh(c + d*x)**8*cosh(c + d*x)**7/d + 128*b**3*sinh
(c + d*x)**6*cosh(c + d*x)**9/(9*d) - 256*b**3*sinh(c + d*x)**4*cosh(c + d*
x)**11/(33*d) + 1024*b**3*sinh(c + d*x)**2*cosh(c + d*x)**13/(429*d) - 2048
*b**3*cosh(c + d*x)**15/(6435*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**3*sinh(
c)**3, True))
```

3.209 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=143

$$\frac{b^2(a+5b)\cosh^9(c+dx)}{3d}-\frac{4b^2(3a+5b)\cosh^7(c+dx)}{7d}+\frac{3b(a+b)(a+5b)\cosh^5(c+dx)}{5d}-\frac{2b(a+b)^2\cosh^3(c+dx)}{d}$$

```
[Out] (a+b)^3*cosh(d*x+c)/d-2*b*(a+b)^2*cosh(d*x+c)^3/d+3/5*b*(a+b)*(a+5*b)*cosh(
d*x+c)^5/d-4/7*b^2*(3*a+5*b)*cosh(d*x+c)^7/d+1/3*b^2*(a+5*b)*cosh(d*x+c)^9/
d-6/11*b^3*cosh(d*x+c)^11/d+1/13*b^3*cosh(d*x+c)^13/d
```

Rubi [A] time = 0.16, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3215, 1090}

$$\frac{b^2(a+5b)\cosh^9(c+dx)}{3d}-\frac{4b^2(3a+5b)\cosh^7(c+dx)}{7d}+\frac{3b(a+b)(a+5b)\cosh^5(c+dx)}{5d}-\frac{2b(a+b)^2\cosh^3(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*Cosh[c + d*x])/d - (2*b*(a + b)^2*Cosh[c + d*x]^3)/d + (3*b*(a +
 b)*(a + 5*b)*Cosh[c + d*x]^5)/(5*d) - (4*b^2*(3*a + 5*b)*Cosh[c + d*x]^7)/
(7*d) + (b^2*(a + 5*b)*Cosh[c + d*x]^9)/(3*d) - (6*b^3*Cosh[c + d*x]^11)/(1
1*d) + (b^3*Cosh[c + d*x]^13)/(13*d)
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \left(a+b-2bx^2+bx^4\right)^3\,dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^3\left(1+\frac{b\left(3a^2+3ab+b^2\right)}{a^3}\right)-6b(a+b)^2x^2+12b^2(a+b)\left(1+\frac{a+}{4}\right.\right.\right.}{}\\ &= \frac{(a+b)^3\cosh(c+dx)}{d}-\frac{2b(a+b)^2\cosh^3(c+dx)}{d}+\frac{3b(a+b)(a+5b}{5}\end{aligned}$$

Mathematica [A] time = 0.90, size = 157, normalized size = 1.10

$$\frac{-15015b\left(1280a^2+1344ab+429b^2\right)\cosh(3(c+dx))+3003b\left(768a^2+1728ab+715b^2\right)\cosh(5(c+dx))+6006}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (60060*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*Cosh[c + d*x] - 15015
*b*(1280*a^2 + 1344*a*b + 429*b^2)*Cosh[3*(c + d*x)] + 3003*b*(768*a^2 + 17
28*a*b + 715*b^2)*Cosh[5*(c + d*x)] - 4290*b^2*(216*a + 143*b)*Cosh[7*(c +
d*x)] + 10010*b^2*(8*a + 13*b)*Cosh[9*(c + d*x)] - 17745*b^3*Cosh[11*(c + d
*x)] + 1155*b^3*Cosh[13*(c + d*x)])/(61501440*d)
```

fricas [B] time = 1.37, size = 594, normalized size = 4.15

$$1155\, b^3 \cosh\left(dx+c\right)^{13} + 15015\, b^3 \cosh\left(dx+c\right) \sinh\left(dx+c\right)^{12} - 17745\, b^3 \cosh\left(dx+c\right)^{11} + 15015\, \Big(22\, b^3 \cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/61501440*(1155*b^3*cosh(d*x + c)^13 + 15015*b^3*cosh(d*x + c)*sinh(d*x +
c)^12 - 17745*b^3*cosh(d*x + c)^11 + 15015*(22*b^3*cosh(d*x + c)^3 - 13*b^3
*cosh(d*x + c))*sinh(d*x + c)^10 + 10010*(8*a*b^2 + 13*b^3)*cosh(d*x + c)^9
 + 45045*(33*b^3*cosh(d*x + c)^5 - 65*b^3*cosh(d*x + c)^3 + 2*(8*a*b^2 + 13
*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 - 4290*(216*a*b^2 + 143*b^3)*cosh(d*x
+ c)^7 + 30030*(66*b^3*cosh(d*x + c)^7 - 273*b^3*cosh(d*x + c)^5 + 28*(8*a*
b^2 + 13*b^3)*cosh(d*x + c)^3 - (216*a*b^2 + 143*b^3)*cosh(d*x + c))*sinh(d
*x + c)^6 + 3003*(768*a^2*b + 1728*a*b^2 + 715*b^3)*cosh(d*x + c)^5 + 15015
*(55*b^3*cosh(d*x + c)^9 - 390*b^3*cosh(d*x + c)^7 + 84*(8*a*b^2 + 13*b^3)*
cosh(d*x + c)^5 - 10*(216*a*b^2 + 143*b^3)*cosh(d*x + c)^3 + (768*a^2*b + 1
728*a*b^2 + 715*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 15015*(1280*a^2*b + 1
344*a*b^2 + 429*b^3)*cosh(d*x + c)^3 + 15015*(6*b^3*cosh(d*x + c)^11 - 65*b
^3*cosh(d*x + c)^9 + 24*(8*a*b^2 + 13*b^3)*cosh(d*x + c)^7 - 6*(216*a*b^2 +
 143*b^3)*cosh(d*x + c)^5 + 2*(768*a^2*b + 1728*a*b^2 + 715*b^3)*cosh(d*x +
 c)^3 - 3*(1280*a^2*b + 1344*a*b^2 + 429*b^3)*cosh(d*x + c))*sinh(d*x + c)^
2 + 60060*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*cosh(d*x + c))/d
```

giac [B] time = 0.35, size = 372, normalized size = 2.60

$$\frac{b^3 e^{(13\, dx+13\, c)}}{106496\, d} - \frac{13\, b^3 e^{(11\, dx+11\, c)}}{90112\, d} - \frac{13\, b^3 e^{(-11\, dx-11\, c)}}{90112\, d} + \frac{b^3 e^{(-13\, dx-13\, c)}}{106496\, d} + \frac{\left(8\, ab^2 + 13\, b^3\right) e^{(9\, dx+9\, c)}}{12288\, d} - \frac{\left(216\, ab^2 + 143\, b^3\right)}{28672\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/106496*b^3*e^(13*d*x + 13*c)/d - 13/90112*b^3*e^(11*d*x + 11*c)/d - 13/90
112*b^3*e^(-11*d*x - 11*c)/d + 1/106496*b^3*e^(-13*d*x - 13*c)/d + 1/12288*
(8*a*b^2 + 13*b^3)*e^(9*d*x + 9*c)/d - 1/28672*(216*a*b^2 + 143*b^3)*e^(7*d
*x + 7*c)/d + 1/40960*(768*a^2*b + 1728*a*b^2 + 715*b^3)*e^(5*d*x + 5*c)/d
- 1/8192*(1280*a^2*b + 1344*a*b^2 + 429*b^3)*e^(3*d*x + 3*c)/d + 1/2048*(10
24*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*e^(d*x + c)/d + 1/2048*(1024*a^
3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*e^(-d*x - c)/d - 1/8192*(1280*a^2*b
+ 1344*a*b^2 + 429*b^3)*e^(-3*d*x - 3*c)/d + 1/40960*(768*a^2*b + 1728*a*b^
2 + 715*b^3)*e^(-5*d*x - 5*c)/d - 1/28672*(216*a*b^2 + 143*b^3)*e^(-7*d*x -
 7*c)/d + 1/12288*(8*a*b^2 + 13*b^3)*e^(-9*d*x - 9*c)/d
```

maple [A] time = 0.05, size = 176, normalized size = 1.23

$$b^3 \left(\frac{1024}{3003} + \frac{\left(\sinh^{12}(dx+c)\right)}{13} - \frac{12\left(\sinh^{10}(dx+c)\right)}{143} + \frac{40\left(\sinh^{8}(dx+c)\right)}{429} - \frac{320\left(\sinh^{6}(dx+c)\right)}{3003} + \frac{128\left(\sinh^{4}(dx+c)\right)}{1001} - \frac{512\left(\sinh^{2}(dx+c)\right)}{3003}\right) \cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(b^3*(1024/3003+1/13*sinh(d*x+c)^12-12/143*sinh(d*x+c)^10+40/429*sinh(d
*x+c)^8-320/3003*sinh(d*x+c)^6+128/1001*sinh(d*x+c)^4-512/3003*sinh(d*x+c)^
2)*cosh(d*x+c)+3*a*b^2*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105
*sinh(d*x+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(8/15+1/5*sinh(d*x
+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*cosh(d*x+c))
```

maxima [B] time = 0.34, size = 399, normalized size = 2.79

$$-\frac{1}{24600576}b^3\left(\frac{\left(3549\,e^{(-2\,dx-2\,c)}-26026\,e^{(-4\,dx-4\,c)}+122694\,e^{(-6\,dx-6\,c)}-429429\,e^{(-8\,dx-8\,c)}+1288287\,e^{(-10\,dx-10\,c}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/24600576*b^3*((3549*e^(-2*d*x - 2*c) - 26026*e^(-4*d*x - 4*c) + 122694*e
^(-6*d*x - 6*c) - 429429*e^(-8*d*x - 8*c) + 1288287*e^(-10*d*x - 10*c) - 51
53148*e^(-12*d*x - 12*c) - 231)*e^(13*d*x + 13*c)/d - (5153148*e^(-d*x - c)
 - 1288287*e^(-3*d*x - 3*c) + 429429*e^(-5*d*x - 5*c) - 122694*e^(-7*d*x -
7*c) + 26026*e^(-9*d*x - 9*c) - 3549*e^(-11*d*x - 11*c) + 231*e^(-13*d*x -
13*c))/d) - 1/53760*a*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) +
8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (3
9690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(
-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/160*a^2*b*(3*e^(5*d*x + 5*c)/d
- 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*
d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^3*cosh(d*x + c)/d
```

mupad [B] time = 1.07, size = 211, normalized size = 1.48

$$\frac{a^3\cosh(c+d\,x)+\frac{3\,a^2\,b\cosh(c+d\,x)^5}{5}-2\,a^2\,b\cosh(c+d\,x)^3+3\,a^2\,b\cosh(c+d\,x)+\frac{a\,b^2\cosh(c+d\,x)^9}{3}-\frac{12\,a\,b^2\cosh(c+d\,x)}{7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)*(a + b*sinh(c + d*x)^4)^3,x)
```

```
[Out] (a^3*cosh(c + d*x) + b^3*cosh(c + d*x) - 2*b^3*cosh(c + d*x)^3 + 3*b^3*cosh
(c + d*x)^5 - (20*b^3*cosh(c + d*x)^7)/7 + (5*b^3*cosh(c + d*x)^9)/3 - (6*b
^3*cosh(c + d*x)^11)/11 + (b^3*cosh(c + d*x)^13)/13 - 4*a*b^2*cosh(c + d*x)
^3 - 2*a^2*b*cosh(c + d*x)^3 + (18*a*b^2*cosh(c + d*x)^5)/5 + (3*a^2*b*cosh
(c + d*x)^5)/5 - (12*a*b^2*cosh(c + d*x)^7)/7 + (a*b^2*cosh(c + d*x)^9)/3 +
 3*a*b^2*cosh(c + d*x) + 3*a^2*b*cosh(c + d*x))/d
```

sympy [A] time = 69.18, size = 377, normalized size = 2.64

$$\begin{cases} \frac{a^3\cosh(c+dx)}{d}+\frac{3a^2b\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{4a^2b\sinh^2(c+dx)\cosh^3(c+dx)}{d}+\frac{8a^2b\cosh^5(c+dx)}{5d}+\frac{3ab^2\sinh^8(c+dx)\cosh(c+dx)}{d}-\frac{8al}{} \\ x\left(a+b\sinh^4(c)\right)^3\sinh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d
 - 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5
/(5*d) + 3*a*b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*a*b**2*sinh(c + d*x)
```

```
**6*cosh(c + d*x)**3/d + 48*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d)
- 192*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*a*b**2*cosh(c +
 d*x)**9/(105*d) + b**3*sinh(c + d*x)**12*cosh(c + d*x)/d - 4*b**3*sinh(c +
 d*x)**10*cosh(c + d*x)**3/d + 8*b**3*sinh(c + d*x)**8*cosh(c + d*x)**5/d -
 64*b**3*sinh(c + d*x)**6*cosh(c + d*x)**7/(7*d) + 128*b**3*sinh(c + d*x)**
4*cosh(c + d*x)**9/(21*d) - 512*b**3*sinh(c + d*x)**2*cosh(c + d*x)**11/(23
1*d) + 1024*b**3*cosh(c + d*x)**13/(3003*d), Ne(d, 0)), (x*(a + b*sinh(c)**
4)**3*sinh(c), True))
```

3.210 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=158

$$-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\left(3a^2+9ab+5b^2\right)\cosh^3(c+dx)}{3d}-\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d}+\frac{b^2(3a+10b)\,\text{cc}}{7d}$$

```
[Out] -a^3*arctanh(cosh(d*x+c))/d-b*(3*a^2+3*a*b+b^2)*cosh(d*x+c)/d+1/3*b*(3*a^2+
9*a*b+5*b^2)*cosh(d*x+c)^3/d-1/5*b^2*(9*a+10*b)*cosh(d*x+c)^5/d+1/7*b^2*(3*
a+10*b)*cosh(d*x+c)^7/d-5/9*b^3*cosh(d*x+c)^9/d+1/11*b^3*cosh(d*x+c)^11/d
```

Rubi [A] time = 0.14, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3215, 1153, 206}

$$\frac{b\left(3a^2+9ab+5b^2\right)\cosh^3(c+dx)}{3d}-\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2(3a+10b)\cos}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) - (b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x])
/d + (b*(3*a^2 + 9*a*b + 5*b^2)*Cosh[c + d*x]^3)/(3*d) - (b^2*(9*a + 10*b)*
Cosh[c + d*x]^5)/(5*d) + (b^2*(3*a + 10*b)*Cosh[c + d*x]^7)/(7*d) - (5*b^3*
Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^11)/(11*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{1-x^2}dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(b\left(3a^2+3ab+b^2\right)-b\left(3a^2+9ab+5b^2\right)x^2+b^2(9a\right.\right.}{}$$

$$= -\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d}+\frac{b\left(3a^2+9ab+5b^2\right)\cosh^3(c}{3d}$$

$$= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d}+\frac{b}{}$$

Mathematica [A] time = 0.37, size = 139, normalized size = 0.88

$$\frac{3548160a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-20790b\left(384a^2+280ab+77b^2\right)\cosh(c+dx)-2079b^2(112a+55b)\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-20790*b*(384*a^2 + 280*a*b + 77*b^2)*Cosh[c + d*x] + 6930*b*(8*a + 5*b)*(
16*a + 11*b)*Cosh[3*(c + d*x)] - 2079*b^2*(112*a + 55*b)*Cosh[5*(c + d*x)]
+ 495*b^2*(48*a + 55*b)*Cosh[7*(c + d*x)] - 4235*b^3*Cosh[9*(c + d*x)] + 31
5*b^3*Cosh[11*(c + d*x)] + 3548160*a^3*Log[Tanh[(c + d*x)/2]])/(3548160*d)
```

fricas [B] time = 0.97, size = 3824, normalized size = 24.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/7096320*(315*b^3*cosh(d*x + c)^22 + 6930*b^3*cosh(d*x + c)*sinh(d*x + c)^
21 + 315*b^3*sinh(d*x + c)^22 - 4235*b^3*cosh(d*x + c)^20 + 385*(189*b^3*co
sh(d*x + c)^2 - 11*b^3)*sinh(d*x + c)^20 + 7700*(63*b^3*cosh(d*x + c)^3 - 1
1*b^3*cosh(d*x + c))*sinh(d*x + c)^19 + 495*(48*a*b^2 + 55*b^3)*cosh(d*x +
c)^18 + 55*(41895*b^3*cosh(d*x + c)^4 - 14630*b^3*cosh(d*x + c)^2 + 432*a*b
^2 + 495*b^3)*sinh(d*x + c)^18 + 330*(25137*b^3*cosh(d*x + c)^5 - 14630*b^3
*cosh(d*x + c)^3 + 27*(48*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 -
 2079*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^16 + 33*(712215*b^3*cosh(d*x + c)^
6 - 621775*b^3*cosh(d*x + c)^4 - 7056*a*b^2 - 3465*b^3 + 2295*(48*a*b^2 + 5
5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 528*(101745*b^3*cosh(d*x + c)^7
- 124355*b^3*cosh(d*x + c)^5 + 765*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^3 - 63
*(112*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 6930*(128*a^2*b + 1
68*a*b^2 + 55*b^3)*cosh(d*x + c)^14 + 330*(305235*b^3*cosh(d*x + c)^8 - 497
420*b^3*cosh(d*x + c)^6 + 4590*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^4 + 2688*a
^2*b + 3528*a*b^2 + 1155*b^3 - 756*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^14 + 4620*(33915*b^3*cosh(d*x + c)^9 - 71060*b^3*cosh(d*x + c)^
7 + 918*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^5 - 252*(112*a*b^2 + 55*b^3)*cosh
(d*x + c)^3 + 21*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^13 - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^12 + 2310*(88
179*b^3*cosh(d*x + c)^10 - 230945*b^3*cosh(d*x + c)^8 + 3978*(48*a*b^2 + 55
*b^3)*cosh(d*x + c)^6 - 1638*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^4 - 3456*a^
2*b - 2520*a*b^2 - 693*b^3 + 273*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^12 + 8*(27776385*b^3*cosh(d*x + c)^11 - 88913825*b^3*
```

```
cosh(d*x + c)^9 + 1969110*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^7 - 1135134*(11
2*a*b^2 + 55*b^3)*cosh(d*x + c)^5 + 315315*(128*a^2*b + 168*a*b^2 + 55*b^3)
*cosh(d*x + c)^3 - 31185*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*si
nh(d*x + c)^11 - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^10 +
22*(9258795*b^3*cosh(d*x + c)^12 - 35565530*b^3*cosh(d*x + c)^10 + 984555*(
48*a*b^2 + 55*b^3)*cosh(d*x + c)^8 - 756756*(112*a*b^2 + 55*b^3)*cosh(d*x +
 c)^6 + 315315*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^4 - 362880*a^
2*b - 264600*a*b^2 - 72765*b^3 - 62370*(384*a^2*b + 280*a*b^2 + 77*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^10 + 220*(712215*b^3*cosh(d*x + c)^13 - 3233230
*b^3*cosh(d*x + c)^11 + 109395*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^9 - 108108
*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^7 + 63063*(128*a^2*b + 168*a*b^2 + 55*b
^3)*cosh(d*x + c)^5 - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^
3 - 945*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 6
930*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^8 + 330*(305235*b^3*cosh
(d*x + c)^14 - 1616615*b^3*cosh(d*x + c)^12 + 65637*(48*a*b^2 + 55*b^3)*cos
h(d*x + c)^10 - 81081*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^8 + 63063*(128*a^2
*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^6 - 31185*(384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^4 + 2688*a^2*b + 3528*a*b^2 + 1155*b^3 - 2835*(384*a^2
*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 2640*(20349*b^3
*cosh(d*x + c)^15 - 124355*b^3*cosh(d*x + c)^13 + 5967*(48*a*b^2 + 55*b^3)*
cosh(d*x + c)^11 - 9009*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^9 + 9009*(128*a^
2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^7 - 6237*(384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^5 - 945*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)
^3 + 21*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 2
079*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^6 + 231*(101745*b^3*cosh(d*x + c)^16
 - 710600*b^3*cosh(d*x + c)^14 + 39780*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^12
 - 72072*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^10 + 90090*(128*a^2*b + 168*a*b
^2 + 55*b^3)*cosh(d*x + c)^8 - 83160*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(
d*x + c)^6 - 18900*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^4 - 1008*
a*b^2 - 495*b^3 + 840*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^6 + 462*(17955*b^3*cosh(d*x + c)^17 - 142120*b^3*cosh(d*x + c)^1
5 + 9180*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^13 - 19656*(112*a*b^2 + 55*b^3)*
cosh(d*x + c)^11 + 30030*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^9 -
 35640*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^7 - 11340*(384*a^2*b
+ 280*a*b^2 + 77*b^3)*cosh(d*x + c)^5 + 840*(128*a^2*b + 168*a*b^2 + 55*b^3
)*cosh(d*x + c)^3 - 27*(112*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^5
- 4235*b^3*cosh(d*x + c)^2 + 495*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^4 + 165*
(13965*b^3*cosh(d*x + c)^18 - 124355*b^3*cosh(d*x + c)^16 + 9180*(48*a*b^2
+ 55*b^3)*cosh(d*x + c)^14 - 22932*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^12 +
42042*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^10 - 62370*(384*a^2*b
+ 280*a*b^2 + 77*b^3)*cosh(d*x + c)^8 - 26460*(384*a^2*b + 280*a*b^2 + 77*b
^3)*cosh(d*x + c)^6 + 2940*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^4
 + 144*a*b^2 + 165*b^3 - 189*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 660*(735*b^3*cosh(d*x + c)^19 - 7315*b^3*cosh(d*x + c)^17 + 612*(
48*a*b^2 + 55*b^3)*cosh(d*x + c)^15 - 1764*(112*a*b^2 + 55*b^3)*cosh(d*x +
c)^13 + 3822*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^11 - 6930*(384*
a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^9 - 3780*(384*a^2*b + 280*a*b^2 +
 77*b^3)*cosh(d*x + c)^7 + 588*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x +
c)^5 - 63*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^3 + 3*(48*a*b^2 + 55*b^3)*cosh
(d*x + c))*sinh(d*x + c)^3 + 315*b^3 + 55*(1323*b^3*cosh(d*x + c)^20 - 1463
0*b^3*cosh(d*x + c)^18 + 1377*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^16 - 4536*(
112*a*b^2 + 55*b^3)*cosh(d*x + c)^14 + 11466*(128*a^2*b + 168*a*b^2 + 55*b^
3)*cosh(d*x + c)^12 - 24948*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^
10 - 17010*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^8 + 3528*(128*a^2
*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^6 - 567*(112*a*b^2 + 55*b^3)*cosh(d*
x + c)^4 - 77*b^3 + 54*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 - 7096320*(a^3*cosh(d*x + c)^11 + 11*a^3*cosh(d*x + c)^10*sinh(d*x + c) +
55*a^3*cosh(d*x + c)^9*sinh(d*x + c)^2 + 165*a^3*cosh(d*x + c)^8*sinh(d*x +
 c)^3 + 330*a^3*cosh(d*x + c)^7*sinh(d*x + c)^4 + 462*a^3*cosh(d*x + c)^6*s
```

```
inh(d*x + c)^5 + 462*a^3*cosh(d*x + c)^5*sinh(d*x + c)^6 + 330*a^3*cosh(d*x
 + c)^4*sinh(d*x + c)^7 + 165*a^3*cosh(d*x + c)^3*sinh(d*x + c)^8 + 55*a^3*
cosh(d*x + c)^2*sinh(d*x + c)^9 + 11*a^3*cosh(d*x + c)*sinh(d*x + c)^10 + a
^3*sinh(d*x + c)^11)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 7096320*(a^3*
cosh(d*x + c)^11 + 11*a^3*cosh(d*x + c)^10*sinh(d*x + c) + 55*a^3*cosh(d*x
+ c)^9*sinh(d*x + c)^2 + 165*a^3*cosh(d*x + c)^8*sinh(d*x + c)^3 + 330*a^3*
cosh(d*x + c)^7*sinh(d*x + c)^4 + 462*a^3*cosh(d*x + c)^6*sinh(d*x + c)^5 +
 462*a^3*cosh(d*x + c)^5*sinh(d*x + c)^6 + 330*a^3*cosh(d*x + c)^4*sinh(d*x
 + c)^7 + 165*a^3*cosh(d*x + c)^3*sinh(d*x + c)^8 + 55*a^3*cosh(d*x + c)^2*
sinh(d*x + c)^9 + 11*a^3*cosh(d*x + c)*sinh(d*x + c)^10 + a^3*sinh(d*x + c)
^11)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 22*(315*b^3*cosh(d*x + c)^21
- 3850*b^3*cosh(d*x + c)^19 + 405*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^17 - 15
12*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^15 + 4410*(128*a^2*b + 168*a*b^2 + 55
*b^3)*cosh(d*x + c)^13 - 11340*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x +
c)^11 - 9450*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^9 + 2520*(128*a
^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^7 - 567*(112*a*b^2 + 55*b^3)*cosh(
d*x + c)^5 - 385*b^3*cosh(d*x + c) + 90*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^3
)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)^10*sinh(d*x + c)
+ 55*d*cosh(d*x + c)^9*sinh(d*x + c)^2 + 165*d*cosh(d*x + c)^8*sinh(d*x + c
)^3 + 330*d*cosh(d*x + c)^7*sinh(d*x + c)^4 + 462*d*cosh(d*x + c)^6*sinh(d*
x + c)^5 + 462*d*cosh(d*x + c)^5*sinh(d*x + c)^6 + 330*d*cosh(d*x + c)^4*si
nh(d*x + c)^7 + 165*d*cosh(d*x + c)^3*sinh(d*x + c)^8 + 55*d*cosh(d*x + c)^
2*sinh(d*x + c)^9 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^1
1)
```

giac [B] time = 0.41, size = 377, normalized size = 2.39

$$\frac{315\, b^3 e^{(11\, dx+11\, c)} - 4235\, b^3 e^{(9\, dx+9\, c)} + 23760\, ab^2 e^{(7\, dx+7\, c)} + 27225\, b^3 e^{(7\, dx+7\, c)} - 232848\, ab^2 e^{(5\, dx+5\, c)} - 114345\, b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/7096320*(315*b^3*e^(11*d*x + 11*c) - 4235*b^3*e^(9*d*x + 9*c) + 23760*a*b
^2*e^(7*d*x + 7*c) + 27225*b^3*e^(7*d*x + 7*c) - 232848*a*b^2*e^(5*d*x + 5*
c) - 114345*b^3*e^(5*d*x + 5*c) + 887040*a^2*b*e^(3*d*x + 3*c) + 1164240*a*
b^2*e^(3*d*x + 3*c) + 381150*b^3*e^(3*d*x + 3*c) - 7983360*a^2*b*e^(d*x + c
) - 5821200*a*b^2*e^(d*x + c) - 1600830*b^3*e^(d*x + c) - 7096320*a^3*log(e
^(d*x + c) + 1) + 7096320*a^3*log(abs(e^(d*x + c) - 1)) - (7983360*a^2*b*e^
(10*d*x + 10*c) + 5821200*a*b^2*e^(10*d*x + 10*c) + 1600830*b^3*e^(10*d*x +
 10*c) - 887040*a^2*b*e^(8*d*x + 8*c) - 1164240*a*b^2*e^(8*d*x + 8*c) - 381
150*b^3*e^(8*d*x + 8*c) + 232848*a*b^2*e^(6*d*x + 6*c) + 114345*b^3*e^(6*d*
x + 6*c) - 23760*a*b^2*e^(4*d*x + 4*c) - 27225*b^3*e^(4*d*x + 4*c) + 4235*b
^3*e^(2*d*x + 2*c) - 315*b^3)*e^(-11*d*x - 11*c))/d
```

maple [A] time = 0.08, size = 148, normalized size = 0.94

$$\frac{-2a^3 \operatorname{arctanh}\left(e^{dx+c}\right) + 3a^2 b \left(-\frac{2}{3} + \frac{\left(\sinh^2(dx+c)\right)}{3}\right) \cosh(dx+c) + 3a\, b^2 \left(-\frac{16}{35} + \frac{\left(\sinh^6(dx+c)\right)}{7} - \frac{6\left(\sinh^4(dx+c)\right)}{35} + \frac{8(\mathrm{si}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c
)+3*a*b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*
cosh(d*x+c)+b^3*(-256/693+1/11*sinh(d*x+c)^10-10/99*sinh(d*x+c)^8+80/693*si
nh(d*x+c)^6-32/231*sinh(d*x+c)^4+128/693*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.34, size = 327, normalized size = 2.07

$$-\frac{1}{1419264}\,b^3\left(\frac{\left(847\,e^{(-2\,dx-2\,c)}-5445\,e^{(-4\,dx-4\,c)}+22869\,e^{(-6\,dx-6\,c)}-76230\,e^{(-8\,dx-8\,c)}+320166\,e^{(-10\,dx-10\,c)}-63\right)e}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/1419264*b^3*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6
*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(1
1*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(
-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*
x - 11*c))/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) +
 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^
(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/8*a^2*b*(
e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d
) + a^3*log(tanh(1/2*d*x + 1/2*c))/d
```

mupad [B] time = 0.65, size = 326, normalized size = 2.06

$$\frac{\mathrm{e}^{-3\,c-3\,d\,x}\,\left(128\,a^2\,b+168\,a\,b^2+55\,b^3\right)}{1024\,d}-\frac{2\,\mathrm{atan}\left(\frac{a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}+\frac{\mathrm{e}^{3\,c+3\,d\,x}\,\left(128\,a^2\,b+168\,a\,b^2+55\,b^3\right)}{1024\,d}-\frac{11\,b^3}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x),x)
```

```
[Out] (exp(- 3*c - 3*d*x)*(168*a*b^2 + 128*a^2*b + 55*b^3))/(1024*d) - (2*atan((a
^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2)
 + (exp(3*c + 3*d*x)*(168*a*b^2 + 128*a^2*b + 55*b^3))/(1024*d) - (11*b^3*e
xp(- 9*c - 9*d*x))/(18432*d) - (11*b^3*exp(9*c + 9*d*x))/(18432*d) + (b^3*e
xp(- 11*c - 11*d*x))/(22528*d) + (b^3*exp(11*c + 11*d*x))/(22528*d) - (3*b*
exp(- c - d*x)*(280*a*b + 384*a^2 + 77*b^2))/(1024*d) + (b^2*exp(- 7*c - 7*
d*x)*(48*a + 55*b))/(14336*d) + (b^2*exp(7*c + 7*d*x)*(48*a + 55*b))/(14336
*d) - (3*b^2*exp(- 5*c - 5*d*x)*(112*a + 55*b))/(10240*d) - (3*b^2*exp(5*c
+ 5*d*x)*(112*a + 55*b))/(10240*d) - (3*b*exp(c + d*x)*(280*a*b + 384*a^2 +
 77*b^2))/(1024*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.211 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=148

$$\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d}+\frac{3b^2(a+2b)\cosh^5(c}{5d}$$

```
[Out] 1/2*a^3*arctanh(cosh(d*x+c))/d+b*(3*a^2+3*a*b+b^2)*cosh(d*x+c)/d-2/3*b^2*(3
*a+2*b)*cosh(d*x+c)^3/d+3/5*b^2*(a+2*b)*cosh(d*x+c)^5/d-4/7*b^3*cosh(d*x+c)
^7/d+1/9*b^3*cosh(d*x+c)^9/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d
```

Rubi [A] time = 0.21, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3215, 1157, 1810, 206}

$$\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{3b^2(a+2b)\cosh^5(c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]
)/d - (2*b^2*(3*a + 2*b)*Cosh[c + d*x]^3)/(3*d) + (3*b^2*(a + 2*b)*Cosh[c +
 d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(9*d
) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-a^3-6a^2b-6ab^2-2b^3+2b\left(3a^2+9a\right.}{}\right.}{}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \left(-2b\left(3a^2+3ab+b^2\right)+\right.\right.}{}$$

$$= \frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d} - \frac{2b^2(3a+2b)\cosh^3(c+dx)}{3d} + \frac{3b}{}$$

$$= \frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d} - \frac{2b^2(}{}$$

Mathematica [A] time = 0.35, size = 155, normalized size = 1.05

$$\frac{-10080a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)-10080a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)-40320a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+1890b\left(128a^2+80a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (1890*b*(128*a^2 + 80*a*b + 21*b^2)*Cosh[c + d*x] - 1260*b^2*(20*a + 7*b)*C
osh[3*(c + d*x)] + 3024*a*b^2*Cosh[5*(c + d*x)] + 2268*b^3*Cosh[5*(c + d*x)
] - 405*b^3*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(c + d*x)] - 10080*a^3*Csch[(
c + d*x)/2]^2 - 40320*a^3*Log[Tanh[(c + d*x)/2]] - 10080*a^3*Sech[(c + d*x)
/2]^2)/(80640*d)
```

fricas [B] time = 1.94, size = 4895, normalized size = 33.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/161280*(35*b^3*cosh(d*x + c)^22 + 770*b^3*cosh(d*x + c)*sinh(d*x + c)^21
+ 35*b^3*sinh(d*x + c)^22 - 475*b^3*cosh(d*x + c)^20 + 5*(1617*b^3*cosh(d*x
 + c)^2 - 95*b^3)*sinh(d*x + c)^20 + 100*(539*b^3*cosh(d*x + c)^3 - 95*b^3*
cosh(d*x + c))*sinh(d*x + c)^19 + (3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^18
+ (256025*b^3*cosh(d*x + c)^4 - 90250*b^3*cosh(d*x + c)^2 + 3024*a*b^2 + 31
13*b^3)*sinh(d*x + c)^18 + 6*(153615*b^3*cosh(d*x + c)^5 - 90250*b^3*cosh(d
*x + c)^3 + 3*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 9*(
3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^16 + 3*(870485*b^3*cosh(d*x + c)^6 - 7
67125*b^3*cosh(d*x + c)^4 - 10416*a*b^2 - 4587*b^3 + 51*(3024*a*b^2 + 3113*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 48*(124355*b^3*cosh(d*x + c)^7 - 1
53425*b^3*cosh(d*x + c)^5 + 17*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^3 - 3*
(3472*a*b^2 + 1529*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 126*(1920*a^2*b +
 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^14 + 6*(1865325*b^3*cosh(d*x + c)^8 -
3068500*b^3*cosh(d*x + c)^6 + 510*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^4 +
 40320*a^2*b + 34104*a*b^2 + 9933*b^3 - 180*(3472*a*b^2 + 1529*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^14 + 4*(4352425*b^3*cosh(d*x + c)^9 - 9205500*b^3*c
osh(d*x + c)^7 + 2142*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^5 - 1260*(3472*
a*b^2 + 1529*b^3)*cosh(d*x + c)^3 + 441*(1920*a^2*b + 1624*a*b^2 + 473*b^3)
```

*cosh(d*x + c))*sinh(d*x + c)^13 - 630*(256*a^3 + 384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^12 + 2*(11316305*b^3*cosh(d*x + c)^10 - 29917875*b^3*c
osh(d*x + c)^8 + 9282*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^6 - 8190*(3472*
a*b^2 + 1529*b^3)*cosh(d*x + c)^4 - 80640*a^3 - 120960*a^2*b - 88200*a*b^2
- 24255*b^3 + 5733*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^12 + 8*(3086265*b^3*cosh(d*x + c)^11 - 9972625*b^3*cosh(d*x + c)
^9 + 3978*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^7 - 4914*(3472*a*b^2 + 1529
*b^3)*cosh(d*x + c)^5 + 5733*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x +
c)^3 - 945*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(
d*x + c)^11 - 630*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^
10 + 2*(11316305*b^3*cosh(d*x + c)^12 - 43879550*b^3*cosh(d*x + c)^10 + 218
79*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^8 - 36036*(3472*a*b^2 + 1529*b^3)*
cosh(d*x + c)^6 + 63063*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^4
- 80640*a^3 - 120960*a^2*b - 88200*a*b^2 - 24255*b^3 - 20790*(256*a^3 + 38
4*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(435242
5*b^3*cosh(d*x + c)^13 - 19945250*b^3*cosh(d*x + c)^11 + 12155*(3024*a*b^2
+ 3113*b^3)*cosh(d*x + c)^9 - 25740*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^7
+ 63063*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^5 - 34650*(256*a
^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^3 - 1575*(256*a^3 + 384*
a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 126*(1920*a^2*
b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^8 + 6*(1865325*b^3*cosh(d*x + c)^14
- 9972625*b^3*cosh(d*x + c)^12 + 7293*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c
)^10 - 19305*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^8 + 63063*(1920*a^2*b +
1624*a*b^2 + 473*b^3)*cosh(d*x + c)^6 - 51975*(256*a^3 + 384*a^2*b + 280*a*
b^2 + 77*b^3)*cosh(d*x + c)^4 + 40320*a^2*b + 34104*a*b^2 + 9933*b^3 - 4725
*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
8 + 48*(124355*b^3*cosh(d*x + c)^15 - 767125*b^3*cosh(d*x + c)^13 + 663*(30
24*a*b^2 + 3113*b^3)*cosh(d*x + c)^11 - 2145*(3472*a*b^2 + 1529*b^3)*cosh(d
x + c)^9 + 9009(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^7 - 1039
5*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^5 - 1575*(256*a^
3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^3 + 21*(1920*a^2*b + 1624
*a*b^2 + 473*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 9*(3472*a*b^2 + 1529*b^3
)*cosh(d*x + c)^6 + 3*(870485*b^3*cosh(d*x + c)^16 - 6137000*b^3*cosh(d*x +
c)^14 + 6188*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^12 - 24024*(3472*a*b^2
+ 1529*b^3)*cosh(d*x + c)^10 + 126126*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*c
osh(d*x + c)^8 - 194040*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x
+ c)^6 - 44100*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^4
- 10416*a*b^2 - 4587*b^3 + 1176*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^6 + 6*(153615*b^3*cosh(d*x + c)^17 - 1227400*b^3*co
sh(d*x + c)^15 + 1428*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^13 - 6552*(3472
*a*b^2 + 1529*b^3)*cosh(d*x + c)^11 + 42042*(1920*a^2*b + 1624*a*b^2 + 473*
b^3)*cosh(d*x + c)^9 - 83160*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cos
h(d*x + c)^7 - 26460*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x +
c)^5 + 1176*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^3 - 9*(3472*a
*b^2 + 1529*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 475*b^3*cosh(d*x + c)^2 +
(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^4 + (256025*b^3*cosh(d*x + c)^18 - 2
301375*b^3*cosh(d*x + c)^16 + 3060*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^14
- 16380*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^12 + 126126*(1920*a^2*b + 16
24*a*b^2 + 473*b^3)*cosh(d*x + c)^10 - 311850*(256*a^3 + 384*a^2*b + 280*a*
b^2 + 77*b^3)*cosh(d*x + c)^8 - 132300*(256*a^3 + 384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^6 + 8820*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x
+ c)^4 + 3024*a*b^2 + 3113*b^3 - 135*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^4 + 4*(13475*b^3*cosh(d*x + c)^19 - 135375*b^3*cosh(d*x +
c)^17 + 204*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^15 - 1260*(3472*a*b^2 + 1
529*b^3)*cosh(d*x + c)^13 + 11466*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(
d*x + c)^11 - 34650*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c
)^9 - 18900*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^7 + 17
64*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^5 - 45*(3472*a*b^2 + 1
529*b^3)*cosh(d*x + c)^3 + (3024*a*b^2 + 3113*b^3)*cosh(d*x + c))*sinh(d*x

```
+ c)^3 + 35*b^3 + (8085*b^3*cosh(d*x + c)^20 - 90250*b^3*cosh(d*x + c)^18 +
 153*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^16 - 1080*(3472*a*b^2 + 1529*b^3
)*cosh(d*x + c)^14 + 11466*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c
)^12 - 41580*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^10 -
28350*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^8 + 3528*(19
20*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^6 - 135*(3472*a*b^2 + 1529*b
^3)*cosh(d*x + c)^4 - 475*b^3 + 6*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 80640*(a^3*cosh(d*x + c)^13 + 13*a^3*cosh(d*x + c)*sinh(d
*x + c)^12 + a^3*sinh(d*x + c)^13 - 2*a^3*cosh(d*x + c)^11 + a^3*cosh(d*x +
 c)^9 + 2*(39*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^11 + 22*(13*a^3*cosh
(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c)^10 + (715*a^3*cosh(d*x + c)^
4 - 110*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^9 + 3*(429*a^3*cosh(d*x +
c)^5 - 110*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^8 + 12*
(143*a^3*cosh(d*x + c)^6 - 55*a^3*cosh(d*x + c)^4 + 3*a^3*cosh(d*x + c)^2)*
sinh(d*x + c)^7 + 12*(143*a^3*cosh(d*x + c)^7 - 77*a^3*cosh(d*x + c)^5 + 7*
a^3*cosh(d*x + c)^3)*sinh(d*x + c)^6 + 3*(429*a^3*cosh(d*x + c)^8 - 308*a^3
*cosh(d*x + c)^6 + 42*a^3*cosh(d*x + c)^4)*sinh(d*x + c)^5 + (715*a^3*cosh(
d*x + c)^9 - 660*a^3*cosh(d*x + c)^7 + 126*a^3*cosh(d*x + c)^5)*sinh(d*x +
c)^4 + 2*(143*a^3*cosh(d*x + c)^10 - 165*a^3*cosh(d*x + c)^8 + 42*a^3*cosh(
d*x + c)^6)*sinh(d*x + c)^3 + 2*(39*a^3*cosh(d*x + c)^11 - 55*a^3*cosh(d*x
+ c)^9 + 18*a^3*cosh(d*x + c)^7)*sinh(d*x + c)^2 + (13*a^3*cosh(d*x + c)^12
 - 22*a^3*cosh(d*x + c)^10 + 9*a^3*cosh(d*x + c)^8)*sinh(d*x + c))*log(cosh
(d*x + c) + sinh(d*x + c) + 1) - 80640*(a^3*cosh(d*x + c)^13 + 13*a^3*cosh(
d*x + c)*sinh(d*x + c)^12 + a^3*sinh(d*x + c)^13 - 2*a^3*cosh(d*x + c)^11 +
 a^3*cosh(d*x + c)^9 + 2*(39*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^11 +
22*(13*a^3*cosh(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c)^10 + (715*a^3
*cosh(d*x + c)^4 - 110*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^9 + 3*(429*
a^3*cosh(d*x + c)^5 - 110*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d
*x + c)^8 + 12*(143*a^3*cosh(d*x + c)^6 - 55*a^3*cosh(d*x + c)^4 + 3*a^3*co
sh(d*x + c)^2)*sinh(d*x + c)^7 + 12*(143*a^3*cosh(d*x + c)^7 - 77*a^3*cosh(
d*x + c)^5 + 7*a^3*cosh(d*x + c)^3)*sinh(d*x + c)^6 + 3*(429*a^3*cosh(d*x +
 c)^8 - 308*a^3*cosh(d*x + c)^6 + 42*a^3*cosh(d*x + c)^4)*sinh(d*x + c)^5 +
 (715*a^3*cosh(d*x + c)^9 - 660*a^3*cosh(d*x + c)^7 + 126*a^3*cosh(d*x + c)
^5)*sinh(d*x + c)^4 + 2*(143*a^3*cosh(d*x + c)^10 - 165*a^3*cosh(d*x + c)^8
 + 42*a^3*cosh(d*x + c)^6)*sinh(d*x + c)^3 + 2*(39*a^3*cosh(d*x + c)^11 - 5
5*a^3*cosh(d*x + c)^9 + 18*a^3*cosh(d*x + c)^7)*sinh(d*x + c)^2 + (13*a^3*c
osh(d*x + c)^12 - 22*a^3*cosh(d*x + c)^10 + 9*a^3*cosh(d*x + c)^8)*sinh(d*x
 + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(385*b^3*cosh(d*x + c)^21
 - 4750*b^3*cosh(d*x + c)^19 + 9*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^17 -
 72*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^15 + 882*(1920*a^2*b + 1624*a*b^2
 + 473*b^3)*cosh(d*x + c)^13 - 3780*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b
^3)*cosh(d*x + c)^11 - 3150*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh
(d*x + c)^9 + 504*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^7 - 27*
(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^5 - 475*b^3*cosh(d*x + c) + 2*(3024*a
*b^2 + 3113*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))/(d*cosh(d*x + c)^13 + 13*d
*cosh(d*x + c)*sinh(d*x + c)^12 + d*sinh(d*x + c)^13 - 2*d*cosh(d*x + c)^11
 + 2*(39*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^11 + 22*(13*d*cosh(d*x + c)^3
 - d*cosh(d*x + c))*sinh(d*x + c)^10 + d*cosh(d*x + c)^9 + (715*d*cosh(d*x
+ c)^4 - 110*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^9 + 3*(429*d*cosh(d*x + c
)^5 - 110*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^8 + 12*(143*
d*cosh(d*x + c)^6 - 55*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x +
c)^7 + 12*(143*d*cosh(d*x + c)^7 - 77*d*cosh(d*x + c)^5 + 7*d*cosh(d*x + c)
^3)*sinh(d*x + c)^6 + 3*(429*d*cosh(d*x + c)^8 - 308*d*cosh(d*x + c)^6 + 42
*d*cosh(d*x + c)^4)*sinh(d*x + c)^5 + (715*d*cosh(d*x + c)^9 - 660*d*cosh(d
*x + c)^7 + 126*d*cosh(d*x + c)^5)*sinh(d*x + c)^4 + 2*(143*d*cosh(d*x + c)
^10 - 165*d*cosh(d*x + c)^8 + 42*d*cosh(d*x + c)^6)*sinh(d*x + c)^3 + 2*(39
*d*cosh(d*x + c)^11 - 55*d*cosh(d*x + c)^9 + 18*d*cosh(d*x + c)^7)*sinh(d*x
 + c)^2 + (13*d*cosh(d*x + c)^12 - 22*d*cosh(d*x + c)^10 + 9*d*cosh(d*x + c
)^8)*sinh(d*x + c))
```

giac [B] time = 0.44, size = 300, normalized size = 2.03

$$\frac{35\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^9-720\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^7+3024\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+6048\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/161280*(35*b^3*(e^(d*x + c) + e^(-d*x - c))^9 - 720*b^3*(e^(d*x + c) + e^
(-d*x - c))^7 + 3024*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 6048*b^3*(e^(d*
x + c) + e^(-d*x - c))^5 - 40320*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 268
80*b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 241920*a^2*b*(e^(d*x + c) + e^(-d*x
 - c)) + 241920*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 80640*b^3*(e^(d*x + c)
 + e^(-d*x - c)) + 40320*a^3*log(e^(d*x + c) + e^(-d*x - c) + 2) - 40320*a^
3*log(e^(d*x + c) + e^(-d*x - c) - 2) - 161280*a^3*(e^(d*x + c) + e^(-d*x -
 c))/((e^(d*x + c) + e^(-d*x - c))^2 - 4))/d
```

maple [A] time = 0.10, size = 130, normalized size = 0.88

$$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\cosh(dx+c)+3a\,b^2\left(\frac{8}{15}+\frac{\left(\sinh^4(dx+c)\right)}{5}-\frac{4\left(\sinh^2(dx+c)\right)}{15}\right)\cosh}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*cosh(d*
x+c)+3*a*b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*(1
28/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4-64/315*sin
h(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.33, size = 334, normalized size = 2.26

$$-\frac{1}{161280}\,b^3\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\frac{39690\,e^{(-d}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) + 1/160*a*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*
x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d
 + 3*e^(-5*d*x - 5*c)/d) + 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2
*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) +
 e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

mupad [B] time = 1.19, size = 326, normalized size = 2.20

$$\frac{\operatorname{atan}\left(\frac{a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\sqrt{a^6}}{\sqrt{-d^2}}-\frac{9\,b^3\,\mathrm{e}^{-7\,c-7\,d\,x}}{3584\,d}-\frac{9\,b^3\,\mathrm{e}^{7\,c+7\,d\,x}}{3584\,d}+\frac{b^3\,\mathrm{e}^{-9\,c-9\,d\,x}}{4608\,d}+\frac{b^3\,\mathrm{e}^{9\,c+9\,d\,x}}{4608\,d}+\frac{3\,b\,\mathrm{e}^{-c-d\,x}\,\left(128\,a^2+80\,a\,b\right.}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^3,x)
```

```
[Out] (atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^
2)^(1/2) - (9*b^3*exp(- 7*c - 7*d*x))/(3584*d) - (9*b^3*exp(7*c + 7*d*x))/(
3584*d) + (b^3*exp(- 9*c - 9*d*x))/(4608*d) + (b^3*exp(9*c + 9*d*x))/(4608*
d) + (3*b*exp(- c - d*x)*(80*a*b + 128*a^2 + 21*b^2))/(256*d) + (3*b^2*exp(
- 5*c - 5*d*x)*(4*a + 3*b))/(640*d) + (3*b^2*exp(5*c + 5*d*x)*(4*a + 3*b))/
(640*d) - (b^2*exp(- 3*c - 3*d*x)*(20*a + 7*b))/(128*d) - (b^2*exp(3*c + 3*
d*x)*(20*a + 7*b))/(128*d) + (3*b*exp(c + d*x)*(80*a*b + 128*a^2 + 21*b^2))
/(256*d) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d
*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.212 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=142

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{b^2(a+b)\,c}{}$$

```
[Out] -3/8*a^2*(a+8*b)*arctanh(cosh(d*x+c))/d-b^2*(3*a+b)*cosh(d*x+c)/d+b^2*(a+b)
*cosh(d*x+c)^3/d-3/5*b^3*cosh(d*x+c)^5/d+1/7*b^3*cosh(d*x+c)^7/d+3/8*a^3*co
th(d*x+c)*csch(d*x+c)/d-1/4*a^3*coth(d*x+c)*csch(d*x+c)^3/d
```

Rubi [A] time = 0.27, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1157, 1814, 1810, 206}

$$-\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d}+\frac{b^2(a+b)\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-3*a^2*(a + 8*b)*ArcTanh[Cosh[c + d*x]])/(8*d) - (b^2*(3*a + b)*Cosh[c + d
*x])/d + (b^2*(a + b)*Cosh[c + d*x]^3)/d - (3*b^3*Cosh[c + d*x]^5)/(5*d) +
(b^3*Cosh[c + d*x]^7)/(7*d) + (3*a^3*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (
a^3*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^3}\,dx,x,\cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-3a^3-12a^2b-12ab^2-4b^3+4b(3a\cdots}{\cdots}\right)}{\cdots}$$

$$= \frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} - \frac{\text{Su}\cdots}{\cdots}$$

$$= \frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} - \frac{\text{Su}\cdots}{\cdots}$$

$$= -\frac{b^2(3a+b)\cosh(c+dx)}{d} + \frac{b^2(a+b)\cosh^3(c+dx)}{d} - \frac{3b^3\cosh^5(c\cdots}{5d}$$

$$= -\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d} - \frac{b^2(3a+b)\cosh(c+dx)}{d} + \frac{b^2\cdots}{\cdots}$$

Mathematica [A] time = 0.39, size = 173, normalized size = 1.22

$$\frac{-35a^3\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)+210a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+35a^3\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)+210a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+840a^3\log\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-35*b^2*(144*a + 35*b)*Cosh[c + d*x] + 35*b^2*(16*a + 7*b)*Cosh[3*(c + d*x
)] - 49*b^3*Cosh[5*(c + d*x)] + 5*b^3*Cosh[7*(c + d*x)] + 210*a^3*Csch[(c +
 d*x)/2]^2 - 35*a^3*Csch[(c + d*x)/2]^4 + 840*a^3*Log[Tanh[(c + d*x)/2]] +
6720*a^2*b*Log[Tanh[(c + d*x)/2]] + 210*a^3*Sech[(c + d*x)/2]^2 + 35*a^3*Se
ch[(c + d*x)/2]^4)/(2240*d)
```

fricas [B] time = 1.71, size = 6441, normalized size = 45.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/4480*(5*b^3*cosh(d*x + c)^22 + 110*b^3*cosh(d*x + c)*sinh(d*x + c)^21 + 5
*b^3*sinh(d*x + c)^22 - 69*b^3*cosh(d*x + c)^20 + 3*(385*b^3*cosh(d*x + c)^
2 - 23*b^3)*sinh(d*x + c)^20 + 20*(385*b^3*cosh(d*x + c)^3 - 69*b^3*cosh(d*
x + c))*sinh(d*x + c)^19 + (560*a*b^2 + 471*b^3)*cosh(d*x + c)^18 + (36575*
b^3*cosh(d*x + c)^4 - 13110*b^3*cosh(d*x + c)^2 + 560*a*b^2 + 471*b^3)*sinh
(d*x + c)^18 + 18*(7315*b^3*cosh(d*x + c)^5 - 4370*b^3*cosh(d*x + c)^3 + (5
60*a*b^2 + 471*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - (7280*a*b^2 + 2519*b^
3)*cosh(d*x + c)^16 + (373065*b^3*cosh(d*x + c)^6 - 334305*b^3*cosh(d*x + c
)^4 - 7280*a*b^2 - 2519*b^3 + 153*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^2)*si
```

nh(d*x + c)^16 + 16*(53295*b^3*cosh(d*x + c)^7 - 66861*b^3*cosh(d*x + c)^5
+ 51*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^3 - (7280*a*b^2 + 2519*b^3)*cosh(d
*x + c))*sinh(d*x + c)^15 + 6*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c
)^14 + 6*(266475*b^3*cosh(d*x + c)^8 - 445740*b^3*cosh(d*x + c)^6 + 510*(56
0*a*b^2 + 471*b^3)*cosh(d*x + c)^4 + 560*a^3 + 3080*a*b^2 + 891*b^3 - 20*(7
280*a*b^2 + 2519*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 4*(621775*b^3*cos
h(d*x + c)^9 - 1337220*b^3*cosh(d*x + c)^7 + 2142*(560*a*b^2 + 471*b^3)*cos
h(d*x + c)^5 - 140*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^3 + 21*(560*a^3 +
3080*a*b^2 + 891*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 14*(880*a^3 + 840*a
*b^2 + 231*b^3)*cosh(d*x + c)^12 + 2*(1616615*b^3*cosh(d*x + c)^10 - 434596
5*b^3*cosh(d*x + c)^8 + 9282*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^6 - 910*(7
280*a*b^2 + 2519*b^3)*cosh(d*x + c)^4 - 6160*a^3 - 5880*a*b^2 - 1617*b^3 +
273*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 24
*(146965*b^3*cosh(d*x + c)^11 - 482885*b^3*cosh(d*x + c)^9 + 1326*(560*a*b^
2 + 471*b^3)*cosh(d*x + c)^7 - 182*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^5
+ 91*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^3 - 7*(880*a^3 + 840*a*
b^2 + 231*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 14*(880*a^3 + 840*a*b^2 +
231*b^3)*cosh(d*x + c)^10 + 2*(1616615*b^3*cosh(d*x + c)^12 - 6374082*b^3*c
osh(d*x + c)^10 + 21879*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^8 - 4004*(7280*
a*b^2 + 2519*b^3)*cosh(d*x + c)^6 + 3003*(560*a^3 + 3080*a*b^2 + 891*b^3)*c
osh(d*x + c)^4 - 6160*a^3 - 5880*a*b^2 - 1617*b^3 - 462*(880*a^3 + 840*a*b^
2 + 231*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(621775*b^3*cosh(d*x + c
)^13 - 2897310*b^3*cosh(d*x + c)^11 + 12155*(560*a*b^2 + 471*b^3)*cosh(d*x
+ c)^9 - 2860*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^7 + 3003*(560*a^3 + 308
0*a*b^2 + 891*b^3)*cosh(d*x + c)^5 - 770*(880*a^3 + 840*a*b^2 + 231*b^3)*co
sh(d*x + c)^3 - 35*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^9 + 6*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^8 + 6*(266475*b^3
*cosh(d*x + c)^14 - 1448655*b^3*cosh(d*x + c)^12 + 7293*(560*a*b^2 + 471*b^
3)*cosh(d*x + c)^10 - 2145*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^8 + 3003*(
560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^6 - 1155*(880*a^3 + 840*a*b^2
 + 231*b^3)*cosh(d*x + c)^4 + 560*a^3 + 3080*a*b^2 + 891*b^3 - 105*(880*a^3
 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(53295*b^3*co
sh(d*x + c)^15 - 334305*b^3*cosh(d*x + c)^13 + 1989*(560*a*b^2 + 471*b^3)*c
osh(d*x + c)^11 - 715*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^9 + 1287*(560*a
^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^7 - 693*(880*a^3 + 840*a*b^2 + 231
*b^3)*cosh(d*x + c)^5 - 105*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^3
 + 3*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - (728
0*a*b^2 + 2519*b^3)*cosh(d*x + c)^6 + (373065*b^3*cosh(d*x + c)^16 - 267444
0*b^3*cosh(d*x + c)^14 + 18564*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^12 - 800
8*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^10 + 18018*(560*a^3 + 3080*a*b^2 +
891*b^3)*cosh(d*x + c)^8 - 12936*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x +
 c)^6 - 2940*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^4 - 7280*a*b^2 -
 2519*b^3 + 168*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 6*(21945*b^3*cosh(d*x + c)^17 - 178296*b^3*cosh(d*x + c)^15 + 1428
*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^13 - 728*(7280*a*b^2 + 2519*b^3)*cosh(
d*x + c)^11 + 2002*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^9 - 1848*
(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^7 - 588*(880*a^3 + 840*a*b^2
+ 231*b^3)*cosh(d*x + c)^5 + 56*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x +
 c)^3 - (7280*a*b^2 + 2519*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 69*b^3*cos
h(d*x + c)^2 + (560*a*b^2 + 471*b^3)*cosh(d*x + c)^4 + (36575*b^3*cosh(d*x
+ c)^18 - 334305*b^3*cosh(d*x + c)^16 + 3060*(560*a*b^2 + 471*b^3)*cosh(d*x
 + c)^14 - 1820*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^12 + 6006*(560*a^3 +
3080*a*b^2 + 891*b^3)*cosh(d*x + c)^10 - 6930*(880*a^3 + 840*a*b^2 + 231*b^
3)*cosh(d*x + c)^8 - 2940*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^6 +
 420*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^4 + 560*a*b^2 + 471*b^3
 - 15*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(1925*b^
3*cosh(d*x + c)^19 - 19665*b^3*cosh(d*x + c)^17 + 204*(560*a*b^2 + 471*b^3)
*cosh(d*x + c)^15 - 140*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^13 + 546*(560
*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^11 - 770*(880*a^3 + 840*a*b^2 +

```
231*b^3)*cosh(d*x + c)^9 - 420*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c
)^7 + 84*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^5 - 5*(7280*a*b^2 +
 2519*b^3)*cosh(d*x + c)^3 + (560*a*b^2 + 471*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 5*b^3 + 3*(385*b^3*cosh(d*x + c)^20 - 4370*b^3*cosh(d*x + c)^18 +
51*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^16 - 40*(7280*a*b^2 + 2519*b^3)*cosh
(d*x + c)^14 + 182*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^12 - 308*
(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^10 - 210*(880*a^3 + 840*a*b^2
 + 231*b^3)*cosh(d*x + c)^8 + 56*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x
+ c)^6 - 5*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^4 - 23*b^3 + 2*(560*a*b^2
+ 471*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 1680*((a^3 + 8*a^2*b)*cosh(d*
x + c)^15 + 15*(a^3 + 8*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^14 + (a^3 + 8*a^
2*b)*sinh(d*x + c)^15 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c)^13 - (4*a^3 + 32*a^
2*b - 105*(a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(35*(a^3 +
 8*a^2*b)*cosh(d*x + c)^3 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^
12 + 6*(a^3 + 8*a^2*b)*cosh(d*x + c)^11 + 3*(455*(a^3 + 8*a^2*b)*cosh(d*x +
 c)^4 + 2*a^3 + 16*a^2*b - 104*(a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x +
c)^11 + 11*(273*(a^3 + 8*a^2*b)*cosh(d*x + c)^5 - 104*(a^3 + 8*a^2*b)*cosh(
d*x + c)^3 + 6*(a^3 + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^10 - 4*(a^3 + 8
*a^2*b)*cosh(d*x + c)^9 + (5005*(a^3 + 8*a^2*b)*cosh(d*x + c)^6 - 2860*(a^3
 + 8*a^2*b)*cosh(d*x + c)^4 - 4*a^3 - 32*a^2*b + 330*(a^3 + 8*a^2*b)*cosh(d
*x + c)^2)*sinh(d*x + c)^9 + 9*(715*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 - 572*(
a^3 + 8*a^2*b)*cosh(d*x + c)^5 + 110*(a^3 + 8*a^2*b)*cosh(d*x + c)^3 - 4*(a
^3 + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^8 + (a^3 + 8*a^2*b)*cosh(d*x + c
)^7 + (6435*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 - 6864*(a^3 + 8*a^2*b)*cosh(d*x
 + c)^6 + 1980*(a^3 + 8*a^2*b)*cosh(d*x + c)^4 + a^3 + 8*a^2*b - 144*(a^3 +
 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + (5005*(a^3 + 8*a^2*b)*cosh(d*x
 + c)^9 - 6864*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 + 2772*(a^3 + 8*a^2*b)*cosh(
d*x + c)^5 - 336*(a^3 + 8*a^2*b)*cosh(d*x + c)^3 + 7*(a^3 + 8*a^2*b)*cosh(d
*x + c))*sinh(d*x + c)^6 + 3*(1001*(a^3 + 8*a^2*b)*cosh(d*x + c)^10 - 1716*
(a^3 + 8*a^2*b)*cosh(d*x + c)^8 + 924*(a^3 + 8*a^2*b)*cosh(d*x + c)^6 - 168
*(a^3 + 8*a^2*b)*cosh(d*x + c)^4 + 7*(a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(
d*x + c)^5 + (1365*(a^3 + 8*a^2*b)*cosh(d*x + c)^11 - 2860*(a^3 + 8*a^2*b)*
cosh(d*x + c)^9 + 1980*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 - 504*(a^3 + 8*a^2*b
)*cosh(d*x + c)^5 + 35*(a^3 + 8*a^2*b)*cosh(d*x + c)^3)*sinh(d*x + c)^4 + (
455*(a^3 + 8*a^2*b)*cosh(d*x + c)^12 - 1144*(a^3 + 8*a^2*b)*cosh(d*x + c)^1
0 + 990*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 - 336*(a^3 + 8*a^2*b)*cosh(d*x + c)
^6 + 35*(a^3 + 8*a^2*b)*cosh(d*x + c)^4)*sinh(d*x + c)^3 + 3*(35*(a^3 + 8*a
^2*b)*cosh(d*x + c)^13 - 104*(a^3 + 8*a^2*b)*cosh(d*x + c)^11 + 110*(a^3 +
8*a^2*b)*cosh(d*x + c)^9 - 48*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 + 8*
a^2*b)*cosh(d*x + c)^5)*sinh(d*x + c)^2 + (15*(a^3 + 8*a^2*b)*cosh(d*x + c)
^14 - 52*(a^3 + 8*a^2*b)*cosh(d*x + c)^12 + 66*(a^3 + 8*a^2*b)*cosh(d*x + c
)^10 - 36*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 + 7*(a^3 + 8*a^2*b)*cosh(d*x + c)
^6)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 1680*((a^3 + 8*
a^2*b)*cosh(d*x + c)^15 + 15*(a^3 + 8*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^14
 + (a^3 + 8*a^2*b)*sinh(d*x + c)^15 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c)^13 -
(4*a^3 + 32*a^2*b - 105*(a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^13 +
 13*(35*(a^3 + 8*a^2*b)*cosh(d*x + c)^3 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c))*
sinh(d*x + c)^12 + 6*(a^3 + 8*a^2*b)*cosh(d*x + c)^11 + 3*(455*(a^3 + 8*a^2
*b)*cosh(d*x + c)^4 + 2*a^3 + 16*a^2*b - 104*(a^3 + 8*a^2*b)*cosh(d*x + c)^
2)*sinh(d*x + c)^11 + 11*(273*(a^3 + 8*a^2*b)*cosh(d*x + c)^5 - 104*(a^3 +
8*a^2*b)*cosh(d*x + c)^3 + 6*(a^3 + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^1
0 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c)^9 + (5005*(a^3 + 8*a^2*b)*cosh(d*x + c)
^6 - 2860*(a^3 + 8*a^2*b)*cosh(d*x + c)^4 - 4*a^3 - 32*a^2*b + 330*(a^3 + 8
*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(715*(a^3 + 8*a^2*b)*cosh(d*x
+ c)^7 - 572*(a^3 + 8*a^2*b)*cosh(d*x + c)^5 + 110*(a^3 + 8*a^2*b)*cosh(d*x
 + c)^3 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^8 + (a^3 + 8*a^2*b
)*cosh(d*x + c)^7 + (6435*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 - 6864*(a^3 + 8*a
^2*b)*cosh(d*x + c)^6 + 1980*(a^3 + 8*a^2*b)*cosh(d*x + c)^4 + a^3 + 8*a^2*
b - 144*(a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + (5005*(a^3 + 8*a
```

```
^2*b)*cosh(d*x + c)^9 - 6864*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 + 2772*(a^3 +
8*a^2*b)*cosh(d*x + c)^5 - 336*(a^3 + 8*a^2*b)*cosh(d*x + c)^3 + 7*(a^3 + 8
*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 3*(1001*(a^3 + 8*a^2*b)*cosh(d*x +
 c)^10 - 1716*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 + 924*(a^3 + 8*a^2*b)*cosh(d*
x + c)^6 - 168*(a^3 + 8*a^2*b)*cosh(d*x + c)^4 + 7*(a^3 + 8*a^2*b)*cosh(d*x
 + c)^2)*sinh(d*x + c)^5 + (1365*(a^3 + 8*a^2*b)*cosh(d*x + c)^11 - 2860*(a
^3 + 8*a^2*b)*cosh(d*x + c)^9 + 1980*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 - 504*
(a^3 + 8*a^2*b)*cosh(d*x + c)^5 + 35*(a^3 + 8*a^2*b)*cosh(d*x + c)^3)*sinh(
d*x + c)^4 + (455*(a^3 + 8*a^2*b)*cosh(d*x + c)^12 - 1144*(a^3 + 8*a^2*b)*c
osh(d*x + c)^10 + 990*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 - 336*(a^3 + 8*a^2*b)
*cosh(d*x + c)^6 + 35*(a^3 + 8*a^2*b)*cosh(d*x + c)^4)*sinh(d*x + c)^3 + 3*
(35*(a^3 + 8*a^2*b)*cosh(d*x + c)^13 - 104*(a^3 + 8*a^2*b)*cosh(d*x + c)^11
 + 110*(a^3 + 8*a^2*b)*cosh(d*x + c)^9 - 48*(a^3 + 8*a^2*b)*cosh(d*x + c)^7
 + 7*(a^3 + 8*a^2*b)*cosh(d*x + c)^5)*sinh(d*x + c)^2 + (15*(a^3 + 8*a^2*b)
*cosh(d*x + c)^14 - 52*(a^3 + 8*a^2*b)*cosh(d*x + c)^12 + 66*(a^3 + 8*a^2*b
)*cosh(d*x + c)^10 - 36*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 + 7*(a^3 + 8*a^2*b)
*cosh(d*x + c)^6)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2
*(55*b^3*cosh(d*x + c)^21 - 690*b^3*cosh(d*x + c)^19 + 9*(560*a*b^2 + 471*b
^3)*cosh(d*x + c)^17 - 8*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^15 + 42*(560
*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^13 - 84*(880*a^3 + 840*a*b^2 + 2
31*b^3)*cosh(d*x + c)^11 - 70*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)
^9 + 24*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^7 - 3*(7280*a*b^2 +
2519*b^3)*cosh(d*x + c)^5 - 69*b^3*cosh(d*x + c) + 2*(560*a*b^2 + 471*b^3)*
cosh(d*x + c)^3)*sinh(d*x + c))/(d*cosh(d*x + c)^15 + 15*d*cosh(d*x + c)*si
nh(d*x + c)^14 + d*sinh(d*x + c)^15 - 4*d*cosh(d*x + c)^13 + (105*d*cosh(d*
x + c)^2 - 4*d)*sinh(d*x + c)^13 + 13*(35*d*cosh(d*x + c)^3 - 4*d*cosh(d*x
+ c))*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^11 + 3*(455*d*cosh(d*x + c)^4 -
104*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^11 + 11*(273*d*cosh(d*x + c)^5 -
 104*d*cosh(d*x + c)^3 + 6*d*cosh(d*x + c))*sinh(d*x + c)^10 - 4*d*cosh(d*x
 + c)^9 + (5005*d*cosh(d*x + c)^6 - 2860*d*cosh(d*x + c)^4 + 330*d*cosh(d*x
 + c)^2 - 4*d)*sinh(d*x + c)^9 + 9*(715*d*cosh(d*x + c)^7 - 572*d*cosh(d*x
+ c)^5 + 110*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c))*sinh(d*x + c)^8 + d*cos
h(d*x + c)^7 + (6435*d*cosh(d*x + c)^8 - 6864*d*cosh(d*x + c)^6 + 1980*d*co
sh(d*x + c)^4 - 144*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + (5005*d*cosh(d
*x + c)^9 - 6864*d*cosh(d*x + c)^7 + 2772*d*cosh(d*x + c)^5 - 336*d*cosh(d*
x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 3*(1001*d*cosh(d*x + c)^10
- 1716*d*cosh(d*x + c)^8 + 924*d*cosh(d*x + c)^6 - 168*d*cosh(d*x + c)^4 +
7*d*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (1365*d*cosh(d*x + c)^11 - 2860*d*co
sh(d*x + c)^9 + 1980*d*cosh(d*x + c)^7 - 504*d*cosh(d*x + c)^5 + 35*d*cosh(
d*x + c)^3)*sinh(d*x + c)^4 + (455*d*cosh(d*x + c)^12 - 1144*d*cosh(d*x + c
)^10 + 990*d*cosh(d*x + c)^8 - 336*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4
)*sinh(d*x + c)^3 + 3*(35*d*cosh(d*x + c)^13 - 104*d*cosh(d*x + c)^11 + 110
*d*cosh(d*x + c)^9 - 48*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5)*sinh(d*x +
 c)^2 + (15*d*cosh(d*x + c)^14 - 52*d*cosh(d*x + c)^12 + 66*d*cosh(d*x + c)
^10 - 36*d*cosh(d*x + c)^8 + 7*d*cosh(d*x + c)^6)*sinh(d*x + c))
```

giac [B] time = 0.49, size = 271, normalized size = 1.91

$$\frac{5\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^7-84\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+560\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+560\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*b^3*(e^(d*x + c) + e^(-d*x - c))^7 - 84*b^3*(e^(d*x + c) + e^(-d*
x - c))^5 + 560*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 560*b^3*(e^(d*x + c)
 + e^(-d*x - c))^3 - 6720*a*b^2*(e^(d*x + c) + e^(-d*x - c)) - 2240*b^3*(e^
(d*x + c) + e^(-d*x - c)) - 840*(a^3 + 8*a^2*b)*log(e^(d*x + c) + e^(-d*x -
```

```
 c) + 2) + 840*(a^3 + 8*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) + 1120*(
3*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 20*a^3*(e^(d*x + c) + e^(-d*x - c)))
/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^2)/d
```

maple [A] time = 0.10, size = 125, normalized size = 0.88

$$\frac{a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{arctanh}\left(e^{dx+c}\right)}{4}\right)-6a^2b\operatorname{arctanh}\left(e^{dx+c}\right)+3ab^2\left(-\frac{2}{3}+\frac{\left(\sinh^2(dx+c)\right)}{3}\right)c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(
d*x+c)))-6*a^2*b*arctanh(exp(d*x+c))+3*a*b^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(
d*x+c)+b^3*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)
*cosh(d*x+c))
```

maxima [B] time = 0.34, size = 340, normalized size = 2.39

$$-\frac{1}{4480}b^3\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/8*a*b^2*(e^(3*d*x + 3*c)/d
 - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) - 1/8*a^3*(3*lo
g(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*
e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*
x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))
) - 3*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

mupad [B] time = 1.15, size = 421, normalized size = 2.96

$$\frac{b^3\,e^{-7\,c-7\,d\,x}}{896\,d}-\frac{7\,b^3\,e^{-5\,c-5\,d\,x}}{640\,d}-\frac{7\,b^3\,e^{5\,c+5\,d\,x}}{640\,d}-\frac{3\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(a^3\,\sqrt{-d^2}+8\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+16\,a^5\,b+64\,a^4\,b^2}}\right)\sqrt{a^6+16\,a^5\,b+64\,a^4\,b^2}}{4\,\sqrt{-d^2}}+\frac{b^3\,e^{7\,c+7\,d\,x}}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^5,x)
```

```
[Out] (b^3*exp(- 7*c - 7*d*x))/(896*d) - (7*b^3*exp(- 5*c - 5*d*x))/(640*d) - (7*
b^3*exp(5*c + 5*d*x))/(640*d) - (3*atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2)
+ 8*a^2*b*(-d^2)^(1/2)))/(d*(16*a^5*b + a^6 + 64*a^4*b^2)^(1/2)))*(16*a^5*b
 + a^6 + 64*a^4*b^2)^(1/2))/(4*(-d^2)^(1/2)) + (b^3*exp(7*c + 7*d*x))/(896*
d) - (6*a^3*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp
(6*c + 6*d*x) - 1)) - (b^2*exp(c + d*x)*(144*a + 35*b))/(128*d) - (4*a^3*ex
p(c + d*x))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x
) + exp(8*c + 8*d*x) + 1)) + (b^2*exp(- 3*c - 3*d*x)*(16*a + 7*b))/(128*d)
+ (b^2*exp(3*c + 3*d*x)*(16*a + 7*b))/(128*d) - (b^2*exp(- c - d*x)*(144*a
+ 35*b))/(128*d) + (3*a^3*exp(c + d*x))/(4*d*(exp(2*c + 2*d*x) - 1)) - (a^3
*exp(c + d*x))/(2*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.213 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=156

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}+\frac{a^2(5a+24b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^2(5a+24}{}$$

```
[Out] 1/16*a^2*(5*a+24*b)*arctanh(cosh(d*x+c))/d+b^2*(3*a+b)*cosh(d*x+c)/d-2/3*b^
3*cosh(d*x+c)^3/d+1/5*b^3*cosh(d*x+c)^5/d-1/16*a^2*(5*a+24*b)*coth(d*x+c)*c
sch(d*x+c)/d+5/24*a^3*coth(d*x+c)*csch(d*x+c)^3/d-1/6*a^3*coth(d*x+c)*csch(
d*x+c)^5/d
```

Rubi [A] time = 0.30, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1157, 1814, 1810, 206}

$$\frac{a^2(5a+24b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^2(5a+24b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (a^2*(5*a + 24*b)*ArcTanh[Cosh[c + d*x]])/(16*d) + (b^2*(3*a + b)*Cosh[c +
d*x])/d - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) - (a^
2*(5*a + 24*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^3*Coth[c + d*x]*C
sch[c + d*x]^3)/(24*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \text{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^4}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\text{csch}^5(c+dx)}{6d} - \frac{\text{Subst}\left(\int \frac{-5a^3-18a^2b-18ab^2-6b^3+6}{}\right.}{}$$

$$= \frac{5a^3\coth(c+dx)\text{csch}^3(c+dx)}{24d} - \frac{a^3\coth(c+dx)\text{csch}^5(c+dx)}{6d} +$$

$$= -\frac{a^2(5a+24b)\coth(c+dx)\text{csch}(c+dx)}{16d} + \frac{5a^3\coth(c+dx)\text{csch}}{24d}$$

$$= -\frac{a^2(5a+24b)\coth(c+dx)\text{csch}(c+dx)}{16d} + \frac{5a^3\coth(c+dx)\text{csch}}{24d}$$

$$= \frac{b^2(3a+b)\cosh(c+dx)}{d} - \frac{2b^3\cosh^3(c+dx)}{3d} + \frac{b^3\cosh^5(c+dx)}{5d}$$

$$= \frac{a^2(5a+24b)\tanh^{-1}(\cosh(c+dx))}{16d} + \frac{b^2(3a+b)\cosh(c+dx)}{d} -$$

Mathematica [A] time = 0.70, size = 223, normalized size = 1.43

$$-\frac{5a^3\text{csch}^6\left(\frac{1}{2}(c+dx)\right) - 30a^3\text{csch}^4\left(\frac{1}{2}(c+dx)\right) + 150a^3\text{csch}^2\left(\frac{1}{2}(c+dx)\right) + 5a^3\text{sech}^6\left(\frac{1}{2}(c+dx)\right) + 30a^3\text{sech}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/1920*(-240*b^2*(24*a + 5*b)*Cosh[c + d*x] + 200*b^3*Cosh[3*(c + d*x)] -
24*b^3*Cosh[5*(c + d*x)] + 150*a^3*Csch[(c + d*x)/2]^2 + 720*a^2*b*Csch[(c
+ d*x)/2]^2 - 30*a^3*Csch[(c + d*x)/2]^4 + 5*a^3*Csch[(c + d*x)/2]^6 + 600*
a^3*Log[Tanh[(c + d*x)/2]] + 2880*a^2*b*Log[Tanh[(c + d*x)/2]] + 150*a^3*Se
ch[(c + d*x)/2]^2 + 720*a^2*b*Sech[(c + d*x)/2]^2 + 30*a^3*Sech[(c + d*x)/2
]^4 + 5*a^3*Sech[(c + d*x)/2]^6)/d
```

fricas [B] time = 2.52, size = 8547, normalized size = 54.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^22 + 66*b^3*cosh(d*x + c)*sinh(d*x + c)^21 + 3*b
^3*sinh(d*x + c)^22 - 43*b^3*cosh(d*x + c)^20 + (693*b^3*cosh(d*x + c)^2 -
```

43*b^3)*sinh(d*x + c)^20 + 20*(231*b^3*cosh(d*x + c)^3 - 43*b^3*cosh(d*x + c))*sinh(d*x + c)^19 + 15*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^18 + 5*(4389*b^3*cosh(d*x + c)^4 - 1634*b^3*cosh(d*x + c)^2 + 144*a*b^2 + 69*b^3)*sinh(d*x + c)^18 + 6*(13167*b^3*cosh(d*x + c)^5 - 8170*b^3*cosh(d*x + c)^3 + 45*(48*a*b^2 + 23*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 15*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^16 + 3*(74613*b^3*cosh(d*x + c)^6 - 69445*b^3*cosh(d*x + c)^4 - 100*a^3 - 480*a^2*b - 1200*a*b^2 - 395*b^3 + 765*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 48*(10659*b^3*cosh(d*x + c)^7 - 13889*b^3*cosh(d*x + c)^5 + 255*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^3 - 5*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 10*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^14 + 10*(95931*b^3*cosh(d*x + c)^8 - 166668*b^3*cosh(d*x + c)^6 + 4590*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^4 + 170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3 - 180*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 20*(74613*b^3*cosh(d*x + c)^9 - 166668*b^3*cosh(d*x + c)^7 + 6426*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^5 - 420*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^3 + 7*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 90*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^12 + 2*(969969*b^3*cosh(d*x + c)^10 - 2708355*b^3*cosh(d*x + c)^8 + 139230*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^6 - 13650*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^4 - 1980*a^3 - 1440*a^2*b - 1800*a*b^2 - 495*b^3 + 455*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 8*(264537*b^3*cosh(d*x + c)^11 - 902785*b^3*cosh(d*x + c)^9 + 59670*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^7 - 8190*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^5 + 455*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^3 - 135*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 90*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^10 + 2*(969969*b^3*cosh(d*x + c)^12 - 3972254*b^3*cosh(d*x + c)^10 + 328185*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^8 - 60060*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^6 + 5005*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^4 - 1980*a^3 - 1440*a^2*b - 1800*a*b^2 - 495*b^3 - 2970*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(74613*b^3*cosh(d*x + c)^13 - 361114*b^3*cosh(d*x + c)^11 + 36465*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^9 - 8580*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^7 + 1001*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^5 - 990*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^3 - 45*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 10*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^8 + 10*(95931*b^3*cosh(d*x + c)^14 - 541671*b^3*cosh(d*x + c)^12 + 65637*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^10 - 19305*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^8 + 3003*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^6 - 4455*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3 - 405*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(31977*b^3*cosh(d*x + c)^15 - 208335*b^3*cosh(d*x + c)^13 + 29835*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^11 - 10725*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^9 + 2145*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^7 - 4455*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^5 - 675*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 5*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 15*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^6 + (223839*b^3*cosh(d*x + c)^16 - 1666680*b^3*cosh(d*x + c)^14 + 278460*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^12 - 120120*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^10 + 30030*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^8 - 83160*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^6 - 18900*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^4 - 300*a^3 - 1440*a^2*b - 3600*a*b^2 - 1185*b^3 + 280*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(39501*b^3*cosh(d*x + c)^17 - 333336*b^3*cosh(d*x + c)^15 + 64260*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^13 - 32760*(20*a^3 + 96*a^2*b + 240*a*b

```
^2 + 79*b^3)*cosh(d*x + c)^11 + 10010*(170*a^3 + 432*a^2*b + 648*a*b^2 + 18
7*b^3)*cosh(d*x + c)^9 - 35640*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh
(d*x + c)^7 - 11340*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^5
 + 280*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^3 - 45*(20
*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 43*b
^3*cosh(d*x + c)^2 + 15*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^4 + 5*(4389*b^3*c
osh(d*x + c)^18 - 41667*b^3*cosh(d*x + c)^16 + 9180*(48*a*b^2 + 23*b^3)*cos
h(d*x + c)^14 - 5460*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)
^12 + 2002*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^10 - 8
910*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^8 - 3780*(44*a^3
+ 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^6 + 140*(170*a^3 + 432*a^2*b
+ 648*a*b^2 + 187*b^3)*cosh(d*x + c)^4 + 144*a*b^2 + 69*b^3 - 45*(20*a^3 +
96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 20*(231*b
^3*cosh(d*x + c)^19 - 2451*b^3*cosh(d*x + c)^17 + 612*(48*a*b^2 + 23*b^3)*c
osh(d*x + c)^15 - 420*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c
)^13 + 182*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^11 - 9
90*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^9 - 540*(44*a^3 +
32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^7 + 28*(170*a^3 + 432*a^2*b + 6
48*a*b^2 + 187*b^3)*cosh(d*x + c)^5 - 15*(20*a^3 + 96*a^2*b + 240*a*b^2 + 7
9*b^3)*cosh(d*x + c)^3 + 3*(48*a*b^2 + 23*b^3)*cosh(d*x + c))*sinh(d*x + c)
^3 + 3*b^3 + (693*b^3*cosh(d*x + c)^20 - 8170*b^3*cosh(d*x + c)^18 + 2295*(
48*a*b^2 + 23*b^3)*cosh(d*x + c)^16 - 1800*(20*a^3 + 96*a^2*b + 240*a*b^2 +
 79*b^3)*cosh(d*x + c)^14 + 910*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)
*cosh(d*x + c)^12 - 5940*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x +
 c)^10 - 4050*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^8 + 280
*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^6 - 225*(20*a^3
+ 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^4 - 43*b^3 + 90*(48*a*b^2 +
23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 30*((5*a^3 + 24*a^2*b)*cosh(d*x
+ c)^17 + 17*(5*a^3 + 24*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^16 + (5*a^3 + 2
4*a^2*b)*sinh(d*x + c)^17 - 6*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^15 - 2*(15*a
^3 + 72*a^2*b - 68*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^15 + 1
0*(68*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 9*(5*a^3 + 24*a^2*b)*cosh(d*x +
c))*sinh(d*x + c)^14 + 15*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^13 + 5*(476*(5*a
^3 + 24*a^2*b)*cosh(d*x + c)^4 + 15*a^3 + 72*a^2*b - 126*(5*a^3 + 24*a^2*b)
*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(476*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^5 - 210*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 + 15*(5*a^3 + 24*a^2*b)*cosh(
d*x + c))*sinh(d*x + c)^12 - 20*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 + 2*(61
88*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^6 - 4095*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^4 - 50*a^3 - 240*a^2*b + 585*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*
x + c)^11 + 22*(884*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 819*(5*a^3 + 24*a^
2*b)*cosh(d*x + c)^5 + 195*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 10*(5*a^3 +
 24*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^10 + 15*(5*a^3 + 24*a^2*b)*cosh(d*x
 + c)^9 + 5*(4862*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 6006*(5*a^3 + 24*a^2
*b)*cosh(d*x + c)^6 + 2145*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 + 15*a^3 + 72
*a^2*b - 220*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 5*(4862*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^9 - 7722*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
7 + 3861*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 - 660*(5*a^3 + 24*a^2*b)*cosh(d
*x + c)^3 + 27*(5*a^3 + 24*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^8 - 6*(5*a^3
 + 24*a^2*b)*cosh(d*x + c)^7 + 2*(9724*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10
- 19305*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 + 12870*(5*a^3 + 24*a^2*b)*cosh(
d*x + c)^6 - 3300*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 - 15*a^3 - 72*a^2*b +
270*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 2*(6188*(5*a^3 +
24*a^2*b)*cosh(d*x + c)^11 - 15015*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^9 + 128
70*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 4620*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^5 + 630*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 21*(5*a^3 + 24*a^2*b)*cosh(
d*x + c))*sinh(d*x + c)^6 + (5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 + (6188*(5*a
^3 + 24*a^2*b)*cosh(d*x + c)^12 - 18018*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10
 + 19305*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 9240*(5*a^3 + 24*a^2*b)*cosh(
d*x + c)^6 + 1890*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 + 5*a^3 + 24*a^2*b - 1
```

26*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*(476*(5*a^3 + 24
*a^2*b)*cosh(d*x + c)^13 - 1638*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 + 2145*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^9 - 1320*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
7 + 378*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 - 42*(5*a^3 + 24*a^2*b)*cosh(d*x
+ c)^3 + (5*a^3 + 24*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^4 + 10*(68*(5*a^3
+ 24*a^2*b)*cosh(d*x + c)^14 - 273*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^12 + 4
29*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10 - 330*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^8 + 126*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^6 - 21*(5*a^3 + 24*a^2*b)*cosh(
d*x + c)^4 + (5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(68*(5
*a^3 + 24*a^2*b)*cosh(d*x + c)^15 - 315*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^13
+ 585*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 - 550*(5*a^3 + 24*a^2*b)*cosh(d*
x + c)^9 + 270*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 63*(5*a^3 + 24*a^2*b)*c
osh(d*x + c)^5 + 5*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (1
7*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^16 - 90*(5*a^3 + 24*a^2*b)*cosh(d*x + c)
^14 + 195*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^12 - 220*(5*a^3 + 24*a^2*b)*cosh
(d*x + c)^10 + 135*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 42*(5*a^3 + 24*a^2*
b)*cosh(d*x + c)^6 + 5*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4)*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) + 1) - 30*((5*a^3 + 24*a^2*b)*cosh(d*x + c
)^17 + 17*(5*a^3 + 24*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^16 + (5*a^3 + 24*a
^2*b)*sinh(d*x + c)^17 - 6*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^15 - 2*(15*a^3
+ 72*a^2*b - 68*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^15 + 10*(
68*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 9*(5*a^3 + 24*a^2*b)*cosh(d*x + c))
*sinh(d*x + c)^14 + 15*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^13 + 5*(476*(5*a^3
+ 24*a^2*b)*cosh(d*x + c)^4 + 15*a^3 + 72*a^2*b - 126*(5*a^3 + 24*a^2*b)*co
sh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(476*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
5 - 210*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 + 15*(5*a^3 + 24*a^2*b)*cosh(d*x
+ c))*sinh(d*x + c)^12 - 20*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 + 2*(6188*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^6 - 4095*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
4 - 50*a^3 - 240*a^2*b + 585*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x +
c)^11 + 22*(884*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 819*(5*a^3 + 24*a^2*b
)*cosh(d*x + c)^5 + 195*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 10*(5*a^3 + 24
*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^10 + 15*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^9 + 5*(4862*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 6006*(5*a^3 + 24*a^2*b)
*cosh(d*x + c)^6 + 2145*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 + 15*a^3 + 72*a^
2*b - 220*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 5*(4862*(5*
a^3 + 24*a^2*b)*cosh(d*x + c)^9 - 7722*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 +
3861*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 - 660*(5*a^3 + 24*a^2*b)*cosh(d*x
+ c)^3 + 27*(5*a^3 + 24*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^8 - 6*(5*a^3 +
24*a^2*b)*cosh(d*x + c)^7 + 2*(9724*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10 - 1
9305*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 + 12870*(5*a^3 + 24*a^2*b)*cosh(d*x
+ c)^6 - 3300*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 - 15*a^3 - 72*a^2*b + 270
*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 2*(6188*(5*a^3 + 24*
a^2*b)*cosh(d*x + c)^11 - 15015*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^9 + 12870*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 4620*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
5 + 630*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 21*(5*a^3 + 24*a^2*b)*cosh(d*x
+ c))*sinh(d*x + c)^6 + (5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 + (6188*(5*a^3
+ 24*a^2*b)*cosh(d*x + c)^12 - 18018*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10 +
19305*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 9240*(5*a^3 + 24*a^2*b)*cosh(d*x
+ c)^6 + 1890*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 + 5*a^3 + 24*a^2*b - 126*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*(476*(5*a^3 + 24*a^
2*b)*cosh(d*x + c)^13 - 1638*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 + 2145*(5*
a^3 + 24*a^2*b)*cosh(d*x + c)^9 - 1320*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 +
378*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 - 42*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^3 + (5*a^3 + 24*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^4 + 10*(68*(5*a^3 +
24*a^2*b)*cosh(d*x + c)^14 - 273*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^12 + 429*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10 - 330*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
8 + 126*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^6 - 21*(5*a^3 + 24*a^2*b)*cosh(d*x
+ c)^4 + (5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(68*(5*a^
3 + 24*a^2*b)*cosh(d*x + c)^15 - 315*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^13 +

585*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 - 550*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^9 + 270*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 63*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 + 5*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (17*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^16 - 90*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^14 + 195*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^12 - 220*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10 + 135*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 42*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^6 + 5*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(33*b^3*cosh(d*x + c)^21 - 430*b^3*cosh(d*x + c)^19 + 135*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^17 - 120*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^15 + 70*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^13 - 540*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^11 - 450*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^9 + 40*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^7 - 45*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^5 - 43*b^3*cosh(d*x + c) + 30*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))/(d*cosh(d*x + c)^17 + 17*d*cosh(d*x + c)*sinh(d*x + c)^16 + d*sinh(d*x + c)^17 - 6*d*cosh(d*x + c)^15 + 2*(68*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^15 + 10*(68*d*cosh(d*x + c)^3 - 9*d*cosh(d*x + c))*sinh(d*x + c)^14 + 15*d*cosh(d*x + c)^13 + 5*(476*d*cosh(d*x + c)^4 - 126*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^13 + 13*(476*d*cosh(d*x + c)^5 - 210*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^12 - 20*d*cosh(d*x + c)^11 + 2*(6188*d*cosh(d*x + c)^6 - 4095*d*cosh(d*x + c)^4 + 585*d*cosh(d*x + c)^2 - 10*d)*sinh(d*x + c)^11 + 22*(884*d*cosh(d*x + c)^7 - 819*d*cosh(d*x + c)^5 + 195*d*cosh(d*x + c)^3 - 10*d*cosh(d*x + c))*sinh(d*x + c)^10 + 15*d*cosh(d*x + c)^9 + 5*(4862*d*cosh(d*x + c)^8 - 6006*d*cosh(d*x + c)^6 + 2145*d*cosh(d*x + c)^4 - 220*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^9 + 5*(4862*d*cosh(d*x + c)^9 - 7722*d*cosh(d*x + c)^7 + 3861*d*cosh(d*x + c)^5 - 660*d*cosh(d*x + c)^3 + 27*d*cosh(d*x + c))*sinh(d*x + c)^8 - 6*d*cosh(d*x + c)^7 + 2*(9724*d*cosh(d*x + c)^10 - 19305*d*cosh(d*x + c)^8 + 12870*d*cosh(d*x + c)^6 - 3300*d*cosh(d*x + c)^4 + 270*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^7 + 2*(6188*d*cosh(d*x + c)^11 - 15015*d*cosh(d*x + c)^9 + 12870*d*cosh(d*x + c)^7 - 4620*d*cosh(d*x + c)^5 + 630*d*cosh(d*x + c)^3 - 21*d*cosh(d*x + c))*sinh(d*x + c)^6 + d*cosh(d*x + c)^5 + (6188*d*cosh(d*x + c)^12 - 18018*d*cosh(d*x + c)^10 + 19305*d*cosh(d*x + c)^8 - 9240*d*cosh(d*x + c)^6 + 1890*d*cosh(d*x + c)^4 - 126*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^5 + 5*(476*d*cosh(d*x + c)^13 - 1638*d*cosh(d*x + c)^11 + 2145*d*cosh(d*x + c)^9 - 1320*d*cosh(d*x + c)^7 + 378*d*cosh(d*x + c)^5 - 42*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 10*(68*d*cosh(d*x + c)^14 - 273*d*cosh(d*x + c)^12 + 429*d*cosh(d*x + c)^10 - 330*d*cosh(d*x + c)^8 + 126*d*cosh(d*x + c)^6 - 21*d*cosh(d*x + c)^4 + d*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(68*d*cosh(d*x + c)^15 - 315*d*cosh(d*x + c)^13 + 585*d*cosh(d*x + c)^11 - 550*d*cosh(d*x + c)^9 + 270*d*cosh(d*x + c)^7 - 63*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (17*d*cosh(d*x + c)^16 - 90*d*cosh(d*x + c)^14 + 195*d*cosh(d*x + c)^12 - 220*d*cosh(d*x + c)^10 + 135*d*cosh(d*x + c)^8 - 42*d*cosh(d*x + c)^6 + 5*d*cosh(d*x + c)^4)*sinh(d*x + c))

giac [B] time = 0.48, size = 321, normalized size = 2.06

$$\frac{3\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5-40\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+720\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+240\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] 1/480*(3*b^3*(e^(d*x + c) + e^(-d*x - c))^5 - 40*b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 720*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 240*b^3*(e^(d*x + c) + e^(-d*x - c)) + 15*(5*a^3 + 24*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 15*(5*a^3 + 24*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 20*(15*a^3*(e^

```
(d*x + c) + e^(-d*x - c))^5 + 72*a^2*b*(e^(d*x + c) + e^(-d*x - c))^5 - 160
*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 576*a^2*b*(e^(d*x + c) + e^(-d*x - c)
)^3 + 528*a^3*(e^(d*x + c) + e^(-d*x - c)) + 1152*a^2*b*(e^(d*x + c) + e^(-
d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^3)/d
```

maple [A] time = 0.10, size = 128, normalized size = 0.82

$$\frac{a^{3}\left(\left(-\frac{\operatorname{csch}(dx+c)^{5}}{6}+\frac{5\operatorname{csch}(dx+c)^{3}}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth\left(dx+c\right)+\frac{5\operatorname{arctanh}\left(e^{dx+c}\right)}{8}\right)+3a^{2}b\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x
+c)+5/8*arctanh(exp(d*x+c)))+3*a^2*b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(
exp(d*x+c))+3*a*b^2*cosh(d*x+c)+b^3*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+
c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.34, size = 390, normalized size = 2.50

$$\frac{1}{480}\,b^{3}\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{3}{2}\,ab^{2}\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/480*b^3*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 3/2*a
*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/48*a^3*(15*log(e^(-d*x - c) + 1)/
d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) +
 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(
-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c)
- 1))) + 3/2*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(
e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c)
- 1)))
```

mupad [B] time = 1.14, size = 633, normalized size = 4.06

$$\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(5\,a^{3}\,\sqrt{-d^{2}}+24\,a^{2}\,b\,\sqrt{-d^{2}}\right)}{d\,\sqrt{25\,a^{6}+240\,a^{5}\,b+576\,a^{4}\,b^{2}}}\right)\,\sqrt{25\,a^{6}+240\,a^{5}\,b+576\,a^{4}\,b^{2}}}{8\,\sqrt{-d^{2}}}-\frac{\frac{4\,e^{5\,c+5\,d\,x}\,\left(8\,a^{3}+9\,b\,a^{2}\right)}{3\,d}-\frac{8\,a^{2}\,b\,e^{3\,c+3\,d\,x}}{d}-\frac{8\,a^{2}\,b\,e^{7\,c+}}{d}}{15\,e^{4\,c+4\,d\,x}-6\,e^{2\,c+2\,d\,x}-20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^7,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*a^3*(-d^2)^(1/2) + 24*a^2*b*(-d^2)^(1/2)))/(d*(24
0*a^5*b + 25*a^6 + 576*a^4*b^2)^(1/2)))*(240*a^5*b + 25*a^6 + 576*a^4*b^2)^
(1/2))/(8*(-d^2)^(1/2)) - ((4*exp(5*c + 5*d*x)*(9*a^2*b + 8*a^3))/(3*d) - (
8*a^2*b*exp(3*c + 3*d*x))/d - (8*a^2*b*exp(7*c + 7*d*x))/d + (2*a^2*b*exp(9
*c + 9*d*x))/d + (2*a^2*b*exp(c + d*x))/d)/(15*exp(4*c + 4*d*x) - 6*exp(2*c
 + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x
) + exp(12*c + 12*d*x) + 1) - (5*b^3*exp(- 3*c - 3*d*x))/(96*d) - (5*b^3*ex
p(3*c + 3*d*x))/(96*d) + (b^3*exp(- 5*c - 5*d*x))/(160*d) + (b^3*exp(5*c +
5*d*x))/(160*d) - (exp(c + d*x)*(24*a^2*b + 5*a^3))/(8*d*(exp(2*c + 2*d*x)
- 1)) - (a^3*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) +
```

```
exp(6*c + 6*d*x) - 1)) + (b^2*exp(c + d*x)*(24*a + 5*b))/(16*d) - (exp(c +
d*x)*(48*a^2*b - 5*a^3))/(12*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
 - (22*a^3*exp(c + d*x))/(3*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*
exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (16*a^3*exp(c + d*x))/(3*d*(5*e
xp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8
*d*x) + exp(10*c + 10*d*x) - 1)) + (b^2*exp(- c - d*x)*(24*a + 5*b))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.214 $\int \operatorname{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=171

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{8d}+\frac{7a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{48d}-\frac{a\left(35a^2+144ab+384b^2\right)\tanh^{-1}(\cosh(c+dx))}{128d}$$

```
[Out] -1/128*a*(35*a^2+144*a*b+384*b^2)*arctanh(cosh(d*x+c))/d-b^3*cosh(d*x+c)/d+
1/3*b^3*cosh(d*x+c)^3/d+1/128*a^2*(35*a+144*b)*coth(d*x+c)*csch(d*x+c)/d-1/
192*a^2*(35*a+144*b)*coth(d*x+c)*csch(d*x+c)^3/d+7/48*a^3*coth(d*x+c)*csch(
d*x+c)^5/d-1/8*a^3*coth(d*x+c)*csch(d*x+c)^7/d
```

Rubi [A] time = 0.33, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1157, 1814, 1153, 206}

$$-\frac{a\left(35a^2+144ab+384b^2\right)\tanh^{-1}(\cosh(c+dx))}{128d}-\frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}^3(c+dx)}{192d}+\frac{a^2(35a+144b)\operatorname{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^9*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(a*(35*a^2 + 144*a*b + 384*b^2)*ArcTanh[Cosh[c + d*x]])/(128*d) - (b^3*Cos
h[c + d*x])/d + (b^3*Cosh[c + d*x]^3)/(3*d) + (a^2*(35*a + 144*b)*Coth[c +
d*x]*Csch[c + d*x])/(128*d) - (a^2*(35*a + 144*b)*Coth[c + d*x]*Csch[c + d*
x]^3)/(192*d) + (7*a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(48*d) - (a^3*Coth[c
+ d*x]*Csch[c + d*x]^7)/(8*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^5}dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{8d} + \frac{\operatorname{Subst}\left(\int \frac{-(a+2b)\left(7a^2+10ab+4b^2\right)+\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{7a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{48d} - \frac{a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{8d} - \cdots$$

$$= -\frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}^3(c+dx)}{192d} + \frac{7a^3\coth(c+dx)\cdots}{48d}$$

$$= \frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}(c+dx)}{128d} - \frac{a^2(35a+144b)\coth\cdots}{19\cdots}$$

$$= \frac{a^2(35a+144b)\coth(c+dx)\operatorname{csch}(c+dx)}{128d} - \frac{a^2(35a+144b)\coth\cdots}{19\cdots}$$

$$= -\frac{b^3\cosh(c+dx)}{d} + \frac{b^3\cosh^3(c+dx)}{3d} + \frac{a^2(35a+144b)\coth(c+\cdots}{128d}$$

$$= -\frac{a\left(35a^2+144ab+384b^2\right)\tanh^{-1}(\cosh(c+dx))}{128d} - \frac{b^3\cosh(c+\cdots}{d}$$

Mathematica [A] time = 1.68, size = 219, normalized size = 1.28

$$\frac{a\left(48\left(35a^2+144ab+384b^2\right)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-3a^2\operatorname{csch}^8\left(\frac{1}{2}(c+dx)\right)+20a^2\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)+3a^2\operatorname{se}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^9*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-4608*b^3*Cosh[c + d*x] + 512*b^3*Cosh[3*(c + d*x)] + a*(12*a*(35*a + 144*
b)*Csch[(c + d*x)/2]^2 - 18*a*(5*a + 16*b)*Csch[(c + d*x)/2]^4 + 20*a^2*Csc
h[(c + d*x)/2]^6 - 3*a^2*Csch[(c + d*x)/2]^8 + 48*(35*a^2 + 144*a*b + 384*b
^2)*Log[Tanh[(c + d*x)/2]] + 12*a*(35*a + 144*b)*Sech[(c + d*x)/2]^2 + 18*a
*(5*a + 16*b)*Sech[(c + d*x)/2]^4 + 20*a^2*Sech[(c + d*x)/2]^6 + 3*a^2*Sech
[(c + d*x)/2]^8))/(6144*d)
```

fricas [B] time = 2.00, size = 10848, normalized size = 63.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(16*b^3*cosh(d*x + c)^22 + 352*b^3*cosh(d*x + c)*sinh(d*x + c)^21 + 1
6*b^3*sinh(d*x + c)^22 - 272*b^3*cosh(d*x + c)^20 + 16*(231*b^3*cosh(d*x +
c)^2 - 17*b^3)*sinh(d*x + c)^20 + 320*(77*b^3*cosh(d*x + c)^3 - 17*b^3*cosh
(d*x + c))*sinh(d*x + c)^19 + 2*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x +
c)^18 + 2*(58520*b^3*cosh(d*x + c)^4 - 25840*b^3*cosh(d*x + c)^2 + 105*a^3
+ 432*a^2*b + 728*b^3)*sinh(d*x + c)^18 + 12*(35112*b^3*cosh(d*x + c)^5 - 2
5840*b^3*cosh(d*x + c)^3 + 3*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c))
*sinh(d*x + c)^17 - 2*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^16 +
2*(596904*b^3*cosh(d*x + c)^6 - 658920*b^3*cosh(d*x + c)^4 - 805*a^3 - 3312
*a^2*b - 1880*b^3 + 153*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^16 + 32*(85272*b^3*cosh(d*x + c)^7 - 131784*b^3*cosh(d*x + c)^5
 + 51*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^3 - (805*a^3 + 3312*a^2
*b + 1880*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 2*(2681*a^3 + 7344*a^2*b +
 2512*b^3)*cosh(d*x + c)^14 + 2*(2558160*b^3*cosh(d*x + c)^8 - 5271360*b^3*
cosh(d*x + c)^6 + 3060*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^4 + 26
81*a^3 + 7344*a^2*b + 2512*b^3 - 120*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^14 + 4*(1989680*b^3*cosh(d*x + c)^9 - 5271360*b^
3*cosh(d*x + c)^7 + 4284*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^5 -
280*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^3 + 7*(2681*a^3 + 7344*
a^2*b + 2512*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 2*(5053*a^3 + 4464*a^2*
b + 1232*b^3)*cosh(d*x + c)^12 + 2*(5173168*b^3*cosh(d*x + c)^10 - 17131920
*b^3*cosh(d*x + c)^8 + 18564*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^
6 - 1820*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^4 - 5053*a^3 - 446
4*a^2*b - 1232*b^3 + 91*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^12 + 8*(1410864*b^3*cosh(d*x + c)^11 - 5710640*b^3*cosh(d*x
+ c)^9 + 7956*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^7 - 1092*(805*a
^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^5 + 91*(2681*a^3 + 7344*a^2*b + 2
512*b^3)*cosh(d*x + c)^3 - 3*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x +
c))*sinh(d*x + c)^11 - 2*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^1
0 + 2*(5173168*b^3*cosh(d*x + c)^12 - 25126816*b^3*cosh(d*x + c)^10 + 43758
*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^8 - 8008*(805*a^3 + 3312*a^2
*b + 1880*b^3)*cosh(d*x + c)^6 + 1001*(2681*a^3 + 7344*a^2*b + 2512*b^3)*co
sh(d*x + c)^4 - 5053*a^3 - 4464*a^2*b - 1232*b^3 - 66*(5053*a^3 + 4464*a^2*
b + 1232*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(1989680*b^3*cosh(d*x +
 c)^13 - 11421280*b^3*cosh(d*x + c)^11 + 24310*(105*a^3 + 432*a^2*b + 728*b
^3)*cosh(d*x + c)^9 - 5720*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^
7 + 1001*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^5 - 110*(5053*a^3
 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^3 - 5*(5053*a^3 + 4464*a^2*b + 1232
*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(2681*a^3 + 7344*a^2*b + 2512*b^3)
*cosh(d*x + c)^8 + 2*(2558160*b^3*cosh(d*x + c)^14 - 17131920*b^3*cosh(d*x
+ c)^12 + 43758*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^10 - 12870*(8
05*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^8 + 3003*(2681*a^3 + 7344*a^2
*b + 2512*b^3)*cosh(d*x + c)^6 - 495*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cos
h(d*x + c)^4 + 2681*a^3 + 7344*a^2*b + 2512*b^3 - 45*(5053*a^3 + 4464*a^2*b
 + 1232*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(170544*b^3*cosh(d*x + c
)^15 - 1317840*b^3*cosh(d*x + c)^13 + 3978*(105*a^3 + 432*a^2*b + 728*b^3)*
cosh(d*x + c)^11 - 1430*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^9 +
 429*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^7 - 99*(5053*a^3 + 44
64*a^2*b + 1232*b^3)*cosh(d*x + c)^5 - 15*(5053*a^3 + 4464*a^2*b + 1232*b^3
)*cosh(d*x + c)^3 + (2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c))*sinh(
d*x + c)^7 - 2*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^6 + 2*(59690
4*b^3*cosh(d*x + c)^16 - 5271360*b^3*cosh(d*x + c)^14 + 18564*(105*a^3 + 43
2*a^2*b + 728*b^3)*cosh(d*x + c)^12 - 8008*(805*a^3 + 3312*a^2*b + 1880*b^3
)*cosh(d*x + c)^10 + 3003*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^
8 - 924*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^6 - 210*(5053*a^3
+ 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^4 - 805*a^3 - 3312*a^2*b - 1880*b^3
```

+ 28*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
4*(105336*b^3*cosh(d*x + c)^17 - 1054272*b^3*cosh(d*x + c)^15 + 4284*(105*a
^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^13 - 2184*(805*a^3 + 3312*a^2*b + 1
880*b^3)*cosh(d*x + c)^11 + 1001*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*
x + c)^9 - 396*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^7 - 126*(50
53*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^5 + 28*(2681*a^3 + 7344*a^2*b
 + 2512*b^3)*cosh(d*x + c)^3 - 3*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^5 - 272*b^3*cosh(d*x + c)^2 + 2*(105*a^3 + 432*a^2*b +
 728*b^3)*cosh(d*x + c)^4 + 2*(58520*b^3*cosh(d*x + c)^18 - 658920*b^3*cosh
(d*x + c)^16 + 3060*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^14 - 1820
*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^12 + 1001*(2681*a^3 + 7344
*a^2*b + 2512*b^3)*cosh(d*x + c)^10 - 495*(5053*a^3 + 4464*a^2*b + 1232*b^3
)*cosh(d*x + c)^8 - 210*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^6
+ 70*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^4 + 105*a^3 + 432*a^2
*b + 728*b^3 - 15*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^2)*sinh(d
x + c)^4 + 8(3080*b^3*cosh(d*x + c)^19 - 38760*b^3*cosh(d*x + c)^17 + 204
*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^15 - 140*(805*a^3 + 3312*a^2
*b + 1880*b^3)*cosh(d*x + c)^13 + 91*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cos
h(d*x + c)^11 - 55*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^9 - 30*
(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^7 + 14*(2681*a^3 + 7344*a^
2*b + 2512*b^3)*cosh(d*x + c)^5 - 5*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(
d*x + c)^3 + (105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
 + 16*b^3 + 2*(1848*b^3*cosh(d*x + c)^20 - 25840*b^3*cosh(d*x + c)^18 + 153
*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^16 - 120*(805*a^3 + 3312*a^2
*b + 1880*b^3)*cosh(d*x + c)^14 + 91*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cos
h(d*x + c)^12 - 66*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^10 - 45
*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^8 + 28*(2681*a^3 + 7344*a
^2*b + 2512*b^3)*cosh(d*x + c)^6 - 15*(805*a^3 + 3312*a^2*b + 1880*b^3)*cos
h(d*x + c)^4 - 136*b^3 + 6*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^2 - 3*((35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^19 + 1
9*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^18 + (35*a^3
 + 144*a^2*b + 384*a*b^2)*sinh(d*x + c)^19 - 8*(35*a^3 + 144*a^2*b + 384*a*
b^2)*cosh(d*x + c)^17 - (280*a^3 + 1152*a^2*b + 3072*a*b^2 - 171*(35*a^3 +
144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^17 + 17*(57*(35*a^3 +
 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 8*(35*a^3 + 144*a^2*b + 384*a*b^2
)*cosh(d*x + c))*sinh(d*x + c)^16 + 28*(35*a^3 + 144*a^2*b + 384*a*b^2)*cos
h(d*x + c)^15 + 4*(969*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 2
45*a^3 + 1008*a^2*b + 2688*a*b^2 - 272*(35*a^3 + 144*a^2*b + 384*a*b^2)*cos
h(d*x + c)^2)*sinh(d*x + c)^15 + 4*(2907*(35*a^3 + 144*a^2*b + 384*a*b^2)*c
osh(d*x + c)^5 - 1360*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 + 10
5*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^14 - 56*(35
*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^13 + 28*(969*(35*a^3 + 144*a^2*
b + 384*a*b^2)*cosh(d*x + c)^6 - 680*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(
d*x + c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 105*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 52*(969*(35*a^3 + 144*a^2*b + 3
84*a*b^2)*cosh(d*x + c)^7 - 952*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x +
 c)^5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 14*(35*a^3 +
 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^12 + 70*(35*a^3 + 144*
a^2*b + 384*a*b^2)*cosh(d*x + c)^11 + 2*(37791*(35*a^3 + 144*a^2*b + 384*a*
b^2)*cosh(d*x + c)^8 - 49504*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)
^6 + 19110*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 1225*a^3 + 50
40*a^2*b + 13440*a*b^2 - 2184*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c
)^2)*sinh(d*x + c)^11 + 22*(4199*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
+ c)^9 - 7072*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 + 3822*(35*a
^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 - 728*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^3 + 35*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^10 - 56*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 +
2*(46189*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 97240*(35*a^3
+ 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^8 + 70070*(35*a^3 + 144*a^2*b + 384*

a*b^2)*cosh(d*x + c)^6 - 20020*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x +
c)^4 - 980*a^3 - 4032*a^2*b - 10752*a*b^2 + 1925*(35*a^3 + 144*a^2*b + 384*
a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 2*(37791*(35*a^3 + 144*a^2*b + 38
4*a*b^2)*cosh(d*x + c)^11 - 97240*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^9 + 90090*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 - 36036*(3
5*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 + 5775*(35*a^3 + 144*a^2*b +
 384*a*b^2)*cosh(d*x + c)^3 - 252*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^8 + 28*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^
7 + 4*(12597*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12 - 38896*(35*
a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 + 45045*(35*a^3 + 144*a^2*b +
 384*a*b^2)*cosh(d*x + c)^8 - 24024*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d
x + c)^6 + 5775(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 245*a^3
 + 1008*a^2*b + 2688*a*b^2 - 504*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^7 + 28*(969*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^13 - 3536*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^11 + 5005*(3
5*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 - 3432*(35*a^3 + 144*a^2*b +
 384*a*b^2)*cosh(d*x + c)^7 + 1155*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*
x + c)^5 - 168*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 + 7*(35*a^3
 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 8*(35*a^3 + 144*
a^2*b + 384*a*b^2)*cosh(d*x + c)^5 + 4*(2907*(35*a^3 + 144*a^2*b + 384*a*b^
2)*cosh(d*x + c)^14 - 12376*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^
12 + 21021*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 18018*(35*a^
3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^8 + 8085*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^6 - 1764*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x +
c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 147*(35*a^3 + 144*a^2*b + 384*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 4*(969*(35*a^3 + 144*a^2*b + 384*a*b^2
)*cosh(d*x + c)^15 - 4760*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^13
 + 9555*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^11 - 10010*(35*a^3 +
 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 + 5775*(35*a^3 + 144*a^2*b + 384*a*
b^2)*cosh(d*x + c)^7 - 1764*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^
5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 10*(35*a^3 + 144
*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^3 + 144*a^2*b +
384*a*b^2)*cosh(d*x + c)^3 + (969*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^16 - 5440*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^14 + 12740*(
35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12 - 16016*(35*a^3 + 144*a^2*
b + 384*a*b^2)*cosh(d*x + c)^10 + 11550*(35*a^3 + 144*a^2*b + 384*a*b^2)*co
sh(d*x + c)^8 - 4704*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^6 + 980
*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 35*a^3 + 144*a^2*b + 38
4*a*b^2 - 80*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c
)^3 + (171*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^17 - 1088*(35*a^3
 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^15 + 2940*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^13 - 4368*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x +
 c)^11 + 3850*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 - 2016*(35*a
^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 + 588*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^5 - 80*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)
^3 + 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (1
9*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^18 - 136*(35*a^3 + 144*a^2
*b + 384*a*b^2)*cosh(d*x + c)^16 + 420*(35*a^3 + 144*a^2*b + 384*a*b^2)*cos
h(d*x + c)^14 - 728*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12 + 770
*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 504*(35*a^3 + 144*a^2*
b + 384*a*b^2)*cosh(d*x + c)^8 + 196*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(
d*x + c)^6 - 40*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 3*(35*a^
3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c
) + sinh(d*x + c) + 1) + 3*((35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^
19 + 19*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^18 + (
35*a^3 + 144*a^2*b + 384*a*b^2)*sinh(d*x + c)^19 - 8*(35*a^3 + 144*a^2*b +
384*a*b^2)*cosh(d*x + c)^17 - (280*a^3 + 1152*a^2*b + 3072*a*b^2 - 171*(35*
a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^17 + 17*(57*(35
*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 8*(35*a^3 + 144*a^2*b + 384

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*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^16 + 28*(35*a^3 + 144*a^2*b + 384*a*b^
2)*cosh(d*x + c)^15 + 4*(969*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)
^4 + 245*a^3 + 1008*a^2*b + 2688*a*b^2 - 272*(35*a^3 + 144*a^2*b + 384*a*b^
2)*cosh(d*x + c)^2)*sinh(d*x + c)^15 + 4*(2907*(35*a^3 + 144*a^2*b + 384*a*
b^2)*cosh(d*x + c)^5 - 1360*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^
3 + 105*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^14 -
56*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^13 + 28*(969*(35*a^3 + 14
4*a^2*b + 384*a*b^2)*cosh(d*x + c)^6 - 680*(35*a^3 + 144*a^2*b + 384*a*b^2)
*cosh(d*x + c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 105*(35*a^3 + 144*a^2*b
 + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 52*(969*(35*a^3 + 144*a^2
*b + 384*a*b^2)*cosh(d*x + c)^7 - 952*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh
(d*x + c)^5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 14*(35
*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^12 + 70*(35*a^3
+ 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^11 + 2*(37791*(35*a^3 + 144*a^2*b +
384*a*b^2)*cosh(d*x + c)^8 - 49504*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*
x + c)^6 + 19110*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 1225*a^
3 + 5040*a^2*b + 13440*a*b^2 - 2184*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c)^11 + 22*(4199*(35*a^3 + 144*a^2*b + 384*a*b^2)*cos
h(d*x + c)^9 - 7072*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 + 3822
*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 - 728*(35*a^3 + 144*a^2*b
 + 384*a*b^2)*cosh(d*x + c)^3 + 35*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*
x + c))*sinh(d*x + c)^10 - 56*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c
)^9 + 2*(46189*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 97240*(3
5*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^8 + 70070*(35*a^3 + 144*a^2*b
+ 384*a*b^2)*cosh(d*x + c)^6 - 20020*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(
d*x + c)^4 - 980*a^3 - 4032*a^2*b - 10752*a*b^2 + 1925*(35*a^3 + 144*a^2*b
+ 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 2*(37791*(35*a^3 + 144*a^2*
b + 384*a*b^2)*cosh(d*x + c)^11 - 97240*(35*a^3 + 144*a^2*b + 384*a*b^2)*co
sh(d*x + c)^9 + 90090*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 - 36
036*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 + 5775*(35*a^3 + 144*a
^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 252*(35*a^3 + 144*a^2*b + 384*a*b^2)*co
sh(d*x + c))*sinh(d*x + c)^8 + 28*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^7 + 4*(12597*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12 - 3889
6*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 + 45045*(35*a^3 + 144*a
^2*b + 384*a*b^2)*cosh(d*x + c)^8 - 24024*(35*a^3 + 144*a^2*b + 384*a*b^2)*
cosh(d*x + c)^6 + 5775*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 2
45*a^3 + 1008*a^2*b + 2688*a*b^2 - 504*(35*a^3 + 144*a^2*b + 384*a*b^2)*cos
h(d*x + c)^2)*sinh(d*x + c)^7 + 28*(969*(35*a^3 + 144*a^2*b + 384*a*b^2)*co
sh(d*x + c)^13 - 3536*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^11 + 5
005*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 - 3432*(35*a^3 + 144*a
^2*b + 384*a*b^2)*cosh(d*x + c)^7 + 1155*(35*a^3 + 144*a^2*b + 384*a*b^2)*c
osh(d*x + c)^5 - 168*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 + 7*(
35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 8*(35*a^3
+ 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 + 4*(2907*(35*a^3 + 144*a^2*b + 38
4*a*b^2)*cosh(d*x + c)^14 - 12376*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^12 + 21021*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 18018*
(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^8 + 8085*(35*a^3 + 144*a^2*b
 + 384*a*b^2)*cosh(d*x + c)^6 - 1764*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(
d*x + c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 147*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 4*(969*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^15 - 4760*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x +
 c)^13 + 9555*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^11 - 10010*(35
*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 + 5775*(35*a^3 + 144*a^2*b +
384*a*b^2)*cosh(d*x + c)^7 - 1764*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 10*(35*a^3
 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^3 + 144*a^
2*b + 384*a*b^2)*cosh(d*x + c)^3 + (969*(35*a^3 + 144*a^2*b + 384*a*b^2)*co
sh(d*x + c)^16 - 5440*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^14 + 1
2740*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12 - 16016*(35*a^3 + 14
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4*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 + 11550*(35*a^3 + 144*a^2*b + 384*a*b
^2)*cosh(d*x + c)^8 - 4704*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^6
 + 980*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 35*a^3 + 144*a^2*
b + 384*a*b^2 - 80*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^3 + (171*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^17 - 1088*(
35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^15 + 2940*(35*a^3 + 144*a^2*b
 + 384*a*b^2)*cosh(d*x + c)^13 - 4368*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh
(d*x + c)^11 + 3850*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 - 2016
*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 + 588*(35*a^3 + 144*a^2*b
 + 384*a*b^2)*cosh(d*x + c)^5 - 80*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*
x + c)^3 + 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^
2 + (19*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^18 - 136*(35*a^3 + 1
44*a^2*b + 384*a*b^2)*cosh(d*x + c)^16 + 420*(35*a^3 + 144*a^2*b + 384*a*b^
2)*cosh(d*x + c)^14 - 728*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12
 + 770*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 504*(35*a^3 + 14
4*a^2*b + 384*a*b^2)*cosh(d*x + c)^8 + 196*(35*a^3 + 144*a^2*b + 384*a*b^2)
*cosh(d*x + c)^6 - 40*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 3*
(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d
*x + c) + sinh(d*x + c) - 1) + 4*(88*b^3*cosh(d*x + c)^21 - 1360*b^3*cosh(d
*x + c)^19 + 9*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^17 - 8*(805*a^
3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^15 + 7*(2681*a^3 + 7344*a^2*b + 25
12*b^3)*cosh(d*x + c)^13 - 6*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x +
c)^11 - 5*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^9 + 4*(2681*a^3
+ 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^7 - 3*(805*a^3 + 3312*a^2*b + 1880*b
^3)*cosh(d*x + c)^5 - 136*b^3*cosh(d*x + c) + 2*(105*a^3 + 432*a^2*b + 728*
b^3)*cosh(d*x + c)^3)*sinh(d*x + c))/(d*cosh(d*x + c)^19 + 19*d*cosh(d*x +
c)*sinh(d*x + c)^18 + d*sinh(d*x + c)^19 - 8*d*cosh(d*x + c)^17 + (171*d*co
sh(d*x + c)^2 - 8*d)*sinh(d*x + c)^17 + 17*(57*d*cosh(d*x + c)^3 - 8*d*cosh
(d*x + c))*sinh(d*x + c)^16 + 28*d*cosh(d*x + c)^15 + 4*(969*d*cosh(d*x + c
)^4 - 272*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^15 + 4*(2907*d*cosh(d*x +
c)^5 - 1360*d*cosh(d*x + c)^3 + 105*d*cosh(d*x + c))*sinh(d*x + c)^14 - 56*
d*cosh(d*x + c)^13 + 28*(969*d*cosh(d*x + c)^6 - 680*d*cosh(d*x + c)^4 + 10
5*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^13 + 52*(969*d*cosh(d*x + c)^7 - 9
52*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 - 14*d*cosh(d*x + c))*sinh(d*x
 + c)^12 + 70*d*cosh(d*x + c)^11 + 2*(37791*d*cosh(d*x + c)^8 - 49504*d*cos
h(d*x + c)^6 + 19110*d*cosh(d*x + c)^4 - 2184*d*cosh(d*x + c)^2 + 35*d)*sin
h(d*x + c)^11 + 22*(4199*d*cosh(d*x + c)^9 - 7072*d*cosh(d*x + c)^7 + 3822*
d*cosh(d*x + c)^5 - 728*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x +
c)^10 - 56*d*cosh(d*x + c)^9 + 2*(46189*d*cosh(d*x + c)^10 - 97240*d*cosh(d
*x + c)^8 + 70070*d*cosh(d*x + c)^6 - 20020*d*cosh(d*x + c)^4 + 1925*d*cosh
(d*x + c)^2 - 28*d)*sinh(d*x + c)^9 + 2*(37791*d*cosh(d*x + c)^11 - 97240*d
*cosh(d*x + c)^9 + 90090*d*cosh(d*x + c)^7 - 36036*d*cosh(d*x + c)^5 + 5775
*d*cosh(d*x + c)^3 - 252*d*cosh(d*x + c))*sinh(d*x + c)^8 + 28*d*cosh(d*x +
 c)^7 + 4*(12597*d*cosh(d*x + c)^12 - 38896*d*cosh(d*x + c)^10 + 45045*d*co
sh(d*x + c)^8 - 24024*d*cosh(d*x + c)^6 + 5775*d*cosh(d*x + c)^4 - 504*d*co
sh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 + 28*(969*d*cosh(d*x + c)^13 - 3536*d*
cosh(d*x + c)^11 + 5005*d*cosh(d*x + c)^9 - 3432*d*cosh(d*x + c)^7 + 1155*d
*cosh(d*x + c)^5 - 168*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)
^6 - 8*d*cosh(d*x + c)^5 + 4*(2907*d*cosh(d*x + c)^14 - 12376*d*cosh(d*x +
c)^12 + 21021*d*cosh(d*x + c)^10 - 18018*d*cosh(d*x + c)^8 + 8085*d*cosh(d*
x + c)^6 - 1764*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x +
 c)^5 + 4*(969*d*cosh(d*x + c)^15 - 4760*d*cosh(d*x + c)^13 + 9555*d*cosh(d
*x + c)^11 - 10010*d*cosh(d*x + c)^9 + 5775*d*cosh(d*x + c)^7 - 1764*d*cosh
(d*x + c)^5 + 245*d*cosh(d*x + c)^3 - 10*d*cosh(d*x + c))*sinh(d*x + c)^4 +
 d*cosh(d*x + c)^3 + (969*d*cosh(d*x + c)^16 - 5440*d*cosh(d*x + c)^14 + 12
740*d*cosh(d*x + c)^12 - 16016*d*cosh(d*x + c)^10 + 11550*d*cosh(d*x + c)^8
 - 4704*d*cosh(d*x + c)^6 + 980*d*cosh(d*x + c)^4 - 80*d*cosh(d*x + c)^2 +
d)*sinh(d*x + c)^3 + (171*d*cosh(d*x + c)^17 - 1088*d*cosh(d*x + c)^15 + 29
40*d*cosh(d*x + c)^13 - 4368*d*cosh(d*x + c)^11 + 3850*d*cosh(d*x + c)^9 -
```

```
2016*d*cosh(d*x + c)^7 + 588*d*cosh(d*x + c)^5 - 80*d*cosh(d*x + c)^3 + 3*d
*cosh(d*x + c))*sinh(d*x + c)^2 + (19*d*cosh(d*x + c)^18 - 136*d*cosh(d*x +
 c)^16 + 420*d*cosh(d*x + c)^14 - 728*d*cosh(d*x + c)^12 + 770*d*cosh(d*x +
 c)^10 - 504*d*cosh(d*x + c)^8 + 196*d*cosh(d*x + c)^6 - 40*d*cosh(d*x + c)
^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

giac [B] time = 0.48, size = 335, normalized size = 1.96

$$\frac{32\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-384\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)-3\left(35\,a^3+144\,a^2b+384\,ab^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/768*(32*b^3*(e^(d*x + c) + e^(-d*x - c))^3 - 384*b^3*(e^(d*x + c) + e^(-d
*x - c)) - 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*log(e^(d*x + c) + e^(-d*x - c
) + 2) + 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*log(e^(d*x + c) + e^(-d*x - c)
- 2) + 4*(105*a^3*(e^(d*x + c) + e^(-d*x - c))^7 + 432*a^2*b*(e^(d*x + c) +
 e^(-d*x - c))^7 - 1540*a^3*(e^(d*x + c) + e^(-d*x - c))^5 - 6336*a^2*b*(e^
(d*x + c) + e^(-d*x - c))^5 + 8176*a^3*(e^(d*x + c) + e^(-d*x - c))^3 + 299
52*a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 - 17856*a^3*(e^(d*x + c) + e^(-d*x
- c)) - 46080*a^2*b*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x -
 c))^2 - 4)^4)/d
```

maple [A] time = 0.14, size = 143, normalized size = 0.84

$$\frac{a^3\left(\left(-\frac{\mathrm{csch}(dx+c)^7}{8}+\frac{7\mathrm{csch}(dx+c)^5}{48}-\frac{35\mathrm{csch}(dx+c)^3}{192}+\frac{35\,\mathrm{csch}(dx+c)}{128}\right)\coth(dx+c)-\frac{35\,\mathrm{arctanh}\left(e^{dx+c}\right)}{64}\right)+3a^2b\left(\left(-\frac{\mathrm{csch}(dx+c}{4}\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*((-1/8*csch(d*x+c)^7+7/48*csch(d*x+c)^5-35/192*csch(d*x+c)^3+35/12
8*csch(d*x+c))*coth(d*x+c)-35/64*arctanh(exp(d*x+c)))+3*a^2*b*((-1/4*csch(d
*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(d*x+c)))-6*a*b^2*arcta
nh(exp(d*x+c))+b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

maxima [B] time = 0.33, size = 463, normalized size = 2.71

$$\frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)-\frac{1}{384}\,a^3\left(\frac{105\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{105\,\log\left(e^{(-dx-c)}-1\right)}{d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) - 1/384*a^3*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*x - c)
 - 1)/d + 2*(105*e^(-d*x - c) - 805*e^(-3*d*x - 3*c) + 2681*e^(-5*d*x - 5*c
) - 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9*c) + 2681*e^(-11*d*x - 11*c)
 - 805*e^(-13*d*x - 13*c) + 105*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x - 2*c)
- 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) - 70*e^(-8*d*x - 8*c) + 56*e^(-
10*d*x - 10*c) - 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) - e^(-16*d*x
- 16*c) - 1))) - 3/8*a^2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c)
- 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*
e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
```

```
 - 6*c) - e^(-8*d*x - 8*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d - log(
e^(-d*x - c) - 1)/d)
```

mupad [B] time = 1.12, size = 759, normalized size = 4.44

$$\frac{b^3\,e^{-3\,c-3\,d\,x}}{24\,d}-\frac{3\,b^3\,e^{-c-d\,x}}{8\,d}-\frac{3\,b^3\,e^{c+d\,x}}{8\,d}+\frac{b^3\,e^{3\,c+3\,d\,x}}{24\,d}-\frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(35\,a^3\,\sqrt{-d^2}+384\,a\,b^2\,\sqrt{-d^2}+144\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{1225\,a^6+10080\,a^5\,b+47616\,a^4\,b^2+110592\,a^3\,b^3+147456\,a^2\,b^4}}\right)\,\sqrt{122}}{64\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^9,x)
```

```
[Out] (b^3*exp(- 3*c - 3*d*x))/(24*d) - (3*b^3*exp(- c - d*x))/(8*d) - (3*b^3*exp
(c + d*x))/(8*d) + (b^3*exp(3*c + 3*d*x))/(24*d) - (atan((exp(d*x)*exp(c)*(
35*a^3*(-d^2)^(1/2) + 384*a*b^2*(-d^2)^(1/2) + 144*a^2*b*(-d^2)^(1/2)))/(d*
(10080*a^5*b + 1225*a^6 + 147456*a^2*b^4 + 110592*a^3*b^3 + 47616*a^4*b^2)^
(1/2)))*(10080*a^5*b + 1225*a^6 + 147456*a^2*b^4 + 110592*a^3*b^3 + 47616*a
^4*b^2)^(1/2))/(64*(-d^2)^(1/2)) + (exp(c + d*x)*(144*a^2*b + 35*a^3))/(64*
d*(exp(2*c + 2*d*x) - 1)) - (exp(c + d*x)*(48*a^2*b + a^3))/(4*d*(6*exp(4*c
 + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)
) - (exp(c + d*x)*(144*a^2*b + 35*a^3))/(96*d*(exp(4*c + 4*d*x) - 2*exp(2*c
 + 2*d*x) + 1)) - (exp(c + d*x)*(432*a^2*b - 7*a^3))/(24*d*(3*exp(2*c + 2*d
*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (170*a^3*exp(c + d*x))/
(3*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*ex
p(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (404*a^3*exp(c + d*x))/(3*d*(15
*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c +
 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (112*a^3*exp(c
+ d*x))/(d*(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x)
- 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(
14*c + 14*d*x) - 1)) - (32*a^3*exp(c + d*x))/(d*(28*exp(4*c + 4*d*x) - 8*ex
p(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c +
10*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x)
 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**9*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.215 $\int \operatorname{csch}^{11}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=189

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{10d}+\frac{9a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{80d}+\frac{3a\left(21a^2+80ab+128b^2\right)\tanh^{-1}(\cosh(c+d$$

[Out] 3/256*a*(21*a^2+80*a*b+128*b^2)*arctanh(cosh(d*x+c))/d+b^3*cosh(d*x+c)/d-3/256*a*(21*a^2+80*a*b+128*b^2)*coth(d*x+c)*csch(d*x+c)/d+1/128*a^2*(21*a+80*b)*coth(d*x+c)*csch(d*x+c)^3/d-1/160*a^2*(21*a+80*b)*coth(d*x+c)*csch(d*x+c)^5/d+9/80*a^3*coth(d*x+c)*csch(d*x+c)^7/d-1/10*a^3*coth(d*x+c)*csch(d*x+c)^9/d

Rubi [A] time = 0.37, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 388, 206}

$$\frac{3a\left(21a^2+80ab+128b^2\right)\tanh^{-1}(\cosh(c+dx))}{256d}-\frac{3a\left(21a^2+80ab+128b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{256d}-\frac{a^2(21a$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^11*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (3*a*(21*a^2 + 80*a*b + 128*b^2)*ArcTanh[Cosh[c + d*x]])/(256*d) + (b^3*Cosh[c + d*x])/d - (3*a*(21*a^2 + 80*a*b + 128*b^2)*Coth[c + d*x]*Csch[c + d*x])/(256*d) + (a^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^3)/(128*d) - (a^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^5)/(160*d) + (9*a^3*Coth[c + d*x]*Csch[c + d*x]^7)/(80*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^9)/(10*d)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^{11}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^6}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{10d} - \frac{\text{Subst}\left(\int \frac{-9a^3-30a^2b-30ab^2-10b^3+10b \ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{9a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{80d} - \frac{a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{10d} + \ldots$$

$$= -\frac{a^2(21a+80b)\coth(c+dx)\operatorname{csch}^5(c+dx)}{160d} + \frac{9a^3\coth(c+dx)\operatorname{csch}\ldots}{80d}$$

$$= \frac{a^2(21a+80b)\coth(c+dx)\operatorname{csch}^3(c+dx)}{128d} - \frac{a^2(21a+80b)\coth(c+\ldots}{160d\ldots}$$

$$= -\frac{3a\left(21a^2+80ab+128b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{256d} + \frac{a^2(21a+\ldots}{\ldots}$$

$$= \frac{b^3\cosh(c+dx)}{d} - \frac{3a\left(21a^2+80ab+128b^2\right)\coth(c+dx)\operatorname{csch}(c+\ldots}{256d}$$

$$= \frac{3a\left(21a^2+80ab+128b^2\right)\tanh^{-1}(\cosh(c+dx))}{256d} + \frac{b^3\cosh(c+dx\ldots}{d}$$

Mathematica [A] time = 2.46, size = 265, normalized size = 1.40

$$\frac{b^3\cosh(c+dx)}{d} - \frac{a\left(60\left(21a^2+80ab+128b^2\right)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+60\left(21a^2+80ab+128b^2\right)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^11*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (b^3*Cosh[c + d*x])/d - (a*(60*(21*a^2 + 80*a*b + 128*b^2)*Csch[(c + d*x)/2
]^2 - 40*a*(7*a + 24*b)*Csch[(c + d*x)/2]^4 + 10*a*(7*a + 16*b)*Csch[(c + d
*x)/2]^6 - 15*a^2*Csch[(c + d*x)/2]^8 + 2*a^2*Csch[(c + d*x)/2]^10 + 240*(2
1*a^2 + 80*a*b + 128*b^2)*Log[Tanh[(c + d*x)/2]] + 60*(21*a^2 + 80*a*b + 12
8*b^2)*Sech[(c + d*x)/2]^2 + 40*a*(7*a + 24*b)*Sech[(c + d*x)/2]^4 + 10*a*(
7*a + 16*b)*Sech[(c + d*x)/2]^6 + 15*a^2*Sech[(c + d*x)/2]^8 + 2*a^2*Sech[(
c + d*x)/2]^10))/(20480*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.53, size = 477, normalized size = 2.52

$$1280\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+15\left(21\,a^3+80\,a^2b+128\,ab^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\left(21\,a^3+80\,a^2b+128\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2560*(1280*b^3*(e^(d*x + c) + e^(-d*x - c)) + 15*(21*a^3 + 80*a^2*b + 128
*a*b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2) - 15*(21*a^3 + 80*a^2*b + 128*a
*b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 4*(315*a^3*(e^(d*x + c) + e^(-d
*x - c))^9 + 1200*a^2*b*(e^(d*x + c) + e^(-d*x - c))^9 + 1920*a*b^2*(e^(d*x
 + c) + e^(-d*x - c))^9 - 5880*a^3*(e^(d*x + c) + e^(-d*x - c))^7 - 22400*a
^2*b*(e^(d*x + c) + e^(-d*x - c))^7 - 30720*a*b^2*(e^(d*x + c) + e^(-d*x -
c))^7 + 43008*a^3*(e^(d*x + c) + e^(-d*x - c))^5 + 163840*a^2*b*(e^(d*x + c
) + e^(-d*x - c))^5 + 184320*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 - 151680*
a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 542720*a^2*b*(e^(d*x + c) + e^(-d*x -
c))^3 - 491520*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 247040*a^3*(e^(d*x +
c) + e^(-d*x - c)) + 675840*a^2*b*(e^(d*x + c) + e^(-d*x - c)) + 491520*a*b
^2*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^5)/d
```

maple [A] time = 0.16, size = 166, normalized size = 0.88

$$a^3\left(\left(-\frac{\operatorname{csch}(dx+c)^9}{10}+\frac{9\operatorname{csch}(dx+c)^7}{80}-\frac{21\operatorname{csch}(dx+c)^5}{160}+\frac{21\operatorname{csch}(dx+c)^3}{128}-\frac{63\operatorname{csch}(dx+c)}{256}\right)\coth(dx+c)+\frac{63\operatorname{arctanh}\left(e^{dx+c}\right)}{128}\right)+3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*((-1/10*csch(d*x+c)^9+9/80*csch(d*x+c)^7-21/160*csch(d*x+c)^5+21/1
28*csch(d*x+c)^3-63/256*csch(d*x+c))*coth(d*x+c)+63/128*arctanh(exp(d*x+c))
)+3*a^2*b*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*
x+c)+5/8*arctanh(exp(d*x+c)))+3*a*b^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh
(exp(d*x+c)))+b^3*cosh(d*x+c))
```

maxima [B] time = 0.34, size = 573, normalized size = 3.03

$$\frac{1}{2}\,b^3\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{1280}\,a^3\left(\frac{315\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{315\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(315\,e^{(-dx-c)}-3045\,e^{(-3\,d}\right.}{d\left(10\,e^{(-2\,dx-2\,c)}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/1280*a^3*(315*log(e^(-d*x - c)
 + 1)/d - 315*log(e^(-d*x - c) - 1)/d + 2*(315*e^(-d*x - c) - 3045*e^(-3*d*
x - 3*c) + 13188*e^(-5*d*x - 5*c) - 33660*e^(-7*d*x - 7*c) + 55970*e^(-9*d*
```

```
x - 9*c) + 55970*e^(-11*d*x - 11*c) - 33660*e^(-13*d*x - 13*c) + 13188*e^(-
15*d*x - 15*c) - 3045*e^(-17*d*x - 17*c) + 315*e^(-19*d*x - 19*c))/(d*(10*e
^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) + 120*e^(-6*d*x - 6*c) - 210*e^(-8*d*
x - 8*c) + 252*e^(-10*d*x - 10*c) - 210*e^(-12*d*x - 12*c) + 120*e^(-14*d*x
 - 14*c) - 45*e^(-16*d*x - 16*c) + 10*e^(-18*d*x - 18*c) - e^(-20*d*x - 20*
c) - 1))) + 1/16*a^2*b*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) -
1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 19
8*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(
-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x -
8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 3/2*a*b^2*(log(e^
(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x -
 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

mupad [B] time = 1.11, size = 1194, normalized size = 6.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^11,x)
```

```
[Out] (b^3*exp(c + d*x))/(2*d) - ((24*exp(5*c + 5*d*x)*(7*a*b^2 + 4*a^2*b))/(5*d)
 - (48*exp(7*c + 7*d*x)*(7*a*b^2 + 8*a^2*b))/(5*d) - (48*exp(11*c + 11*d*x)
*(7*a*b^2 + 8*a^2*b))/(5*d) + (24*exp(13*c + 13*d*x)*(7*a*b^2 + 4*a^2*b))/(
5*d) + (4*exp(9*c + 9*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(5*d) - (48*a
*b^2*exp(3*c + 3*d*x))/(5*d) - (48*a*b^2*exp(15*c + 15*d*x))/(5*d) + (6*a*b
^2*exp(17*c + 17*d*x))/(5*d) + (6*a*b^2*exp(c + d*x))/(5*d))/(45*exp(4*c +
4*d*x) - 10*exp(2*c + 2*d*x) - 120*exp(6*c + 6*d*x) + 210*exp(8*c + 8*d*x)
- 252*exp(10*c + 10*d*x) + 210*exp(12*c + 12*d*x) - 120*exp(14*c + 14*d*x)
+ 45*exp(16*c + 16*d*x) - 10*exp(18*c + 18*d*x) + exp(20*c + 20*d*x) + 1) +
 (b^3*exp(- c - d*x))/(2*d) + (3*atan((exp(d*x)*exp(c)*(21*a^3*(-d^2)^(1/2)
 + 128*a*b^2*(-d^2)^(1/2) + 80*a^2*b*(-d^2)^(1/2)))/(d*(3360*a^5*b + 441*a^
6 + 16384*a^2*b^4 + 20480*a^3*b^3 + 11776*a^4*b^2)^(1/2)))*(3360*a^5*b + 44
1*a^6 + 16384*a^2*b^4 + 20480*a^3*b^3 + 11776*a^4*b^2)^(1/2))/(128*(-d^2)^(
1/2)) - (exp(c + d*x)*(208*a^2*b + a^3))/(5*d*(5*exp(2*c + 2*d*x) - 10*exp(
4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x
) - 1)) - (exp(c + d*x)*(80*a^2*b + 21*a^3))/(80*d*(3*exp(2*c + 2*d*x) - 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (3*exp(c + d*x)*(464*a^2*b - 3*
a^3))/(40*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) +
 exp(8*c + 8*d*x) + 1)) - (3*exp(c + d*x)*(128*a*b^2 + 80*a^2*b + 21*a^3))/
(128*d*(exp(2*c + 2*d*x) - 1)) - (1032*a^3*exp(c + d*x))/(5*d*(7*exp(2*c +
2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d*x) +
21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) - 1)) + (
exp(c + d*x)*(400*a^2*b - 1536*a*b^2 + 105*a^3))/(320*d*(exp(4*c + 4*d*x) -
 2*exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(32*a^2*b + 209*a^3))/(5*d*(15*
exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c +
8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (176*a^3*exp(c +
 d*x))/(d*(28*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) +
 70*exp(8*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) - 8*ex
p(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (256*a^3*exp(c + d*x))/(5*d*(
9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(8*
c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*c +
 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**11*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.216 $\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=220

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^{11}(c+dx)}{12d}+\frac{11a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{120d}-\frac{a\left(77a^2+280ab+384b^2\right)\coth(c+dx)\operatorname{csch}}{512d}$$

```
[Out] -1/1024*(231*a^3+840*a^2*b+1152*a*b^2+1024*b^3)*arctanh(cosh(d*x+c))/d+3/10
24*a*(77*a^2+280*a*b+384*b^2)*coth(d*x+c)*csch(d*x+c)/d-1/512*a*(77*a^2+280
*a*b+384*b^2)*coth(d*x+c)*csch(d*x+c)^3/d+7/640*a^2*(11*a+40*b)*coth(d*x+c)
*csch(d*x+c)^5/d-3/320*a^2*(11*a+40*b)*coth(d*x+c)*csch(d*x+c)^7/d+11/120*a
^3*coth(d*x+c)*csch(d*x+c)^9/d-1/12*a^3*coth(d*x+c)*csch(d*x+c)^11/d
```

Rubi [A] time = 0.40, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 385, 206}

$$-\frac{\left(840a^2b+231a^3+1152ab^2+1024b^3\right)\tanh^{-1}(\cosh(c+dx))}{1024d}-\frac{a\left(77a^2+280ab+384b^2\right)\coth(c+dx)\operatorname{csch}^3(}{512d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^13*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*ArcTanh[Cosh[c + d*x]])/(10
24*d) + (3*a*(77*a^2 + 280*a*b + 384*b^2)*Coth[c + d*x]*Csch[c + d*x])/(102
4*d) - (a*(77*a^2 + 280*a*b + 384*b^2)*Coth[c + d*x]*Csch[c + d*x]^3)/(512*
d) + (7*a^2*(11*a + 40*b)*Coth[c + d*x]*Csch[c + d*x]^5)/(640*d) - (3*a^2*(
11*a + 40*b)*Coth[c + d*x]*Csch[c + d*x]^7)/(320*d) + (11*a^3*Coth[c + d*x]
*Csch[c + d*x]^9)/(120*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^11)/(12*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
```

```
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^7}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}^{11}(c+dx)}{12d} + \frac{\operatorname{Subst}\left(\int \frac{-11a^3-36a^2b-36ab^2-12b^3+}{}\right.}{}$$

$$= \frac{11a^3\coth(c+dx)\operatorname{csch}^{9}(c+dx)}{120d} - \frac{a^3\coth(c+dx)\operatorname{csch}^{11}(c+dx)}{12d} -$$

$$= -\frac{3a^2(11a+40b)\coth(c+dx)\operatorname{csch}^{7}(c+dx)}{320d} + \frac{11a^3\coth(c+dx)\operatorname{cs}}{120d}$$

$$= \frac{7a^2(11a+40b)\coth(c+dx)\operatorname{csch}^{5}(c+dx)}{640d} - \frac{3a^2(11a+40b)\coth(}{320}$$

$$= -\frac{a\left(77a^2+280ab+384b^2\right)\coth(c+dx)\operatorname{csch}^{3}(c+dx)}{512d} + \frac{7a^2(11a}{}$$

$$= \frac{3a\left(77a^2+280ab+384b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{1024d} - \frac{a\left(77a^2+\right.}{}$$

$$= -\frac{\left(231a^3+840a^2b+1152ab^2+1024b^3\right)\tanh^{-1}(\cosh(c+dx))}{1024d} + \frac{3}{}$$

Mathematica [A] time = 2.04, size = 246, normalized size = 1.12

$$\frac{2a\left(750629a^2+2074200ab+1422720b^2\right)\cosh(3(c+dx))\operatorname{csch}^{12}(c+dx) - 9a\left(77099a^2+280360ab+246400b^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^13*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-30*a*(76555*a^2 + 75816*a*b + 45696*b^2)*Coth[c + d*x]*Csch[c + d*x]^11 +
 2*a*(750629*a^2 + 2074200*a*b + 1422720*b^2)*Cosh[3*(c + d*x)]*Csch[c + d*
x]^12 - 9*a*(77099*a^2 + 280360*a*b + 246400*b^2)*Cosh[5*(c + d*x)]*Csch[c
+ d*x]^12 + 63*a*(3421*a^2 + 12440*a*b + 14720*b^2)*Cosh[7*(c + d*x)]*Csch[
c + d*x]^12 - 525*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[9*(c + d*x)]*Csch[c +
 d*x]^12 + 45*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[11*(c + d*x)]*Csch[c + d*
```

```
x]^12 + 15360*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*Log[Tanh[(c + d
*x)/2]])/(15728640*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.52, size = 537, normalized size = 2.44

$$-\frac{15\left(231\,a^{3}+840\,a^{2}b+1152\,ab^{2}+1024\,b^{3}\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\left(231\,a^{3}+840\,a^{2}b+1152\,ab^{2}+10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/30720*(15*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*log(e^(d*x + c)
+ e^(-d*x - c) + 2) - 15*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*log(
e^(d*x + c) + e^(-d*x - c) - 2) - 4*(3465*a^3*(e^(d*x + c) + e^(-d*x - c))^
11 + 12600*a^2*b*(e^(d*x + c) + e^(-d*x - c))^11 + 17280*a*b^2*(e^(d*x + c)
 + e^(-d*x - c))^11 - 78540*a^3*(e^(d*x + c) + e^(-d*x - c))^9 - 285600*a^2
*b*(e^(d*x + c) + e^(-d*x - c))^9 - 391680*a*b^2*(e^(d*x + c) + e^(-d*x - c
))^9 + 731808*a^3*(e^(d*x + c) + e^(-d*x - c))^7 + 2661120*a^2*b*(e^(d*x +
c) + e^(-d*x - c))^7 + 3502080*a*b^2*(e^(d*x + c) + e^(-d*x - c))^7 - 35608
32*a^3*(e^(d*x + c) + e^(-d*x - c))^5 - 12948480*a^2*b*(e^(d*x + c) + e^(-d
*x - c))^5 - 15482880*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 9391360*a^3*(e
^(d*x + c) + e^(-d*x - c))^3 + 32839680*a^2*b*(e^(d*x + c) + e^(-d*x - c))^
3 + 33914880*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12180480*a^3*(e^(d*x +
c) + e^(-d*x - c)) - 34283520*a^2*b*(e^(d*x + c) + e^(-d*x - c)) - 29491200
*a*b^2*(e^(d*x + c) + e^(-d*x - c)))/((e^(d*x + c) + e^(-d*x - c))^2 - 4)^6
)/d
```

maple [A] time = 0.14, size = 202, normalized size = 0.92

$$\frac{a^{3}\left(\left(-\frac{\mathrm{csch}(dx+c)^{11}}{12}+\frac{11\mathrm{csch}(dx+c)^{9}}{120}-\frac{33\mathrm{csch}(dx+c)^{7}}{320}+\frac{77\mathrm{csch}(dx+c)^{5}}{640}-\frac{77\mathrm{csch}(dx+c)^{3}}{512}+\frac{231\,\mathrm{csch}(dx+c)}{1024}\right)\coth\left(dx+c\right)-\frac{231}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*((-1/12*csch(d*x+c)^11+11/120*csch(d*x+c)^9-33/320*csch(d*x+c)^7+7
7/640*csch(d*x+c)^5-77/512*csch(d*x+c)^3+231/1024*csch(d*x+c))*coth(d*x+c)-
231/512*arctanh(exp(d*x+c)))+3*a^2*b*((-1/8*csch(d*x+c)^7+7/48*csch(d*x+c)^
5-35/192*csch(d*x+c)^3+35/128*csch(d*x+c))*coth(d*x+c)-35/64*arctanh(exp(d*
x+c)))+3*a*b^2*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctan
h(exp(d*x+c)))-2*b^3*arctanh(exp(d*x+c)))
```

maxima [B] time = 0.34, size = 720, normalized size = 3.27

$$-\frac{1}{15360}\,a^{3}\left(\frac{3465\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3465\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3465\,e^{(-dx-c)}-40425\,e^{(-3\,dx-3\,c)}+215523\,e^{(}\right.}{d\left(12\,e^{(-2\,dx-2\,c)}-66\,e^{(-4\,d}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/15360*a^3*(3465*log(e^(-d*x - c) + 1)/d - 3465*log(e^(-d*x - c) - 1)/d +
 2*(3465*e^(-d*x - c) - 40425*e^(-3*d*x - 3*c) + 215523*e^(-5*d*x - 5*c) -
693891*e^(-7*d*x - 7*c) + 1501258*e^(-9*d*x - 9*c) - 2296650*e^(-11*d*x - 1
1*c) - 2296650*e^(-13*d*x - 13*c) + 1501258*e^(-15*d*x - 15*c) - 693891*e^(
-17*d*x - 17*c) + 215523*e^(-19*d*x - 19*c) - 40425*e^(-21*d*x - 21*c) + 34
65*e^(-23*d*x - 23*c))/(d*(12*e^(-2*d*x - 2*c) - 66*e^(-4*d*x - 4*c) + 220*
e^(-6*d*x - 6*c) - 495*e^(-8*d*x - 8*c) + 792*e^(-10*d*x - 10*c) - 924*e^(-
12*d*x - 12*c) + 792*e^(-14*d*x - 14*c) - 495*e^(-16*d*x - 16*c) + 220*e^(-
18*d*x - 18*c) - 66*e^(-20*d*x - 20*c) + 12*e^(-22*d*x - 22*c) - e^(-24*d*x
 - 24*c) - 1))) - 1/128*a^2*b*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*
x - c) - 1)/d + 2*(105*e^(-d*x - c) - 805*e^(-3*d*x - 3*c) + 2681*e^(-5*d*x
 - 5*c) - 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9*c) + 2681*e^(-11*d*x -
 11*c) - 805*e^(-13*d*x - 13*c) + 105*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x -
 2*c) - 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) - 70*e^(-8*d*x - 8*c) + 5
6*e^(-10*d*x - 10*c) - 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) - e^(-1
6*d*x - 16*c) - 1))) - 3/8*a*b^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x
 - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c
) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(
-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - b^3*(log(e^(-d*x - c) + 1)/d - lo
g(e^(-d*x - c) - 1)/d)
```

mupad [B] time = 1.10, size = 1314, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^13,x)
```

```
[Out] (3*exp(c + d*x)*(384*a*b^2 + 280*a^2*b + 77*a^3))/(512*d*(exp(2*c + 2*d*x)
- 1)) - (5632*a^3*exp(c + d*x))/(3*d*(11*exp(2*c + 2*d*x) - 55*exp(4*c + 4*
d*x) + 165*exp(6*c + 6*d*x) - 330*exp(8*c + 8*d*x) + 462*exp(10*c + 10*d*x)
 - 462*exp(12*c + 12*d*x) + 330*exp(14*c + 14*d*x) - 165*exp(16*c + 16*d*x)
 + 55*exp(18*c + 18*d*x) - 11*exp(20*c + 20*d*x) + exp(22*c + 22*d*x) - 1))
 - (1024*a^3*exp(c + d*x))/(3*d*(66*exp(4*c + 4*d*x) - 12*exp(2*c + 2*d*x)
- 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c + 10*d*x) + 92
4*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) + 495*exp(16*c + 16*d*x) - 22
0*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c + 22*d*x) + exp(
24*c + 24*d*x) + 1)) - (exp(c + d*x)*(2424*a^2*b + a^3))/(6*d*(15*exp(4*c +
 4*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) -
6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (atan((exp(d*x)*exp(c)*(2
31*a^3*(-d^2)^(1/2) + 1024*b^3*(-d^2)^(1/2) + 1152*a*b^2*(-d^2)^(1/2) + 840
*a^2*b*(-d^2)^(1/2)))/(d*(2359296*a*b^5 + 388080*a^5*b + 53361*a^6 + 104857
6*b^6 + 3047424*a^2*b^4 + 2408448*a^3*b^3 + 1237824*a^4*b^2)^(1/2)))*(23592
96*a*b^5 + 388080*a^5*b + 53361*a^6 + 1048576*b^6 + 3047424*a^2*b^4 + 24084
48*a^3*b^3 + 1237824*a^4*b^2)^(1/2))/(512*(-d^2)^(1/2)) - (exp(c + d*x)*(10
200*a^2*b - 11*a^3))/(60*d*(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)) - (exp(c +
d*x)*(384*a*b^2 + 280*a^2*b + 77*a^3))/(256*d*(exp(4*c + 4*d*x) - 2*exp(2*c
 + 2*d*x) + 1)) + (exp(c + d*x)*(280*a^2*b - 5760*a*b^2 + 77*a^3))/(320*d*(
3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (42*exp(
c + d*x)*(8*a^2*b + 15*a^3))/(d*(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) +
 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(
12*c + 12*d*x) + exp(14*c + 14*d*x) - 1)) - (23488*a^3*exp(c + d*x))/(5*d*(
9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(8*
c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*c +
 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1)) - (3*exp(c + d*x
)*(640*a*b^2 + 40*a^2*b + 11*a^3))/(160*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c +
 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (4*exp(c + d*x)*(12
```

```
0*a^2*b + 3361*a^3))/(5*d*(28*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*x) - 56*ex
p(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28*exp(12*c
+ 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (20864*a^3*ex
p(c + d*x))/(5*d*(45*exp(4*c + 4*d*x) - 10*exp(2*c + 2*d*x) - 120*exp(6*c +
 6*d*x) + 210*exp(8*c + 8*d*x) - 252*exp(10*c + 10*d*x) + 210*exp(12*c + 12
*d*x) - 120*exp(14*c + 14*d*x) + 45*exp(16*c + 16*d*x) - 10*exp(18*c + 18*d
*x) + exp(20*c + 20*d*x) + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**13*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.217 $\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=255

$$\frac{b\left(1920a^2+12312ab+10579b^2\right)\sinh(c+dx)\cosh^5(c+dx)}{3840d}-\frac{b\left(4992a^2+10728ab+5549b^2\right)\sinh(c+dx)\cosh}{3072d}$$

```
[Out] -1/2048*(1024*a^3+1920*a^2*b+1512*a*b^2+429*b^3)*x+1/2048*(1024*a^3+4224*a^
2*b+4632*a*b^2+1619*b^3)*cosh(d*x+c)*sinh(d*x+c)/d-1/3072*b*(4992*a^2+10728
*a*b+5549*b^2)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/3840*b*(1920*a^2+12312*a*b+105
79*b^2)*cosh(d*x+c)^5*sinh(d*x+c)/d-1/4480*b^2*(6888*a+11821*b)*cosh(d*x+c)
^7*sinh(d*x+c)/d+1/1680*b^2*(504*a+2593*b)*cosh(d*x+c)^9*sinh(d*x+c)/d-85/1
68*b^3*cosh(d*x+c)^11*sinh(d*x+c)/d+1/14*b^3*cosh(d*x+c)^13*sinh(d*x+c)/d
```

Rubi [A] time = 0.56, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3217, 1257, 1814, 1157, 385, 206}

$$\frac{b\left(1920a^2+12312ab+10579b^2\right)\sinh(c+dx)\cosh^5(c+dx)}{3840d}-\frac{b\left(4992a^2+10728ab+5549b^2\right)\sinh(c+dx)\cosh}{3072d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*x)/2048 + ((1024*a^3 + 422
4*a^2*b + 4632*a*b^2 + 1619*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(2048*d) - (b
*(4992*a^2 + 10728*a*b + 5549*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(3072*d)
+ (b*(1920*a^2 + 12312*a*b + 10579*b^2)*Cosh[c + d*x]^5*Sinh[c + d*x])/(384
0*d) - (b^2*(6888*a + 11821*b)*Cosh[c + d*x]^7*Sinh[c + d*x])/(4480*d) + (b
^2*(504*a + 2593*b)*Cosh[c + d*x]^9*Sinh[c + d*x])/(1680*d) - (85*b^3*Cosh[
c + d*x]^11*Sinh[c + d*x])/(168*d) + (b^3*Cosh[c + d*x]^13*Sinh[c + d*x])/(
14*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-2ax^2+(a+b)x^4\right)^3}{\left(1-x^2\right)^8}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^{13}(c+dx)\sinh(c+dx)}{14d} + \frac{\text{Subst}\left(\int \frac{-b^3+14\left(a^3-b^3\right)x^2-14\left(5a^3\right.}{}\right.}{}$$

$$= -\frac{85b^3\cosh^{11}(c+dx)\sinh(c+dx)}{168d} + \frac{b^3\cosh^{13}(c+dx)\sinh(c+}{14d}$$

$$= \frac{b^2(504a+2593b)\cosh^9(c+dx)\sinh(c+dx)}{1680d} - \frac{85b^3\cosh^{11}(c+}{16}$$

$$= -\frac{b^2(6888a+11821b)\cosh^7(c+dx)\sinh(c+dx)}{4480d} + \frac{b^2(504a+2}{}$$

$$= \frac{b\left(1920a^2+12312ab+10579b^2\right)\cosh^5(c+dx)\sinh(c+dx)}{3840d} - \frac{b}{}$$

$$= -\frac{b\left(4992a^2+10728ab+5549b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{3072d} +$$

$$= \frac{\left(1024a^3+4224a^2b+4632ab^2+1619b^3\right)\cosh(c+dx)\sinh(c+}{2048d}$$

$$= -\frac{\left(1024a^3+1920a^2b+1512ab^2+429b^3\right)x}{2048} + \frac{\left(1024a^3+4224a^2\right.}{}$$

Mathematica [A] time = 0.66, size = 189, normalized size = 0.74

$$\frac{-105b\left(2304a^2+2880ab+1001b^2\right)\sinh(4(c+dx))+35b\left(768a^2+2160ab+1001b^2\right)\sinh(6(c+dx))-840\left(10\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-840*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*(c + d*x) + 105*(4096*
a^3 + 11520*a^2*b + 10080*a*b^2 + 3003*b^3)*Sinh[2*(c + d*x)] - 105*b*(2304
*a^2 + 2880*a*b + 1001*b^2)*Sinh[4*(c + d*x)] + 35*b*(768*a^2 + 2160*a*b +
1001*b^2)*Sinh[6*(c + d*x)] - 105*b^2*(120*a + 91*b)*Sinh[8*(c + d*x)] + 21
*b^2*(48*a + 91*b)*Sinh[10*(c + d*x)] - 245*b^3*Sinh[12*(c + d*x)] + 15*b^3
*Sinh[14*(c + d*x)])/(1720320*d)
```

fricas [B] time = 0.74, size = 627, normalized size = 2.46

$$\frac{105\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{13}+210\left(13\,b^3\cosh\left(dx+c\right)^3-7\,b^3\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{11}+35\left(429\,b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/860160*(105*b^3*cosh(d*x + c)*sinh(d*x + c)^13 + 210*(13*b^3*cosh(d*x + c
)^3 - 7*b^3*cosh(d*x + c))*sinh(d*x + c)^11 + 35*(429*b^3*cosh(d*x + c)^5 -
 770*b^3*cosh(d*x + c)^3 + 3*(48*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh(d*x +
c)^9 + 60*(429*b^3*cosh(d*x + c)^7 - 1617*b^3*cosh(d*x + c)^5 + 21*(48*a*b^
2 + 91*b^3)*cosh(d*x + c)^3 - 7*(120*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh(d*
x + c)^7 + 21*(715*b^3*cosh(d*x + c)^9 - 4620*b^3*cosh(d*x + c)^7 + 126*(48
*a*b^2 + 91*b^3)*cosh(d*x + c)^5 - 140*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^3
 + 5*(768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 7
0*(39*b^3*cosh(d*x + c)^11 - 385*b^3*cosh(d*x + c)^9 + 18*(48*a*b^2 + 91*b^
3)*cosh(d*x + c)^7 - 42*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^5 + 5*(768*a^2*b
 + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 - 3*(2304*a^2*b + 2880*a*b^2 + 10
01*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 420*(1024*a^3 + 1920*a^2*b + 1512*
a*b^2 + 429*b^3)*d*x + 105*(b^3*cosh(d*x + c)^13 - 14*b^3*cosh(d*x + c)^11
+ (48*a*b^2 + 91*b^3)*cosh(d*x + c)^9 - 4*(120*a*b^2 + 91*b^3)*cosh(d*x + c
)^7 + (768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c)^5 - 2*(2304*a^2*b +
 2880*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 + (4096*a^3 + 11520*a^2*b + 10080*a
*b^2 + 3003*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.34, size = 401, normalized size = 1.57

$$\frac{b^3e^{(14\,dx+14\,c)}}{229376\,d}-\frac{7\,b^3e^{(12\,dx+12\,c)}}{98304\,d}+\frac{7\,b^3e^{(-12\,dx-12\,c)}}{98304\,d}-\frac{b^3e^{(-14\,dx-14\,c)}}{229376\,d}-\frac{1}{2048}\left(1024\,a^3+1920\,a^2b+1512\,ab^2+429\,b^3\right)x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/229376*b^3*e^(14*d*x + 14*c)/d - 7/98304*b^3*e^(12*d*x + 12*c)/d + 7/9830
4*b^3*e^(-12*d*x - 12*c)/d - 1/229376*b^3*e^(-14*d*x - 14*c)/d - 1/2048*(10
24*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*x + 1/163840*(48*a*b^2 + 91*b^3
)*e^(10*d*x + 10*c)/d - 1/32768*(120*a*b^2 + 91*b^3)*e^(8*d*x + 8*c)/d + 1/
98304*(768*a^2*b + 2160*a*b^2 + 1001*b^3)*e^(6*d*x + 6*c)/d - 1/32768*(2304
*a^2*b + 2880*a*b^2 + 1001*b^3)*e^(4*d*x + 4*c)/d + 1/32768*(4096*a^3 + 115
20*a^2*b + 10080*a*b^2 + 3003*b^3)*e^(2*d*x + 2*c)/d - 1/32768*(4096*a^3 +
11520*a^2*b + 10080*a*b^2 + 3003*b^3)*e^(-2*d*x - 2*c)/d + 1/32768*(2304*a^
2*b + 2880*a*b^2 + 1001*b^3)*e^(-4*d*x - 4*c)/d - 1/98304*(768*a^2*b + 2160
```

```
*a*b^2 + 1001*b^3)*e^(-6*d*x - 6*c)/d + 1/32768*(120*a*b^2 + 91*b^3)*e^(-8*
d*x - 8*c)/d - 1/163840*(48*a*b^2 + 91*b^3)*e^(-10*d*x - 10*c)/d
```

maple [A] time = 0.15, size = 240, normalized size = 0.94

$$b^3\left(\left(\frac{\left(\sinh^{13}(dx+c)\right)}{14}-\frac{13\left(\sinh^{11}(dx+c)\right)}{168}+\frac{143\left(\sinh^{9}(dx+c)\right)}{1680}-\frac{429\left(\sinh^{7}(dx+c)\right)}{4480}+\frac{143\left(\sinh^{5}(dx+c)\right)}{1280}-\frac{143\left(\sinh^{3}(dx+c)\right)}{1024}+\frac{429\sinh(}{204}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(b^3*((1/14*sinh(d*x+c)^13-13/168*sinh(d*x+c)^11+143/1680*sinh(d*x+c)^9
-429/4480*sinh(d*x+c)^7+143/1280*sinh(d*x+c)^5-143/1024*sinh(d*x+c)^3+429/2
048*sinh(d*x+c))*cosh(d*x+c)-429/2048*d*x-429/2048*c)+3*a*b^2*((1/10*sinh(d
*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/128*sinh(d*x+c)^3+63/256
*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+3*a^2*b*((1/6*sinh(d*x+c)^5-
5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a^3*(1/2*
cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

maxima [A] time = 0.33, size = 442, normalized size = 1.73

$$-\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{3440640}b^3\left(\frac{\left(245e^{(-2dx-2c)}-1911e^{(-4dx-4c)}+9555e^{(-6dx-6c)}-35035e^{(}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/3440640*b^3*((2
45*e^(-2*d*x - 2*c) - 1911*e^(-4*d*x - 4*c) + 9555*e^(-6*d*x - 6*c) - 35035
*e^(-8*d*x - 8*c) + 105105*e^(-10*d*x - 10*c) - 315315*e^(-12*d*x - 12*c) -
 15)*e^(14*d*x + 14*c)/d + 720720*(d*x + c)/d + (315315*e^(-2*d*x - 2*c) -
105105*e^(-4*d*x - 4*c) + 35035*e^(-6*d*x - 6*c) - 9555*e^(-8*d*x - 8*c) +
1911*e^(-10*d*x - 10*c) - 245*e^(-12*d*x - 12*c) + 15*e^(-14*d*x - 14*c))/d
) - 3/20480*a*b^2*((25*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*
d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c
)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c)
- 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 1/128*a^2*b*((9*e^(-2*d*
x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (
45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

mupad [B] time = 0.79, size = 393, normalized size = 1.54

$$\frac{e^{6c+6dx}\left(768a^2b+2160ab^2+1001b^3\right)}{98304d}-\frac{e^{-6c-6dx}\left(768a^2b+2160ab^2+1001b^3\right)}{98304d}-x\left(\frac{a^3}{2}+\frac{15a^2b}{16}+\frac{189a}{256}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2*(a + b*sinh(c + d*x)^4)^3,x)
```

```
[Out] (exp(6*c + 6*d*x)*(2160*a*b^2 + 768*a^2*b + 1001*b^3))/(98304*d) - (exp(- 6
*c - 6*d*x)*(2160*a*b^2 + 768*a^2*b + 1001*b^3))/(98304*d) - x*((189*a*b^2)
/256 + (15*a^2*b)/16 + a^3/2 + (429*b^3)/2048) + (exp(- 4*c - 4*d*x)*(2880*
a*b^2 + 2304*a^2*b + 1001*b^3))/(32768*d) - (exp(4*c + 4*d*x)*(2880*a*b^2 +
 2304*a^2*b + 1001*b^3))/(32768*d) - (exp(- 2*c - 2*d*x)*(10080*a*b^2 + 115
20*a^2*b + 4096*a^3 + 3003*b^3))/(32768*d) + (exp(2*c + 2*d*x)*(10080*a*b^2
 + 11520*a^2*b + 4096*a^3 + 3003*b^3))/(32768*d) + (7*b^3*exp(- 12*c - 12*d
*x))/(98304*d) - (7*b^3*exp(12*c + 12*d*x))/(98304*d) - (b^3*exp(- 14*c - 1
4*d*x))/(229376*d) + (b^3*exp(14*c + 14*d*x))/(229376*d) - (b^2*exp(- 10*c
```

```
- 10*d*x)*(48*a + 91*b))/(163840*d) + (b^2*exp(10*c + 10*d*x)*(48*a + 91*b)
)/(163840*d) + (b^2*exp(- 8*c - 8*d*x)*(120*a + 91*b))/(32768*d) - (b^2*exp
(8*c + 8*d*x)*(120*a + 91*b))/(32768*d)
```

sympy [**A**] time = 104.16, size = 877, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 15*a**2*b*x*sinh(c + d*x)**6/16 - 45*a**2*
b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 45*a**2*b*x*sinh(c + d*x)**2*cos
h(c + d*x)**4/16 - 15*a**2*b*x*cosh(c + d*x)**6/16 + 33*a**2*b*sinh(c + d*x
)**5*cosh(c + d*x)/(16*d) - 5*a**2*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d
) + 15*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + 189*a*b**2*x*sinh(c +
 d*x)**10/256 - 945*a*b**2*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 945*a*
b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 945*a*b**2*x*sinh(c + d*x)**
4*cosh(c + d*x)**6/128 + 945*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256
 - 189*a*b**2*x*cosh(c + d*x)**10/256 + 579*a*b**2*sinh(c + d*x)**9*cosh(c
+ d*x)/(256*d) - 711*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 63*
a*b**2*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 441*a*b**2*sinh(c + d*x)*
*3*cosh(c + d*x)**7/(128*d) + 189*a*b**2*sinh(c + d*x)*cosh(c + d*x)**9/(25
6*d) + 429*b**3*x*sinh(c + d*x)**14/2048 - 3003*b**3*x*sinh(c + d*x)**12*co
sh(c + d*x)**2/2048 + 9009*b**3*x*sinh(c + d*x)**10*cosh(c + d*x)**4/2048 -
 15015*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**6/2048 + 15015*b**3*x*sinh(c
+ d*x)**6*cosh(c + d*x)**8/2048 - 9009*b**3*x*sinh(c + d*x)**4*cosh(c + d*x
)**10/2048 + 3003*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**12/2048 - 429*b**3
*x*cosh(c + d*x)**14/2048 + 1619*b**3*sinh(c + d*x)**13*cosh(c + d*x)/(2048
*d) - 4511*b**3*sinh(c + d*x)**11*cosh(c + d*x)**3/(1536*d) + 171457*b**3*s
inh(c + d*x)**9*cosh(c + d*x)**5/(30720*d) - 429*b**3*sinh(c + d*x)**7*cosh
(c + d*x)**7/(70*d) + 40469*b**3*sinh(c + d*x)**5*cosh(c + d*x)**9/(10240*d
) - 715*b**3*sinh(c + d*x)**3*cosh(c + d*x)**11/(512*d) + 429*b**3*sinh(c +
 d*x)*cosh(c + d*x)**13/(2048*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**3*sinh(
c)**2, True))
```

3.218 $\int \left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=211

$$\frac{b\left(1152a^2+3912ab+2279b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{1536d}-\frac{b\left(1920a^2+2232ab+793b^2\right)\sinh(c+dx)\cosh(c}{1024d}$$

```
[Out] 1/1024*(1024*a^3+1152*a^2*b+840*a*b^2+231*b^3)*x-1/1024*b*(1920*a^2+2232*a*
b+793*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/1536*b*(1152*a^2+3912*a*b+2279*b^2)*
cosh(d*x+c)^3*sinh(d*x+c)/d-1/1920*b^2*(3000*a+3481*b)*cosh(d*x+c)^5*sinh(d
*x+c)/d+3/320*b^2*(40*a+139*b)*cosh(d*x+c)^7*sinh(d*x+c)/d-61/120*b^3*cosh(
d*x+c)^9*sinh(d*x+c)/d+1/12*b^3*cosh(d*x+c)^11*sinh(d*x+c)/d
```

Rubi [A] time = 0.39, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3209, 1157, 1814, 385, 206}

$$\frac{b\left(1152a^2+3912ab+2279b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{1536d}-\frac{b\left(1920a^2+2232ab+793b^2\right)\sinh(c+dx)\cosh(c}{1024d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*x)/1024 - (b*(1920*a^2 + 223
2*a*b + 793*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(1024*d) + (b*(1152*a^2 + 391
2*a*b + 2279*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(1536*d) - (b^2*(3000*a +
3481*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(1920*d) + (3*b^2*(40*a + 139*b)*Cos
h[c + d*x]^7*Sinh[c + d*x])/(320*d) - (61*b^3*Cosh[c + d*x]^9*Sinh[c + d*x]
)/(120*d) + (b^3*Cosh[c + d*x]^11*Sinh[c + d*x])/(12*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
```

```
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{\left(1-x^2\right)^7}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b^3\cosh^{11}(c+dx)\sinh(c+dx)}{12d}-\frac{\text{Subst}\left(\int \frac{-12a^3+b^3+12\left(5a^3+b^3\right)x^2-12\left(10a^3+3a^2b-b^3\right)x^4+}{(1}\right.}{}$$

$$=-\frac{61b^3\cosh^9(c+dx)\sinh(c+dx)}{120d}+\frac{b^3\cosh^{11}(c+dx)\sinh(c+dx)}{12d}+\frac{\text{Subst}\left(\int \frac{3}{-}\right.}{}$$

$$=\frac{3b^2(40a+139b)\cosh^7(c+dx)\sinh(c+dx)}{320d}-\frac{61b^3\cosh^9(c+dx)\sinh(c+dx)}{120d}+$$

$$=-\frac{b^2(3000a+3481b)\cosh^5(c+dx)\sinh(c+dx)}{1920d}+\frac{3b^2(40a+139b)\cosh^7(c+dx)}{320d}$$

$$=\frac{b\left(1152a^2+3912ab+2279b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{1536d}-\frac{b^2(3000a+3481b)\,\text{c}}{}$$

$$=-\frac{b\left(1920a^2+2232ab+793b^2\right)\cosh(c+dx)\sinh(c+dx)}{1024d}+\frac{b\left(1152a^2+3912ab\right.}{}$$

$$=\frac{\left(1024a^3+1152a^2b+840ab^2+231b^3\right)x}{1024}-\frac{b\left(1920a^2+2232ab+793b^2\right)\cosh(c+}{1024d}$$

Mathematica [A] time = 0.39, size = 156, normalized size = 0.74

$$\frac{-720b\left(128a^2+112ab+33b^2\right)\sinh(2(c+dx))+45b\left(256a^2+448ab+165b^2\right)\sinh(4(c+dx))+120\left(1024a^3+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (120*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*(c + d*x) - 720*b*(128*a
^2 + 112*a*b + 33*b^2)*Sinh[2*(c + d*x)] + 45*b*(256*a^2 + 448*a*b + 165*b^
2)*Sinh[4*(c + d*x)] - 40*b^2*(96*a + 55*b)*Sinh[6*(c + d*x)] + 45*b^2*(8*a
 + 11*b)*Sinh[8*(c + d*x)] - 72*b^3*Sinh[10*(c + d*x)] + 5*b^3*Sinh[12*(c +
 d*x)])/(122880*d)
```

fricas [B] time = 1.05, size = 461, normalized size = 2.18

$$\frac{15\,b^3\cosh(dx+c)\sinh(dx+c)^{11}+5\left(55\,b^3\cosh(dx+c)^3-36\,b^3\cosh(dx+c)\right)\sinh(dx+c)^9+90\left(11\,b^3\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/30720*(15*b^3*cosh(d*x + c)*sinh(d*x + c)^11 + 5*(55*b^3*cosh(d*x + c)^3
- 36*b^3*cosh(d*x + c))*sinh(d*x + c)^9 + 90*(11*b^3*cosh(d*x + c)^5 - 24*b
^3*cosh(d*x + c)^3 + (8*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 6*
(165*b^3*cosh(d*x + c)^7 - 756*b^3*cosh(d*x + c)^5 + 105*(8*a*b^2 + 11*b^3)
*cosh(d*x + c)^3 - 10*(96*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 +
5*(55*b^3*cosh(d*x + c)^9 - 432*b^3*cosh(d*x + c)^7 + 126*(8*a*b^2 + 11*b^3
)*cosh(d*x + c)^5 - 40*(96*a*b^2 + 55*b^3)*cosh(d*x + c)^3 + 9*(256*a^2*b +
 448*a*b^2 + 165*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(1024*a^3 + 1152*
a^2*b + 840*a*b^2 + 231*b^3)*d*x + 15*(b^3*cosh(d*x + c)^11 - 12*b^3*cosh(d
*x + c)^9 + 6*(8*a*b^2 + 11*b^3)*cosh(d*x + c)^7 - 4*(96*a*b^2 + 55*b^3)*co
sh(d*x + c)^5 + 3*(256*a^2*b + 448*a*b^2 + 165*b^3)*cosh(d*x + c)^3 - 24*(1
28*a^2*b + 112*a*b^2 + 33*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.14, size = 327, normalized size = 1.55

$$\frac{b^3e^{(12\,dx+12\,c)}}{49152\,d}-\frac{3\,b^3e^{(10\,dx+10\,c)}}{10240\,d}+\frac{3\,b^3e^{(-10\,dx-10\,c)}}{10240\,d}-\frac{b^3e^{(-12\,dx-12\,c)}}{49152\,d}+\frac{1}{1024}\left(1024\,a^3+1152\,a^2b+840\,ab^2+231\,b^3\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/49152*b^3*e^(12*d*x + 12*c)/d - 3/10240*b^3*e^(10*d*x + 10*c)/d + 3/10240
*b^3*e^(-10*d*x - 10*c)/d - 1/49152*b^3*e^(-12*d*x - 12*c)/d + 1/1024*(1024
*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*x + 3/16384*(8*a*b^2 + 11*b^3)*e^(
8*d*x + 8*c)/d - 1/6144*(96*a*b^2 + 55*b^3)*e^(6*d*x + 6*c)/d + 3/16384*(25
6*a^2*b + 448*a*b^2 + 165*b^3)*e^(4*d*x + 4*c)/d - 3/1024*(128*a^2*b + 112*
a*b^2 + 33*b^3)*e^(2*d*x + 2*c)/d + 3/1024*(128*a^2*b + 112*a*b^2 + 33*b^3)
*e^(-2*d*x - 2*c)/d - 3/16384*(256*a^2*b + 448*a*b^2 + 165*b^3)*e^(-4*d*x -
 4*c)/d + 1/6144*(96*a*b^2 + 55*b^3)*e^(-6*d*x - 6*c)/d - 3/16384*(8*a*b^2
+ 11*b^3)*e^(-8*d*x - 8*c)/d
```

maple [A] time = 0.13, size = 193, normalized size = 0.91

$$\frac{b^3\left(\left(\frac{\left(\sinh^{11}(dx+c)\right)}{12}-\frac{11\left(\sinh^{9}(dx+c)\right)}{120}+\frac{33\left(\sinh^{7}(dx+c)\right)}{320}-\frac{77\left(\sinh^{5}(dx+c)\right)}{640}+\frac{77\left(\sinh^{3}(dx+c)\right)}{512}-\frac{231\sinh(dx+c)}{1024}\right)\cosh(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(b^3*((1/12*sinh(d*x+c)^11-11/120*sinh(d*x+c)^9+33/320*sinh(d*x+c)^7-77
/640*sinh(d*x+c)^5+77/512*sinh(d*x+c)^3-231/1024*sinh(d*x+c))*cosh(d*x+c)+2
31/1024*d*x+231/1024*c)+3*a*b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/1
92*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+3*a^2
*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^3*(d*x
+c))
```

maxima [A] time = 0.33, size = 344, normalized size = 1.63

$$\frac{3}{64}\,a^2b\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+a^3x-\frac{1}{245760}\,b^3\left(\frac{\left(72\,e^{(-2\,dx-2\,c)}-495\,e^{(-4\,dx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) + a^3*x - 1/245760*b^3*((72*e^(-2*d*x - 2*c) -
 495*e^(-4*d*x - 4*c) + 2200*e^(-6*d*x - 6*c) - 7425*e^(-8*d*x - 8*c) + 237
60*e^(-10*d*x - 10*c) - 5)*e^(12*d*x + 12*c)/d - 55440*(d*x + c)/d - (23760
*e^(-2*d*x - 2*c) - 7425*e^(-4*d*x - 4*c) + 2200*e^(-6*d*x - 6*c) - 495*e^(
-8*d*x - 8*c) + 72*e^(-10*d*x - 10*c) - 5*e^(-12*d*x - 12*c))/d) - 1/2048*a
*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) -
3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4
*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d)
```

mupad [B] time = 0.56, size = 210, normalized size = 1.00

$$\frac{\frac{7425\,b^3\,\sinh(4\,c+4\,d\,x)}{8} - 2970\,b^3\,\sinh\left(2\,c+2\,d\,x\right) - 275\,b^3\,\sinh\left(6\,c+6\,d\,x\right) + \frac{495\,b^3\,\sinh(8\,c+8\,d\,x)}{8} - 9\,b^3\,\sinh\left(10\,c+10\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3,x)
```

```
[Out] ((7425*b^3*sinh(4*c + 4*d*x))/8 - 2970*b^3*sinh(2*c + 2*d*x) - 275*b^3*sinh
(6*c + 6*d*x) + (495*b^3*sinh(8*c + 8*d*x))/8 - 9*b^3*sinh(10*c + 10*d*x) +
 (5*b^3*sinh(12*c + 12*d*x))/8 - 10080*a*b^2*sinh(2*c + 2*d*x) - 11520*a^2*
b*sinh(2*c + 2*d*x) + 2520*a*b^2*sinh(4*c + 4*d*x) + 1440*a^2*b*sinh(4*c +
4*d*x) - 480*a*b^2*sinh(6*c + 6*d*x) + 45*a*b^2*sinh(8*c + 8*d*x) + 15360*a
^3*d*x + 3465*b^3*d*x + 12600*a*b^2*d*x + 17280*a^2*b*d*x)/(15360*d)
```

sympy [A] time = 49.88, size = 666, normalized size = 3.16

$$\begin{cases} a^3x + \frac{9a^2bx\sinh^4(c+dx)}{8} - \frac{9a^2bx\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{9a^2bx\cosh^4(c+dx)}{8} + \frac{15a^2b\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{9a^2b\sinh(c+dx)\cosh}{8d} \\ x\left(a+b\sinh^4(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*x + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*x*sinh(c + d*x
)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*a**2*b*sinh(c
+ d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*
d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*cosh
(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 105*a
*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x)**
8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*sinh
(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh(c
+ d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 231
*b**3*x*sinh(c + d*x)**12/1024 - 693*b**3*x*sinh(c + d*x)**10*cosh(c + d*x)
**2/512 + 3465*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**4/1024 - 1155*b**3*x*
sinh(c + d*x)**6*cosh(c + d*x)**6/256 + 3465*b**3*x*sinh(c + d*x)**4*cosh(c
 + d*x)**8/1024 - 693*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**10/512 + 231*b
**3*x*cosh(c + d*x)**12/1024 + 793*b**3*sinh(c + d*x)**11*cosh(c + d*x)/(10
24*d) - 7337*b**3*sinh(c + d*x)**9*cosh(c + d*x)**3/(3072*d) + 9273*b**3*si
nh(c + d*x)**7*cosh(c + d*x)**5/(2560*d) - 7623*b**3*sinh(c + d*x)**5*cosh(
c + d*x)**7/(2560*d) + 1309*b**3*sinh(c + d*x)**3*cosh(c + d*x)**9/(1024*d)
 - 231*b**3*sinh(c + d*x)*cosh(c + d*x)**11/(1024*d), Ne(d, 0)), (x*(a + b*
sinh(c)**4)**3, True))
```

3.219 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=181

$$-\frac{a^3\coth(c+dx)}{d}+\frac{b\left(384a^2+528ab+193b^2\right)\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{3}{256}bx\left(128a^2+80ab+21b^2\right)+\frac{b^2(8}{}$$

```
[Out] -3/256*b*(128*a^2+80*a*b+21*b^2)*x-a^3*coth(d*x+c)/d+1/256*b*(384*a^2+528*a
*b+193*b^2)*cosh(d*x+c)*sinh(d*x+c)/d-1/128*b^2*(208*a+149*b)*cosh(d*x+c)^3
*sinh(d*x+c)/d+1/160*b^2*(80*a+171*b)*cosh(d*x+c)^5*sinh(d*x+c)/d-41/80*b^3
*cosh(d*x+c)^7*sinh(d*x+c)/d+1/10*b^3*cosh(d*x+c)^9*sinh(d*x+c)/d
```

Rubi [A] time = 0.44, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3217, 1259, 1805, 453, 206}

$$\frac{b\left(384a^2+528ab+193b^2\right)\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{3}{256}bx\left(128a^2+80ab+21b^2\right)-\frac{a^3\coth(c+dx)}{d}+\frac{b^2(80}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-3*b*(128*a^2 + 80*a*b + 21*b^2)*x)/256 - (a^3*Coth[c + d*x])/d + (b*(384*
a^2 + 528*a*b + 193*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) - (b^2*(208*a
 + 149*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*d) + (b^2*(80*a + 171*b)*Cosh
[c + d*x]^5*Sinh[c + d*x])/(160*d) - (41*b^3*Cosh[c + d*x]^7*Sinh[c + d*x])
/(80*d) + (b^3*Cosh[c + d*x]^9*Sinh[c + d*x])/(10*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^2\left(1-x^2\right)^6}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^9(c+dx)\sinh(c+dx)}{10d} - \frac{\operatorname{Subst}\left(\int \frac{-10a^3+\left(50a^3+b^3\right)x^2-10\left(10a^3+3\right.}{}\right.}{}$$

$$= -\frac{41b^3\cosh^7(c+dx)\sinh(c+dx)}{80d} + \frac{b^3\cosh^9(c+dx)\sinh(c+dx)}{10d}$$

$$= \frac{b^2(80a+171b)\cosh^5(c+dx)\sinh(c+dx)}{160d} - \frac{41b^3\cosh^7(c+dx)\sin}{80d}$$

$$= -\frac{b^2(208a+149b)\cosh^3(c+dx)\sinh(c+dx)}{128d} + \frac{b^2(80a+171b)\cos}{}$$

$$= \frac{b\left(384a^2+528ab+193b^2\right)\cosh(c+dx)\sinh(c+dx)}{256d} - \frac{b^2(208a+}{}$$

$$= -\frac{a^3\coth(c+dx)}{d} + \frac{b\left(384a^2+528ab+193b^2\right)\cosh(c+dx)\sinh(c}{256d}$$

$$= -\frac{3}{256}b\left(128a^2+80ab+21b^2\right)x - \frac{a^3\coth(c+dx)}{d} + \frac{b\left(384a^2+52\right.}{}$$

Mathematica [A] time = 0.78, size = 134, normalized size = 0.74

$$\frac{-10240a^3\coth(c+dx)-120b\left(128a^2+80ab+21b^2\right)(c+dx)+60b\left(128a^2+120ab+35b^2\right)\sinh(2(c+dx))-1}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-120*b*(128*a^2 + 80*a*b + 21*b^2)*(c + d*x) - 10240*a^3*Coth[c + d*x] + 6
0*b*(128*a^2 + 120*a*b + 35*b^2)*Sinh[2*(c + d*x)] - 120*b^2*(12*a + 5*b)*S
inh[4*(c + d*x)] + 10*b^2*(16*a + 15*b)*Sinh[6*(c + d*x)] - 25*b^3*Sinh[8*(
c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)
```

fricas [B] time = 0.46, size = 474, normalized size = 2.62

$$2\,b^3\cosh(dx+c)^{11} + 22\,b^3\cosh(dx+c)\sinh(dx+c)^{10} - 27\,b^3\cosh(dx+c)^9 + 3\left(110\,b^3\cosh(dx+c)^3 - 81\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/20480*(2*b^3*cosh(d*x + c)^11 + 22*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 2
7*b^3*cosh(d*x + c)^9 + 3*(110*b^3*cosh(d*x + c)^3 - 81*b^3*cosh(d*x + c))*
sinh(d*x + c)^8 + 5*(32*a*b^2 + 35*b^3)*cosh(d*x + c)^7 + 7*(132*b^3*cosh(d
*x + c)^5 - 324*b^3*cosh(d*x + c)^3 + 5*(32*a*b^2 + 35*b^3)*cosh(d*x + c))*
sinh(d*x + c)^6 - 50*(32*a*b^2 + 15*b^3)*cosh(d*x + c)^5 + (660*b^3*cosh(d*
x + c)^7 - 3402*b^3*cosh(d*x + c)^5 + 175*(32*a*b^2 + 35*b^3)*cosh(d*x + c)
^3 - 250*(32*a*b^2 + 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 60*(128*a^2*b
 + 144*a*b^2 + 45*b^3)*cosh(d*x + c)^3 + (110*b^3*cosh(d*x + c)^9 - 972*b^3
*cosh(d*x + c)^7 + 105*(32*a*b^2 + 35*b^3)*cosh(d*x + c)^5 - 500*(32*a*b^2
+ 15*b^3)*cosh(d*x + c)^3 + 180*(128*a^2*b + 144*a*b^2 + 45*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^2 - 20*(1024*a^3 + 384*a^2*b + 360*a*b^2 + 105*b^3)*cosh
(d*x + c) + 80*(256*a^3 - 3*(128*a^2*b + 80*a*b^2 + 21*b^3)*d*x)*sinh(d*x +
 c))/(d*sinh(d*x + c))
```

giac [B] time = 0.40, size = 355, normalized size = 1.96

$$2\,b^3e^{(10\,dx+10\,c)} - 25\,b^3e^{(8\,dx+8\,c)} + 160\,ab^2e^{(6\,dx+6\,c)} + 150\,b^3e^{(6\,dx+6\,c)} - 1440\,ab^2e^{(4\,dx+4\,c)} - 600\,b^3e^{(4\,dx+4\,c)} + 768$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/20480*(2*b^3*e^(10*d*x + 10*c) - 25*b^3*e^(8*d*x + 8*c) + 160*a*b^2*e^(6*
d*x + 6*c) + 150*b^3*e^(6*d*x + 6*c) - 1440*a*b^2*e^(4*d*x + 4*c) - 600*b^3
*e^(4*d*x + 4*c) + 7680*a^2*b*e^(2*d*x + 2*c) + 7200*a*b^2*e^(2*d*x + 2*c)
+ 2100*b^3*e^(2*d*x + 2*c) - 240*(128*a^2*b + 80*a*b^2 + 21*b^3)*(d*x + c)
- 40960*a^3/(e^(2*d*x + 2*c) - 1) + (35072*a^2*b*e^(10*d*x + 10*c) + 21920*
a*b^2*e^(10*d*x + 10*c) + 5754*b^3*e^(10*d*x + 10*c) - 7680*a^2*b*e^(8*d*x
+ 8*c) - 7200*a*b^2*e^(8*d*x + 8*c) - 2100*b^3*e^(8*d*x + 8*c) + 1440*a*b^2
*e^(6*d*x + 6*c) + 600*b^3*e^(6*d*x + 6*c) - 160*a*b^2*e^(4*d*x + 4*c) - 15
0*b^3*e^(4*d*x + 4*c) + 25*b^3*e^(2*d*x + 2*c) - 2*b^3)*e^(-10*d*x - 10*c))
/d
```

maple [A] time = 0.08, size = 163, normalized size = 0.90

$$-a^3\coth(dx+c) + 3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + 3a\,b^2\left(\left(\frac{\left(\sinh^5(dx+c)\right)}{6} - \frac{5\left(\sinh^3(dx+c)\right)}{24} + \frac{5\sinh(dx+c)}{16}\right)\cosh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3
*a*b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)
-5/16*d*x-5/16*c)+b^3*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d
*x+c)^5-21/128*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/
256*c))
```

maxima [A] time = 0.33, size = 284, normalized size = 1.57

$$-\frac{3}{8}\,a^2b\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) - \frac{1}{20480}\,b^3\left(\frac{\left(25\,e^{(-2\,dx-2\,c)} - 150\,e^{(-4\,dx-4\,c)} + 600\,e^{(-6\,dx-6\,c)} - 2100\,e^{(-8\,dx-8\right.}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/20480*b^3*((2
5*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(
-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x
 - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c)
 + 2*e^(-10*d*x - 10*c))/d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d
*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) -
 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1
))
```

mupad [B] time = 1.09, size = 265, normalized size = 1.46

$$\frac{5\,b^3\,{\mathrm{e}}^{-8\,c-8\,d\,x}}{4096\,d}-\frac{2\,a^3}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}-1\right)}-\frac{5\,b^3\,{\mathrm{e}}^{8\,c+8\,d\,x}}{4096\,d}-\frac{b^3\,{\mathrm{e}}^{-10\,c-10\,d\,x}}{10240\,d}+\frac{b^3\,{\mathrm{e}}^{10\,c+10\,d\,x}}{10240\,d}-\frac{3\,b\,x\,\left(128\,a^2+80\,a\,b+21\,b^2\right)}{256}-\frac{3\,b\,\mathrm{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^2,x)
```

```
[Out] (5*b^3*exp(- 8*c - 8*d*x))/(4096*d) - (2*a^3)/(d*(exp(2*c + 2*d*x) - 1)) -
(5*b^3*exp(8*c + 8*d*x))/(4096*d) - (b^3*exp(- 10*c - 10*d*x))/(10240*d) +
(b^3*exp(10*c + 10*d*x))/(10240*d) - (3*b*x*(80*a*b + 128*a^2 + 21*b^2))/25
6 - (3*b*exp(- 2*c - 2*d*x)*(120*a*b + 128*a^2 + 35*b^2))/(1024*d) + (3*b*e
xp(2*c + 2*d*x)*(120*a*b + 128*a^2 + 35*b^2))/(1024*d) + (3*b^2*exp(- 4*c -
 4*d*x)*(12*a + 5*b))/(512*d) - (3*b^2*exp(4*c + 4*d*x)*(12*a + 5*b))/(512*
d) - (b^2*exp(- 6*c - 6*d*x)*(16*a + 15*b))/(2048*d) + (b^2*exp(6*c + 6*d*x
)*(16*a + 15*b))/(2048*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.220 $\int \operatorname{csch}^4(c+dx) \left(a+b\sinh^4(c+dx)\right)^3 \, dx$

Optimal. Leaf size=161

$$-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}+\frac{1}{128}bx\left(384a^2+144ab+35b^2\right)+\frac{b^2(144a+163b)\sinh(c+dx)\cosh^3(c+$$

```
[Out] 1/128*b*(384*a^2+144*a*b+35*b^2)*x+a^3*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/
d-3/128*b^2*(80*a+31*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*b^2*(144*a+163*b)*c
osh(d*x+c)^3*sinh(d*x+c)/d-25/48*b^3*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b^3*co
sh(d*x+c)^7*sinh(d*x+c)/d
```

Rubi [A] time = 0.38, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3217, 1259, 1805, 1261, 207}

$$\frac{1}{128}bx\left(384a^2+144ab+35b^2\right)-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}+\frac{b^2(144a+163b)\sinh(c+dx)\cosh^3(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (b*(384*a^2 + 144*a*b + 35*b^2)*x)/128 + (a^3*Coth[c + d*x])/d - (a^3*Coth[
c + d*x]^3)/(3*d) - (3*b^2*(80*a + 31*b)*Cosh[c + d*x]*Sinh[c + d*x])/(128*
d) + (b^2*(144*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) - (25*b^3*
Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^3*Cosh[c + d*x]^7*Sinh[c + d*x])
/(8*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^4\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^7(c+dx)\sinh(c+dx)}{8d} + \frac{\operatorname{Subst}\left(\int \frac{8a^3-40a^3x^2+\left(80a^3+24a^2b-b^3\right)x}{}\right.}{}$$

$$= -\frac{25b^3\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^3\cosh^7(c+dx)\sinh(c+dx)}{8d}$$

$$= \frac{b^2(144a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d} - \frac{25b^3\cosh^5(c+dx)\,s}{48d}$$

$$= -\frac{3b^2(80a+31b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b^2(144a+163b)\cosh}{19}$$

$$= -\frac{3b^2(80a+31b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b^2(144a+163b)\cosh}{19}$$

$$= \frac{a^3\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{3b^2(80a+31b)\cosh(c+dx)\,s}{128d}$$

$$= \frac{1}{128}b\left(384a^2+144ab+35b^2\right)x + \frac{a^3\coth(c+dx)}{d} - \frac{a^3\coth^3(c+d}{3d}$$

Mathematica [A] time = 0.78, size = 131, normalized size = 0.81

$$\frac{b\left(9216a^2c+9216a^2dx-96b(24a+7b)\sinh(2(c+dx))+24b(12a+7b)\sinh(4(c+dx))+3456abc+3456abdx\right.}{3072}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-1024*a^3*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + b*(9216*a^2*c + 3456*a*b*
c + 840*b^2*c + 9216*a^2*d*x + 3456*a*b*d*x + 840*b^2*d*x - 96*b*(24*a + 7*
b)*Sinh[2*(c + d*x)] + 24*b*(12*a + 7*b)*Sinh[4*(c + d*x)] - 32*b^2*Sinh[6*
(c + d*x)] + 3*b^2*Sinh[8*(c + d*x)]))/(3072*d)
```

fricas [B] time = 0.99, size = 567, normalized size = 3.52

$$\frac{3b^3\cosh(dx+c)^{11}+33b^3\cosh(dx+c)\sinh(dx+c)^{10}-41b^3\cosh(dx+c)^9+9\left(55b^3\cosh(dx+c)^3-41b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/6144*(3*b^3*cosh(d*x + c)^11 + 33*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 41
*b^3*cosh(d*x + c)^9 + 9*(55*b^3*cosh(d*x + c)^3 - 41*b^3*cosh(d*x + c))*si
nh(d*x + c)^8 + 3*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^7 + 21*(66*b^3*cosh(d*x
 + c)^5 - 164*b^3*cosh(d*x + c)^3 + (96*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh
(d*x + c)^6 - 3*(1056*a*b^2 + 425*b^3)*cosh(d*x + c)^5 + 3*(330*b^3*cosh(d*
x + c)^7 - 1722*b^3*cosh(d*x + c)^5 + 35*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^
3 - 5*(1056*a*b^2 + 425*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(512*a^3 +
972*a*b^2 + 319*b^3)*cosh(d*x + c)^3 - 16*(256*a^3 - 3*(384*a^2*b + 144*a*b
^2 + 35*b^3)*d*x)*sinh(d*x + c)^3 + 3*(55*b^3*cosh(d*x + c)^9 - 492*b^3*cos
h(d*x + c)^7 + 21*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^5 - 10*(1056*a*b^2 + 42
5*b^3)*cosh(d*x + c)^3 + 8*(512*a^3 + 972*a*b^2 + 319*b^3)*cosh(d*x + c))*s
inh(d*x + c)^2 - 24*(512*a^3 + 204*a*b^2 + 63*b^3)*cosh(d*x + c) + 48*(256*
a^3 - 3*(384*a^2*b + 144*a*b^2 + 35*b^3)*d*x - (256*a^3 - 3*(384*a^2*b + 14
4*a*b^2 + 35*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3 +
 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

giac [A] time = 0.45, size = 285, normalized size = 1.77

$$3\,b^3e^{(8\,dx+8\,c)} - 32\,b^3e^{(6\,dx+6\,c)} + 288\,ab^2e^{(4\,dx+4\,c)} + 168\,b^3e^{(4\,dx+4\,c)} - 2304\,ab^2e^{(2\,dx+2\,c)} - 672\,b^3e^{(2\,dx+2\,c)} + 48\,(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/6144*(3*b^3*e^(8*d*x + 8*c) - 32*b^3*e^(6*d*x + 6*c) + 288*a*b^2*e^(4*d*x
 + 4*c) + 168*b^3*e^(4*d*x + 4*c) - 2304*a*b^2*e^(2*d*x + 2*c) - 672*b^3*e^
(2*d*x + 2*c) + 48*(384*a^2*b + 144*a*b^2 + 35*b^3)*(d*x + c) - (19200*a^2*
b*e^(8*d*x + 8*c) + 7200*a*b^2*e^(8*d*x + 8*c) + 1750*b^3*e^(8*d*x + 8*c) -
 2304*a*b^2*e^(6*d*x + 6*c) - 672*b^3*e^(6*d*x + 6*c) + 288*a*b^2*e^(4*d*x
+ 4*c) + 168*b^3*e^(4*d*x + 4*c) - 32*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-8*d*
x - 8*c) - 8192*(3*a^3*e^(2*d*x + 2*c) - a^3)/(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [A] time = 0.10, size = 137, normalized size = 0.85

$$\frac{a^3\left(\frac{2}{3} - \frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c) + 3a^2b\,(dx+c) + 3a\,b^2\left(\left(\frac{(\sinh^3(dx+c))}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(d*x+c)+3*a*b^2*((1/4*
sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b^3*((1/8*sinh(d*
x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x
+c)+35/128*d*x+35/128*c))
```

maxima [A] time = 0.34, size = 282, normalized size = 1.75

$$\frac{3}{64}\,ab^2\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) + 3\,a^2bx - \frac{1}{6144}\,b^3\left(\frac{\left(32\,e^{(-2\,dx-2\,c)} - 168\,e^{(-4\,dx}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) + 3*a^2*b*x - 1/6144*b^3*((32*e^(-2*d*x - 2*c)
 - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 168
0*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x
 - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*
d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*
x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 1.07, size = 269, normalized size = 1.67

$$x\left(3a^2b+\frac{9ab^2}{8}+\frac{35b^3}{128}\right)-\frac{4a^3}{3d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}+\frac{b^3e^{-6c-6dx}}{192d}-\frac{b^3e^{6c+6dx}}{192d}-\frac{b^3e^{-8c-8dx}}{2048d}+\frac{b^3e^{8c+8dx}}{2048d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^4,x)
```

```
[Out] x*((9*a*b^2)/8 + 3*a^2*b + (35*b^3)/128) - (4*a^3)/(3*d*(exp(4*c + 4*d*x) -
 2*exp(2*c + 2*d*x) + 1)) + (b^3*exp(- 6*c - 6*d*x))/(192*d) - (b^3*exp(6*c
 + 6*d*x))/(192*d) - (b^3*exp(- 8*c - 8*d*x))/(2048*d) + (b^3*exp(8*c + 8*d
*x))/(2048*d) - (8*a^3*exp(2*c + 2*d*x))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4
*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (b^2*exp(- 4*c - 4*d*x)*(12*a + 7*b)
)/(256*d) + (b^2*exp(4*c + 4*d*x)*(12*a + 7*b))/(256*d) + (b^2*exp(- 2*c -
2*d*x)*(24*a + 7*b))/(64*d) - (b^2*exp(2*c + 2*d*x)*(24*a + 7*b))/(64*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.221 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=148

$$-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}-\frac{a^2(a+3b)\coth(c+dx)}{d}+\frac{b^2(24a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}$$

```
[Out] -1/16*b^2*(24*a+5*b)*x-a^2*(a+3*b)*coth(d*x+c)/d+2/3*a^3*coth(d*x+c)^3/d-1/
5*a^3*coth(d*x+c)^5/d+1/16*b^2*(24*a+11*b)*cosh(d*x+c)*sinh(d*x+c)/d-13/24*
b^3*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b^3*cosh(d*x+c)^5*sinh(d*x+c)/d
```

Rubi [A] time = 0.36, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3217, 1259, 1805, 1802, 207}

$$-\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}+\frac{b^2(24a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(b^2*(24*a + 5*b)*x)/16 - (a^2*(a + 3*b)*Coth[c + d*x])/d + (2*a^3*Coth[c
+ d*x]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) + (b^2*(24*a + 11*b)*Cosh[c +
 d*x]*Sinh[c + d*x])/(16*d) - (13*b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d)
 + (b^3*Cosh[c + d*x]^5*Sinh[c + d*x])/(6*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^6\left(1-x^2\right)^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^5(c+dx)\sinh(c+dx)}{6d} - \frac{\operatorname{Subst}\left(\int \frac{-6a^3+30a^3x^2-6a^2(10a+3b)x^4+(}{}\right.}{}$$

$$= -\frac{13b^3\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b^3\cosh^5(c+dx)\sinh(c+dx)}{6d}$$

$$= \frac{b^2(24a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b^3\cosh^3(c+dx)\sinh}{24d}$$

$$= \frac{b^2(24a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b^3\cosh^3(c+dx)\sinh}{24d}$$

$$= -\frac{a^2(a+3b)\coth(c+dx)}{d} + \frac{2a^3\coth^3(c+dx)}{3d} - \frac{a^3\coth^5(c+dx)}{5d} +$$

$$= -\frac{1}{16}b^2(24a+5b)x - \frac{a^2(a+3b)\coth(c+dx)}{d} + \frac{2a^3\coth^3(c+dx)}{3d} -$$

Mathematica [A] time = 1.08, size = 110, normalized size = 0.74

$$\frac{5b^2(9(16a+5b)\sinh(2(c+dx)) - 288ac - 288adx - 9b\sinh(4(c+dx)) + b\sinh(6(c+dx)) - 60bc - 60bdx) - 64}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-64*a^2*Coth[c + d*x]*(8*a + 45*b - 4*a*Csch[c + d*x]^2 + 3*a*Csch[c + d*x
]^4) + 5*b^2*(-288*a*c - 60*b*c - 288*a*d*x - 60*b*d*x + 9*(16*a + 5*b)*Sin
h[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)]))/(960*d)
```

fricas [B] time = 0.88, size = 768, normalized size = 5.19

$$\frac{5\,b^3\cosh\left(dx+c\right)^{11} + 55\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{10} - 70\,b^3\cosh\left(dx+c\right)^9 + 15\left(55\,b^3\cosh\left(dx+c\right)^3 - 42\,b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/1920*(5*b^3*cosh(d*x + c)^11 + 55*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 70
*b^3*cosh(d*x + c)^9 + 15*(55*b^3*cosh(d*x + c)^3 - 42*b^3*cosh(d*x + c))*s
inh(d*x + c)^8 + 20*(36*a*b^2 + 25*b^3)*cosh(d*x + c)^7 + 70*(33*b^3*cosh(d
```

```
*x + c)^5 - 84*b^3*cosh(d*x + c)^3 + 2*(36*a*b^2 + 25*b^3)*cosh(d*x + c))*s
inh(d*x + c)^6 - (1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 1625*b^3)*cosh(d*x +
 c)^5 + 8*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*sinh(d*x + c)^5
 + 5*(330*b^3*cosh(d*x + c)^7 - 1764*b^3*cosh(d*x + c)^5 + 140*(36*a*b^2 +
25*b^3)*cosh(d*x + c)^3 - (1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 1625*b^3)*c
osh(d*x + c))*sinh(d*x + c)^4 + 20*(256*a^3 + 864*a^2*b + 324*a*b^2 + 125*b
^3)*cosh(d*x + c)^3 - 40*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x -
 2*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^3 + 5*(55*b^3*cosh(d*x + c)^9 - 504*b^3*cosh(d*x + c)^7 + 84*(36*a
*b^2 + 25*b^3)*cosh(d*x + c)^5 - 2*(1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 16
25*b^3)*cosh(d*x + c)^3 + 12*(256*a^3 + 864*a^2*b + 324*a*b^2 + 125*b^3)*co
sh(d*x + c))*sinh(d*x + c)^2 - 10*(1024*a^3 + 1152*a^2*b + 360*a*b^2 + 131*
b^3)*cosh(d*x + c) + 40*((128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*
cosh(d*x + c)^4 + 256*a^3 + 1440*a^2*b - 30*(24*a*b^2 + 5*b^3)*d*x - 3*(128
*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c
))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*
cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

giac [**B**] time = 0.46, size = 286, normalized size = 1.93

$$\frac{5\,b^3e^{(6\,dx+6\,c)} - 45\,b^3e^{(4\,dx+4\,c)} + 720\,ab^2e^{(2\,dx+2\,c)} + 225\,b^3e^{(2\,dx+2\,c)} - 120\left(24\,ab^2 + 5\,b^3\right)(dx + c) + 5\left(528\,ab^2e^{(6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*b^3*e^(6*d*x + 6*c) - 45*b^3*e^(4*d*x + 4*c) + 720*a*b^2*e^(2*d*x
 + 2*c) + 225*b^3*e^(2*d*x + 2*c) - 120*(24*a*b^2 + 5*b^3)*(d*x + c) + 5*(5
28*a*b^2*e^(6*d*x + 6*c) + 110*b^3*e^(6*d*x + 6*c) - 144*a*b^2*e^(4*d*x + 4
*c) - 45*b^3*e^(4*d*x + 4*c) + 9*b^3*e^(2*d*x + 2*c) - b^3)*e^(-6*d*x - 6*c
) - 256*(45*a^2*b*e^(8*d*x + 8*c) - 180*a^2*b*e^(6*d*x + 6*c) + 80*a^3*e^(4
*d*x + 4*c) + 270*a^2*b*e^(4*d*x + 4*c) - 40*a^3*e^(2*d*x + 2*c) - 180*a^2*
b*e^(2*d*x + 2*c) + 8*a^3 + 45*a^2*b)/(e^(2*d*x + 2*c) - 1)^5)/d
```

maple [**A**] time = 0.09, size = 126, normalized size = 0.85

$$\frac{a^3\left(-\frac{8}{15} - \frac{\operatorname{csch}(dx+c)^4}{5} + \frac{4\operatorname{csch}(dx+c)^2}{15}\right)\coth\left(dx + c\right) - 3a^2b\coth\left(dx + c\right) + 3a\,b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-3*a^2*b*c
oth(d*x+c)+3*a*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^3*((1/6*si
nh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*
c))
```

maxima [**B**] time = 0.34, size = 359, normalized size = 2.43

$$-\frac{3}{8}\,ab^2\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) - \frac{1}{384}\,b^3\left(\frac{\left(9\,e^{(-2\,dx-2\,c)} - 45\,e^{(-4\,dx-4\,c)} - 1\right)e^{(6\,dx+6\,c)}}{d} + \frac{120\,(dx + c)}{d} + \frac{45\,e^{(}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b^3*((9*e
^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c
```

```
)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) - 16
/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x
- 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 1.03, size = 511, normalized size = 3.45

$$\frac{\frac{6a^2b}{5d}-\frac{2e^{2c+2dx}\left(8a^3+9ba^2\right)}{5d}+\frac{18a^2be^{4c+4dx}}{5d}-\frac{6a^2be^{6c+6dx}}{5d}}{6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1}-\frac{\frac{2\left(8a^3+9ba^2\right)}{15d}-\frac{12a^2be^{2c+2dx}}{5d}+\frac{6a^2be^{4c+4dx}}{5d}}{3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1}-\frac{\frac{6a^2b}{5d}+\frac{4e^{4c+4dx}}{}}{5e^{2c+2dx}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^6,x)
```

```
[Out] ((6*a^2*b)/(5*d) - (2*exp(2*c + 2*d*x)*(9*a^2*b + 8*a^3))/(5*d) + (18*a^2*b
*exp(4*c + 4*d*x))/(5*d) - (6*a^2*b*exp(6*c + 6*d*x))/(5*d))/(6*exp(4*c + 4
*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (
(2*(9*a^2*b + 8*a^3))/(15*d) - (12*a^2*b*exp(2*c + 2*d*x))/(5*d) + (6*a^2*b
*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) - 1) - ((6*a^2*b)/(5*d) + (4*exp(4*c + 4*d*x)*(9*a^2*b + 8*a^3))
/(5*d) - (24*a^2*b*exp(2*c + 2*d*x))/(5*d) - (24*a^2*b*exp(6*c + 6*d*x))/(5
*d) + (6*a^2*b*exp(8*c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c +
4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1)
 - (b^2*x*(24*a + 5*b))/16 + (3*b^3*exp(- 4*c - 4*d*x))/(128*d) - (3*b^3*ex
p(4*c + 4*d*x))/(128*d) - (b^3*exp(- 6*c - 6*d*x))/(384*d) + (b^3*exp(6*c +
 6*d*x))/(384*d) - (3*b^2*exp(- 2*c - 2*d*x)*(16*a + 5*b))/(128*d) + (3*b^2
*exp(2*c + 2*d*x)*(16*a + 5*b))/(128*d) - (12*a^2*b)/(5*d*(exp(2*c + 2*d*x)
 - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.222 $\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=133

$$-\frac{a^3\coth^7(c+dx)}{7d}+\frac{3a^3\coth^5(c+dx)}{5d}-\frac{a^2(a+b)\coth^3(c+dx)}{d}+\frac{a^2(a+3b)\coth(c+dx)}{d}+\frac{3}{8}b^2x(8a+b)+\frac{b^3\sin}{}$$

```
[Out] 3/8*b^2*(8*a+b)*x+a^2*(a+3*b)*coth(d*x+c)/d-a^2*(a+b)*coth(d*x+c)^3/d+3/5*a
^3*coth(d*x+c)^5/d-1/7*a^3*coth(d*x+c)^7/d-5/8*b^3*cosh(d*x+c)*sinh(d*x+c)/
d+1/4*b^3*cosh(d*x+c)^3*sinh(d*x+c)/d
```

Rubi [A] time = 0.29, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3217, 1259, 1805, 1802, 207}

$$-\frac{a^2(a+b)\coth^3(c+dx)}{d}+\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^7(c+dx)}{7d}+\frac{3a^3\coth^5(c+dx)}{5d}+\frac{3}{8}b^2x(8a+b)+\frac{b^3\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^8*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (3*b^2*(8*a + b)*x)/8 + (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^2*(a + b)*Coth
[c + d*x]^3)/d + (3*a^3*Coth[c + d*x]^5)/(5*d) - (a^3*Coth[c + d*x]^7)/(7*d
) - (5*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c
 + d*x])/(4*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^8\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{4a^3-20a^3x^2+4a^2(10a+3b)x^4-4a}{}\right.}{}$$

$$= -\frac{5b^3\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d} -$$

$$= -\frac{5b^3\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d} -$$

$$= \frac{a^2(a+3b)\coth(c+dx)}{d} - \frac{a^2(a+b)\coth^3(c+dx)}{d} + \frac{3a^3\coth^5(c+}{5d}$$

$$= \frac{3}{8}b^2(8a+b)x + \frac{a^2(a+3b)\coth(c+dx)}{d} - \frac{a^2(a+b)\coth^3(c+dx)}{d}$$

Mathematica [A] time = 0.72, size = 106, normalized size = 0.80

$$\frac{35b^2(12(8a+b)(c+dx)-8b\sinh(2(c+dx))+b\sinh(4(c+dx)))-32a^2\coth(c+dx)\left((8a+35b)\operatorname{csch}^2(c+dx)\right.}{1120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^8*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-32*a^2*Coth[c + d*x]*(-2*(8*a + 35*b) + (8*a + 35*b)*Csch[c + d*x]^2 - 6*
a*Csch[c + d*x]^4 + 5*a*Csch[c + d*x]^6) + 35*b^2*(12*(8*a + b)*(c + d*x) -
 8*b*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]))/(1120*d)
```

fricas [B] time = 0.73, size = 928, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(35*b^3*cosh(d*x + c)^11 + 385*b^3*cosh(d*x + c)*sinh(d*x + c)^10 -
525*b^3*cosh(d*x + c)^9 + 525*(11*b^3*cosh(d*x + c)^3 - 9*b^3*cosh(d*x + c)
)*sinh(d*x + c)^8 + (1024*a^3 + 4480*a^2*b + 2695*b^3)*cosh(d*x + c)^7 - 8*
(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*sinh(d*x + c)^7 + 7*(2310*b
^3*cosh(d*x + c)^5 - 6300*b^3*cosh(d*x + c)^3 + (1024*a^3 + 4480*a^2*b + 26
95*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 7*(1024*a^3 + 4480*a^2*b + 975*b^3
)*cosh(d*x + c)^5 + 56*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x - 3*(
128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x +
```

```
c)^5 + 35*(330*b^3*cosh(d*x + c)^7 - 1890*b^3*cosh(d*x + c)^5 + (1024*a^3 +
 4480*a^2*b + 2695*b^3)*cosh(d*x + c)^3 - (1024*a^3 + 4480*a^2*b + 975*b^3)
*cosh(d*x + c))*sinh(d*x + c)^4 + 42*(512*a^3 + 1600*a^2*b + 215*b^3)*cosh(
d*x + c)^3 - 56*(5*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x
 + c)^4 + 384*a^3 + 1680*a^2*b - 315*(8*a*b^2 + b^3)*d*x - 10*(128*a^3 + 56
0*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(27
5*b^3*cosh(d*x + c)^9 - 2700*b^3*cosh(d*x + c)^7 + 3*(1024*a^3 + 4480*a^2*b
 + 2695*b^3)*cosh(d*x + c)^5 - 10*(1024*a^3 + 4480*a^2*b + 975*b^3)*cosh(d*
x + c)^3 + 18*(512*a^3 + 1600*a^2*b + 215*b^3)*cosh(d*x + c))*sinh(d*x + c)
^2 - 70*(512*a^3 + 576*a^2*b + 63*b^3)*cosh(d*x + c) - 56*((128*a^3 + 560*a
^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 5*(128*a^3 + 560*a^2*b -
105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 640*a^3 - 2800*a^2*b + 525*(8*a*
b^2 + b^3)*d*x + 9*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x
 + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^7 + 7*(3*d*cosh(d*x + c)^2 - d)*si
nh(d*x + c)^5 + 7*(5*d*cosh(d*x + c)^4 - 10*d*cosh(d*x + c)^2 + 3*d)*sinh(d
*x + c)^3 + 7*(d*cosh(d*x + c)^6 - 5*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^
2 - 5*d)*sinh(d*x + c))
```

giac [B] time = 0.47, size = 253, normalized size = 1.90

$$\frac{35\,b^3e^{(4\,dx+4\,c)} - 280\,b^3e^{(2\,dx+2\,c)} + 840\left(8\,ab^2 + b^3\right)(dx+c) - 35\left(144\,ab^2e^{(4\,dx+4\,c)} + 18\,b^3e^{(4\,dx+4\,c)} - 8\,b^3e^{(2\,dx+2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2240*(35*b^3*e^(4*d*x + 4*c) - 280*b^3*e^(2*d*x + 2*c) + 840*(8*a*b^2 + b
^3)*(d*x + c) - 35*(144*a*b^2*e^(4*d*x + 4*c) + 18*b^3*e^(4*d*x + 4*c) - 8*
b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c) - 256*(105*a^2*b*e^(10*d*x + 10
*c) - 455*a^2*b*e^(8*d*x + 8*c) + 280*a^3*e^(6*d*x + 6*c) + 770*a^2*b*e^(6*
d*x + 6*c) - 168*a^3*e^(4*d*x + 4*c) - 630*a^2*b*e^(4*d*x + 4*c) + 56*a^3*e
^(2*d*x + 2*c) + 245*a^2*b*e^(2*d*x + 2*c) - 8*a^3 - 35*a^2*b)/(e^(2*d*x +
2*c) - 1)^7)/d
```

maple [A] time = 0.15, size = 121, normalized size = 0.91

$$\frac{a^3\left(\frac{16}{35} - \frac{\operatorname{csch}(dx+c)^6}{7} + \frac{6\operatorname{csch}(dx+c)^4}{35} - \frac{8\operatorname{csch}(dx+c)^2}{35}\right)\coth(dx+c) + 3a^2b\left(\frac{2}{3} - \frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c) + 3a\,b^2\,(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(16/35-1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*co
th(d*x+c)+3*a^2*b*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a*b^2*(d*x+c)+b^3*(
(1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

maxima [B] time = 0.35, size = 537, normalized size = 4.04

$$\frac{1}{64}\,b^3\left(24\,x + \frac{e^{(4\,dx+4\,c)}}{d} - \frac{8\,e^{(2\,dx+2\,c)}}{d} + \frac{8\,e^{(-2\,dx-2\,c)}}{d} - \frac{e^{(-4\,dx-4\,c)}}{d}\right) + 3\,ab^2x + \frac{32}{35}\,a^3\left(\frac{}{d\left(7\,e^{(-2\,dx-2\,c)} - 21\,e^{(-4\,dx-4\,c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3*a*b^2*x + 32/35*a^3*(7*e^(-2*d*x - 2*c)/(d*(
```

```
7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d
*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*
c) - 1)) - 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^
(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(
-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x -
8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) -
1)) - 1/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c)
- 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-
14*d*x - 14*c) - 1))) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c)
- 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) -
3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 1.03, size = 749, normalized size = 5.63

$$\frac{\frac{32\,a^2\,b}{35\,d}-\frac{16\,e^{2\,c+2\,d\,x}\,\left(8\,a^3+9\,b\,a^2\right)}{35\,d}+\frac{192\,a^2\,b\,e^{4\,c+4\,d\,x}}{35\,d}-\frac{16\,a^2\,b\,e^{6\,c+6\,d\,x}}{7\,d}}{5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1}-\frac{\frac{16\,e^{6\,c+6\,d\,x}\,\left(8\,a^3+9\,b\,a^2\right)}{7\,d}+\frac{24\,a^2\,b\,e^{2\,c+2\,d\,x}}{7\,d}-96}{7\,e^{2\,c+2\,d\,x}-21\,e^{4\,c+4\,d\,x}+35\,e^{6\,c+6\,d\,x}-35\,e^{8\,c+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^8,x)
```

```
[Out] ((32*a^2*b)/(35*d) - (16*exp(2*c + 2*d*x)*(9*a^2*b + 8*a^3))/(35*d) + (192*
a^2*b*exp(4*c + 4*d*x))/(35*d) - (16*a^2*b*exp(6*c + 6*d*x))/(7*d))/(5*exp(
2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*
x) + exp(10*c + 10*d*x) - 1) - ((16*exp(6*c + 6*d*x)*(9*a^2*b + 8*a^3))/(7*
d) + (24*a^2*b*exp(2*c + 2*d*x))/(7*d) - (96*a^2*b*exp(4*c + 4*d*x))/(7*d)
- (96*a^2*b*exp(8*c + 8*d*x))/(7*d) + (24*a^2*b*exp(10*c + 10*d*x))/(7*d))/
(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*
c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d
*x) - 1) - ((4*a^2*b)/(7*d) + (8*exp(4*c + 4*d*x)*(9*a^2*b + 8*a^3))/(7*d)
- (32*a^2*b*exp(2*c + 2*d*x))/(7*d) - (64*a^2*b*exp(6*c + 6*d*x))/(7*d) + (
20*a^2*b*exp(8*c + 8*d*x))/(7*d))/(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x)
 - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(1
2*c + 12*d*x) + 1) + ((32*a^2*b)/(35*d) - (8*a^2*b*exp(2*c + 2*d*x))/(7*d))
/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((4*(9*
a^2*b + 8*a^3))/(35*d) - (96*a^2*b*exp(2*c + 2*d*x))/(35*d) + (12*a^2*b*exp
(4*c + 4*d*x))/(7*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c
+ 6*d*x) + exp(8*c + 8*d*x) + 1) + (3*b^2*x*(8*a + b))/8 + (b^3*exp(- 2*c -
 2*d*x))/(8*d) - (b^3*exp(2*c + 2*d*x))/(8*d) - (b^3*exp(- 4*c - 4*d*x))/(6
4*d) + (b^3*exp(4*c + 4*d*x))/(64*d) - (4*a^2*b)/(7*d*(exp(4*c + 4*d*x) - 2
*exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**8*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.223 $\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=140

$$-\frac{a^3\coth^9(c+dx)}{9d}+\frac{4a^3\coth^7(c+dx)}{7d}-\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{3a^2(2a+b)\coth^5(c+dx)}{5d}+\frac{2a^2(2a+$$

```
[Out] -1/2*b^3*x-a*(a^2+3*a*b+3*b^2)*coth(d*x+c)/d+2/3*a^2*(2*a+3*b)*coth(d*x+c)^
3/d-3/5*a^2*(2*a+b)*coth(d*x+c)^5/d+4/7*a^3*coth(d*x+c)^7/d-1/9*a^3*coth(d*
x+c)^9/d+1/2*b^3*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3217, 1259, 1802, 207}

$$-\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{3a^2(2a+b)\coth^5(c+dx)}{5d}+\frac{2a^2(2a+3b)\coth^3(c+dx)}{3d}-\frac{a^3\coth^9(c+dx)}{9d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^10*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(b^3*x)/2 - (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d + (2*a^2*(2*a + 3*b)
*Coth[c + d*x]^3)/(3*d) - (3*a^2*(2*a + b)*Coth[c + d*x]^5)/(5*d) + (4*a^3*
Coth[c + d*x]^7)/(7*d) - (a^3*Coth[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]*S
inh[c + d*x])/(2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^{10}\left(1-x^2\right)^2}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-2a^3+10a^3x^2-2a^2(10a+3b)x^4+2}{}\right.}{}$$

$$= \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int\left(-\frac{2a^3}{x^{10}}+\frac{8a^3}{x^8}-\frac{6a^2(2a+b)}{x^6}+\right.\right.}{}$$

$$= -\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d} + \frac{2a^2(2a+3b)\coth^3(c+dx)}{3d} -$$

$$= -\frac{b^3x}{2} - \frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d} + \frac{2a^2(2a+3b)\coth^3(c+}{3d}$$

Mathematica [A] time = 0.61, size = 115, normalized size = 0.82

$$\frac{315b^3(\sinh(2(c+dx))-2(c+dx))-4a\coth(c+dx)\left(35a^2\operatorname{csch}^8(c+dx)-40a^2\operatorname{csch}^6(c+dx)+128a^2+3a(16a\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^10*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-4*a*Coth[c + d*x]*(128*a^2 + 504*a*b + 945*b^2 - 4*a*(16*a + 63*b)*Csch[c
 + d*x]^2 + 3*a*(16*a + 63*b)*Csch[c + d*x]^4 - 40*a^2*Csch[c + d*x]^6 + 35
*a^2*Csch[c + d*x]^8) + 315*b^3*(-2*(c + d*x) + Sinh[2*(c + d*x)]))/(1260*d
)
```

fricas [B] time = 0.91, size = 1314, normalized size = 9.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/2520*(315*b^3*cosh(d*x + c)^11 + 3465*b^3*cosh(d*x + c)*sinh(d*x + c)^10
- (1024*a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^9 - 4*(315*
b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*sinh(d*x + c)^9 + 9*(5775*b^3*
cosh(d*x + c)^3 - (1024*a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^8 + 9*(1024*a^3 + 4032*a^2*b + 5880*a*b^2 + 1225*b^3)*c
osh(d*x + c)^7 + 36*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2 - 4*(3
15*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
 c)^7 + 21*(6930*b^3*cosh(d*x + c)^5 - 4*(1024*a^3 + 4032*a^2*b + 7560*a*b^
2 + 2835*b^3)*cosh(d*x + c)^3 + 3*(1024*a^3 + 4032*a^2*b + 5880*a*b^2 + 122
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 9*(4096*a^3 + 16128*a^2*b + 16800*a
*b^2 + 2625*b^3)*cosh(d*x + c)^5 - 36*(1260*b^3*d*x + 14*(315*b^3*d*x - 256
*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 - 1024*a^3 - 4032*a^2*b - 7
560*a*b^2 - 21*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x +
 c)^2)*sinh(d*x + c)^5 + 9*(11550*b^3*cosh(d*x + c)^7 - 14*(1024*a^3 + 4032
*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^5 + 35*(1024*a^3 + 4032*a^2*b
 + 5880*a*b^2 + 1225*b^3)*cosh(d*x + c)^3 - 5*(4096*a^3 + 16128*a^2*b + 168
00*a*b^2 + 2625*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 42*(2048*a^3 + 6144*a
^2*b + 5040*a*b^2 + 675*b^3)*cosh(d*x + c)^3 - 12*(28*(315*b^3*d*x - 256*a^
3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^6 - 8820*b^3*d*x - 105*(315*b^3*
```

```
d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 + 7168*a^3 + 28224
*a^2*b + 52920*a*b^2 + 120*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(1925*b^3*cosh(d*x + c)^9 - 4*(1024*
a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^7 + 21*(1024*a^3 +
4032*a^2*b + 5880*a*b^2 + 1225*b^3)*cosh(d*x + c)^5 - 10*(4096*a^3 + 16128*
a^2*b + 16800*a*b^2 + 2625*b^3)*cosh(d*x + c)^3 + 14*(2048*a^3 + 6144*a^2*b
 + 5040*a*b^2 + 675*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 126*(1024*a^3 + 1
152*a^2*b + 840*a*b^2 + 105*b^3)*cosh(d*x + c) - 36*((315*b^3*d*x - 256*a^3
 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^8 - 7*(315*b^3*d*x - 256*a^3 - 10
08*a^2*b - 1890*a*b^2)*cosh(d*x + c)^6 + 4410*b^3*d*x + 20*(315*b^3*d*x - 2
56*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 - 3584*a^3 - 14112*a^2*b
- 26460*a*b^2 - 28*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^9 + 9*(4*d*cosh(d*x + c)^2 - d)*
sinh(d*x + c)^7 + 9*(14*d*cosh(d*x + c)^4 - 21*d*cosh(d*x + c)^2 + 4*d)*sin
h(d*x + c)^5 + 3*(28*d*cosh(d*x + c)^6 - 105*d*cosh(d*x + c)^4 + 120*d*cosh
(d*x + c)^2 - 28*d)*sinh(d*x + c)^3 + 9*(d*cosh(d*x + c)^8 - 7*d*cosh(d*x +
 c)^6 + 20*d*cosh(d*x + c)^4 - 28*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c))
```

giac [B] time = 0.51, size = 360, normalized size = 2.57

$$-\frac{1260\,(dx+c)b^3 - 315\,b^3e^{(2\,dx+2\,c)} - 315\left(2\,b^3e^{(2\,dx+2\,c)} - b^3\right)e^{(-2\,dx-2\,c)} + \frac{16\left(945\,ab^2e^{(16\,dx+16\,c)} - 7560\,ab^2e^{(14\,dx+14\,c)} + 504\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/2520*(1260*(d*x + c)*b^3 - 315*b^3*e^(2*d*x + 2*c) - 315*(2*b^3*e^(2*d*x
 + 2*c) - b^3)*e^(-2*d*x - 2*c) + 16*(945*a*b^2*e^(16*d*x + 16*c) - 7560*a*
b^2*e^(14*d*x + 14*c) + 5040*a^2*b*e^(12*d*x + 12*c) + 26460*a*b^2*e^(12*d*
x + 12*c) - 22680*a^2*b*e^(10*d*x + 10*c) - 52920*a*b^2*e^(10*d*x + 10*c) +
 16128*a^3*e^(8*d*x + 8*c) + 40824*a^2*b*e^(8*d*x + 8*c) + 66150*a*b^2*e^(8
*d*x + 8*c) - 10752*a^3*e^(6*d*x + 6*c) - 37296*a^2*b*e^(6*d*x + 6*c) - 529
20*a*b^2*e^(6*d*x + 6*c) + 4608*a^3*e^(4*d*x + 4*c) + 18144*a^2*b*e^(4*d*x
+ 4*c) + 26460*a*b^2*e^(4*d*x + 4*c) - 1152*a^3*e^(2*d*x + 2*c) - 4536*a^2*
b*e^(2*d*x + 2*c) - 7560*a*b^2*e^(2*d*x + 2*c) + 128*a^3 + 504*a^2*b + 945*
a*b^2)/(e^(2*d*x + 2*c) - 1)^9)/d
```

maple [A] time = 0.13, size = 130, normalized size = 0.93

$$\frac{a^3\left(-\frac{128}{315} - \frac{\mathrm{csch}(dx+c)^8}{9} + \frac{8\mathrm{csch}(dx+c)^6}{63} - \frac{16\mathrm{csch}(dx+c)^4}{105} + \frac{64\mathrm{csch}(dx+c)^2}{315}\right)\coth(dx+c) + 3a^2b\left(-\frac{8}{15} - \frac{\mathrm{csch}(dx+c)^4}{5} + \frac{4\mathrm{csch}\cdots}{\cdots}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(-128/315-1/9*csch(d*x+c)^8+8/63*csch(d*x+c)^6-16/105*csch(d*x+c)^
4+64/315*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-8/15-1/5*csch(d*x+c)^4+4/15*c
sch(d*x+c)^2)*coth(d*x+c)-3*a*b^2*coth(d*x+c)+b^3*(1/2*cosh(d*x+c)*sinh(d*x
+c)-1/2*d*x-1/2*c))
```

maxima [B] time = 0.38, size = 842, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 256/315*a^3*(9*e^
(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x
- 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12
*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1
)) - 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*
e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-1
2*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x -
 18*c) - 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x -
4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c)
- 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^
(-18*d*x - 18*c) - 1)) - 126*e^(-8*d*x - 8*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e
^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d
*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x -
 16*c) + e^(-18*d*x - 18*c) - 1)) - 1/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x
 - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*
c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) +
 e^(-18*d*x - 18*c) - 1))) - 16/5*a^2*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))) + 6*a*b^2/(d*(e^(-
2*d*x - 2*c) - 1))
```

mupad [B] time = 1.08, size = 1500, normalized size = 10.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^10,x)
```

```
[Out] ((2*a*b^2)/(3*d) - (2*exp(2*c + 2*d*x)*(7*a*b^2 + 4*a^2*b))/(3*d) + (2*exp(
4*c + 4*d*x)*(7*a*b^2 + 8*a^2*b))/d + (10*exp(8*c + 8*d*x)*(7*a*b^2 + 8*a^2
*b))/(3*d) - (2*exp(10*c + 10*d*x)*(7*a*b^2 + 4*a^2*b))/d - (2*exp(6*c + 6*
d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(9*d) + (14*a*b^2*exp(12*c + 12*d*x
))/(3*d) - (2*a*b^2*exp(14*c + 14*d*x))/(3*d))/(28*exp(4*c + 4*d*x) - 8*exp
(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c + 1
0*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x)
+ 1) - ((2*(105*a*b^2 + 144*a^2*b + 128*a^3))/(315*d) - (8*exp(2*c + 2*d*x)
*(7*a*b^2 + 8*a^2*b))/(21*d) + (4*exp(4*c + 4*d*x)*(7*a*b^2 + 4*a^2*b))/(7*
d) - (8*a*b^2*exp(6*c + 6*d*x))/(3*d) + (2*a*b^2*exp(8*c + 8*d*x))/(3*d))/(
5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c
+ 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(7*a*b^2 + 4*a^2*b))/(21*d) - (4*a
*b^2*exp(2*c + 2*d*x))/(3*d) + (2*a*b^2*exp(4*c + 4*d*x))/(3*d))/(3*exp(2*c
 + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((2*a*b^2)/(3*d) +
 (8*exp(4*c + 4*d*x)*(7*a*b^2 + 4*a^2*b))/(3*d) - (16*exp(6*c + 6*d*x)*(7*a
*b^2 + 8*a^2*b))/(3*d) - (16*exp(10*c + 10*d*x)*(7*a*b^2 + 8*a^2*b))/(3*d)
+ (8*exp(12*c + 12*d*x)*(7*a*b^2 + 4*a^2*b))/(3*d) + (4*exp(8*c + 8*d*x)*(1
05*a*b^2 + 144*a^2*b + 128*a^3))/(9*d) - (16*a*b^2*exp(2*c + 2*d*x))/(3*d)
- (16*a*b^2*exp(14*c + 14*d*x))/(3*d) + (2*a*b^2*exp(16*c + 16*d*x))/(3*d))
/(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(
8*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*c
 + 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1) + ((2*(7*a*b^2
+ 8*a^2*b))/(21*d) + (20*exp(4*c + 4*d*x)*(7*a*b^2 + 8*a^2*b))/(21*d) - (20
*exp(6*c + 6*d*x)*(7*a*b^2 + 4*a^2*b))/(21*d) - (2*exp(2*c + 2*d*x)*(105*a*
b^2 + 144*a^2*b + 128*a^3))/(63*d) + (10*a*b^2*exp(8*c + 8*d*x))/(3*d) - (2
*a*b^2*exp(10*c + 10*d*x))/(3*d))/(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x)
 - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(1
2*c + 12*d*x) + 1) + ((2*(7*a*b^2 + 8*a^2*b))/(21*d) - (2*exp(2*c + 2*d*x)*
(7*a*b^2 + 4*a^2*b))/(7*d) + (2*a*b^2*exp(4*c + 4*d*x))/d - (2*a*b^2*exp(6*
```

```
c + 6*d*x))/(3*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6
*d*x) + exp(8*c + 8*d*x) + 1) - (b^3*x)/2 - ((2*(7*a*b^2 + 4*a^2*b))/(21*d)
 - (4*exp(2*c + 2*d*x)*(7*a*b^2 + 8*a^2*b))/(7*d) - (40*exp(6*c + 6*d*x)*(7
*a*b^2 + 8*a^2*b))/(21*d) + (10*exp(8*c + 8*d*x)*(7*a*b^2 + 4*a^2*b))/(7*d)
 + (2*exp(4*c + 4*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(21*d) - (4*a*b^2
*exp(10*c + 10*d*x))/d + (2*a*b^2*exp(12*c + 12*d*x))/(3*d))/(7*exp(2*c + 2
*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d*x) + 2
1*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) - 1) - (b^
3*exp(- 2*c - 2*d*x))/(8*d) + (b^3*exp(2*c + 2*d*x))/(8*d) - (4*a*b^2)/(3*d
*(exp(2*c + 2*d*x) - 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**10*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.224 $\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=147

$$-\frac{a^3\coth^{11}(c+dx)}{11d}+\frac{5a^3\coth^9(c+dx)}{9d}-\frac{a\left(5a^2+9ab+3b^2\right)\coth^3(c+dx)}{3d}+\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{a^2(\ldots}{}$$

```
[Out] b^3*x+a*(a^2+3*a*b+3*b^2)*coth(d*x+c)/d-1/3*a*(5*a^2+9*a*b+3*b^2)*coth(d*x+
c)^3/d+1/5*a^2*(10*a+9*b)*coth(d*x+c)^5/d-1/7*a^2*(10*a+3*b)*coth(d*x+c)^7/
d+5/9*a^3*coth(d*x+c)^9/d-1/11*a^3*coth(d*x+c)^11/d
```

Rubi [A] time = 0.15, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3217, 1261, 207}

$$-\frac{a\left(5a^2+9ab+3b^2\right)\coth^3(c+dx)}{3d}+\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{a^2(10a+3b)\coth^7(c+dx)}{7d}+\frac{a^2(10a+9b)\ldots}{5\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^12*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] b^3*x + (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d - (a*(5*a^2 + 9*a*b + 3*b
^2)*Coth[c + d*x]^3)/(3*d) + (a^2*(10*a + 9*b)*Coth[c + d*x]^5)/(5*d) - (a^
2*(10*a + 3*b)*Coth[c + d*x]^7)/(7*d) + (5*a^3*Coth[c + d*x]^9)/(9*d) - (a^
3*Coth[c + d*x]^11)/(11*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^{12}(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^{12}} - \frac{5a^3}{x^{10}} + \frac{a^2(10a+3b)}{x^8} - \frac{a^2(10a+9b)}{x^6} + \frac{a(5a^2+9ab+3b^2)}{x^4} - \frac{a(\cdots}{}\right.\right.}{d}$$
$$= \frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d} - \frac{a\left(5a^2+9ab+3b^2\right)\coth^3(c\cdots}{3d}$$
$$= b^3x + \frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d} - \frac{a\left(5a^2+9ab+3b^2\right)\text{co}\cdots}{3d}$$

Mathematica [A] time = 6.10, size = 239, normalized size = 1.63

$$-\frac{a^3\coth(c+dx)\operatorname{csch}^{10}(c+dx)}{11d} + \frac{10a^3\coth(c+dx)\operatorname{csch}^8(c+dx)}{99d} + \frac{\operatorname{csch}^3(c+dx)\left(-640a^3\cosh(c+dx) - 237\cdots\right.}{34\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^12*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (b^3*(c + d*x))/d + (2*(640*a^3*Cosh[c + d*x] + 2376*a^2*b*Cosh[c + d*x] +
3465*a*b^2*Cosh[c + d*x])*Csch[c + d*x])/(3465*d) + ((-640*a^3*Cosh[c + d*x
] - 2376*a^2*b*Cosh[c + d*x] - 3465*a*b^2*Cosh[c + d*x])*Csch[c + d*x]^3)/(
3465*d) + (2*(80*a^3*Cosh[c + d*x] + 297*a^2*b*Cosh[c + d*x])*Csch[c + d*x]
^5)/(1155*d) + ((-80*a^3*Cosh[c + d*x] - 297*a^2*b*Cosh[c + d*x])*Csch[c +
d*x]^7)/(693*d) + (10*a^3*Coth[c + d*x]*Csch[c + d*x]^8)/(99*d) - (a^3*Coth
[c + d*x]*Csch[c + d*x]^10)/(11*d)
```

fricas [B] time = 0.54, size = 1607, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(2*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^11 + 22*(640*a^
3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + (3465*b^3*d*x
 - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*sinh(d*x + c)^11 - 22*(640*a^3 + 237
6*a^2*b + 3465*a*b^2)*cosh(d*x + c)^9 - 11*(3465*b^3*d*x - 1280*a^3 - 4752*
a^2*b - 6930*a*b^2 - 5*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^9 + 66*(5*(640*a^3 + 2376*a^2*b + 3465*a*b^2
)*cosh(d*x + c)^3 - 3*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c))*si
nh(d*x + c)^8 + 110*(640*a^3 + 2376*a^2*b + 3087*a*b^2)*cosh(d*x + c)^7 + 1
1*(17325*b^3*d*x + 30*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*c
osh(d*x + c)^4 - 6400*a^3 - 23760*a^2*b - 34650*a*b^2 - 36*(3465*b^3*d*x -
1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 154*
(6*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^5 - 12*(640*a^3 + 2376
*a^2*b + 3465*a*b^2)*cosh(d*x + c)^3 + 5*(640*a^3 + 2376*a^2*b + 3087*a*b^2
)*cosh(d*x + c))*sinh(d*x + c)^6 - 330*(640*a^3 + 2376*a^2*b + 2415*a*b^2)*
cosh(d*x + c)^5 + 33*(14*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2
)*cosh(d*x + c)^6 - 17325*b^3*d*x - 42*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*
b - 6930*a*b^2)*cosh(d*x + c)^4 + 6400*a^3 + 23760*a^2*b + 34650*a*b^2 + 35
*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(
d*x + c)^5 + 22*(30*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^7 - 1
26*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^5 + 175*(640*a^3 + 237
```

```
6*a^2*b + 3087*a*b^2)*cosh(d*x + c)^3 - 75*(640*a^3 + 2376*a^2*b + 2415*a*b
^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 660*(640*a^3 + 1872*a^2*b + 1533*a*b^2
)*cosh(d*x + c)^3 + 11*(15*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b
^2)*cosh(d*x + c)^8 - 84*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2
)*cosh(d*x + c)^6 + 103950*b^3*d*x + 175*(3465*b^3*d*x - 1280*a^3 - 4752*a^
2*b - 6930*a*b^2)*cosh(d*x + c)^4 - 38400*a^3 - 142560*a^2*b - 207900*a*b^2
 - 150*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 22*(5*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^
9 - 36*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^7 + 105*(640*a^3 +
 2376*a^2*b + 3087*a*b^2)*cosh(d*x + c)^5 - 150*(640*a^3 + 2376*a^2*b + 241
5*a*b^2)*cosh(d*x + c)^3 + 90*(640*a^3 + 1872*a^2*b + 1533*a*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^2 - 4620*(128*a^3 + 144*a^2*b + 105*a*b^2)*cosh(d*x + c
) + 11*((3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^1
0 - 9*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^8 +
 35*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^6 - 1
45530*b^3*d*x - 75*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh
(d*x + c)^4 + 53760*a^3 + 199584*a^2*b + 291060*a*b^2 + 90*(3465*b^3*d*x -
1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh
(d*x + c)^11 + 11*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^9 + 11*(30*d*cosh
(d*x + c)^4 - 36*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^7 + 33*(14*d*cosh(d
*x + c)^6 - 42*d*cosh(d*x + c)^4 + 35*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c
)^5 + 11*(15*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 175*d*cosh(d*x + c)
^4 - 150*d*cosh(d*x + c)^2 + 30*d)*sinh(d*x + c)^3 + 11*(d*cosh(d*x + c)^10
 - 9*d*cosh(d*x + c)^8 + 35*d*cosh(d*x + c)^6 - 75*d*cosh(d*x + c)^4 + 90*d
*cosh(d*x + c)^2 - 42*d)*sinh(d*x + c))
```

giac [B] time = 0.50, size = 359, normalized size = 2.44

$$3465\,(dx+c)b^3 - \frac{4\left(10395\,ab^2e^{(18\,dx+18\,c)} - 86625\,ab^2e^{(16\,dx+16\,c)} + 83160\,a^2be^{(14\,dx+14\,c)} + 318780\,ab^2e^{(14\,dx+14\,c)} - 382536\,a^2be^{(12\,dx+12\,c)} - 679140\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/3465*(3465*(d*x + c)*b^3 - 4*(10395*a*b^2*e^(18*d*x + 18*c) - 86625*a*b^2
*e^(16*d*x + 16*c) + 83160*a^2*b*e^(14*d*x + 14*c) + 318780*a*b^2*e^(14*d*x
 + 14*c) - 382536*a^2*b*e^(12*d*x + 12*c) - 679140*a*b^2*e^(12*d*x + 12*c)
+ 295680*a^3*e^(10*d*x + 10*c) + 715176*a^2*b*e^(10*d*x + 10*c) + 921690*a*
b^2*e^(10*d*x + 10*c) - 211200*a^3*e^(8*d*x + 8*c) - 700920*a^2*b*e^(8*d*x
+ 8*c) - 824670*a*b^2*e^(8*d*x + 8*c) + 105600*a^3*e^(6*d*x + 6*c) + 392040
*a^2*b*e^(6*d*x + 6*c) + 485100*a*b^2*e^(6*d*x + 6*c) - 35200*a^3*e^(4*d*x
+ 4*c) - 130680*a^2*b*e^(4*d*x + 4*c) - 180180*a*b^2*e^(4*d*x + 4*c) + 7040
*a^3*e^(2*d*x + 2*c) + 26136*a^2*b*e^(2*d*x + 2*c) + 38115*a*b^2*e^(2*d*x +
 2*c) - 640*a^3 - 2376*a^2*b - 3465*a*b^2)/(e^(2*d*x + 2*c) - 1)^11)/d
```

maple [A] time = 0.13, size = 145, normalized size = 0.99

$$\frac{a^3\left(\frac{256}{693} - \frac{\operatorname{csch}(dx+c)^{10}}{11} + \frac{10\operatorname{csch}(dx+c)^8}{99} - \frac{80\operatorname{csch}(dx+c)^6}{693} + \frac{32\operatorname{csch}(dx+c)^4}{231} - \frac{128\operatorname{csch}(dx+c)^2}{693}\right)\coth(dx+c) + 3a^2b\left(\frac{16}{35} - \frac{\operatorname{csch}(dx}{7}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(256/693-1/11*csch(d*x+c)^10+10/99*csch(d*x+c)^8-80/693*csch(d*x+c
)^6+32/231*csch(d*x+c)^4-128/693*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(16/35-
1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*coth(d*x+c)+3*a*b^
2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b^3*(d*x+c))
```

maxima [B] time = 0.36, size = 1291, normalized size = 8.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] b^3*x + 512/693*a^3*(11*e^(-2*d*x - 2*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4
*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x
- 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x
- 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c
) - 1)) - 55*e^(-4*d*x - 4*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c)
 + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 4
62*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 5
5*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 1
65*e^(-6*d*x - 6*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(
-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*
d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d
*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 330*e^(-8*d
*x - 8*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6
*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c
) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c)
 - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 462*e^(-10*d*x - 10*c
)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 33
0*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*
e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^
(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 1/(d*(11*e^(-2*d*x - 2*c) - 5
5*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-
10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-
16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x
 - 22*c) - 1))) + 96/35*a^2*b*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) -
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) - 21*e^(-4*d
*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) - 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) - 1/(d*(7*e^(-2*d
*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c)
 + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)))
 + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
+ e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 0.98, size = 1955, normalized size = 13.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^12,x)
```

```
[Out] ((64*a*b^2)/(165*d) - (32*exp(2*c + 2*d*x)*(7*a*b^2 + 4*a^2*b))/(55*d) + (1
28*exp(4*c + 4*d*x)*(7*a*b^2 + 8*a^2*b))/(55*d) + (64*exp(8*c + 8*d*x)*(7*a
*b^2 + 8*a^2*b))/(11*d) - (224*exp(10*c + 10*d*x)*(7*a*b^2 + 4*a^2*b))/(55*
d) - (32*exp(6*c + 6*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(99*d) + (1792
*a*b^2*exp(12*c + 12*d*x))/(165*d) - (96*a*b^2*exp(14*c + 14*d*x))/(55*d))/
(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(8
*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*c
+ 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1) - ((4*(105*a*b^2
```

```
 + 144*a^2*b + 128*a^3))/(693*d) - (32*exp(2*c + 2*d*x)*(7*a*b^2 + 8*a^2*b)
)/(77*d) + (8*exp(4*c + 4*d*x)*(7*a*b^2 + 4*a^2*b))/(11*d) - (128*a*b^2*exp
(6*c + 6*d*x))/(33*d) + (12*a*b^2*exp(8*c + 8*d*x))/(11*d))/(15*exp(4*c + 4
*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*
exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - ((4*(7*a*b^2 + 4*a^2*b))/(55
*d) - (32*exp(2*c + 2*d*x)*(7*a*b^2 + 8*a^2*b))/(55*d) - (32*exp(6*c + 6*d*
x)*(7*a*b^2 + 8*a^2*b))/(11*d) + (28*exp(8*c + 8*d*x)*(7*a*b^2 + 4*a^2*b))/
(11*d) + (4*exp(4*c + 4*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(33*d) - (4
48*a*b^2*exp(10*c + 10*d*x))/(55*d) + (84*a*b^2*exp(12*c + 12*d*x))/(55*d))
/(28*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8
*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c +
14*d*x) + exp(16*c + 16*d*x) + 1) - ((4*(7*a*b^2 + 4*a^2*b))/(55*d) - (64*a
*b^2*exp(2*c + 2*d*x))/(55*d) + (36*a*b^2*exp(4*c + 4*d*x))/(55*d))/(6*exp(
4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) +
 1) - ((96*exp(6*c + 6*d*x)*(7*a*b^2 + 4*a^2*b))/(11*d) - (192*exp(8*c + 8*
d*x)*(7*a*b^2 + 8*a^2*b))/(11*d) - (192*exp(12*c + 12*d*x)*(7*a*b^2 + 8*a^2
*b))/(11*d) + (96*exp(14*c + 14*d*x)*(7*a*b^2 + 4*a^2*b))/(11*d) + (16*exp(
10*c + 10*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(11*d) + (24*a*b^2*exp(2*
c + 2*d*x))/(11*d) - (192*a*b^2*exp(4*c + 4*d*x))/(11*d) - (192*a*b^2*exp(1
6*c + 16*d*x))/(11*d) + (24*a*b^2*exp(18*c + 18*d*x))/(11*d))/(11*exp(2*c +
 2*d*x) - 55*exp(4*c + 4*d*x) + 165*exp(6*c + 6*d*x) - 330*exp(8*c + 8*d*x)
 + 462*exp(10*c + 10*d*x) - 462*exp(12*c + 12*d*x) + 330*exp(14*c + 14*d*x)
 - 165*exp(16*c + 16*d*x) + 55*exp(18*c + 18*d*x) - 11*exp(20*c + 20*d*x) +
 exp(22*c + 22*d*x) - 1) - ((12*a*b^2)/(55*d) + (144*exp(4*c + 4*d*x)*(7*a*
b^2 + 4*a^2*b))/(55*d) - (384*exp(6*c + 6*d*x)*(7*a*b^2 + 8*a^2*b))/(55*d)
- (576*exp(10*c + 10*d*x)*(7*a*b^2 + 8*a^2*b))/(55*d) + (336*exp(12*c + 12*
d*x)*(7*a*b^2 + 4*a^2*b))/(55*d) + (8*exp(8*c + 8*d*x)*(105*a*b^2 + 144*a^2
*b + 128*a^3))/(11*d) - (192*a*b^2*exp(2*c + 2*d*x))/(55*d) - (768*a*b^2*ex
p(14*c + 14*d*x))/(55*d) + (108*a*b^2*exp(16*c + 16*d*x))/(55*d))/(45*exp(4
*c + 4*d*x) - 10*exp(2*c + 2*d*x) - 120*exp(6*c + 6*d*x) + 210*exp(8*c + 8*
d*x) - 252*exp(10*c + 10*d*x) + 210*exp(12*c + 12*d*x) - 120*exp(14*c + 14*
d*x) + 45*exp(16*c + 16*d*x) - 10*exp(18*c + 18*d*x) + exp(20*c + 20*d*x) +
 1) + b^3*x + ((32*(7*a*b^2 + 8*a^2*b))/(385*d) + (96*exp(4*c + 4*d*x)*(7*a
*b^2 + 8*a^2*b))/(77*d) - (16*exp(6*c + 6*d*x)*(7*a*b^2 + 4*a^2*b))/(11*d)
- (8*exp(2*c + 2*d*x)*(105*a*b^2 + 144*a^2*b + 128*a^3))/(231*d) + (64*a*b^
2*exp(8*c + 8*d*x))/(11*d) - (72*a*b^2*exp(10*c + 10*d*x))/(55*d))/(7*exp(2
*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d*
x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) - 1)
 + ((32*(7*a*b^2 + 8*a^2*b))/(385*d) - (16*exp(2*c + 2*d*x)*(7*a*b^2 + 4*a^
2*b))/(55*d) + (128*a*b^2*exp(4*c + 4*d*x))/(55*d) - (48*a*b^2*exp(6*c + 6*
d*x))/(55*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*
x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) + ((64*a*b^2)/(165*d) - (
24*a*b^2*exp(2*c + 2*d*x))/(55*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x)
 + exp(6*c + 6*d*x) - 1) - (12*a*b^2)/(55*d*(exp(4*c + 4*d*x) - 2*exp(2*c +
 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**12*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.225 $\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=144

$$-\frac{a^3\coth^{13}(c+dx)}{13d}+\frac{6a^3\coth^{11}(c+dx)}{11d}-\frac{a^2(5a+b)\coth^9(c+dx)}{3d}+\frac{4a^2(5a+3b)\coth^7(c+dx)}{7d}-\frac{3a(a+b)(5a}{}$$

```
[Out] -(a+b)^3*coth(d*x+c)/d+2*a*(a+b)^2*coth(d*x+c)^3/d-3/5*a*(a+b)*(5*a+b)*coth
(d*x+c)^5/d+4/7*a^2*(5*a+3*b)*coth(d*x+c)^7/d-1/3*a^2*(5*a+b)*coth(d*x+c)^9
/d+6/11*a^3*coth(d*x+c)^11/d-1/13*a^3*coth(d*x+c)^13/d
```

Rubi [A] time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3217, 1108}

$$-\frac{a^2(5a+b)\coth^9(c+dx)}{3d}+\frac{4a^2(5a+3b)\coth^7(c+dx)}{7d}-\frac{a^3\coth^{13}(c+dx)}{13d}+\frac{6a^3\coth^{11}(c+dx)}{11d}-\frac{3a(a+b)(5a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^14*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*a*(a + b)^2*Coth[c + d*x]^3)/d - (3*a*(
a + b)*(5*a + b)*Coth[c + d*x]^5)/(5*d) + (4*a^2*(5*a + 3*b)*Coth[c + d*x]^
7)/(7*d) - (a^2*(5*a + b)*Coth[c + d*x]^9)/(3*d) + (6*a^3*Coth[c + d*x]^11)
/(11*d) - (a^3*Coth[c + d*x]^13)/(13*d)
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^{14}}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{14}}-\frac{6a^3}{x^{12}}+\frac{3a^2(5a+b)}{x^{10}}-\frac{4a^2(5a+3b)}{x^8}+\frac{3a(a+b)(5a+b)}{x^6}-\frac{6a(a+}{x^4}\right.\right.}{d}\\ &= -\frac{(a+b)^3\coth(c+dx)}{d}+\frac{2a(a+b)^2\coth^3(c+dx)}{d}-\frac{3a(a+b)(}{}\end{aligned}$$

Mathematica [B] time = 3.24, size = 350, normalized size = 2.43

$$-\frac{\operatorname{csch}^{13}(c+dx)\left(3660800a^3\cosh(5(c+dx))-1464320a^3\cosh(7(c+dx))+399360a^3\cosh(9(c+dx))-665\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^14*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/61501440*((8580*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[c + d
*x] - 6435*(1024*a^3 + 2944*a^2*b + 2408*a*b^2 + 693*b^3)*Cosh[3*(c + d*x)]
 + 3660800*a^3*Cosh[5*(c + d*x)] + 13087360*a^2*b*Cosh[5*(c + d*x)] + 13093
080*a*b^2*Cosh[5*(c + d*x)] + 4129125*b^3*Cosh[5*(c + d*x)] - 1464320*a^3*C
osh[7*(c + d*x)] - 5234944*a^2*b*Cosh[7*(c + d*x)] - 6390384*a*b^2*Cosh[7*(
c + d*x)] - 2312310*b^3*Cosh[7*(c + d*x)] + 399360*a^3*Cosh[9*(c + d*x)] +
1427712*a^2*b*Cosh[9*(c + d*x)] + 1873872*a*b^2*Cosh[9*(c + d*x)] + 810810*
b^3*Cosh[9*(c + d*x)] - 66560*a^3*Cosh[11*(c + d*x)] - 237952*a^2*b*Cosh[11
*(c + d*x)] - 312312*a*b^2*Cosh[11*(c + d*x)] - 165165*b^3*Cosh[11*(c + d*x
)] + 5120*a^3*Cosh[13*(c + d*x)] + 18304*a^2*b*Cosh[13*(c + d*x)] + 24024*a
*b^2*Cosh[13*(c + d*x)] + 15015*b^3*Cosh[13*(c + d*x)])*Csch[c + d*x]^13)/d
```

fricas [B] time = 0.65, size = 2323, normalized size = 16.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -4/15015*((2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^1
2 - 48*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 +
 (2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*sinh(d*x + c)^12 - 52*(6
40*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^10 - 2*(16640*a^
3 + 59488*a^2*b + 78078*a*b^2 + 90090*b^3 - 33*(2560*a^3 + 9152*a^2*b + 120
12*a*b^2 + 15015*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 - 40*(22*(640*a^3 +
 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^3 - 13*(640*a^3 + 2288*a^2*b + 3003
*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 78*(2560*a^3 + 9152*a^2*b + 13552*
a*b^2 + 12705*b^3)*cosh(d*x + c)^8 + 3*(165*(2560*a^3 + 9152*a^2*b + 12012*
a*b^2 + 15015*b^3)*cosh(d*x + c)^4 + 66560*a^3 + 237952*a^2*b + 352352*a*b^
2 + 330330*b^3 - 780*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^8 - 96*(33*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh
(d*x + c)^5 - 65*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^3 + 52*(
320*a^3 + 1144*a^2*b + 1309*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 572*(12
80*a^3 + 4576*a^2*b + 7581*a*b^2 + 5775*b^3)*cosh(d*x + c)^6 + 4*(231*(2560
*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^6 - 2730*(640*a^
3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^4 - 183040*a^3 - 6543
68*a^2*b - 1084083*a*b^2 - 825825*b^3 + 546*(2560*a^3 + 9152*a^2*b + 13552*
a*b^2 + 12705*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 24*(132*(640*a^3 + 22
88*a^2*b + 3003*a*b^2)*cosh(d*x + c)^7 - 546*(640*a^3 + 2288*a^2*b + 3003*a
*b^2)*cosh(d*x + c)^5 + 1456*(320*a^3 + 1144*a^2*b + 1309*a*b^2)*cosh(d*x +
 c)^3 - 143*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*cosh(d*x + c))*sinh(d*x +
c)^5 + 143*(12800*a^3 + 53824*a^2*b + 79884*a*b^2 + 51975*b^3)*cosh(d*x + c
)^4 + (495*(2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^
8 - 10920*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^6 +
5460*(2560*a^3 + 9152*a^2*b + 13552*a*b^2 + 12705*b^3)*cosh(d*x + c)^4 + 18
30400*a^3 + 7696832*a^2*b + 11423412*a*b^2 + 7432425*b^3 - 8580*(1280*a^3 +
 4576*a^2*b + 7581*a*b^2 + 5775*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 16*
(55*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^9 - 390*(640*a^3 + 22
88*a^2*b + 3003*a*b^2)*cosh(d*x + c)^7 + 2184*(320*a^3 + 1144*a^2*b + 1309*
a*b^2)*cosh(d*x + c)^5 - 715*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*cosh(d*x
+ c)^3 + 143*(3200*a^3 + 9424*a^2*b + 6489*a*b^2)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4392960*a^3 + 10323456*a^2*b + 12108096*a*b^2 + 6936930*b^3 - 3432*
(960*a^3 + 4664*a^2*b + 5859*a*b^2 + 3465*b^3)*cosh(d*x + c)^2 + 6*(11*(256
0*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^10 - 390*(640*a
^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^8 + 364*(2560*a^3 +
9152*a^2*b + 13552*a*b^2 + 12705*b^3)*cosh(d*x + c)^6 - 1430*(1280*a^3 + 45
```

```
76*a^2*b + 7581*a*b^2 + 5775*b^3)*cosh(d*x + c)^4 - 549120*a^3 - 2667808*a^
2*b - 3351348*a*b^2 - 1981980*b^3 + 143*(12800*a^3 + 53824*a^2*b + 79884*a*
b^2 + 51975*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 8*(6*(640*a^3 + 2288*a^
2*b + 3003*a*b^2)*cosh(d*x + c)^11 - 65*(640*a^3 + 2288*a^2*b + 3003*a*b^2)
*cosh(d*x + c)^9 + 624*(320*a^3 + 1144*a^2*b + 1309*a*b^2)*cosh(d*x + c)^7
- 429*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*cosh(d*x + c)^5 + 286*(3200*a^3
+ 9424*a^2*b + 6489*a*b^2)*cosh(d*x + c)^3 - 858*(960*a^3 + 1528*a^2*b + 90
3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^14 + 14*d*cosh(d*x
+ c)*sinh(d*x + c)^13 + d*sinh(d*x + c)^14 - 14*d*cosh(d*x + c)^12 + 7*(13*
d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^12 + 4*(91*d*cosh(d*x + c)^3 - 36*d*
cosh(d*x + c))*sinh(d*x + c)^11 + 91*d*cosh(d*x + c)^10 + 7*(143*d*cosh(d*x
 + c)^4 - 132*d*cosh(d*x + c)^2 + 13*d)*sinh(d*x + c)^10 + 2*(1001*d*cosh(d
*x + c)^5 - 1320*d*cosh(d*x + c)^3 + 325*d*cosh(d*x + c))*sinh(d*x + c)^9 -
 364*d*cosh(d*x + c)^8 + 7*(429*d*cosh(d*x + c)^6 - 990*d*cosh(d*x + c)^4 +
 585*d*cosh(d*x + c)^2 - 52*d)*sinh(d*x + c)^8 + 8*(429*d*cosh(d*x + c)^7 -
 1188*d*cosh(d*x + c)^5 + 975*d*cosh(d*x + c)^3 - 208*d*cosh(d*x + c))*sinh
(d*x + c)^7 + 1001*d*cosh(d*x + c)^6 + 7*(429*d*cosh(d*x + c)^8 - 1848*d*co
sh(d*x + c)^6 + 2730*d*cosh(d*x + c)^4 - 1456*d*cosh(d*x + c)^2 + 143*d)*si
nh(d*x + c)^6 + 2*(1001*d*cosh(d*x + c)^9 - 4752*d*cosh(d*x + c)^7 + 8190*d
*cosh(d*x + c)^5 - 5824*d*cosh(d*x + c)^3 + 1287*d*cosh(d*x + c))*sinh(d*x
+ c)^5 - 2002*d*cosh(d*x + c)^4 + 7*(143*d*cosh(d*x + c)^10 - 990*d*cosh(d*
x + c)^8 + 2730*d*cosh(d*x + c)^6 - 3640*d*cosh(d*x + c)^4 + 2145*d*cosh(d*
x + c)^2 - 286*d)*sinh(d*x + c)^4 + 4*(91*d*cosh(d*x + c)^11 - 660*d*cosh(d
*x + c)^9 + 1950*d*cosh(d*x + c)^7 - 2912*d*cosh(d*x + c)^5 + 2145*d*cosh(d
*x + c)^3 - 572*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3003*d*cosh(d*x + c)^2 +
 7*(13*d*cosh(d*x + c)^12 - 132*d*cosh(d*x + c)^10 + 585*d*cosh(d*x + c)^8
- 1456*d*cosh(d*x + c)^6 + 2145*d*cosh(d*x + c)^4 - 1716*d*cosh(d*x + c)^2
+ 429*d)*sinh(d*x + c)^2 + 2*(7*d*cosh(d*x + c)^13 - 72*d*cosh(d*x + c)^11
+ 325*d*cosh(d*x + c)^9 - 832*d*cosh(d*x + c)^7 + 1287*d*cosh(d*x + c)^5 -
1144*d*cosh(d*x + c)^3 + 429*d*cosh(d*x + c))*sinh(d*x + c) - 1716*d)
```

giac [B] time = 0.52, size = 563, normalized size = 3.91

$$-\frac{2\left(15015\,b^3e^{(24\,dx+24\,c)} - 180180\,b^3e^{(22\,dx+22\,c)} + 240240\,ab^2e^{(20\,dx+20\,c)} + 990990\,b^3e^{(20\,dx+20\,c)} - 2042040\,ab^2e^{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -2/15015*(15015*b^3*e^(24*d*x + 24*c) - 180180*b^3*e^(22*d*x + 22*c) + 2402
40*a*b^2*e^(20*d*x + 20*c) + 990990*b^3*e^(20*d*x + 20*c) - 2042040*a*b^2*e
^(18*d*x + 18*c) - 3303300*b^3*e^(18*d*x + 18*c) + 2306304*a^2*b*e^(16*d*x
+ 16*c) + 7711704*a*b^2*e^(16*d*x + 16*c) + 7432425*b^3*e^(16*d*x + 16*c) -
 10762752*a^2*b*e^(14*d*x + 14*c) - 17008992*a*b^2*e^(14*d*x + 14*c) - 1189
1880*b^3*e^(14*d*x + 14*c) + 8785920*a^3*e^(12*d*x + 12*c) + 20646912*a^2*b
*e^(12*d*x + 12*c) + 24216192*a*b^2*e^(12*d*x + 12*c) + 13873860*b^3*e^(12*
d*x + 12*c) - 6589440*a^3*e^(10*d*x + 10*c) - 21250944*a^2*b*e^(10*d*x + 10
*c) - 23207184*a*b^2*e^(10*d*x + 10*c) - 11891880*b^3*e^(10*d*x + 10*c) + 3
660800*a^3*e^(8*d*x + 8*c) + 13087360*a^2*b*e^(8*d*x + 8*c) + 15135120*a*b^
2*e^(8*d*x + 8*c) + 7432425*b^3*e^(8*d*x + 8*c) - 1464320*a^3*e^(6*d*x + 6*
c) - 5234944*a^2*b*e^(6*d*x + 6*c) - 6630624*a*b^2*e^(6*d*x + 6*c) - 330330
0*b^3*e^(6*d*x + 6*c) + 399360*a^3*e^(4*d*x + 4*c) + 1427712*a^2*b*e^(4*d*x
 + 4*c) + 1873872*a*b^2*e^(4*d*x + 4*c) + 990990*b^3*e^(4*d*x + 4*c) - 6656
0*a^3*e^(2*d*x + 2*c) - 237952*a^2*b*e^(2*d*x + 2*c) - 312312*a*b^2*e^(2*d*
x + 2*c) - 180180*b^3*e^(2*d*x + 2*c) + 5120*a^3 + 18304*a^2*b + 24024*a*b^
2 + 15015*b^3)/(d*(e^(2*d*x + 2*c) - 1)^13)
```

maple [A] time = 0.13, size = 177, normalized size = 1.23

$$a^3\left(-\frac{1024}{3003}-\frac{\operatorname{csch}(dx+c)^{12}}{13}+\frac{12\operatorname{csch}(dx+c)^{10}}{143}-\frac{40\operatorname{csch}(dx+c)^8}{429}+\frac{320\operatorname{csch}(dx+c)^6}{3003}-\frac{128\operatorname{csch}(dx+c)^4}{1001}+\frac{512\operatorname{csch}(dx+c)^2}{3003}\right)\coth(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(-1024/3003-1/13*csch(d*x+c)^12+12/143*csch(d*x+c)^10-40/429*csch(
d*x+c)^8+320/3003*csch(d*x+c)^6-128/1001*csch(d*x+c)^4+512/3003*csch(d*x+c)
^2)*coth(d*x+c)+3*a^2*b*(-128/315-1/9*csch(d*x+c)^8+8/63*csch(d*x+c)^6-16/1
05*csch(d*x+c)^4+64/315*csch(d*x+c)^2)*coth(d*x+c)+3*a*b^2*(-8/15-1/5*csch(
d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-b^3*coth(d*x+c))
```

maxima [B] time = 0.37, size = 1916, normalized size = 13.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -2048/3003*a^3*(13*e^(-2*d*x - 2*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x
- 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10
*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x -
 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x -
22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 78*e^(-4*d*x - 4
*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) -
715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) +
1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c)
- 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) +
e^(-26*d*x - 26*c) - 1)) + 286*e^(-6*d*x - 6*c)/(d*(13*e^(-2*d*x - 2*c) - 7
8*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(
-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*
e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e
^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 715*
e^(-8*d*x - 8*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*
d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d
*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18
*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d
*x - 24*c) + e^(-26*d*x - 26*c) - 1)) + 1287*e^(-10*d*x - 10*c)/(d*(13*e^(-
2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x -
 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x
 - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*
x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26
*c) - 1)) - 1716*e^(-12*d*x - 12*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x
- 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10
*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x -
 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x -
22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 1/(d*(13*e^(-2*d
*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*
c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x -
14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x -
 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c)
 - 1))) - 256/105*a^2*b*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(
-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x
 - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 1
6*c) + e^(-18*d*x - 18*c) - 1)) - 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c
) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*
```

```
e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2
*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8
*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c
) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 126*e^(-8*d*x - 8*c)/
(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^
(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14
*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 1/(d*(9*e^
(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x
- 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 1
4*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1))) - 16/5*a*b^2*(5*e^(
-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)
/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) - 1))) + 2*b^3/(d*(e^(-2*d*x - 2*c) - 1))
```

mupad [B] time = 1.14, size = 3138, normalized size = 21.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^14,x)
```

```
[Out] ((6*b^3*exp(4*c + 4*d*x))/(13*d) - (2*b^3*exp(6*c + 6*d*x))/(13*d) + (2*b^2
*(96*a + 55*b))/(715*d) - (6*b^2*exp(2*c + 2*d*x)*(8*a + 11*b))/(143*d))/(6
*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1) - ((2*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(3003*d) - (1
2*b^3*exp(10*c + 10*d*x))/(13*d) + (2*b^3*exp(12*c + 12*d*x))/(13*d) - (4*b
*exp(2*c + 2*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(143*d) + (2*b*exp(4*c + 4*
d*x)*(448*a*b + 256*a^2 + 165*b^2))/(143*d) + (30*b^2*exp(8*c + 8*d*x)*(8*a
 + 11*b))/(143*d) - (8*b^2*exp(6*c + 6*d*x)*(96*a + 55*b))/(143*d))/(7*exp(
2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d
*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) - 1
) - ((8*exp(6*c + 6*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(14
3*d) - (18*b^3*exp(16*c + 16*d*x))/(13*d) + (2*b^3*exp(18*c + 18*d*x))/(13*
d) - (2*b^2*(96*a + 55*b))/(715*d) - (24*b*exp(4*c + 4*d*x)*(112*a*b + 128*
a^2 + 33*b^2))/(143*d) - (84*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 + 33*b^2
))/(143*d) + (6*b*exp(2*c + 2*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(715*d) +
 (84*b*exp(10*c + 10*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(715*d) + (72*b^2*
exp(14*c + 14*d*x)*(8*a + 11*b))/(143*d) - (168*b^2*exp(12*c + 12*d*x)*(96*
a + 55*b))/(715*d))/(45*exp(4*c + 4*d*x) - 10*exp(2*c + 2*d*x) - 120*exp(6*
c + 6*d*x) + 210*exp(8*c + 8*d*x) - 252*exp(10*c + 10*d*x) + 210*exp(12*c +
 12*d*x) - 120*exp(14*c + 14*d*x) + 45*exp(16*c + 16*d*x) - 10*exp(18*c + 1
8*d*x) + exp(20*c + 20*d*x) + 1) - ((2*b*(448*a*b + 256*a^2 + 165*b^2))/(21
45*d) - (8*b^3*exp(6*c + 6*d*x))/(13*d) + (2*b^3*exp(8*c + 8*d*x))/(13*d) +
 (12*b^2*exp(4*c + 4*d*x)*(8*a + 11*b))/(143*d) - (8*b^2*exp(2*c + 2*d*x)*(
96*a + 55*b))/(715*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6
*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) + ((2*b*(112*a*b
 + 128*a^2 + 33*b^2))/(429*d) - (2*exp(2*c + 2*d*x)*(840*a*b^2 + 1152*a^2*b
 + 1024*a^3 + 231*b^3))/(429*d) + (14*b^3*exp(12*c + 12*d*x))/(13*d) - (2*b
^3*exp(14*c + 14*d*x))/(13*d) + (14*b*exp(4*c + 4*d*x)*(112*a*b + 128*a^2 +
 33*b^2))/(143*d) - (14*b*exp(6*c + 6*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(
429*d) - (42*b^2*exp(10*c + 10*d*x)*(8*a + 11*b))/(143*d) + (14*b^2*exp(8*c
 + 8*d*x)*(96*a + 55*b))/(143*d))/(28*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*x)
 - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28*e
xp(12*c + 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1) - ((2*b^
3)/(13*d) + (8*exp(12*c + 12*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*
b^3))/(13*d) - (24*b^3*exp(2*c + 2*d*x))/(13*d) - (24*b^3*exp(22*c + 22*d*x
```

```
))/(13*d) + (2*b^3*exp(24*c + 24*d*x))/(13*d) - (48*b*exp(10*c + 10*d*x)*(1
12*a*b + 128*a^2 + 33*b^2))/(13*d) - (48*b*exp(14*c + 14*d*x)*(112*a*b + 12
8*a^2 + 33*b^2))/(13*d) + (6*b*exp(8*c + 8*d*x)*(448*a*b + 256*a^2 + 165*b^
2))/(13*d) + (6*b*exp(16*c + 16*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(13*d)
+ (12*b^2*exp(4*c + 4*d*x)*(8*a + 11*b))/(13*d) + (12*b^2*exp(20*c + 20*d*x
)*(8*a + 11*b))/(13*d) - (8*b^2*exp(6*c + 6*d*x)*(96*a + 55*b))/(13*d) - (8
*b^2*exp(18*c + 18*d*x)*(96*a + 55*b))/(13*d))/(13*exp(2*c + 2*d*x) - 78*ex
p(4*c + 4*d*x) + 286*exp(6*c + 6*d*x) - 715*exp(8*c + 8*d*x) + 1287*exp(10*
c + 10*d*x) - 1716*exp(12*c + 12*d*x) + 1716*exp(14*c + 14*d*x) - 1287*exp(
16*c + 16*d*x) + 715*exp(18*c + 18*d*x) - 286*exp(20*c + 20*d*x) + 78*exp(2
2*c + 22*d*x) - 13*exp(24*c + 24*d*x) + exp(26*c + 26*d*x) - 1) - ((2*b^3*e
xp(4*c + 4*d*x))/(13*d) - (4*b^3*exp(2*c + 2*d*x))/(13*d) + (2*b^2*(8*a + 1
1*b))/(143*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x)
- 1) + ((2*b*(112*a*b + 128*a^2 + 33*b^2))/(429*d) + (10*b^3*exp(8*c + 8*d*
x))/(13*d) - (2*b^3*exp(10*c + 10*d*x))/(13*d) - (2*b*exp(2*c + 2*d*x)*(448
*a*b + 256*a^2 + 165*b^2))/(429*d) - (20*b^2*exp(6*c + 6*d*x)*(8*a + 11*b))
/(143*d) + (4*b^2*exp(4*c + 4*d*x)*(96*a + 55*b))/(143*d))/(15*exp(4*c + 4*
d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*e
xp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - ((2*b*(448*a*b + 256*a^2 + 16
5*b^2))/(2145*d) + (8*exp(4*c + 4*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 +
 231*b^3))/(429*d) - (16*b^3*exp(14*c + 14*d*x))/(13*d) + (2*b^3*exp(16*c +
 16*d*x))/(13*d) - (16*b*exp(2*c + 2*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(42
9*d) - (112*b*exp(6*c + 6*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(429*d) + (28*
b*exp(8*c + 8*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(429*d) + (56*b^2*exp(12*
c + 12*d*x)*(8*a + 11*b))/(143*d) - (112*b^2*exp(10*c + 10*d*x)*(96*a + 55*
b))/(715*d))/(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x
) - 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) +
 36*exp(14*c + 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1) + (
(2*b^3)/(13*d) - (4*exp(10*c + 10*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 +
 231*b^3))/(13*d) + (22*b^3*exp(20*c + 20*d*x))/(13*d) - (2*b^3*exp(22*c +
22*d*x))/(13*d) + (20*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(13*
d) + (28*b*exp(12*c + 12*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(13*d) - (2*b*e
xp(6*c + 6*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(13*d) - (4*b*exp(14*c + 14*
d*x)*(448*a*b + 256*a^2 + 165*b^2))/(13*d) - (2*b^2*exp(2*c + 2*d*x)*(8*a +
 11*b))/(13*d) - (10*b^2*exp(18*c + 18*d*x)*(8*a + 11*b))/(13*d) + (2*b^2*e
xp(4*c + 4*d*x)*(96*a + 55*b))/(13*d) + (6*b^2*exp(16*c + 16*d*x)*(96*a + 5
5*b))/(13*d))/(66*exp(4*c + 4*d*x) - 12*exp(2*c + 2*d*x) - 220*exp(6*c + 6*
d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c + 10*d*x) + 924*exp(12*c + 12*d*
x) - 792*exp(14*c + 14*d*x) + 495*exp(16*c + 16*d*x) - 220*exp(18*c + 18*d*
x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c + 22*d*x) + exp(24*c + 24*d*x) + 1
) - ((20*exp(8*c + 8*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(1
43*d) - (20*b^3*exp(18*c + 18*d*x))/(13*d) + (2*b^3*exp(20*c + 20*d*x))/(13
*d) + (2*b^2*(8*a + 11*b))/(143*d) - (80*b*exp(6*c + 6*d*x)*(112*a*b + 128*
a^2 + 33*b^2))/(143*d) - (168*b*exp(10*c + 10*d*x)*(112*a*b + 128*a^2 + 33*
b^2))/(143*d) + (6*b*exp(4*c + 4*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(143*d
) + (28*b*exp(12*c + 12*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(143*d) + (90*b
^2*exp(16*c + 16*d*x)*(8*a + 11*b))/(143*d) - (4*b^2*exp(2*c + 2*d*x)*(96*a
 + 55*b))/(143*d) - (48*b^2*exp(14*c + 14*d*x)*(96*a + 55*b))/(143*d))/(11*
exp(2*c + 2*d*x) - 55*exp(4*c + 4*d*x) + 165*exp(6*c + 6*d*x) - 330*exp(8*c
 + 8*d*x) + 462*exp(10*c + 10*d*x) - 462*exp(12*c + 12*d*x) + 330*exp(14*c
+ 14*d*x) - 165*exp(16*c + 16*d*x) + 55*exp(18*c + 18*d*x) - 11*exp(20*c +
20*d*x) + exp(22*c + 22*d*x) - 1) - (4*b^3)/(13*d*(exp(2*c + 2*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**14*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.226 $\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=182

$$-\frac{a^3\coth^{15}(c+dx)}{15d}+\frac{7a^3\coth^{13}(c+dx)}{13d}-\frac{a\left(35a^2+30ab+3b^2\right)\coth^7(c+dx)}{7d}-\frac{3a^2(7a+b)\coth^{11}(c+dx)}{11d}+\frac{5a^2(\cdots}{}$$

```
[Out] (a+b)^3*coth(d*x+c)/d-1/3*(a+b)^2*(7*a+b)*coth(d*x+c)^3/d+3/5*a*(a+b)*(7*a+
3*b)*coth(d*x+c)^5/d-1/7*a*(35*a^2+30*a*b+3*b^2)*coth(d*x+c)^7/d+5/9*a^2*(7
*a+3*b)*coth(d*x+c)^9/d-3/11*a^2*(7*a+b)*coth(d*x+c)^11/d+7/13*a^3*coth(d*x
+c)^13/d-1/15*a^3*coth(d*x+c)^15/d
```

Rubi [A] time = 0.16, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3217, 1261}

$$-\frac{a\left(35a^2+30ab+3b^2\right)\coth^7(c+dx)}{7d}-\frac{3a^2(7a+b)\coth^{11}(c+dx)}{11d}+\frac{5a^2(7a+3b)\coth^9(c+dx)}{9d}-\frac{a^3\coth^{15}(c+d\cdots}{15d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^16*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*Coth[c + d*x])/d - ((a + b)^2*(7*a + b)*Coth[c + d*x]^3)/(3*d) +
 (3*a*(a + b)*(7*a + 3*b)*Coth[c + d*x]^5)/(5*d) - (a*(35*a^2 + 30*a*b + 3*
b^2)*Coth[c + d*x]^7)/(7*d) + (5*a^2*(7*a + 3*b)*Coth[c + d*x]^9)/(9*d) - (
3*a^2*(7*a + b)*Coth[c + d*x]^11)/(11*d) + (7*a^3*Coth[c + d*x]^13)/(13*d)
- (a^3*Coth[c + d*x]^15)/(15*d)
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)\left(a-2ax^2+(a+b)x^4\right)^3}{x^{16}}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{16}}-\frac{7a^3}{x^{14}}+\frac{3a^2(7a+b)}{x^{12}}-\frac{5a^2(7a+3b)}{x^{10}}+\frac{a\left(35a^2+30ab+3b^2\right)}{x^8}+\frac{3a(\cdots}{}\right.\right.}{d}\\ &= \frac{(a+b)^3\coth(c+dx)}{d}-\frac{(a+b)^2(7a+b)\coth^3(c+dx)}{3d}+\frac{3a(a+b\cdots}{}\end{aligned}$$

Mathematica [B] time = 4.66, size = 404, normalized size = 2.22

$$-\frac{\text{csch}^{15}(c+dx)\left(21525504a^3\cosh(5(c+dx))-9784320a^3\cosh(7(c+dx))+3261440a^3\cosh(9(c+dx))-7\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^16*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/369008640*((45045*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[c +
 d*x] - 5005*(7168*a^3 + 20352*a^2*b + 16632*a*b^2 + 4785*b^3)*Cosh[3*(c +
d*x)] + 21525504*a^3*Cosh[5*(c + d*x)] + 74954880*a^2*b*Cosh[5*(c + d*x)] +
 74162088*a*b^2*Cosh[5*(c + d*x)] + 23288265*b^3*Cosh[5*(c + d*x)] - 978432
0*a^3*Cosh[7*(c + d*x)] - 34070400*a^2*b*Cosh[7*(c + d*x)] - 39999960*a*b^2
*Cosh[7*(c + d*x)] - 14189175*b^3*Cosh[7*(c + d*x)] + 3261440*a^3*Cosh[9*(c
 + d*x)] + 11356800*a^2*b*Cosh[9*(c + d*x)] + 14054040*a*b^2*Cosh[9*(c + d*
x)] + 5720715*b^3*Cosh[9*(c + d*x)] - 752640*a^3*Cosh[11*(c + d*x)] - 26208
00*a^2*b*Cosh[11*(c + d*x)] - 3243240*a*b^2*Cosh[11*(c + d*x)] - 1486485*b^
3*Cosh[11*(c + d*x)] + 107520*a^3*Cosh[13*(c + d*x)] + 374400*a^2*b*Cosh[13
*(c + d*x)] + 463320*a*b^2*Cosh[13*(c + d*x)] + 225225*b^3*Cosh[13*(c + d*x
)] - 7168*a^3*Cosh[15*(c + d*x)] - 24960*a^2*b*Cosh[15*(c + d*x)] - 30888*a
*b^2*Cosh[15*(c + d*x)] - 15015*b^3*Cosh[15*(c + d*x)])*Csch[c + d*x]^15)/d
```

fricas [B] time = 0.87, size = 2967, normalized size = 16.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 8/45045*((3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^1
3 + 13*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)*sin
h(d*x + c)^12 - 2*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*sinh(d*x
 + c)^13 - 15*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x +
 c)^11 + 6*(8960*a^3 + 31200*a^2*b + 38610*a*b^2 + 65065*b^3 - 26*(1792*a^3
 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 +
 11*(26*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^3
- 15*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c))*sinh
(d*x + c)^10 + 210*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x
 + c)^9 - 10*(143*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x
 + c)^4 + 37632*a^3 + 131040*a^2*b + 216216*a*b^2 + 234234*b^3 - 165*(1792*
a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9
 + 9*(143*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^
5 - 275*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^3
+ 210*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c))*sinh(d
*x + c)^8 - 182*(8960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x
 + c)^7 - 4*(858*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x
+ c)^6 - 2475*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c
)^4 - 407680*a^3 - 1419600*a^2*b - 2918916*a*b^2 - 2147145*b^3 + 3780*(896*
a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7
+ 2*(858*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^7
 - 3465*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^5
+ 8820*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c)^3 - 63
7*(8960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x + c))*sinh(d*
x + c)^6 + 1365*(3584*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*cosh(d*x +
c)^5 - 6*(429*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c
)^8 - 2310*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^6
 + 8820*(896*a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^4 + 81
5360*a^3 + 3800160*a^2*b + 6396390*a*b^2 + 3578575*b^3 - 1274*(4480*a^3 + 1
5600*a^2*b + 32076*a*b^2 + 23595*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*
```

(143*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^9 - 9
90*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^7 + 529
2*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c)^5 - 1274*(8
960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x + c)^3 + 1365*(35
84*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*cosh(d*x + c))*sinh(d*x + c)^4
 - 429*(25088*a^3 + 24000*a^2*b + 3492*a*b^2 - 10395*b^3)*cosh(d*x + c)^3 -
 2*(286*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c)^10 -
 2475*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^8 + 17
640*(896*a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^6 - 6370*(
4480*a^3 + 15600*a^2*b + 32076*a*b^2 + 23595*b^3)*cosh(d*x + c)^4 - 5381376
*a^3 - 32329440*a^2*b - 40980654*a*b^2 - 19324305*b^3 + 13650*(1792*a^3 + 8
352*a^2*b + 14058*a*b^2 + 7865*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 3*(2
6*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^11 - 275
*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^9 + 2520*
(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c)^7 - 1274*(896
0*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x + c)^5 + 4550*(3584
*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*cosh(d*x + c)^3 - 429*(25088*a^3
 + 24000*a^2*b + 3492*a*b^2 - 10395*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 2
860*(1792*a^3 - 1248*a^2*b - 108*a*b^2 + 693*b^3)*cosh(d*x + c) - 2*(13*(17
92*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c)^12 - 165*(1792*
a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^10 + 1890*(896*a^3
 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^8 - 1274*(4480*a^3 + 1
5600*a^2*b + 32076*a*b^2 + 23595*b^3)*cosh(d*x + c)^6 + 6825*(1792*a^3 + 83
52*a^2*b + 14058*a*b^2 + 7865*b^3)*cosh(d*x + c)^4 + 20500480*a^3 + 5491200
0*a^2*b + 59304960*a*b^2 + 25765740*b^3 - 1287*(12544*a^3 + 75360*a^2*b + 9
5526*a*b^2 + 45045*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^17
 + 17*d*cosh(d*x + c)*sinh(d*x + c)^16 + d*sinh(d*x + c)^17 - 15*d*cosh(d*x
 + c)^15 + (136*d*cosh(d*x + c)^2 - 15*d)*sinh(d*x + c)^15 + 5*(136*d*cosh(
d*x + c)^3 - 45*d*cosh(d*x + c))*sinh(d*x + c)^14 + 104*d*cosh(d*x + c)^13
+ (2380*d*cosh(d*x + c)^4 - 1575*d*cosh(d*x + c)^2 + 106*d)*sinh(d*x + c)^1
3 + 13*(476*d*cosh(d*x + c)^5 - 525*d*cosh(d*x + c)^3 + 104*d*cosh(d*x + c)
)*sinh(d*x + c)^12 - 440*d*cosh(d*x + c)^11 + (12376*d*cosh(d*x + c)^6 - 20
475*d*cosh(d*x + c)^4 + 8268*d*cosh(d*x + c)^2 - 470*d)*sinh(d*x + c)^11 +
11*(1768*d*cosh(d*x + c)^7 - 4095*d*cosh(d*x + c)^5 + 2704*d*cosh(d*x + c)^
3 - 440*d*cosh(d*x + c))*sinh(d*x + c)^10 + 1260*d*cosh(d*x + c)^9 + 5*(486
2*d*cosh(d*x + c)^8 - 15015*d*cosh(d*x + c)^6 + 15158*d*cosh(d*x + c)^4 - 5
170*d*cosh(d*x + c)^2 + 294*d)*sinh(d*x + c)^9 + (24310*d*cosh(d*x + c)^9 -
 96525*d*cosh(d*x + c)^7 + 133848*d*cosh(d*x + c)^5 - 72600*d*cosh(d*x + c)
^3 + 11340*d*cosh(d*x + c))*sinh(d*x + c)^8 - 2548*d*cosh(d*x + c)^7 + (194
48*d*cosh(d*x + c)^10 - 96525*d*cosh(d*x + c)^8 + 181896*d*cosh(d*x + c)^6
 - 155100*d*cosh(d*x + c)^4 + 52920*d*cosh(d*x + c)^2 - 3458*d)*sinh(d*x + c
)^7 + (12376*d*cosh(d*x + c)^11 - 75075*d*cosh(d*x + c)^9 + 178464*d*cosh(d
*x + c)^7 - 203280*d*cosh(d*x + c)^5 + 105840*d*cosh(d*x + c)^3 - 17836*d*c
osh(d*x + c))*sinh(d*x + c)^6 + 3640*d*cosh(d*x + c)^5 + (6188*d*cosh(d*x +
 c)^12 - 45045*d*cosh(d*x + c)^10 + 136422*d*cosh(d*x + c)^8 - 217140*d*cos
h(d*x + c)^6 + 185220*d*cosh(d*x + c)^4 - 72618*d*cosh(d*x + c)^2 + 6370*d)
*sinh(d*x + c)^5 + 5*(476*d*cosh(d*x + c)^13 - 4095*d*cosh(d*x + c)^11 + 14
872*d*cosh(d*x + c)^9 - 29040*d*cosh(d*x + c)^7 + 31752*d*cosh(d*x + c)^5 -
 17836*d*cosh(d*x + c)^3 + 3640*d*cosh(d*x + c))*sinh(d*x + c)^4 - 3432*d*c
osh(d*x + c)^3 + (680*d*cosh(d*x + c)^14 - 6825*d*cosh(d*x + c)^12 + 30316*
d*cosh(d*x + c)^10 - 77550*d*cosh(d*x + c)^8 + 123480*d*cosh(d*x + c)^6 - 1
21030*d*cosh(d*x + c)^4 + 63700*d*cosh(d*x + c)^2 - 9438*d)*sinh(d*x + c)^3
 + (136*d*cosh(d*x + c)^15 - 1575*d*cosh(d*x + c)^13 + 8112*d*cosh(d*x + c)
^11 - 24200*d*cosh(d*x + c)^9 + 45360*d*cosh(d*x + c)^7 - 53508*d*cosh(d*x
+ c)^5 + 36400*d*cosh(d*x + c)^3 - 10296*d*cosh(d*x + c))*sinh(d*x + c)^2 +
 1430*d*cosh(d*x + c) + (17*d*cosh(d*x + c)^16 - 225*d*cosh(d*x + c)^14 + 1
378*d*cosh(d*x + c)^12 - 5170*d*cosh(d*x + c)^10 + 13230*d*cosh(d*x + c)^8
 - 24206*d*cosh(d*x + c)^6 + 31850*d*cosh(d*x + c)^4 - 28314*d*cosh(d*x + c)
^2 + 11440*d)*sinh(d*x + c))

giac [**B**] time = 0.54, size = 621, normalized size = 3.41

$$-\frac{4\left(45045\,b^{3}e^{(26\,dx+26\,c)}-555555\,b^{3}e^{(24\,dx+24\,c)}+1081080\,ab^{2}e^{(22\,dx+22\,c)}+3153150\,b^{3}e^{(22\,dx+22\,c)}-9297288\,ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -4/45045*(45045*b^3*e^(26*d*x + 26*c) - 555555*b^3*e^(24*d*x + 24*c) + 1081
080*a*b^2*e^(22*d*x + 22*c) + 3153150*b^3*e^(22*d*x + 22*c) - 9297288*a*b^2
*e^(20*d*x + 20*c) - 10900890*b^3*e^(20*d*x + 20*c) + 11531520*a^2*b*e^(18*
d*x + 18*c) + 35675640*a*b^2*e^(18*d*x + 18*c) + 25600575*b^3*e^(18*d*x + 1
8*c) - 54362880*a^2*b*e^(16*d*x + 16*c) - 80463240*a*b^2*e^(16*d*x + 16*c)
- 43108065*b^3*e^(16*d*x + 16*c) + 46126080*a^3*e^(14*d*x + 14*c) + 1062547
20*a^2*b*e^(14*d*x + 14*c) + 118301040*a*b^2*e^(14*d*x + 14*c) + 53513460*b
^3*e^(14*d*x + 14*c) - 35875840*a^3*e^(12*d*x + 12*c) - 113393280*a^2*b*e^(
12*d*x + 12*c) - 118918800*a*b^2*e^(12*d*x + 12*c) - 49549500*b^3*e^(12*d*x
 + 12*c) + 21525504*a^3*e^(10*d*x + 10*c) + 74954880*a^2*b*e^(10*d*x + 10*c
) + 83459376*a*b^2*e^(10*d*x + 10*c) + 34189155*b^3*e^(10*d*x + 10*c) - 978
4320*a^3*e^(8*d*x + 8*c) - 34070400*a^2*b*e^(8*d*x + 8*c) - 41081040*a*b^2*
e^(8*d*x + 8*c) - 17342325*b^3*e^(8*d*x + 8*c) + 3261440*a^3*e^(6*d*x + 6*c
) + 11356800*a^2*b*e^(6*d*x + 6*c) + 14054040*a*b^2*e^(6*d*x + 6*c) + 62762
70*b^3*e^(6*d*x + 6*c) - 752640*a^3*e^(4*d*x + 4*c) - 2620800*a^2*b*e^(4*d*
x + 4*c) - 3243240*a*b^2*e^(4*d*x + 4*c) - 1531530*b^3*e^(4*d*x + 4*c) + 10
7520*a^3*e^(2*d*x + 2*c) + 374400*a^2*b*e^(2*d*x + 2*c) + 463320*a*b^2*e^(2
*d*x + 2*c) + 225225*b^3*e^(2*d*x + 2*c) - 7168*a^3 - 24960*a^2*b - 30888*a
*b^2 - 15015*b^3)/(d*(e^(2*d*x + 2*c) - 1)^15)
```

maple [**A**] time = 0.14, size = 218, normalized size = 1.20

$$\frac{a^{3}\left(\frac{2048}{6435}-\frac{\operatorname{csch}(dx+c)^{14}}{15}+\frac{14\operatorname{csch}(dx+c)^{12}}{195}-\frac{56\operatorname{csch}(dx+c)^{10}}{715}+\frac{112\operatorname{csch}(dx+c)^{8}}{1287}-\frac{128\operatorname{csch}(dx+c)^{6}}{1287}+\frac{256\operatorname{csch}(dx+c)^{4}}{2145}-\frac{1024\operatorname{csch}(dx\right.}{6435}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(2048/6435-1/15*csch(d*x+c)^14+14/195*csch(d*x+c)^12-56/715*csch(d
*x+c)^10+112/1287*csch(d*x+c)^8-128/1287*csch(d*x+c)^6+256/2145*csch(d*x+c)
^4-1024/6435*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(256/693-1/11*csch(d*x+c)^1
0+10/99*csch(d*x+c)^8-80/693*csch(d*x+c)^6+32/231*csch(d*x+c)^4-128/693*csc
h(d*x+c)^2)*coth(d*x+c)+3*a*b^2*(16/35-1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4
-8/35*csch(d*x+c)^2)*coth(d*x+c)+b^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c))
```

maxima [**B**] time = 0.38, size = 2731, normalized size = 15.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 4096/6435*a^3*(15*e^(-2*d*x - 2*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x
- 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 1
0*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x
- 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d
*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*
x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 105*e^(-4*d*x - 4*c)/(d*(15*e^(-2*d*
x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8
```

```
*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x -
 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*
x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d
*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 455*e^(-6*d
*x - 6*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x -
6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x -
12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x
 - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d
*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x -
30*c) - 1)) - 1365*e^(-8*d*x - 8*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x
 - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x -
10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x
 - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*
d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d
*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 3003*e^(-10*d*x - 10*c)/(d*(15*e^(-
2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x
 - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d
*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-2
0*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-
26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 5005*e^
(-12*d*x - 12*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6
*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12
*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(
-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e
^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30
*d*x - 30*c) - 1)) + 6435*e^(-14*d*x - 14*c)/(d*(15*e^(-2*d*x - 2*c) - 105*
e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-
10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e
^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 136
5*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15
*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 1/(d*(15*e^(-2*d*x - 2*c)
 - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 300
3*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) -
6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c)
 + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c
) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1))) + 512/231*a^2*b*(11*e
^(-2*d*x - 2*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d
*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x
 - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x -
 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 55*e^(-4*d*x -
4*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) -
 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 3
30*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11
*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 165*e^(-6*d*x - 6*c)/(d*(1
1*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8
*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*
d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*
x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 330*e^(-8*d*x - 8*c)/(d*(11*e^(-2*d*
x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c
) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c
) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c)
+ e^(-22*d*x - 22*c) - 1)) + 462*e^(-10*d*x - 10*c)/(d*(11*e^(-2*d*x - 2*c)
 - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*
e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*
e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22
*d*x - 22*c) - 1)) - 1/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*
e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-
12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-1
8*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1))) + 96/35*a
```

```
*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*
e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) - 1)) - 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x
 - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) +
 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) +
35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6
*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x -
12*c) + e^(-14*d*x - 14*c) - 1)) - 1/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x
- 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
- 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1))) + 4/3*b^3*(3*e^(-2*d*x -
 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))
- 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

mupad [B] time = 1.22, size = 3823, normalized size = 21.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^16,x)
```

```
[Out] ((32*b^3)/(455*d) - (8*b^3*exp(2*c + 2*d*x))/(105*d))/(3*exp(2*c + 2*d*x) -
 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - ((8*exp(8*c + 8*d*x)*(840*a*b
^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(39*d) - (352*b^3*exp(18*c + 18*d*x)
)/(91*d) + (44*b^3*exp(20*c + 20*d*x))/(105*d) + (4*b^2*(8*a + 11*b))/(455*
d) - (64*b*exp(6*c + 6*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(91*d) - (128*b*e
xp(10*c + 10*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(65*d) + (4*b*exp(4*c + 4*d
*x)*(448*a*b + 256*a^2 + 165*b^2))/(91*d) + (24*b*exp(12*c + 12*d*x)*(448*a
*b + 256*a^2 + 165*b^2))/(65*d) + (132*b^2*exp(16*c + 16*d*x)*(8*a + 11*b))
/(91*d) - (32*b^2*exp(2*c + 2*d*x)*(96*a + 55*b))/(1365*d) - (64*b^2*exp(14
*c + 14*d*x)*(96*a + 55*b))/(91*d))/(66*exp(4*c + 4*d*x) - 12*exp(2*c + 2*d
*x) - 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c + 10*d*x)
+ 924*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) + 495*exp(16*c + 16*d*x)
- 220*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c + 22*d*x) +
exp(24*c + 24*d*x) + 1) + ((192*b^3*exp(4*c + 4*d*x))/(455*d) - (16*b^3*exp
(6*c + 6*d*x))/(105*d) + (32*b^2*(96*a + 55*b))/(15015*d) - (16*b^2*exp(2*c
 + 2*d*x)*(8*a + 11*b))/(455*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x)
+ 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((4*
(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(6435*d) - (96*b^3*exp(10*c
+ 10*d*x))/(65*d) + (4*b^3*exp(12*c + 12*d*x))/(15*d) - (64*b*exp(2*c + 2*d
*x)*(112*a*b + 128*a^2 + 33*b^2))/(2145*d) + (12*b*exp(4*c + 4*d*x)*(448*a*
b + 256*a^2 + 165*b^2))/(715*d) + (4*b^2*exp(8*c + 8*d*x)*(8*a + 11*b))/(13
*d) - (32*b^2*exp(6*c + 6*d*x)*(96*a + 55*b))/(429*d))/(28*exp(4*c + 4*d*x)
 - 8*exp(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(
10*c + 10*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c +
16*d*x) + 1) - ((32*exp(6*c + 6*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 2
31*b^3))/(429*d) - (288*b^3*exp(16*c + 16*d*x))/(91*d) + (8*b^3*exp(18*c +
18*d*x))/(21*d) - (32*b^2*(96*a + 55*b))/(15015*d) - (192*b*exp(4*c + 4*d*x
)*(112*a*b + 128*a^2 + 33*b^2))/(1001*d) - (128*b*exp(8*c + 8*d*x)*(112*a*b
 + 128*a^2 + 33*b^2))/(143*d) + (8*b*exp(2*c + 2*d*x)*(448*a*b + 256*a^2 +
165*b^2))/(1001*d) + (144*b*exp(10*c + 10*d*x)*(448*a*b + 256*a^2 + 165*b^2
))/(715*d) + (96*b^2*exp(14*c + 14*d*x)*(8*a + 11*b))/(91*d) - (64*b^2*exp(
12*c + 12*d*x)*(96*a + 55*b))/(143*d))/(11*exp(2*c + 2*d*x) - 55*exp(4*c +
4*d*x) + 165*exp(6*c + 6*d*x) - 330*exp(8*c + 8*d*x) + 462*exp(10*c + 10*d*
x) - 462*exp(12*c + 12*d*x) + 330*exp(14*c + 14*d*x) - 165*exp(16*c + 16*d*
x) + 55*exp(18*c + 18*d*x) - 11*exp(20*c + 20*d*x) + exp(22*c + 22*d*x) - 1
) - ((32*exp(14*c + 14*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/
(15*d) + (8*b^3*exp(2*c + 2*d*x))/(15*d) - (32*b^3*exp(4*c + 4*d*x))/(5*d)
- (32*b^3*exp(24*c + 24*d*x))/(5*d) + (8*b^3*exp(26*c + 26*d*x))/(15*d) - (
64*b*exp(12*c + 12*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(5*d) - (64*b*exp(16*
c + 16*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(5*d) + (8*b*exp(10*c + 10*d*x)*(
```

```
448*a*b + 256*a^2 + 165*b^2))/(5*d) + (8*b*exp(18*c + 18*d*x)*(448*a*b + 25
6*a^2 + 165*b^2))/(5*d) + (16*b^2*exp(6*c + 6*d*x)*(8*a + 11*b))/(5*d) + (1
6*b^2*exp(22*c + 22*d*x)*(8*a + 11*b))/(5*d) - (32*b^2*exp(8*c + 8*d*x)*(96
*a + 55*b))/(15*d) - (32*b^2*exp(20*c + 20*d*x)*(96*a + 55*b))/(15*d))/(15*
exp(2*c + 2*d*x) - 105*exp(4*c + 4*d*x) + 455*exp(6*c + 6*d*x) - 1365*exp(8
*c + 8*d*x) + 3003*exp(10*c + 10*d*x) - 5005*exp(12*c + 12*d*x) + 6435*exp(
14*c + 14*d*x) - 6435*exp(16*c + 16*d*x) + 5005*exp(18*c + 18*d*x) - 3003*e
xp(20*c + 20*d*x) + 1365*exp(22*c + 22*d*x) - 455*exp(24*c + 24*d*x) + 105*
exp(26*c + 26*d*x) - 15*exp(28*c + 28*d*x) + exp(30*c + 30*d*x) - 1) - ((4*
b*(448*a*b + 256*a^2 + 165*b^2))/(5005*d) - (64*b^3*exp(6*c + 6*d*x))/(91*d
) + (4*b^3*exp(8*c + 8*d*x))/(21*d) + (8*b^2*exp(4*c + 4*d*x)*(8*a + 11*b))
/(91*d) - (32*b^2*exp(2*c + 2*d*x)*(96*a + 55*b))/(3003*d))/(15*exp(4*c + 4
*d*x) - 6*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*
exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) + ((64*b*(112*a*b + 128*a^2 +
33*b^2))/(15015*d) - (32*exp(2*c + 2*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^
3 + 231*b^3))/(6435*d) + (128*b^3*exp(12*c + 12*d*x))/(65*d) - (32*b^3*exp(
14*c + 14*d*x))/(105*d) + (256*b*exp(4*c + 4*d*x)*(112*a*b + 128*a^2 + 33*b
^2))/(2145*d) - (32*b*exp(6*c + 6*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(715*
d) - (32*b^2*exp(10*c + 10*d*x)*(8*a + 11*b))/(65*d) + (64*b^2*exp(8*c + 8*
d*x)*(96*a + 55*b))/(429*d))/(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84
*exp(6*c + 6*d*x) - 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(
12*c + 12*d*x) + 36*exp(14*c + 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c +
18*d*x) - 1) - ((4*b^3)/(105*d) + (16*exp(12*c + 12*d*x)*(840*a*b^2 + 1152*
a^2*b + 1024*a^3 + 231*b^3))/(15*d) - (32*b^3*exp(2*c + 2*d*x))/(35*d) - (1
92*b^3*exp(22*c + 22*d*x))/(35*d) + (52*b^3*exp(24*c + 24*d*x))/(105*d) - (
192*b*exp(10*c + 10*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(35*d) - (256*b*exp(
14*c + 14*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(35*d) + (4*b*exp(8*c + 8*d*x)
*(448*a*b + 256*a^2 + 165*b^2))/(7*d) + (36*b*exp(16*c + 16*d*x)*(448*a*b +
 256*a^2 + 165*b^2))/(35*d) + (24*b^2*exp(4*c + 4*d*x)*(8*a + 11*b))/(35*d)
 + (88*b^2*exp(20*c + 20*d*x)*(8*a + 11*b))/(35*d) - (64*b^2*exp(6*c + 6*d*
x)*(96*a + 55*b))/(105*d) - (32*b^2*exp(18*c + 18*d*x)*(96*a + 55*b))/(21*d
))/(91*exp(4*c + 4*d*x) - 14*exp(2*c + 2*d*x) - 364*exp(6*c + 6*d*x) + 1001
*exp(8*c + 8*d*x) - 2002*exp(10*c + 10*d*x) + 3003*exp(12*c + 12*d*x) - 343
2*exp(14*c + 14*d*x) + 3003*exp(16*c + 16*d*x) - 2002*exp(18*c + 18*d*x) +
1001*exp(20*c + 20*d*x) - 364*exp(22*c + 22*d*x) + 91*exp(24*c + 24*d*x) -
14*exp(26*c + 26*d*x) + exp(28*c + 28*d*x) + 1) - ((4*b^3*exp(4*c + 4*d*x))
/(35*d) - (96*b^3*exp(2*c + 2*d*x))/(455*d) + (4*b^2*(8*a + 11*b))/(455*d))
/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c +
8*d*x) + 1) + ((64*b*(112*a*b + 128*a^2 + 33*b^2))/(15015*d) + (96*b^3*exp(
8*c + 8*d*x))/(91*d) - (8*b^3*exp(10*c + 10*d*x))/(35*d) - (24*b*exp(2*c +
2*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(5005*d) - (16*b^2*exp(6*c + 6*d*x)*(
8*a + 11*b))/(91*d) + (32*b^2*exp(4*c + 4*d*x)*(96*a + 55*b))/(1001*d))/(7*
exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*c +
 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x)
 - 1) - ((4*b*(448*a*b + 256*a^2 + 165*b^2))/(5005*d) + (16*exp(4*c + 4*d*x
)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(715*d) - (1152*b^3*exp(14
*c + 14*d*x))/(455*d) + (12*b^3*exp(16*c + 16*d*x))/(35*d) - (192*b*exp(2*c
 + 2*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(5005*d) - (256*b*exp(6*c + 6*d*x)*
(112*a*b + 128*a^2 + 33*b^2))/(715*d) + (72*b*exp(8*c + 8*d*x)*(448*a*b + 2
56*a^2 + 165*b^2))/(715*d) + (48*b^2*exp(12*c + 12*d*x)*(8*a + 11*b))/(65*d
) - (192*b^2*exp(10*c + 10*d*x)*(96*a + 55*b))/(715*d))/(45*exp(4*c + 4*d*x
) - 10*exp(2*c + 2*d*x) - 120*exp(6*c + 6*d*x) + 210*exp(8*c + 8*d*x) - 252
*exp(10*c + 10*d*x) + 210*exp(12*c + 12*d*x) - 120*exp(14*c + 14*d*x) + 45*
exp(16*c + 16*d*x) - 10*exp(18*c + 18*d*x) + exp(20*c + 20*d*x) + 1) + ((32
*b^3)/(455*d) - (32*exp(10*c + 10*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 +
 231*b^3))/(65*d) + (2112*b^3*exp(20*c + 20*d*x))/(455*d) - (16*b^3*exp(22*
c + 22*d*x))/(35*d) + (192*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 + 33*b^2))
/(91*d) + (256*b*exp(12*c + 12*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(65*d) -
(16*b*exp(6*c + 6*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(91*d) - (288*b*exp(1
```

```
4*c + 14*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(455*d) - (48*b^2*exp(2*c + 2*
d*x)*(8*a + 11*b))/(455*d) - (176*b^2*exp(18*c + 18*d*x)*(8*a + 11*b))/(91*
d) + (64*b^2*exp(4*c + 4*d*x)*(96*a + 55*b))/(455*d) + (96*b^2*exp(16*c + 1
6*d*x)*(96*a + 55*b))/(91*d))/(13*exp(2*c + 2*d*x) - 78*exp(4*c + 4*d*x) +
286*exp(6*c + 6*d*x) - 715*exp(8*c + 8*d*x) + 1287*exp(10*c + 10*d*x) - 171
6*exp(12*c + 12*d*x) + 1716*exp(14*c + 14*d*x) - 1287*exp(16*c + 16*d*x) +
715*exp(18*c + 18*d*x) - 286*exp(20*c + 20*d*x) + 78*exp(22*c + 22*d*x) - 1
3*exp(24*c + 24*d*x) + exp(26*c + 26*d*x) - 1) - (4*b^3)/(105*d*(exp(4*c +
4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**16*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.227 $\int \operatorname{csch}^{18}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=221

$$-\frac{a^3\coth^{17}(c+dx)}{17d}+\frac{8a^3\coth^{15}(c+dx)}{15d}-\frac{a\left(70a^2+45ab+3b^2\right)\coth^9(c+dx)}{9d}+\frac{4a\left(14a^2+15ab+3b^2\right)\coth^7(c+\ldots}{7d}$$

```
[Out] -(a+b)^3*coth(d*x+c)/d+2/3*(a+b)^2*(4*a+b)*coth(d*x+c)^3/d-1/5*(a+b)*(28*a^
2+17*a*b+b^2)*coth(d*x+c)^5/d+4/7*a*(14*a^2+15*a*b+3*b^2)*coth(d*x+c)^7/d-1
/9*a*(70*a^2+45*a*b+3*b^2)*coth(d*x+c)^9/d+2/11*a^2*(28*a+9*b)*coth(d*x+c)^
11/d-1/13*a^2*(28*a+3*b)*coth(d*x+c)^13/d+8/15*a^3*coth(d*x+c)^15/d-1/17*a^
3*coth(d*x+c)^17/d
```

Rubi [A] time = 0.19, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3217, 1261}

$$-\frac{a\left(70a^2+45ab+3b^2\right)\coth^9(c+dx)}{9d}+\frac{4a\left(14a^2+15ab+3b^2\right)\coth^7(c+dx)}{7d}-\frac{(a+b)\left(28a^2+17ab+b^2\right)\coth^5(\ldots}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^18*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*(a + b)^2*(4*a + b)*Coth[c + d*x]^3)/(3
*d) - ((a + b)*(28*a^2 + 17*a*b + b^2)*Coth[c + d*x]^5)/(5*d) + (4*a*(14*a^
2 + 15*a*b + 3*b^2)*Coth[c + d*x]^7)/(7*d) - (a*(70*a^2 + 45*a*b + 3*b^2)*C
oth[c + d*x]^9)/(9*d) + (2*a^2*(28*a + 9*b)*Coth[c + d*x]^11)/(11*d) - (a^2
*(28*a + 3*b)*Coth[c + d*x]^13)/(13*d) + (8*a^3*Coth[c + d*x]^15)/(15*d) -
(a^3*Coth[c + d*x]^17)/(17*d)
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{csch}^{18}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(1-x^2\right)^2\left(a-2ax^2+(a+b)x^4\right)^3}{x^{18}}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\operatorname{Subst}\left(\int\left(\frac{a^3}{x^{18}}-\frac{8a^3}{x^{16}}+\frac{a^2(28a+3b)}{x^{14}}-\frac{2a^2(28a+9b)}{x^{12}}+\frac{a\left(70a^2+45ab+3b^2\right)}{x^{10}}-\frac{4a\ldots}{\ldots}\right.\right.}{\ldots}$$

$$=-\frac{(a+b)^3\coth(c+dx)}{d}+\frac{2(a+b)^2(4a+b)\coth^3(c+dx)}{3d}-\frac{(a+b\ldots}{\ldots}$$

Mathematica [B] time = 6.19, size = 494, normalized size = 2.24

$$\frac{\operatorname{csch}^{17}(c+dx)\left(-697016320a^3\cosh(c+dx)+557613056a^3\cosh(3(c+dx))-354844672a^3\cosh(5(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^18*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-697016320*a^3*Cosh[c + d*x] - 784143360*a^2*b*Cosh[c + d*x] - 571771200*
a*b^2*Cosh[c + d*x] - 157237080*b^3*Cosh[c + d*x] + 557613056*a^3*Cosh[3*(c
 + d*x)] + 1568286720*a^2*b*Cosh[3*(c + d*x)] + 1280767488*a*b^2*Cosh[3*(c
+ d*x)] + 368384016*b^3*Cosh[3*(c + d*x)] - 354844672*a^3*Cosh[5*(c + d*x)]
 - 1211857920*a^2*b*Cosh[5*(c + d*x)] - 1189284096*a*b^2*Cosh[5*(c + d*x)]
- 372263892*b^3*Cosh[5*(c + d*x)] + 177422336*a^3*Cosh[7*(c + d*x)] + 60592
8960*a^2*b*Cosh[7*(c + d*x)] + 692659968*a*b^2*Cosh[7*(c + d*x)] + 24228804
6*b^3*Cosh[7*(c + d*x)] - 68239360*a^3*Cosh[9*(c + d*x)] - 233049600*a^2*b*
Cosh[9*(c + d*x)] - 277717440*a*b^2*Cosh[9*(c + d*x)] - 108738630*b^3*Cosh[
9*(c + d*x)] + 19496960*a^3*Cosh[11*(c + d*x)] + 66585600*a^2*b*Cosh[11*(c
+ d*x)] + 79347840*a*b^2*Cosh[11*(c + d*x)] + 33693660*b^3*Cosh[11*(c + d*x
)] - 3899392*a^3*Cosh[13*(c + d*x)] - 13317120*a^2*b*Cosh[13*(c + d*x)] - 1
5869568*a*b^2*Cosh[13*(c + d*x)] - 6942936*b^3*Cosh[13*(c + d*x)] + 487424*
a^3*Cosh[15*(c + d*x)] + 1664640*a^2*b*Cosh[15*(c + d*x)] + 1983696*a*b^2*C
osh[15*(c + d*x)] + 867867*b^3*Cosh[15*(c + d*x)] - 28672*a^3*Cosh[17*(c +
d*x)] - 97920*a^2*b*Cosh[17*(c + d*x)] - 116688*a*b^2*Cosh[17*(c + d*x)] -
51051*b^3*Cosh[17*(c + d*x)])*Csch[c + d*x]^17)/(6273146880*d)
```

fricas [B] time = 0.78, size = 3585, normalized size = 16.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -16/765765*((28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x
+ c)^14 - 14*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh(d*x
 + c)*sinh(d*x + c)^13 + (28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b
^3)*sinh(d*x + c)^14 - 34*(14336*a^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b
^3)*cosh(d*x + c)^12 - (487424*a^3 + 1664640*a^2*b + 1983696*a*b^2 + 724924
2*b^3 - 91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^12 - 4*(91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 4
59459*b^3)*cosh(d*x + c)^3 - 204*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81
081*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + 17*(229376*a^3 + 783360*a^2*b +
1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^10 + (1001*(28672*a^3 + 97920*a^
2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^4 + 3899392*a^3 + 13317120*a
^2*b + 30572256*a*b^2 + 43750707*b^3 - 2244*(14336*a^3 + 48960*a^2*b + 5834
4*a*b^2 + 213213*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 - 2*(1001*(28672*a^
3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh(d*x + c)^5 - 7480*(7168*a
^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*cosh(d*x + c)^3 + 85*(229376*a^
3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^3)*cosh(d*x + c))*sinh(d*x + c)^
9 - 68*(286720*a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x +
 c)^8 + (3003*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*
x + c)^6 - 16830*(14336*a^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(
d*x + c)^4 - 19496960*a^3 - 66585600*a^2*b - 206771136*a*b^2 - 163975812*b^
3 + 765*(229376*a^3 + 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 - 8*(429*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 -
459459*b^3)*cosh(d*x + c)^7 - 6732*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 -
81081*b^3)*cosh(d*x + c)^5 + 255*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 -
 1756755*b^3)*cosh(d*x + c)^3 - 272*(71680*a^3 + 244800*a^2*b - 176748*a*b^
2 - 351351*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 17*(4014080*a^3 + 23592960
```

*a^2*b + 45412224*a*b^2 + 25138113*b^3)*cosh(d*x + c)^6 + (3003*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^8 - 31416*(14336*a^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(d*x + c)^6 + 3570*(229376*a^3 + 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^4 + 68239360*a^3 + 401080320*a^2*b + 772007808*a*b^2 + 427347921*b^3 - 1904*(286720*a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 2*(1001*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh(d*x + c)^9 - 26928*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*cosh(d*x + c)^7 + 2142*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^3)*cosh(d*x + c)^5 - 7616*(71680*a^3 + 244800*a^2*b - 176748*a*b^2 - 351351*b^3)*cosh(d*x + c)^3 + 51*(4014080*a^3 + 3824640*a^2*b - 12739584*a*b^2 - 11594583*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 442*(401408*a^3 + 3176640*a^2*b + 4162488*a*b^2 + 1857471*b^3)*cosh(d*x + c)^4 + (1001*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^10 - 16830*(14336*a^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(d*x + c)^8 + 3570*(229376*a^3 + 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^6 - 4760*(286720*a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x + c)^4 - 177422336*a^3 - 1404074880*a^2*b - 1839819696*a*b^2 - 821002182*b^3 + 255*(4014080*a^3 + 23592960*a^2*b + 45412224*a*b^2 + 25138113*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh(d*x + c)^11 - 3740*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*cosh(d*x + c)^9 + 510*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^3)*cosh(d*x + c)^7 - 3808*(71680*a^3 + 244800*a^2*b - 176748*a*b^2 - 351351*b^3)*cosh(d*x + c)^5 + 85*(4014080*a^3 + 3824640*a^2*b - 12739584*a*b^2 - 11594583*b^3)*cosh(d*x + c)^3 - 884*(200704*a^3 - 217440*a^2*b - 480876*a*b^2 - 297297*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 557613056*a^3 - 1736317440*a^2*b - 1775057856*a*b^2 - 680611932*b^3 + 221*(4759552*a^3 + 12643200*a^2*b + 13669392*a*b^2 + 5435199*b^3)*cosh(d*x + c)^2 + (91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^12 - 2244*(14336*a^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(d*x + c)^10 + 765*(229376*a^3 + 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^8 - 1904*(286720*a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x + c)^6 + 255*(4014080*a^3 + 23592960*a^2*b + 45412224*a*b^2 + 25138113*b^3)*cosh(d*x + c)^4 + 1051860992*a^3 + 2794147200*a^2*b + 3020935632*a*b^2 + 1201178979*b^3 - 2652*(401408*a^3 + 3176640*a^2*b + 4162488*a*b^2 + 1857471*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*(7*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh(d*x + c)^13 - 408*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*cosh(d*x + c)^11 + 85*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^3)*cosh(d*x + c)^9 - 1088*(71680*a^3 + 244800*a^2*b - 176748*a*b^2 - 351351*b^3)*cosh(d*x + c)^7 + 51*(4014080*a^3 + 3824640*a^2*b - 12739584*a*b^2 - 11594583*b^3)*cosh(d*x + c)^5 - 1768*(200704*a^3 - 217440*a^2*b - 480876*a*b^2 - 297297*b^3)*cosh(d*x + c)^3 - 5967*(57344*a^3 + 62080*a^2*b + 64944*a*b^2 + 33033*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^20 + 20*d*cosh(d*x + c)*sinh(d*x + c)^19 + d*sinh(d*x + c)^20 - 17*d*cosh(d*x + c)^18 + (190*d*cosh(d*x + c)^2 - 17*d)*sinh(d*x + c)^18 + 6*(190*d*cosh(d*x + c)^3 - 51*d*cosh(d*x + c))*sinh(d*x + c)^17 + 136*d*cosh(d*x + c)^16 + 17*(285*d*cosh(d*x + c)^4 - 153*d*cosh(d*x + c)^2 + 8*d)*sinh(d*x + c)^16 + 272*(57*d*cosh(d*x + c)^5 - 51*d*cosh(d*x + c)^3 + 8*d*cosh(d*x + c))*sinh(d*x + c)^15 - 681*d*cosh(d*x + c)^14 + 3*(12920*d*cosh(d*x + c)^6 - 17340*d*cosh(d*x + c)^4 + 5440*d*cosh(d*x + c)^2 - 227*d)*sinh(d*x + c)^14 + 2*(38760*d*cosh(d*x + c)^7 - 72828*d*cosh(d*x + c)^5 + 38080*d*cosh(d*x + c)^3 - 4753*d*cosh(d*x + c))*sinh(d*x + c)^13 + 2397*d*cosh(d*x + c)^12 + (125970*d*cosh(d*x + c)^8 - 315588*d*cosh(d*x + c)^6 + 247520*d*cosh(d*x + c)^4 - 61971*d*cosh(d*x + c)^2 + 2397*d)*sinh(d*x + c)^12 + 4*(41990*d*cosh(d*x + c)^9 - 135252*d*cosh(d*x + c)^7 + 148512*d*cosh(d*x + c)^5 - 61789*d*cosh(d*x + c)^3 + 7089*d*cosh(d*x + c))*sinh(d*x + c)^11 - 6324*d*cosh(d*x + c)^10 + (184756*d*cosh(d*x + c)^10 - 743886*d*cosh(d*x + c)^8 + 1089088*d*cosh(d*x + c)^6 - 681681*d*cosh(d*x + c)^4 + 158202*d*cosh(d*x + c)^2 - 6324*d)*sinh(d*x + c)^10 + 2*(83980*d*cosh(d*x + c)^11 - 413270*d*cosh(d*x + c)^

```
9 + 777920*d*cosh(d*x + c)^7 - 679679*d*cosh(d*x + c)^5 + 259930*d*cosh(d*x
 + c)^3 - 30260*d*cosh(d*x + c))*sinh(d*x + c)^9 + 13056*d*cosh(d*x + c)^8
+ 3*(41990*d*cosh(d*x + c)^12 - 247962*d*cosh(d*x + c)^10 + 583440*d*cosh(d
*x + c)^8 - 681681*d*cosh(d*x + c)^6 + 395505*d*cosh(d*x + c)^4 - 94860*d*c
osh(d*x + c)^2 + 4352*d)*sinh(d*x + c)^8 + 8*(9690*d*cosh(d*x + c)^13 - 676
26*d*cosh(d*x + c)^11 + 194480*d*cosh(d*x + c)^9 - 291291*d*cosh(d*x + c)^7
 + 233937*d*cosh(d*x + c)^5 - 90780*d*cosh(d*x + c)^3 + 11696*d*cosh(d*x +
c))*sinh(d*x + c)^7 - 21828*d*cosh(d*x + c)^6 + (38760*d*cosh(d*x + c)^14 -
 315588*d*cosh(d*x + c)^12 + 1089088*d*cosh(d*x + c)^10 - 2045043*d*cosh(d*
x + c)^8 + 2214828*d*cosh(d*x + c)^6 - 1328040*d*cosh(d*x + c)^4 + 365568*d
*cosh(d*x + c)^2 - 21828*d)*sinh(d*x + c)^6 + 2*(7752*d*cosh(d*x + c)^15 -
72828*d*cosh(d*x + c)^13 + 297024*d*cosh(d*x + c)^11 - 679679*d*cosh(d*x +
c)^9 + 935748*d*cosh(d*x + c)^7 - 762552*d*cosh(d*x + c)^5 + 327488*d*cosh(
d*x + c)^3 - 51204*d*cosh(d*x + c))*sinh(d*x + c)^5 + 30498*d*cosh(d*x + c)
^4 + (4845*d*cosh(d*x + c)^16 - 52020*d*cosh(d*x + c)^14 + 247520*d*cosh(d*
x + c)^12 - 681681*d*cosh(d*x + c)^10 + 1186515*d*cosh(d*x + c)^8 - 1328040
*d*cosh(d*x + c)^6 + 913920*d*cosh(d*x + c)^4 - 327420*d*cosh(d*x + c)^2 +
30498*d)*sinh(d*x + c)^4 + 4*(285*d*cosh(d*x + c)^17 - 3468*d*cosh(d*x + c)
^15 + 19040*d*cosh(d*x + c)^13 - 61789*d*cosh(d*x + c)^11 + 129965*d*cosh(d
*x + c)^9 - 181560*d*cosh(d*x + c)^7 + 163744*d*cosh(d*x + c)^5 - 85340*d*c
osh(d*x + c)^3 + 18122*d*cosh(d*x + c))*sinh(d*x + c)^3 - 36686*d*cosh(d*x
+ c)^2 + (190*d*cosh(d*x + c)^18 - 2601*d*cosh(d*x + c)^16 + 16320*d*cosh(d
*x + c)^14 - 61971*d*cosh(d*x + c)^12 + 158202*d*cosh(d*x + c)^10 - 284580*
d*cosh(d*x + c)^8 + 365568*d*cosh(d*x + c)^6 - 327420*d*cosh(d*x + c)^4 + 1
82988*d*cosh(d*x + c)^2 - 36686*d)*sinh(d*x + c)^2 + 2*(10*d*cosh(d*x + c)^
19 - 153*d*cosh(d*x + c)^17 + 1088*d*cosh(d*x + c)^15 - 4753*d*cosh(d*x + c
)^13 + 14178*d*cosh(d*x + c)^11 - 30260*d*cosh(d*x + c)^9 + 46784*d*cosh(d*
x + c)^7 - 51204*d*cosh(d*x + c)^5 + 36244*d*cosh(d*x + c)^3 - 11934*d*cosh
(d*x + c))*sinh(d*x + c) + 19448*d)
```

giac [B] time = 0.53, size = 679, normalized size = 3.07

$$-\frac{16\left(510510\,b^{3}e^{(28\,dx+28\,c)} - 6381375\,b^{3}e^{(26\,dx+26\,c)} + 14702688\,ab^{2}e^{(24\,dx+24\,c)} + 36807771\,b^{3}e^{(24\,dx+24\,c)} - 12742\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -16/765765*(510510*b^3*e^(28*d*x + 28*c) - 6381375*b^3*e^(26*d*x + 26*c) +
14702688*a*b^2*e^(24*d*x + 24*c) + 36807771*b^3*e^(24*d*x + 24*c) - 1274232
96*a*b^2*e^(22*d*x + 22*c) - 129771642*b^3*e^(22*d*x + 22*c) + 168030720*a^
2*b*e^(20*d*x + 20*c) + 494290368*a*b^2*e^(20*d*x + 20*c) + 312227916*b^3*e
^(20*d*x + 20*c) - 798145920*a^2*b*e^(18*d*x + 18*c) - 1132457040*a*b^2*e^(
18*d*x + 18*c) - 541906365*b^3*e^(18*d*x + 18*c) + 697016320*a^3*e^(16*d*x
+ 16*c) + 1582289280*a^2*b*e^(16*d*x + 16*c) + 1704228240*a*b^2*e^(16*d*x +
 16*c) + 699143445*b^3*e^(16*d*x + 16*c) - 557613056*a^3*e^(14*d*x + 14*c)
- 1736317440*a^2*b*e^(14*d*x + 14*c) - 1775057856*a*b^2*e^(14*d*x + 14*c) -
 680611932*b^3*e^(14*d*x + 14*c) + 354844672*a^3*e^(12*d*x + 12*c) + 121185
7920*a^2*b*e^(12*d*x + 12*c) + 1316707392*a*b^2*e^(12*d*x + 12*c) + 5020355
34*b^3*e^(12*d*x + 12*c) - 177422336*a^3*e^(10*d*x + 10*c) - 605928960*a^2*
b*e^(10*d*x + 10*c) - 707362656*a*b^2*e^(10*d*x + 10*c) - 279095817*b^3*e^(
10*d*x + 10*c) + 68239360*a^3*e^(8*d*x + 8*c) + 233049600*a^2*b*e^(8*d*x +
8*c) + 277717440*a*b^2*e^(8*d*x + 8*c) + 115120005*b^3*e^(8*d*x + 8*c) - 19
496960*a^3*e^(6*d*x + 6*c) - 66585600*a^2*b*e^(6*d*x + 6*c) - 79347840*a*b^
2*e^(6*d*x + 6*c) - 34204170*b^3*e^(6*d*x + 6*c) + 3899392*a^3*e^(4*d*x + 4
*c) + 13317120*a^2*b*e^(4*d*x + 4*c) + 15869568*a*b^2*e^(4*d*x + 4*c) + 694
2936*b^3*e^(4*d*x + 4*c) - 487424*a^3*e^(2*d*x + 2*c) - 1664640*a^2*b*e^(2*
d*x + 2*c) - 1983696*a*b^2*e^(2*d*x + 2*c) - 867867*b^3*e^(2*d*x + 2*c) + 2
```

```
8672*a^3 + 97920*a^2*b + 116688*a*b^2 + 51051*b^3)/(d*(e^(2*d*x + 2*c) - 1)
^17)
```

maple [A] time = 0.14, size = 258, normalized size = 1.17

$$a^3\left(-\frac{32768}{109395}-\frac{\operatorname{csch}(dx+c)^{16}}{17}+\frac{16\operatorname{csch}(dx+c)^{14}}{255}-\frac{224\operatorname{csch}(dx+c)^{12}}{3315}+\frac{896\operatorname{csch}(dx+c)^{10}}{12155}-\frac{1792\operatorname{csch}(dx+c)^{8}}{21879}+\frac{2048\operatorname{csch}(dx+c)^{6}}{21879}-\frac{4096\operatorname{csc}}{36}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(-32768/109395-1/17*csch(d*x+c)^16+16/255*csch(d*x+c)^14-224/3315*
csch(d*x+c)^12+896/12155*csch(d*x+c)^10-1792/21879*csch(d*x+c)^8+2048/21879
*csch(d*x+c)^6-4096/36465*csch(d*x+c)^4+16384/109395*csch(d*x+c)^2)*coth(d*
x+c)+3*a^2*b*(-1024/3003-1/13*csch(d*x+c)^12+12/143*csch(d*x+c)^10-40/429*c
sch(d*x+c)^8+320/3003*csch(d*x+c)^6-128/1001*csch(d*x+c)^4+512/3003*csch(d*
x+c)^2)*coth(d*x+c)+3*a*b^2*(-128/315-1/9*csch(d*x+c)^8+8/63*csch(d*x+c)^6-
16/105*csch(d*x+c)^4+64/315*csch(d*x+c)^2)*coth(d*x+c)+b^3*(-8/15-1/5*csch(
d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))
```

maxima [B] time = 0.39, size = 3719, normalized size = 16.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -65536/109395*a^3*(17*e^(-2*d*x - 2*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*
d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x
 - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-
16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 1237
6*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) -
680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e
^(-34*d*x - 34*c) - 1)) - 136*e^(-4*d*x - 4*c)/(d*(17*e^(-2*d*x - 2*c) - 13
6*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^
(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24
310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c
) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x -
26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 3
2*c) + e^(-34*d*x - 34*c) - 1)) + 680*e^(-6*d*x - 6*c)/(d*(17*e^(-2*d*x - 2
*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) +
 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14
*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*
x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-2
6*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32
*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) - 2380*e^(-8*d*x - 8*c)/(d*(17*e^(-
2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x
 - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14
*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*
e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2
380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) -
17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) + 6188*e^(-10*d*x - 10*c)/
(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 238
0*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 1
9448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*
c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x
- 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x
 - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) - 12376*e^(-12*
d*x - 12*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x
```

- 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) + 19448*e^(-14*d*x - 14*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) - 24310*e^(-16*d*x - 16*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) - 1/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1))) - 2048/1001*a^2*b*(13*e^(-2*d*x - 2*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 78*e^(-4*d*x - 4*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) + 286*e^(-6*d*x - 6*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 715*e^(-8*d*x - 8*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) + 1287*e^(-10*d*x - 10*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 1716*e^(-12*d*x - 12*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 1/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1))) - 256/105*a*b^2*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 36*e^(-4*d*x - 4*c)/(d*(9*e

```
^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x
 - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x -
14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) + 84*e^(-6*d*x - 6*
c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126
*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(
-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 126*e^(
-8*d*x - 8*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x -
 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*
c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)
) - 1/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) -
126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*
e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1))) - 16/
15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(
-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x -
 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) - 1)))
```

mupad [B] time = 1.31, size = 4490, normalized size = 20.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^18,x)
```

```
[Out] ((24*b^3)/(595*d) - (64*exp(10*c + 10*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a
^3 + 231*b^3))/(85*d) + (6864*b^3*exp(20*c + 20*d*x))/(595*d) - (104*b^3*ex
p(22*c + 22*d*x))/(85*d) + (48*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 + 33*b
^2))/(17*d) + (576*b*exp(12*c + 12*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(85*d
) - (24*b*exp(6*c + 6*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(119*d) - (144*b*
exp(14*c + 14*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(119*d) - (48*b^2*exp(2*c
 + 2*d*x)*(8*a + 11*b))/(595*d) - (528*b^2*exp(18*c + 18*d*x)*(8*a + 11*b))
/(119*d) + (16*b^2*exp(4*c + 4*d*x)*(96*a + 55*b))/(119*d) + (264*b^2*exp(1
6*c + 16*d*x)*(96*a + 55*b))/(119*d))/(91*exp(4*c + 4*d*x) - 14*exp(2*c + 2
*d*x) - 364*exp(6*c + 6*d*x) + 1001*exp(8*c + 8*d*x) - 2002*exp(10*c + 10*d
*x) + 3003*exp(12*c + 12*d*x) - 3432*exp(14*c + 14*d*x) + 3003*exp(16*c + 1
6*d*x) - 2002*exp(18*c + 18*d*x) + 1001*exp(20*c + 20*d*x) - 364*exp(22*c +
 22*d*x) + 91*exp(24*c + 24*d*x) - 14*exp(26*c + 26*d*x) + exp(28*c + 28*d*
x) + 1) + ((24*b^3)/(595*d) - (4*b^3*exp(2*c + 2*d*x))/(85*d))/(6*exp(4*c +
 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) -
 ((64*exp(8*c + 8*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(221*
d) - (1056*b^3*exp(18*c + 18*d*x))/(119*d) + (88*b^3*exp(20*c + 20*d*x))/(8
5*d) + (48*b^2*(8*a + 11*b))/(7735*d) - (192*b*exp(6*c + 6*d*x)*(112*a*b +
128*a^2 + 33*b^2))/(221*d) - (3456*b*exp(10*c + 10*d*x)*(112*a*b + 128*a^2
+ 33*b^2))/(1105*d) + (72*b*exp(4*c + 4*d*x)*(448*a*b + 256*a^2 + 165*b^2))
/(1547*d) + (144*b*exp(12*c + 12*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(221*d
) + (4752*b^2*exp(16*c + 16*d*x)*(8*a + 11*b))/(1547*d) - (32*b^2*exp(2*c +
 2*d*x)*(96*a + 55*b))/(1547*d) - (2112*b^2*exp(14*c + 14*d*x)*(96*a + 55*b
))/(1547*d))/(13*exp(2*c + 2*d*x) - 78*exp(4*c + 4*d*x) + 286*exp(6*c + 6*d
*x) - 715*exp(8*c + 8*d*x) + 1287*exp(10*c + 10*d*x) - 1716*exp(12*c + 12*d
*x) + 1716*exp(14*c + 14*d*x) - 1287*exp(16*c + 16*d*x) + 715*exp(18*c + 18
*d*x) - 286*exp(20*c + 20*d*x) + 78*exp(22*c + 22*d*x) - 13*exp(24*c + 24*d
*x) + exp(26*c + 26*d*x) - 1) + ((48*b^3*exp(4*c + 4*d*x))/(119*d) - (8*b^3
*exp(6*c + 6*d*x))/(51*d) + (8*b^2*(96*a + 55*b))/(4641*d) - (48*b^2*exp(2*
c + 2*d*x)*(8*a + 11*b))/(1547*d))/(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x
) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(
12*c + 12*d*x) + 1) - ((64*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(
109395*d) - (192*b^3*exp(10*c + 10*d*x))/(85*d) + (112*b^3*exp(12*c + 12*d*
x))/(255*d) - (384*b*exp(2*c + 2*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(12155*
```

d) + (48*b*exp(4*c + 4*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(2431*d) + (96*b
^2*exp(8*c + 8*d*x)*(8*a + 11*b))/(221*d) - (64*b^2*exp(6*c + 6*d*x)*(96*a
+ 55*b))/(663*d))/(9*exp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c +
6*d*x) - 126*exp(8*c + 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d
*x) + 36*exp(14*c + 14*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1
) - ((64*exp(6*c + 6*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(6
63*d) - (792*b^3*exp(16*c + 16*d*x))/(119*d) + (44*b^3*exp(18*c + 18*d*x))/
(51*d) - (8*b^2*(96*a + 55*b))/(4641*d) - (48*b*exp(4*c + 4*d*x)*(112*a*b +
 128*a^2 + 33*b^2))/(221*d) - (288*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 +
33*b^2))/(221*d) + (12*b*exp(2*c + 2*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(1
547*d) + (72*b*exp(10*c + 10*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(221*d) +
(3168*b^2*exp(14*c + 14*d*x)*(8*a + 11*b))/(1547*d) - (176*b^2*exp(12*c + 1
2*d*x)*(96*a + 55*b))/(221*d))/(66*exp(4*c + 4*d*x) - 12*exp(2*c + 2*d*x) -
 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c + 10*d*x) + 924
*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) + 495*exp(16*c + 16*d*x) - 220
*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c + 22*d*x) + exp(2
4*c + 24*d*x) + 1) - ((64*exp(14*c + 14*d*x)*(840*a*b^2 + 1152*a^2*b + 1024
*a^3 + 231*b^3))/(17*d) + (4*b^3*exp(2*c + 2*d*x))/(17*d) - (72*b^3*exp(4*c
 + 4*d*x))/(17*d) - (312*b^3*exp(24*c + 24*d*x))/(17*d) + (28*b^3*exp(26*c
+ 26*d*x))/(17*d) - (336*b*exp(12*c + 12*d*x)*(112*a*b + 128*a^2 + 33*b^2))
/(17*d) - (432*b*exp(16*c + 16*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(17*d) +
(36*b*exp(10*c + 10*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(17*d) + (60*b*exp(
18*c + 18*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(17*d) + (48*b^2*exp(6*c + 6*
d*x)*(8*a + 11*b))/(17*d) + (144*b^2*exp(22*c + 22*d*x)*(8*a + 11*b))/(17*d
) - (40*b^2*exp(8*c + 8*d*x)*(96*a + 55*b))/(17*d) - (88*b^2*exp(20*c + 20*
d*x)*(96*a + 55*b))/(17*d))/(120*exp(4*c + 4*d*x) - 16*exp(2*c + 2*d*x) - 5
60*exp(6*c + 6*d*x) + 1820*exp(8*c + 8*d*x) - 4368*exp(10*c + 10*d*x) + 800
8*exp(12*c + 12*d*x) - 11440*exp(14*c + 14*d*x) + 12870*exp(16*c + 16*d*x)
- 11440*exp(18*c + 18*d*x) + 8008*exp(20*c + 20*d*x) - 4368*exp(22*c + 22*d
*x) + 1820*exp(24*c + 24*d*x) - 560*exp(26*c + 26*d*x) + 120*exp(28*c + 28*
d*x) - 16*exp(30*c + 30*d*x) + exp(32*c + 32*d*x) + 1) - ((12*b*(448*a*b +
256*a^2 + 165*b^2))/(17017*d) - (96*b^3*exp(6*c + 6*d*x))/(119*d) + (4*b^3*
exp(8*c + 8*d*x))/(17*d) + (144*b^2*exp(4*c + 4*d*x)*(8*a + 11*b))/(1547*d)
 - (16*b^2*exp(2*c + 2*d*x)*(96*a + 55*b))/(1547*d))/(7*exp(2*c + 2*d*x) -
21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) - 35*exp(8*c + 8*d*x) + 21*exp(10
*c + 10*d*x) - 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) - 1) + ((48*b*(112
*a*b + 128*a^2 + 33*b^2))/(12155*d) - (64*exp(2*c + 2*d*x)*(840*a*b^2 + 115
2*a^2*b + 1024*a^3 + 231*b^3))/(12155*d) + (288*b^3*exp(12*c + 12*d*x))/(85
*d) - (48*b^3*exp(14*c + 14*d*x))/(85*d) + (1728*b*exp(4*c + 4*d*x)*(112*a*
b + 128*a^2 + 33*b^2))/(12155*d) - (144*b*exp(6*c + 6*d*x)*(448*a*b + 256*a
^2 + 165*b^2))/(2431*d) - (864*b^2*exp(10*c + 10*d*x)*(8*a + 11*b))/(1105*d
) + (48*b^2*exp(8*c + 8*d*x)*(96*a + 55*b))/(221*d))/(45*exp(4*c + 4*d*x) -
 10*exp(2*c + 2*d*x) - 120*exp(6*c + 6*d*x) + 210*exp(8*c + 8*d*x) - 252*ex
p(10*c + 10*d*x) + 210*exp(12*c + 12*d*x) - 120*exp(14*c + 14*d*x) + 45*exp
(16*c + 16*d*x) - 10*exp(18*c + 18*d*x) + exp(20*c + 20*d*x) + 1) - ((4*b^3
)/(255*d) + (448*exp(12*c + 12*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 23
1*b^3))/(255*d) - (48*b^3*exp(2*c + 2*d*x))/(85*d) - (1248*b^3*exp(22*c + 2
2*d*x))/(85*d) + (364*b^3*exp(24*c + 24*d*x))/(255*d) - (672*b*exp(10*c + 1
0*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(85*d) - (1152*b*exp(14*c + 14*d*x)*(1
12*a*b + 128*a^2 + 33*b^2))/(85*d) + (12*b*exp(8*c + 8*d*x)*(448*a*b + 256*
a^2 + 165*b^2))/(17*d) + (36*b*exp(16*c + 16*d*x)*(448*a*b + 256*a^2 + 165*
b^2))/(17*d) + (48*b^2*exp(4*c + 4*d*x)*(8*a + 11*b))/(85*d) + (528*b^2*exp
(20*c + 20*d*x)*(8*a + 11*b))/(85*d) - (32*b^2*exp(6*c + 6*d*x)*(96*a + 55*
b))/(51*d) - (176*b^2*exp(18*c + 18*d*x)*(96*a + 55*b))/(51*d))/(15*exp(2*c
 + 2*d*x) - 105*exp(4*c + 4*d*x) + 455*exp(6*c + 6*d*x) - 1365*exp(8*c + 8*
d*x) + 3003*exp(10*c + 10*d*x) - 5005*exp(12*c + 12*d*x) + 6435*exp(14*c +
14*d*x) - 6435*exp(16*c + 16*d*x) + 5005*exp(18*c + 18*d*x) - 3003*exp(20*c
 + 20*d*x) + 1365*exp(22*c + 22*d*x) - 455*exp(24*c + 24*d*x) + 105*exp(26*
c + 26*d*x) - 15*exp(28*c + 28*d*x) + exp(30*c + 30*d*x) - 1) - ((128*exp(1

```
6*c + 16*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(17*d) + (32*b
^3*exp(4*c + 4*d*x))/(17*d) - (384*b^3*exp(6*c + 6*d*x))/(17*d) - (384*b^3*
exp(26*c + 26*d*x))/(17*d) + (32*b^3*exp(28*c + 28*d*x))/(17*d) - (768*b*ex
p(14*c + 14*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(17*d) - (768*b*exp(18*c + 1
8*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(17*d) + (96*b*exp(12*c + 12*d*x)*(448
*a*b + 256*a^2 + 165*b^2))/(17*d) + (96*b*exp(20*c + 20*d*x)*(448*a*b + 256
*a^2 + 165*b^2))/(17*d) + (192*b^2*exp(8*c + 8*d*x)*(8*a + 11*b))/(17*d) +
(192*b^2*exp(24*c + 24*d*x)*(8*a + 11*b))/(17*d) - (128*b^2*exp(10*c + 10*d
*x)*(96*a + 55*b))/(17*d) - (128*b^2*exp(22*c + 22*d*x)*(96*a + 55*b))/(17*
d))/(17*exp(2*c + 2*d*x) - 136*exp(4*c + 4*d*x) + 680*exp(6*c + 6*d*x) - 23
80*exp(8*c + 8*d*x) + 6188*exp(10*c + 10*d*x) - 12376*exp(12*c + 12*d*x) +
19448*exp(14*c + 14*d*x) - 24310*exp(16*c + 16*d*x) + 24310*exp(18*c + 18*d
*x) - 19448*exp(20*c + 20*d*x) + 12376*exp(22*c + 22*d*x) - 6188*exp(24*c +
 24*d*x) + 2380*exp(26*c + 26*d*x) - 680*exp(28*c + 28*d*x) + 136*exp(30*c
+ 30*d*x) - 17*exp(32*c + 32*d*x) + exp(34*c + 34*d*x) - 1) - ((8*b^3*exp(4
*c + 4*d*x))/(85*d) - (96*b^3*exp(2*c + 2*d*x))/(595*d) + (48*b^2*(8*a + 11
*b))/(7735*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d
*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) + ((48*b*(112*a*b + 128*
a^2 + 33*b^2))/(12155*d) + (24*b^3*exp(8*c + 8*d*x))/(17*d) - (28*b^3*exp(1
0*c + 10*d*x))/(85*d) - (12*b*exp(2*c + 2*d*x)*(448*a*b + 256*a^2 + 165*b^2
))/(2431*d) - (48*b^2*exp(6*c + 6*d*x)*(8*a + 11*b))/(221*d) + (8*b^2*exp(4
*c + 4*d*x)*(96*a + 55*b))/(221*d))/(28*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*
x) - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28
*exp(12*c + 12*d*x) - 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1) - ((12
*b*(448*a*b + 256*a^2 + 165*b^2))/(17017*d) + (64*exp(4*c + 4*d*x)*(840*a*b
^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(2431*d) - (576*b^3*exp(14*c + 14*d*
x))/(119*d) + (12*b^3*exp(16*c + 16*d*x))/(17*d) - (96*b*exp(2*c + 2*d*x)*(
112*a*b + 128*a^2 + 33*b^2))/(2431*d) - (1152*b*exp(6*c + 6*d*x)*(112*a*b +
 128*a^2 + 33*b^2))/(2431*d) + (360*b*exp(8*c + 8*d*x)*(448*a*b + 256*a^2 +
 165*b^2))/(2431*d) + (288*b^2*exp(12*c + 12*d*x)*(8*a + 11*b))/(221*d) - (
96*b^2*exp(10*c + 10*d*x)*(96*a + 55*b))/(221*d))/(11*exp(2*c + 2*d*x) - 55
*exp(4*c + 4*d*x) + 165*exp(6*c + 6*d*x) - 330*exp(8*c + 8*d*x) + 462*exp(1
0*c + 10*d*x) - 462*exp(12*c + 12*d*x) + 330*exp(14*c + 14*d*x) - 165*exp(1
6*c + 16*d*x) + 55*exp(18*c + 18*d*x) - 11*exp(20*c + 20*d*x) + exp(22*c +
22*d*x) - 1) - (4*b^3)/(255*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + ex
p(6*c + 6*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**18*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.228 $\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=248

$$-\frac{a^3\coth^{19}(c+dx)}{19d}+\frac{9a^3\coth^{17}(c+dx)}{17d}-\frac{3a\left(42a^2+21ab+b^2\right)\coth^{11}(c+dx)}{11d}+\frac{a\left(42a^2+35ab+5b^2\right)\coth^9(\cdots}{3d}$$

```
[Out] (a+b)^3*coth(d*x+c)/d-(a+b)^2*(3*a+b)*coth(d*x+c)^3/d+3/5*(a+b)*(12*a^2+9*a
*b+b^2)*coth(d*x+c)^5/d-1/7*(84*a^3+105*a^2*b+30*a*b^2+b^3)*coth(d*x+c)^7/d
+1/3*a*(42*a^2+35*a*b+5*b^2)*coth(d*x+c)^9/d-3/11*a*(42*a^2+21*a*b+b^2)*cot
h(d*x+c)^11/d+21/13*a^2*(4*a+b)*coth(d*x+c)^13/d-1/5*a^2*(12*a+b)*coth(d*x+
c)^15/d+9/17*a^3*coth(d*x+c)^17/d-1/19*a^3*coth(d*x+c)^19/d
```

Rubi [A] time = 0.22, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3217, 1261}

$$-\frac{3a\left(42a^2+21ab+b^2\right)\coth^{11}(c+dx)}{11d}+\frac{a\left(42a^2+35ab+5b^2\right)\coth^9(c+dx)}{3d}-\frac{\left(105a^2b+84a^3+30ab^2+b^3\right)\cdots}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^20*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*Coth[c + d*x])/d - ((a + b)^2*(3*a + b)*Coth[c + d*x]^3)/d + (3*
(a + b)*(12*a^2 + 9*a*b + b^2)*Coth[c + d*x]^5)/(5*d) - ((84*a^3 + 105*a^2*
b + 30*a*b^2 + b^3)*Coth[c + d*x]^7)/(7*d) + (a*(42*a^2 + 35*a*b + 5*b^2)*C
oth[c + d*x]^9)/(3*d) - (3*a*(42*a^2 + 21*a*b + b^2)*Coth[c + d*x]^11)/(11*
d) + (21*a^2*(4*a + b)*Coth[c + d*x]^13)/(13*d) - (a^2*(12*a + b)*Coth[c +
d*x]^15)/(5*d) + (9*a^3*Coth[c + d*x]^17)/(17*d) - (a^3*Coth[c + d*x]^19)/(
19*d)
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3\left(a-2ax^2+(a+b)x^4\right)^3}{x^{20}}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{20}}-\frac{9a^3}{x^{18}}+\frac{3a^2(12a+b)}{x^{16}}-\frac{21a^2(4a+b)}{x^{14}}+\frac{3a\left(42a^2+21ab+b^2\right)}{x^{12}}-\cdots\right.\right.}{\cdots}\\ &= \frac{(a+b)^3\coth(c+dx)}{d}-\frac{(a+b)^2(3a+b)\coth^3(c+dx)}{d}+\frac{3(a+\cdots}{\cdots}\end{aligned}$$

Mathematica [B] time = 6.18, size = 548, normalized size = 2.21

$$\frac{\operatorname{csch}^{19}(c+dx)\left(-7945986048a^3\cosh(c+dx)+6501261312a^3\cosh(3(c+dx))-4334174208a^3\cosh(5(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^20*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-7945986048*a^3*Cosh[c + d*x] - 8939234304*a^2*b*Cosh[c + d*x] - 65181916
80*a*b^2*Cosh[c + d*x] - 1792502712*b^3*Cosh[c + d*x] + 6501261312*a^3*Cosh
[3*(c + d*x)] + 18149354496*a^2*b*Cosh[3*(c + d*x)] + 14814072000*a*b^2*Cos
h[3*(c + d*x)] + 4260103848*b^3*Cosh[3*(c + d*x)] - 4334174208*a^3*Cosh[5*(
c + d*x)] - 14582690304*a^2*b*Cosh[5*(c + d*x)] - 14221509120*a*b^2*Cosh[5*
(c + d*x)] - 4440518082*b^3*Cosh[5*(c + d*x)] + 2333786112*a^3*Cosh[7*(c +
d*x)] + 7852217856*a^2*b*Cosh[7*(c + d*x)] + 8803791360*a*b^2*Cosh[7*(c + d
*x)] + 3047642598*b^3*Cosh[7*(c + d*x)] - 1000194048*a^3*Cosh[9*(c + d*x)]
- 3365236224*a^2*b*Cosh[9*(c + d*x)] - 3906077760*a*b^2*Cosh[9*(c + d*x)] -
 1489040982*b^3*Cosh[9*(c + d*x)] + 333398016*a^3*Cosh[11*(c + d*x)] + 1121
745408*a^2*b*Cosh[11*(c + d*x)] + 1302025920*a*b^2*Cosh[11*(c + d*x)] + 527
386002*b^3*Cosh[11*(c + d*x)] - 83349504*a^3*Cosh[13*(c + d*x)] - 280436352
*a^2*b*Cosh[13*(c + d*x)] - 325506480*a*b^2*Cosh[13*(c + d*x)] - 134271423*
b^3*Cosh[13*(c + d*x)] + 14708736*a^3*Cosh[15*(c + d*x)] + 49488768*a^2*b*C
osh[15*(c + d*x)] + 57442320*a*b^2*Cosh[15*(c + d*x)] + 23694957*b^3*Cosh[1
5*(c + d*x)] - 1634304*a^3*Cosh[17*(c + d*x)] - 5498752*a^2*b*Cosh[17*(c +
d*x)] - 6382480*a*b^2*Cosh[17*(c + d*x)] - 2632773*b^3*Cosh[17*(c + d*x)] +
 86016*a^3*Cosh[19*(c + d*x)] + 289408*a^2*b*Cosh[19*(c + d*x)] + 335920*a*
b^2*Cosh[19*(c + d*x)] + 138567*b^3*Cosh[19*(c + d*x)])*Csch[c + d*x]^19)/(
79459860480*d)
```

fricas [B] time = 0.55, size = 4259, normalized size = 17.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 64/4849845*((43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*
x + c)^15 + 15*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh
(d*x + c)*sinh(d*x + c)^14 - 2*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 124
7103*b^3)*sinh(d*x + c)^15 - 19*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 -
1538823*b^3)*cosh(d*x + c)^13 + 2*(408576*a^3 + 1374688*a^2*b + 1595620*a*b
^2 + 15935205*b^3 - 105*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(35*(43008*a^3 + 144704*a^2*b + 1
67960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^3 - 19*(43008*a^3 + 144704*a^2*b +
 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 + 57*(129024*a
^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^11 - 6*(455*(
21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^4 + 1225
728*a^3 + 4124064*a^2*b + 17719780*a*b^2 + 31639465*b^3 - 494*(21504*a^3 +
72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 +
 11*(273*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x +
 c)^5 - 494*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*
x + c)^3 + 57*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh
(d*x + c))*sinh(d*x + c)^10 - 969*(43008*a^3 + 144704*a^2*b - 529880*a*b^2
- 586443*b^3)*cosh(d*x + c)^9 - 2*(5005*(21504*a^3 + 72352*a^2*b + 83980*a*
b^2 + 1247103*b^3)*cosh(d*x + c)^6 - 13585*(21504*a^3 + 72352*a^2*b + 83980
*a*b^2 + 838695*b^3)*cosh(d*x + c)^4 - 20837376*a^3 - 70109088*a^2*b - 4194
80100*a*b^2 - 351267345*b^3 + 3135*(64512*a^3 + 217056*a^2*b + 932620*a*b^2
 + 1665235*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(715*(43008*a^3 + 1447
04*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^7 - 2717*(43008*a^3 +
```

144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^5 + 1045*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^3 - 969*(43008*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 6783*(24576*a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x + c)^7 - 6*(2145*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^8 - 10868*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d*x + c)^6 + 6270*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^3)*cosh(d*x + c)^4 + 27783168*a^3 + 248673824*a^2*b + 549145220*a*b^2 + 303230785*b^3 - 11628*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + (5005*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^9 - 32604*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^7 + 26334*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^5 - 81396*(43008*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c)^3 + 47481*(24576*a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 323*(1548288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3)*cosh(d*x + c)^5 - 2*(3003*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^10 - 24453*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d*x + c)^8 + 26334*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^3)*cosh(d*x + c)^6 - 122094*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^3)*cosh(d*x + c)^4 - 250048512*a^3 - 3065771296*a^2*b - 4040697700*a*b^2 - 1763542209*b^3 + 20349*(86016*a^3 + 769888*a^2*b + 1700140*a*b^2 + 938795*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (1365*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^11 - 13585*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^9 + 18810*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^7 - 122094*(43008*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c)^5 + 237405*(24576*a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x + c)^3 - 1615*(1548288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 323*(8687616*a^3 + 15456448*a^2*b + 15194920*a*b^2 + 6026163*b^3)*cosh(d*x + c)^3 - 2*(455*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^12 - 5434*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d*x + c)^10 + 9405*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^3)*cosh(d*x + c)^8 - 81396*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^3)*cosh(d*x + c)^6 + 33915*(86016*a^3 + 769888*a^2*b + 1700140*a*b^2 + 938795*b^3)*cosh(d*x + c)^4 + 2569943040*a^3 + 6422325280*a^2*b + 6933472780*a*b^2 + 2675035935*b^3 - 3230*(774144*a^3 + 9491552*a^2*b + 12509900*a*b^2 + 5459883*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (105*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^13 - 1482*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^11 + 3135*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^9 - 34884*(43008*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c)^7 + 142443*(24576*a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x + c)^5 - 3230*(1548288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3)*cosh(d*x + c)^3 - 969*(8687616*a^3 + 15456448*a^2*b + 15194920*a*b^2 + 6026163*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 12597*(86016*a^3 + 215488*a^2*b + 179720*a*b^2 + 65703*b^3)*cosh(d*x + c) - 2*(15*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^14 - 247*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d*x + c)^12 + 627*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^3)*cosh(d*x + c)^10 - 8721*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^3)*cosh(d*x + c)^8 + 6783*(86016*a^3 + 769888*a^2*b + 1700140*a*b^2 + 938795*b^3)*cosh(d*x + c)^6 - 1615*(774144*a^3 + 9491552*a^2*b + 12509900*a*b^2 + 5459883*b^3)*cosh(d*x + c)^4 - 2708858880*a^3 - 8648596320*a^2*b - 8918927940*a*b^2 - 3269488365*b^3 + 4845*(1591296*a^3 + 3976672*a^2*b + 4293172*a*b^2 + 1656369*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^23 + 23*d*cosh(d*x + c)*sinh(d*x + c)^22 + d*sinh(d*x + c)^23 - 19*d*cosh(d*x + c)^21 + (253*d*cosh(d*x + c)^2 - 19*d)*sinh(d*x + c)^21 + 7*(253*d*cosh(d*x + c)^3 - 57*d*cosh(d*x + c))*sinh(d*x + c)^20 + 171*d*cosh(d*x + c)^19 + (8855*d*cosh(d*x + c)^4 - 3990*d*cosh(d*x

```
 + c)^2 + 171*d)*sinh(d*x + c)^19 + 19*(1771*d*cosh(d*x + c)^5 - 1330*d*cos
h(d*x + c)^3 + 171*d*cosh(d*x + c))*sinh(d*x + c)^18 - 969*d*cosh(d*x + c)^
17 + 57*(1771*d*cosh(d*x + c)^6 - 1995*d*cosh(d*x + c)^4 + 513*d*cosh(d*x +
 c)^2 - 17*d)*sinh(d*x + c)^17 + 969*(253*d*cosh(d*x + c)^7 - 399*d*cosh(d*
x + c)^5 + 171*d*cosh(d*x + c)^3 - 17*d*cosh(d*x + c))*sinh(d*x + c)^16 + 3
875*d*cosh(d*x + c)^15 + (490314*d*cosh(d*x + c)^8 - 1031016*d*cosh(d*x + c
)^6 + 662796*d*cosh(d*x + c)^4 - 131784*d*cosh(d*x + c)^2 + 3877*d)*sinh(d*
x + c)^15 + (817190*d*cosh(d*x + c)^9 - 2209320*d*cosh(d*x + c)^7 + 1988388
*d*cosh(d*x + c)^5 - 658920*d*cosh(d*x + c)^3 + 58125*d*cosh(d*x + c))*sinh
(d*x + c)^14 - 11609*d*cosh(d*x + c)^13 + (1144066*d*cosh(d*x + c)^10 - 386
6310*d*cosh(d*x + c)^8 + 4639572*d*cosh(d*x + c)^6 - 2306220*d*cosh(d*x + c
)^4 + 407085*d*cosh(d*x + c)^2 - 11647*d)*sinh(d*x + c)^13 + 13*(104006*d*c
osh(d*x + c)^11 - 429590*d*cosh(d*x + c)^9 + 662796*d*cosh(d*x + c)^7 - 461
244*d*cosh(d*x + c)^5 + 135625*d*cosh(d*x + c)^3 - 11609*d*cosh(d*x + c))*s
inh(d*x + c)^12 + 26961*d*cosh(d*x + c)^11 + (1352078*d*cosh(d*x + c)^12 -
6701604*d*cosh(d*x + c)^10 + 12924522*d*cosh(d*x + c)^8 - 11992344*d*cosh(d
*x + c)^6 + 5292105*d*cosh(d*x + c)^4 - 908466*d*cosh(d*x + c)^2 + 27303*d)
*sinh(d*x + c)^11 + (1144066*d*cosh(d*x + c)^13 - 6701604*d*cosh(d*x + c)^1
1 + 15796638*d*cosh(d*x + c)^9 - 18845112*d*cosh(d*x + c)^7 + 11636625*d*co
sh(d*x + c)^5 - 3320174*d*cosh(d*x + c)^3 + 296571*d*cosh(d*x + c))*sinh(d*
x + c)^10 - 49419*d*cosh(d*x + c)^9 + (817190*d*cosh(d*x + c)^14 - 5584670*
d*cosh(d*x + c)^12 + 15796638*d*cosh(d*x + c)^10 - 23556390*d*cosh(d*x + c)
^8 + 19404385*d*cosh(d*x + c)^6 - 8327605*d*cosh(d*x + c)^4 + 1501665*d*cos
h(d*x + c)^2 - 51357*d)*sinh(d*x + c)^9 + 3*(163438*d*cosh(d*x + c)^15 - 12
88770*d*cosh(d*x + c)^13 + 4308174*d*cosh(d*x + c)^11 - 7852130*d*cosh(d*x
 + c)^9 + 8311875*d*cosh(d*x + c)^7 - 4980261*d*cosh(d*x + c)^5 + 1482855*d*
cosh(d*x + c)^3 - 148257*d*cosh(d*x + c))*sinh(d*x + c)^8 + 71706*d*cosh(d*
x + c)^7 + 3*(81719*d*cosh(d*x + c)^16 - 736440*d*cosh(d*x + c)^14 + 287211
6*d*cosh(d*x + c)^12 - 6281704*d*cosh(d*x + c)^10 + 8316165*d*cosh(d*x + c)
^8 - 6662084*d*cosh(d*x + c)^6 + 3003330*d*cosh(d*x + c)^4 - 616284*d*cosh(
d*x + c)^2 + 26486*d)*sinh(d*x + c)^7 + (100947*d*cosh(d*x + c)^17 - 103101
6*d*cosh(d*x + c)^15 + 4639572*d*cosh(d*x + c)^13 - 11992344*d*cosh(d*x + c
)^11 + 19394375*d*cosh(d*x + c)^9 - 19921044*d*cosh(d*x + c)^7 + 12455982*d
*cosh(d*x + c)^5 - 4151196*d*cosh(d*x + c)^3 + 501942*d*cosh(d*x + c))*sinh
(d*x + c)^6 - 80750*d*cosh(d*x + c)^5 + (33649*d*cosh(d*x + c)^18 - 386631*
d*cosh(d*x + c)^16 + 1988388*d*cosh(d*x + c)^14 - 5996172*d*cosh(d*x + c)^1
2 + 11642631*d*cosh(d*x + c)^10 - 14989689*d*cosh(d*x + c)^8 + 12613986*d*c
osh(d*x + c)^6 - 6470982*d*cosh(d*x + c)^4 + 1668618*d*cosh(d*x + c)^2 - 10
4006*d)*sinh(d*x + c)^5 + (8855*d*cosh(d*x + c)^19 - 113715*d*cosh(d*x + c)
^17 + 662796*d*cosh(d*x + c)^15 - 2306220*d*cosh(d*x + c)^13 + 5289375*d*co
sh(d*x + c)^11 - 8300435*d*cosh(d*x + c)^9 + 8897130*d*cosh(d*x + c)^7 - 62
26794*d*cosh(d*x + c)^5 + 2509710*d*cosh(d*x + c)^3 - 403750*d*cosh(d*x + c
))*sinh(d*x + c)^4 + 65246*d*cosh(d*x + c)^3 + (1771*d*cosh(d*x + c)^20 - 2
5270*d*cosh(d*x + c)^18 + 165699*d*cosh(d*x + c)^16 - 658920*d*cosh(d*x + c
)^14 + 1764035*d*cosh(d*x + c)^12 - 3331042*d*cosh(d*x + c)^10 + 4504995*d*
cosh(d*x + c)^8 - 4313988*d*cosh(d*x + c)^6 + 2781030*d*cosh(d*x + c)^4 - 1
040060*d*cosh(d*x + c)^2 + 119510*d)*sinh(d*x + c)^3 + (253*d*cosh(d*x + c)
^21 - 3990*d*cosh(d*x + c)^19 + 29241*d*cosh(d*x + c)^17 - 131784*d*cosh(d*
x + c)^15 + 406875*d*cosh(d*x + c)^13 - 905502*d*cosh(d*x + c)^11 + 1482855
*d*cosh(d*x + c)^9 - 1779084*d*cosh(d*x + c)^7 + 1505826*d*cosh(d*x + c)^5
 - 807500*d*cosh(d*x + c)^3 + 195738*d*cosh(d*x + c))*sinh(d*x + c)^2 - 2519
4*d*cosh(d*x + c) + (23*d*cosh(d*x + c)^22 - 399*d*cosh(d*x + c)^20 + 3249*
d*cosh(d*x + c)^18 - 16473*d*cosh(d*x + c)^16 + 58155*d*cosh(d*x + c)^14 -
151411*d*cosh(d*x + c)^12 + 300333*d*cosh(d*x + c)^10 - 462213*d*cosh(d*x +
 c)^8 + 556206*d*cosh(d*x + c)^6 - 520030*d*cosh(d*x + c)^4 + 358530*d*cosh
(d*x + c)^2 - 125970*d)*sinh(d*x + c))
```

giac [B] time = 0.57, size = 737, normalized size = 2.97

$$-\frac{32\left(4849845\,b^3e^{(30\,dx+30\,c)}-61108047\,b^3e^{(28\,dx+28\,c)}+155195040\,ab^2e^{(26\,dx+26\,c)}+355978623\,b^3e^{(26\,dx+26\,c)}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -32/4849845*(4849845*b^3*e^(30*d*x + 30*c) - 61108047*b^3*e^(28*d*x + 28*c)
 + 155195040*a*b^2*e^(26*d*x + 26*c) + 355978623*b^3*e^(26*d*x + 26*c) - 13
52413920*a*b^2*e^(24*d*x + 24*c) - 1270797957*b^3*e^(24*d*x + 24*c) + 18623
40480*a^2*b*e^(22*d*x + 22*c) + 5287716720*a*b^2*e^(22*d*x + 22*c) + 310653
3573*b^3*e^(22*d*x + 22*c) - 8897848960*a^2*b*e^(20*d*x + 20*c) - 122567130
40*a*b^2*e^(20*d*x + 20*c) - 5504019807*b^3*e^(20*d*x + 20*c) + 7945986048*
a^3*e^(18*d*x + 18*c) + 17837083264*a^2*b*e^(18*d*x + 18*c) + 18774904720*a
*b^2*e^(18*d*x + 18*c) + 7296522519*b^3*e^(18*d*x + 18*c) - 6501261312*a^3*
e^(16*d*x + 16*c) - 20011694976*a^2*b*e^(16*d*x + 16*c) - 20101788720*a*b^2
*e^(16*d*x + 16*c) - 7366637421*b^3*e^(16*d*x + 16*c) + 4334174208*a^3*e^(1
4*d*x + 14*c) + 14582690304*a^2*b*e^(14*d*x + 14*c) + 15573923040*a*b^2*e^(
14*d*x + 14*c) + 5711316039*b^3*e^(14*d*x + 14*c) - 2333786112*a^3*e^(12*d*
x + 12*c) - 7852217856*a^2*b*e^(12*d*x + 12*c) - 8958986400*a*b^2*e^(12*d*x
 + 12*c) - 3403621221*b^3*e^(12*d*x + 12*c) + 1000194048*a^3*e^(10*d*x + 10
*c) + 3365236224*a^2*b*e^(10*d*x + 10*c) + 3906077760*a*b^2*e^(10*d*x + 10*
c) + 1550149029*b^3*e^(10*d*x + 10*c) - 333398016*a^3*e^(8*d*x + 8*c) - 112
1745408*a^2*b*e^(8*d*x + 8*c) - 1302025920*a*b^2*e^(8*d*x + 8*c) - 53223584
7*b^3*e^(8*d*x + 8*c) + 83349504*a^3*e^(6*d*x + 6*c) + 280436352*a^2*b*e^(6
*d*x + 6*c) + 325506480*a*b^2*e^(6*d*x + 6*c) + 134271423*b^3*e^(6*d*x + 6*
c) - 14708736*a^3*e^(4*d*x + 4*c) - 49488768*a^2*b*e^(4*d*x + 4*c) - 574423
20*a*b^2*e^(4*d*x + 4*c) - 23694957*b^3*e^(4*d*x + 4*c) + 1634304*a^3*e^(2*
d*x + 2*c) + 5498752*a^2*b*e^(2*d*x + 2*c) + 6382480*a*b^2*e^(2*d*x + 2*c)
+ 2632773*b^3*e^(2*d*x + 2*c) - 86016*a^3 - 289408*a^2*b - 335920*a*b^2 - 1
38567*b^3)/(d*(e^(2*d*x + 2*c) - 1)^19)
```

maple [A] time = 0.14, size = 298, normalized size = 1.20

$$\frac{a^3\left(\frac{65536}{230945}-\frac{\mathrm{csch}(dx+c)^{18}}{19}+\frac{18\mathrm{csch}(dx+c)^{16}}{323}-\frac{96\mathrm{csch}(dx+c)^{14}}{1615}+\frac{1344\mathrm{csch}(dx+c)^{12}}{20995}-\frac{16128\mathrm{csch}(dx+c)^{10}}{230945}+\frac{3584\mathrm{csch}(dx+c)^{8}}{46189}-\frac{409}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(65536/230945-1/19*csch(d*x+c)^18+18/323*csch(d*x+c)^16-96/1615*cs
ch(d*x+c)^14+1344/20995*csch(d*x+c)^12-16128/230945*csch(d*x+c)^10+3584/461
89*csch(d*x+c)^8-4096/46189*csch(d*x+c)^6+24576/230945*csch(d*x+c)^4-32768/
230945*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(2048/6435-1/15*csch(d*x+c)^14+14
/195*csch(d*x+c)^12-56/715*csch(d*x+c)^10+112/1287*csch(d*x+c)^8-128/1287*c
sch(d*x+c)^6+256/2145*csch(d*x+c)^4-1024/6435*csch(d*x+c)^2)*coth(d*x+c)+3*
a*b^2*(256/693-1/11*csch(d*x+c)^10+10/99*csch(d*x+c)^8-80/693*csch(d*x+c)^6
+32/231*csch(d*x+c)^4-128/693*csch(d*x+c)^2)*coth(d*x+c)+b^3*(16/35-1/7*csc
h(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*coth(d*x+c))
```

maxima [B] time = 0.40, size = 4883, normalized size = 19.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 131072/230945*a^3*(19*e^(-2*d*x - 2*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*
d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*
x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(
-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 755
82*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c)
 - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32
*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) -
 1)) - 171*e^(-4*d*x - 4*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c)
+ 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) -
 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 1
6*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d
*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^
(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e
^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) + 969*
e^(-6*d*x - 6*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6
*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-
12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 9237
8*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c)
- 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x -
28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x -
 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) - 3876*e^(-8*d*x
- 8*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c
) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 1
2*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d
*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^
(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 38
76*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 1
9*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) + 11628*e^(-10*d*x - 10*c)/
(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 387
6*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) +
50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18
*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*
x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-
30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-3
6*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) - 27132*e^(-12*d*x - 12*c)/(d*(19*
e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8
*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e
^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 9
2378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*
c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x
- 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x -
 36*c) + e^(-38*d*x - 38*c) - 1)) + 50388*e^(-14*d*x - 14*c)/(d*(19*e^(-2*d
*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x -
8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d
*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^
(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27
132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c)
 - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c)
+ e^(-38*d*x - 38*c) - 1)) - 75582*e^(-16*d*x - 16*c)/(d*(19*e^(-2*d*x - 2*
c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) +
11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14
*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*
x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(
-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*
e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-3
8*d*x - 38*c) - 1)) + 92378*e^(-18*d*x - 18*c)/(d*(19*e^(-2*d*x - 2*c) - 17
1*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e
^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 7
5582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*
```

c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) - 1/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1))) + 4096/2145*a^2*b*(15*e^(-2*d*x - 2*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 105*e^(-4*d*x - 4*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 455*e^(-6*d*x - 6*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 1365*e^(-8*d*x - 8*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 3003*e^(-10*d*x - 10*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 5005*e^(-12*d*x - 12*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 6435*e^(-14*d*x - 14*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 1/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1))) + 512/231*a*b^2*(11*e^(-2*d*x - 2*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 55*e^(-4*d*x - 4*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*

```
x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x
- 20*c) + e^(-22*d*x - 22*c) - 1)) + 165*e^(-6*d*x - 6*c)/(d*(11*e^(-2*d*x
- 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c)
+ 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c)
- 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) +
e^(-22*d*x - 22*c) - 1)) - 330*e^(-8*d*x - 8*c)/(d*(11*e^(-2*d*x - 2*c) - 5
5*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-
10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-
16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x
 - 22*c) - 1)) + 462*e^(-10*d*x - 10*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*
d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x -
 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x -
 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c)
 - 1)) - 1/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x -
6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*
c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c
) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1))) + 32/35*b^3*(7*e^(-2*
d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*
c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e
^(-14*d*x - 14*c) - 1)) - 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e
^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
 - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) + 35*e^(-6*d*x -
 6*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) -
35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14
*d*x - 14*c) - 1)) - 1/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^
(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x
 - 12*c) + e^(-14*d*x - 14*c) - 1)))
```

mupad [B] time = 1.42, size = 5190, normalized size = 20.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^4)^3/sinh(c + d*x)^20,x)
```

```
[Out] ((512*b*(112*a*b + 128*a^2 + 33*b^2))/(138567*d) + (1792*b^3*exp(8*c + 8*d*
x))/(969*d) - (448*b^3*exp(10*c + 10*d*x))/(969*d) - (64*b*exp(2*c + 2*d*x)
*(448*a*b + 256*a^2 + 165*b^2))/(12597*d) - (256*b^2*exp(6*c + 6*d*x)*(8*a
+ 11*b))/(969*d) + (512*b^2*exp(4*c + 4*d*x)*(96*a + 55*b))/(12597*d))/(9*e
xp(2*c + 2*d*x) - 36*exp(4*c + 4*d*x) + 84*exp(6*c + 6*d*x) - 126*exp(8*c +
 8*d*x) + 126*exp(10*c + 10*d*x) - 84*exp(12*c + 12*d*x) + 36*exp(14*c + 14
*d*x) - 9*exp(16*c + 16*d*x) + exp(18*c + 18*d*x) - 1) - ((8*b*(448*a*b + 2
56*a^2 + 165*b^2))/(12597*d) + (128*exp(4*c + 4*d*x)*(840*a*b^2 + 1152*a^2*
b + 1024*a^3 + 231*b^3))/(4199*d) - (2816*b^3*exp(14*c + 14*d*x))/(323*d) +
 (440*b^3*exp(16*c + 16*d*x))/(323*d) - (512*b*exp(2*c + 2*d*x)*(112*a*b +
128*a^2 + 33*b^2))/(12597*d) - (2560*b*exp(6*c + 6*d*x)*(112*a*b + 128*a^2
+ 33*b^2))/(4199*d) + (880*b*exp(8*c + 8*d*x)*(448*a*b + 256*a^2 + 165*b^2)
)/(4199*d) + (704*b^2*exp(12*c + 12*d*x)*(8*a + 11*b))/(323*d) - (2816*b^2*
exp(10*c + 10*d*x)*(96*a + 55*b))/(4199*d))/(66*exp(4*c + 4*d*x) - 12*exp(2
*c + 2*d*x) - 220*exp(6*c + 6*d*x) + 495*exp(8*c + 8*d*x) - 792*exp(10*c +
10*d*x) + 924*exp(12*c + 12*d*x) - 792*exp(14*c + 14*d*x) + 495*exp(16*c +
16*d*x) - 220*exp(18*c + 18*d*x) + 66*exp(20*c + 20*d*x) - 12*exp(22*c + 22
*d*x) + exp(24*c + 24*d*x) + 1) - ((512*exp(18*c + 18*d*x)*(840*a*b^2 + 115
2*a^2*b + 1024*a^3 + 231*b^3))/(19*d) + (128*b^3*exp(6*c + 6*d*x))/(19*d) -
 (1536*b^3*exp(8*c + 8*d*x))/(19*d) - (1536*b^3*exp(28*c + 28*d*x))/(19*d)
+ (128*b^3*exp(30*c + 30*d*x))/(19*d) - (3072*b*exp(16*c + 16*d*x)*(112*a*b
 + 128*a^2 + 33*b^2))/(19*d) - (3072*b*exp(20*c + 20*d*x)*(112*a*b + 128*a^
2 + 33*b^2))/(19*d) + (384*b*exp(14*c + 14*d*x)*(448*a*b + 256*a^2 + 165*b^
2))/(19*d) + (384*b*exp(22*c + 22*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(19*d
) + (768*b^2*exp(10*c + 10*d*x)*(8*a + 11*b))/(19*d) + (768*b^2*exp(26*c +
```

26*d*x)*(8*a + 11*b))/(19*d) - (512*b^2*exp(12*c + 12*d*x)*(96*a + 55*b))/(
19*d) - (512*b^2*exp(24*c + 24*d*x)*(96*a + 55*b))/(19*d))/(19*exp(2*c + 2*
d*x) - 171*exp(4*c + 4*d*x) + 969*exp(6*c + 6*d*x) - 3876*exp(8*c + 8*d*x)
+ 11628*exp(10*c + 10*d*x) - 27132*exp(12*c + 12*d*x) + 50388*exp(14*c + 14
*d*x) - 75582*exp(16*c + 16*d*x) + 92378*exp(18*c + 18*d*x) - 92378*exp(20*
c + 20*d*x) + 75582*exp(22*c + 22*d*x) - 50388*exp(24*c + 24*d*x) + 27132*e
xp(26*c + 26*d*x) - 11628*exp(28*c + 28*d*x) + 3876*exp(30*c + 30*d*x) - 96
9*exp(32*c + 32*d*x) + 171*exp(34*c + 34*d*x) - 19*exp(36*c + 36*d*x) + exp
(38*c + 38*d*x) - 1) + ((128*b^3)/(4845*d) - (1792*exp(10*c + 10*d*x)*(840*
a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(1615*d) + (128128*b^3*exp(20*c +
 20*d*x))/(4845*d) - (2912*b^3*exp(22*c + 22*d*x))/(969*d) + (3584*b*exp(8*
c + 8*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(969*d) + (3584*b*exp(12*c + 12*d*
x)*(112*a*b + 128*a^2 + 33*b^2))/(323*d) - (224*b*exp(6*c + 6*d*x)*(448*a*b
 + 256*a^2 + 165*b^2))/(969*d) - (704*b*exp(14*c + 14*d*x)*(448*a*b + 256*a
^2 + 165*b^2))/(323*d) - (64*b^2*exp(2*c + 2*d*x)*(8*a + 11*b))/(969*d) - (
9152*b^2*exp(18*c + 18*d*x)*(8*a + 11*b))/(969*d) + (128*b^2*exp(4*c + 4*d*
x)*(96*a + 55*b))/(969*d) + (1408*b^2*exp(16*c + 16*d*x)*(96*a + 55*b))/(32
3*d))/(15*exp(2*c + 2*d*x) - 105*exp(4*c + 4*d*x) + 455*exp(6*c + 6*d*x) -
1365*exp(8*c + 8*d*x) + 3003*exp(10*c + 10*d*x) - 5005*exp(12*c + 12*d*x) +
 6435*exp(14*c + 14*d*x) - 6435*exp(16*c + 16*d*x) + 5005*exp(18*c + 18*d*x
) - 3003*exp(20*c + 20*d*x) + 1365*exp(22*c + 22*d*x) - 455*exp(24*c + 24*d
*x) + 105*exp(26*c + 26*d*x) - 15*exp(28*c + 28*d*x) + exp(30*c + 30*d*x) -
 1) + ((128*b^3)/(4845*d) - (32*b^3*exp(2*c + 2*d*x))/(969*d))/(5*exp(2*c +
 2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) +
exp(10*c + 10*d*x) - 1) - ((128*exp(8*c + 8*d*x)*(840*a*b^2 + 1152*a^2*b +
1024*a^3 + 231*b^3))/(323*d) - (18304*b^3*exp(18*c + 18*d*x))/(969*d) + (22
88*b^3*exp(20*c + 20*d*x))/(969*d) + (32*b^2*(8*a + 11*b))/(6783*d) - (1024
*b*exp(6*c + 6*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(969*d) - (1536*b*exp(10*
c + 10*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(323*d) + (16*b*exp(4*c + 4*d*x)*
(448*a*b + 256*a^2 + 165*b^2))/(323*d) + (352*b*exp(12*c + 12*d*x)*(448*a*b
 + 256*a^2 + 165*b^2))/(323*d) + (13728*b^2*exp(16*c + 16*d*x)*(8*a + 11*b)
)/(2261*d) - (128*b^2*exp(2*c + 2*d*x)*(96*a + 55*b))/(6783*d) - (5632*b^2*
exp(14*c + 14*d*x)*(96*a + 55*b))/(2261*d))/(91*exp(4*c + 4*d*x) - 14*exp(2
*c + 2*d*x) - 364*exp(6*c + 6*d*x) + 1001*exp(8*c + 8*d*x) - 2002*exp(10*c
+ 10*d*x) + 3003*exp(12*c + 12*d*x) - 3432*exp(14*c + 14*d*x) + 3003*exp(16
*c + 16*d*x) - 2002*exp(18*c + 18*d*x) + 1001*exp(20*c + 20*d*x) - 364*exp(
22*c + 22*d*x) + 91*exp(24*c + 24*d*x) - 14*exp(26*c + 26*d*x) + exp(28*c +
 28*d*x) + 1) + ((128*b^3*exp(4*c + 4*d*x))/(323*d) - (160*b^3*exp(6*c + 6*
d*x))/(969*d) + (128*b^2*(96*a + 55*b))/(88179*d) - (64*b^2*exp(2*c + 2*d*x
)*(8*a + 11*b))/(2261*d))/(7*exp(2*c + 2*d*x) - 21*exp(4*c + 4*d*x) + 35*ex
p(6*c + 6*d*x) - 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) - 7*exp(12*c +
 12*d*x) + exp(14*c + 14*d*x) - 1) - ((128*(840*a*b^2 + 1152*a^2*b + 1024*a
^3 + 231*b^3))/(230945*d) - (5376*b^3*exp(10*c + 10*d*x))/(1615*d) + (224*b
^3*exp(12*c + 12*d*x))/(323*d) - (1536*b*exp(2*c + 2*d*x)*(112*a*b + 128*a^
2 + 33*b^2))/(46189*d) + (96*b*exp(4*c + 4*d*x)*(448*a*b + 256*a^2 + 165*b^
2))/(4199*d) + (192*b^2*exp(8*c + 8*d*x)*(8*a + 11*b))/(323*d) - (512*b^2*e
xp(6*c + 6*d*x)*(96*a + 55*b))/(4199*d))/(45*exp(4*c + 4*d*x) - 10*exp(2*c
+ 2*d*x) - 120*exp(6*c + 6*d*x) + 210*exp(8*c + 8*d*x) - 252*exp(10*c + 10*
d*x) + 210*exp(12*c + 12*d*x) - 120*exp(14*c + 14*d*x) + 45*exp(16*c + 16*d
*x) - 10*exp(18*c + 18*d*x) + exp(20*c + 20*d*x) + 1) - ((512*exp(6*c + 6*d
x)(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(4199*d) - (4224*b^3*exp
(16*c + 16*d*x))/(323*d) + (1760*b^3*exp(18*c + 18*d*x))/(969*d) - (128*b^2
*(96*a + 55*b))/(88179*d) - (1024*b*exp(4*c + 4*d*x)*(112*a*b + 128*a^2 + 3
3*b^2))/(4199*d) - (7680*b*exp(8*c + 8*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(
4199*d) + (32*b*exp(2*c + 2*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(4199*d) +
(2112*b*exp(10*c + 10*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(4199*d) + (8448*
b^2*exp(14*c + 14*d*x)*(8*a + 11*b))/(2261*d) - (5632*b^2*exp(12*c + 12*d*x
)*(96*a + 55*b))/(4199*d))/(13*exp(2*c + 2*d*x) - 78*exp(4*c + 4*d*x) + 286
*exp(6*c + 6*d*x) - 715*exp(8*c + 8*d*x) + 1287*exp(10*c + 10*d*x) - 1716*e

xp(12*c + 12*d*x) + 1716*exp(14*c + 14*d*x) - 1287*exp(16*c + 16*d*x) + 715 *exp(18*c + 18*d*x) - 286*exp(20*c + 20*d*x) + 78*exp(22*c + 22*d*x) - 13*e xp(24*c + 24*d*x) + exp(26*c + 26*d*x) - 1) - ((2048*exp(14*c + 14*d*x)*(84 0*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(323*d) + (128*b^3*exp(2*c + 2* d*x))/(969*d) - (1024*b^3*exp(4*c + 4*d*x))/(323*d) - (46592*b^3*exp(24*c + 24*d*x))/(969*d) + (4480*b^3*exp(26*c + 26*d*x))/(969*d) - (28672*b*exp(12 *c + 12*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(969*d) - (15360*b*exp(16*c + 16 *d*x)*(112*a*b + 128*a^2 + 33*b^2))/(323*d) + (896*b*exp(10*c + 10*d*x)*(44 8*a*b + 256*a^2 + 165*b^2))/(323*d) + (7040*b*exp(18*c + 18*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(969*d) + (2560*b^2*exp(6*c + 6*d*x)*(8*a + 11*b))/(96 9*d) + (6656*b^2*exp(22*c + 22*d*x)*(8*a + 11*b))/(323*d) - (2560*b^2*exp(8 *c + 8*d*x)*(96*a + 55*b))/(969*d) - (11264*b^2*exp(20*c + 20*d*x)*(96*a + 55*b))/(969*d))/(17*exp(2*c + 2*d*x) - 136*exp(4*c + 4*d*x) + 680*exp(6*c + 6*d*x) - 2380*exp(8*c + 8*d*x) + 6188*exp(10*c + 10*d*x) - 12376*exp(12*c + 12*d*x) + 19448*exp(14*c + 14*d*x) - 24310*exp(16*c + 16*d*x) + 24310*exp (18*c + 18*d*x) - 19448*exp(20*c + 20*d*x) + 12376*exp(22*c + 22*d*x) - 618 8*exp(24*c + 24*d*x) + 2380*exp(26*c + 26*d*x) - 680*exp(28*c + 28*d*x) + 1 36*exp(30*c + 30*d*x) - 17*exp(32*c + 32*d*x) + exp(34*c + 34*d*x) - 1) - ((8*b*(448*a*b + 256*a^2 + 165*b^2))/(12597*d) - (896*b^3*exp(6*c + 6*d*x))/ (969*d) + (280*b^3*exp(8*c + 8*d*x))/(969*d) + (32*b^2*exp(4*c + 4*d*x)*(8* a + 11*b))/(323*d) - (128*b^2*exp(2*c + 2*d*x)*(96*a + 55*b))/(12597*d))/(2 8*exp(4*c + 4*d*x) - 8*exp(2*c + 2*d*x) - 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) - 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) - 8*exp(14*c + 14* d*x) + exp(16*c + 16*d*x) + 1) + ((512*b*(112*a*b + 128*a^2 + 33*b^2))/(138 567*d) - (256*exp(2*c + 2*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(46189*d) + (1792*b^3*exp(12*c + 12*d*x))/(323*d) - (320*b^3*exp(14*c + 14*d*x))/(323*d) + (7680*b*exp(4*c + 4*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(46189*d) - (320*b*exp(6*c + 6*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(4199*d) - (384*b^2*exp(10*c + 10*d*x)*(8*a + 11*b))/(323*d) + (1280*b^2*exp(8*c + 8 *d*x)*(96*a + 55*b))/(4199*d))/(11*exp(2*c + 2*d*x) - 55*exp(4*c + 4*d*x) + 165*exp(6*c + 6*d*x) - 330*exp(8*c + 8*d*x) + 462*exp(10*c + 10*d*x) - 462 *exp(12*c + 12*d*x) + 330*exp(14*c + 14*d*x) - 165*exp(16*c + 16*d*x) + 55* exp(18*c + 18*d*x) - 11*exp(20*c + 20*d*x) + exp(22*c + 22*d*x) - 1) - ((8* b^3)/(969*d) + (896*exp(12*c + 12*d*x)*(840*a*b^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(323*d) - (128*b^3*exp(2*c + 2*d*x))/(323*d) - (11648*b^3*exp(22 *c + 22*d*x))/(323*d) + (3640*b^3*exp(24*c + 24*d*x))/(969*d) - (3584*b*exp (10*c + 10*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(323*d) - (7680*b*exp(14*c + 14*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(323*d) + (280*b*exp(8*c + 8*d*x)*(44 8*a*b + 256*a^2 + 165*b^2))/(323*d) + (1320*b*exp(16*c + 16*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(323*d) + (160*b^2*exp(4*c + 4*d*x)*(8*a + 11*b))/(323 *d) + (4576*b^2*exp(20*c + 20*d*x)*(8*a + 11*b))/(323*d) - (640*b^2*exp(6*c + 6*d*x)*(96*a + 55*b))/(969*d) - (7040*b^2*exp(18*c + 18*d*x)*(96*a + 55* b))/(969*d))/(120*exp(4*c + 4*d*x) - 16*exp(2*c + 2*d*x) - 560*exp(6*c + 6* d*x) + 1820*exp(8*c + 8*d*x) - 4368*exp(10*c + 10*d*x) + 8008*exp(12*c + 12 *d*x) - 11440*exp(14*c + 14*d*x) + 12870*exp(16*c + 16*d*x) - 11440*exp(18* c + 18*d*x) + 8008*exp(20*c + 20*d*x) - 4368*exp(22*c + 22*d*x) + 1820*exp(24*c + 24*d*x) - 560*exp(26*c + 26*d*x) + 120*exp(28*c + 28*d*x) - 16*exp(3 0*c + 30*d*x) + exp(32*c + 32*d*x) + 1) - ((256*exp(16*c + 16*d*x)*(840*a*b ^2 + 1152*a^2*b + 1024*a^3 + 231*b^3))/(19*d) + (64*b^3*exp(4*c + 4*d*x))/(57*d) - (1024*b^3*exp(6*c + 6*d*x))/(57*d) - (3584*b^3*exp(26*c + 26*d*x))/ (57*d) + (320*b^3*exp(28*c + 28*d*x))/(57*d) - (4096*b*exp(14*c + 14*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(57*d) - (5120*b*exp(18*c + 18*d*x)*(112*a*b + 128*a^2 + 33*b^2))/(57*d) + (448*b*exp(12*c + 12*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(57*d) + (704*b*exp(20*c + 20*d*x)*(448*a*b + 256*a^2 + 165*b^2))/(57*d) + (640*b^2*exp(8*c + 8*d*x)*(8*a + 11*b))/(57*d) + (1664*b^2*exp(2 4*c + 24*d*x)*(8*a + 11*b))/(57*d) - (512*b^2*exp(10*c + 10*d*x)*(96*a + 55 *b))/(57*d) - (1024*b^2*exp(22*c + 22*d*x)*(96*a + 55*b))/(57*d))/(153*exp(4*c + 4*d*x) - 18*exp(2*c + 2*d*x) - 816*exp(6*c + 6*d*x) + 3060*exp(8*c + 8*d*x) - 8568*exp(10*c + 10*d*x) + 18564*exp(12*c + 12*d*x) - 31824*exp(14*

```
c + 14*d*x) + 43758*exp(16*c + 16*d*x) - 48620*exp(18*c + 18*d*x) + 43758*e
xp(20*c + 20*d*x) - 31824*exp(22*c + 22*d*x) + 18564*exp(24*c + 24*d*x) - 8
568*exp(26*c + 26*d*x) + 3060*exp(28*c + 28*d*x) - 816*exp(30*c + 30*d*x) +
 153*exp(32*c + 32*d*x) - 18*exp(34*c + 34*d*x) + exp(36*c + 36*d*x) + 1) -
 ((80*b^3*exp(4*c + 4*d*x))/(969*d) - (128*b^3*exp(2*c + 2*d*x))/(969*d) +
(32*b^2*(8*a + 11*b))/(6783*d))/(15*exp(4*c + 4*d*x) - 6*exp(2*c + 2*d*x) -
 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c + 10*d*x) + exp(12*
c + 12*d*x) + 1) - (8*b^3)/(969*d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x)
- 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**20*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.229 $\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh^3(c+dx)}{3bd}+\frac{\cosh(c+dx)}{bd}$$

```
[Out] cosh(d*x+c)/b/d-1/3*cosh(d*x+c)^3/b/d-1/2*a*arctan(b^(1/4)*cosh(d*x+c)/(a^(
1/2)-b^(1/2))^(1/2))/b^(7/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*a*arctanh(b^(1/4
)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/b^(7/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1170, 1166, 205, 208}

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh^3(c+dx)}{3bd}+\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(a*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a
] - Sqrt[b]]*b^(7/4)*d) + (a*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(7/4)*d) + Cosh[c + d*x]/(b*d) - C
osh[c + d*x]^3/(3*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\frac{\mathrm{Subst}\left(\int \frac{(1-x^2)^3}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\mathrm{Subst}\left(\int \left(-\frac{1}{b}+\frac{x^2}{b}+\frac{a-ax^2}{b(a-b+2bx^2-bx^4)}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\cosh(c+dx)}{bd}-\frac{\cosh^3(c+dx)}{3bd}-\frac{\mathrm{Subst}\left(\int \frac{a-ax^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{bd}$$
$$= \frac{\cosh(c+dx)}{bd}-\frac{\cosh^3(c+dx)}{3bd}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2bd}+$$
$$= -\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{7/4}d}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{7/4}d}+\frac{\cosh(c+dx)}{bd}-\frac{\cosh^3(c+d}{3bd}$$

Mathematica [C] time = 0.35, size = 390, normalized size = 2.64

$$-3a\mathrm{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+d\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (18*Cosh[c + d*x] - 2*Cosh[3*(c + d*x)] - 3*a*RootSum[b - 4*b*#1^2 - 16*a*#
1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2]
 - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#
1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
 d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-C
osh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x
)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#
1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(24*b*d)
```

fricas [B] time = 0.75, size = 1617, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/24*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6
+ 3*(5*cosh(d*x + c)^2 - 3)*sinh(d*x + c)^4 - 9*cosh(d*x + c)^4 + 4*(5*cosh
(d*x + c)^3 - 9*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 - 18*
```

```
cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(
d*x + c)^2*sinh(d*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d
*x + c)^3)*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4
)) + a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c
)*sinh(d*x + c) + a^3*sinh(d*x + c)^2 + a^3 + 2*(a^2*b^2*d*cosh(d*x + c) +
a^2*b^2*d*sinh(d*x + c) - ((a*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^5 - b^6)*
d^3*sinh(d*x + c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3
 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)
*d^2))) + 6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*
b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d*x + c)^3)*sqrt(-((a*b^3 - b^
4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2)
)*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*
x + c)^2 + a^3 - 2*(a^2*b^2*d*cosh(d*x + c) + a^2*b^2*d*sinh(d*x + c) - ((a
*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^5 - b^6)*d^3*sinh(d*x + c))*sqrt(a^5/(
(a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^
7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) - 6*(b*d*cosh(d*x + c
)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x +
c)^2 + b*d*sinh(d*x + c)^3)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*
a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*
a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2 + a^3 + 2*(a^2*b^2*d*
cosh(d*x + c) + a^2*b^2*d*sinh(d*x + c) + ((a*b^5 - b^6)*d^3*cosh(d*x + c)
+ (a*b^5 - b^6)*d^3*sinh(d*x + c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)
))*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)
/((a*b^3 - b^4)*d^2))) + 6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sin
h(d*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d*x + c)^3)*sqr
t(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b
^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c)
 + a^3*sinh(d*x + c)^2 + a^3 - 2*(a^2*b^2*d*cosh(d*x + c) + a^2*b^2*d*sinh(
d*x + c) + ((a*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^5 - b^6)*d^3*sinh(d*x +
c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)*d^2*sqrt
(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) - 9*cosh
(d*x + c)^2 + 6*(cosh(d*x + c)^5 - 6*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sin
h(d*x + c) + 1)/(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d*x + c)
+ 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d*x + c)^3)
```

giac [B] time = 0.53, size = 553, normalized size = 3.74

$$-\frac{12\left(\left(\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,a^2+8\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,ab\right)b^2+\left(\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b^2+8\sqrt{-b^2+\sqrt{ab}\,b}\,ab^3\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{b^4-\sqrt{b^8+(ab^3-b^4)b^4}}{b^4}}}\right)}{a^2b^5+7ab^6-8b^7}+\frac{6\left(\sqrt{b^2+\sqrt{\cdots}}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/24*(12*((sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 8*sqrt(a*b)*sqrt(-b^2
+ sqrt(a*b)*b)*a*b)*b^2 + (sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^2 + 8*sqrt(-b^2 +
 sqrt(a*b)*b)*a*b^3)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-
(b^4 - sqrt(b^8 + (a*b^3 - b^4)*b^4))/b^4))/(a^2*b^5 + 7*a*b^6 - 8*b^7) + 6
*(sqrt(b^2 + sqrt(a*b)*b)*a^2*b^3*abs(b) - sqrt(a*b)*sqrt(b^2 + sqrt(a*b)*b
)*a*b^4 - (sqrt(b^2 + sqrt(a*b)*b)*a^2*b + sqrt(a*b)*sqrt(b^2 + sqrt(a*b)*b
)*a^2)*b^2*abs(b) + (sqrt(b^2 + sqrt(a*b)*b)*a^2*b^2 + sqrt(a*b)*sqrt(b^2 +
 sqrt(a*b)*b)*a*b^2)*b^2)*log(2*sqrt((b^4 + sqrt(b^8 + (a*b^3 - b^4)*b^4))/
b^4) + e^(d*x + c) + e^(-d*x - c))/(a^2*b^6 - a*b^7) - 6*(sqrt(b^2 + sqrt(a
*b)*b)*a*b^2*abs(b) - sqrt(a*b)*sqrt(b^2 + sqrt(a*b)*b)*a*b^2)*log(abs(-2*s
qrt((b^4 + sqrt(b^8 + (a*b^3 - b^4)*b^4))/b^4) + e^(d*x + c) + e^(-d*x - c)
))/(a*b^5 - b^6) + (b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*b^2*(e^(d*x + c
) + e^(-d*x - c)))/b^3)/d
```

maple [B] time = 0.13, size = 270, normalized size = 1.82

$$-\frac{a\sqrt{ab}\,\arctan\left(\frac{2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{2d\,b^{2}\sqrt{-ab+\sqrt{ab}\,a}}-\frac{a\sqrt{ab}\,\arctan\left(\frac{-2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{2d\,b^{2}\sqrt{-ab-\sqrt{ab}\,a}}+\frac{1}{3db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/2/d*a/b^2*(a*b)^(1/2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*
d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/2/d*a/b^2*(
a*b)^(1/2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*
a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/3/d/b/(tanh(1/2*d*x+1/2*
c)-1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)-1
/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/
(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(e^{(6\,dx+6\,c)}-9\,e^{(4\,dx+4\,c)}-9\,e^{(2\,dx+2\,c)}+1\right)e^{(-3\,dx-3\,c)}}{24\,bd}-\frac{1}{128}\int\frac{256\left(ae^{(7\,dx+7\,c)}-3\,ae^{(5\,dx+5\,c)}+3\,ae^{(3\,c}\right.}{b^{2}e^{(8\,dx+8\,c)}-4\,b^{2}e^{(6\,dx+6\,c)}-4\,b^{2}e^{(2\,dx+2\,c)}+b^{2}-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/24*(e^(6*d*x + 6*c) - 9*e^(4*d*x + 4*c) - 9*e^(2*d*x + 2*c) + 1)*e^(-3*d
*x - 3*c)/(b*d) - 1/128*integrate(256*(a*e^(7*d*x + 7*c) - 3*a*e^(5*d*x + 5
*c) + 3*a*e^(3*d*x + 3*c) - a*e^(d*x + c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(
6*d*x + 6*c) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*
c))*e^(4*d*x)), x)
```

mupad [B] time = 9.80, size = 1124, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log(((((4194304*a^8*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^11*(a - b)^2)
 - (8388608*a^7*d^3*exp(c + d*x)*(a + b)*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b^7
*d^2*(a - b)))^(1/2))/(b^10*(a - b)))*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b^7*d^
2*(a - b)))^(1/2))/4 + (2097152*a^9*d*exp(c + d*x))/(b^13*(a - b)))*(-((a^5
*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (262144*a^10*(exp(2*c
+ 2*d*x) + 1)*(a + b))/(b^15*(a - b)^2))*(((a^5*b^7)^(1/2) + a^2*b^4)/(16*(
b^8*d^2 - a*b^7*d^2)))^(1/2) - log(((((4194304*a^8*d^2*(exp(2*c + 2*d*x) +
 1)*(3*a + b))/(b^11*(a - b)^2) + (8388608*a^7*d^3*exp(c + d*x)*(a + b)*(-(
(a^5*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/(b^10*(a - b)))*(-((a^
5*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (2097152*a^9*d*exp(c
+ d*x))/(b^13*(a - b)))*(-((a^5*b^7)^(1/2) + a^2*b^4)/(b^7*d^2*(a - b)))^(1
/2))/4 - (262144*a^10*(exp(2*c + 2*d*x) + 1)*(a + b))/(b^15*(a - b)^2))*(((
a^5*b^7)^(1/2) + a^2*b^4)/(16*(b^8*d^2 - a*b^7*d^2)))^(1/2) + log(((((4194
304*a^8*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^11*(a - b)^2) - (8388608*a
^7*d^3*exp(c + d*x)*(a + b)*(((a^5*b^7)^(1/2) - a^2*b^4)/(b^7*d^2*(a - b)))
^(1/2))/(b^10*(a - b)))*(((a^5*b^7)^(1/2) - a^2*b^4)/(b^7*d^2*(a - b)))^(1/
2))/4 + (2097152*a^9*d*exp(c + d*x))/(b^13*(a - b)))*(((a^5*b^7)^(1/2) - a^
2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (262144*a^10*(exp(2*c + 2*d*x) + 1)*(a
```

```
 + b))/(b^15*(a - b)^2))*(-((a^5*b^7)^(1/2) - a^2*b^4)/(16*(b^8*d^2 - a*b^7
*d^2)))^(1/2) - log((((((4194304*a^8*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/
(b^11*(a - b)^2) + (8388608*a^7*d^3*exp(c + d*x)*(a + b)*(((a^5*b^7)^(1/2)
- a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/(b^10*(a - b)))*(((a^5*b^7)^(1/2) - a^
2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (2097152*a^9*d*exp(c + d*x))/(b^13*(a
- b)))*(((a^5*b^7)^(1/2) - a^2*b^4)/(b^7*d^2*(a - b)))^(1/2))/4 - (262144*a
^10*(exp(2*c + 2*d*x) + 1)*(a + b))/(b^15*(a - b)^2))*(-((a^5*b^7)^(1/2) -
a^2*b^4)/(16*(b^8*d^2 - a*b^7*d^2)))^(1/2) + (3*exp(c + d*x))/(8*b*d) + (3*
exp(- c - d*x))/(8*b*d) - exp(- 3*c - 3*d*x)/(24*b*d) - exp(3*c + 3*d*x)/(2
4*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.230 $\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh(c+dx)}{bd}$$

```
[Out] -cosh(d*x+c)/b/d+1/2*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*a^
(1/2)/b^(5/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*arctanh(b^(1/4)*cosh(d*x+c)/(a^
(1/2)+b^(1/2))^(1/2))*a^(1/2)/b^(5/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1170, 1093, 205, 208}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (Sqrt[a]*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[S
qrt[a] - Sqrt[b]]*b^(5/4)*d) + (Sqrt[a]*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqr
t[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)*d) - Cosh[c + d*x
]/(b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{a}{b\left(a-b+2bx^2-bx^4\right)}\right)dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)}{bd}+\frac{a\,\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{bd}$$

$$= -\frac{\cosh(c+dx)}{bd}-\frac{\sqrt{a}\ \text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2\sqrt{b}\,d}+\frac{\sqrt{a}\ \text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{}$$

$$= \frac{\sqrt{a}\ \tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\ b^{5/4}d}+\frac{\sqrt{a}\ \tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\ b^{5/4}d}-\frac{\cosh(c+dx)}{bd}$$

Mathematica [C] time = 0.27, size = 235, normalized size = 1.69

$$-\frac{a\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&,\frac{2\#1^3\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\text{co}\right.}{}\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/2*(2*Cosh[c + d*x] + a*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b
*#1^6 + b*#1^8 & , (-(c*#1) - d*x*#1 - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1 + c*#1^3 + d*x*#
1^3 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#1^6)
 & ])/(b*d)
```

fricas [B] time = 0.56, size = 1247, normalized size = 8.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*((b*d*cosh(d*x + c) + b*d*sinh(d*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(
a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))*log(a^2*cosh
(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2
 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) - ((a*b^4 - b^5)*d^3*co
sh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^2*b^5 - 2*a*b^6
 + b^7)*d^4)))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)
*d^4)) + a)/((a*b^2 - b^3)*d^2))) - (b*d*cosh(d*x + c) + b*d*sinh(d*x + c))
*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((
```

```
a*b^2 - b^3)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x +
 c) + a^2*sinh(d*x + c)^2 + a^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d
*x + c) - ((a*b^4 - b^5)*d^3*cosh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c
))*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)))*sqrt(-((a*b^2 - b^3)*d^2*sqrt
(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) + (b*d*cos
h(d*x + c) + b*d*sinh(d*x + c))*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5
- 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(a^2*cosh(d*x + c)^2 +
2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2 + 2*(a^2*b*d*
cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a*b^4 - b^5)*d^3*cosh(d*x + c) +
(a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)))
*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a
*b^2 - b^3)*d^2))) - (b*d*cosh(d*x + c) + b*d*sinh(d*x + c))*sqrt(((a*b^2 -
 b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2
))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d
*x + c)^2 + a^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a*b^
4 - b^5)*d^3*cosh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)))*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 -
2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))) - 2*cosh(d*x + c)^2 - 4*cos
h(d*x + c)*sinh(d*x + c) - 2*sinh(d*x + c)^2 - 2)/(b*d*cosh(d*x + c) + b*d*
sinh(d*x + c))
```

giac [B] time = 0.52, size = 418, normalized size = 3.01

$$\frac{2\left(\sqrt{ab}\,\sqrt{-b^2+\sqrt{ab}\,b}\,ab^2+8\,\sqrt{ab}\,\sqrt{-b^2+\sqrt{ab}\,b}\,b^3+\left(\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b+8\,\sqrt{-b^2+\sqrt{ab}\,b}\,ab^2\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{b^2-\sqrt{b^4+(ab-b^2)b^2}}{b^2}}}\right)}{a^2b^4+7\,ab^5-8\,b^6}+\frac{\left(\sqrt{b^2+\sqrt{ab}\,b}\,a^2 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/4*(2*(sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^2 + 8*sqrt(a*b)*sqrt(-b^2 +
sqrt(a*b)*b)*b^3 + (sqrt(-b^2 + sqrt(a*b)*b)*a^2*b + 8*sqrt(-b^2 + sqrt(a*b
)*b)*a*b^2)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(b^2 - sq
rt(b^4 + (a*b - b^2)*b^2))/b^2))/(a^2*b^4 + 7*a*b^5 - 8*b^6) + (sqrt(b^2 +
sqrt(a*b)*b)*a^2*b*abs(b) + sqrt(a*b)*sqrt(b^2 + sqrt(a*b)*b)*a*b^2)*log(2*
sqrt((b^2 + sqrt(b^4 + (a*b - b^2)*b^2))/b^2) + e^(d*x + c) + e^(-d*x - c))
/(a^2*b^4 - a*b^5) - (sqrt(b^2 + sqrt(a*b)*b)*a^2*b*abs(b) - sqrt(a*b)*sqrt
(b^2 + sqrt(a*b)*b)*a*b^2)*log(abs(-2*sqrt((b^2 + sqrt(b^4 + (a*b - b^2)*b^
2))/b^2) + e^(d*x + c) + e^(-d*x - c)))/(a^2*b^4 - a*b^5) - 2*(e^(d*x + c)
+ e^(-d*x - c))/b)/d
```

maple [A] time = 0.09, size = 175, normalized size = 1.26

$$\frac{a\arctan\left(\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{2db\sqrt{-ab+\sqrt{ab}\,a}}-\frac{a\arctan\left(\frac{-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{2db\sqrt{-ab-\sqrt{ab}\,a}}-\frac{1}{db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{1}{db\left(\tanh\left(\frac{d \cdots}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] 1/2/d*a/b/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+
4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/2/d*a/b/(-a*b-(a*b)^(1/2)*
a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a
*b)^(1/2)*a)^(1/2))-1/d/b/(tanh(1/2*d*x+1/2*c)+1)+1/d/b/(tanh(1/2*d*x+1/2*c
)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(e^{(2\,dx+2\,c)}+1\right)e^{(-dx-c)}}{2\,bd}-\frac{1}{32}\int\frac{256\left(ae^{(5\,dx+5\,c)}-ae^{(3\,dx+3\,c)}\right)}{b^2e^{(8\,dx+8\,c)}-4\,b^2e^{(6\,dx+6\,c)}-4\,b^2e^{(2\,dx+2\,c)}+b^2-2\left(8\,abe^{(4\,c)}-3\,b^2e^{(4\,c)}\right)e^{(4\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/32*integrate(256*(a*e^(5*
d*x + 5*c) - a*e^(3*d*x + 3*c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c
) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*
x)), x)
```

mupad [B] time = 7.85, size = 1046, normalized size = 7.53

$$\ln\left(\frac{\left(\frac{\left(\frac{4194304\,a^6\,d^2\,\left(e^{2\,c+2\,d\,x}+1\right)\,\left(3\,a+b\right)}{b^9\,{\left(a-b\right)}^2}+\frac{16777216\,a^6\,d^3\,e^{c+d\,x}\,\sqrt{-\frac{\sqrt{a^3\,b^5}+a\,b^3}{b^5\,d^2\,\left(a-b\right)}}}{b^8\,\left(a-b\right)}\right)\,\sqrt{-\frac{\sqrt{a^3\,b^5}+a\,b^3}{b^5\,d^2\,\left(a-b\right)}}}{4}-\frac{2097152\,a^7\,d\,e^{c+d\,x}}{b^{11}\,\left(a-b\right)}\right)\,\sqrt{-\frac{\sqrt{a^3\,b^5}+a\,b^3}{b^5\,d^2\,\left(a-b\right)}}}{4}-\frac{26214}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log(((((4194304*a^6*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^9*(a - b)^2)
 + (16777216*a^6*d^3*exp(c + d*x)*(-((a^3*b^5)^(1/2) + a*b^3)/(b^5*d^2*(a -
b)))^(1/2))/(b^8*(a - b)))*(-((a^3*b^5)^(1/2) + a*b^3)/(b^5*d^2*(a - b)))^(
1/2))/4 - (2097152*a^7*d*exp(c + d*x))/(b^11*(a - b)))*(-((a^3*b^5)^(1/2) +
 a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (262144*a^7*(exp(2*c + 2*d*x) + 1)*(a
 + b))/(b^12*(a - b)^2))*(((a^3*b^5)^(1/2) + a*b^3)/(16*(b^6*d^2 - a*b^5*d^
2)))^(1/2) - log(((((4194304*a^6*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^
9*(a - b)^2) - (16777216*a^6*d^3*exp(c + d*x)*(-((a^3*b^5)^(1/2) + a*b^3)/(
b^5*d^2*(a - b)))^(1/2))/(b^8*(a - b)))*(-((a^3*b^5)^(1/2) + a*b^3)/(b^5*d^
2*(a - b)))^(1/2))/4 + (2097152*a^7*d*exp(c + d*x))/(b^11*(a - b)))*(-((a^3
*b^5)^(1/2) + a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (262144*a^7*(exp(2*c + 2
*d*x) + 1)*(a + b))/(b^12*(a - b)^2))*(((a^3*b^5)^(1/2) + a*b^3)/(16*(b^6*d
^2 - a*b^5*d^2)))^(1/2) - log(((((4194304*a^6*d^2*(exp(2*c + 2*d*x) + 1)*(
3*a + b))/(b^9*(a - b)^2) - (16777216*a^6*d^3*exp(c + d*x)*(((a^3*b^5)^(1/2
) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/(b^8*(a - b)))*(((a^3*b^5)^(1/2) - a*b
^3)/(b^5*d^2*(a - b)))^(1/2))/4 + (2097152*a^7*d*exp(c + d*x))/(b^11*(a - b
)))*(((a^3*b^5)^(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (262144*a^7*(e
xp(2*c + 2*d*x) + 1)*(a + b))/(b^12*(a - b)^2))*(-((a^3*b^5)^(1/2) - a*b^3)
/(16*(b^6*d^2 - a*b^5*d^2)))^(1/2) + log(((((4194304*a^6*d^2*(exp(2*c + 2*
d*x) + 1)*(3*a + b))/(b^9*(a - b)^2) + (16777216*a^6*d^3*exp(c + d*x)*(((a^
3*b^5)^(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/(b^8*(a - b)))*(((a^3*b^5)^
(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (2097152*a^7*d*exp(c + d*x))/(
b^11*(a - b)))*(((a^3*b^5)^(1/2) - a*b^3)/(b^5*d^2*(a - b)))^(1/2))/4 - (26
2144*a^7*(exp(2*c + 2*d*x) + 1)*(a + b))/(b^12*(a - b)^2))*(-((a^3*b^5)^(1/
2) - a*b^3)/(16*(b^6*d^2 - a*b^5*d^2)))^(1/2) - exp(c + d*x)/(2*b*d) - exp(
- c - d*x)/(2*b*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.231 $\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

```
[Out] -1/2*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(3/4)/d/(a^(1/2)
-b^(1/2))^(1/2)+1/2*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/b^
(3/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3215, 1166, 205, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[Sqrt[a] -
Sqrt[b]]*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b
]]]/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c\right.}{2d}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/4}d}$$

Mathematica [C] time = 0.18, size = 365, normalized size = 3.17

$$-\frac{\text{RootSum}\left[\#1^8 b-4\#1^6 b-16\#1^4 a+6\#1^4 b-4\#1^2 b+b\&, \frac{2\#1^6\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/8*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-
c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*
#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*x)/
2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2
- 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Co
sh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log
[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ]/d
```

fricas [B] time = 1.14, size = 975, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a
*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c) - ((a*b^2 - b^3)*d^3*
cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4
 + b^5)*d^4)))*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4
)) + 1)/((a*b - b^2)*d^2)) + 1) - 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(b*d*cosh(d*x + c) + b*d*s
inh(d*x + c) - ((a*b^2 - b^3)*d^3*cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*
x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)))*sqrt(-((a*b - b^2)*d^2*sqr
t(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2)) + 1) + 1/4*sqr
t(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2
)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c) + ((a*b^2 - b^3)*d^3*cosh(d*x
 + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*
d^4)))*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/(
(a*b - b^2)*d^2)) + 1) - 1/4*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b
^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x +
```

```
c) + ((a*b^2 - b^3)*d^3*cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)))*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2)) + 1)
```

giac [B] time = 0.37, size = 311, normalized size = 2.70

$$-\frac{\frac{\left(\sqrt{-b^2-\sqrt{ab}\,b}\,ab+8\sqrt{-b^2-\sqrt{ab}\,b}\,b^2-\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,a-8\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,b\right)|b|\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{b+\sqrt{(a-b)b+b^2}}{b}}}\right)}{a^2b^3+7ab^4-8b^5}+\frac{\left(\sqrt{-b^2+\sqrt{ab}\,b}\,ab+8\sqrt{-b^2+\sqrt{ab}\cdots}\right.}{\cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*((sqrt(-b^2 - sqrt(a*b)*b)*a*b + 8*sqrt(-b^2 - sqrt(a*b)*b)*b^2 - sqrt
(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a - 8*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b)*
abs(b)*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(b + sqrt((a - b)*b +
b^2))/b))/(a^2*b^3 + 7*a*b^4 - 8*b^5) + (sqrt(-b^2 + sqrt(a*b)*b)*a*b + 8*s
qrt(-b^2 + sqrt(a*b)*b)*b^2 + sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a + 8*sqrt
(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b)*abs(b)*arctan(1/2*(e^(d*x + c) + e^(-d*x
- c))/sqrt(-(b - sqrt((a - b)*b + b^2))/b))/(a^2*b^3 + 7*a*b^4 - 8*b^5))/d
```

maple [A] time = 0.06, size = 142, normalized size = 1.23

$$-\frac{\sqrt{ab}\;\arctan\left(\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\;a}}\right)}{2db\sqrt{-ab+\sqrt{ab}\;a}}-\frac{\sqrt{ab}\;\arctan\left(\frac{-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\;a}}\right)}{2db\sqrt{-ab-\sqrt{ab}\;a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/2/d*(a*b)^(1/2)/b/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+
1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/2/d*(a*b)^(1/2)
/b/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b
)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sinh(d*x + c)^3/(b*sinh(d*x + c)^4 - a), x)
```

mupad [B] time = 6.16, size = 975, normalized size = 8.48

$$\ln\left(\frac{\left(\frac{\left(\frac{4194304\,a^4\,d^2\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)\,(3\,a+b)}{b^7\,(a-b)^2}-\frac{8388608\,a^4\,d^3\,\mathrm{e}^{c+d\,x}\,(a+b)\,\sqrt{-\frac{b^2-\sqrt{a\,b^3}}{b^3\,d^2\,(a-b)}}}{b^7\,(a-b)}\right)\sqrt{-\frac{b^2-\sqrt{a\,b^3}}{b^3\,d^2\,(a-b)}}}{4}+\frac{2097152\,a^4\,d\,\mathrm{e}^{c+d\,x}}{b^8\,(a-b)}\right)\sqrt{-\frac{b^2-\sqrt{a\,b^3}}{b^3\,d^2\,(a-b)}}}{4}-\frac{2621}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log(((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^7*(a - b)^2)
- (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*(a
 - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*(a - b)))^(1
/2))/4 + (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-(b^2 - (a*b^3)^(1/2)
)/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c + 2*d*x) + 1)*(a + b))
/(b^9*(a - b)^2))*((b^2 - (a*b^3)^(1/2))/(16*(b^4*d^2 - a*b^3*d^2)))^(1/2)
- log(((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^7*(a - b)^2
) + (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*
(a - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 - (a*b^3)^(1/2))/(b^3*d^2*(a - b)))^
(1/2))/4 - (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-(b^2 - (a*b^3)^(1/
2))/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c + 2*d*x) + 1)*(a + b
))/(b^9*(a - b)^2))*((b^2 - (a*b^3)^(1/2))/(16*(b^4*d^2 - a*b^3*d^2)))^(1/2
) + log(((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^7*(a - b)
^2) - (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 + (a*b^3)^(1/2))/(b^3*d^
2*(a - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 + (a*b^3)^(1/2))/(b^3*d^2*(a - b))
)^(1/2))/4 + (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-(b^2 + (a*b^3)^(
1/2))/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c + 2*d*x) + 1)*(a +
 b))/(b^9*(a - b)^2))*((b^2 + (a*b^3)^(1/2))/(16*(b^4*d^2 - a*b^3*d^2)))^(1
/2) - log(((((4194304*a^4*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^7*(a -
b)^2) + (8388608*a^4*d^3*exp(c + d*x)*(a + b)*(-(b^2 + (a*b^3)^(1/2))/(b^3*
d^2*(a - b)))^(1/2))/(b^7*(a - b)))*(-(b^2 + (a*b^3)^(1/2))/(b^3*d^2*(a - b
)))^(1/2))/4 - (2097152*a^4*d*exp(c + d*x))/(b^8*(a - b)))*(-(b^2 + (a*b^3)
^(1/2))/(b^3*d^2*(a - b)))^(1/2))/4 - (262144*a^4*(exp(2*c + 2*d*x) + 1)*(a
 + b))/(b^9*(a - b)^2))*((b^2 + (a*b^3)^(1/2))/(16*(b^4*d^2 - a*b^3*d^2)))^
(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.232 $\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\,d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\,d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] 1/2*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(1/4)/d/a^(1/2)/(
a^(1/2)-b^(1/2))^(1/2)+1/2*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1
/2))/b^(1/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3215, 1093, 205, 208}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\,d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}\,d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqr
t[a] - Sqrt[b]]*b^(1/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2\sqrt{a}\,d} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x\right.}{2\sqrt{a}\,d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt[4]{b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt[4]{b}\,d}$$

Mathematica [C] time = 0.17, size = 221, normalized size = 1.77

$$-\frac{\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^3\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\right.}{}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-
(c*#1) - d*x*#1 - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1 + c*#1^3 + d*x*#1^3 + 2*Log[-Cosh[(c
+ d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#1^6) & ]/d
```

fricas [B] time = 0.94, size = 979, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/(
(a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 + 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) - ((a^2*b - a*b^2)*
d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a
^2*b^2 + a*b^3)*d^4)))*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 +
a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2)) + 1) - 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt
(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(a*d*cosh(d*x
+ c) + a*d*sinh(d*x + c) - ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*
b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(-((
a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*
d^2)) + 1) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*
d^4)) - 1)/((a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*
x + c) + sinh(d*x + c)^2 + 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) + ((a^2
*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/(
(a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2
*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) + 1) - 1/4*sqrt(((a^2 - a*b
)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2))*log
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(a*d
*cosh(d*x + c) + a*d*sinh(d*x + c) + ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (
a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4))
```

```
)*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^
2 - a*b)*d^2)) + 1)
```

giac [B] time = 0.31, size = 329, normalized size = 2.63

$$\frac{\frac{\left(\sqrt{-b^2-\sqrt{ab}\,b}\,a^2b+8\sqrt{-b^2-\sqrt{ab}\,b}\,ab^2-\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,ab-8\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,b^2\right)|b|\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{b+\sqrt{(a-b)b+b^2}}{b}}}\right)}{a^3b^3+7a^2b^4-8ab^5}+\frac{\left(\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b+8\sqrt{-b^2+\cdots}\right.}{\cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*((sqrt(-b^2 - sqrt(a*b)*b)*a^2*b + 8*sqrt(-b^2 - sqrt(a*b)*b)*a*b^2 - s
qrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b - 8*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b
)*b^2)*abs(b)*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(b + sqrt((a -
b)*b + b^2))/b))/(a^3*b^3 + 7*a^2*b^4 - 8*a*b^5) + (sqrt(-b^2 + sqrt(a*b)*b
)*a^2*b + 8*sqrt(-b^2 + sqrt(a*b)*b)*a*b^2 + sqrt(a*b)*sqrt(-b^2 + sqrt(a*b
)*b)*a*b + 8*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*abs(b)*arctan(1/2*(e^(
d*x + c) + e^(-d*x - c))/sqrt(-(b - sqrt((a - b)*b + b^2))/b))/(a^3*b^3 + 7
*a^2*b^4 - 8*a*b^5))/d
```

maple [A] time = 0.10, size = 126, normalized size = 1.01

$$\frac{\arctan\left(\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{2d\sqrt{-ab+\sqrt{ab}\,a}}-\frac{\arctan\left(\frac{-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{2d\sqrt{-ab-\sqrt{ab}\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] 1/2/d/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a
*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/2/d/(-a*b-(a*b)^(1/2)*a)^(1/2)
*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2
)*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh\left(dx+c\right)}{b\sinh\left(dx+c\right)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sinh(d*x + c)/(b*sinh(d*x + c)^4 - a), x)
```

mupad [B] time = 8.15, size = 1007, normalized size = 8.06

$$\ln\left(\frac{\left(\frac{\left(\frac{4194304\,a^2\,d^2\,\left(e^{2\,c+2\,d\,x}+1\right)\,(3\,a+b)}{b^5\,(a-b)^2}+\frac{16777216\,a^3\,d^3\,e^{c+d\,x}\,\sqrt{-\frac{a\,b-\sqrt{a^3\,b}}{a^2\,b\,d^2\,(a-b)}}}{b^5\,(a-b)}\right)\sqrt{-\frac{a\,b-\sqrt{a^3\,b}}{a^2\,b\,d^2\,(a-b)}}}{4}-\frac{2097152\,a^2\,d\,e^{c+d\,x}}{b^6\,(a-b)}\right)\sqrt{-\frac{a\,b-\sqrt{a^3\,b}}{a^2\,b\,d^2\,(a-b)}}}{4}-\frac{262\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^5*(a - b)^2)
+ (16777216*a^3*d^3*exp(c + d*x)*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a - b)
))^(1/2))/(b^5*(a - b)))*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a - b)))^(1/2)
)/4 - (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(-(a*b - (a^3*b)^(1/2))/(
a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c + 2*d*x) + 1)*(a + b))/(b
^6*(a - b)^2))*(-(a*b - (a^3*b)^(1/2))/(16*(a^3*b*d^2 - a^2*b^2*d^2)))^(1/2
) - log((((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^5*(a - b)
^2) - (16777216*a^3*d^3*exp(c + d*x)*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a
- b)))^(1/2))/(b^5*(a - b)))*(-(a*b - (a^3*b)^(1/2))/(a^2*b*d^2*(a - b)))^(
1/2))/4 + (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(-(a*b - (a^3*b)^(1/2
))/(a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c + 2*d*x) + 1)*(a + b)
)/(b^6*(a - b)^2))*(-(a*b - (a^3*b)^(1/2))/(16*(a^3*b*d^2 - a^2*b^2*d^2)))^
(1/2) - log((((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^5*(a
- b)^2) - (16777216*a^3*d^3*exp(c + d*x)*(-(a*b + (a^3*b)^(1/2))/(a^2*b*d^2
*(a - b)))^(1/2))/(b^5*(a - b)))*(-(a*b + (a^3*b)^(1/2))/(a^2*b*d^2*(a - b)
))^(1/2))/4 + (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(-(a*b + (a^3*b)^
(1/2))/(a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c + 2*d*x) + 1)*(a
+ b))/(b^6*(a - b)^2))*(-(a*b + (a^3*b)^(1/2))/(16*(a^3*b*d^2 - a^2*b^2*d^2
)))^(1/2) + log((((((4194304*a^2*d^2*(exp(2*c + 2*d*x) + 1)*(3*a + b))/(b^5
*(a - b)^2) + (16777216*a^3*d^3*exp(c + d*x)*(-(a*b + (a^3*b)^(1/2))/(a^2*b
*d^2*(a - b)))^(1/2))/(b^5*(a - b)))*(-(a*b + (a^3*b)^(1/2))/(a^2*b*d^2*(a
- b)))^(1/2))/4 - (2097152*a^2*d*exp(c + d*x))/(b^6*(a - b)))*(-(a*b + (a^3
*b)^(1/2))/(a^2*b*d^2*(a - b)))^(1/2))/4 - (262144*a*(exp(2*c + 2*d*x) + 1)
*(a + b))/(b^6*(a - b)^2))*(-(a*b + (a^3*b)^(1/2))/(16*(a^3*b*d^2 - a^2*b^2
*d^2)))^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.233 $\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

```
[Out] -arctanh(cosh(d*x+c))/a/d-1/2*b^(1/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b
^(1/2))^(1/2))/a/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cosh
(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3215, 1170, 207, 1166, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a*Sqr
t[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c + d*x]]/(a*d) + (b^(1/4)*ArcTanh[(
b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a*Sqrt[Sqrt[a] + Sqrt[b
]]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(-\frac{1}{a(-1+x^2)}+\frac{b-bx^2}{a(a-b+2bx^2-bx^4)}\right)dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(c+dx)\right)}{ad} - \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{ad}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{b\operatorname{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2ad} + \frac{b\operatorname{Subst}}{}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

Mathematica [C] time = 0.25, size = 385, normalized size = 2.83

$$\frac{8\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - b\operatorname{RootSum}\left[\#1^8 b - 4\#1^6 b - 16\#1^4 a + 6\#1^4 b - 4\#1^2 b + b\&, \frac{2\#1^6\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (8*Log[Tanh[(c + d*x)/2]] - b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#1^2 + 3*d*x*#1^2
 + 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Si
nh[(c + d*x)/2]*#1]*#1^2 - 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-Cosh[(c + d*x)/2]
 - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 +
c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c
+ d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 -
 3*b*#1^5 + b*#1^7) & ])/(8*a*d)
```

fricas [B] time = 2.22, size = 1067, normalized size = 7.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) +
 b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2 + 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c) -
((a^4 - a^3*b)*d^3*cosh(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt(b/
((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2
*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)) + b) - a*d*sqrt(-((a^3 -
a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)
)*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)
^2 - 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c) - ((a^4 - a^3*b)*d^3*cosh
(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt(b/((a^5 - 2*a^4*b + a^3*b
^2)*d^4)))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4))
 + b)/((a^3 - a^2*b)*d^2)) + b) + a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5
- 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2
+ 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*(a*b*d*cosh(d*x +
 c) + a*b*d*sinh(d*x + c) + ((a^4 - a^3*b)*d^3*cosh(d*x + c) + (a^4 - a^3*b
)*d^3*sinh(d*x + c))*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*sqrt(((a^3 -
a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)
) + b) - a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)
) - b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(
d*x + c) + b*sinh(d*x + c)^2 - 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c)
 + ((a^4 - a^3*b)*d^3*cosh(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt
(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 -
 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)) + b) - 4*log(cosh(d*x +
 c) + sinh(d*x + c) + 1) + 4*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(a*d)
```

giac [B] time = 0.26, size = 415, normalized size = 3.05

$$\frac{\left(\left(\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,a+8\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,b\right)|a||b|-\left(\sqrt{-b^2-\sqrt{ab}\,b}\,a^2b+8\sqrt{-b^2-\sqrt{ab}\,b}\,ab^2\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{ab+\sqrt{a^2b^2+(a^2-ab)ab}}{ab}}}\right)}{a^4b^2+7a^3b^3-8a^2b^4}-\frac{\left(\left(\sqrt{ab}\sqrt{-b^2+\sqrt{\cdots}}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*(((sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a + 8*sqrt(a*b)*sqrt(-b^2 - sqrt(
a*b)*b)*b)*abs(a)*abs(b) - (sqrt(-b^2 - sqrt(a*b)*b)*a^2*b + 8*sqrt(-b^2 -
sqrt(a*b)*b)*a*b^2)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(
a*b + sqrt(a^2*b^2 + (a^2 - a*b)*a*b))/(a*b)))/(a^4*b^2 + 7*a^3*b^3 - 8*a^2
*b^4) - ((sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a + 8*sqrt(a*b)*sqrt(-b^2 + sq
rt(a*b)*b)*b)*abs(a)*abs(b) + (sqrt(-b^2 + sqrt(a*b)*b)*a^2*b + 8*sqrt(-b^2
 + sqrt(a*b)*b)*a*b^2)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt
(-(a*b - sqrt(a^2*b^2 + (a^2 - a*b)*a*b))/(a*b)))/(a^4*b^2 + 7*a^3*b^3 - 8*
a^2*b^4) - log(e^(d*x + c) + e^(-d*x - c) + 2)/a + log(e^(d*x + c) + e^(-d*
x - c) - 2)/a)/d
```

maple [A] time = 0.12, size = 159, normalized size = 1.17

$$-\frac{\sqrt{ab}\,\arctan\left(\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{2da\sqrt{-ab+\sqrt{ab}\,a}}-\frac{\sqrt{ab}\,\arctan\left(\frac{-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{2da\sqrt{-ab-\sqrt{ab}\,a}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x)
```

[Out] -1/2/d*(a*b)^(1/2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/2/d*(a*b)^(1/2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/d/a*ln(tanh(1/2*d*x+1/2*c))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad}+\frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad}-2\int\frac{be^{(7\,dx+7\,c)}-3\,be^{(5\,dx+5\,c)}+3\,be^{(3\,dx+3\,c)}-}{abe^{(8\,dx+8\,c)}-4\,abe^{(6\,dx+6\,c)}-4\,abe^{(2\,dx+2\,c)}+ab-2\left(8\,a^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d) - 2*integrate((b*e^(7*d*x + 7*c) - 3*b*e^(5*d*x + 5*c) + 3*b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*x)), x)

mupad [B] time = 9.19, size = 1243, normalized size = 9.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)),x)

[Out] log(((((4294967296*a*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b - 49*a^2 + 15*b^2))/(b^6*(a - b)^3) + (8589934592*a^2*d^3*exp(c + d*x)*(3*a*b + 16*a^2 - 15*b^2)*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/(b^7*(a - b)^2))*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 - (2147483648*d*exp(c + d*x)*(17*a - 15*b))/(b^6*(a - b)^2))*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(3*a*b + 16*a^2 - 15*b^2))/(a*b^6*(a - b)^3))*(-(a^2*b + (a^5*b)^(1/2))/(16*(a^5*d^2 - a^4*b*d^2)))^(1/2) - log(((((4294967296*a*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b - 49*a^2 + 15*b^2))/(b^6*(a - b)^3) - (8589934592*a^2*d^3*exp(c + d*x)*(3*a*b + 16*a^2 - 15*b^2)*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/(b^7*(a - b)^2))*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 + (2147483648*d*exp(c + d*x)*(17*a - 15*b))/(b^6*(a - b)^2))*(-(a^2*b + (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(3*a*b + 16*a^2 - 15*b^2))/(a*b^6*(a - b)^3))*(-(a^2*b + (a^5*b)^(1/2))/(16*(a^5*d^2 - a^4*b*d^2)))^(1/2) - (2*atan((exp(d*x)*exp(c)*(65536*a^2*(-a^2*d^2)^(1/2) + 50625*b^2*(-a^2*d^2)^(1/2) - 115200*a*b*(-a^2*d^2)^(1/2)))/(65536*a^3*d + 50625*a*b^2*d - 115200*a^2*b*d)))/(-a^2*d^2)^(1/2) - log(((((4294967296*a*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b - 49*a^2 + 15*b^2))/(b^6*(a - b)^3) - (8589934592*a^2*d^3*exp(c + d*x)*(3*a*b + 16*a^2 - 15*b^2)*(-(a^2*b - (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/(b^7*(a - b)^2))*(-(a^2*b - (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 + (2147483648*d*exp(c + d*x)*(17*a - 15*b))/(b^6*(a - b)^2))*(-(a^2*b - (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(3*a*b + 16*a^2 - 15*b^2))/(a*b^6*(a - b)^3))*(-(a^2*b - (a^5*b)^(1/2))/(16*(a^5*d^2 - a^4*b*d^2)))^(1/2) + log(((((4294967296*a*d^2*(exp(2*c + 2*d*x) + 1)*(26*a*b - 49*a^2 + 15*b^2))/(b^6*(a - b)^3) + (8589934592*a^2*d^3*exp(c + d*x)*(3*a*b + 16*a^2 - 15*b^2)*(-(a^2*b - (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/(b^7*(a - b)^2))*(-(a^2*b - (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 - (2147483648*d*exp(c + d*x)*(17*a - 15*b))/(b^6*(a - b)^2))*(-(a^2*b - (a^5*b)^(1/2))/(a^4*d^2*(a - b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(3*a*b + 16*a^2 - 15*b^2))/(a*b^6*(a - b)^3))*(-(a^2*b - (a^5*b)^(1/2))/(16*(a^5*d^2 - a^4*b*d^2)))^(1/2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.234 $\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(\cosh(c+dx)+1)}+\frac{\tanh^{-1}}{}$$

```
[Out] 1/2*arctanh(cosh(d*x+c))/a/d+1/4/a/d/(1-cosh(d*x+c))-1/4/a/d/(1+cosh(d*x+c)
)+1/2*b^(3/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(3/2)/d
/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^(3/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b
^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3215, 1170, 207, 1093, 205, 208}

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(\cosh(c+dx)+1)}+\frac{\tanh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(3/2
)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + ArcTanh[Cosh[c + d*x]]/(2*a*d) + (b^(3/4)*Ar
cTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqr
t[a] + Sqrt[b]]*d) + 1/(4*a*d*(1 - Cosh[c + d*x])) - 1/(4*a*d*(1 + Cosh[c +
 d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+2bx^2-bx^4)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}-\frac{1}{2a(-1+x^2)}+\frac{b}{a(a-b+2bx^2-bx^4)}\right)dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(1+\cosh(c+dx))}-\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(c+dx\right.}{2ad}$$

$$= \frac{\tanh^{-1}(\cosh(c+dx))}{2ad}+\frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(1+\cosh(c+dx))}-\frac{b^{3/2}\,\text{Subs}}{}$$

$$= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}\,d}+\frac{}{4a}$$

Mathematica [C] time = 0.39, size = 265, normalized size = 1.44

$$-\frac{4b\,\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&,\frac{2\#1^3\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -1/8*(Csch[(c + d*x)/2]^2 + 4*Log[Tanh[(c + d*x)/2]] + 4*b*RootSum[b - 4*b*
#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(c*#1) - d*x*#1 - 2*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
+ d*x)/2]*#1]*#1 + c*#1^3 + d*x*#1^3 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^3)/(-b - 8*a*#1^
2 + 3*b*#1^2 - 3*b*#1^4 + b*#1^6) & ] + Sech[(c + d*x)/2]^2)/(a*d)
```

fricas [B] time = 1.02, size = 1954, normalized size = 10.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*cosh(d*x + c)^3 + 12*cosh(d*x + c)*sinh(d*x + c)^2 + 4*sinh(d*x + c
)^3 - (a*d*cosh(d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh
(d*x + c)^4 - 2*a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(
d*x + c)^2 + a*d + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c
))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2
)/((a^4 - a^3*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d
*x + c) + b^2*sinh(d*x + c)^2 + b^2 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*si
nh(d*x + c) - ((a^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x
 + c))*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*
sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))) + (a
*d*cosh(d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c
)^4 - 2*a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)
^2 + a*d + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(
-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4
- a^3*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c)
+ b^2*sinh(d*x + c)^2 + b^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x +
 c) - ((a^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x + c))*s
qrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3
/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))) - (a*d*cosh(
d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c)^4 - 2*
a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^2 + a*d
 + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 -
a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*
d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sin
h(d*x + c)^2 + b^2 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a
^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x + c))*sqrt(b^3/(
(a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2
*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) + (a*d*cosh(d*x + c)^4
 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c)^4 - 2*a*d*cosh(d
*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^2 + a*d + 4*(a*d*
cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*d^2
*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(
b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)
^2 + b^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a^5 - a^4*b
)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x + c))*sqrt(b^3/((a^7 - 2*a
^6*b + a^5*b^2)*d^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a
^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) - 2*(cosh(d*x + c)^4 + 4*cosh(d*
x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d
*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*
x + c) + 1)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*(cosh(d*x + c)^4 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))
*sinh(d*x + c) + 1)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*cosh(d*x
+ c)^2 + 1)*sinh(d*x + c) + 4*cosh(d*x + c))/(a*d*cosh(d*x + c)^4 + 4*a*d*c
osh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c)^4 - 2*a*d*cosh(d*x + c)^2
+ 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^2 + a*d + 4*(a*d*cosh(d*x +
 c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 0.27, size = 463, normalized size = 2.52

$$\frac{2\left(\left(\sqrt{-b^2-\sqrt{ab}\,b}\,ab+8\sqrt{-b^2-\sqrt{ab}\,b}\,b^2\right)|a||b|-\left(\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,ab+8\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\,b^2\right)|b|\right)\arctan\left(\frac{e^{(dx+c)}+e^{(-dx-c)}}{2\sqrt{-\frac{ab+\sqrt{a^2b^2+(a^2-ab)ab}}{ab}}}\right)}{(a^3b^2+7a^2b^3-8ab^4)|a|}+\frac{2\left(\left(\sqrt{-b^2+\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/4*(2*((sqrt(-b^2 - sqrt(a*b)*b)*a*b + 8*sqrt(-b^2 - sqrt(a*b)*b)*b^2)*abs
(a)*abs(b) - (sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b + 8*sqrt(a*b)*sqrt(-b^
2 - sqrt(a*b)*b)*b^2)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(
-(a*b + sqrt(a^2*b^2 + (a^2 - a*b)*a*b))/(a*b)))/((a^3*b^2 + 7*a^2*b^3 - 8*
a*b^4)*abs(a)) + 2*((sqrt(-b^2 + sqrt(a*b)*b)*a*b + 8*sqrt(-b^2 + sqrt(a*b)
*b)*b^2)*abs(a)*abs(b) + (sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b + 8*sqrt(a
*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x
 - c))/sqrt(-(a*b - sqrt(a^2*b^2 + (a^2 - a*b)*a*b))/(a*b)))/((a^3*b^2 + 7*
a^2*b^3 - 8*a*b^4)*abs(a)) + log(e^(d*x + c) + e^(-d*x - c) + 2)/a - log(e^
(d*x + c) + e^(-d*x - c) - 2)/a - 4*(e^(d*x + c) + e^(-d*x - c))/(((e^(d*x
+ c) + e^(-d*x - c))^2 - 4)*a))/d
```

maple [A] time = 0.14, size = 190, normalized size = 1.03

$$\frac{\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{b\arctan\left(\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}-2a}{4\sqrt{-ab+\sqrt{ab}\,a}}\right)}{2da\sqrt{-ab+\sqrt{ab}\,a}}-\frac{b\arctan\left(\frac{-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\sqrt{ab}+2a}{4\sqrt{-ab-\sqrt{ab}\,a}}\right)}{2da\sqrt{-ab-\sqrt{ab}\,a}}-\frac{1}{8da\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2+1/2/d*b/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1
/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2)
)-1/2/d*b/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2
*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-1/8/d/a/tanh(1/2*d*x+1/2*
c)^2-1/2/d/a*ln(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{e^{(3\,dx+3\,c)}+e^{(dx+c)}}{ade^{(4\,dx+4\,c)}-2\,ade^{(2\,dx+2\,c)}+ad}+\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{2\,ad}-\frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{2\,ad}-8\int\frac{}{abe^{(8\,dx+8\,c)}-4\,abe^{(6\,dx+6\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*
c) + a*d) + 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) - 1/2*log((e^(d*x + c)
- 1)*e^(-c))/(a*d) - 8*integrate((b*e^(5*d*x + 5*c) - b*e^(3*d*x + 3*c))/(a
*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c) - 4*a*b*e^(2*d*x + 2*c) + a*b -
2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*x)), x)
```

mupad [B] time = 12.21, size = 1517, normalized size = 8.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^3*(a - b*sinh(c + d*x)^4)),x)
```

```
[Out] atan((exp(d*x)*exp(c)*(256*a^6*(-a^2*d^2)^(1/2) + b^6*(-a^2*d^2)^(1/2) + 96
*a^2*b^4*(-a^2*d^2)^(1/2) - 288*a^3*b^3*(-a^2*d^2)^(1/2) + 512*a^4*b^2*(-a^
2*d^2)^(1/2) - 16*a*b^5*(-a^2*d^2)^(1/2) - 512*a^5*b*(-a^2*d^2)^(1/2)))/(25
6*a^7*d - 16*a^2*b^5*d + 96*a^3*b^4*d - 288*a^4*b^3*d + 512*a^5*b^2*d + a*b
^6*d - 512*a^6*b*d))/(-a^2*d^2)^(1/2) - log(((((8589934592*d^3*exp(c + d*x
)*(8*a^2 - 7*a*b + 3*b^2)*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))^
(1/2))/(b^5*(a - b)^2) - (4294967296*d^2*(exp(2*c + 2*d*x) + 1)*(2*a*b^2 -
7*a^2*b + 12*a^3 + b^3))/(a^2*b^4*(a - b)^3))*(-((a^7*b^3)^(1/2) + a^3*b^2)
/(a^6*d^2*(a - b)))^(1/2))/4 - (4294967296*d*exp(c + d*x)*(2*a^2 - 2*a*b +
```

```
b^2))/(a^3*b^4*(a - b)^2))*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))
^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(4*a^3 - a*b^2 + b^3))/(a^5*b
^3*(a - b)^3))*(-((a^7*b^3)^(1/2) + a^3*b^2)/(16*(a^7*d^2 - a^6*b*d^2)))^(1
/2) + log((268435456*(exp(2*c + 2*d*x) + 1)*(4*a^3 - a*b^2 + b^3))/(a^5*b^3
*(a - b)^3) - (((((8589934592*d^3*exp(c + d*x)*(8*a^2 - 7*a*b + 3*b^2)*(-((
a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/(b^5*(a - b)^2) + (4294
967296*d^2*(exp(2*c + 2*d*x) + 1)*(2*a*b^2 - 7*a^2*b + 12*a^3 + b^3))/(a^2*
b^4*(a - b)^3))*(-((a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/4 -
 (4294967296*d*exp(c + d*x)*(2*a^2 - 2*a*b + b^2))/(a^3*b^4*(a - b)^2))*(-(
(a^7*b^3)^(1/2) + a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/4)*(-((a^7*b^3)^(1/2)
+ a^3*b^2)/(16*(a^7*d^2 - a^6*b*d^2)))^(1/2) - log((((((8589934592*d^3*exp(
c + d*x)*(8*a^2 - 7*a*b + 3*b^2)*(((a^7*b^3)^(1/2) - a^3*b^2)/(a^6*d^2*(a -
 b)))^(1/2))/(b^5*(a - b)^2) - (4294967296*d^2*(exp(2*c + 2*d*x) + 1)*(2*a*
b^2 - 7*a^2*b + 12*a^3 + b^3))/(a^2*b^4*(a - b)^3))*(((a^7*b^3)^(1/2) - a^3
*b^2)/(a^6*d^2*(a - b)))^(1/2))/4 - (4294967296*d*exp(c + d*x)*(2*a^2 - 2*a
*b + b^2))/(a^3*b^4*(a - b)^2))*(((a^7*b^3)^(1/2) - a^3*b^2)/(a^6*d^2*(a -
b)))^(1/2))/4 + (268435456*(exp(2*c + 2*d*x) + 1)*(4*a^3 - a*b^2 + b^3))/(a
^5*b^3*(a - b)^3))*(((a^7*b^3)^(1/2) - a^3*b^2)/(16*(a^7*d^2 - a^6*b*d^2)))
^(1/2) + log((268435456*(exp(2*c + 2*d*x) + 1)*(4*a^3 - a*b^2 + b^3))/(a^5*
b^3*(a - b)^3) - (((((8589934592*d^3*exp(c + d*x)*(8*a^2 - 7*a*b + 3*b^2)*(
((a^7*b^3)^(1/2) - a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/(b^5*(a - b)^2) + (42
94967296*d^2*(exp(2*c + 2*d*x) + 1)*(2*a*b^2 - 7*a^2*b + 12*a^3 + b^3))/(a^
2*b^4*(a - b)^3))*(((a^7*b^3)^(1/2) - a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/4
- (4294967296*d*exp(c + d*x)*(2*a^2 - 2*a*b + b^2))/(a^3*b^4*(a - b)^2))*((
(a^7*b^3)^(1/2) - a^3*b^2)/(a^6*d^2*(a - b)))^(1/2))/4)*(((a^7*b^3)^(1/2) -
 a^3*b^2)/(16*(a^7*d^2 - a^6*b*d^2)))^(1/2) - exp(c + d*x)/(a*d*(exp(2*c +
2*d*x) - 1)) - (2*exp(c + d*x))/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x)
 + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.235 $\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=175

$$-\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(\tanh(c+dx)+$$

```
[Out] 1/2*x/b-1/2*a^(3/4)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/b^
(3/2)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*a^(3/4)*arctanh((a^(1/2)+b^(1/2))^(1/2)
*tanh(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)+b^(1/2))^(1/2)-1/4/b/d/(1-tanh(d*x
+c))+1/4/b/d/(1+tanh(d*x+c))
```

Rubi [A] time = 0.26, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3217, 1287, 207, 1130, 208}

$$-\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(\tanh(c+dx)+$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] x/(2*b) - (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]
)/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)*d) + (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] +
Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/2)*d) - 1
/(4*b*d*(1 - Tanh[c + d*x])) + 1/(4*b*d*(1 + Tanh[c + d*x]))
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2(a-2ax^2+(a-b)x^4)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{4b(-1+x)^2}-\frac{1}{4b(1+x)^2}-\frac{1}{2b(-1+x^2)}+\frac{ax^2}{b(a-2ax^2+(a-b)x^4)}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(1+\tanh(c+dx))}-\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c\right.}{2bd}$$

$$= \frac{x}{2b}-\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(1+\tanh(c+dx))}+\frac{\left(a\left(\sqrt{a}+\sqrt{b}\right)\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{x}{2b}-\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/2}d}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/2}d}-\frac{}{4bd(1}$$

Mathematica [A] time = 0.89, size = 158, normalized size = 0.90

$$\frac{\frac{2a\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{2a\tan^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}+2\sqrt{b}(c+dx)-\sqrt{b}\sinh(2(c+dx))}{4b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (2*Sqrt[b]*(c + d*x) + (2*a*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt
[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (2*a*ArcTanh[((Sqrt[a
] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sq
rt[b]] - Sqrt[b]*Sinh[2*(c + d*x)])/(4*b^(3/2)*d)
```

fricas [B] time = 1.47, size = 1441, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*d*x*cosh(d*x + c)^2 - cosh(d*x + c)^4 - 4*cosh(d*x + c)*sinh(d*x + c
)^3 - sinh(d*x + c)^4 + 2*(2*d*x - 3*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*(
b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)
^2)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2
)/((a*b^3 - b^4)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d
```

```
*x + c) + a^2*sinh(d*x + c)^2 + 2*(a^2*b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5
- 2*a*b^6 + b^7)*d^4)) - a^2 + 2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*
a*b^6 + b^7)*d^4)) - a^2*b*d)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 -
2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + 2*(b*d*cosh(d*x + c)^2
+ 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)*sqrt(((a*b^3 - b
^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2
))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d
*x + c)^2 + 2*(a^2*b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4
)) - a^2 - 2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) -
 a^2*b*d)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))
 + a^2)/((a*b^3 - b^4)*d^2))) + 2*(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c
)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((
a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(a^2*cosh(d*x
 + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 - 2*(a^2*
b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2 + 2*((a*b^
4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2*b*d)*sqrt(-((a
*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 -
b^4)*d^2))) - 2*(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) +
b*d*sinh(d*x + c)^2)*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6
+ b^7)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*co
sh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 - 2*(a^2*b^2 - a*b^3)*d^2*s
qrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2 - 2*((a*b^4 - b^5)*d^3*sqrt(
a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2*b*d)*sqrt(-((a*b^3 - b^4)*d^2*sq
rt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) + 4*(2
*d*x*cosh(d*x + c) - cosh(d*x + c)^3)*sinh(d*x + c) + 1)/(b*d*cosh(d*x + c)
^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, replacing 0 by ` u`, a substitution
 variable should perhaps be purged.Not invertible Error: Bad Argument Value
```

maple [C] time = 0.09, size = 223, normalized size = 1.27

$$-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^4-\_R^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}\right)}{db}-\frac{1}{2db\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{2db\left(\tanh\left(\frac{}{}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/d*a/b*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1
/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/
2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)-1/2/d/b*ln(
tanh(1/2*d*x+1/2*c)-1)+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/b/(tanh(1/2*
d*x+1/2*c)+1)+1/2/d/b*ln(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(4\,dxe^{(2\,dx+2\,c)}-e^{(4\,dx+4\,c)}+1\right)e^{(-2\,dx-2\,c)}}{8\,bd}-\frac{1}{64}\int\frac{256\left(ae^{(6\,dx+6\,c)}-2\,ae^{(4\,dx+4\,c)}+ae^{(2\,dx+2\,c)}\right)}{b^2e^{(8\,dx+8\,c)}-4\,b^2e^{(6\,dx+6\,c)}-4\,b^2e^{(2\,dx+2\,c)}+b^2-2\left(8\,abe^{(4\,c)}-3\,b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/8*(4*d*x*e^(2*d*x + 2*c) - e^(4*d*x + 4*c) + 1)*e^(-2*d*x - 2*c)/(b*d) -
1/64*integrate(256*(a*e^(6*d*x + 6*c) - 2*a*e^(4*d*x + 4*c) + a*e^(2*d*x +
2*c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c) - 4*b^2*e^(2*d*x + 2*c)
+ b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*x)), x)
```

mupad [B] time = 11.24, size = 2191, normalized size = 12.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log(((((((4194304*a^6*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*
b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c
 + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(b^12*(a - b)^2) - (16777216*a^
6*d^3*(((a^3*b^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2)*(40*a*b^2 - 35*
b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*exp(2*
c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^11*(a - b)))*(((a^3*b^7)^(1/2)
 + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (2097152*a^7*d*(256*a^2*b - 256*a
*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x) + 756*a*b
^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(b^14*(a - b)))*(((a^3*b
^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (524288*a^8*(185*a*b^2 -
 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c
+ 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(b^15
*(a - b)^2))*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*(b^7*d^2 - a*b^6*d^2)))^(1/2
) - log(((((((4194304*a^6*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*
a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp
(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(b^12*(a - b)^2) + (1677721
6*a^6*d^3*(((a^3*b^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2)*(40*a*b^2 -
 35*b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*ex
p(2*c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^11*(a - b)))*(((a^3*b^7)^(
1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 + (2097152*a^7*d*(256*a^2*b - 2
56*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x) + 756
*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(b^14*(a - b)))*(((a
^3*b^7)^(1/2) + a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (524288*a^8*(185*a*b
^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(
2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(
b^15*(a - b)^2))*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*(b^7*d^2 - a*b^6*d^2)))^
(1/2) + log(((((((4194304*a^6*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 +
930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b
*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(b^12*(a - b)^2) - (167
77216*a^6*d^3*(-((a^3*b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2)*(40*a*
b^2 - 35*b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b
^2*exp(2*c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^11*(a - b)))*(-((a^3*
b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (2097152*a^7*d*(256*a^2
*b - 256*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x)
 + 756*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(b^14*(a - b))
)*(-((a^3*b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (524288*a^8*(
185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b
^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d
*x)))/(b^15*(a - b)^2))*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*(b^7*d^2 - a*b^6*d
^2)))^(1/2) - log(((((((4194304*a^6*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 -
b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768
*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(b^12*(a - b)^2)
+ (16777216*a^6*d^3*(-((a^3*b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2)*
(40*a*b^2 - 35*b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 3
```

```
26*a*b^2*exp(2*c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^11*(a - b)))*(-
((a^3*b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 + (2097152*a^7*d*(2
56*a^2*b - 256*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c +
2*d*x) + 756*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(b^14*(a
 - b)))*(-((a^3*b^7)^(1/2) - a^2*b^3)/(b^6*d^2*(a - b)))^(1/2))/4 - (524288
*a^8*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x)
- 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c
 + 2*d*x)))/(b^15*(a - b)^2))*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*(b^7*d^2 - a
*b^6*d^2)))^(1/2) + x/(2*b) + exp(- 2*c - 2*d*x)/(8*b*d) - exp(2*c + 2*d*x)
/(8*b*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.236 $\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x}{b}$$

```
[Out] -x/b+1/2*a^(1/4)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/b/d/(
a^(1/2)-b^(1/2))^(1/2)+1/2*a^(1/4)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x
+c)/a^(1/4))/b/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1287, 207, 1166, 208}

$$\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(x/b) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])
/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b*d) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]
]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a-2ax^2+(a-b)x^4)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b(-1+x^2)} + \frac{a(1-x^2)}{b(a-2ax^2+(a-b)x^4)}\right) dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{bd} + \frac{a\,\text{Subst}\left(\int \frac{1-x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{bd}$$
$$= -\frac{x}{b} - \frac{\left(\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)\right)\text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2bd} - \frac{\left(a\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\right.}{}$$
$$= -\frac{x}{b} + \frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,bd} + \frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,bd}$$

Mathematica [A] time = 0.47, size = 143, normalized size = 1.13

$$\frac{\frac{\sqrt{a}\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}} - 2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (-2*(c + d*x) - (Sqrt[a]*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a
 + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[a]*ArcTanh[((Sqrt[
a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*S
qrt[b]])/(2*b*d)
```

fricas [B] time = 1.17, size = 1009, normalized size = 7.94

$$-\frac{b\sqrt{\frac{(ab^2-b^3)d^2\sqrt{\frac{a}{(a^2b^3-2ab^4+b^5)d^4}}+a}{(ab^2-b^3)d^2}}\log\left(2\left(ab-b^2\right)d^2\sqrt{\frac{a}{(a^2b^3-2ab^4+b^5)d^4}} + \cosh(dx+c)^2 + 2\cosh(dx+c)\sinh(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(b*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a
)/((a*b^2 - b^3)*d^2))*log(2*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b
^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - b*d)*sq
```

```
rt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a)/((a*b^2
- b^3)*d^2)) - 1) - b*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 +
b^5)*d^4)) + a)/((a*b^2 - b^3)*d^2))*log(2*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3
 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 - 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d
^4)) - b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
 + a)/((a*b^2 - b^3)*d^2)) - 1) - b*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-2*(a*b - b^2)*d^2*
sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 -
2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a
*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2)) - 1) + b*sqrt(-((a*b^2 - b^3)*d
^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-2
*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*((a*b^2 - b^3)*d^3*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(
a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2)) - 1) + 4*x)/b
```

giac [A] time = 0.53, size = 13, normalized size = 0.10

$$-\frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -(d*x + c)/(b*d)
```

maple [C] time = 0.07, size = 144, normalized size = 1.13

$$-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\left(\_R^6-3\_R^4+3\_R^2-1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}\right)}{4db}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{db}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/4/d*a/b*sum((_R^6-3*_R^4+3*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R
*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*
a*_Z^2+a))+1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-16\,a\int\frac{e^{(4\,dx+4\,c)}}{b^2e^{(8\,dx+8\,c)}-4\,b^2e^{(6\,dx+6\,c)}-4\,b^2e^{(2\,dx+2\,c)}+b^2-2\left(8\,abe^{(4\,c)}-3\,b^2e^{(4\,c)}\right)e^{(4\,dx)}}\,dx-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -16*a*integrate(e^(4*d*x + 4*c)/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c
) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*
x)), x) - x/b
```

mupad [B] time = 11.05, size = 1861, normalized size = 14.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((524288*a^3*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp
(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*
b*exp(2*c + 2*d*x)))/(b^8*(a - b)) + (1048576*a^3*d^3*((a*b^2 - (a*b^5)^(1/
2))/(b^4*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3
*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))
/(b^7*(a - b)))*((a*b^2 - (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (262
144*a^4*d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(
2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^9*(a - b)))*((a*b^2 - (a*b^5)^
(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (32768*a^4*(128*a*b - 128*a^2 - 15*b^2
 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2*c + 2*d*x) - 304*a*b*exp(2*c + 2
*d*x)))/(b^10*(a - b)))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(b^5*d^2 - a*b^4*d^2)
))^(1/2) - log((((((524288*a^3*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 +
256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x)
 - 240*a^2*b*exp(2*c + 2*d*x)))/(b^8*(a - b)) - (1048576*a^3*d^3*((a*b^2 -
(a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*
b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c
 + 2*d*x)))/(b^7*(a - b)))*((a*b^2 - (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2
))/4 - (262144*a^4*d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) +
31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^9*(a - b)))*((a*b^2
 - (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (32768*a^4*(128*a*b - 128*a
^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2*c + 2*d*x) - 304*a*b*
exp(2*c + 2*d*x)))/(b^10*(a - b)))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(b^5*d^2 -
 a*b^4*d^2)))^(1/2) - x/b - log((((((524288*a^3*d^2*(31*a*b^2 - 128*a^2*b +
 128*a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2
*exp(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d*x)))/(b^8*(a - b)) - (1048576*a
^3*d^3*((a*b^2 + (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^
2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) +
 48*a^2*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b^2 + (a*b^5)^(1/2))/(b^4*d
^2*(a - b)))^(1/2))/4 - (262144*a^4*d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*ex
p(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^9*
(a - b)))*((a*b^2 + (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (32768*a^4
*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2*c +
2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(b^10*(a - b)))*(-(a*b^2 + (a*b^5)^(1/2
))/(16*(b^5*d^2 - a*b^4*d^2)))^(1/2) + log((((((524288*a^3*d^2*(31*a*b^2 -
128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x)
 + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d*x)))/(b^8*(a - b)) +
 (1048576*a^3*d^3*((a*b^2 + (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2)*(45*a*b
^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c
 + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b^2 + (a*b^5)^(1
/2))/(b^4*d^2*(a - b)))^(1/2))/4 + (262144*a^4*d*(72*a*b - 64*a^2 - 9*b^2 +
 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d
*x)))/(b^9*(a - b)))*((a*b^2 + (a*b^5)^(1/2))/(b^4*d^2*(a - b)))^(1/2))/4 +
 (32768*a^4*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2
*exp(2*c + 2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(b^10*(a - b)))*(-(a*b^2 + (
a*b^5)^(1/2))/(16*(b^5*d^2 - a*b^4*d^2)))^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.237 $\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\,d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\,d\sqrt{\sqrt{a}-\sqrt{b}}}$$

```
[Out] -1/2*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(1/4)/d/b^(1/2)
/(a^(1/2)-b^(1/2))^(1/2)+1/2*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^
(1/4))/a^(1/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3217, 1130, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\,d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}\,d\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[S
qrt[a] - Sqrt[b]]*Sqrt[b]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*
x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{1}{-a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2d} + \frac{\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{}{-a-\sqrt{a}}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\,\sqrt{b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\,\sqrt{b}\,d}$$

Mathematica [A] time = 0.36, size = 127, normalized size = 1.02

$$\frac{\frac{\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{2\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/Sqr
t[-a + Sqrt[a]*Sqrt[b]] + ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[
a + Sqrt[a]*Sqrt[b]]]/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[b]*d)
```

fricas [B] time = 1.02, size = 975, normalized size = 7.80

$$-\frac{1}{4}\sqrt{\frac{\left(ab-b^2\right)d^2\sqrt{\frac{1}{\left(a^3b-2\,a^2b^2+ab^3\right)d^4}}+1}{\left(ab-b^2\right)d^2}}\,\log\left(2\left(a^2-ab\right)d^2\sqrt{\frac{1}{\left(a^3b-2\,a^2b^2+ab^3\right)d^4}}+\cosh\left(dx+c\right)^2+2\,\cosh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/(
(a*b - b^2)*d^2))*log(2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)
*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
+ 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - a*d)*s
qrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a*b -
 b^2)*d^2)) - 1) + 1/4*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a
*b^3)*d^4)) + 1)/((a*b - b^2)*d^2))*log(2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b -
2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2 - 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b
^3)*d^4)) - a*d)*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*
d^4)) + 1)/((a*b - b^2)*d^2)) - 1) + 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^
3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2))*log(-2*(a^2 - a*b)*d
^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x
 + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3
*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b
- 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2)) - 1) - 1/4*sqrt(-((a*b -
 b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2))
*log(-2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*((a^2*b - a*
```

```
b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(-((a*b - b^2
)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2)) - 1
)
```

giac [A] time = 0.35, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
[Out] 0
```

maple [C] time = 0.08, size = 94, normalized size = 0.75

$$-\frac{\sum_{\_R=\mathrm{RootOf}\left(a\,\_Z^8-4a\,\_Z^6+(6a-16b)\,\_Z^4-4a\,\_Z^2+a\right)}\frac{\left(\_R^4-\_R^2\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x)
[Out] -1/d*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d
*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sinh\left(dx+c\right)^2}{b\sinh\left(dx+c\right)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
[Out] -integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^4 - a), x)
```

mupad [B] time = 12.90, size = 1859, normalized size = 14.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4),x)
[Out] log(((((((262144*a^2*d^2*(102*a*b - 128*a^2 - 22*b^2 - 272*a^2*exp(2*c + 2*d
*x) + 19*b^2*exp(2*c + 2*d*x) + 189*a*b*exp(2*c + 2*d*x)))/(b^6*(a - b)) -
(131072*a^2*d^3*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2)*(119*a*b^
2 - 136*a^2*b + b^3 - 1024*a^3*exp(2*c + 2*d*x) + 9*b^3*exp(2*c + 2*d*x) -
809*a*b^2*exp(2*c + 2*d*x) + 1808*a^2*b*exp(2*c + 2*d*x)))/(b^6*(a - b)))*(
(a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 + (32768*a*d*(120*a^2*b
 - 129*a*b^2 + b^3 - 1024*a^3*exp(2*c + 2*d*x) - b^3*exp(2*c + 2*d*x) + 201
*a*b^2*exp(2*c + 2*d*x) + 816*a^2*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b
 + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 - (16384*a*(106*a*b - 128*a
^2 - 2*b^2 + 240*a^2*exp(2*c + 2*d*x) + 3*b^2*exp(2*c + 2*d*x) - 275*a*b*ex
p(2*c + 2*d*x)))/(b^7*(a - b)))*(-(a*b + (a*b^3)^(1/2))/(16*(a*b^3*d^2 - a^
2*b^2*d^2)))^(1/2) - log((((((262144*a^2*d^2*(102*a*b - 128*a^2 - 22*b^2 -
272*a^2*exp(2*c + 2*d*x) + 19*b^2*exp(2*c + 2*d*x) + 189*a*b*exp(2*c + 2*d*
x)))/(b^6*(a - b)) + (131072*a^2*d^3*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a -
 b)))^(1/2)*(119*a*b^2 - 136*a^2*b + b^3 - 1024*a^3*exp(2*c + 2*d*x) + 9*b^
3*exp(2*c + 2*d*x) - 809*a*b^2*exp(2*c + 2*d*x) + 1808*a^2*b*exp(2*c + 2*d*
```

```
x)))/(b^6*(a - b)))*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 -
(32768*a*d*(120*a^2*b - 129*a*b^2 + b^3 - 1024*a^3*exp(2*c + 2*d*x) - b^3*e
xp(2*c + 2*d*x) + 201*a*b^2*exp(2*c + 2*d*x) + 816*a^2*b*exp(2*c + 2*d*x)))
/(b^7*(a - b)))*((a*b + (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 - (163
84*a*(106*a*b - 128*a^2 - 2*b^2 + 240*a^2*exp(2*c + 2*d*x) + 3*b^2*exp(2*c
+ 2*d*x) - 275*a*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*(-(a*b + (a*b^3)^(1/2)
)/(16*(a*b^3*d^2 - a^2*b^2*d^2)))^(1/2) + log((((((262144*a^2*d^2*(102*a*b
- 128*a^2 - 22*b^2 - 272*a^2*exp(2*c + 2*d*x) + 19*b^2*exp(2*c + 2*d*x) + 1
89*a*b*exp(2*c + 2*d*x)))/(b^6*(a - b)) - (131072*a^2*d^3*((a*b - (a*b^3)^(
1/2))/(a*b^2*d^2*(a - b)))^(1/2)*(119*a*b^2 - 136*a^2*b + b^3 - 1024*a^3*ex
p(2*c + 2*d*x) + 9*b^3*exp(2*c + 2*d*x) - 809*a*b^2*exp(2*c + 2*d*x) + 1808
*a^2*b*exp(2*c + 2*d*x)))/(b^6*(a - b)))*((a*b - (a*b^3)^(1/2))/(a*b^2*d^2*
(a - b)))^(1/2))/4 + (32768*a*d*(120*a^2*b - 129*a*b^2 + b^3 - 1024*a^3*exp
(2*c + 2*d*x) - b^3*exp(2*c + 2*d*x) + 201*a*b^2*exp(2*c + 2*d*x) + 816*a^2
*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b - (a*b^3)^(1/2))/(a*b^2*d^2*(a -
 b)))^(1/2))/4 - (16384*a*(106*a*b - 128*a^2 - 2*b^2 + 240*a^2*exp(2*c + 2*
d*x) + 3*b^2*exp(2*c + 2*d*x) - 275*a*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*(
-(a*b - (a*b^3)^(1/2))/(16*(a*b^3*d^2 - a^2*b^2*d^2)))^(1/2) - log((((((262
144*a^2*d^2*(102*a*b - 128*a^2 - 22*b^2 - 272*a^2*exp(2*c + 2*d*x) + 19*b^2
*exp(2*c + 2*d*x) + 189*a*b*exp(2*c + 2*d*x)))/(b^6*(a - b)) + (131072*a^2*
d^3*((a*b - (a*b^3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2)*(119*a*b^2 - 136*a^2*
b + b^3 - 1024*a^3*exp(2*c + 2*d*x) + 9*b^3*exp(2*c + 2*d*x) - 809*a*b^2*ex
p(2*c + 2*d*x) + 1808*a^2*b*exp(2*c + 2*d*x)))/(b^6*(a - b)))*((a*b - (a*b^
3)^(1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 - (32768*a*d*(120*a^2*b - 129*a*b^2
 + b^3 - 1024*a^3*exp(2*c + 2*d*x) - b^3*exp(2*c + 2*d*x) + 201*a*b^2*exp(2
*c + 2*d*x) + 816*a^2*b*exp(2*c + 2*d*x)))/(b^7*(a - b)))*((a*b - (a*b^3)^(
1/2))/(a*b^2*d^2*(a - b)))^(1/2))/4 - (16384*a*(106*a*b - 128*a^2 - 2*b^2 +
 240*a^2*exp(2*c + 2*d*x) + 3*b^2*exp(2*c + 2*d*x) - 275*a*b*exp(2*c + 2*d*
x)))/(b^7*(a - b)))*(-(a*b - (a*b^3)^(1/2))/(16*(a*b^3*d^2 - a^2*b^2*d^2)))
^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.238 $\int \frac{1}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] 1/2*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)
-b^(1/2))^(1/2)+1/2*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^
(3/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3209, 1166, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[c + d*x]^4)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sq
rt[a] - Sqrt[b]]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/
4)]/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{-a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2d} - \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{}{-a-\sqrt{}}\right.}{}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

Mathematica [A] time = 0.25, size = 128, normalized size = 1.11

$$\frac{\frac{\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-1),x]
```

```
[Out] (-(ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/S
qrt[-a + Sqrt[a]*Sqrt[b]]) + ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sq
rt[a + Sqrt[a]*Sqrt[b]]]/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*d)
```

fricas [B] time = 3.68, size = 975, normalized size = 8.48

$$-\frac{1}{4}\sqrt{\frac{\left(a^2-ab\right)d^2\sqrt{\frac{b}{\left(a^5-2\,a^4b+a^3b^2\right)d^4}}+1}{\left(a^2-ab\right)d^2}}\,\log\left(2\left(a^3-a^2b\right)d^2\sqrt{\frac{b}{\left(a^5-2\,a^4b+a^3b^2\right)d^4}}+b\cosh\left(dx+c\right)^2+2\,b\cosh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a
^2 - a*b)*d^2))*log(2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d
^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
)^2 + 2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)
*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a^2 -
 a*b)*d^2)) - b) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b
^2)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*
a^4*b + a^3*b^2)*d^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
) + b*sinh(d*x + c)^2 - 2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b
^2)*d^4)) - a*b*d)*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*
d^4)) + 1)/((a^2 - a*b)*d^2)) - b) + 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^
5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-2*(a^3 - a^2*b)*d
^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*((a^4 - a^3*b)*d^3*sqrt(b/((a
^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5
- 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2)) - b) - 1/4*sqrt(-((a^2 -
 a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*l
og(-2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - 2*((a^4 -
```

```
a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(-((a^2 - a
*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2)) - b
)
```

giac [A] time = 0.16, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 0
```

maple [C] time = 0.08, size = 102, normalized size = 0.89

$$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_Z^{2}+a\right)} \frac{\left(-\_R^{6}+3\_R^{4}-3\_R^{2}+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{7}a-3\_R^{5}a+3\_R^{3}a-8\_R^{3}b-\_Ra}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] 1/4/d*sum((-_R^6+3*_R^4-3*_R^2+1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*
ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z
^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b\sinh\left(dx+c\right)^{4}-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sinh(d*x + c)^4 - a), x)
```

mupad [B] time = 10.10, size = 1787, normalized size = 15.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sinh(c + d*x)^4),x)
```

```
[Out] log((((((524288*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*c
 + 2*d*x) + b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*ex
p(2*c + 2*d*x)))/(b^5*(a - b)) + (1048576*a*d^3*((a^2 - (a^3*b)^(1/2))/(a^3
*d^2*(a - b)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c
 + 2*d*x) - 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a
 - b)))*((a^2 - (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 + (262144*d*(72*
a*b - 64*a^2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) -
 288*a*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*((a^2 - (a^3*b)^(1/2))/(a^3*d^2*
(a - b)))^(1/2))/4 + (32768*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c +
 2*d*x) + 29*b^2*exp(2*c + 2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(a*b^5*(a -
b)))*((a^2 - (a^3*b)^(1/2))/(16*(a^4*d^2 - a^3*b*d^2)))^(1/2) - log((((((52
4288*d^2*(31*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) +
 b^3*exp(2*c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d
*x)))/(b^5*(a - b)) - (1048576*a*d^3*((a^2 - (a^3*b)^(1/2))/(a^3*d^2*(a - b
)))^(1/2)*(45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) -
```

```
 50*a*b^2*exp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*((a
^2 - (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 - (262144*d*(72*a*b - 64*a^
2 - 9*b^2 + 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*ex
p(2*c + 2*d*x)))/(b^5*(a - b)))*((a^2 - (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(
1/2))/4 + (32768*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 2
9*b^2*exp(2*c + 2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(a*b^5*(a - b)))*((a^2
- (a^3*b)^(1/2))/(16*(a^4*d^2 - a^3*b*d^2)))^(1/2) - log(((((524288*d^2*(3
1*a*b^2 - 128*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*
c + 2*d*x) + 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d*x)))/(b^5*
(a - b)) - (1048576*a*d^3*((a^2 + (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2)*(
45*a*b^2 - 104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*e
xp(2*c + 2*d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*((a^2 + (a^3*b
)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 - (262144*d*(72*a*b - 64*a^2 - 9*b^2 +
 256*a^2*exp(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d
*x)))/(b^5*(a - b)))*((a^2 + (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 + (
32768*(128*a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2
*c + 2*d*x) - 304*a*b*exp(2*c + 2*d*x)))/(a*b^5*(a - b)))*((a^2 + (a^3*b)^(
1/2))/(16*(a^4*d^2 - a^3*b*d^2)))^(1/2) + log(((((524288*d^2*(31*a*b^2 - 1
28*a^2*b + 128*a^3 - b^3 + 256*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x)
+ 21*a*b^2*exp(2*c + 2*d*x) - 240*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)) +
(1048576*a*d^3*((a^2 + (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2)*(45*a*b^2 -
104*a^2*b + 64*a^3 - 3*b^3 + 4*b^3*exp(2*c + 2*d*x) - 50*a*b^2*exp(2*c + 2*
d*x) + 48*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*((a^2 + (a^3*b)^(1/2))/(a
^3*d^2*(a - b)))^(1/2))/4 + (262144*d*(72*a*b - 64*a^2 - 9*b^2 + 256*a^2*ex
p(2*c + 2*d*x) + 31*b^2*exp(2*c + 2*d*x) - 288*a*b*exp(2*c + 2*d*x)))/(b^5*
(a - b)))*((a^2 + (a^3*b)^(1/2))/(a^3*d^2*(a - b)))^(1/2))/4 + (32768*(128*
a*b - 128*a^2 - 15*b^2 + 256*a^2*exp(2*c + 2*d*x) + 29*b^2*exp(2*c + 2*d*x)
 - 304*a*b*exp(2*c + 2*d*x)))/(a*b^5*(a - b)))*((a^2 + (a^3*b)^(1/2))/(16*(
a^4*d^2 - a^3*b*d^2)))^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.239 $\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth(c+dx)}{ad}$$

```
[Out] -coth(d*x+c)/a/d-1/2*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*b
^(1/2)/a^(5/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*arctanh((a^(1/2)+b^(1/2))^(1/2
)*tanh(d*x+c)/a^(1/4))*b^(1/2)/a^(5/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1130, 208}

$$-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(5
/4)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + (Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*
Tanh[c + d*x])/a^(1/4)])/(2*a^(5/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Coth[c + d
*x]/(a*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2\left(a-2ax^2+(a-b)x^4\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{bx^2}{a\left(a-2ax^2+(a-b)x^4\right)}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\coth(c+dx)}{ad}+\frac{b\,\text{Subst}\left(\int \frac{x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{ad}$$
$$= -\frac{\coth(c+dx)}{ad}+\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\,\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2ad}+$$
$$= -\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{\coth(c+dx)}{ad}$$

Mathematica [A] time = 0.81, size = 143, normalized size = 1.03

$$\frac{\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\,\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}+a}}+\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\,\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}-a}}-2\coth(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] ((Sqrt[b]*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt
[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]*ArcTanh[((Sqrt[a] + Sqrt[b])*T
anh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - 2*Cot
h[c + d*x])/(2*a*d)
```

fricas [B] time = 1.14, size = 1305, normalized size = 9.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d
*x + c)^2 - a*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2
)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x
+ c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a
^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 + 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 -
 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7
 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - (a*d*cosh(d*x + c)
^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(((
a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2
*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2
*sinh(d*x + c)^2 + 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*
d^4)) - b^2 - 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)
) - a^2*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^
4)) + b)/((a^3 - a^2*b)*d^2))) - (a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)
```

```
*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b
^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(
d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - 2*(a
^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 + 2*((a^5 -
 a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(-((a^
3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b
)*d^2))) + (a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*s
inh(d*x + c)^2 - a*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a
^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cos
h(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - 2*(a^4 - a^3*b)*d^2*sqrt(b
^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 - 2*((a^5 - a^4*b)*d^3*sqrt(b^3/(
(a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^
3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))) + 8)/(a*d*cos
h(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*
d)
```

giac [A] time = 0.22, size = 21, normalized size = 0.15

$$-\frac{2}{ad\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -2/(a*d*(e^(2*d*x + 2*c) - 1))
```

maple [C] time = 0.12, size = 135, normalized size = 0.97

$$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_Z^{2}+a\right)}\frac{\left(\_R^{4}-\_R^{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{7}a-3\_R^{5}a+3\_R^{3}a-8\_R^{3}b-\_Ra}\right)}{da}-\frac{1}{2da\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/d/a*b*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R^
3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*
a-16*b)*_Z^4-4*a*_Z^2+a))-1/2/d/a/tanh(1/2*d*x+1/2*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{ade^{(2\,dx+2\,c)}-ad}-4\int\frac{be^{(6\,dx+6\,c)}-2\,be^{(4\,dx+4\,c)}+be^{(2\,dx+2\,c)}}{abe^{(8\,dx+8\,c)}-4\,abe^{(6\,dx+6\,c)}-4\,abe^{(2\,dx+2\,c)}+ab-2\left(8\,a^{2}e^{(4\,c)}-3\,abe^{(4\,c)}\right)e^{(4\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(6*d*x + 6*c) - 2*b*e^(4*
d*x + 4*c) + b*e^(2*d*x + 2*c))/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c
) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*
x)), x)
```

mupad [B] time = 11.29, size = 2128, normalized size = 15.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)),x)
```

```
[Out] log((((((4194304*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2
+ b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2
*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(a^2*b^4*(a - b)^2) - (16777216*d^3
*(((a^5*b^3)^(1/2) + a^3*b)/(a^5*d^2*(a - b)))^(1/2)*(40*a*b^2 - 35*b^3 + 5
12*a^3*exp(2*c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*exp(2*c + 2*d
*x) - 896*a^2*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*(((a^5*b^3)^(1/2) + a^3*b
)/(a^5*d^2*(a - b)))^(1/2))/4 - (2097152*d*(256*a^2*b - 256*a*b^2 - 5*b^3 -
 1024*a^3*exp(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x) + 756*a*b^2*exp(2*c + 2
*d*x) + 256*a^2*b*exp(2*c + 2*d*x)))/(a^3*b^4*(a - b)))*(((a^5*b^3)^(1/2) +
 a^3*b)/(a^5*d^2*(a - b)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*
a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*
b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(a^4*b^3*(a - b)^2))*(
((a^5*b^3)^(1/2) + a^3*b)/(16*(a^6*d^2 - a^5*b*d^2)))^(1/2) - log((((((4194
304*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c
 + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*
a^2*b^2*exp(2*c + 2*d*x)))/(a^2*b^4*(a - b)^2) + (16777216*d^3*(((a^5*b^3)^
(1/2) + a^3*b)/(a^5*d^2*(a - b)))^(1/2)*(40*a*b^2 - 35*b^3 + 512*a^3*exp(2*
c + 2*d*x) + 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*exp(2*c + 2*d*x) - 896*a^2
*b*exp(2*c + 2*d*x)))/(b^5*(a - b)))*(((a^5*b^3)^(1/2) + a^3*b)/(a^5*d^2*(a
 - b)))^(1/2))/4 + (2097152*d*(256*a^2*b - 256*a*b^2 - 5*b^3 - 1024*a^3*exp
(2*c + 2*d*x) + 6*b^3*exp(2*c + 2*d*x) + 756*a*b^2*exp(2*c + 2*d*x) + 256*a
^2*b*exp(2*c + 2*d*x)))/(a^3*b^4*(a - b)))*(((a^5*b^3)^(1/2) + a^3*b)/(a^5*
d^2*(a - b)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3
- 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c +
 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(a^4*b^3*(a - b)^2))*(((a^5*b^3)^(1
/2) + a^3*b)/(16*(a^6*d^2 - a^5*b*d^2)))^(1/2) + log((((((4194304*d^2*(512*
a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 6
27*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2
*c + 2*d*x)))/(a^2*b^4*(a - b)^2) - (16777216*d^3*(-((a^5*b^3)^(1/2) - a^3*
b)/(a^5*d^2*(a - b)))^(1/2)*(40*a*b^2 - 35*b^3 + 512*a^3*exp(2*c + 2*d*x) +
 64*b^3*exp(2*c + 2*d*x) + 326*a*b^2*exp(2*c + 2*d*x) - 896*a^2*b*exp(2*c +
 2*d*x)))/(b^5*(a - b)))*(-((a^5*b^3)^(1/2) - a^3*b)/(a^5*d^2*(a - b)))^(1/
2))/4 - (2097152*d*(256*a^2*b - 256*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*
x) + 6*b^3*exp(2*c + 2*d*x) + 756*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*
c + 2*d*x)))/(a^3*b^4*(a - b)))*(-((a^5*b^3)^(1/2) - a^3*b)/(a^5*d^2*(a - b
)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3
*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) +
2048*a^2*b*exp(2*c + 2*d*x)))/(a^4*b^3*(a - b)^2))*(-((a^5*b^3)^(1/2) - a^3
*b)/(16*(a^6*d^2 - a^5*b*d^2)))^(1/2) - log((((((4194304*d^2*(512*a^4 - 118
4*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*
exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*
x)))/(a^2*b^4*(a - b)^2) + (16777216*d^3*(-((a^5*b^3)^(1/2) - a^3*b)/(a^5*d
^2*(a - b)))^(1/2)*(40*a*b^2 - 35*b^3 + 512*a^3*exp(2*c + 2*d*x) + 64*b^3*e
xp(2*c + 2*d*x) + 326*a*b^2*exp(2*c + 2*d*x) - 896*a^2*b*exp(2*c + 2*d*x)))
/(b^5*(a - b)))*(-((a^5*b^3)^(1/2) - a^3*b)/(a^5*d^2*(a - b)))^(1/2))/4 + (
2097152*d*(256*a^2*b - 256*a*b^2 - 5*b^3 - 1024*a^3*exp(2*c + 2*d*x) + 6*b^
3*exp(2*c + 2*d*x) + 756*a*b^2*exp(2*c + 2*d*x) + 256*a^2*b*exp(2*c + 2*d*x
)))/(a^3*b^4*(a - b)))*(-((a^5*b^3)^(1/2) - a^3*b)/(a^5*d^2*(a - b)))^(1/2)
)/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c
+ 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*
b*exp(2*c + 2*d*x)))/(a^4*b^3*(a - b)^2))*(-((a^5*b^3)^(1/2) - a^3*b)/(16*(
a^6*d^2 - a^5*b*d^2)))^(1/2) - 2/(a*d*(exp(2*c + 2*d*x) - 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.240 $\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth^3(c+dx)}{3ad}+\frac{\coth(c+dx)}{ad}$$

```
[Out] coth(d*x+c)/a/d-1/3*coth(d*x+c)^3/a/d+1/2*b*arctanh((a^(1/2)-b^(1/2))^(1/2)
*tanh(d*x+c)/a^(1/4))/a^(7/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b*arctanh((a^(1
/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(7/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1166, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth^3(c+dx)}{3ad}+\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (b*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqr
t[Sqrt[a] - Sqrt[b]]*d) + (b*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x]
)/a^(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) + Coth[c + d*x]/(a*d) - C
oth[c + d*x]^3/(3*a*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4\left(a-2ax^2+(a-b)x^4\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{1}{ax^2}+\frac{b-bx^2}{a\left(a-2ax^2+(a-b)x^4\right)}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}+\frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{ad}$$

$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)b\right)\operatorname{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, t\right.}{2a^{3/2}d}$$

$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}+\frac{\coth(c+dx)}{ad}-$$

Mathematica [A] time = 2.28, size = 165, normalized size = 1.11

$$\frac{\frac{3b\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{3b\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}+4\sqrt{a}\coth(c+dx)-2\sqrt{a}\coth(c+dx)\operatorname{csch}^2(c+dx)}{6a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] ((-3*b*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]
]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (3*b*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + 4*Sqrt[a]*Co
th[c + d*x] - 2*Sqrt[a]*Coth[c + d*x]*Csch[c + d*x]^2)/(6*a^(3/2)*d)
```

fricas [B] time = 3.49, size = 2206, normalized size = 14.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*d*si
nh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*sin
h(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*cos
h(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x + c
)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a*d*cosh(d*x
+ c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5
/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(b^4*cosh(
d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b^4*sinh(d*x + c)^2 - b^4
+ 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 2*(a^
2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt
(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4
- a^3*b)*d^2))) - 3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c
)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^
```

```
2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3
 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*
cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a
*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*
d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*l
og(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b^4*sinh(d*x +
 c)^2 - b^4 + 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d
^4)) - 2*(a^2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)
*d^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) +
 b^2)/((a^4 - a^3*b)*d^2))) + 3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*
sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*co
sh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cos
h(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c
)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x
+ c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((
a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^2)/((a^4 - a
^3*b)*d^2))*log(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b
^4*sinh(d*x + c)^2 - b^4 - 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b
 + a^7*b^2)*d^4)) + 2*(a^2*b^3*d + (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8
*b + a^7*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^
7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) - 3*(a*d*cosh(d*x + c)^6 + 6*a*d*
cosh(d*x + c)*sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4
 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2
+ 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*
d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*
(a*d*cosh(d*x + c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) -
b^2)/((a^4 - a^3*b)*d^2))*log(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sin
h(d*x + c) + b^4*sinh(d*x + c)^2 - b^4 - 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/(
(a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 2*(a^2*b^3*d + (a^7 - a^6*b)*d^3*sqrt(b^5
/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9
- 2*a^8*b + a^7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) - 48*cosh(d*x + c)^
2 - 96*cosh(d*x + c)*sinh(d*x + c) - 48*sinh(d*x + c)^2 + 16)/(a*d*cosh(d*x
 + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d
*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*
cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x +
 c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x
 + c))*sinh(d*x + c))
```

giac [A] time = 0.21, size = 34, normalized size = 0.23

$$-\frac{4\left(3e^{(2dx+2c)}-1\right)}{3ad\left(e^{(2dx+2c)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
[Out] -4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*(e^(2*d*x + 2*c) - 1)^3)
```

maple [C] time = 0.16, size = 179, normalized size = 1.21

$$-\frac{\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}+\frac{3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}-4a\_Z^{6}+(6a-16b)\_Z^{4}-4a\_Z^{2}+a\right)}\frac{\left(\_R^{6}-3\_R^{4}+3\_R^{2}-1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^{7}a-3\_R^{5}a+3\_R^{3}a-8\_R^{3}b-\_Ra}\right.}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)-1/4/d/a*b*sum((
_R^6-3*_R^4+3*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*
d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/24/
d/a/tanh(1/2*d*x+1/2*c)^3+3/8/d/a/tanh(1/2*d*x+1/2*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-16\,b\int\frac{e^{(4\,dx+4\,c)}}{abe^{(8\,dx+8\,c)}-4\,abe^{(6\,dx+6\,c)}-4\,abe^{(2\,dx+2\,c)}+ab-2\left(8\,a^2e^{(4\,c)}-3\,abe^{(4\,c)}\right)e^{(4\,dx)}}\,dx-\frac{4\,(}{3\left(ade^{(6\,dx+6\,c)}-3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -16*b*integrate(e^(4*d*x + 4*c)/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c
) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*
x)), x) - 4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*e^(6*d*x + 6*c) - 3*a*d*e^(4*d*x
 + 4*c) + 3*a*d*e^(2*d*x + 2*c) - a*d)
```

mupad [B] time = 12.33, size = 2178, normalized size = 14.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a - b*sinh(c + d*x)^4)),x)
```

```
[Out] log((((((4194304*d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2
+ b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2
*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(a^4*b^2*(a - b)^2) + (8388608*d^3*
(((a^7*b^5)^(1/2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2)*(181*a*b^2 - 432*a^2*
b + 256*a^3 + 5*b^3 - 512*a^3*exp(2*c + 2*d*x) - 6*b^3*exp(2*c + 2*d*x) - 6
22*a*b^2*exp(2*c + 2*d*x) + 1152*a^2*b*exp(2*c + 2*d*x)))/(a^2*b^3*(a - b))
)*(((a^7*b^5)^(1/2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 + (2097152*d*(17
6*a*b - 256*a^2 + 75*b^2 + 1536*a^2*exp(2*c + 2*d*x) - 134*b^2*exp(2*c + 2*
d*x) - 1408*a*b*exp(2*c + 2*d*x)))/(a^5*b*(a - b)))*(((a^7*b^5)^(1/2) + a^4
*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^
3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^
2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*d*x)))/(a^7*(a - b)^2))*(((a^7*
b^5)^(1/2) + a^4*b^2)/(16*(a^8*d^2 - a^7*b*d^2)))^(1/2) - log((((((4194304*
d^2*(512*a^4 - 1184*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2
*d*x) + 627*a*b^3*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*
b^2*exp(2*c + 2*d*x)))/(a^4*b^2*(a - b)^2) - (8388608*d^3*(((a^7*b^5)^(1/2)
 + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2)*(181*a*b^2 - 432*a^2*b + 256*a^3 + 5*b
^3 - 512*a^3*exp(2*c + 2*d*x) - 6*b^3*exp(2*c + 2*d*x) - 622*a*b^2*exp(2*c
+ 2*d*x) + 1152*a^2*b*exp(2*c + 2*d*x)))/(a^2*b^3*(a - b)))*(((a^7*b^5)^(1/
2) + a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 - (2097152*d*(176*a*b - 256*a^2 +
 75*b^2 + 1536*a^2*exp(2*c + 2*d*x) - 134*b^2*exp(2*c + 2*d*x) - 1408*a*b*e
xp(2*c + 2*d*x)))/(a^5*b*(a - b)))*(((a^7*b^5)^(1/2) + a^4*b^2)/(a^7*d^2*(a
 - b)))^(1/2))/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024
*a^3*exp(2*c + 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x
) + 2048*a^2*b*exp(2*c + 2*d*x)))/(a^7*(a - b)^2))*(((a^7*b^5)^(1/2) + a^4*
b^2)/(16*(a^8*d^2 - a^7*b*d^2)))^(1/2) - log((((((4194304*d^2*(512*a^4 - 11
84*a^3*b - 253*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3
*exp(2*c + 2*d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d
*x)))/(a^4*b^2*(a - b)^2) - (8388608*d^3*(-((a^7*b^5)^(1/2) - a^4*b^2)/(a^7
*d^2*(a - b)))^(1/2)*(181*a*b^2 - 432*a^2*b + 256*a^3 + 5*b^3 - 512*a^3*exp
(2*c + 2*d*x) - 6*b^3*exp(2*c + 2*d*x) - 622*a*b^2*exp(2*c + 2*d*x) + 1152*
a^2*b*exp(2*c + 2*d*x)))/(a^2*b^3*(a - b)))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(
```

```
a^7*d^2*(a - b)))^(1/2))/4 - (2097152*d*(176*a*b - 256*a^2 + 75*b^2 + 1536*
a^2*exp(2*c + 2*d*x) - 134*b^2*exp(2*c + 2*d*x) - 1408*a*b*exp(2*c + 2*d*x)
))/(a^5*b*(a - b)))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))
/4 - (524288*(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c +
 2*d*x) - 35*b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b
*exp(2*c + 2*d*x)))/(a^7*(a - b)^2))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^8
*d^2 - a^7*b*d^2)))^(1/2) + log(((((4194304*d^2*(512*a^4 - 1184*a^3*b - 25
3*a*b^3 - b^4 + 930*a^2*b^2 + b^4*exp(2*c + 2*d*x) + 627*a*b^3*exp(2*c + 2*
d*x) + 768*a^3*b*exp(2*c + 2*d*x) - 1392*a^2*b^2*exp(2*c + 2*d*x)))/(a^4*b^
2*(a - b)^2) + (8388608*d^3*(-((a^7*b^5)^(1/2) - a^4*b^2)/(a^7*d^2*(a - b))
)^(1/2)*(181*a*b^2 - 432*a^2*b + 256*a^3 + 5*b^3 - 512*a^3*exp(2*c + 2*d*x)
 - 6*b^3*exp(2*c + 2*d*x) - 622*a*b^2*exp(2*c + 2*d*x) + 1152*a^2*b*exp(2*c
 + 2*d*x)))/(a^2*b^3*(a - b)))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(a^7*d^2*(a -
b)))^(1/2))/4 + (2097152*d*(176*a*b - 256*a^2 + 75*b^2 + 1536*a^2*exp(2*c +
 2*d*x) - 134*b^2*exp(2*c + 2*d*x) - 1408*a*b*exp(2*c + 2*d*x)))/(a^5*b*(a
- b)))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(a^7*d^2*(a - b)))^(1/2))/4 - (524288*
(185*a*b^2 - 464*a^2*b + 256*a^3 + 24*b^3 - 1024*a^3*exp(2*c + 2*d*x) - 35*
b^3*exp(2*c + 2*d*x) - 988*a*b^2*exp(2*c + 2*d*x) + 2048*a^2*b*exp(2*c + 2*
d*x)))/(a^7*(a - b)^2))*(-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^8*d^2 - a^7*b*
d^2)))^(1/2) - 4/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - 8/(3*a
*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.241 $\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=235

$$-\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{a\cosh(c+dx)\left(a \cdots\right.}{4b^2d(a-b)\left(a-b\cosh^4(\cdots\right.}$$

```
[Out] cosh(d*x+c)/b^2/d+1/4*a*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/(a-b)/b^2/d/(a-b+
2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)-1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-
b^(1/2))^(1/2))*a^(1/2)*(5*a^(1/2)-6*b^(1/2))/b^(9/4)/d/(a^(1/2)-b^(1/2))^(
3/2)-1/8*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*a^(1/2)*(5*a^
(1/2)+6*b^(1/2))/b^(9/4)/d/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A] time = 0.49, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3215, 1205, 1676, 1166, 205, 208}

$$\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt{a}\left(5\sqrt{a} \cdots\right.}{8 \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(Sqrt[a]*(5*Sqrt[a] - 6*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[
a] - Sqrt[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(9/4)*d) - (Sqrt[a]*(5*Sqrt[
a] + 6*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(
8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(9/4)*d) + Cosh[c + d*x]/(b^2*d) + (a*Cosh[c
+ d*x]*(a + b - b*Cosh[c + d*x]^2))/(4*(a - b)*b^2*d*(a - b + 2*b*Cosh[c +
d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
```

```
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2a\left(a+\frac{a^2}{b}-4b\right)-\cdots}{a-b\cdots}\cdots\right)}{\cdots}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8a(a-b)}{b}+\cdots\right)\cdots\right)}{\cdots}$$

$$= \frac{\cosh(c+dx)}{b^2d} + \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\cdots}{\cdots}$$

$$= \frac{\cosh(c+dx)}{b^2d} + \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\left(\sqrt{a}\,\cdots\right.}{\cdots}$$

$$= -\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{9/4}d} - \frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+d\cdots}{\sqrt{\sqrt{a}+\sqrt{b}}}\cdots\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{9/4}d}\cdots$$

Mathematica [C] time = 0.97, size = 615, normalized size = 2.62

$$\frac{a\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&,\frac{2\#1^6b\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\#1^6bc+\#1^6bdx+40\#1^4a\log\left(-\cdots\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (32*Cosh[c + d*x] + (32*a*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/((
a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (a*Root
Sum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b
*d*x - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#
1 - Sinh[(c + d*x)/2]*#1] - 20*a*c*#1^2 + 27*b*c*#1^2 - 20*a*d*x*#1^2 + 27*
b*d*x*#1^2 - 40*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*
x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 54*b*Log[-Cosh[(c + d*x)/2] - Sinh[
(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 20*a*c*#
1^4 - 27*b*c*#1^4 + 20*a*d*x*#1^4 - 27*b*d*x*#1^4 + 40*a*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^
4 - 54*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
- Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(32*
b^2*d)
```

fricas [B] time = 0.93, size = 7664, normalized size = 32.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*(a*b - b^2)*cosh(d*x + c)^10 + 80*(a*b - b^2)*cosh(d*x + c)*sinh(d*
x + c)^9 + 8*(a*b - b^2)*sinh(d*x + c)^10 - 8*(2*a*b - 3*b^2)*cosh(d*x + c)
^8 + 8*(45*(a*b - b^2)*cosh(d*x + c)^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^8 + 6
4*(15*(a*b - b^2)*cosh(d*x + c)^3 - (2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^7 - 8*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^6 + 8*(210*(a*b - b^2)*
cosh(d*x + c)^4 - 28*(2*a*b - 3*b^2)*cosh(d*x + c)^2 - 20*a^2 + 17*a*b - 2*
b^2)*sinh(d*x + c)^6 + 16*(126*(a*b - b^2)*cosh(d*x + c)^5 - 28*(2*a*b - 3*
b^2)*cosh(d*x + c)^3 - 3*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^5 - 8*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^4 + 8*(210*(a*b - b^2)*c
osh(d*x + c)^6 - 70*(2*a*b - 3*b^2)*cosh(d*x + c)^4 - 15*(20*a^2 - 17*a*b +
 2*b^2)*cosh(d*x + c)^2 - 20*a^2 + 17*a*b - 2*b^2)*sinh(d*x + c)^4 + 32*(30
*(a*b - b^2)*cosh(d*x + c)^7 - 14*(2*a*b - 3*b^2)*cosh(d*x + c)^5 - 5*(20*a
^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^3 - (20*a^2 - 17*a*b + 2*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 - 8*(2*a*b - 3*b^2)*cosh(d*x + c)^2 + 8*(45*(a*b - b^2
)*cosh(d*x + c)^8 - 28*(2*a*b - 3*b^2)*cosh(d*x + c)^6 - 15*(20*a^2 - 17*a*
b + 2*b^2)*cosh(d*x + c)^4 - 6*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^2 -
2*a*b + 3*b^2)*sinh(d*x + c)^2 + ((a*b^3 - b^4)*d*cosh(d*x + c)^9 + 9*(a*b^
3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a*b^3 - b^4)*d*sinh(d*x + c)^9
- 4*(a*b^3 - b^4)*d*cosh(d*x + c)^7 + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^2
- (a*b^3 - b^4)*d)*sinh(d*x + c)^7 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos
h(d*x + c)^5 + 28*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh
(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 42*(a*
b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x +
 c)^5 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^3 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x
+ c)^5 - 70*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b
^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a*b^3 - b^4)*d*cosh(d*x + c)^
6 - 35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d
*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*cosh(
d*x + c) + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^7 - 21*(a*b^3 - b^4)*d*cosh(d
*x + c)^5 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 -
 b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a*b^3 - b^4)*d*cosh(d*x + c)^8
 - 28*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 10*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d
*cosh(d*x + c)^4 - 12*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a*b^3 - b^4)*d)*si
nh(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7
 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a
```

^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4))
+ 15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2)
)*log(-625*a^5 + 2625*a^4*b - 3684*a^3*b^2 + 1728*a^2*b^3 - (625*a^5 - 2625
*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cosh(d*x + c)^2 - 2*(625*a^5 - 2625*a
^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c) - (625*a^5
- 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*sinh(d*x + c)^2 + 2*((125*a^5*b
^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d*cosh(d*x + c) + (125*a^5*b^
2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d*sinh(d*x + c) - 2*((2*a^4*b^
7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*cosh(d*x + c) + (2*a^4
*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sinh(d*x + c))*sqrt
((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*
b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^1
5)*d^4)))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 -
3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*
b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 1
5*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2)))
- ((a*b^3 - b^4)*d*cosh(d*x + c)^9 + 9*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d
*x + c)^8 + (a*b^3 - b^4)*d*sinh(d*x + c)^9 - 4*(a*b^3 - b^4)*d*cosh(d*x +
c)^7 + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c
)^7 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^5 + 28*(3*(a*b^3 - b
^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*
(63*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 42*(a*b^3 - b^4)*d*cosh(d*x + c)^2 -
(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^5 - 4*(a*b^3 - b^4)*d*cosh(
d*x + c)^3 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 70*(a*b^3 - b^4)*d*cos
h(d*x + c)^3 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x +
c)^4 + 4*(21*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 35*(a*b^3 - b^4)*d*cosh(d*x
+ c)^4 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4
)*d)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*cosh(d*x + c) + 4*(9*(a*b^3 - b^4)*d
*cosh(d*x + c)^7 - 21*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 5*(8*a^2*b^2 - 11*a
*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x
+ c)^2 + (9*(a*b^3 - b^4)*d*cosh(d*x + c)^8 - 28*(a*b^3 - b^4)*d*cosh(d*x
+ c)^6 - 10*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 - 12*(a*b^3 -
b^4)*d*cosh(d*x + c)^2 + (a*b^3 - b^4)*d)*sinh(d*x + c))*sqrt(-((a^3*b^4 -
3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 -
6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*
b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)
/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-625*a^5 + 2625*a^4*b - 3
684*a^3*b^2 + 1728*a^2*b^3 - (625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^
2*b^3)*cosh(d*x + c)^2 - 2*(625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*
b^3)*cosh(d*x + c)*sinh(d*x + c) - (625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1
728*a^2*b^3)*sinh(d*x + c)^2 - 2*((125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4
- 336*a^2*b^5)*d*cosh(d*x + c) + (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 -
336*a^2*b^5)*d*sinh(d*x + c) - 2*((2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11
*a*b^10 + 3*b^11)*d^3*cosh(d*x + c) + (2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 -
11*a*b^10 + 3*b^11)*d^3*sinh(d*x + c))*sqrt((625*a^7 - 3450*a^6*b + 7161*a
^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11
- 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)))*sqrt(-((a^3*b^4 - 3*a
^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 662
4*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^1
2 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)/((
a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + ((a*b^3 - b^4)*d*cosh(d*x + c
)^9 + 9*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a*b^3 - b^4)*d*sin
h(d*x + c)^9 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^7 + 4*(9*(a*b^3 - b^4)*d*cos
h(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^7 - 2*(8*a^2*b^2 - 11*a*b^3 +
3*b^4)*d*cosh(d*x + c)^5 + 28*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (a*b^3
- b^4)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x +
c)^4 - 42*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*
d)*sinh(d*x + c)^5 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^3 + 2*(63*(a*b^3 - b^4
)*d*cosh(d*x + c)^5 - 70*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 5*(8*a^2*b^2 - 1

1*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a*b^3 - b^4)*d*c
osh(d*x + c)^6 - 35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 5*(8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^3 + (a*b^3 -
b^4)*d*cosh(d*x + c) + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^7 - 21*(a*b^3 -
b^4)*d*cosh(d*x + c)^5 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3
- 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a*b^3 - b^4)*d*co
sh(d*x + c)^8 - 28*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 10*(8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d*cosh(d*x + c)^4 - 12*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a*b^3
- b^4)*d)*sinh(d*x + c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*s
qrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a
^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 +
b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6
- b^7)*d^2))*log(-625*a^5 + 2625*a^4*b - 3684*a^3*b^2 + 1728*a^2*b^3 - (62
5*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cosh(d*x + c)^2 - 2*(625*
a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)
- (625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*sinh(d*x + c)^2 + 2
*((125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d*cosh(d*x + c) +
(125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d*sinh(d*x + c) +
2*((2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*cosh(d*x +
c) + (2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sinh(d*
x + c))*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3
*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*
a*b^14 + b^15)*d^4)))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt(
(625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b
^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15
)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b
^7)*d^2))) - ((a*b^3 - b^4)*d*cosh(d*x + c)^9 + 9*(a*b^3 - b^4)*d*cosh(d*x
+ c)*sinh(d*x + c)^8 + (a*b^3 - b^4)*d*sinh(d*x + c)^9 - 4*(a*b^3 - b^4)*d*
cosh(d*x + c)^7 + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*s
inh(d*x + c)^7 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^5 + 28*(3
*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x
+ c)^6 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 42*(a*b^3 - b^4)*d*cosh(d*
x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^5 - 4*(a*b^3 - b
^4)*d*cosh(d*x + c)^3 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 70*(a*b^3 -
b^4)*d*cosh(d*x + c)^3 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))
*sinh(d*x + c)^4 + 4*(21*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 35*(a*b^3 - b^4)
*d*cosh(d*x + c)^4 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (
a*b^3 - b^4)*d)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*cosh(d*x + c) + 4*(9*(a*b
^3 - b^4)*d*cosh(d*x + c)^7 - 21*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 5*(8*a^2
*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c
))*sinh(d*x + c)^2 + (9*(a*b^3 - b^4)*d*cosh(d*x + c)^8 - 28*(a*b^3 - b^4)*
d*cosh(d*x + c)^6 - 10*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 - 1
2*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a*b^3 - b^4)*d)*sinh(d*x + c))*sqrt(((
a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*
a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11
- 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b -
36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-625*a^5 + 2625
*a^4*b - 3684*a^3*b^2 + 1728*a^2*b^3 - (625*a^5 - 2625*a^4*b + 3684*a^3*b^2
- 1728*a^2*b^3)*cosh(d*x + c)^2 - 2*(625*a^5 - 2625*a^4*b + 3684*a^3*b^2 -
1728*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c) - (625*a^5 - 2625*a^4*b + 3684*a
^3*b^2 - 1728*a^2*b^3)*sinh(d*x + c)^2 - 2*((125*a^5*b^2 - 520*a^4*b^3 + 72
3*a^3*b^4 - 336*a^2*b^5)*d*cosh(d*x + c) + (125*a^5*b^2 - 520*a^4*b^3 + 723
*a^3*b^4 - 336*a^2*b^5)*d*sinh(d*x + c) + 2*((2*a^4*b^7 - 9*a^3*b^8 + 15*a^
2*b^9 - 11*a*b^10 + 3*b^11)*d^3*cosh(d*x + c) + (2*a^4*b^7 - 9*a^3*b^8 + 15
*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sinh(d*x + c))*sqrt((625*a^7 - 3450*a^6*
b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15
*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)))*sqrt(((a^3*
b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*
b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 2

```
0*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*
a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + 8*a*b - 8*b^2 + 16*(
5*(a*b - b^2)*cosh(d*x + c)^9 - 4*(2*a*b - 3*b^2)*cosh(d*x + c)^7 - 3*(20*a
^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^5 - 2*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x
 + c)^3 - (2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 - b^4)*d*co
sh(d*x + c)^9 + 9*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a*b^3 -
b^4)*d*sinh(d*x + c)^9 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^7 + 4*(9*(a*b^3 -
b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^7 - 2*(8*a^2*b^2 -
11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^5 + 28*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^3
 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a*b^3 - b^4)*d*c
osh(d*x + c)^4 - 42*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3
 + 3*b^4)*d)*sinh(d*x + c)^5 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^3 + 2*(63*(a
*b^3 - b^4)*d*cosh(d*x + c)^5 - 70*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 5*(8*a
^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a*b^3
- b^4)*d*cosh(d*x + c)^6 - 35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 5*(8*a^2*b^
2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^3
+ (a*b^3 - b^4)*d*cosh(d*x + c) + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^7 - 21
*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(
d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a*b^3 -
 b^4)*d*cosh(d*x + c)^8 - 28*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 10*(8*a^2*b^
2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 - 12*(a*b^3 - b^4)*d*cosh(d*x + c)^
2 + (a*b^3 - b^4)*d)*sinh(d*x + c))
```

giac [B]   time = 1.34, size = 1078, normalized size = 4.59

$$-\frac{\left(\left(\sqrt{ab}\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^2+8\,\sqrt{ab}\,\sqrt{-b^2+\sqrt{ab}\,b}\,ab\right)\left(ab^2-b^3\right)^2|b|+\left(5\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^4b^2+28\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^3b^3-89\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b^4+56\,\sqrt{-b^2+\sqrt{ab}\,b}\,a\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 8*sqrt(a*b)*sqrt(-b^2 + sq
rt(a*b)*b)*a*b)*(a*b^2 - b^3)^2*abs(b) + (5*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^
2 + 28*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3 - 89*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b
^4 + 56*sqrt(-b^2 + sqrt(a*b)*b)*a*b^5)*abs(-a*b^2 + b^3)*abs(b) - (5*sqrt(
a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^4 + 24*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*
b)*a^3*b^5 - 111*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^6 + 130*sqrt(a*b)
*sqrt(-b^2 + sqrt(a*b)*b)*a*b^7 - 48*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^8
)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a*b^3 - b^4 + sqrt
((a^2*b^2 - 2*a*b^3 + b^4)*(a*b^3 - b^4) + (a*b^3 - b^4)^2))/(a*b^3 - b^4))
)/((a^4*b^6 + 5*a^3*b^7 - 21*a^2*b^8 + 23*a*b^9 - 8*b^10)*abs(-a*b^2 + b^3)
) - ((sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2 + 8*sqrt(a*b)*sqrt(-b^2 - sqrt
(a*b)*b)*a*b)*(a*b^2 - b^3)^2*abs(b) - (5*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^2
+ 28*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 - 89*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^4
 + 56*sqrt(-b^2 - sqrt(a*b)*b)*a*b^5)*abs(-a*b^2 + b^3)*abs(b) - (5*sqrt(a*
b)*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^4 + 24*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)
*a^3*b^5 - 111*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^6 + 130*sqrt(a*b)*s
qrt(-b^2 - sqrt(a*b)*b)*a*b^7 - 48*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^8)*
abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a*b^3 - b^4 - sqrt((
a^2*b^2 - 2*a*b^3 + b^4)*(a*b^3 - b^4) + (a*b^3 - b^4)^2))/(a*b^3 - b^4)))/
((a^4*b^6 + 5*a^3*b^7 - 21*a^2*b^8 + 23*a*b^9 - 8*b^10)*abs(-a*b^2 + b^3))
- 4*(a*b*(e^(d*x + c) + e^(-d*x - c))^3 - 4*a^2*(e^(d*x + c) + e^(-d*x - c)
) - 4*a*b*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4
- 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)*(a*b^2 - b^3)) - 4*(e^(
d*x + c) + e^(-d*x - c))/b^2)/d
```

maple [B] time = 0.13, size = 1191, normalized size = 5.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a
-b)*tanh(1/2*d*x+1/2*c)^6+1/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^6+3/2/d*a^2/b^2/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*
d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4-4/d*a
/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)
^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2
*d*x+1/2*c)^4-3/2/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2-1/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*t
anh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4
-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2+1/2/d*a^2/b^2/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)+1/8/d*a/b^2/(a
-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b
)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-5/8/d*a^2/b^2/(a-b)/(-
a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2
)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+3/4/d*a/b/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/
2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/
2)*a)^(1/2))+1/8/d*a/b^2/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^
(1/2)+5/8/d*a^2/b^2/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/
2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-3/4/d*a/b/(
a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a
*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-1/d/b^2/(tanh(1/2*d*x+1/2*c)-1)+
1/d/b^2/(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ab - b^2 + \left(abe^{(10\,c)} - b^2e^{(10\,c)}\right)e^{(10\,dx)} - \left(2\,abe^{(8\,c)} - 3\,b^2e^{(8\,c)}\right)e^{(8\,dx)} - \left(20\,a^2e^{(6\,c)} - 17\,abe^{(6\,c)} + 2\,b^2e^{(6\,c)}\right)e^{(6\,dx)} - (}{2\left(\left(ab^3de^{(9\,c)} - b^4de^{(9\,c)}\right)e^{(9\,dx)} - 4\left(ab^3de^{(7\,c)} - b^4de^{(7\,c)}\right)e^{(7\,dx)} - 2\left(8\,a^2b^2de^{(5\,c)} - 11\,ab^3de^{(5\,c)} + 3\,b^4de^{(5\,c)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b - b^2 + (a*b*e^(10*c) - b^2*e^(10*c))*e^(10*d*x) - (2*a*b*e^(8*c)
- 3*b^2*e^(8*c))*e^(8*d*x) - (20*a^2*e^(6*c) - 17*a*b*e^(6*c) + 2*b^2*e^(6*
c))*e^(6*d*x) - (20*a^2*e^(4*c) - 17*a*b*e^(4*c) + 2*b^2*e^(4*c))*e^(4*d*x)
 - (2*a*b*e^(2*c) - 3*b^2*e^(2*c))*e^(2*d*x))/((a*b^3*d*e^(9*c) - b^4*d*e^(
9*c))*e^(9*d*x) - 4*(a*b^3*d*e^(7*c) - b^4*d*e^(7*c))*e^(7*d*x) - 2*(8*a^2*
b^2*d*e^(5*c) - 11*a*b^3*d*e^(5*c) + 3*b^4*d*e^(5*c))*e^(5*d*x) - 4*(a*b^3*
d*e^(3*c) - b^4*d*e^(3*c))*e^(3*d*x) + (a*b^3*d*e^c - b^4*d*e^c)*e^(d*x)) +
 1/512*integrate(256*(a*b*e^(7*d*x + 7*c) - a*b*e^(d*x + c) + (20*a^2*e^(5*
c) - 27*a*b*e^(5*c))*e^(5*d*x) - (20*a^2*e^(3*c) - 27*a*b*e^(3*c))*e^(3*d*x
))/(a*b^3 - b^4 + (a*b^3*e^(8*c) - b^4*e^(8*c))*e^(8*d*x) - 4*(a*b^3*e^(6*c
) - b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b^2*e^(4*c) - 11*a*b^3*e^(4*c) + 3*b^
4*e^(4*c))*e^(4*d*x) - 4*(a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh (c + d\, x)^9}{\left(a - b \sinh (c + d\, x)^4\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^2,x)
[Out] int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**9/(a-b*sinh(d*x+c)**4)**2,x)
[Out] Timed out
```

3.242 $\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cosh(c+dx)\left(2-\cosh\right.}{4bd(a-b)\left(a-b\cosh^4(c+dx)+\right.}$$

```
[Out] -1/4*a*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cosh(d*x+c)^2-b*cos
h(d*x+c)^4)+1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a^(1
/2)-4*b^(1/2))/b^(7/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh(b^(1/4)*cosh(d
*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(3*a^(1/2)+4*b^(1/2))/b^(7/4)/d/(a^(1/2)+b^(
1/2))^(3/2)
```

Rubi [A] time = 0.37, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1205, 1166, 205, 208}

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cosh(c+dx)\left(2-\cosh\right.}{4bd(a-b)\left(a-b\cosh^4(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((3*Sqrt[a] - 4*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(7/4)*d) - ((3*Sqrt[a] + 4*Sqrt[b])*A
rcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(Sqrt[a] + Sqrt
[b])^(3/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*
b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{4a(a-2b)-2a(3}{a-b+2bx^2-}\right.}{8a(}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\left(3a-\sqrt{a}\sqrt{b}-4b\right)\text{Su}}{}$$

$$= \frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{7/4}d} - \frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{7/4}d} - \frac{}{4(a}$$

Mathematica [C] time = 0.70, size = 737, normalized size = 3.51

$$-\frac{\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{-6\#1^6a\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-c\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/32*((-16*a*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh
[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6
*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (3*a*c - 4*b*c + 3*a*d*x - 4*b*d*x + 6*a*Lo
g[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
 d*x)/2]*#1] - 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 5*a*c*#1^2 + 12*b*c*#1^2 - 5*a*d*x*#1^2
 + 12*b*d*x*#1^2 - 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 24*b*Log[-Cosh[(c + d*x)/2] -
 Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 5*
a*c*#1^4 - 12*b*c*#1^4 + 5*a*d*x*#1^4 - 12*b*d*x*#1^4 + 10*a*Log[-Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]
*#1^4 - 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]
*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 3*a*c*#1^6 + 4*b*c*#1^6 - 3*a*d*x*#1^6 +
 4*b*d*x*#1^6 - 6*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
```

```
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 + 8*b*Log[-Cosh[(c + d*x)/2] - Sinh
[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1)
 - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/((a - b)*b*d)
```

fricas [B] time = 0.91, size = 6266, normalized size = 29.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a*cosh(d*x + c)^7 + 56*a*cosh(d*x + c)*sinh(d*x + c)^6 + 8*a*sinh(
d*x + c)^7 - 40*a*cosh(d*x + c)^5 + 8*(21*a*cosh(d*x + c)^2 - 5*a)*sinh(d*x
 + c)^5 + 40*(7*a*cosh(d*x + c)^3 - 5*a*cosh(d*x + c))*sinh(d*x + c)^4 - 40
*a*cosh(d*x + c)^3 + 40*(7*a*cosh(d*x + c)^4 - 10*a*cosh(d*x + c)^2 - a)*si
nh(d*x + c)^3 + 8*(21*a*cosh(d*x + c)^5 - 50*a*cosh(d*x + c)^3 - 15*a*cosh(
d*x + c))*sinh(d*x + c)^2 - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b
^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(
a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*
b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x +
 c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 -
b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 -
4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 -
10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2
- b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2
 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*
cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 +
 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*s
qrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b +
1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9
 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16
*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-81*a^3 + 405*a^2*b
- 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cosh(d*x
 + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cosh(d*x + c)*sinh(d
*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*sinh(d*x + c)^2 + 2*(2
*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*cosh(d*x + c) + 2*(9*a^3*b^
2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*sinh(d*x + c) - ((3*a^4*b^5 - 14*a^3*
b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*cosh(d*x + c) + (3*a^4*b^5 - 14*a^
3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*sinh(d*x + c))*sqrt((81*a^5 - 52
2*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 +
15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)))*sqrt(-((a^
3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*
b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3
*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/((a
^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cosh(d*x + c)
^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh
(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh
(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*
b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 -
b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)
^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*s
inh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*c
osh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 +
 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x +
c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*
d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*
((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a
^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))
```

```
*sinh(d*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a
^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5
*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*
a^2 - 15*a*b + 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-81
*a^3 + 405*a^2*b - 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*b + 680*a*b^2 -
384*b^3)*cosh(d*x + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cos
h(d*x + c)*sinh(d*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*sinh(
d*x + c)^2 - 2*(2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*cosh(d*x +
 c) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*sinh(d*x + c) - ((3*
a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*cosh(d*x + c) + (
3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*sinh(d*x + c))*
sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b
^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*
d^4)))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*
a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15
*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*
a*b + 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - ((a*b^2 - b^3
)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*
b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*
b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*
b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c
)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3
)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*
b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*
(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a
^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b
^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 1
1*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*
b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(
d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)
*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6
)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/(
(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 +
b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^
6)*d^2))*log(-81*a^3 + 405*a^2*b - 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*
b + 680*a*b^2 - 384*b^3)*cosh(d*x + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^
2 - 384*b^3)*cosh(d*x + c)*sinh(d*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2
- 384*b^3)*sinh(d*x + c)^2 + 2*(2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b
^5)*d*cosh(d*x + c) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*sinh
(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*c
osh(d*x + c) + (3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3
*sinh(d*x + c))*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 57
6*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6
*a*b^12 + b^13)*d^4)))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt
((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 -
 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)
) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)))
+ ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d
*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x +
c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c
)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)
*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (
8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x
 + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*
x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 -
 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d
*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2
```

```
 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3
- (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^3*b^3 - 3*a^2*b^4
+ 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3
 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^1
1 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4
 + 3*a*b^5 - b^6)*d^2))*log(-81*a^3 + 405*a^2*b - 680*a*b^2 + 384*b^3 - (81
*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cosh(d*x + c)^2 - 2*(81*a^3 - 405*a
^2*b + 680*a*b^2 - 384*b^3)*cosh(d*x + c)*sinh(d*x + c) - (81*a^3 - 405*a^2
*b + 680*a*b^2 - 384*b^3)*sinh(d*x + c)^2 - 2*(2*(9*a^3*b^2 - 47*a^2*b^3 +
82*a*b^4 - 48*b^5)*d*cosh(d*x + c) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 -
 48*b^5)*d*sinh(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8
 + 5*b^9)*d^3*cosh(d*x + c) + (3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b
^8 + 5*b^9)*d^3*sinh(d*x + c))*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 13
92*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 +
15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5
- b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b
^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^
12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5
 - b^6)*d^2))) + 8*a*cosh(d*x + c) + 8*(7*a*cosh(d*x + c)^6 - 25*a*cosh(d*x
 + c)^4 - 15*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/((a*b^2 - b^3)*d*cosh(d*
x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*
d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*
d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^
2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*
b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*
x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3
)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^
3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a
*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(
d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3
*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*
d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5
- (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x
 + c))*sinh(d*x + c))
```

giac [B] time = 1.15, size = 1002, normalized size = 4.77

$$\left(\left(3\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,a^2+20\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,ab-32\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\,b^2\right)\left(ab-b^2\right)^2|b|+2\left(\sqrt{-b^2+\sqrt{ab}\,b}\,a^3b^2+5\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b^3-22\sqrt{-b^2+\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*(((3*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 20*sqrt(a*b)*sqrt(-b^2 +
sqrt(a*b)*b)*a*b - 32*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*(a*b - b^2)^2
*abs(b) + 2*(sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^2 + 5*sqrt(-b^2 + sqrt(a*b)*b)*
a^2*b^3 - 22*sqrt(-b^2 + sqrt(a*b)*b)*a*b^4 + 16*sqrt(-b^2 + sqrt(a*b)*b)*b
^5)*abs(-a*b + b^2)*abs(b) - (sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3 +
6*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^4 - 15*sqrt(a*b)*sqrt(-b^2 + sqr
t(a*b)*b)*a*b^5 + 8*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^6)*abs(b))*arctan(
1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a*b^2 - b^3 + sqrt((a^2*b - 2*a*b^2
 + b^3)*(a*b^2 - b^3) + (a*b^2 - b^3)^2))/(a*b^2 - b^3)))/((a^4*b^5 + 5*a^3
*b^6 - 21*a^2*b^7 + 23*a*b^8 - 8*b^9)*abs(-a*b + b^2)) + ((3*sqrt(a*b)*sqrt
(-b^2 - sqrt(a*b)*b)*a^2 + 20*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b - 32*s
qrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^2)*(a*b - b^2)^2*abs(b) + 2*(sqrt(-b^2
```

```
- sqrt(a*b)*b)*a^3*b^2 + 5*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^3 - 22*sqrt(-b^2
- sqrt(a*b)*b)*a*b^4 + 16*sqrt(-b^2 - sqrt(a*b)*b)*b^5)*abs(-a*b + b^2)*abs
(b) - (sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 + 6*sqrt(a*b)*sqrt(-b^2 -
 sqrt(a*b)*b)*a^2*b^4 - 15*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^5 + 8*sqr
t(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^6)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-
d*x - c))/sqrt(-(a*b^2 - b^3 - sqrt((a^2*b - 2*a*b^2 + b^3)*(a*b^2 - b^3) +
 (a*b^2 - b^3)^2))/(a*b^2 - b^3)))/((a^4*b^5 + 5*a^3*b^6 - 21*a^2*b^7 + 23*
a*b^8 - 8*b^9)*abs(-a*b + b^2)) - 4*(a*(e^(d*x + c) + e^(-d*x - c))^3 - 8*a
*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4 - 8*b*(e^
(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)*(a*b - b^2)))/d
```

maple [B] time = 0.27, size = 1200, normalized size = 5.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d/b^2/(tanh(1/2*d*x+1/2*c)^4-4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2-2*t
anh(1/2*d*x+1/2*c)^2+1)/(a-b)*tanh(1/2*d*x+1/2*c)^2*(a*b)^(1/2)+1/2/a/d/b/(
tanh(1/2*d*x+1/2*c)^4-4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+
1/2*c)^2+1)/(a-b)*tanh(1/2*d*x+1/2*c)^2*(a*b)^(1/2)-1/4/d/b/(tanh(1/2*d*x+1
/2*c)^4-4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)^2+1)/(a
-b)*tanh(1/2*d*x+1/2*c)^2-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4-4*(a*b)^(1/2)/a*ta
nh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)^2+1)/(a-b)+1/4/d/b^2/(tanh(1/2*d*
x+1/2*c)^4-4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)^2+1)
/(a-b)*(a*b)^(1/2)+3/8/d*a/b^2/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*
(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*
(a*b)^(1/2)-1/2/d/b/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/
2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)
+1/8/d*a/b/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2
*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/4/d/b^2/(tanh(1/2*
d*x+1/2*c)^4-2*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2+
1)/(a-b)*tanh(1/2*d*x+1/2*c)^2*(a*b)^(1/2)-1/2/a/d/b/(tanh(1/2*d*x+1/2*c)^4
-2*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2+1)/(a-b)*tan
h(1/2*d*x+1/2*c)^2*(a*b)^(1/2)-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4-2*tanh(1/2*d*
x+1/2*c)^2+4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2+1)/(a-b)*tanh(1/2*d*x+1/2*
c)^2-1/4/d/b^2/(tanh(1/2*d*x+1/2*c)^4-2*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)
/a*tanh(1/2*d*x+1/2*c)^2+1)/(a-b)*(a*b)^(1/2)-1/4/d/b/(tanh(1/2*d*x+1/2*c)^
4-2*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2+1)/(a-b)+3/
8/d*a/b^2/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c
)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1/2/d/b/(a
-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b
)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1/8/d*a/b/(a-b)/(-a*b+
(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*
a)/(-a*b+(a*b)^(1/2)*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ae^{(7\,dx+7\,c)} - 5\,ae^{(5\,dx+5\,c)} - 5\,ae^{(3\,dx+3\,c)} + ae^{(dx+c)}}{2\left(ab^2d - b^3d + \left(ab^2de^{(8\,c)} - b^3de^{(8\,c)}\right)e^{(8\,dx)} - 4\left(ab^2de^{(6\,c)} - b^3de^{(6\,c)}\right)e^{(6\,dx)} - 2\left(8\,a^2bde^{(4\,c)} - 11\,ab^2de^{(4\,c)} + 3\,b^3d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*e^(7*d*x + 7*c) - 5*a*e^(5*d*x + 5*c) - 5*a*e^(3*d*x + 3*c) + a*e^(
d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d*x) -
4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c) - 11*a
*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^3*d*e^
```

```
(2*c))*e^(2*d*x)) + 1/128*integrate(64*((3*a*e^(7*c) - 4*b*e^(7*c))*e^(7*d*
x) - (5*a*e^(5*c) - 12*b*e^(5*c))*e^(5*d*x) + (5*a*e^(3*c) - 12*b*e^(3*c))*
e^(3*d*x) - (3*a*e^c - 4*b*e^c)*e^(d*x))/(a*b^2 - b^3 + (a*b^2*e^(8*c) - b^
3*e^(8*c))*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^2
*b*e^(4*c) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e^(2*c)
 - b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(c+d\,x\right)^7}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^2,x)
[Out] int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4)**2,x)
[Out] Timed out
```

3.243 $\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=217

$$-\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+\right.}{4bd(a-b)\left(a-b\cosh^4(c+dx)+2b\,\mathrm{c}\right.}$$

```
[Out] 1/4*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cosh(d*x+c)^2-b*co
sh(d*x+c)^4)-1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(a^(1/
2)-2*b^(1/2))/b^(5/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh(b^(1/4)
*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+2*b^(1/2))/b^(5/4)/d/a^(1/2)
/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A] time = 0.29, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1205, 1166, 205, 208}

$$-\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+\right.}{4bd(a-b)\left(a-b\cosh^4(c+dx)+2b\,\mathrm{c}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(5/4)*d) - ((Sqrt[a] + 2*Sqrt[
b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*Sqrt[a]*(S
qrt[a] + Sqrt[b])^(3/2)*b^(5/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c + d*x
]^2))/(4*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2a(a-3b)+2a}{a-b+2bx^2-b}\right.}{8a(a}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\left(\sqrt{a}-2\sqrt{b}\right)\text{Subst}}{8\sqrt{}}$$

$$= -\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{5/4}d} - \frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{5/4}d} + \frac{}{4}$$

Mathematica [C] time = 0.58, size = 597, normalized size = 2.75

$$\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6b\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((32*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2
*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b
*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 4*a*c*
#1^2 + 11*b*c*#1^2 - 4*a*d*x*#1^2 + 11*b*d*x*#1^2 - 8*a*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2
 + 22*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1]*#1^2 + 4*a*c*#1^4 - 11*b*c*#1^4 + 4*a*d*x*#1^4 - 11*
b*d*x*#1^4 + 8*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x
)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 22*b*Log[-Cosh[(c + d*x)/2] - Sinh[(
c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^6
+ b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*
b*#1^5 + b*#1^7) & ])/(32*(a - b)*b*d)
```

fricas [B] time = 0.84, size = 6250, normalized size = 28.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^7 + 56*b*cosh(d*x + c)*sinh(d*x + c)^6 + 8*b*sinh(d
*x + c)^7 - 8*(4*a + b)*cosh(d*x + c)^5 + 8*(21*b*cosh(d*x + c)^2 - 4*a - b
)*sinh(d*x + c)^5 + 40*(7*b*cosh(d*x + c)^3 - (4*a + b)*cosh(d*x + c))*sinh
(d*x + c)^4 - 8*(4*a + b)*cosh(d*x + c)^3 + 8*(35*b*cosh(d*x + c)^4 - 10*(4
*a + b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c)^3 + 8*(21*b*cosh(d*x + c)^
5 - 10*(4*a + b)*cosh(d*x + c)^3 - 3*(4*a + b)*cosh(d*x + c))*sinh(d*x + c)
^2 + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sin
h(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x
 + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x
+ c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b
^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 +
2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2
- (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(
d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh
(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^
4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sin
h(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*
b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)
^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4*b^2 - 3*a^3*b
^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 +
64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^
2*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b
^4 - a*b^5)*d^2))*log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b +
 28*a*b^2 - 32*b^3)*cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)
*cosh(d*x + c)*sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*sinh(d*x
 + c)^2 + 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*cosh(d*x + c) +
(a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*sinh(d*x + c) - 2*((a^4*b^5 -
 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 +
3*a^2*b^7 - a*b^8)*d^3*sinh(d*x + c))*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 8
0*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*
b^9 - 6*a^2*b^10 + a*b^11)*d^4)))*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 -
a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5
 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*
d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))
) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh
(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x
+ c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x +
 c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^
3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 -
 (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d
*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(
d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4
 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh
(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b
^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^
3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4*b^2 - 3*a^3*b^
3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 6
4*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2
*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^
```

4 - a*b^5)*d^2))*log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cosh(d*x + c)*sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*sinh(d*x + c)^2 - 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*cosh(d*x + c) + (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*sinh(d*x + c) - 2*((a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sinh(d*x + c))*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)))*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))) + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cosh(d*x + c)*sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*sinh(d*x + c)^2 + 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*cosh(d*x + c) + (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*sinh(d*x + c) + 2*((a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sinh(d*x + c))*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)))*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b +

```
28*a*b^2 - 32*b^3)*cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)
*cosh(d*x + c)*sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*sinh(d*x
 + c)^2 - 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*cosh(d*x + c) +
(a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*sinh(d*x + c) + 2*((a^4*b^5 -
 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 +
3*a^2*b^7 - a*b^8)*d^3*sinh(d*x + c))*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 8
0*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*
b^9 - 6*a^2*b^10 + a*b^11)*d^4)))*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 -
 a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^
5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)
*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2)
)) + 8*b*cosh(d*x + c) + 8*(7*b*cosh(d*x + c)^6 - 5*(4*a + b)*cosh(d*x + c)
^4 - 3*(4*a + b)*cosh(d*x + c)^2 + b)*sinh(d*x + c))/((a*b^2 - b^3)*d*cosh(
d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3
)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3
)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*
b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(
a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(
d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b
^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 -
b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11
*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cos
h(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 +
 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3
)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^
5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d
*x + c))*sinh(d*x + c))
```

giac [B] time = 0.97, size = 1033, normalized size = 4.76

$$\left(\left(\sqrt{ab}\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^2+8\,\sqrt{ab}\,\sqrt{-b^2+\sqrt{ab}\,b}\,ab\right)\left(ab-b^2\right)^2|b|-\left(\sqrt{-b^2+\sqrt{ab}\,b}\,a^4b+4\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^3b^2-29\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b^3+24\,\sqrt{-b^2+\sqrt{ab}\,b}\,ab^4\right)|-ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*(((sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 8*sqrt(a*b)*sqrt(-b^2 + sqr
t(a*b)*b)*a*b)*(a*b - b^2)^2*abs(b) - (sqrt(-b^2 + sqrt(a*b)*b)*a^4*b + 4*s
qrt(-b^2 + sqrt(a*b)*b)*a^3*b^2 - 29*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^3 + 24*
sqrt(-b^2 + sqrt(a*b)*b)*a*b^4)*abs(-a*b + b^2)*abs(b) - (sqrt(a*b)*sqrt(-b
^2 + sqrt(a*b)*b)*a^4*b^2 + 4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3 -
27*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^4 + 38*sqrt(a*b)*sqrt(-b^2 + sq
rt(a*b)*b)*a*b^5 - 16*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^6)*abs(b))*arcta
n(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a*b^2 - b^3 + sqrt((a^2*b - 2*a*b
^2 + b^3)*(a*b^2 - b^3) + (a*b^2 - b^3)^2))/(a*b^2 - b^3)))/((a^5*b^4 + 5*a
^4*b^5 - 21*a^3*b^6 + 23*a^2*b^7 - 8*a*b^8)*abs(-a*b + b^2)) + ((sqrt(a*b)*
sqrt(-b^2 - sqrt(a*b)*b)*a^2 + 8*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b)*(a
*b - b^2)^2*abs(b) - (sqrt(-b^2 - sqrt(a*b)*b)*a^4*b + 4*sqrt(-b^2 - sqrt(a
*b)*b)*a^3*b^2 - 29*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^3 + 24*sqrt(-b^2 - sqrt(
a*b)*b)*a*b^4)*abs(-a*b + b^2)*abs(b) - (sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)
*a^4*b^2 + 4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 - 27*sqrt(a*b)*sqrt
(-b^2 - sqrt(a*b)*b)*a^2*b^4 + 38*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^5
- 16*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^6)*abs(b))*arctan(1/2*(e^(d*x + c
) + e^(-d*x - c))/sqrt(-(a*b^2 - b^3 - sqrt((a^2*b - 2*a*b^2 + b^3)*(a*b^2
- b^3) + (a*b^2 - b^3)^2))/(a*b^2 - b^3)))/((a^5*b^4 + 5*a^4*b^5 - 21*a^3*b
```

```
^6 + 23*a^2*b^7 - 8*a*b^8)*abs(-a*b + b^2)) + 4*(b*(e^(d*x + c) + e^(-d*x -
 c))^3 - 4*a*(e^(d*x + c) + e^(-d*x - c)) - 4*b*(e^(d*x + c) + e^(-d*x - c)
))/((b*(e^(d*x + c) + e^(-d*x - c))^4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2
- 16*a + 16*b)*(a*b - b^2)))/d
```

maple [B] time = 0.11, size = 1116, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*
x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*
tanh(1/2*d*x+1/2*c)^6+1/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*
a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c
)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^6+3/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*t
anh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4
-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4-4/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4-3/
2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tan
h(1/2*d*x+1/2*c)^2-1/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2+1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)*a/b/(a-b)+1/8/d/b/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*a
rctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a
)^(1/2))*(a*b)^(1/2)-1/8/d*a/b/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*
(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+1
/4/d/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a
+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+1/8/d/b/(a-b)/(-a*b-(a*b)^(
1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a
*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+1/8/d*a/b/(a-b)/(-a*b-(a*b)^(1/2)*a)^(
1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^
(1/2)*a)^(1/2))-1/4/d/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(
1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(4\,ae^{(5\,c)} + be^{(5\,c)}\right)e^{(5\,dx)} + \left(4\,ae^{(3\,c)} + be^{(3\,c)}\right)e^{(3\,dx)} - be^{(7\,dx+7\,c)} - be^{(d}}{2\left(ab^{2}d - b^{3}d + \left(ab^{2}de^{(8\,c)} - b^{3}de^{(8\,c)}\right)e^{(8\,dx)} - 4\left(ab^{2}de^{(6\,c)} - b^{3}de^{(6\,c)}\right)e^{(6\,dx)} - 2\left(8\,a^{2}bde^{(4\,c)} - 11\,ab^{2}de^{(4\,c)} + 3\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*e^(5*c) + b*e^(5*c))*e^(5*d*x) + (4*a*e^(3*c) + b*e^(3*c))*e^(3*
d*x) - b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*
c) - b^3*d*e^(8*c))*e^(8*d*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*
x) - 2*(8*a^2*b*d*e^(4*c) - 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x)
 - 4*(a*b^2*d*e^(2*c) - b^3*d*e^(2*c))*e^(2*d*x)) + 1/32*integrate(16*((4*a
*e^(5*c) - 11*b*e^(5*c))*e^(5*d*x) - (4*a*e^(3*c) - 11*b*e^(3*c))*e^(3*d*x)
 + b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a*b^2 - b^3 + (a*b^2*e^(8*c) - b^3*e
^(8*c))*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*
e^(4*c) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e^(2*c) -
b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+dx)^5}{\left(a-b\sinh(c+dx)^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^2,x)
[Out] int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4)**2,x)
[Out] Timed out
```

3.244 $\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=186

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

```
[Out] -1/4*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*
x+c)^4)-1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(3/4)/d/a
^(1/2)/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(
1/2))^(1/2))/b^(3/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A] time = 0.21, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1178, 1166, 205, 208}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a
] - Sqrt[b])^(3/2)*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a
] + Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/4)*d) - (Cosh[c + d
*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Co
sh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-4ab+2abx^2}{a-b+2bx^2-bx^4} \cdots\right.}{8a(a- \cdots}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2} \cdots\right.}{8\sqrt{a}\left(\sqrt{a} \cdots\right.}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/4}d} - \frac{\cosh(c+dx)\,\cdots}{4(a-b)d\left(a-b+2b\cos \cdots\right.}$$

Mathematica [C] time = 0.51, size = 422, normalized size = 2.27

$$-\frac{\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cos \cdots\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/32*((16*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2
*(c + d*x)] + b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b
*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(
c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 7*c*#1^2 + 7*d
*x*#1^2 + 14*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]
*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 7*c*#1^4 - 7*d*x*#1^4 - 14*Log[-Cosh[(c
+ d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
 Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3
*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/((a - b)*d)
```

fricas [B] time = 0.64, size = 5238, normalized size = 28.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*cosh(d*x + c)^7 + 56*cosh(d*x + c)*sinh(d*x + c)^6 + 8*sinh(d*x +
c)^7 + 8*(21*cosh(d*x + c)^2 - 5)*sinh(d*x + c)^5 - 40*cosh(d*x + c)^5 + 40
*(7*cosh(d*x + c)^3 - 5*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(7*cosh(d*x + c
)^4 - 10*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^3 - 40*cosh(d*x + c)^3 + 8*(21*
cosh(d*x + c)^5 - 50*cosh(d*x + c)^3 - 15*cosh(d*x + c))*sinh(d*x + c)^2 -
((a*b - b^2)*d*cosh(d*x + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c
)^7 + (a*b - b^2)*d*sinh(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(
7*(a*b - b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2
 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 -
 3*(a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(
d*x + c)^4 - 30*(a*b - b^2)*d*cosh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)
*sinh(d*x + c)^4 - 4*(a*b - b^2)*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cos
h(d*x + c)^5 - 10*(a*b - b^2)*d*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*
(a*b - b^2)*d*cosh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^
2 - (a*b - b^2)*d)*sinh(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(
d*x + c)^7 - 3*(a*b - b^2)*d*cosh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*c
osh(d*x + c)^3 - (a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 -
 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)
) + 3*a + b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*c
osh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d
*x + c)^2 + 2*(2*(a^2*b + 3*a*b^2)*d*cosh(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*
sinh(d*x + c) - ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*cosh(d*x + c
) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sinh(d*x + c))*sqrt((a^2
+ 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b
^7 - 6*a^2*b^8 + a*b^9)*d^4)))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^
4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a
^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b
^2 + 3*a^2*b^3 - a*b^4)*d^2)) + a + 3*b) + ((a*b - b^2)*d*cosh(d*x + c)^8 +
 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x + c
)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2
- (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x +
c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*co
sh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a*b - b^2)
*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^5 - 10*(a*b - b^2)*d*
cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*(a*b - b^2)*d*cosh(d*x + c)^4 -
3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)
^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(d*x + c)^7 - 3*(a*b - b^2)*d*cos
h(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^3 - (a*b - b^2)*d*c
osh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4
*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cosh(d*x + c)^2 + 2*(a + 3*b)*cos
h(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d*x + c)^2 - 2*(2*(a^2*b + 3*a*b^
2)*d*cosh(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*sinh(d*x + c) - ((a^5*b^2 - 2*a^
4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*cosh(d*x + c) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2
*b^5 - a*b^6)*d^3*sinh(d*x + c))*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a
^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)))*s
qrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2
)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8
+ a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) +
a + 3*b) - ((a*b - b^2)*d*cosh(d*x + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*s
inh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x +
 c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6
 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d
*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b
```

```
^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*cosh(d*x + c)^2 - (8*a^2 - 11*a*b
+ 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a*b - b^2)*d*cosh(d*x + c)^2 + 8*(7*(a*b -
 b^2)*d*cosh(d*x + c)^5 - 10*(a*b - b^2)*d*cosh(d*x + c)^3 - (8*a^2 - 11*a*
b + 3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b - b^2)*d*cosh(d*x +
 c)^6 - 15*(a*b - b^2)*d*cosh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*cos
h(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b
^2)*d*cosh(d*x + c)^7 - 3*(a*b - b^2)*d*cosh(d*x + c)^5 - (8*a^2 - 11*a*b +
 3*b^2)*d*cosh(d*x + c)^3 - (a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqr
t(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/(
(a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a
*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((
a + 3*b)*cosh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c) + (a + 3
*b)*sinh(d*x + c)^2 + 2*(2*(a^2*b + 3*a*b^2)*d*cosh(d*x + c) + 2*(a^2*b + 3
*a*b^2)*d*sinh(d*x + c) + ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*co
sh(d*x + c) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sinh(d*x + c))*
sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6
+ 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b
^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b
^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) + a + 3*b) + ((a*b - b^2)*d*cosh(d*x
 + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*d*sin
h(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*
x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*co
sh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 - 3*(a*b - b^2)*d*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(d*x + c)^4 - 30*(a*b -
b^2)*d*cosh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a
*b - b^2)*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^5 - 10*(a*b
- b^2)*d*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c))*sinh(d
*x + c)^3 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*(a*b - b^2)*d*cosh(d*x
+ c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh
(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(d*x + c)^7 - 3*(a*b - b
^2)*d*cosh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^3 - (a*b -
 b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
- a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5
- 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cosh(d*x + c)^2 + 2*(a +
3*b)*cosh(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d*x + c)^2 - 2*(2*(a^2*b
+ 3*a*b^2)*d*cosh(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*sinh(d*x + c) + ((a^5*b^
2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*cosh(d*x + c) + (a^5*b^2 - 2*a^4*b^3
 + 2*a^2*b^5 - a*b^6)*d^3*sinh(d*x + c))*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b
^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*
d^4)))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b
+ 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a
^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d
^2)) + a + 3*b) + 8*(7*cosh(d*x + c)^6 - 25*cosh(d*x + c)^4 - 15*cosh(d*x +
 c)^2 + 1)*sinh(d*x + c) + 8*cosh(d*x + c))/((a*b - b^2)*d*cosh(d*x + c)^8
+ 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x +
c)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2
 - (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x +
 c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))
*sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*c
osh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a*b - b^2
)*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^5 - 10*(a*b - b^2)*d
*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*(a*b - b^2)*d*cosh(d*x + c)^4 -
 3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c
)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(d*x + c)^7 - 3*(a*b - b^2)*d*co
sh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^3 - (a*b - b^2)*d*
cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 0.73, size = 854, normalized size = 4.59

$$-\frac{\left(\left(\sqrt{ab}\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^2+8\,\sqrt{ab}\,\sqrt{-b^2+\sqrt{ab}\,b}\,ab\right)(a-b)^2|b|+2\left(\sqrt{-b^2+\sqrt{ab}\,b}\,a^3b+7\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b^2-8\,\sqrt{-b^2+\sqrt{ab}\,b}\,ab^3\right)|-a+b||b|+\left(\sqrt{ab}\,\sqrt{-b^2+\cdots}\right.}{\left(a^5b^3+5\,a^4b^4-21\,a^3b^5+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 8*sqrt(a*b)*sqrt(-b^2 + sq
rt(a*b)*b)*a*b)*(a - b)^2*abs(b) + 2*(sqrt(-b^2 + sqrt(a*b)*b)*a^3*b + 7*sq
rt(-b^2 + sqrt(a*b)*b)*a^2*b^2 - 8*sqrt(-b^2 + sqrt(a*b)*b)*a*b^3)*abs(-a +
 b)*abs(b) + (sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b + 6*sqrt(a*b)*sqrt(-
b^2 + sqrt(a*b)*b)*a^2*b^2 - 15*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^3 +
8*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^4)*abs(b))*arctan(1/2*(e^(d*x + c) +
 e^(-d*x - c))/sqrt(-(a*b - b^2 + sqrt((a^2 - 2*a*b + b^2)*(a*b - b^2) + (a
*b - b^2)^2))/(a*b - b^2)))/((a^5*b^3 + 5*a^4*b^4 - 21*a^3*b^5 + 23*a^2*b^6
 - 8*a*b^7)*abs(-a + b)) + ((sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2 + 8*sqr
t(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b)*(a - b)^2*abs(b) + 2*(sqrt(-b^2 - sqrt
(a*b)*b)*a^3*b + 7*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^2 - 8*sqrt(-b^2 - sqrt(a*
b)*b)*a*b^3)*abs(-a + b)*abs(b) + (sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b
 + 6*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^2 - 15*sqrt(a*b)*sqrt(-b^2 -
sqrt(a*b)*b)*a*b^3 + 8*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^4)*abs(b))*arct
an(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a*b - b^2 - sqrt((a^2 - 2*a*b +
b^2)*(a*b - b^2) + (a*b - b^2)^2))/(a*b - b^2)))/((a^5*b^3 + 5*a^4*b^4 - 21
*a^3*b^5 + 23*a^2*b^6 - 8*a*b^7)*abs(-a + b)) + 4*((e^(d*x + c) + e^(-d*x -
 c))^3 - 8*e^(d*x + c) - 8*e^(-d*x - c))/((b*(e^(d*x + c) + e^(-d*x - c))^4
 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)*(a - b)))/d
```

maple [B] time = 0.10, size = 767, normalized size = 4.12

$$-\frac{\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\left(\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-4\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+6\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-16b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(\tanh^2\left(\frac{dx}{2}\cdots\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh
(1/2*d*x+1/2*c)^6-3/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+
6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^
2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4+4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*
d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(
1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^4*b+5/2/d/(tanh(1/2*d*x+1
/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2
*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2-1/2/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)-1/8/d/b/(
a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*
b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1/8/d/(a-b)/(-a*b+(a*
b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/
(-a*b+(a*b)^(1/2)*a)^(1/2))+1/8/d/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1
/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2
```

```
))-1/8/d/b/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2
*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{e^{(7\,dx+7\,c)}-5\,e^{(5\,dx+5\,c)}-5\,e^{(3\,dx+3\,c)}+e^{(dx+c)}}{2\left(abd-b^2d+\left(abde^{(8\,c)}-b^2de^{(8\,c)}\right)e^{(8\,dx)}-4\left(abde^{(6\,c)}-b^2de^{(6\,c)}\right)e^{(6\,dx)}-2\left(8\,a^2de^{(4\,c)}-11\,abde^{(4\,c)}+3\,b^2de^{(4\,c)}\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^(7*d*x + 7*c) - 5*e^(5*d*x + 5*c) - 5*e^(3*d*x + 3*c) + e^(d*x + c)
)/(a*b*d - b^2*d + (a*b*d*e^(8*c) - b^2*d*e^(8*c))*e^(8*d*x) - 4*(a*b*d*e^(
6*c) - b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*d*e^(4*c) - 11*a*b*d*e^(4*c) + 3
*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a*b*d*e^(2*c) - b^2*d*e^(2*c))*e^(2*d*x)) -
1/8*integrate(4*(e^(7*d*x + 7*c) - 7*e^(5*d*x + 5*c) + 7*e^(3*d*x + 3*c) -
e^(d*x + c))/(a*b - b^2 + (a*b*e^(8*c) - b^2*e^(8*c))*e^(8*d*x) - 4*(a*b*e^
(6*c) - b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^2*e^(4*c) - 11*a*b*e^(4*c) + 3*b^2*
e^(4*c))*e^(4*d*x) - 4*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh\left(c+d\,x\right)^3}{{\left(a-b\,\sinh\left(c+d\,x\right)^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.245 $\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=221

$$\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\cosh(c+dx)\left(a-b\cosh^2\right.}{4ad(a-b)\left(a-b\cosh^4(c+dx)+\right.}$$

```
[Out] 1/4*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*co
sh(d*x+c)^4)+1/8*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a^(
1/2)-2*b^(1/2))/a^(3/2)/b^(1/4)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh(b^(1/
4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(3*a^(1/2)+2*b^(1/2))/a^(3/2)/b^(1/
4)/d/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A] time = 0.30, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3215, 1092, 1166, 205, 208}

$$\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\cosh(c+dx)\left(a-b\cosh^2\right.}{4ad(a-b)\left(a-b\cosh^4(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((3*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*a^(3/2)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(1/4)*d) + ((3*Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(3/2)
*(Sqrt[a] + Sqrt[b])^(3/2)*b^(1/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c +
d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2-2(\cdots}{a-b+\cdots}\right.}{\cdots}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\left(\left(3\sqrt{a}-2\sqrt{b}\right)\sqrt{b}\right)\text{Su}\cdots}{8\cdots}$$

$$= \frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt[4]{b}\,d} + \frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt[4]{b}\,d} + \frac{\cdots}{4a(\cdots}$$

Mathematica [C] time = 0.38, size = 597, normalized size = 2.70

$$\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6b\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((32*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2
*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b
*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 12*a*c
*#1^2 - 5*b*c*#1^2 + 12*a*d*x*#1^2 - 5*b*d*x*#1^2 + 24*a*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^
2 - 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
- Sinh[(c + d*x)/2]*#1]*#1^2 - 12*a*c*#1^4 + 5*b*c*#1^4 - 12*a*d*x*#1^4 + 5
*b*d*x*#1^4 - 24*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 10*b*Log[-Cosh[(c + d*x)/2] - Sinh
[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^
6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
 d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 -
3*b*#1^5 + b*#1^7) & ])/(32*a*(a - b)*d)
```

fricas [B] time = 0.88, size = 6018, normalized size = 27.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

```
[Out] 1/16*(8*b*cosh(d*x + c)^7 + 56*b*cosh(d*x + c)*sinh(d*x + c)^6 + 8*b*sinh(d
*x + c)^7 - 8*(4*a + b)*cosh(d*x + c)^5 + 8*(21*b*cosh(d*x + c)^2 - 4*a - b
)*sinh(d*x + c)^5 + 40*(7*b*cosh(d*x + c)^3 - (4*a + b)*cosh(d*x + c))*sinh
(d*x + c)^4 - 8*(4*a + b)*cosh(d*x + c)^3 + 8*(35*b*cosh(d*x + c)^4 - 10*(4
*a + b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c)^3 + 8*(21*b*cosh(d*x + c)^
5 - 10*(4*a + b)*cosh(d*x + c)^3 - 3*(4*a + b)*cosh(d*x + c))*sinh(d*x + c)
^2 + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)
*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*
cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*
d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(
7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2
)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(
a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 -
 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a
^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x
 + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*
b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3
 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a
^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^
5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5
*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c
)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c)*sinh(d*x + c) + (81*a^2 -
81*a*b + 20*b^2)*sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 + 2*((27*a^4 -
24*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d*s
inh(d*x + c) - 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d
^3*cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*
d^3*sinh(d*x + c))*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15
*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*sqrt(-((a^
6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^
9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^
7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*
d^2))) - ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2
)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b
^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 +
 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a
*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 -
 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)
^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 1
5*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh
(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((
a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8
*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x +
 c))*sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((
81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 1
5*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3
*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cosh(d*x
 + c)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c)*sinh(d*x + c) + (81*a^
2 - 81*a*b + 20*b^2)*sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 - 2*((27*a^
4 - 24*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)
*d*sinh(d*x + c) - 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^
```

```
5)*d^3*cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b
^5)*d^3*sinh(d*x + c))*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2
+ 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*sqrt(-
((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/
((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^
3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b
^3)*d^2))) + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a
*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b -
 a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)
^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b
 - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)
^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x
+ c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2
)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6
 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*
cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d +
8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5
- (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d
*x + c))*sinh(d*x + c))*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqr
t((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4
+ 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6
- 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cosh(
d*x + c)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c)*sinh(d*x + c) + (81
*a^2 - 81*a*b + 20*b^2)*sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 + 2*((27
*a^4 - 24*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b
^2)*d*sinh(d*x + c) + 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3
*b^5)*d^3*cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^
3*b^5)*d^3*sinh(d*x + c))*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b
^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*sqr
t(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2
)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 +
a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3
*b^3)*d^2))) - ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cos
h(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b
 - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x +
c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*
x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2
*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x +
c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*
x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b
^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)
^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*
d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d
+ 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^
5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh
(d*x + c))*sinh(d*x + c))*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*s
qrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^
4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^
6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cos
h(d*x + c)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c)*sinh(d*x + c) + (
81*a^2 - 81*a*b + 20*b^2)*sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 - 2*((
27*a^4 - 24*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2
*b^2)*d*sinh(d*x + c) + 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a
^3*b^5)*d^3*cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 +
a^3*b^5)*d^3*sinh(d*x + c))*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8
*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*s
```

```
qrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b
^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6
+ a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a
^3*b^3)*d^2))) + 8*b*cosh(d*x + c) + 8*(7*b*cosh(d*x + c)^6 - 5*(4*a + b)*c
osh(d*x + c)^4 - 3*(4*a + b)*cosh(d*x + c)^2 + b)*sinh(d*x + c))/((a^2*b -
a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^
7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6
 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c
)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^
2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 +
 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x +
c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*
d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*
b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh
(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d
*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*
b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*co
sh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3
*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 0.54, size = 1050, normalized size = 4.75

$$\left(\left(\sqrt{ab}\,\sqrt{-b^2+\sqrt{ab}\,b}\,ab+8\,\sqrt{ab}\,\sqrt{-b^2+\sqrt{ab}\,b}\,b^2\right)\left(a^2-ab\right)^2|b|-\left(3\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^4b+20\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^3b^2-31\,\sqrt{-b^2+\sqrt{ab}\,b}\,a^2b^3+8\,\sqrt{-b^2+\sqrt{ab}\,b}\,a \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b + 8*sqrt(a*b)*sqrt(-b^2 + sq
rt(a*b)*b)*b^2)*(a^2 - a*b)^2*abs(b) - (3*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b +
20*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^2 - 31*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^3 +
 8*sqrt(-b^2 + sqrt(a*b)*b)*a*b^4)*abs(-a^2 + a*b)*abs(b) + (3*sqrt(a*b)*sq
rt(-b^2 + sqrt(a*b)*b)*a^5*b + 16*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^
2 - 57*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3 + 54*sqrt(a*b)*sqrt(-b^2
+ sqrt(a*b)*b)*a^2*b^4 - 16*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^5)*abs(b
))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^2*b - a*b^2 + sqrt((a^3
 - 2*a^2*b + a*b^2)*(a^2*b - a*b^2) + (a^2*b - a*b^2)^2))/(a^2*b - a*b^2)))
/((a^6*b^3 + 5*a^5*b^4 - 21*a^4*b^5 + 23*a^3*b^6 - 8*a^2*b^7)*abs(-a^2 + a*
b)) + ((sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b + 8*sqrt(a*b)*sqrt(-b^2 - sq
rt(a*b)*b)*b^2)*(a^2 - a*b)^2*abs(b) - (3*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b +
20*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^2 - 31*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^3 +
 8*sqrt(-b^2 - sqrt(a*b)*b)*a*b^4)*abs(-a^2 + a*b)*abs(b) + (3*sqrt(a*b)*sq
rt(-b^2 - sqrt(a*b)*b)*a^5*b + 16*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^
2 - 57*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 + 54*sqrt(a*b)*sqrt(-b^2
- sqrt(a*b)*b)*a^2*b^4 - 16*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^5)*abs(b
))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^2*b - a*b^2 - sqrt((a^3
 - 2*a^2*b + a*b^2)*(a^2*b - a*b^2) + (a^2*b - a*b^2)^2))/(a^2*b - a*b^2)))
/((a^6*b^3 + 5*a^5*b^4 - 21*a^4*b^5 + 23*a^3*b^6 - 8*a^2*b^7)*abs(-a^2 + a*
b)) - 4*(b*(e^(d*x + c) + e^(-d*x - c))^3 - 4*a*(e^(d*x + c) + e^(-d*x - c)
) - 4*b*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4 -
8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)*(a^2 - a*b)))/d
```

maple [B] time = 0.14, size = 1112, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh
(1/2*d*x+1/2*c)^6+1/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)/(a-b)/a*tanh(1/2*d*x+1/2*c)^6*b+3/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(
1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*t
anh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4-4/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^4*b-3/
2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/
2*d*x+1/2*c)^2-1/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tan
h(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a
)/a/(a-b)*tanh(1/2*d*x+1/2*c)^2*b+1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)/(a-b)+1/8/d/(a-b)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arcta
n(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1
/2))*(a*b)^(1/2)+3/8/d/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(
1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/4/d/(a-
b)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*
b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*b+1/8/d/(a-b)/a/(-a*b-(a*b)^(1/2)
*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(
a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-3/8/d/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arc
tan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)
^(1/2))+1/4/d/(a-b)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*
x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(4\,ae^{(5\,c)}+be^{(5\,c)}\right)e^{(5\,dx)}+\left(4\,ae^{(3\,c)}+be^{(3\,c)}\right)e^{(3\,dx)}-be^{(7\,dx+7\,c)}-be^{(dx}}{2\left(a^2bd-ab^2d+\left(a^2bde^{(8\,c)}-ab^2de^{(8\,c)}\right)e^{(8\,dx)}-4\left(a^2bde^{(6\,c)}-ab^2de^{(6\,c)}\right)e^{(6\,dx)}-2\left(8\,a^3de^{(4\,c)}-11\,a^2bde^{(4\,c)}+3\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*e^(5*c) + b*e^(5*c))*e^(5*d*x) + (4*a*e^(3*c) + b*e^(3*c))*e^(3*
d*x) - b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(
8*c) - a*b^2*d*e^(8*c))*e^(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e
^(6*d*x) - 2*(8*a^3*d*e^(4*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(
4*d*x) - 4*(a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(-
((12*a*e^(5*c) - 5*b*e^(5*c))*e^(5*d*x) - (12*a*e^(3*c) - 5*b*e^(3*c))*e^(3
*d*x) - b*e^(7*d*x + 7*c) + b*e^(d*x + c))/(a^2*b - a*b^2 + (a^2*b*e^(8*c)
- a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*c))*e^(6*d*x) -
2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(4*d*x) - 4*(a^2*b
*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(c+d\,x\right)}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.246 $\int \frac{\mathrm{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=325

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tan}{}$$

```
[Out] -arctanh(cosh(d*x+c))/a^2/d-1/4*b*cosh(d*x+c)*(2-cosh(d*x+c)^2)/a/(a-b)/d/(
a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)-1/8*b^(1/4)*arctan(b^(1/4)*cosh(d*x+
c)/(a^(1/2)-b^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*b^(1/4)*a
rctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)+b^(1
/2))^(3/2)-1/2*b^(1/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/
a^2/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1
/2)+b^(1/2))^(1/2))/a^2/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3215, 1238, 207, 1178, 1166, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(3/
2)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/S
qrt[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c
 + d*x]]/(a^2*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
Sqrt[b]]])/(8*a^(3/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1
/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] + Sqrt[b]]
*d) - (b*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*C
osh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2(-1+x^2)}+\frac{b-bx^2}{a\left(a-b+2bx^2-bx^4\right)^2}+\frac{b-bx^2}{a^2\left(a-b+2bx^2-bx^4\right)}\right)dx, x, \cosh(c+dx\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(c+dx)\right)}{a^2 d} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx\right)}{a^2 d}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+\right)}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2 d}$$

Mathematica [C] time = 0.88, size = 761, normalized size = 2.34

$$-\frac{b\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&,\frac{10\#1^6a\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)+5\#1^6ac+5\#1^6adx-8\#1^6b\log\cdots}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((16*a*b*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/((a - b)*(8*a - 3*b + 4*b*
Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + 32*Log[Tanh[(c + d*x)/2]] - (b*
RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-5*a*c
 + 4*b*c - 5*a*d*x + 4*b*d*x - 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 8*b*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 1
9*a*c*#1^2 - 12*b*c*#1^2 + 19*a*d*x*#1^2 - 12*b*d*x*#1^2 + 38*a*Log[-Cosh[(
c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*
#1]*#1^2 - 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 19*a*c*#1^4 + 12*b*c*#1^4 - 19*a*d*x*
#1^4 + 12*b*d*x*#1^4 - 38*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Co
sh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 24*b*Log[-Cosh[(c + d*x)/
2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4
+ 5*a*c*#1^6 - 4*b*c*#1^6 + 5*a*d*x*#1^6 - 4*b*d*x*#1^6 + 10*a*Log[-Cosh[(c
 + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#
1]*#1^6 - 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^
5 + b*#1^7) & ])/(a - b))/(32*a^2*d)
```

fricas [B] time = 1.56, size = 7793, normalized size = 23.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a*b*cosh(d*x + c)^7 + 56*a*b*cosh(d*x + c)*sinh(d*x + c)^6 + 8*a*b
*sinh(d*x + c)^7 - 40*a*b*cosh(d*x + c)^5 + 8*(21*a*b*cosh(d*x + c)^2 - 5*a
*b)*sinh(d*x + c)^5 - 40*a*b*cosh(d*x + c)^3 + 40*(7*a*b*cosh(d*x + c)^3 -
5*a*b*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(7*a*b*cosh(d*x + c)^4 - 10*a*b*c
osh(d*x + c)^2 - a*b)*sinh(d*x + c)^3 + 8*a*b*cosh(d*x + c) + 8*(21*a*b*cos
h(d*x + c)^5 - 50*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x + c))*sinh(d*x + c)
^2 + ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4*(a^3*b - a^2
*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b
 - a^2*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*
x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b - a^2*b^2)*d
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4
 - 30*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*
d)*sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b -
a^2*b^2)*d*cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (8*a^
4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b -
a^2*b^2)*d*cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 3*(8*
a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d
*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 -
3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*co
sh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a
^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 12
41*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*
```

b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^
3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-625*a^3*b + 1125*a^2*b
^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)
*cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*cosh(
d*x + c)*sinh(d*x + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*s
inh(d*x + c)^2 + 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d*
cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d*sinh
(d*x + c) - ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*
cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d
^3*sinh(d*x + c))*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4
 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a
^8*b^5 + a^7*b^6)*d^4)))*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*s
qrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 -
 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d
^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)
*d^2))) - ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d*cosh
(d*x + c)*sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4*(a^3*b
- a^2*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (
a^3*b - a^2*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*co
sh(d*x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b - a^2*b
^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^4 - 30*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*
b^2)*d)*sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^3
*b - a^2*b^2)*d*cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 -
(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3
*b - a^2*b^2)*d*cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 -
3*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*s
inh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(d*x + c)
^7 - 3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)
*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt
(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2
 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 +
16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-625*a^3*b + 1125*
a^2*b^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128
*b^4)*cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*
cosh(d*x + c)*sinh(d*x + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b
^4)*sinh(d*x + c)^2 - 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^
4)*d*cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d
*sinh(d*x + c) - ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)
*d^3*cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b
^4)*d^3*sinh(d*x + c))*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*
a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4
- 6*a^8*b^5 + a^7*b^6)*d^4)))*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*
d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a
^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b
^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4
*b^3)*d^2))) + ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d
*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4*(a
^3*b - a^2*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^
2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)
*d*cosh(d*x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b -
a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*cosh(
d*x + c)^4 - 30*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3
*a^2*b^2)*d)*sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(7
*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)
^3 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7
*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)
^4 - 3*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)

```
*d)*sinh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(d*x
 + c)^7 - 3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2
*b^2)*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3
*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 -
 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^
2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-625*a^3*b + 1
125*a^2*b^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 -
 128*b^4)*cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b
^4)*cosh(d*x + c)*sinh(d*x + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 1
28*b^4)*sinh(d*x + c)^2 + 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^
2*b^4)*d*cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^
4)*d*sinh(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*
b^4)*d^3*cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a
^6*b^4)*d^3*sinh(d*x + c))*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 -
464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*
b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^
3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/
((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^
7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 -
a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2
)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4
*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x +
c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b
^2)*d*cosh(d*x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b
 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*co
sh(d*x + c)^4 - 30*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b
+ 3*a^2*b^2)*d)*sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8
*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^3 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4
*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^4 - 3*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b
^2)*d)*sinh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(
d*x + c)^7 - 3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*
a^2*b^2)*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x +
c))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*
a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^
2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a
*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-625*a^3*b
+ 1125*a^2*b^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^
3 - 128*b^4)*cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 12
8*b^4)*cosh(d*x + c)*sinh(d*x + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3
- 128*b^4)*sinh(d*x + c)^2 - 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16
*a^2*b^4)*d*cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2
*b^4)*d*sinh(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a
^6*b^4)*d^3*cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 +
3*a^6*b^4)*d^3*sinh(d*x + c))*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3
 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a
^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4
*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^
5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 +
 a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2
 - a^4*b^3)*d^2))) + 16*((a*b - b^2)*cosh(d*x + c)^8 + 8*(a*b - b^2)*cosh(d
*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*sinh(d*x + c)^8 - 4*(a*b - b^2)*cosh(
d*x + c)^6 + 4*(7*(a*b - b^2)*cosh(d*x + c)^2 - a*b + b^2)*sinh(d*x + c)^6
+ 8*(7*(a*b - b^2)*cosh(d*x + c)^3 - 3*(a*b - b^2)*cosh(d*x + c))*sinh(d*x
+ c)^5 - 2*(8*a^2 - 11*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a*b - b^2)*cos
h(d*x + c)^4 - 30*(a*b - b^2)*cosh(d*x + c)^2 - 8*a^2 + 11*a*b - 3*b^2)*sin
h(d*x + c)^4 + 8*(7*(a*b - b^2)*cosh(d*x + c)^5 - 10*(a*b - b^2)*cosh(d*x +
```

```
 c)^3 - (8*a^2 - 11*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b -
b^2)*cosh(d*x + c)^2 + 4*(7*(a*b - b^2)*cosh(d*x + c)^6 - 15*(a*b - b^2)*co
sh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*cosh(d*x + c)^2 - a*b + b^2)*sin
h(d*x + c)^2 + a*b - b^2 + 8*((a*b - b^2)*cosh(d*x + c)^7 - 3*(a*b - b^2)*c
osh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*cosh(d*x + c)^3 - (a*b - b^2)*cos
h(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 16*((a*
b - b^2)*cosh(d*x + c)^8 + 8*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
*b - b^2)*sinh(d*x + c)^8 - 4*(a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(a*b - b^2
)*cosh(d*x + c)^2 - a*b + b^2)*sinh(d*x + c)^6 + 8*(7*(a*b - b^2)*cosh(d*x
+ c)^3 - 3*(a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(8*a^2 - 11*a*b +
 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a*b - b^2)*cosh(d*x + c)^4 - 30*(a*b - b^2
)*cosh(d*x + c)^2 - 8*a^2 + 11*a*b - 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a*b - b
^2)*cosh(d*x + c)^5 - 10*(a*b - b^2)*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b - b^2)*cosh(d*x + c)^2 + 4*(7*
(a*b - b^2)*cosh(d*x + c)^6 - 15*(a*b - b^2)*cosh(d*x + c)^4 - 3*(8*a^2 - 1
1*a*b + 3*b^2)*cosh(d*x + c)^2 - a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2 + 8
*((a*b - b^2)*cosh(d*x + c)^7 - 3*(a*b - b^2)*cosh(d*x + c)^5 - (8*a^2 - 11
*a*b + 3*b^2)*cosh(d*x + c)^3 - (a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) - 1) + 8*(7*a*b*cosh(d*x + c)^6 - 25*a*b*c
osh(d*x + c)^4 - 15*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^3*b - a^2
*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)
^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sin
h(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a
^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b - a^2*b^2)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 30*(a^3*b - a^2
*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d)*sinh(d*x + c)^4
 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*
x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (8*a^4 - 11*a^3*b + 3*a
^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*
x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^4 - 11*a^3*b + 3
*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^2 + (a^3*b
 - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 3*(a^3*b - a^2*b^2
)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^3 - (a
^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [B] time = 0.40, size = 1112, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*(((5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 36*sqrt(a*b)*sqrt(-b^2 +
sqrt(a*b)*b)*a*b - 32*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*(a^3 - a^2*b)
^2*abs(b) - 2*(3*sqrt(-b^2 + sqrt(a*b)*b)*a^5*b + 19*sqrt(-b^2 + sqrt(a*b)*
b)*a^4*b^2 - 38*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3 + 16*sqrt(-b^2 + sqrt(a*b)
*b)*a^2*b^4)*abs(-a^3 + a^2*b)*abs(b) + (sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)
*a^7*b + 6*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^6*b^2 - 15*sqrt(a*b)*sqrt(-
b^2 + sqrt(a*b)*b)*a^5*b^3 + 8*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^4)*
abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^3*b - a^2*b^2 + sq
rt((a^4 - 2*a^3*b + a^2*b^2)*(a^3*b - a^2*b^2) + (a^3*b - a^2*b^2)^2))/(a^3
*b - a^2*b^2)))/((a^8*b^2 + 5*a^7*b^3 - 21*a^6*b^4 + 23*a^5*b^5 - 8*a^4*b^6
)*abs(-a^3 + a^2*b)) - ((5*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2 + 36*sqrt
(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b - 32*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*
b^2)*(a^3 - a^2*b)^2*abs(b) + 2*(3*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b + 19*sqrt
(-b^2 - sqrt(a*b)*b)*a^4*b^2 - 38*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 + 16*sqr
t(-b^2 - sqrt(a*b)*b)*a^2*b^4)*abs(-a^3 + a^2*b)*abs(b) + (sqrt(a*b)*sqrt(-
b^2 - sqrt(a*b)*b)*a^7*b + 6*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^6*b^2 - 1
5*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b^3 + 8*sqrt(a*b)*sqrt(-b^2 - sqrt
```

```
(a*b)*b)*a^4*b^4)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^
3*b - a^2*b^2 - sqrt((a^4 - 2*a^3*b + a^2*b^2)*(a^3*b - a^2*b^2) + (a^3*b -
 a^2*b^2)^2))/(a^3*b - a^2*b^2)))/((a^8*b^2 + 5*a^7*b^3 - 21*a^6*b^4 + 23*a
^5*b^5 - 8*a^4*b^6)*abs(-a^3 + a^2*b)) - 4*(b*(e^(d*x + c) + e^(-d*x - c))^
3 - 8*b*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4 -
8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)*(a^2 - a*b)) - 4*log(e^(d
*x + c) + e^(-d*x - c) + 2)/a^2 + 4*log(e^(d*x + c) + e^(-d*x - c) - 2)/a^2
)/d
```

maple [B] time = 0.18, size = 966, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)/a*ta
nh(1/2*d*x+1/2*c)^6*b-3/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^4*b+4/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)
^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+
1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4+5/2/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x
+1/2*c)^2*b-1/2/d/a*b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)/(a-b)-5/8/d/(a-b)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*
d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+1
/2/d/a^2*b/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*
c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1/8/d/(a-
b)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*
b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*b-5/8/d/(a-b)/a/(-a*b-(a*b)^(1/2)
*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(
a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+1/2/d/a^2*b/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/
2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1
/2)*a)^(1/2))*(a*b)^(1/2)+1/8/d/(a-b)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1
/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2
))*b+1/d/a^2*ln(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{be^{(7\,dx+7\,c)} - 5\,be^{(5\,dx+5\,c)} - 5\,be^{(3\,dx+3\,c)} + be^{(dx+c)}}{2\left(a^2bd - ab^2d + \left(a^2bde^{(8\,c)} - ab^2de^{(8\,c)}\right)e^{(8\,dx)} - 4\left(a^2bde^{(6\,c)} - ab^2de^{(6\,c)}\right)e^{(6\,dx)} - 2\left(8\,a^3de^{(4\,c)} - 11\,a^2bde^{(4\,c)} + 3\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*e^(7*d*x + 7*c) - 5*b*e^(5*d*x + 5*c) - 5*b*e^(3*d*x + 3*c) + b*e^(
d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^(8*d*x
) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4*c) -
11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c) - a*
b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log((e^
(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/4*((5*a*b*e^(7*c) - 4*b^2*e^
(7*c))*e^(7*d*x) - (19*a*b*e^(5*c) - 12*b^2*e^(5*c))*e^(5*d*x) + (19*a*b*e^
(3*c) - 12*b^2*e^(3*c))*e^(3*d*x) - (5*a*b*e^c - 4*b^2*e^c)*e^(d*x))/(a^3*b
 - a^2*b^2 + (a^3*b*e^(8*c) - a^2*b^2*e^(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c)
 - a^2*b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^4*e^(4*c) - 11*a^3*b*e^(4*c) + 3*a^2
```

```
*b^2*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(c+d\,x\right)\,{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^2),x)
[Out] int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4)**2,x)
[Out] Timed out
```

3.247 $\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=320

$$\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\right.}{2b^2d}$$

```
[Out] x/b^2+1/8*a^(1/4)*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/b^(3
/2)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*a^(1/4)*arctanh((a^(1/2)+b^(1/2))^(1/2)*t
anh(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)+b^(1/2))^(3/2)-1/2*a^(1/4)*arctanh((
a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/b^2/d/(a^(1/2)-b^(1/2))^(1/2)-1
/2*a^(1/4)*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/b^2/d/(a^(1
/2)+b^(1/2))^(1/2)-1/4*tanh(d*x+c)/(a-b)/b/d+1/4*tanh(d*x+c)^5/b/d/(a-2*a*t
anh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A] time = 0.46, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3217, 1313, 1275, 12, 1122, 1166, 208, 1287, 207}

$$\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\right.}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] x/b^2 - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/
(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tanh[c + d*x])/a^(1/4)])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - (a^(1
/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[
a] + Sqrt[b]]*b^2*d) - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d
*x])/a^(1/4)])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tanh[c + d*x]/(4*(
a - b)*b*d) + Tanh[c + d*x]^5/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tan
h[c + d*x]^4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
```

```
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1)
)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{(1-x^2)\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(a-ax^2)}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{bd} - \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a-2ax^2+(a-b)x^4\right)}\,dx,\right.}{bd}$$

$$= \frac{\tanh^5(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2abx^4}{a-2ax^2+(a-b)x^4}\,dx,\right.}{8ab^2d}$$

$$= \frac{\tanh^5(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(\right.}{b^2d}$$

$$= \frac{x}{b^2} - \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\tanh^5(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\left(\sqrt{a}\,\left(\right.\right.}{}$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d} - \frac{\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^2d} - \frac{\tanh(c}{4(a-}$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d} + \frac{\sqrt[4]{a}\,\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} - \frac{\sqrt[4]{a}\,\tan}{}$$

Mathematica [A] time = 4.78, size = 262, normalized size = 0.82

$$\frac{-\frac{\sqrt{a}\left(4\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{a}\left(4\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{2ab(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{(a-b)(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b)}}{8b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (8*(c + d*x) + (Sqrt[a]*(4*Sqrt[a] - 5*Sqrt[b])*ArcTan[((Sqrt[a] - Sqrt[b])
*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a +
 Sqrt[a]*Sqrt[b]]) - (Sqrt[a]*(4*Sqrt[a] + 5*Sqrt[b])*ArcTanh[((Sqrt[a] + S
qrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqr
t[a + Sqrt[a]*Sqrt[b]]) + (2*a*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)])
)/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])))/(8*b
^2*d)
```

fricas [B] time = 1.41, size = 6944, normalized size = 21.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*(a*b - b^2)*d*x*cosh(d*x + c)^8 + 128*(a*b - b^2)*d*x*cosh(d*x + c
)*sinh(d*x + c)^7 + 16*(a*b - b^2)*d*x*sinh(d*x + c)^8 - 8*(8*(a*b - b^2)*d
```

```
*x - a*b)*cosh(d*x + c)^6 + 8*(56*(a*b - b^2)*d*x*cosh(d*x + c)^2 - 8*(a*b
- b^2)*d*x + a*b)*sinh(d*x + c)^6 + 16*(56*(a*b - b^2)*d*x*cosh(d*x + c)^3
- 3*(8*(a*b - b^2)*d*x - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(4*(8*a^2
- 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x + c)^4 + 8*(140*(a*b - b^2)
*d*x*cosh(d*x + c)^4 - 4*(8*a^2 - 11*a*b + 3*b^2)*d*x - 15*(8*(a*b - b^2)*d
*x - a*b)*cosh(d*x + c)^2 - 8*a^2 + 3*a*b)*sinh(d*x + c)^4 + 32*(28*(a*b -
b^2)*d*x*cosh(d*x + c)^5 - 5*(8*(a*b - b^2)*d*x - a*b)*cosh(d*x + c)^3 - (4
*(8*a^2 - 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x + c))*sinh(d*x + c)
^3 + 16*(a*b - b^2)*d*x - 8*(8*(a*b - b^2)*d*x + 5*a*b)*cosh(d*x + c)^2 + 8
*(56*(a*b - b^2)*d*x*cosh(d*x + c)^6 - 15*(8*(a*b - b^2)*d*x - a*b)*cosh(d*
x + c)^4 - 8*(a*b - b^2)*d*x - 6*(4*(8*a^2 - 11*a*b + 3*b^2)*d*x + 8*a^2 -
3*a*b)*cosh(d*x + c)^2 - 5*a*b)*sinh(d*x + c)^2 + ((a*b^3 - b^4)*d*cosh(d*x
 + c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^3 - b^4)*d
*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d
*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 3*(a
*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*cosh(d
*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*
b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + 8*(7*(a*b^3 -
 b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (8*a^2*b^2 -
 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^3 - b^4)*d*
cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 3*(8*a^2*b^2 - 11*a*
b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a*b^3
- b^4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(a*b^3 - b^4)*d*cosh(d*x
+ c)^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d
*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)
*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((
a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b
^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6
- b^7)*d^2))*log(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b
^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)
/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12
+ b^13)*d^4)) + 128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3 - (128*a^3 - 664
*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)^2 - 2*(128*a^3 - 664*a^2*b + 1
125*a*b^2 - 625*b^3)*cosh(d*x + c)*sinh(d*x + c) - (128*a^3 - 664*a^2*b + 1
125*a*b^2 - 625*b^3)*sinh(d*x + c)^2 + 2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2
*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450
*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15
*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4
- 125*b^5)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5
 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b
^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a
^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) - (
(a*b^3 - b^4)*d*cosh(d*x + c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x
+ c)^7 + (a*b^3 - b^4)*d*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^
6 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6
 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*
d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3
5*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8
*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*
x + c)^2 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d
*x + c)^3 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^
4 - 3*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*s
inh(d*x + c)^2 + (a*b^3 - b^4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(
a*b^3 - b^4)*d*cosh(d*x + c)^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x
+ c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^3*b^4 - 3*
a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450
*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15
```

*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^
4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a
^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1
450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 +
 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 128*a^3 - 664*a^2*b + 1125*a*b^2 -
625*b^3 - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)^2 - 2*
(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)*sinh(d*x + c) -
(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*sinh(d*x + c)^2 - 2*(2*(2*a^4*
b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4
*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^
4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + (24*a^3*b^2 -
127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6
 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*
b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b
^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3
*a*b^6 - b^7)*d^2))) - ((a*b^3 - b^4)*d*cosh(d*x + c)^8 + 8*(a*b^3 - b^4)*d
*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^3 - b^4)*d*sinh(d*x + c)^8 - 4*(a*b^3
 - b^4)*d*cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 -
 b^4)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)
^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 30*(a*b^3 - b^4
)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^4 - 4
*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 1
0*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 15*(a*b^3
 - b^4)*d*cosh(d*x + c)^4 - 3*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c
)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d + 8*((a*b^3 - b^4)
*d*cosh(d*x + c)^7 - 3*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - (8*a^2*b^2 - 11*a*
b^3 + 3*b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x +
c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*
b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4
*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2
*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-2*(16*a^4*
b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*sqrt((64*a^5 - 464*
a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15
*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 128*a^3 - 6
64*a^2*b + 1125*a*b^2 - 625*b^3 - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b
^3)*cosh(d*x + c)^2 - 2*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d
*x + c)*sinh(d*x + c) - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*sinh(d
x + c)^2 + 2(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^
3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6
*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13
)*d^4)) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d)*sqrt(((a^3*b^
4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2
 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^1
0 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((
a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + ((a*b^3 - b^4)*d*cosh(d*x + c
)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^3 - b^4)*d*sin
h(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d*cos
h(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b^3 +
 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3
 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*cosh(d*x +
 c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)
*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + 8*(7*(a*b^3 - b^4
)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (8*a^2*b^2 - 11*
a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^3 - b^4)*d*cosh
(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 3*(8*a^2*b^2 - 11*a*b^3
+ 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a*b^3 - b^
4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(a*b^3 - b^4)*d*cosh(d*x + c)

```
^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cos
h(d*x + c))*sinh(d*x + c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*
sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b
^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*
d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7
)*d^2))*log(-2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*
d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a
^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^
13)*d^4)) + 128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3 - (128*a^3 - 664*a^2
*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)^2 - 2*(128*a^3 - 664*a^2*b + 1125*
a*b^2 - 625*b^3)*cosh(d*x + c)*sinh(d*x + c) - (128*a^3 - 664*a^2*b + 1125*
a*b^2 - 625*b^3)*sinh(d*x + c)^2 - 2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7
 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2
*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2
*b^11 - 6*a*b^12 + b^13)*d^4)) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 12
5*b^5)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 46
4*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 +
15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 -
47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + 8*a*b
+ 16*(8*(a*b - b^2)*d*x*cosh(d*x + c)^7 - 3*(8*(a*b - b^2)*d*x - a*b)*cosh(
d*x + c)^5 - 2*(4*(8*a^2 - 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x +
c)^3 - (8*(a*b - b^2)*d*x + 5*a*b)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 -
b^4)*d*cosh(d*x + c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 +
(a*b^3 - b^4)*d*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^6 + 4*(7*
(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6 - 2*(8*a
^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*
x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^3
- b^4)*d*cosh(d*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2
- 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^2
+ 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d*x + c)^3
 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(
a*b^3 - b^4)*d*cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 3*(8*
a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x +
 c)^2 + (a*b^3 - b^4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(a*b^3 - b
^4)*d*cosh(d*x + c)^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 -
(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [A] time = 1.98, size = 149, normalized size = 0.47

$$\frac{\frac{abe^{(6\,dx+6\,c)}-8\,a^2e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}-5\,abe^{(2\,dx+2\,c)}+ab}{(ab^2-b^3)\left(be^{(8\,dx+8\,c)}-4\,be^{(6\,dx+6\,c)}-16\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}-4\,be^{(2\,dx+2\,c)}+b\right)}+\frac{2\,(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a*b*e^(6*d*x + 6*c) - 8*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) -
 5*a*b*e^(2*d*x + 2*c) + a*b)/((a*b^2 - b^3)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*
d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*
c) + b)) + 2*(d*x + c)/b^2)/d
```

maple [C] time = 0.11, size = 574, normalized size = 1.79

$$-\frac{a\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2db\left(\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-4\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+6\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-16b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(\tanh^2\left(\frac{d}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*
x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*
tanh(1/2*d*x+1/2*c)^7+5/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^5+5/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8
*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/
2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/2/d*a/b/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x
+1/2*c)+1/16/d*a/b^2/(a-b)*sum(((4*a-5*b)*_R^6+(-12*a+19*b)*_R^4+(12*a-19*b
)*_R^2-4*a+5*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/
2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/d/b^2*ln(
tanh(1/2*d*x+1/2*c)-1)+1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\left(abde^{(8c)} - b^2de^{(8c)}\right)xe^{(8dx)} + ab + 2\left(abd - b^2d\right)x + \left(abe^{(6c)} - 8\left(abde^{(6c)} - b^2de^{(6c)}\right)x\right)e^{(6dx)} - \left(8a^2e^{(4c)} - 3abe^{\cdots}\right.}{2\left(ab^3d - b^4d + \left(ab^3de^{(8c)} - b^4de^{(8c)}\right)e^{(8dx)} - 4\left(ab^3de^{(6c)} - b^4de^{(6c)}\right)e^{(6dx)} - 2\left(8a^2b^2d\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*b*d*e^(8*c) - b^2*d*e^(8*c))*x*e^(8*d*x) + a*b + 2*(a*b*d - b^2*d
)*x + (a*b*e^(6*c) - 8*(a*b*d*e^(6*c) - b^2*d*e^(6*c))*x)*e^(6*d*x) - (8*a^
2*e^(4*c) - 3*a*b*e^(4*c) + 4*(8*a^2*d*e^(4*c) - 11*a*b*d*e^(4*c) + 3*b^2*d
*e^(4*c))*x)*e^(4*d*x) - (5*a*b*e^(2*c) + 8*(a*b*d*e^(2*c) - b^2*d*e^(2*c))
*x)*e^(2*d*x))/(a*b^3*d - b^4*d + (a*b^3*d*e^(8*c) - b^4*d*e^(8*c))*e^(8*d*
x) - 4*(a*b^3*d*e^(6*c) - b^4*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b^2*d*e^(4*c)
 - 11*a*b^3*d*e^(4*c) + 3*b^4*d*e^(4*c))*e^(4*d*x) - 4*(a*b^3*d*e^(2*c) - b
^4*d*e^(2*c))*e^(2*d*x)) + 1/256*integrate(256*(a*b*e^(6*d*x + 6*c) + a*b*e
^(2*d*x + 2*c) + 2*(8*a^2*e^(4*c) - 11*a*b*e^(4*c))*e^(4*d*x))/(a*b^3 - b^4
 + (a*b^3*e^(8*c) - b^4*e^(8*c))*e^(8*d*x) - 4*(a*b^3*e^(6*c) - b^4*e^(6*c)
)*e^(6*d*x) - 2*(8*a^2*b^2*e^(4*c) - 11*a*b^3*e^(4*c) + 3*b^4*e^(4*c))*e^(4
*d*x) - 4*(a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(c+d\,x\right)^8}{{\left(a-b\,\sinh\left(c+d\,x\right)^4\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**8/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.248 $\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=233

$$\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh(c+dx)}{4bd(a-b)}+\frac{}{4bd\,((}$$

```
[Out] 1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)-3*b^(1/
2))/a^(1/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh((a^(1/2)+b^(1/2))
^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)+3*b^(1/2))/a^(1/4)/b^(3/2)/d/(a^(1/2
)+b^(1/2))^(3/2)+1/4*tanh(d*x+c)/(a-b)/b/d+1/4*sech(d*x+c)^2*tanh(d*x+c)^3/
b/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A] time = 0.34, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1120, 1279, 1166, 208}

$$\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh(c+dx)}{4bd(a-b)}+\frac{}{4bd\,((}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((2*Sqrt[a] - 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(8*a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - ((2*Sqrt[a] + 3*S
qrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(1/4
)*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) + Tanh[c + d*x]/(4*(a - b)*b*d) + (S
ech[c + d*x]^2*Tanh[c + d*x]^3)/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*T
anh[c + d*x]^4))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}^2(c+dx)\tanh^3(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(6a-2ax^2\right)}{a-2ax^2+(a-b)x^4}\,dx, x \cdots\right.}{8abd}$$

$$= \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\operatorname{sech}^2(c+dx)\tanh^3(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\operatorname{sech}^2(c+dx)\tanh^3(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\left(a-\frac{2\sqrt{a}(a-\cdots}{\sqrt{b}}\cdots\right.}{\cdots}$$

$$= \frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} - \frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh\cdots}{\sqrt[4]{a}}\right.}{8\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/2}d}$$

Mathematica [A] time = 2.61, size = 238, normalized size = 1.02

$$\frac{\frac{\sqrt{b}\left(-\sqrt{a}\sqrt{b}-2a+3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{b}\left(\sqrt{a}\sqrt{b}-2a+3b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}} - \frac{4b\sinh(2(c+dx))(-2a+b\cosh(2(c+dx))-b)}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}\cdots}{8b^2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((Sqrt[b]*(-2*a + Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c
 + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]
*(-2*a - Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])
/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (4*b*(-2*a - b + b
*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] -
 b*Cosh[4*(c + d*x)]))/(8*(a - b)*b^2*d)
```

fricas [B] time = 1.36, size = 6045, normalized size = 25.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*(2*a - b)*cosh(d*x + c)^6 + 48*(2*a - b)*cosh(d*x + c)*sinh(d*x +
c)^5 + 8*(2*a - b)*sinh(d*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*
(2*a - b)*cosh(d*x + c)^2 - 8*a + 3*b)*sinh(d*x + c)^4 + 32*(5*(2*a - b)*co
sh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(2*a + 3*b)*
cosh(d*x + c)^2 + 8*(15*(2*a - b)*cosh(d*x + c)^4 - 6*(8*a - 3*b)*cosh(d*x
+ c)^2 - 2*a - 3*b)*sinh(d*x + c)^2 - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*
(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x +
c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x +
c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*
cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30
*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x
 + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x
 + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 -
15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(
d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2
 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b -
11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d
*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90
*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7
 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^
4 + 3*a*b^5 - b^6)*d^2))*log(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*
b^4 + 9*a*b^5)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 +
15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 -
 81*a*b + 81*b^2)*cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*cosh(d*x +
 c)*sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*sinh(d*x + c)^2 - 20*a^2 + 8
1*a*b - 81*b^2 + 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^
7)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 -
 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 2*(5*a^3*b - 19*a^2*b
^2 + 18*a*b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25
*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 1
5*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 -
 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(
a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c
)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c
)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*c
osh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*c
osh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*
(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x
+ c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x
+ c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 1
5*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d
*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2
- b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 1
1*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*
a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7
- 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4
 + 3*a*b^5 - b^6)*d^2))*log(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b
^4 + 9*a*b^5)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 1
```

```
5*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 -
81*a*b + 81*b^2)*cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*cosh(d*x +
c)*sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*sinh(d*x + c)^2 - 20*a^2 + 81
*a*b - 81*b^2 - 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7
)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 -
20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 2*(5*a^3*b - 19*a^2*b^
2 + 18*a*b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*
a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15
*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 -
3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a
*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)
^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)
^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*co
sh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(
a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x +
 c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x +
 c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d
*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15
*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*
x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 -
 b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11
*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x
 + c))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*
b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 -
6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 +
 3*a*b^5 - b^6)*d^2))*log(-2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^
4 + 9*a*b^5)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15
*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 - 8
1*a*b + 81*b^2)*cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*cosh(d*x + c
)*sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*sinh(d*x + c)^2 - 20*a^2 + 81*
a*b - 81*b^2 + 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)
*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 2
0*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 2*(5*a^3*b - 19*a^2*b^2
 + 18*a*b^3)*d)*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^
2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a
^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*
a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b
^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8
 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2
 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh
(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh
(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*
b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c
)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c
)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(
a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x
+ c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b
^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a
*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x +
 c))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b
+ 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*
a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3
*a*b^5 - b^6)*d^2))*log(-2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4
+ 9*a*b^5)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a
^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 - 81*
a*b + 81*b^2)*cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*cosh(d*x + c)*
sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*sinh(d*x + c)^2 - 20*a^2 + 81*a*
```

```
b - 81*b^2 - 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d
^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*
a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 2*(5*a^3*b - 19*a^2*b^2 +
 18*a*b^3)*d)*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2
- 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3
*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^
2*b^4 + 3*a*b^5 - b^6)*d^2))) + 16*(3*(2*a - b)*cosh(d*x + c)^5 - 2*(8*a -
3*b)*cosh(d*x + c)^3 - (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 8*b)/((a*
b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c
)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 +
 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 -
2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cos
h(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*
b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*
b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^
2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)
^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(
a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*
a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c
)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3
)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b
^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [A] time = 1.38, size = 153, normalized size = 0.66

$$-\frac{2\,ae^{(6\,dx+6\,c)}-be^{(6\,dx+6\,c)}-8\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-2\,ae^{(2\,dx+2\,c)}-3\,be^{(2\,dx+2\,c)}+b}{2\left(ab-b^2\right)\left(be^{(8\,dx+8\,c)}-4\,be^{(6\,dx+6\,c)}-16\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}-4\,be^{(2\,dx+2\,c)}+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*e^(6*d*x + 6*c) - b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e
^(4*d*x + 4*c) - 2*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + b)/((a*b - b^2
)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(
4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)*d)
```

maple [C] time = 0.09, size = 716, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x
+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*t
anh(1/2*d*x+1/2*c)^7-1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*
c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+
1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^5-2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^5-1/2/d*a/b/(tanh(1/
2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*t
anh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)
^3-2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh
(1/2*d*x+1/2*c)^3+1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)-1/16/d/b/(a-b)*sum((-_R^6*a+(-5*a+12*b
)*_R^4+(5*a-12*b)*_R^2+a)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(
1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,ae^{(6\,c)} - be^{(6\,c)}\right)e^{(6\,dx)} - \left(8\,ae^{(4\,c)} - 3\,be^{(4\,c)}\right)e^{(4\,dx)} - \left(2\,ae^{(2\,c)} + 3\,be^{(2\,c)}\right)e^{(2\,d}}{2\left(ab^{2}d - b^{3}d + \left(ab^{2}de^{(8\,c)} - b^{3}de^{(8\,c)}\right)e^{(8\,dx)} - 4\left(ab^{2}de^{(6\,c)} - b^{3}de^{(6\,c)}\right)e^{(6\,dx)} - 2\left(8\,a^{2}bde^{(4\,c)} - 11\,ab^{2}de^{(4\,c)} + 3\,b^{3}d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - (8*a*e^(4*c) - 3*b*e^(4*c))*e^(
4*d*x) - (2*a*e^(2*c) + 3*b*e^(2*c))*e^(2*d*x) + b)/(a*b^2*d - b^3*d + (a*b
^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c
))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c) - 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))
*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^3*d*e^(2*c))*e^(2*d*x)) + 1/64*integrat
e(64*((2*a*e^(6*c) - 3*b*e^(6*c))*e^(6*d*x) + (2*a*e^(2*c) - 3*b*e^(2*c))*e
^(2*d*x) + 6*b*e^(4*d*x + 4*c))/(a*b^2 - b^3 + (a*b^2*e^(8*c) - b^3*e^(8*c)
)*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*e^(4*c
) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e^(2*c) - b^3*e^
(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(c+d\,x\right)^{6}}{\left(a-b\,\sinh\left(c+d\,x\right)^{4}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.249 $\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh^5(c+dx)}{4ad\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}$$

```
[Out] 1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)
-b^(1/2))^(3/2)/b^(1/2)-1/8*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(
1/4))/a^(3/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(3/2)-1/4*tanh(d*x+c)/a/(a-b)/d+1
/4*tanh(d*x+c)^5/a/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A] time = 0.25, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3217, 1275, 12, 1122, 1166, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh^5(c+dx)}{4ad\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a
] - Sqrt[b])^(3/2)*Sqrt[b]*d) - ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d
*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - Tanh[c + d*
x]/(4*a*(a - b)*d) + Tanh[c + d*x]^5/(4*a*d*(a - 2*a*Tanh[c + d*x]^2 + (a -
 b)*Tanh[c + d*x]^4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4(1-x^2)}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2bx^4}{a-2ax^2+(a-b)x^4}\,dx,\right.}{8abd}$$

$$= \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{x^4}{a-2ax^2+(a-b)x^4}\,dx, x\right.}{4ad}$$

$$= -\frac{\tanh(c+dx)}{4a(a-b)d} + \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\tanh(c+dx)}{4a(a-b)d} + \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\left(\sqrt{a}+\sqrt{b}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}\,d} - \frac{\tanh(c+dx)}{4a(a-b)d} + \frac{}{4ad}$$

Mathematica [A] time = 4.21, size = 225, normalized size = 1.15

$$-\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}} - \frac{2(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}}{8d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -1/8*(((Sqrt[a] + Sqrt[b])*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[
-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((Sq
rt[a] - Sqrt[b])*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[
a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (2*(-6*Sinh[2*(
c + d*x)] + Sinh[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh
[4*(c + d*x)]))/((a - b)*d)
```

fricas [B] time = 0.70, size = 5658, normalized size = 29.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^6 + 48*b*cosh(d*x + c)*sinh(d*x + c)^5 + 8*b*sinh(d
*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^2 - 8*a +
 3*b)*sinh(d*x + c)^4 + 32*(5*b*cosh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c)
)*sinh(d*x + c)^3 - 40*b*cosh(d*x + c)^2 + 8*(15*b*cosh(d*x + c)^4 - 6*(8*a
 - 3*b)*cosh(d*x + c)^2 - 5*b)*sinh(d*x + c)^2 + ((a*b^2 - b^3)*d*cosh(d*x
+ c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*
sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*
cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2
+ 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^
2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x
+ c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*
d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)
*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b
^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*
x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b
^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d
+ 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 -
(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x +
 c))*sinh(d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt(
(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^
5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b
^3 - a*b^4)*d^2))*log(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a
^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^
5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*co
sh(d*x + c)*sinh(d*x + c) + (3*a + b)*sinh(d*x + c)^2 + 2*(2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*
b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - (
3*a^3 + 4*a^2*b + a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d
^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4
 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 +
 3*a^2*b^3 - a*b^4)*d^2)) - 3*a - b) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8
*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x +
 c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x +
 c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d
*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3
0*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*
x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*
x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3
)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 -
 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh
(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^
2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b -
 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(
d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6
*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*
```

```
a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4
)*d^2))*log(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a^2 + 6*a*b
 + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*
b^6 + a^3*b^7)*d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*cosh(d*x + c
)*sinh(d*x + c) + (3*a + b)*sinh(d*x + c)^2 - 2*(2*(a^6*b - 3*a^5*b^2 + 3*a
^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a
^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - (3*a^3 + 4*
a^2*b + a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9
*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*
b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d^2)) - 3*a - b) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 -
b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*
(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a
*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x
+ c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 -
 b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 -
 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 -
 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d
*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2
 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^
2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d
*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2
+ 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*
sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^
2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 +
 a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*l
og(-2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/
((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^
3*b^7)*d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*cosh(d*x + c)*sinh(d
*x + c) + (3*a + b)*sinh(d*x + c)^2 + 2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 -
 a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a^3 + 4*a^2*b +
a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 +
6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6
*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^
4)*d^2)) - 3*a - b) + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*
cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2
 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 -
b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4
+ 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*s
inh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d
*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b
^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*
b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)
*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*
b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d
*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3
)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(
(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^
9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^
7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-2*(
a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b
 - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*
d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*cosh(d*x + c)*sinh(d*x + c)
 + (3*a + b)*sinh(d*x + c)^2 - 2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^
4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6
*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a^3 + 4*a^2*b + a*b^2)*
```

```
d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b +
 b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^
6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)
) - 3*a - b) + 16*(3*b*cosh(d*x + c)^5 - 2*(8*a - 3*b)*cosh(d*x + c)^3 - 5*
b*cosh(d*x + c))*sinh(d*x + c) + 8*b)/((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*
(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x +
c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x +
c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*
cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30
*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x
 + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x
 + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 -
15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(
d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2
 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b -
11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d
*x + c))
```

giac [A] time = 1.18, size = 128, normalized size = 0.66

$$\frac{be^{(6\,dx+6\,c)} - 8\,ae^{(4\,dx+4\,c)} + 3\,be^{(4\,dx+4\,c)} - 5\,be^{(2\,dx+2\,c)} + b}{2\left(ab - b^2\right)\left(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 5*b*e^
(2*d*x + 2*c) + b)/((a*b - b^2)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) -
16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)*d)
```

maple [C] time = 0.09, size = 490, normalized size = 2.51

$$-\frac{\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\left(\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-4\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+6\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-16b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh
(1/2*d*x+1/2*c)^7+5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+
6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^
2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^5+5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/
2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tan
h(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/2/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)-1/16/d/(
a-b)*sum((_R^6-7*_R^4+7*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln
(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2
+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(8\,ae^{(4\,c)} - 3\,be^{(4\,c)}\right)e^{(4\,dx)} - be^{(6\,dx+6\,c)} + 5\,be^{(2\,dx+2\,c)} - b}{2\left(ab^2d - b^3d + \left(ab^2de^{(8\,c)} - b^3de^{(8\,c)}\right)e^{(8\,dx)} - 4\left(ab^2de^{(6\,c)} - b^3de^{(6\,c)}\right)e^{(6\,dx)} - 2\left(8\,a^2bde^{(4\,c)} - 11\,ab^2de^{(4\,c)} + 3\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((8*a*e^(4*c) - 3*b*e^(4*c))*e^(4*d*x) - b*e^(6*d*x + 6*c) + 5*b*e^(2*
d*x + 2*c) - b)/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d
*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c)
- 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^
3*d*e^(2*c))*e^(2*d*x)) + 1/16*integrate(16*(e^(6*d*x + 6*c) - 6*e^(4*d*x +
 4*c) + e^(2*d*x + 2*c))/(a*b - b^2 + (a*b*e^(8*c) - b^2*e^(8*c))*e^(8*d*x)
 - 4*(a*b*e^(6*c) - b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^2*e^(4*c) - 11*a*b*e^(4
*c) + 3*b^2*e^(4*c))*e^(4*d*x) - 4*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh (c+d\,x)^4}{\left(a-b\sinh (c+d\,x)^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.250 $\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=220

$$-\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh(c+dx)\,(}{4ad(a-b)\left((a-b)\tan\right.}$$

```
[Out] -1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)-b^(1/2
))/a^(5/4)/d/(a^(1/2)-b^(1/2))^(3/2)/b^(1/2)+1/8*arctanh((a^(1/2)+b^(1/2))^
(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)+b^(1/2))/a^(5/4)/d/b^(1/2)/(a^(1/2)+b
^(1/2))^(3/2)+1/4*tanh(d*x+c)*(a-(a+b)*tanh(d*x+c)^2)/a/(a-b)/d/(a-2*a*tanh
(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A] time = 0.30, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1333, 1166, 208}

$$-\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh(c+dx)\,(}{4ad(a-b)\left((a-b)\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(
1/4)])/(8*a^(5/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) + ((2*Sqrt[a] + Sqrt
[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(5/4)*(
Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) + (Tanh[c + d*x]*(a - (a + b)*Tanh[c +
d*x]^2))/(4*a*(a - b)*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)
)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1-x^2\right)^2}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{\frac{2a^2b}{a-b}-\frac{2a(3a-b)b}{a-b}\cdots}{a-2ax^2+(a-b)\cdots}\right.}{8a\cdots}$$

$$= \frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\left(2a-\sqrt{a}\sqrt{b}-b\right)\text{Su}\cdots}{\cdots}$$

$$= -\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d} + \frac{\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(\cdots}{\sqrt[4]{a}}\right.}{8a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}\,d}$$

Mathematica [A] time = 2.09, size = 253, normalized size = 1.15

$$\frac{\frac{\sqrt{a}\left(-\sqrt{a}\sqrt{b}+2a-b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}+2a-b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{4\sqrt{a}\sinh(2(c+dx))(2a-b\cosh(2(c+dx))+b)}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}}{8a^{3/2}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((Sqrt[a]*(2*a + Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c +
d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (
Sqrt[a]*(2*a - Sqrt[a]*Sqrt[b] - b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d
*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (4*S
qrt[a]*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*
Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))/(8*a^(3/2)*(a - b)*d)
```

fricas [B] time = 1.37, size = 6525, normalized size = 29.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*(2*a - b)*cosh(d*x + c)^6 + 48*(2*a - b)*cosh(d*x + c)*sinh(d*x +
c)^5 + 8*(2*a - b)*sinh(d*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*
```

(2*a - b)*cosh(d*x + c)^2 - 8*a + 3*b)*sinh(d*x + c)^4 + 32*(5*(2*a - b)*co
sh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(2*a + 3*b)*
cosh(d*x + c)^2 + 8*(15*(2*a - b)*cosh(d*x + c)^4 - 6*(8*a - 3*b)*cosh(d*x
+ c)^2 - 2*a - 3*b)*sinh(d*x + c)^2 + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 +
8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(
d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*c
osh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b +
 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a
^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*co
sh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b +
3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a
^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8
*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3
 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)
^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b -
a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 -
 (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^5*b - 3*a^4*b^2
+ 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3
+ b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6
*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3
- a^2*b^4)*d^2))*log(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4
)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a
^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))
 + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*
cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cosh(d*x + c)*sinh(
d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*sinh(d*x + c)^2 + 2*((3*a^8*
b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^
3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*
a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 2*(8*a^5 - 5*a^4*b + a^
3*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4
 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b
^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b
^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))) - ((a^2*b - a*b^2)*d*
cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*
b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(
a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(
8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh
(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a
^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8
*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*
x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*co
sh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x
 + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2
)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x +
c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d
*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((
a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a
^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 +
15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^
4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7
*a^4*b^3 + a^3*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b
^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^
6 + a^5*b^7)*d^4)) + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b
 + 9*a*b^2 - b^3)*cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*c
osh(d*x + c)*sinh(d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*sinh(d*x +
 c)^2 - 2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sq
rt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2

```
+ 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 2*(8*
a^5 - 5*a^4*b + a^3*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4
)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a
^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))
 - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))) - (
(a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(
d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d
*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sin
h(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2
*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x
 + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*co
sh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b
- a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a
^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^
2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*
b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*
a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(
d*x + c))*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4
- 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^
3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^
2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(-2*(4*a^7 - 13*a^6*
b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*
b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*
a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 -
(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b
+ 9*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2
 - b^3)*sinh(d*x + c)^2 + 2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4
 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^
11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*
b^7)*d^4)) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d)*sqrt(((a^5*b - 3*a^4*b^2 + 3*
a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^
4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6
 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^
2*b^4)*d^2))) + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*co
sh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b
- a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*
b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x +
 c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^
2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x +
 c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d
*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*
b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c
)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)
*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d
 + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)
^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cos
h(d*x + c))*sinh(d*x + c))*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*
d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^1
0*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) +
 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(-2
*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*sqrt((64*a^4 - 8
0*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 -
 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 32*a^3 - 28*a^2*b +
 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cosh(d*x + c)^2 - 2*(3
2*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c) - (32*a^3 - 2
8*a^2*b + 9*a*b^2 - b^3)*sinh(d*x + c)^2 - 2*((3*a^8*b - 10*a^7*b^2 + 12*a^
```

```
6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*
a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 -
 6*a^6*b^6 + a^5*b^7)*d^4)) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d)*sqrt(((a^5*b
 - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^
2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^
7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2
 + 3*a^3*b^3 - a^2*b^4)*d^2))) + 16*(3*(2*a - b)*cosh(d*x + c)^5 - 2*(8*a -
 3*b)*cosh(d*x + c)^3 - (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 8*b)/((a
^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*
x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x
 + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(
d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b
 - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x +
 c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh
(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b -
a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2
*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a
*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2
- (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^
2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^
2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*
x + c))
```

giac [A] time = 0.76, size = 152, normalized size = 0.69

$$-\frac{2\,ae^{(6\,dx+6\,c)}-be^{(6\,dx+6\,c)}-8\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-2\,ae^{(2\,dx+2\,c)}-3\,be^{(2\,dx+2\,c)}+b}{2\left(a^2-ab\right)\left(be^{(8\,dx+8\,c)}-4\,be^{(6\,dx+6\,c)}-16\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}-4\,be^{(2\,dx+2\,c)}+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*e^(6*d*x + 6*c) - b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e
^(4*d*x + 4*c) - 2*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + b)/((a^2 - a*b
)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(
4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)*d)
```

maple [C] time = 0.11, size = 708, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(
1/2*d*x+1/2*c)^7-1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^5-2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^5*b-1/2/d/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*
d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3-2/d/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d
*x+1/2*c)^3*b+1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*ta
nh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+
a)/(a-b)*tanh(1/2*d*x+1/2*c)-1/16/d/a/(a-b)*sum((-_R^6*a+(11*a-4*b)*_R^4+(-
```

```
11*a+4*b)*_R^2+a)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+
1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(2ae^{(6c)} - be^{(6c)})e^{(6dx)} - (8ae^{(4c)} - 3be^{(4c)})e^{(4dx)} - (2ae^{(2c)} + 3be^{(2c)})e^{(}}{2(a^2bd - ab^2d + (a^2bde^{(8c)} - ab^2de^{(8c)})e^{(8dx)} - 4(a^2bde^{(6c)} - ab^2de^{(6c)})e^{(6dx)} - 2(8a^3de^{(4c)} - 11a^2bde^{(4c)} + 3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - (8*a*e^(4*c) - 3*b*e^(4*c))*e^(
4*d*x) - (2*a*e^(2*c) + 3*b*e^(2*c))*e^(2*d*x) + b)/(a^2*b*d - a*b^2*d + (a
^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*
e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^
(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) - 1/4*i
ntegrate(4*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - 2*(4*a*e^(4*c) - b*e^(4*c
))*e^(4*d*x) + (2*a*e^(2*c) - b*e^(2*c))*e^(2*d*x))/(a^2*b - a*b^2 + (a^2*b
*e^(8*c) - a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*c))*e^(
6*d*x) - 2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(4*d*x) -
 4*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c + d\,x)^2}{\left(a - b\sinh(c + d\,x)^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.251 $\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tanh(c+}{4ad(a-b)\left((a-b)\tan\right.}$$

```
[Out] 1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(4*a^(1/2)-3*b^(1/
2))/a^(7/4)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh((a^(1/2)+b^(1/2))^(1/2)*t
anh(d*x+c)/a^(1/4))*(4*a^(1/2)+3*b^(1/2))/a^(7/4)/d/(a^(1/2)+b^(1/2))^(3/2)
-1/4*b*tanh(d*x+c)*(1-2*tanh(d*x+c)^2)/a/(a-b)/d/(a-2*a*tanh(d*x+c)^2+(a-b)
*tanh(d*x+c)^4)
```

Rubi [A] time = 0.25, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3209, 1205, 1166, 208}

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tanh(c+}{4ad(a-b)\left((a-b)\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[c + d*x]^4)^(-2),x]
```

```
[Out] ((4*Sqrt[a] - 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(8*a^(7/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((4*Sqrt[a] + 3*Sqrt[b])*
ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(7/4)*(Sqrt[
a] + Sqrt[b])^(3/2)*d) - (b*Tanh[c + d*x]*(1 - 2*Tanh[c + d*x]^2))/(4*a*(a
- b)*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
```

```
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{2a(4a-3b)b}{a-b}+}{a-2ax^2+}\right.}{}$$

$$= -\frac{b\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\left(4a-\sqrt{a}\sqrt{b}-3b\right)}{}$$

$$= \frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh}{\sqrt[4]{a}}\right.}{8a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}$$

Mathematica [A] time = 2.87, size = 230, normalized size = 1.10

$$\frac{\frac{\left(-\sqrt{a}\sqrt{b}+4a-3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\left(\sqrt{a}\sqrt{b}+4a-3b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{2\sqrt{a}\,b(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}}{8a^{3/2}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-2),x]
```

```
[Out] (-(((4*a + Sqrt[a]*Sqrt[b] - 3*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x]
)/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]]) + ((4*a - Sqrt[a
]*Sqrt[b] - 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[
a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (2*Sqrt[a]*b*(-6*Sinh[2*(c + d*x)
] + Sinh[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c +
d*x)]))/(8*a^(3/2)*(a - b)*d)
```

fricas [B] time = 1.26, size = 6522, normalized size = 31.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^6 + 48*b*cosh(d*x + c)*sinh(d*x + c)^5 + 8*b*sinh(d
*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^2 - 8*a +
 3*b)*sinh(d*x + c)^4 + 32*(5*b*cosh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c)
```

)*sinh(d*x + c)^3 - 40*b*cosh(d*x + c)^2 + 8*(15*b*cosh(d*x + c)^4 - 6*(8*a - 3*b)*cosh(d*x + c)^2 - 5*b)*sinh(d*x + c)^2 - ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4 + 2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)^2 - 2*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)*sinh(d*x + c) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*sinh(d*x + c)^2 + 2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + (120*a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4 + 2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)^2 - 2*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)*sinh(d*x + c) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*sinh(d*x + c)^2 - 2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b

```
^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15
*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + (120*a^5*b - 217*a^4*b^2 + 132*a^3*
b^3 - 27*a^2*b^4)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt(
(576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a
^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))
 - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) +
 ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sin
h(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh
(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*s
inh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a
^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d
*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*
cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*
b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*
(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x
 + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b
 - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c
)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b -
a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 1
1*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sin
h(d*x + c))*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b
 - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15
*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2
- 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(384*a^3*
b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4 - 2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 -
43*a^5*b^3 + 9*a^4*b^4)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 -
 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9
*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 -
81*b^4)*cosh(d*x + c)^2 - 2*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*
cosh(d*x + c)*sinh(d*x + c) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4
)*sinh(d*x + c)^2 + 2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^
4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/
((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^
7*b^6)*d^4)) - (120*a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d)*sqrt
(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2
+ 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a
^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2
)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) - ((a^2*b - a*b^2)*d*cosh(d
*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*
b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b
- a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3
- 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x +
 c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 -
 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)
^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x
 + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4
*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^
4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*s
inh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 -
 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(
d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^6 - 3
*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2
*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 +
15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 -
3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(384*a^3*b - 680*a^2*b^2 + 405*a*b^
3 - 81*b^4 - 2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^
2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^1
3 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6
```

```
)*d^4)) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)^2 -
2*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)*sinh(d*x + c
) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*sinh(d*x + c)^2 - 2*(2*(
2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1
392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^1
1*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - (120*a^5*b
- 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b
^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4
 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a
^8*b^5 + a^7*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*
b^2 - a^3*b^3)*d^2))) + 16*(3*b*cosh(d*x + c)^5 - 2*(8*a - 3*b)*cosh(d*x +
c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c) + 8*b)/((a^2*b - a*b^2)*d*cosh(d*x
+ c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2
)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a
*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 1
1*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)
^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*
b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11
*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2
+ 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x +
c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7
*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 -
 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh
(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*
(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x
 + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [**A**] time = 0.23, size = 127, normalized size = 0.60

$$\frac{be^{(6\,dx+6\,c)} - 8\,ae^{(4\,dx+4\,c)} + 3\,be^{(4\,dx+4\,c)} - 5\,be^{(2\,dx+2\,c)} + b}{2\left(a^2 - ab\right)\left(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 5*b*e^
(2*d*x + 2*c) + b)/((a^2 - a*b)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) -
16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)*d)
```

maple [**C**] time = 0.11, size = 534, normalized size = 2.54

$$-\frac{b\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\left(\left(\tanh^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-4\left(\tanh^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+6\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-16b\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)*b/a/(a-b)*
tanh(1/2*d*x+1/2*c)^7+5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^5*b+5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)
^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^3*b-1/2/d/(tanh
(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*
b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)*b/a/(a-b)*tanh(1/2*d*x
```

```
+1/2*c)-1/16/d/a/(a-b)*sum(((4*a-3*b)*_R^6+(-12*a+5*b)*_R^4+(-5*b+12*a)*_R^
2-4*a+3*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-
_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(8\,ae^{(4\,c)}-3\,be^{(4\,c)}\right)e^{(4\,dx)}-be^{(6\,dx+6\,c)}+5\,be^{(2\,dx+2\,c)}-b}{2\left(a^2bd-ab^2d+\left(a^2bde^{(8\,c)}-ab^2de^{(8\,c)}\right)e^{(8\,dx)}-4\left(a^2bde^{(6\,c)}-ab^2de^{(6\,c)}\right)e^{(6\,dx)}-2\left(8\,a^3de^{(4\,c)}-11\,a^2bde^{(4\,c)}+3\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((8*a*e^(4*c) - 3*b*e^(4*c))*e^(4*d*x) - b*e^(6*d*x + 6*c) + 5*b*e^(2*
d*x + 2*c) - b)/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^
(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4
*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c
) - a*b^2*d*e^(2*c))*e^(2*d*x)) + integrate(-(2*(8*a*e^(4*c) - 5*b*e^(4*c))
*e^(4*d*x) - b*e^(6*d*x + 6*c) - b*e^(2*d*x + 2*c))/(a^2*b - a*b^2 + (a^2*b
*e^(8*c) - a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*c))*e^(
6*d*x) - 2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(4*d*x) -
 4*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a-b\sinh\left(c+d\,x\right)^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sinh(c + d*x)^4)^2,x)
```

```
[Out] int(1/(a - b*sinh(c + d*x)^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.252 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=237

$$-\frac{\sqrt{b}\left(6\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\left(6\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{b\,\mathrm{ta}}{4a^2d(a-b)}$$

```
[Out] -coth(d*x+c)/a^2/d-1/8*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))
*(6*a^(1/2)-5*b^(1/2))*b^(1/2)/a^(9/4)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctan
h((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*b^(1/2)*(6*a^(1/2)+5*b^(1/2)
)/a^(9/4)/d/(a^(1/2)+b^(1/2))^(3/2)+1/4*b*tanh(d*x+c)*(a-(a+b)*tanh(d*x+c)^
2)/a^2/(a-b)/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A] time = 0.53, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3217, 1334, 1664, 1166, 208}

$$-\frac{\sqrt{b}\left(6\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\left(6\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{b\,\mathrm{ta}}{4a^2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((6*Sqrt[a] - 5*Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
 d*x])/a^(1/4)])/(8*a^(9/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((6*Sqrt[a] + 5*
Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/
(8*a^(9/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) - Coth[c + d*x]/(a^2*d) + (b*Tanh[c
 + d*x]*(a - (a + b)*Tanh[c + d*x]^2))/(4*a^2*(a - b)*d*(a - 2*a*Tanh[c + d
*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1334

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^2\left(a-2ax^2+(a-b)x^4\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}-\frac{\text{Subst}\left(\int \frac{-8ab+\frac{2a(8a-7}{a-}}{x^2(a-}\right.}{}$$

$$=\frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}-\frac{\text{Subst}\left(\int\left(-\frac{8b}{x^2}+\frac{}{(a-}\right.\right.}{}$$

$$=-\frac{\coth(c+dx)}{a^2d}+\frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}-\frac{b\,\text{S}}{}$$

$$=-\frac{\coth(c+dx)}{a^2d}+\frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)}+\frac{((}{}$$

$$=-\frac{\left(6\sqrt{a}-5\sqrt{b}\right)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d}+\frac{\left(6\sqrt{a}+5\sqrt{b}\right)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{}}}{}\right.}{8a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/}}$$

Mathematica [A] time = 1.92, size = 272, normalized size = 1.15

$$\frac{\frac{\left(6a\sqrt{b}+5\sqrt{a}b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{\left(6a\sqrt{b}-5\sqrt{a}b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}-a}}+\frac{4\sqrt{a}b\sinh(2(c+dx))(2a-b\cosh(2(c+dx))+b)}{(a-b)(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b)}-8}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (((6*a*Sqrt[b] - 5*Sqrt[a]*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sq
rt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqrt[b]])
```

```
 + ((6*a*Sqrt[b] + 5*Sqrt[a]*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])
/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sqrt[a]*Sqrt[b]]
) - 8*Sqrt[a]*Coth[c + d*x] + (4*Sqrt[a]*b*(2*a + b - b*Cosh[2*(c + d*x)])*
Sinh[2*(c + d*x)])/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(
c + d*x)]))))/(8*a^(5/2)*d)
```

fricas [B] time = 1.08, size = 8824, normalized size = 37.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*(6*a*b - 5*b^2)*cosh(d*x + c)^8 + 64*(6*a*b - 5*b^2)*cosh(d*x + c)
*sinh(d*x + c)^7 + 8*(6*a*b - 5*b^2)*sinh(d*x + c)^8 - 16*(13*a*b - 10*b^2)
*cosh(d*x + c)^6 + 16*(14*(6*a*b - 5*b^2)*cosh(d*x + c)^2 - 13*a*b + 10*b^2
)*sinh(d*x + c)^6 + 32*(14*(6*a*b - 5*b^2)*cosh(d*x + c)^3 - 3*(13*a*b - 10
*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 16*(32*a^2 - 47*a*b + 15*b^2)*cosh(d
*x + c)^4 + 16*(35*(6*a*b - 5*b^2)*cosh(d*x + c)^4 - 15*(13*a*b - 10*b^2)*c
osh(d*x + c)^2 - 32*a^2 + 47*a*b - 15*b^2)*sinh(d*x + c)^4 + 64*(7*(6*a*b -
 5*b^2)*cosh(d*x + c)^5 - 5*(13*a*b - 10*b^2)*cosh(d*x + c)^3 - (32*a^2 - 4
7*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(7*a*b - 10*b^2)*cosh(d
*x + c)^2 + 16*(14*(6*a*b - 5*b^2)*cosh(d*x + c)^6 - 15*(13*a*b - 10*b^2)*c
osh(d*x + c)^4 - 6*(32*a^2 - 47*a*b + 15*b^2)*cosh(d*x + c)^2 - 7*a*b + 10*
b^2)*sinh(d*x + c)^2 + ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^10 + 10*(a^3*b -
a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*sinh(d*x + c
)^10 - 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*cos
h(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b +
 5*a^2*b^2)*d*cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 -
 (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b
^2)*d*cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 1
3*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*
d*cosh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 70*(a^3*b -
 a^2*b^2)*d*cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 175*(a^
3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh
(d*x + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^4 + 5*(a^3*b
- a^2*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 -
35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d
*cosh(d*x + c)^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + (45*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140*(a^3*b - a^2*b^2)*d
*cosh(d*x + c)^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 12
*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(a^3*b - a^2*b^2)*d)*
sinh(d*x + c)^2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^9 - 20*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3*b + 5*a^
2*b^2)*d*cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)
^3 + 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^7 - 3*a^
6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2
*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3
+ 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)
/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(1728*a^3*b^2 - 3684*a^2*b
^3 + 2625*a*b^4 - 625*b^5 + 2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b
^3 + 25*a^5*b^4)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 345
0*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*
b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b
^4 - 625*b^5)*cosh(d*x + c)^2 - 2*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4
 - 625*b^5)*cosh(d*x + c)*sinh(d*x + c) - (1728*a^3*b^2 - 3684*a^2*b^3 + 26
25*a*b^4 - 625*b^5)*sinh(d*x + c)^2 + 2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 -
 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b
^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 +
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15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^3 - 50*a^3*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^10 + 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^10 - 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 175*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^4 + 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 12*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(a^3*b - a^2*b^2)*d)*sinh(d*x + c)^2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^9 - 20*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^3 + 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5 + 2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cosh(d*x + c)^2 - 2*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cosh(d*x + c)*sinh(d*x + c) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*sinh(d*x + c)^2 - 2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^3 - 50*a^3*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^10 + 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^10 - 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 175*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^4 + 5*(a^3*b - a^2*b^

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2)*d*cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 35*(a^3*
b - a^2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*
x + c)^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ (45*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140*(a^3*b - a^2*b^2)*d*cosh(d*
x + c)^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 12*(8*a^4
- 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(a^3*b - a^2*b^2)*d)*sinh(d*x
 + c)^2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^9 -
20*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d
*cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^3 + 5*(
a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^7 - 3*a^6*b + 3*a
^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 34
50*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11
*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 -
3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(1728*a^3*b^2 - 3684*a^2*b^3 + 2625
*a*b^4 - 625*b^5 - 2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a
^5*b^4)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 +
 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a
^10*b^5 + a^9*b^6)*d^4)) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*
b^5)*cosh(d*x + c)^2 - 2*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^
5)*cosh(d*x + c)*sinh(d*x + c) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4
- 625*b^5)*sinh(d*x + c)^2 + 2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b
^3 + 5*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450
*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b
^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^3
 - 50*a^3*b^4)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((230
4*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*
a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^
4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*
d^2))) + ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^10 + 10*(a^3*b - a^2*b^2)*d*cos
h(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^10 - 5*(a^3*
b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 -
 (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*
cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (a^3*b - a^2*
b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x
 + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 13*a^3*b + 5*a^
2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c
)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 70*(a^3*b - a^2*b^2)*d*co
sh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 175*(a^3*b - a^2*b^2)
*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 +
(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^4 + 5*(a^3*b - a^2*b^2)*d*c
osh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 35*(a^3*b - a^
2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)
^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*
(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140*(a^3*b - a^2*b^2)*d*cosh(d*x + c)
^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 12*(8*a^4 - 13*a
^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(a^3*b - a^2*b^2)*d)*sinh(d*x + c)^
2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^9 - 20*(a^
3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(
d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^3 + 5*(a^3*b
- a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2
 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b
^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 -
 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*
b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4
 - 625*b^5 - 2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4
)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b
^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^
5 + a^9*b^6)*d^4)) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*c
```

```
osh(d*x + c)^2 - 2*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos
h(d*x + c)*sinh(d*x + c) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*
b^5)*sinh(d*x + c)^2 - 2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5
*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6
 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6
*a^10*b^5 + a^9*b^6)*d^4)) - 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^3 - 50*
a^3*b^4)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*
b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b
 + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) +
36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2)))
 + 32*a*b - 40*b^2 + 32*(2*(6*a*b - 5*b^2)*cosh(d*x + c)^7 - 3*(13*a*b - 10
*b^2)*cosh(d*x + c)^5 - 2*(32*a^2 - 47*a*b + 15*b^2)*cosh(d*x + c)^3 - (7*a
*b - 10*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b - a^2*b^2)*d*cosh(d*x +
c)^10 + 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2
*b^2)*d*sinh(d*x + c)^10 - 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^
3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2
*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)
*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2
*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x
 + c)^2 - (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13
*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x
+ c)^5 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a
^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(
d*x + c)^6 - 175*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b
 + 5*a^2*b^2)*d*cosh(d*x + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*
x + c)^4 + 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*
d*cosh(d*x + c)^7 - 35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*
a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cos
h(d*x + c))*sinh(d*x + c)^3 + (45*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140
*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*
cosh(d*x + c)^4 + 12*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(
a^3*b - a^2*b^2)*d)*sinh(d*x + c)^2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a
^2*b^2)*d*cosh(d*x + c)^9 - 20*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a
^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*
b^2)*d*cosh(d*x + c)^3 + 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)
)
```

giac [A] time = 0.39, size = 238, normalized size = 1.00

$$-\frac{6\,abe^{(8\,dx+8\,c)} - 5\,b^2e^{(8\,dx+8\,c)} - 26\,abe^{(6\,dx+6\,c)} + 20\,b^2e^{(6\,dx+6\,c)} - 64\,a^2e^{(4\,dx+4\,c)} + 94\,abe^{(4\,dx+4\,c)} - 30\,b^2e^{(4\,dx+4\,c)} - }{2\,(a^3 - a^2b)\left(be^{(10\,dx+10\,c)} - 5\,be^{(8\,dx+8\,c)} - 16\,ae^{(6\,dx+6\,c)} + 10\,be^{(6\,dx+6\,c)} + 16\,ae^{(4\,dx+4\,c)} - 10\,be^{(4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(6*a*b*e^(8*d*x + 8*c) - 5*b^2*e^(8*d*x + 8*c) - 26*a*b*e^(6*d*x + 6*c
) + 20*b^2*e^(6*d*x + 6*c) - 64*a^2*e^(4*d*x + 4*c) + 94*a*b*e^(4*d*x + 4*c
) - 30*b^2*e^(4*d*x + 4*c) - 14*a*b*e^(2*d*x + 2*c) + 20*b^2*e^(2*d*x + 2*c
) + 4*a*b - 5*b^2)/((a^3 - a^2*b)*(b*e^(10*d*x + 10*c) - 5*b*e^(8*d*x + 8*c
) - 16*a*e^(6*d*x + 6*c) + 10*b*e^(6*d*x + 6*c) + 16*a*e^(4*d*x + 4*c) - 10
*b*e^(4*d*x + 4*c) + 5*b*e^(2*d*x + 2*c) - b)*d)
```

maple [C] time = 0.17, size = 765, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/a^2*tanh(1/2*d*x+1/2*c)+1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)*b/a/(a-b)*tanh(1/2*d*x+1/2*c)^7-1/2/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^5*b-2/
d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x
+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*t
anh(1/2*d*x+1/2*c)^5-1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^3*b-2/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^
8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1
/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/2/d/(tan
h(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16
*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)*b/a/(a-b)*tanh(1/2*d*
x+1/2*c)-1/16/d/a^2*b/(a-b)*sum((-_R^6*a+(27*a-20*b)*_R^4+(-27*a+20*b)*_R^2
+a)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=
RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/2/d/a^2/tanh(1/2*d*x+
1/2*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\,ab-5\,b^2+\left(6\,abe^{(8\,c)}-5\,b^2e^{(8\,c)}\right)e^{(8\,dx)}-2\left(13\,abe^{(6\,c)}-10\,b^2e^{(6\,c)}\right)e^{(6\,dx)}-2}{2\left(a^3bd-a^2b^2d-\left(a^3bde^{(10\,c)}-a^2b^2de^{(10\,c)}\right)e^{(10\,dx)}+5\left(a^3bde^{(8\,c)}-a^2b^2de^{(8\,c)}\right)e^{(8\,dx)}+2\left(8\,a^4de^{(6\,c)}-13\,a^3bde^{(6}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b - 5*b^2 + (6*a*b*e^(8*c) - 5*b^2*e^(8*c))*e^(8*d*x) - 2*(13*a*b*
e^(6*c) - 10*b^2*e^(6*c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 47*a*b*e^(4*c) +
15*b^2*e^(4*c))*e^(4*d*x) - 2*(7*a*b*e^(2*c) - 10*b^2*e^(2*c))*e^(2*d*x))/(
a^3*b*d - a^2*b^2*d - (a^3*b*d*e^(10*c) - a^2*b^2*d*e^(10*c))*e^(10*d*x) +
5*(a^3*b*d*e^(8*c) - a^2*b^2*d*e^(8*c))*e^(8*d*x) + 2*(8*a^4*d*e^(6*c) - 13
*a^3*b*d*e^(6*c) + 5*a^2*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^4*d*e^(4*c) - 13
*a^3*b*d*e^(4*c) + 5*a^2*b^2*d*e^(4*c))*e^(4*d*x) - 5*(a^3*b*d*e^(2*c) - a^
2*b^2*d*e^(2*c))*e^(2*d*x)) - 4*integrate(1/4*((6*a*b*e^(6*c) - 5*b^2*e^(6*
c))*e^(6*d*x) - 2*(8*a*b*e^(4*c) - 5*b^2*e^(4*c))*e^(4*d*x) + (6*a*b*e^(2*c
) - 5*b^2*e^(2*c))*e^(2*d*x))/(a^3*b - a^2*b^2 + (a^3*b*e^(8*c) - a^2*b^2*e
^(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c) - a^2*b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^
4*e^(4*c) - 11*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2
*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\mathrm{sinh}\left(c+d\,x\right)}^2\,{\left(a-b\,{\mathrm{sinh}\left(c+d\,x\right)}^4\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^2),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.253 $\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=315

$$-\frac{\cosh(c+dx)\left(9a^2-2b(2a-5b)\cosh^2(c+dx)-11ab-10b^2\right)}{32b^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{\left(-14\sqrt{a}\sqrt{b}+5a+12b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\,b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+$$

```
[Out] 1/8*a*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/(a-b)/b^2/d/(a-b+2*b*cosh(d*x+c)^2-
b*cosh(d*x+c)^4)^2-1/32*cosh(d*x+c)*(9*a^2-11*a*b-10*b^2-2*(2*a-5*b)*b*cosh
(d*x+c)^2)/(a-b)^2/b^2/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)+1/64*arcta
n(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(5*a+12*b-14*a^(1/2)*b^(1/2)
)/b^(9/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctanh(b^(1/4)*cosh(d*x+c
)/(a^(1/2)+b^(1/2))^(1/2))*(5*a+12*b+14*a^(1/2)*b^(1/2))/b^(9/4)/d/a^(1/2)/
(a^(1/2)+b^(1/2))^(5/2)
```

Rubi [A] time = 0.58, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3215, 1205, 1678, 1166, 205, 208}

$$-\frac{\cosh(c+dx)\left(9a^2-2b(2a-5b)\cosh^2(c+dx)-11ab-10b^2\right)}{32b^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right.}{8b^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt
[a] - Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(9/4)*d) + ((5*a +
 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(9/4)*d) + (a*Cosh[c + d
*x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b^2*d*(a - b + 2*b*Cosh[c + d*x
]^2 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(
2*a - 5*b)*b*Cosh[c + d*x]^2))/(32*(a - b)^2*b^2*d*(a - b + 2*b*Cosh[c + d*
x]^2 - b*Cosh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{2a\left(a^2+ab-8\right.}{b}\right.}{}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(9a^2\right.}{32(a-b)^2b^2d\left(a\right.}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(9a^2\right.}{32(a-b)^2b^2d\left(a\right.}$$

$$= \frac{\left(5a-14\sqrt{a}\sqrt{b}+12b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{9/4}d} + \frac{\left(5a+14\sqrt{a}\sqrt{b}+12b\right)\tanh^{-1}\left(\right.}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b}$$

Mathematica [C] time = 1.74, size = 1021, normalized size = 3.24

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cosh[c + d*x]*(-9*a^2 + 13*a*b + 5*b^2 + (2*a - 5*b)*b*Cosh[2*(c + d*x
)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a
- b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[
2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 - RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*a*b*c + 5*b^2*c - 2*a*b*d*x + 5*b^2*d*
x - 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
 - Sinh[(c + d*x)/2]*#1] + 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2
] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 10*a^2*c*#1^2 + 28*a*b*c
*#1^2 - 39*b^2*c*#1^2 - 10*a^2*d*x*#1^2 + 28*a*b*d*x*#1^2 - 39*b^2*d*x*#1^2
 - 20*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
 - Sinh[(c + d*x)/2]*#1]*#1^2 + 56*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d
*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 78*b^2*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^2 + 10*a^2*c*#1^4 - 28*a*b*c*#1^4 + 39*b^2*c*#1^4 + 10*a^2*d*x*#
1^4 - 28*a*b*d*x*#1^4 + 39*b^2*d*x*#1^4 + 20*a^2*Log[-Cosh[(c + d*x)/2] - S
inh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 56*a
*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh
[(c + d*x)/2]*#1]*#1^4 + 78*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
+ Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 2*a*b*c*#1^6 - 5*b^2*
c*#1^6 + 2*a*b*d*x*#1^6 - 5*b^2*d*x*#1^6 + 4*a*b*Log[-Cosh[(c + d*x)/2] - S
inh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 10*b
^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh
[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7)
 & ])/(128*(a - b)^2*b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 2.04, size = 1089, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*((5*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b + 25*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b
^2 - 98*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^3 + 176*sqrt(-b^2 - sqrt(a*b)*b)*a*b
^4 - sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b - 7*sqrt(a*b)*sqrt(-b^2 - sqr
t(a*b)*b)*a^2*b^2 - 4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^3 - 96*sqrt(a*
b)*sqrt(-b^2 - sqrt(a*b)*b)*b^4)*abs(b)*arctan(1/2*(e^(d*x + c) + e^(-d*x -
 c))/sqrt(-(a^2*b^3 - 2*a*b^4 + b^5 + sqrt((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 -
 b^5)*(a^2*b^3 - 2*a*b^4 + b^5) + (a^2*b^3 - 2*a*b^4 + b^5)^2))/(a^2*b^3 -
2*a*b^4 + b^5)))/(a^5*b^5 + 5*a^4*b^6 - 21*a^3*b^7 + 23*a^2*b^8 - 8*a*b^9)
+ (5*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b + 25*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^2 -
 98*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^3 + 176*sqrt(-b^2 + sqrt(a*b)*b)*a*b^4 -
 sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b - 7*sqrt(a*b)*sqrt(-b^2 + sqrt(a*
b)*b)*a^2*b^2 - 4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^3 - 96*sqrt(a*b)*s
qrt(-b^2 + sqrt(a*b)*b)*b^4)*abs(b)*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))
/sqrt(-(a^2*b^3 - 2*a*b^4 + b^5 - sqrt((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5
)*(a^2*b^3 - 2*a*b^4 + b^5) + (a^2*b^3 - 2*a*b^4 + b^5)^2))/(a^2*b^3 - 2*a*
```

```
b^4 + b^5)))/(a^5*b^5 + 5*a^4*b^6 - 21*a^3*b^7 + 23*a^2*b^8 - 8*a*b^9) - 8*
(2*a*b^2*(e^(d*x + c) + e^(-d*x - c))^7 - 5*b^3*(e^(d*x + c) + e^(-d*x - c)
)^7 - 18*a^2*b*(e^(d*x + c) + e^(-d*x - c))^5 + 6*a*b^2*(e^(d*x + c) + e^(-
d*x - c))^5 + 60*b^3*(e^(d*x + c) + e^(-d*x - c))^5 + 144*a^2*b*(e^(d*x + c
) + e^(-d*x - c))^3 - 96*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 240*b^3*(e^
(d*x + c) + e^(-d*x - c))^3 + 160*a^3*(e^(d*x + c) + e^(-d*x - c)) - 640*a^
2*b*(e^(d*x + c) + e^(-d*x - c)) + 160*a*b^2*(e^(d*x + c) + e^(-d*x - c)) +
 320*b^3*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4 -
 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)^2*(a^2*b^2 - 2*a*b^3 + b
^4)))/d
```

maple [B] time = 0.17, size = 3542, normalized size = 11.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -1/16/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tan
h(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(
1/2)-1/16/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*
tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b
)^(1/2)-13/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*
a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-35/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)
^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^12+85/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2
*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a
^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+105/16/d/(tanh(1/2*d*x+1/2*c)^8
*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/
2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^10*a^3-407/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tan
h(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a
)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a^2-175/16/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x
+1/2*c)^8*a^3+865/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a^2+11/64/d/b/(a^2-2*a*b+b^
2)*a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a
*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+175/16/d/(tanh(1/2*d*x+1/2*c)^8*
a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2
*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c
)^6*a^3-849/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*a^2-105/16/d/(tanh(1/2*d*x+1/2*c)
^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+
1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*
x+1/2*c)^4+383/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*ta
nh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+
a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+35/16/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)*tanh(1/2*
d*x+1/2*c)^2-77/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*t
anh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a
+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+5/16/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)*tanh(1/2*
d*x+1/2*c)^14-11/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
```

```
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14-11/64/d/b/(a^2-2*a*b+b^
2)*a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*
b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-5/64/d/b^2/(a^2-2*a*b+b^2)*a^2/(-
a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/
2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+163/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh
(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*
tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a-106/d
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^8*a-20/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*
c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+
1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10+88/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^8+20/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)
^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)+11/16/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)+18
9/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^6*a-31/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-3/4/d/(tanh(
1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b
*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^2+3/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a
+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)
^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+5/32/d/b/(a^2-2*a*b+b^2)
/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(
1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+5/32/d/b/(a^2-2*a*b+b^2)/
(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(
1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+5/64/d/b^2/(a^2-2*a*b+b^2
)*a^2/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a
*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+3/16/d/(a^2-2*a*b+b^2)/(-a*b+(a*
b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/
(-a*b+(a*b)^(1/2)*a)^(1/2))-3/16/d/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/
2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1
/2)*a)^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh (c+d\,x)^9}{\left(a-b\sinh (c+d\,x)^4\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^9/(a - b*sinh(c + d*x)^4)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**9/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.254 $\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=290

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cosh(c+dx)\left(-3(a-3b)\cosh^2(c\right.}{32bd(a-b)^2\left(a-b\cosh^4(c+dx)+\right.}$$

```
[Out] -1/8*a*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cosh(d*x+c)^2-b*cos
h(d*x+c)^4)^2+1/32*cosh(d*x+c)*(5*a-17*b-3*(a-3*b)*cosh(d*x+c)^2)/(a-b)^2/b
/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)+3/64*arctan(b^(1/4)*cosh(d*x+c)/
(a^(1/2)-b^(1/2))^(1/2))*(a^(1/2)-2*b^(1/2))/b^(7/4)/d/a^(1/2)/(a^(1/2)-b^(
1/2))^(5/2)-3/64*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(a^(1
/2)+2*b^(1/2))/b^(7/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(5/2)
```

Rubi [A] time = 0.47, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3215, 1205, 1178, 1166, 205, 208}

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cosh(c+dx)\left(-3(a-3b)\cosh^2(c\right.}{32bd(a-b)^2\left(a-b\cosh^4(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (3*(Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(7/4)*d) - (3*(Sqrt[a] + 2*S
qrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*Sqrt[
a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*
x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2)
+ (Cosh[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cosh[c + d*x]^2))/(32*(a - b)^2*
b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
```

```
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a(a-4b)\cdots}{(a-b+2\cdots}\right.}{}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+dx)\cdots}{32(a-b)^2bd\left(a-\cdots\right.}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+dx)\cdots}{32(a-b)^2bd\left(a-\cdots\right.}$$

$$= \frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{7/4}d} - \cdots$$

Mathematica [C] time = 1.33, size = 802, normalized size = 2.77

$$-\frac{32\cosh(c+dx)(-7a+25b+3(a-3b)\cosh(2(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))} - 3\text{RootSum}\left[b\#1^8-4b\#1^6-16a\#1^4+6b\#1^4-4b\#1^2+b\&, \frac{-ac\#1^6+3b\cdots}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cosh[c + d*x]*(-7*a + 25*b + 3*(a - 3*b)*Cosh[2*(c + d*x)]))/(8*a - 3
*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a - b)*(-5*Cosh
[c + d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cos
h[4*(c + d*x)])^2 - 3*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^
6 + b*#1^8 & , (a*c - 3*b*c + a*d*x - 3*b*d*x + 2*a*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 6*b*Lo
g[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
 d*x)/2]*#1] - 3*a*c*#1^2 + 17*b*c*#1^2 - 3*a*d*x*#1^2 + 17*b*d*x*#1^2 - 6*
a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[
(c + d*x)/2]*#1]*#1^2 + 34*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a*c*#1^4 - 17*b*c*#1^4
 + 3*a*d*x*#1^4 - 17*b*d*x*#1^4 + 6*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 34*b*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1]*#1^4 - a*c*#1^6 + 3*b*c*#1^6 - a*d*x*#1^6 + 3*b*d*x*#1^6 - 2*a*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^6 + 6*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*
b*#1^5 + b*#1^7) & ])/(256*(a - b)^2*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.64, size = 1506, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/64*(3*((sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3 + 5*sqrt(a*b)*sqrt(-b^2 -
 sqrt(a*b)*b)*a^2*b - 24*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^2)*(a^2*b -
 2*a*b^2 + b^3)^2*abs(b) - (sqrt(-b^2 - sqrt(a*b)*b)*a^5*b^2 + sqrt(-b^2 -
sqrt(a*b)*b)*a^4*b^3 - 45*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^4 + 83*sqrt(-b^2 -
 sqrt(a*b)*b)*a^2*b^5 - 40*sqrt(-b^2 - sqrt(a*b)*b)*a*b^6)*abs(a^2*b - 2*a*
b^2 + b^3)*abs(b) - 2*(sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b^4 + 4*sqrt(
a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^5 - 26*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*
b)*a^3*b^6 + 44*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^7 - 31*sqrt(a*b)*s
qrt(-b^2 - sqrt(a*b)*b)*a*b^8 + 8*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^9)*a
bs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^2*b^2 - 2*a*b^3 + b
^4 + sqrt((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*(a^2*b^2 - 2*a*b^3 + b^4) + (
a^2*b^2 - 2*a*b^3 + b^4)^2))/(a^2*b^2 - 2*a*b^3 + b^4)))/((a^7*b^5 + 3*a^6*
b^6 - 30*a^5*b^7 + 70*a^4*b^8 - 75*a^3*b^9 + 39*a^2*b^10 - 8*a*b^11)*abs(a^
2*b - 2*a*b^2 + b^3)) + 3*((sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3 + 5*sqrt
(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b - 24*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b
)*a*b^2)*(a^2*b - 2*a*b^2 + b^3)^2*abs(b) - (sqrt(-b^2 + sqrt(a*b)*b)*a^5*b
^2 + sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^3 - 45*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^4
 + 83*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^5 - 40*sqrt(-b^2 + sqrt(a*b)*b)*a*b^6)
*abs(a^2*b - 2*a*b^2 + b^3)*abs(b) - 2*(sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*
a^5*b^4 + 4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^5 - 26*sqrt(a*b)*sqrt(
```

```
-b^2 + sqrt(a*b)*b)*a^3*b^6 + 44*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^7
 - 31*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^8 + 8*sqrt(a*b)*sqrt(-b^2 + sq
rt(a*b)*b)*b^9)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^2*
b^2 - 2*a*b^3 + b^4 - sqrt((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*(a^2*b^2 - 2
*a*b^3 + b^4) + (a^2*b^2 - 2*a*b^3 + b^4)^2))/(a^2*b^2 - 2*a*b^3 + b^4)))/(
(a^7*b^5 + 3*a^6*b^6 - 30*a^5*b^7 + 70*a^4*b^8 - 75*a^3*b^9 + 39*a^2*b^10 -
 8*a*b^11)*abs(a^2*b - 2*a*b^2 + b^3)) - 4*(3*a*b*(e^(d*x + c) + e^(-d*x -
c))^7 - 9*b^2*(e^(d*x + c) + e^(-d*x - c))^7 - 44*a*b*(e^(d*x + c) + e^(-d*
x - c))^5 + 140*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 16*a^2*(e^(d*x + c) +
e^(-d*x - c))^3 + 288*a*b*(e^(d*x + c) + e^(-d*x - c))^3 - 688*b^2*(e^(d*x
+ c) + e^(-d*x - c))^3 - 192*a^2*(e^(d*x + c) + e^(-d*x - c)) - 896*a*b*(e^
(d*x + c) + e^(-d*x - c)) + 1088*b^2*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(
d*x + c) + e^(-d*x - c))^4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16
*b)^2*(a^2*b - 2*a*b^2 + b^3)))/d
```

maple [B] time = 0.16, size = 2563, normalized size = 8.84

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 3/64/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh
(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1
/2)+3/64/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*t
anh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)
^(1/2)+15/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2
*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a
/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-3/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12
+2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a
*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a^2-35/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a^2-32/d
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a
*b+b^2)*tanh(1/2*d*x+1/2*c)^8-1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x
+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2
*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)+5/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(
1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*t
anh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*a^2-25/
8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-111/8/d/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)^10*a+13/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a+10/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^10+8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8-42/d/(tanh(1/2*d*x+1/2*c)^8*a-4*t
anh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4
-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/8
```

```
/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)
^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2
*a*b+b^2)+95/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*a-27/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)
^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1
9/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-3/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14-9/64/d/b/(a^
2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2
*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-9/64/d/b/(a^2
-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2
*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+8/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^4-3/32/d/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/
4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))
+3/32/d/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*
d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(a*b*e^(15*c) - 3*b^2*e^(15*c))*e^(15*d*x) - (23*a*b*e^(13*c) - 77*
b^2*e^(13*c))*e^(13*d*x) + (16*a^2*e^(11*c) + 131*a*b*e^(11*c) - 177*b^2*e^
(11*c))*e^(11*d*x) - (144*a^2*e^(9*c) + 367*a*b*e^(9*c) - 109*b^2*e^(9*c))*
e^(9*d*x) - (144*a^2*e^(7*c) + 367*a*b*e^(7*c) - 109*b^2*e^(7*c))*e^(7*d*x)
 + (16*a^2*e^(5*c) + 131*a*b*e^(5*c) - 177*b^2*e^(5*c))*e^(5*d*x) - (23*a*b
*e^(3*c) - 77*b^2*e^(3*c))*e^(3*d*x) + 3*(a*b*e^c - 3*b^2*e^c)*e^(d*x))/(a^
2*b^3*d - 2*a*b^4*d + b^5*d + (a^2*b^3*d*e^(16*c) - 2*a*b^4*d*e^(16*c) + b^
5*d*e^(16*c))*e^(16*d*x) - 8*(a^2*b^3*d*e^(14*c) - 2*a*b^4*d*e^(14*c) + b^5
*d*e^(14*c))*e^(14*d*x) - 4*(8*a^3*b^2*d*e^(12*c) - 23*a^2*b^3*d*e^(12*c) +
 22*a*b^4*d*e^(12*c) - 7*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^3*b^2*d*e^(10
*c) - 39*a^2*b^3*d*e^(10*c) + 30*a*b^4*d*e^(10*c) - 7*b^5*d*e^(10*c))*e^(10
*d*x) + 2*(128*a^4*b*d*e^(8*c) - 352*a^3*b^2*d*e^(8*c) + 355*a^2*b^3*d*e^(8
*c) - 166*a*b^4*d*e^(8*c) + 35*b^5*d*e^(8*c))*e^(8*d*x) + 8*(16*a^3*b^2*d*e
^(6*c) - 39*a^2*b^3*d*e^(6*c) + 30*a*b^4*d*e^(6*c) - 7*b^5*d*e^(6*c))*e^(6*
d*x) - 4*(8*a^3*b^2*d*e^(4*c) - 23*a^2*b^3*d*e^(4*c) + 22*a*b^4*d*e^(4*c) -
 7*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^2*b^3*d*e^(2*c) - 2*a*b^4*d*e^(2*c) + b^
5*d*e^(2*c))*e^(2*d*x)) + 1/128*integrate(24*((a*e^(7*c) - 3*b*e^(7*c))*e^(
7*d*x) - (3*a*e^(5*c) - 17*b*e^(5*c))*e^(5*d*x) + (3*a*e^(3*c) - 17*b*e^(3*
c))*e^(3*d*x) - (a*e^c - 3*b*e^c)*e^(d*x))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*
b^2*e^(8*c) - 2*a*b^3*e^(8*c) + b^4*e^(8*c))*e^(8*d*x) - 4*(a^2*b^2*e^(6*c)
 - 2*a*b^3*e^(6*c) + b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^3*b*e^(4*c) - 19*a^2*b
^2*e^(4*c) + 14*a*b^3*e^(4*c) - 3*b^4*e^(4*c))*e^(4*d*x) - 4*(a^2*b^2*e^(2*
c) - 2*a*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(c+d\,x\right)^7}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^7/(a - b*sinh(c + d*x)^4)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.255 $\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$-\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(10\sqrt{a}\sqrt{b}+3a+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cosh(c+dx)\left(a^2+\right.}{32abd(a-b)^2\left(a-\right.}$$

```
[Out] 1/8*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cosh(d*x+c)^2-b*co
sh(d*x+c)^4)^2-1/32*cosh(d*x+c)*(a^2-11*a*b-2*b^2+2*b*(2*a+b)*cosh(d*x+c)^2
)/a/(a-b)^2/b/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)-1/64*arctan(b^(1/4)
*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a+4*b-10*a^(1/2)*b^(1/2))/a^(3/2)/
b^(5/4)/d/(a^(1/2)-b^(1/2))^(5/2)-1/64*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)
+b^(1/2))^(1/2))*(3*a+4*b+10*a^(1/2)*b^(1/2))/a^(3/2)/b^(5/4)/d/(a^(1/2)+b^
(1/2))^(5/2)
```

Rubi [A] time = 0.50, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3215, 1205, 1178, 1166, 205, 208}

$$-\frac{\cosh(c+dx)\left(a^2+2b(2a+b)\cosh^2(c+dx)-11ab-2b^2\right)}{32abd(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(10\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((3*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt
[a] - Sqrt[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(5/4)*d) - ((3*a +
 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + S
qrt[b]]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(5/4)*d) + (Cosh[c + d*x]
*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 -
 b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a +
b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*C
osh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{2a(a-7b)+1}{\left(a-b+2bx^2-\right.}\right.}{16a}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(a^2-\right.}{32a(a-b)^2bd\left(a-\right.}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(a^2-\right.}{32a(a-b)^2bd\left(a-\right.}$$

$$= -\frac{\left(3a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{5/4}d} - \frac{\left(3a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\right.}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b}$$

Mathematica [C] time = 1.96, size = 1019, normalized size = 3.26

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/128*((32*Cosh[c + d*x]*(a^2 - 9*a*b - b^2 + b*(2*a + b)*Cosh[2*(c + d*x)
]))/(a*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) - (512*(a
 - b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh
[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 + RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (2*a*b*c + b^2*c + 2*a*b*d*x + b^2*d*x +
4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - S
inh[(c + d*x)/2]*#1] + 2*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a^2*c*#1^2 - 32*a*b*c*#1^2
+ 5*b^2*c*#1^2 + 6*a^2*d*x*#1^2 - 32*a*b*d*x*#1^2 + 5*b^2*d*x*#1^2 + 12*a^2
*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(
c + d*x)/2]*#1]*#1^2 - 64*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 10*b^2*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^2 - 6*a^2*c*#1^4 + 32*a*b*c*#1^4 - 5*b^2*c*#1^4 - 6*a^2*d*x*#1^4 + 32*a*b
*d*x*#1^4 - 5*b^2*d*x*#1^4 - 12*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 64*a*b*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1]*#1^4 - 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 2*a*b*c*#1^6 - b^2*c*#1^6 - 2*a*b*
d*x*#1^6 - b^2*d*x*#1^6 - 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
+ Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 2*b^2*Log[-Cosh[(c +
d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*
#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ]/a)/((a - b)^2
*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.43, size = 1793, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/64*((2*(a^3*b - 2*a^2*b^2 + a*b^3)^2*(2*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*
b)*a^2 + 17*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b + 8*sqrt(a*b)*sqrt(-b^2
- sqrt(a*b)*b)*b^2)*abs(b) + (3*sqrt(-b^2 - sqrt(a*b)*b)*a^6*b + sqrt(-b^2
- sqrt(a*b)*b)*a^5*b^2 - 145*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^3 + 291*sqrt(-b
^2 - sqrt(a*b)*b)*a^3*b^4 - 166*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^5 + 16*sqrt(
-b^2 - sqrt(a*b)*b)*a*b^6)*abs(a^3*b - 2*a^2*b^2 + a*b^3)*abs(b) - (3*sqrt(
a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^8*b^2 - sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*
a^7*b^3 - 126*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^6*b^4 + 486*sqrt(a*b)*sq
rt(-b^2 - sqrt(a*b)*b)*a^5*b^5 - 769*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^4
*b^6 + 603*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^7 - 228*sqrt(a*b)*sqrt(
-b^2 - sqrt(a*b)*b)*a^2*b^8 + 32*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^9)*
abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^3*b^2 - 2*a^2*b^3
+ a*b^4 + sqrt((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*(a^3*b^2 - 2*a^2*b^3
 + a*b^4) + (a^3*b^2 - 2*a^2*b^3 + a*b^4)^2))/(a^3*b^2 - 2*a^2*b^3 + a*b^4)
))/((a^8*b^4 + 3*a^7*b^5 - 30*a^6*b^6 + 70*a^5*b^7 - 75*a^4*b^8 + 39*a^3*b^
9 - 8*a^2*b^10)*abs(a^3*b - 2*a^2*b^2 + a*b^3)) - (2*(a^3*b - 2*a^2*b^2 + a
*b^3)^2*(2*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 17*sqrt(a*b)*sqrt(-b^2
```

```
+ sqrt(a*b)*b)*a*b + 8*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*abs(b) - (3*
sqrt(-b^2 + sqrt(a*b)*b)*a^6*b + sqrt(-b^2 + sqrt(a*b)*b)*a^5*b^2 - 145*sqr
t(-b^2 + sqrt(a*b)*b)*a^4*b^3 + 291*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^4 - 166*
sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^5 + 16*sqrt(-b^2 + sqrt(a*b)*b)*a*b^6)*abs(a
^3*b - 2*a^2*b^2 + a*b^3)*abs(b) - (3*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^
8*b^2 - sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^7*b^3 - 126*sqrt(a*b)*sqrt(-b^
2 + sqrt(a*b)*b)*a^6*b^4 + 486*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^5*b^5 -
 769*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^6 + 603*sqrt(a*b)*sqrt(-b^2 +
 sqrt(a*b)*b)*a^3*b^7 - 228*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^8 + 32
*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^9)*abs(b))*arctan(1/2*(e^(d*x + c)
+ e^(-d*x - c))/sqrt(-(a^3*b^2 - 2*a^2*b^3 + a*b^4 - sqrt((a^4*b - 3*a^3*b^
2 + 3*a^2*b^3 - a*b^4)*(a^3*b^2 - 2*a^2*b^3 + a*b^4) + (a^3*b^2 - 2*a^2*b^3
 + a*b^4)^2))/(a^3*b^2 - 2*a^2*b^3 + a*b^4)))/((a^8*b^4 + 3*a^7*b^5 - 30*a^
6*b^6 + 70*a^5*b^7 - 75*a^4*b^8 + 39*a^3*b^9 - 8*a^2*b^10)*abs(a^3*b - 2*a^
2*b^2 + a*b^3)) - 8*(2*a*b^2*(e^(d*x + c) + e^(-d*x - c))^7 + b^3*(e^(d*x +
 c) + e^(-d*x - c))^7 + 2*a^2*b*(e^(d*x + c) + e^(-d*x - c))^5 - 38*a*b^2*(
e^(d*x + c) + e^(-d*x - c))^5 - 12*b^3*(e^(d*x + c) + e^(-d*x - c))^5 - 80*
a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 + 224*a*b^2*(e^(d*x + c) + e^(-d*x - c
))^3 + 48*b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 96*a^3*(e^(d*x + c) + e^(-d*
x - c)) + 384*a^2*b*(e^(d*x + c) + e^(-d*x - c)) - 416*a*b^2*(e^(d*x + c) +
 e^(-d*x - c)) - 64*b^3*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^
(-d*x - c))^4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)^2*(a^3*b
- 2*a^2*b^2 + a*b^3)))/d
```

maple [B] time = 0.17, size = 3511, normalized size = 11.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -99/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+
1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+21/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(
1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*t
anh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-63
/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a^2+105/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tan
h(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4
*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a^2+2
4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+3/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)+3/64/d/b/(a^2-2*a*b+b^2)*a/(-a*b
-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+
2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-105/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*a^2+63/1
6/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-21/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/
2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tan
h(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-3/16/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^14-3/64/d/b/(a^2-2*a*b+b^2)*a/(-a*b+(a*b)^(1/2
)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(
a*b)^(1/2)*a)^(1/2))+225/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c
```

```
)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1
/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a-183/16/d/(tanh(1/2*
d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tan
h(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*
d*x+1/2*c)^8*a-17/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10+6/d/(tanh(1/2*d*x+1/2*c)^8*
a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2
*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
8+113/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x
+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+3/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)-9/16/d/(tanh(1/2*d*x+1/2*c
)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x
+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c
)^6*a+87/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2
*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a
/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-37/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*t
anh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4
-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+13/
16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a
*b+b^2)*tanh(1/2*d*x+1/2*c)^14+1/16/d/b/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a
)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b
)^(1/2)*a)^(1/2))*(a*b)^(1/2)+1/16/d/b/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)
^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b
)^(1/2)*a)^(1/2))*(a*b)^(1/2)-15/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+13/64/d/(a^2-
2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a
+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-13/64/d/(a^2-2*a*b+b^2)/(-a
*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2
)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/
2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tan
h(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+3/2/d/(t
anh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-
16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^12+1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c
)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1
/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+1/16/d*b/(a^2-2*a*b+
b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*
(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-4/d/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)
^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)
^10+1/32/d/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(
1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/
2)+1/32/d/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(
1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/
2)-1/16/d*b/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh
(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+4/d/(tan
h(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16
*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((2*a*b^2*e^(15*c) + b^3*e^(15*c))*e^(15*d*x) + (2*a^2*b*e^(13*c) - 24*
a*b^2*e^(13*c) - 5*b^3*e^(13*c))*e^(13*d*x) - (70*a^2*b*e^(11*c) - 76*a*b^2
*e^(11*c) - 9*b^3*e^(11*c))*e^(11*d*x) + (96*a^3*e^(9*c) + 164*a^2*b*e^(9*c
) - 54*a*b^2*e^(9*c) - 5*b^3*e^(9*c))*e^(9*d*x) + (96*a^3*e^(7*c) + 164*a^2
*b*e^(7*c) - 54*a*b^2*e^(7*c) - 5*b^3*e^(7*c))*e^(7*d*x) - (70*a^2*b*e^(5*c
) - 76*a*b^2*e^(5*c) - 9*b^3*e^(5*c))*e^(5*d*x) + (2*a^2*b*e^(3*c) - 24*a*b
^2*e^(3*c) - 5*b^3*e^(3*c))*e^(3*d*x) + (2*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a
^3*b^3*d - 2*a^2*b^4*d + a*b^5*d + (a^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*
c) + a*b^5*d*e^(16*c))*e^(16*d*x) - 8*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(
14*c) + a*b^5*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3
*d*e^(12*c) + 22*a^2*b^4*d*e^(12*c) - 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(1
6*a^4*b^2*d*e^(10*c) - 39*a^3*b^3*d*e^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*
b^5*d*e^(10*c))*e^(10*d*x) + 2*(128*a^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c)
 + 355*a^3*b^3*d*e^(8*c) - 166*a^2*b^4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8
*d*x) + 8*(16*a^4*b^2*d*e^(6*c) - 39*a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*
c) - 7*a*b^5*d*e^(6*c))*e^(6*d*x) - 4*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e
^(4*c) + 22*a^2*b^4*d*e^(4*c) - 7*a*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d
*e^(2*c) - 2*a^2*b^4*d*e^(2*c) + a*b^5*d*e^(2*c))*e^(2*d*x)) + 1/32*integra
te(4*((2*a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (6*a^2*e^(5*c) - 32*a*b*e^(
5*c) + 5*b^2*e^(5*c))*e^(5*d*x) - (6*a^2*e^(3*c) - 32*a*b*e^(3*c) + 5*b^2*e
^(3*c))*e^(3*d*x) - (2*a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b^2 - 2*a^2*b^3 + a
*b^4 + (a^3*b^2*e^(8*c) - 2*a^2*b^3*e^(8*c) + a*b^4*e^(8*c))*e^(8*d*x) - 4*
(a^3*b^2*e^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^4*
b*e^(4*c) - 19*a^3*b^2*e^(4*c) + 14*a^2*b^3*e^(4*c) - 3*a*b^4*e^(4*c))*e^(4
*d*x) - 4*(a^3*b^2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c))*e^(2*d*x)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(c+d\,x\right)^5}{{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^5/(a - b*sinh(c + d*x)^4)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.256 $\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=288

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cosh(c+dx)\left(-\left((5a+b)\cosh^2(\right.\right.}{32ad(a-b)^2\left(a-b\cosh^4(c+dx)+\right.}$$

```
[Out] -1/8*cosh(d*x+c)*(2-cosh(d*x+c)^2)/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*
x+c)^4)^2-1/32*cosh(d*x+c)*(11*a+b-(5*a+b)*cosh(d*x+c)^2)/a/(a-b)^2/d/(a-b+
2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)-1/64*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)
-b^(1/2))^(1/2))*(5*a^(1/2)-2*b^(1/2))/a^(3/2)/b^(3/4)/d/(a^(1/2)-b^(1/2))^
(5/2)+1/64*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(5*a^(1/2)+
2*b^(1/2))/a^(3/2)/b^(3/4)/d/(a^(1/2)+b^(1/2))^(5/2)
```

Rubi [A] time = 0.52, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1178, 1166, 205, 208}

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cosh(c+dx)\left(-(5a+b)\cosh^2(\right.}{32ad(a-b)^2\left(a-b\cosh^4(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqr
t[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/4)*d) + ((5*Sqrt[a] + 2*
Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*a^(3
/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/4)*d) - (Cosh[c + d*x]*(2 - Cosh[c + d*x
]^2))/(8*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - (
Cosh[c + d*x]*(11*a + b - (5*a + b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*d*(a
- b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
```

```
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-12ab+1\ldots}{\left(a-b+2bx^2\ldots\right.}\ldots\right)}{16\ldots}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\ldots}{32a(a-b)^2d\left(a-\ldots\right.}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\ldots}{32a(a-b)^2d\left(a-\ldots\right.}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/4}d} + \frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/4}d} - \ldots$$

Mathematica [C] time = 1.32, size = 802, normalized size = 2.78

$$\frac{\frac{32\cosh(c+dx)(-17a-b+(5a+b)\cosh(2(c+dx)))}{a(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))} + \frac{\text{RootSum}\left[b\#1^8-4b\#1^6-16a\#1^4+6b\#1^4-4b\#1^2+b\&, \frac{-5ac\#1^6-bc\#1^6-5adx\#1^6-bdx\#1^6-10a\log\left(\#1\cosh\ldots\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cosh[c + d*x]*(-17*a - b + (5*a + b)*Cosh[2*(c + d*x)]))/(a*(8*a - 3*b
 + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (512*(a - b)*(-5*Cosh[c
+ d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4
*(c + d*x)])^2 + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b
*#1^8 & , (5*a*c + b*c + 5*a*d*x + b*d*x + 10*a*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*b*Log[-C
osh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x
)/2]*#1] - 47*a*c*#1^2 + 5*b*c*#1^2 - 47*a*d*x*#1^2 + 5*b*d*x*#1^2 - 94*a*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
```

```
+ d*x)/2]*#1]*#1^2 + 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh
[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 47*a*c*#1^4 - 5*b*c*#1^4 +
47*a*d*x*#1^4 - 5*b*d*x*#1^4 + 94*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 10*b*Log[-Cosh[(c
 + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#
1]*#1^4 - 5*a*c*#1^6 - b*c*#1^6 - 5*a*d*x*#1^6 - b*d*x*#1^6 - 10*a*Log[-Cos
h[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/
2]*#1]*#1^6 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*
x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b
*#1^5 + b*#1^7) & ]/a)/(256*(a - b)^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.05, size = 1576, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/64*(((5*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2 + 41*sqrt(a*b)*sqrt(-b^2
 - sqrt(a*b)*b)*a*b + 8*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^2)*(a^3 - 2*a^2
*b + a*b^2)^2*abs(b) + (13*sqrt(-b^2 - sqrt(a*b)*b)*a^5*b + 77*sqrt(-b^2 -
sqrt(a*b)*b)*a^4*b^2 - 201*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^3 + 119*sqrt(-b^2
 - sqrt(a*b)*b)*a^2*b^4 - 8*sqrt(-b^2 - sqrt(a*b)*b)*a*b^5)*abs(a^3 - 2*a^2
*b + a*b^2)*abs(b) + 2*(4*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^7*b + 15*sqr
t(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^6*b^2 - 108*sqrt(a*b)*sqrt(-b^2 - sqrt(a*
b)*b)*a^5*b^3 + 202*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^4 - 168*sqrt(a
*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b^5 + 63*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b
)*a^2*b^6 - 8*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^7)*abs(b))*arctan(1/2*
(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^3*b - 2*a^2*b^2 + a*b^3 + sqrt((a^4 -
 3*a^3*b + 3*a^2*b^2 - a*b^3)*(a^3*b - 2*a^2*b^2 + a*b^3) + (a^3*b - 2*a^2*
b^2 + a*b^3)^2))/(a^3*b - 2*a^2*b^2 + a*b^3)))/((a^8*b^3 + 3*a^7*b^4 - 30*a
^6*b^5 + 70*a^5*b^6 - 75*a^4*b^7 + 39*a^3*b^8 - 8*a^2*b^9)*abs(a^3 - 2*a^2*
b + a*b^2)) + ((5*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2 + 41*sqrt(a*b)*sqr
t(-b^2 + sqrt(a*b)*b)*a*b + 8*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^2)*(a^3
 - 2*a^2*b + a*b^2)^2*abs(b) + (13*sqrt(-b^2 + sqrt(a*b)*b)*a^5*b + 77*sqrt(
-b^2 + sqrt(a*b)*b)*a^4*b^2 - 201*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^3 + 119*sq
rt(-b^2 + sqrt(a*b)*b)*a^2*b^4 - 8*sqrt(-b^2 + sqrt(a*b)*b)*a*b^5)*abs(a^3
 - 2*a^2*b + a*b^2)*abs(b) + 2*(4*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^7*b +
 15*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^6*b^2 - 108*sqrt(a*b)*sqrt(-b^2 +
sqrt(a*b)*b)*a^5*b^3 + 202*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b^4 - 168
*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^5 + 63*sqrt(a*b)*sqrt(-b^2 + sqrt
(a*b)*b)*a^2*b^6 - 8*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a*b^7)*abs(b))*arct
an(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^3*b - 2*a^2*b^2 + a*b^3 - sqrt
((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(a^3*b - 2*a^2*b^2 + a*b^3) + (a^3*b -
 2*a^2*b^2 + a*b^3)^2))/(a^3*b - 2*a^2*b^2 + a*b^3)))/((a^8*b^3 + 3*a^7*b^4
 - 30*a^6*b^5 + 70*a^5*b^6 - 75*a^4*b^7 + 39*a^3*b^8 - 8*a^2*b^9)*abs(a^3 -
 2*a^2*b + a*b^2)) + 4*(5*a*b*(e^(d*x + c) + e^(-d*x - c))^7 + b^2*(e^(d*x
+ c) + e^(-d*x - c))^7 - 84*a*b*(e^(d*x + c) + e^(-d*x - c))^5 - 12*b^2*(e^
(d*x + c) + e^(-d*x - c))^5 - 144*a^2*(e^(d*x + c) + e^(-d*x - c))^3 + 480*
a*b*(e^(d*x + c) + e^(-d*x - c))^3 + 48*b^2*(e^(d*x + c) + e^(-d*x - c))^3
+ 1216*a^2*(e^(d*x + c) + e^(-d*x - c)) - 1152*a*b*(e^(d*x + c) + e^(-d*x -
```

```
c)) - 64*b^2*(e^(d*x + c) + e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c)
)^4 - 8*b*(e^(d*x + c) + e^(-d*x - c))^2 - 16*a + 16*b)^2*(a^3 - 2*a^2*b +
a*b^2)))/d
```

maple [B] time = 0.13, size = 2916, normalized size = 10.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b
+b^2)/a*tanh(1/2*d*x+1/2*c)^12*b^2+32/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*b^3-4/d/
(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*
a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^4*b^2+1/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+
1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*
d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-104/d/(tanh(1/
2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*t
anh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*t
anh(1/2*d*x+1/2*c)^8-5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2*a/(a^2-2*a*b+b^2)+15/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a-175/8/d/(tan
h(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16
*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^8*a-219/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)
^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10+275/4/d/(tanh(1/2*d*
x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(
1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*
d*x+1/2*c)^8-533/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*t
anh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a
+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+55/2/d/(tanh(1/2*d*x+1/2*c)^8
*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/
2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6
*a-141/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*
x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a
^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+11/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(
1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*t
anh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/2/d/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^14-5/64/d/b/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2
)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2
)*a)^(1/2))*(a*b)^(1/2)-5/64/d/b/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)
*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2
)*a)^(1/2))*(a*b)^(1/2)+79/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*
c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+
1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/4/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*b-1/8/d/(a^2
-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*
a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+1/8/d/(a^2-2*a*b+b^2)/(-a*
b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)
+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
```

```
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+29/4/d/(t
anh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-
16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^12-17/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*
c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+
1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/32/d*b/(a^2-2*a*b
+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4
*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+18/d/(tanh(1/2*d*x+1/2*c)^8*a
-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*
c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*
c)^10-1/64/d/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tan
h(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(
1/2)-1/64/d/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tan
h(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(
1/2)+1/32/d*b/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*ta
nh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+14/d/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b
+b^2)*tanh(1/2*d*x+1/2*c)^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((5*a*b*e^(15*c) + b^2*e^(15*c))*e^(15*d*x) - (49*a*b*e^(13*c) + 5*b^
2*e^(13*c))*e^(13*d*x) - 3*(48*a^2*e^(11*c) - 55*a*b*e^(11*c) - 3*b^2*e^(11
*c))*e^(11*d*x) + (784*a^2*e^(9*c) - 377*a*b*e^(9*c) - 5*b^2*e^(9*c))*e^(9*
d*x) + (784*a^2*e^(7*c) - 377*a*b*e^(7*c) - 5*b^2*e^(7*c))*e^(7*d*x) - 3*(4
8*a^2*e^(5*c) - 55*a*b*e^(5*c) - 3*b^2*e^(5*c))*e^(5*d*x) - (49*a*b*e^(3*c)
 + 5*b^2*e^(3*c))*e^(3*d*x) + (5*a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b^2*d - 2
*a^2*b^3*d + a*b^4*d + (a^3*b^2*d*e^(16*c) - 2*a^2*b^3*d*e^(16*c) + a*b^4*d
*e^(16*c))*e^(16*d*x) - 8*(a^3*b^2*d*e^(14*c) - 2*a^2*b^3*d*e^(14*c) + a*b^
4*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b*d*e^(12*c) - 23*a^3*b^2*d*e^(12*c) +
22*a^2*b^3*d*e^(12*c) - 7*a*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b*d*e^(1
0*c) - 39*a^3*b^2*d*e^(10*c) + 30*a^2*b^3*d*e^(10*c) - 7*a*b^4*d*e^(10*c))*
e^(10*d*x) + 2*(128*a^5*d*e^(8*c) - 352*a^4*b*d*e^(8*c) + 355*a^3*b^2*d*e^(
8*c) - 166*a^2*b^3*d*e^(8*c) + 35*a*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^4*b*
d*e^(6*c) - 39*a^3*b^2*d*e^(6*c) + 30*a^2*b^3*d*e^(6*c) - 7*a*b^4*d*e^(6*c)
)*e^(6*d*x) - 4*(8*a^4*b*d*e^(4*c) - 23*a^3*b^2*d*e^(4*c) + 22*a^2*b^3*d*e^
(4*c) - 7*a*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^2*d*e^(2*c) - 2*a^2*b^3*d*e
^(2*c) + a*b^4*d*e^(2*c))*e^(2*d*x)) - 1/8*integrate(1/2*((5*a*e^(7*c) + b*
e^(7*c))*e^(7*d*x) - (47*a*e^(5*c) - 5*b*e^(5*c))*e^(5*d*x) + (47*a*e^(3*c)
 - 5*b*e^(3*c))*e^(3*d*x) - (5*a*e^c + b*e^c)*e^(d*x))/(a^3*b - 2*a^2*b^2 +
 a*b^3 + (a^3*b*e^(8*c) - 2*a^2*b^2*e^(8*c) + a*b^3*e^(8*c))*e^(8*d*x) - 4*
(a^3*b*e^(6*c) - 2*a^2*b^2*e^(6*c) + a*b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^4*e^
(4*c) - 19*a^3*b*e^(4*c) + 14*a^2*b^2*e^(4*c) - 3*a*b^3*e^(4*c))*e^(4*d*x)
- 4*(a^3*b*e^(2*c) - 2*a^2*b^2*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^3}{\left(a-b\sinh(c+d\,x)^4\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^3/(a - b*sinh(c + d*x)^4)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.257 $\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$\frac{3\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{3\left(10\sqrt{a}\sqrt{b}+7a+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cosh(c+dx)\left((7a\right.}{32a^2d(a-b)^2\left(a\right.}$$

```
[Out] 1/8*cosh(d*x+c)*(a+b-b*cosh(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*co
sh(d*x+c)^4)^2+1/32*cosh(d*x+c)*((7*a-3*b)*(a+2*b)-6*(2*a-b)*b*cosh(d*x+c)^
2)/a^2/(a-b)^2/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)+3/64*arctan(b^(1/4
)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(7*a+4*b-10*a^(1/2)*b^(1/2))/a^(5/2)
/b^(1/4)/d/(a^(1/2)-b^(1/2))^(5/2)+3/64*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2
)+b^(1/2))^(1/2))*(7*a+4*b+10*a^(1/2)*b^(1/2))/a^(5/2)/b^(1/4)/d/(a^(1/2)+b
^(1/2))^(5/2)
```

Rubi [A] time = 0.46, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3215, 1092, 1178, 1166, 205, 208}

$$\frac{\cosh(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cosh^2(c+dx)\right)}{32a^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{3\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (3*(7*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqr
t[a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(1/4)*d) + (3*(7*
a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(1/4)*d) + (Cosh[c + d
*x]*(a + b - b*Cosh[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^
2 - b*Cosh[c + d*x]^4)^2) + (Cosh[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a
- b)*b*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 -
 b*Cosh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b}{(}\right.}{}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+dx)\left((7a\right.}{32a^2(a-b)^2d\left(a-\right.}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+dx)\left((7a\right.}{32a^2(a-b)^2d\left(a-\right.}$$

$$= \frac{3\left(7a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt[4]{b}\,d} + \frac{3\left(7a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}$$

Mathematica [C] time = 1.42, size = 1018, normalized size = 3.25

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cosh[c + d*x]*(7*a^2 + 5*a*b - 3*b^2 + 3*b*(-2*a + b)*Cosh[2*(c + d*x)
]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a -
```

```
 b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2
*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 + 3*RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*a*b*c + b^2*c - 2*a*b*d*x + b^2*d*x -
 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
Sinh[(c + d*x)/2]*#1] + 2*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 14*a^2*c*#1^2 - 12*a*b*c*#1^
2 + 5*b^2*c*#1^2 + 14*a^2*d*x*#1^2 - 12*a*b*d*x*#1^2 + 5*b^2*d*x*#1^2 + 28*
a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sin
h[(c + d*x)/2]*#1]*#1^2 - 24*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
 + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 10*b^2*Log[-Cosh[(c
+ d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
]*#1^2 - 14*a^2*c*#1^4 + 12*a*b*c*#1^4 - 5*b^2*c*#1^4 - 14*a^2*d*x*#1^4 + 1
2*a*b*d*x*#1^4 - 5*b^2*d*x*#1^4 - 28*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 24*a*b*Log[-
Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*
x)/2]*#1]*#1^4 - 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 2*a*b*c*#1^6 - b^2*c*#1^6 + 2
*a*b*d*x*#1^6 - b^2*d*x*#1^6 + 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 2*b^2*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(128*a^
2*(a - b)^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.87, size = 1125, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*(3*(7*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b + 51*sqrt(-b^2 - sqrt(a*b)*b)*a^3
*b^2 - 38*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b^3 + 16*sqrt(-b^2 - sqrt(a*b)*b)*a*
b^4 - 11*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^3*b - 77*sqrt(a*b)*sqrt(-b^2
- sqrt(a*b)*b)*a^2*b^2 + 84*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a*b^3 - 32*s
qrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^4)*abs(b)*arctan(1/2*(e^(d*x + c) + e^(
-d*x - c))/sqrt(-(a^4*b - 2*a^3*b^2 + a^2*b^3 + sqrt((a^5 - 3*a^4*b + 3*a^3
*b^2 - a^2*b^3)*(a^4*b - 2*a^3*b^2 + a^2*b^3) + (a^4*b - 2*a^3*b^2 + a^2*b^
3)^2))/(a^4*b - 2*a^3*b^2 + a^2*b^3)))/(a^7*b^3 + 5*a^6*b^4 - 21*a^5*b^5 +
23*a^4*b^6 - 8*a^3*b^7) + 3*(7*sqrt(-b^2 + sqrt(a*b)*b)*a^4*b + 51*sqrt(-b^
2 + sqrt(a*b)*b)*a^3*b^2 - 38*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^3 + 16*sqrt(-b
^2 + sqrt(a*b)*b)*a*b^4 + 11*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b + 77*
sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b^2 - 84*sqrt(a*b)*sqrt(-b^2 + sqrt(
a*b)*b)*a*b^3 + 32*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^4)*abs(b)*arctan(1/
2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^4*b - 2*a^3*b^2 + a^2*b^3 - sqrt((a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*(a^4*b - 2*a^3*b^2 + a^2*b^3) + (a^4*b
- 2*a^3*b^2 + a^2*b^3)^2))/(a^4*b - 2*a^3*b^2 + a^2*b^3)))/(a^7*b^3 + 5*a^6
*b^4 - 21*a^5*b^5 + 23*a^4*b^6 - 8*a^3*b^7) + 8*(6*a*b^2*(e^(d*x + c) + e^(
-d*x - c))^7 - 3*b^3*(e^(d*x + c) + e^(-d*x - c))^7 - 14*a^2*b*(e^(d*x + c)
 + e^(-d*x - c))^5 - 70*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 36*b^3*(e^(d
*x + c) + e^(-d*x - c))^5 - 16*a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 + 352*a
*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 144*b^3*(e^(d*x + c) + e^(-d*x - c))^
```

```
3 + 352*a^3*(e^(d*x + c) + e^(-d*x - c)) + 128*a^2*b*(e^(d*x + c) + e^(-d*x
 - c)) - 672*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 192*b^3*(e^(d*x + c) + e^
(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4 - 8*b*(e^(d*x + c) + e^(-d*
x - c))^2 - 16*a + 16*b)^2*(a^4 - 2*a^3*b + a^2*b^2)))/d
```

maple [B] time = 0.18, size = 3512, normalized size = 11.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -3/32/d/a^2/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1
/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2
)*b-3/32/d/a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^
(1/2)*b+19/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/
(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^12*b^2-40/d/(tanh(1/2*d*x+1/2*c)^8*a-
4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c
)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
8*b^3+1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d
*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4*b^2+77/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)
^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+
118/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+11/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(
1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*t
anh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)+12/d/(tanh(1/2*d*x+1/2*c)^8*a
-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*
c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)
^10*b^3-231/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a+385/16/d/(tanh(1/2*d*x+1/2*c)^8*
a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2
*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*
a+857/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*
x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a
^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10-1231/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*t
anh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4
-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+831
/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-385/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*a+231/16/d/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^4-77/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-11/16/d/(tanh(1/2*
d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tan
h(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^14-209/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a
+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)
^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-5/16/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*b+21/64/d/(a^2-2*
```

```
a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4
*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-21/64/d/(a^2-2*a*b+b^2)/(-a*b
-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+
2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+37/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/
2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tan
h(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14-283/16/d
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^12+3/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+27/64/d*b/(a^2-2
*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2
*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-197/4/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d
*x+1/2*c)^10+3/16/d/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4
*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*
(a*b)^(1/2)+3/16/d/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*
(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*
(a*b)^(1/2)-27/64/d*b/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1
/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2)
)+37/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+
1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/
(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+3/16/d/a^2/(a^2-2*a*b+b^2)/(-a*b+(a*b
)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(
-a*b+(a*b)^(1/2)*a)^(1/2))*b^2+5/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*b^2-12/d/(tan
h(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16
*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^6*b^3-3/16/d/a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^
(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)
^(1/2)*a)^(1/2))*b^2-5/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(2*a*b^2*e^(15*c) - b^3*e^(15*c))*e^(15*d*x) - (14*a^2*b*e^(13*c) +
28*a*b^2*e^(13*c) - 15*b^3*e^(13*c))*e^(13*d*x) - (86*a^2*b*e^(11*c) - 128*
a*b^2*e^(11*c) + 27*b^3*e^(11*c))*e^(11*d*x) + (352*a^3*e^(9*c) - 60*a^2*b*
e^(9*c) - 106*a*b^2*e^(9*c) + 15*b^3*e^(9*c))*e^(9*d*x) + (352*a^3*e^(7*c)
- 60*a^2*b*e^(7*c) - 106*a*b^2*e^(7*c) + 15*b^3*e^(7*c))*e^(7*d*x) - (86*a^
2*b*e^(5*c) - 128*a*b^2*e^(5*c) + 27*b^3*e^(5*c))*e^(5*d*x) - (14*a^2*b*e^(
3*c) + 28*a*b^2*e^(3*c) - 15*b^3*e^(3*c))*e^(3*d*x) + 3*(2*a*b^2*e^c - b^3*
e^c)*e^(d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(16*c) -
2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*x) - 8*(a^4*b^2*d*e^(14*
c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^(14*d*x) - 4*(8*a^5*b*d*e
^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^(12*c) - 7*a^2*b^4*d*e^(12
*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^4*b^2*d*e^(10*c) + 30*a^3*b
^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a^6*d*e^(8*c) - 3
52*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 166*a^3*b^3*d*e^(8*c) + 35*a^2
*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(6*c) - 39*a^4*b^2*d*e^(6*c) +
30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*e^(6*d*x) - 4*(8*a^5*b*d*e^(4*c
) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(4*c) - 7*a^2*b^4*d*e^(4*c))*e^(4
```

```
*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) + a^2*b^4*d*e^(2*c))*e^(
2*d*x)) + 1/2*integrate(3/4*((2*a*b*e^(7*c) - b^2*e^(7*c))*e^(7*d*x) - (14*
a^2*e^(5*c) - 12*a*b*e^(5*c) + 5*b^2*e^(5*c))*e^(5*d*x) + (14*a^2*e^(3*c) -
 12*a*b*e^(3*c) + 5*b^2*e^(3*c))*e^(3*d*x) - (2*a*b*e^c - b^2*e^c)*e^(d*x))
/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^4*b*e^(8*c) - 2*a^3*b^2*e^(8*c) + a^2*b^
3*e^(8*c))*e^(8*d*x) - 4*(a^4*b*e^(6*c) - 2*a^3*b^2*e^(6*c) + a^2*b^3*e^(6*
c))*e^(6*d*x) - 2*(8*a^5*e^(4*c) - 19*a^4*b*e^(4*c) + 14*a^3*b^2*e^(4*c) -
3*a^2*b^3*e^(4*c))*e^(4*d*x) - 4*(a^4*b*e^(2*c) - 2*a^3*b^2*e^(2*c) + a^2*b
^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh (c+d\,x)}{{\left(a-b\,{\sinh \left(c+d\,x\right)}^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)/(a - b*sinh(c + d*x)^4)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.258 $\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=617

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\sqrt[4]{b}\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\cdots$$

```
[Out] -arctanh(cosh(d*x+c))/a^3/d-1/8*b*cosh(d*x+c)*(2-cosh(d*x+c)^2)/a/(a-b)/d/(
a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)^2-1/4*b*cosh(d*x+c)*(2-cosh(d*x+c)^2
)/a^2/(a-b)/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*x+c)^4)-1/32*b*cosh(d*x+c)*(1
1*a+b-(5*a+b)*cosh(d*x+c)^2)/a^2/(a-b)^2/d/(a-b+2*b*cosh(d*x+c)^2-b*cosh(d*
x+c)^4)-1/64*b^(1/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(5
*a^(1/2)-2*b^(1/2))/a^(5/2)/d/(a^(1/2)-b^(1/2))^(5/2)-1/8*b^(1/4)*arctan(b^
(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(5/2)/d/(a^(1/2)-b^(1/2))^(3/2
)+1/8*b^(1/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(5/2)/
d/(a^(1/2)+b^(1/2))^(3/2)+1/64*b^(1/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)
+b^(1/2))^(1/2))*(5*a^(1/2)+2*b^(1/2))/a^(5/2)/d/(a^(1/2)+b^(1/2))^(5/2)-1/
2*b^(1/4)*arctan(b^(1/4)*cosh(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^3/d/(a^(1/2
)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cosh(d*x+c)/(a^(1/2)+b^(1/2))^
(1/2))/a^3/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3215, 1238, 207, 1178, 1166, 205, 208}

$$-\frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4a^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{b\cosh(c+dx)\left(-(5a+b)\cosh^2(c+dx)+11a+\cdots\right)}{32a^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx\cdots\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[
a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (b^(1/4)*ArcTan[
(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(5/2)*(Sqrt[a] - Sqr
t[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqr
t[b]]])/(2*a^3*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c + d*x]]/(a^3*d)
+ (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(
5/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x]
)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^3*Sqrt[Sqrt[a] + Sqrt[b]]*d) + ((5*Sqrt[a]
 + 2*Sqrt[b])*b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b
]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b*Cosh[c + d*x]*(2 - Cosh[
c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^
4)^2) - (b*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*a^2*(a - b)*d*(a - b + 2
*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)) - (b*Cosh[c + d*x]*(11*a + b - (5*
a + b)*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 -
 b*Cosh[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\text{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^3(-1+x^2)} + \frac{b-bx^2}{a(a-b+2bx^2-bx^4)^3} + \frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)^2} + \frac{b-bx^2}{a^3(a-b+2bx^2-bx^4)}\right) dx,\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(c+dx)\right)}{a^3 d} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{a^3 d}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d} - \frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2} d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d} +$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2} d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2} d} - \frac{\sqrt[4]{b}\tan^{-1}}{2a^3\sqrt{}}$$

Mathematica [C] time = 5.62, size = 1189, normalized size = 1.93

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*a*b*Cosh[c + d*x]*(-41*a + 23*b + (13*a - 7*b)*Cosh[2*(c + d*x)]))/((a
 - b)^2*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (512*a
^2*b*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/((a - b)*(-8*a + 3*b - 4*b*Cos
h[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2) + 256*Log[Tanh[(c + d*x)/2]] - (b*
RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-45*a^
2*c + 71*a*b*c - 32*b^2*c - 45*a^2*d*x + 71*a*b*d*x - 32*b^2*d*x - 90*a^2*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
+ d*x)/2]*#1] + 142*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 64*b^2*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 199*a^2*c*
#1^2 - 253*a*b*c*#1^2 + 96*b^2*c*#1^2 + 199*a^2*d*x*#1^2 - 253*a*b*d*x*#1^2
 + 96*b^2*d*x*#1^2 + 398*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 506*a*b*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^2 + 192*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 199*a^2*c*#1^4 + 253*a*b*c*#1^4 - 96*b^
2*c*#1^4 - 199*a^2*d*x*#1^4 + 253*a*b*d*x*#1^4 - 96*b^2*d*x*#1^4 - 398*a^2*
Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
 + d*x)/2]*#1]*#1^4 + 506*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
```

```
Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 192*b^2*Log[-Cosh[(c +
d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*
#1^4 + 45*a^2*c*#1^6 - 71*a*b*c*#1^6 + 32*b^2*c*#1^6 + 45*a^2*d*x*#1^6 - 71
*a*b*d*x*#1^6 + 32*b^2*d*x*#1^6 + 90*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 142*a*b*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1]*#1^6 + 64*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[
(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^
3 - 3*b*#1^5 + b*#1^7) & ])/(a - b)^2)/(256*a^3*d)
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**B**] time = 0.65, size = 1781, normalized size = 2.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*(((a^5 - 2*a^4*b + a^3*b^2)^2*(45*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a
^3 + 289*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^2*b - 536*sqrt(a*b)*sqrt(-b^2
 - sqrt(a*b)*b)*a*b^2 + 256*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*b^3)*abs(b)
- (61*sqrt(-b^2 - sqrt(a*b)*b)*a^8*b + 285*sqrt(-b^2 - sqrt(a*b)*b)*a^7*b^2
 - 1369*sqrt(-b^2 - sqrt(a*b)*b)*a^6*b^3 + 1895*sqrt(-b^2 - sqrt(a*b)*b)*a^
5*b^4 - 1128*sqrt(-b^2 - sqrt(a*b)*b)*a^4*b^5 + 256*sqrt(-b^2 - sqrt(a*b)*b
)*a^3*b^6)*abs(a^5 - 2*a^4*b + a^3*b^2)*abs(b) + 2*(8*sqrt(a*b)*sqrt(-b^2 -
 sqrt(a*b)*b)*a^12*b + 27*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^11*b^2 - 228
*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^10*b^3 + 482*sqrt(a*b)*sqrt(-b^2 - sq
rt(a*b)*b)*a^9*b^4 - 468*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^8*b^5 + 219*s
qrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a^7*b^6 - 40*sqrt(a*b)*sqrt(-b^2 - sqrt(a
*b)*b)*a^6*b^7)*abs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^5*
b - 2*a^4*b^2 + a^3*b^3 + sqrt((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*(a^5*b
 - 2*a^4*b^2 + a^3*b^3) + (a^5*b - 2*a^4*b^2 + a^3*b^3)^2))/(a^5*b - 2*a^4*
b^2 + a^3*b^3)))/((a^12*b^2 + 3*a^11*b^3 - 30*a^10*b^4 + 70*a^9*b^5 - 75*a^
8*b^6 + 39*a^7*b^7 - 8*a^6*b^8)*abs(a^5 - 2*a^4*b + a^3*b^2)) - ((a^5 - 2*a
^4*b + a^3*b^2)^2*(45*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^3 + 289*sqrt(a*b
)*sqrt(-b^2 + sqrt(a*b)*b)*a^2*b - 536*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a
*b^2 + 256*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*b^3)*abs(b) + (61*sqrt(-b^2 +
 sqrt(a*b)*b)*a^8*b + 285*sqrt(-b^2 + sqrt(a*b)*b)*a^7*b^2 - 1369*sqrt(-b^2
 + sqrt(a*b)*b)*a^6*b^3 + 1895*sqrt(-b^2 + sqrt(a*b)*b)*a^5*b^4 - 1128*sqrt
(-b^2 + sqrt(a*b)*b)*a^4*b^5 + 256*sqrt(-b^2 + sqrt(a*b)*b)*a^3*b^6)*abs(a^
5 - 2*a^4*b + a^3*b^2)*abs(b) + 2*(8*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^1
2*b + 27*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^11*b^2 - 228*sqrt(a*b)*sqrt(-
b^2 + sqrt(a*b)*b)*a^10*b^3 + 482*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^9*b^
4 - 468*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^8*b^5 + 219*sqrt(a*b)*sqrt(-b^
2 + sqrt(a*b)*b)*a^7*b^6 - 40*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a^6*b^7)*a
bs(b))*arctan(1/2*(e^(d*x + c) + e^(-d*x - c))/sqrt(-(a^5*b - 2*a^4*b^2 + a
^3*b^3 - sqrt((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*(a^5*b - 2*a^4*b^2 + a^
3*b^3) + (a^5*b - 2*a^4*b^2 + a^3*b^3)^2))/(a^5*b - 2*a^4*b^2 + a^3*b^3)))/
((a^12*b^2 + 3*a^11*b^3 - 30*a^10*b^4 + 70*a^9*b^5 - 75*a^8*b^6 + 39*a^7*b^
7 - 8*a^6*b^8)*abs(a^5 - 2*a^4*b + a^3*b^2)) - 4*(13*a*b^2*(e^(d*x + c) + e
^(-d*x - c))^7 - 7*b^3*(e^(d*x + c) + e^(-d*x - c))^7 - 212*a*b^2*(e^(d*x +
 c) + e^(-d*x - c))^5 + 116*b^3*(e^(d*x + c) + e^(-d*x - c))^5 - 272*a^2*b*
```

```
(e^(d*x + c) + e^(-d*x - c))^3 + 1248*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3
- 592*b^3*(e^(d*x + c) + e^(-d*x - c))^3 + 2240*a^2*b*(e^(d*x + c) + e^(-d*
x - c)) - 3200*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 960*b^3*(e^(d*x + c) +
e^(-d*x - c)))/((b*(e^(d*x + c) + e^(-d*x - c))^4 - 8*b*(e^(d*x + c) + e^(-
d*x - c))^2 - 16*a + 16*b)^2*(a^4 - 2*a^3*b + a^2*b^2)) - 32*log(e^(d*x + c
) + e^(-d*x - c) + 2)/a^3 + 32*log(e^(d*x + c) + e^(-d*x - c) - 2)/a^3)/d
```

maple [B] time = 0.23, size = 3159, normalized size = 5.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 71/64/d/a^2/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1
/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2
)*b+71/64/d/a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*t
anh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)
^(1/2)*b-1/2/d*b^2/a^3/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/
4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))
*(a*b)^(1/2)-1/2/d*b^2/a^3/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arcta
n(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(
1/2))*(a*b)^(1/2)+43/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a
+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)
^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^12*b^2-216/d/(tanh(1/2*d*x+
1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/
2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*
d*x+1/2*c)^8*b^3+185/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a
+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)
^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4*b^2+537/4/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^8+26/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*t
anh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a
+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*b^3+13/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^10-315/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+50/d/(tanh(1/2*d*x+1/2*c)^8*a-4*t
anh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4
-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-257
/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a
*b+b^2)*tanh(1/2*d*x+1/2*c)^4+3/4/d*b^2/a/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)-9/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)
^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*b-1/d/(tanh(1/2*d*x+1/2*c
)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x
+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2
*c)^14+5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*
d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/
(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+10/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh
(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*
tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+1/d/a^
3*ln(tanh(1/2*d*x+1/2*c))+96/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*
d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(
1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8-8/d*b^3/a^2/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
```

```
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)^12+1/4/d*b/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)
*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2
)*a)^(1/2))-327/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*ta
nh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+
a)^2*b^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^10-45/64/d/(a^2-2*a*b+b^2)/a
/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(
1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-45/64/d/(a^2-2*a*b+b^2)/a
/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^
(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1/4/d*b/(a^2-2*a*b+b^2)/
a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^
(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1161/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*
tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^
4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
6+5/32/d/a^2/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(
1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*b^2-16/d*
b^3/a^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-53/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*b^2+70/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^6*b^3-5/32/d/a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)
^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b
)^(1/2)*a)^(1/2))*b^2+5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((13*a*b^2*e^(15*c) - 7*b^3*e^(15*c))*e^(15*d*x) - (121*a*b^2*e^(13*c
) - 67*b^3*e^(13*c))*e^(13*d*x) - (272*a^2*b*e^(11*c) - 461*a*b^2*e^(11*c)
+ 159*b^3*e^(11*c))*e^(11*d*x) + (1424*a^2*b*e^(9*c) - 1121*a*b^2*e^(9*c) +
 99*b^3*e^(9*c))*e^(9*d*x) + (1424*a^2*b*e^(7*c) - 1121*a*b^2*e^(7*c) + 99*
b^3*e^(7*c))*e^(7*d*x) - (272*a^2*b*e^(5*c) - 461*a*b^2*e^(5*c) + 159*b^3*e
^(5*c))*e^(5*d*x) - (121*a*b^2*e^(3*c) - 67*b^3*e^(3*c))*e^(3*d*x) + (13*a*
b^2*e^c - 7*b^3*e^c)*e^(d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b
^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*x) - 8*(
a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^(14*d*x)
- 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^(12*c) - 7
*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^4*b^2*d*e^(
10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a
^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 166*a^3*b^3*d*
e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(6*c) - 39*a^4*
b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*e^(6*d*x) - 4*(
8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(4*c) - 7*a^2*b^4
*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) + a^2*b^
4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^3*d) + log((e^(d
*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/32*((45*a^2*b*e^(7*c) - 71*a*b
^2*e^(7*c) + 32*b^3*e^(7*c))*e^(7*d*x) - (199*a^2*b*e^(5*c) - 253*a*b^2*e^(
5*c) + 96*b^3*e^(5*c))*e^(5*d*x) + (199*a^2*b*e^(3*c) - 253*a*b^2*e^(3*c) +
 96*b^3*e^(3*c))*e^(3*d*x) - (45*a^2*b*e^c - 71*a*b^2*e^c + 32*b^3*e^c)*e^(
d*x))/(a^5*b - 2*a^4*b^2 + a^3*b^3 + (a^5*b*e^(8*c) - 2*a^4*b^2*e^(8*c) + a
^3*b^3*e^(8*c))*e^(8*d*x) - 4*(a^5*b*e^(6*c) - 2*a^4*b^2*e^(6*c) + a^3*b^3*
```

```
e^(6*c))*e^(6*d*x) - 2*(8*a^6*e^(4*c) - 19*a^5*b*e^(4*c) + 14*a^4*b^2*e^(4*
c) - 3*a^3*b^3*e^(4*c))*e^(4*d*x) - 4*(a^5*b*e^(2*c) - 2*a^4*b^2*e^(2*c) +
a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(c+d\,x\right)\,{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)*(a - b*sinh(c + d*x)^4)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.259 $\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=319

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\ \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\ \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tanh^3(c+dx)}{32abd(a-b)}+\frac{\ }{8ad}$$

```
[Out] -1/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)-5*b^(
1/2))/a^(3/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctanh((a^(1/2)+b^(1/
2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)+5*b^(1/2))/a^(3/4)/b^(3/2)/d/(a^(
1/2)+b^(1/2))^(5/2)-1/32*(a+5*b)*tanh(d*x+c)/a/(a-b)^2/b/d-1/32*tanh(d*x+c)
^3/a/(a-b)/b/d+1/8*tanh(d*x+c)^9/a/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)
^4)^2-1/32*sech(d*x+c)^2*tanh(d*x+c)^5/a/b/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tan
h(d*x+c)^4)
```

Rubi [A] time = 0.49, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3217, 1275, 12, 1120, 1279, 1166, 208}

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\ \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\ \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\mathrm{t}}{8ad\left((a-b)\tanh^4(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((2*Sqrt[a] - 5*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a
^(1/4)])/(64*a^(3/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((2*Sqrt[a] + 5
*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(
3/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - ((a + 5*b)*Tanh[c + d*x])/(32*a
*(a - b)^2*b*d) - Tanh[c + d*x]^3/(32*a*(a - b)*b*d) + Tanh[c + d*x]^9/(8*a
*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (Sech[c + d*x]^
2*Tanh[c + d*x]^5)/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*
x]^4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^8(1-x^2)}{(a-2ax^2+(a-b)x^4)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int -\frac{2bx^8}{(a-2ax^2+(a-b)x\ldots}\right.}{16a\ldots}$$

$$= \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^8}{(a-2ax^2+(a-b)x^4)\ldots}\right.}{8a\ldots}$$

$$= \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{sech}^2(c+\ldots}{32abd\left(a-2a\tanh^2(c+\ldots\right.}$$

$$= -\frac{\tanh^3(c+dx)}{32a(a-b)bd} + \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\ldots}{32ab\ldots}$$

$$= -\frac{(a+5b)\tanh(c+dx)}{32a(a-b)^2bd} - \frac{\tanh^3(c+dx)}{32a(a-b)bd} + \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\ldots\right.}$$

$$= -\frac{(a+5b)\tanh(c+dx)}{32a(a-b)^2bd} - \frac{\tanh^3(c+dx)}{32a(a-b)bd} + \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\ldots\right.}$$

$$= -\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} + \frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\ldots}{\ldots}\right.}{64a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3\ldots}}$$

Mathematica [A] time = 4.09, size = 331, normalized size = 1.04

$$\frac{\frac{\left(2a^{3/2}\sqrt{b}-8\sqrt{a}b^{3/2}+ab+5b^2\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{b}\left(2\sqrt{a}-5\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{8b\sinh(2(c+dx))((5b-\ldots}{8a+4b\cosh(2(c+d\ldots}}{64b^2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (((2*Sqrt[a] - 5*Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*Sqrt[b]*ArcTan[((Sqrt[a] -
Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqr
t[a]*Sqrt[b]]) + ((2*a^(3/2)*Sqrt[b] + a*b - 8*Sqrt[a]*b^(3/2) + 5*b^2)*Arc
Tanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[
a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + (8*b*(5*a - 14*b + (-2*a + 5*b)*Cosh[2*(c +
 d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c
 + d*x)]) + (64*a*(a - b)*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(-8
*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*(a - b)^2*b^
2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 3.35, size = 389, normalized size = 1.22

$$-\frac{ab^2e^{(14\,dx+14\,c)} - 4\,b^3e^{(14\,dx+14\,c)} - 32\,a^2be^{(12\,dx+12\,c)} + 58\,ab^2e^{(12\,dx+12\,c)} + b^3e^{(12\,dx+12\,c)} + 144\,a^2be^{(10\,dx+10\,c)} - 219}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/8*(a*b^2*e^(14*d*x + 14*c) - 4*b^3*e^(14*d*x + 14*c) - 32*a^2*b*e^(12*d*
x + 12*c) + 58*a*b^2*e^(12*d*x + 12*c) + b^3*e^(12*d*x + 12*c) + 144*a^2*b*
e^(10*d*x + 10*c) - 219*a*b^2*e^(10*d*x + 10*c) + 60*b^3*e^(10*d*x + 10*c)
+ 256*a^3*e^(8*d*x + 8*c) - 832*a^2*b*e^(8*d*x + 8*c) + 550*a*b^2*e^(8*d*x
+ 8*c) - 175*b^3*e^(8*d*x + 8*c) + 112*a^2*b*e^(6*d*x + 6*c) - 533*a*b^2*e^
(6*d*x + 6*c) + 220*b^3*e^(6*d*x + 6*c) - 32*a^2*b*e^(4*d*x + 4*c) + 158*a*
b^2*e^(4*d*x + 4*c) - 141*b^3*e^(4*d*x + 4*c) - 17*a*b^2*e^(2*d*x + 2*c) +
44*b^3*e^(2*d*x + 2*c) + 2*a*b^2 - 5*b^3)/((a^2*b^2 - 2*a*b^3 + b^4)*(b*e^(
8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x +
4*c) - 4*b*e^(2*d*x + 2*c) + b)^2*d)
```

maple [C] time = 0.15, size = 2236, normalized size = 7.01

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -1/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+5/16/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^3+49/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-9/16/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^5*a^2-165/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)
^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a+9/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^5+5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/
b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*a^2+377/16/d/(tanh(1/2*d*x+1/2*c)^8
*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/
2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7
*a-49/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9*a^2+377/16
```

```
/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)
^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^9*a-49/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/
2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*
x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-9/16/d/(tanh(1/2*
d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tan
h(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^11*a^2-165/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)
^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11*a+9/d/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*
d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^11+5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13+49/16/d/(tanh(1/2*d*x+1/2*c)
^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+
1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^13-1/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*
d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^
2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15-5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-
4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c
)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15
-1/128/d/b/(a^2-2*a*b+b^2)*sum(((a+5*b)*_R^6+(5*a-47*b)*_R^4+(-5*a+47*b)*_R
^2-a-5*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_
R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*a*b^2 - 5*b^3 + (a*b^2*e^(14*c) - 4*b^3*e^(14*c))*e^(14*d*x) - (32*
a^2*b*e^(12*c) - 58*a*b^2*e^(12*c) - b^3*e^(12*c))*e^(12*d*x) + 3*(48*a^2*b
*e^(10*c) - 73*a*b^2*e^(10*c) + 20*b^3*e^(10*c))*e^(10*d*x) + (256*a^3*e^(8
*c) - 832*a^2*b*e^(8*c) + 550*a*b^2*e^(8*c) - 175*b^3*e^(8*c))*e^(8*d*x) +
(112*a^2*b*e^(6*c) - 533*a*b^2*e^(6*c) + 220*b^3*e^(6*c))*e^(6*d*x) - (32*a
^2*b*e^(4*c) - 158*a*b^2*e^(4*c) + 141*b^3*e^(4*c))*e^(4*d*x) - (17*a*b^2*e
^(2*c) - 44*b^3*e^(2*c))*e^(2*d*x))/(a^2*b^4*d - 2*a*b^5*d + b^6*d + (a^2*b
^4*d*e^(16*c) - 2*a*b^5*d*e^(16*c) + b^6*d*e^(16*c))*e^(16*d*x) - 8*(a^2*b^
4*d*e^(14*c) - 2*a*b^5*d*e^(14*c) + b^6*d*e^(14*c))*e^(14*d*x) - 4*(8*a^3*b
^3*d*e^(12*c) - 23*a^2*b^4*d*e^(12*c) + 22*a*b^5*d*e^(12*c) - 7*b^6*d*e^(12
*c))*e^(12*d*x) + 8*(16*a^3*b^3*d*e^(10*c) - 39*a^2*b^4*d*e^(10*c) + 30*a*b
^5*d*e^(10*c) - 7*b^6*d*e^(10*c))*e^(10*d*x) + 2*(128*a^4*b^2*d*e^(8*c) - 3
52*a^3*b^3*d*e^(8*c) + 355*a^2*b^4*d*e^(8*c) - 166*a*b^5*d*e^(8*c) + 35*b^6
*d*e^(8*c))*e^(8*d*x) + 8*(16*a^3*b^3*d*e^(6*c) - 39*a^2*b^4*d*e^(6*c) + 30
*a*b^5*d*e^(6*c) - 7*b^6*d*e^(6*c))*e^(6*d*x) - 4*(8*a^3*b^3*d*e^(4*c) - 23
*a^2*b^4*d*e^(4*c) + 22*a*b^5*d*e^(4*c) - 7*b^6*d*e^(4*c))*e^(4*d*x) - 8*(a
^2*b^4*d*e^(2*c) - 2*a*b^5*d*e^(2*c) + b^6*d*e^(2*c))*e^(2*d*x)) - 1/256*in
tegrate(64*((a*e^(6*c) - 4*b*e^(6*c))*e^(6*d*x) + (a*e^(2*c) - 4*b*e^(2*c))
*e^(2*d*x) + 18*b*e^(4*d*x + 4*c))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2*e^(8
*c) - 2*a*b^3*e^(8*c) + b^4*e^(8*c))*e^(8*d*x) - 4*(a^2*b^2*e^(6*c) - 2*a*b
^3*e^(6*c) + b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^3*b*e^(4*c) - 19*a^2*b^2*e^(4*
c) + 14*a*b^3*e^(4*c) - 3*b^4*e^(4*c))*e^(4*d*x) - 4*(a^2*b^2*e^(2*c) - 2*a
*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh\left(c+d\,x\right)^8}{{\left(a-b\,\sinh\left(c+d\,x\right)^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^8/(a - b*sinh(c + d*x)^4)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**8/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.260 $\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=345

$$\frac{\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tanh}{32ab}$$

```
[Out] 1/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(4*a+3*b-10*a^(1/
2)*b^(1/2))/a^(5/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(5/2)-1/64*arctanh((a^(1/2)
+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(4*a+3*b+10*a^(1/2)*b^(1/2))/a^(5/4)/b
^(3/2)/d/(a^(1/2)+b^(1/2))^(5/2)+1/8*tanh(d*x+c)*(a*(a+3*b)-(a^2+6*a*b+b^2)
*tanh(d*x+c)^2)/(a-b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2+1/32*
tanh(d*x+c)*(2*a*(a^2-a*b-8*b^2)/(a-b)^3-(2*a^2+15*a*b+3*b^2)*tanh(d*x+c)^2
/(a-b)^2)/a/b/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A] time = 0.73, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 208}

$$\frac{\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tanh}{32ab}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
 d*x])/a^(1/4)])/(64*a^(5/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) - ((4*a +
 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/
a^(1/4)])/(64*a^(5/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) + (Tanh[c + d*x]
*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2
*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((2*a*(a^
2 - a*b - 8*b^2))/(a - b)^3 - ((2*a^2 + 15*a*b + 3*b^2)*Tanh[c + d*x]^2)/(a
 - b)^2))/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(1-x^2\right)^2}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^3b(a+3b)}{(a-b)^3}+\frac{2a}{}}{}\right.}{}$$

$$= \frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\tanh(c+dx)\left(\frac{2a\left(a^2-\right.}{(a-}\right.}{32abd\left(a-2a\tanh^2\right.}$$

$$= \frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\tanh(c+dx)\left(\frac{2a\left(a^2-\right.}{(a-}\right.}{32abd\left(a-2a\tanh^2\right.}$$

$$= \frac{\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} - \frac{\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}}{64a^{5/4}\left(\sqrt{a}+\sqrt{b}\right.}$$

Mathematica [A] time = 3.37, size = 351, normalized size = 1.02

$$-\frac{\frac{4\sinh(2(c+dx))\left(4a^2+3b(a+b)\cosh(2(c+dx))-19ab-3b^2\right)}{ab(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b)}+\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{ab^{3/2}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-10\sqrt{\right.}{}}{}}{64d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -1/64*(((Sqrt[a] + Sqrt[b])^2*(4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((Sqr
t[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a*Sqrt[-a + Sq
rt[a]*Sqrt[b]]*b^(3/2)) + ((Sqrt[a] - Sqrt[b])^2*(4*a + 10*Sqrt[a]*Sqrt[b]
+ 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]
]])/(a*Sqrt[a + Sqrt[a]*Sqrt[b]]*b^(3/2)) + (4*(4*a^2 - 19*a*b - 3*b^2 + 3*
b*(a + b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a*b*(8*a - 3*b + 4*b*Cosh[
2*(c + d*x)] - b*Cosh[4*(c + d*x)])) - (128*(a - b)*(2*a + b - b*Cosh[2*(c
+ d*x)])*Sinh[2*(c + d*x)])/(b*(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh
[4*(c + d*x)])^2))/((a - b)^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 2.54, size = 451, normalized size = 1.31

$$\frac{4\,a^2be^{(14\,dx+14\,c)}-13\,ab^2e^{(14\,dx+14\,c)}+3\,b^3e^{(14\,dx+14\,c)}-24\,a^2be^{(12\,dx+12\,c)}+99\,ab^2e^{(12\,dx+12\,c)}-21\,b^3e^{(12\,dx+12\,c)}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/16*(4*a^2*b*e^(14*d*x + 14*c) - 13*a*b^2*e^(14*d*x + 14*c) + 3*b^3*e^(14*
d*x + 14*c) - 24*a^2*b*e^(12*d*x + 12*c) + 99*a*b^2*e^(12*d*x + 12*c) - 21*
b^3*e^(12*d*x + 12*c) + 64*a^3*e^(10*d*x + 10*c) + 68*a^2*b*e^(10*d*x + 10*
c) - 225*a*b^2*e^(10*d*x + 10*c) + 63*b^3*e^(10*d*x + 10*c) - 384*a^3*e^(8*
d*x + 8*c) - 96*a^2*b*e^(8*d*x + 8*c) + 183*a*b^2*e^(8*d*x + 8*c) - 105*b^3
*e^(8*d*x + 8*c) - 64*a^3*e^(6*d*x + 6*c) - 452*a^2*b*e^(6*d*x + 6*c) + 9*a
*b^2*e^(6*d*x + 6*c) + 105*b^3*e^(6*d*x + 6*c) + 120*a^2*b*e^(4*d*x + 4*c)
- 87*a*b^2*e^(4*d*x + 4*c) - 63*b^3*e^(4*d*x + 4*c) - 4*a^2*b*e^(2*d*x + 2*
c) + 37*a*b^2*e^(2*d*x + 2*c) + 21*b^3*e^(2*d*x + 2*c) - 3*a*b^2 - 3*b^3)/(
(a^3*b - 2*a^2*b^2 + a*b^3)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a
*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2*d)
```

maple [C] time = 0.16, size = 2681, normalized size = 7.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2
```

```
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*a^2-5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9*a^2+9/8/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+
b^2)*tanh(1/2*d*x+1/2*c)^11*a^2-5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13+1/8/d/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b
+b^2)*tanh(1/2*d*x+1/2*c)^15+1/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+
1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*
d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-5/8/d/(tanh(1
/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*
tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)^3+9/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a^2+12/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^7+12/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*t
anh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a
+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-1/64/d/a/b/(a^2-2*a*b+b^2
)*sum((a*(-a-2*b)*_R^6+(-5*a^2+32*a*b-6*b^2)*_R^4+(5*a^2-32*a*b+6*b^2)*_R^2
+a^2+2*a*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)
-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))+1/4/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^13+1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/4/d/(tanh(1/2*d*x+1/2*c)^
8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1
/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c
)-3/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+19/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a-35/4/d/(tanh
(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*
b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^5-27/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*a+41/2/d/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*
d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^7-27/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2
/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9*a+41/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*
tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^
4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+19
/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b
+b^2)*tanh(1/2*d*x+1/2*c)^11*a-35/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11-3/d/(tanh(1
/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*
tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh
(1/2*d*x+1/2*c)^13+1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a
+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)
```

```
^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*a*b^2 + 3*b^3 - (4*a^2*b*e^(14*c) - 13*a*b^2*e^(14*c) + 3*b^3*e^(1
4*c))*e^(14*d*x) + 3*(8*a^2*b*e^(12*c) - 33*a*b^2*e^(12*c) + 7*b^3*e^(12*c)
)*e^(12*d*x) - (64*a^3*e^(10*c) + 68*a^2*b*e^(10*c) - 225*a*b^2*e^(10*c) +
63*b^3*e^(10*c))*e^(10*d*x) + 3*(128*a^3*e^(8*c) + 32*a^2*b*e^(8*c) - 61*a*
b^2*e^(8*c) + 35*b^3*e^(8*c))*e^(8*d*x) + (64*a^3*e^(6*c) + 452*a^2*b*e^(6*
c) - 9*a*b^2*e^(6*c) - 105*b^3*e^(6*c))*e^(6*d*x) - 3*(40*a^2*b*e^(4*c) - 2
9*a*b^2*e^(4*c) - 21*b^3*e^(4*c))*e^(4*d*x) + (4*a^2*b*e^(2*c) - 37*a*b^2*e
^(2*c) - 21*b^3*e^(2*c))*e^(2*d*x))/(a^3*b^3*d - 2*a^2*b^4*d + a*b^5*d + (a
^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*c) + a*b^5*d*e^(16*c))*e^(16*d*x) - 8
*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(14*c) + a*b^5*d*e^(14*c))*e^(14*d*x)
- 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3*d*e^(12*c) + 22*a^2*b^4*d*e^(12*c) -
 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b^2*d*e^(10*c) - 39*a^3*b^3*d*e
^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*b^5*d*e^(10*c))*e^(10*d*x) + 2*(128*a
^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c) + 355*a^3*b^3*d*e^(8*c) - 166*a^2*b^
4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8*d*x) + 8*(16*a^4*b^2*d*e^(6*c) - 39*
a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*c) - 7*a*b^5*d*e^(6*c))*e^(6*d*x) - 4
*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e^(4*c) + 22*a^2*b^4*d*e^(4*c) - 7*a*b
^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d*e^(2*c) - 2*a^2*b^4*d*e^(2*c) + a*b^
5*d*e^(2*c))*e^(2*d*x)) + 1/64*integrate(8*((4*a^2*e^(6*c) - 13*a*b*e^(6*c)
 + 3*b^2*e^(6*c))*e^(6*d*x) + 6*(7*a*b*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + (
4*a^2*e^(2*c) - 13*a*b*e^(2*c) + 3*b^2*e^(2*c))*e^(2*d*x))/(a^3*b^2 - 2*a^2
*b^3 + a*b^4 + (a^3*b^2*e^(8*c) - 2*a^2*b^3*e^(8*c) + a*b^4*e^(8*c))*e^(8*d
*x) - 4*(a^3*b^2*e^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c))*e^(6*d*x) - 2
*(8*a^4*b*e^(4*c) - 19*a^3*b^2*e^(4*c) + 14*a^2*b^3*e^(4*c) - 3*a*b^4*e^(4*
c))*e^(4*d*x) - 4*(a^3*b^2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c))*e^(
2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^6}{\left(a-b\sinh(c+d\,x)^4\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^6/(a - b*sinh(c + d*x)^4)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.261 $\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=314

$$\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tanh(c+dx)\left(\frac{9a^2-24a\ldots}{(a-b\ldots}\right.}{32ad\left((a-b)\tanh^4(c+\ldots\right.}$$

[Out] 3/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)-b^(1/2))/a^(7/4)/d/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)-3/64*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(2*a^(1/2)+b^(1/2))/a^(7/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(5/2)-1/8*b*tanh(d*x+c)*(3*a+b-4*(a+b)*tanh(d*x+c)^2)/(a-b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2-1/32*tanh(d*x+c)*((9*a^2-24*a*b-b^2)/(a-b)^3-(17*a+3*b)*tanh(d*x+c)^2/(a-b)^2)/a/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)

Rubi [A] time = 0.65, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 208}

$$-\frac{\tanh(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}-\frac{(17a+3b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32ad\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(2\sqrt{a}+\sqrt{b}\ldots\right.}{64a^{7}\ldots}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] (3*(2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(7/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - (3*(2*Sqrt[a] + Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(7/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tanh[c + d*x]*(3*a + b - 4*(a + b)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (Tanh[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 - ((17*a + 3*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
```

```
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(1-x^2\right)^3}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^2b^2(\cdots}{(a-b\cdots}}{\cdots}\right.}{\cdots}$$

$$= -\frac{b\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\tanh(c+dx\cdots}{32ad\left(a-2a\tan\cdots\right.}$$

$$= -\frac{b\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\tanh(c+dx\cdots}{32ad\left(a-2a\tan\cdots\right.}$$

$$= \frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d} - \frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\cdots}{\sqrt[4]{\cdots}}\right.}{64a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b\cdots}}$$

Mathematica [A] time = 4.88, size = 316, normalized size = 1.01

$$\frac{-\frac{3\left(2a^{3/2}-3a\sqrt{b}+b^{3/2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{a^{3/2}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{3\left(2a^{3/2}+3a\sqrt{b}-b^{3/2}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{a^{3/2}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}}+\frac{8\sinh(2(c+dx))((2a+b)\cosh(2(c+dx))-7}{a(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}}{64d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-3*(2*a^(3/2) + 3*a*Sqrt[b] - b^(3/2))*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c
 + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[-a + Sqrt[a]*Sqrt[b]]*S
qrt[b]) - (3*(2*a^(3/2) - 3*a*Sqrt[b] + b^(3/2))*ArcTanh[((Sqrt[a] + Sqrt[b
])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[a + Sqrt[a]*Sqr
t[b]]*Sqrt[b]) + (8*(-7*a - 2*b + (2*a + b)*Cosh[2*(c + d*x)])*Sinh[2*(c +
d*x)])/(a*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (64*
(a - b)*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[
2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*(a - b)^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.71, size = 362, normalized size = 1.15

$$3\,ab^2e^{(14\,dx+14\,c)}-30\,ab^2e^{(12\,dx+12\,c)}+3\,b^3e^{(12\,dx+12\,c)}-80\,a^2be^{(10\,dx+10\,c)}+111\,ab^2e^{(10\,dx+10\,c)}-16\,b^3e^{(10\,dx+10\,c)}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*a*b^2*e^(14*d*x + 14*c) - 30*a*b^2*e^(12*d*x + 12*c) + 3*b^3*e^(12*d
*x + 12*c) - 80*a^2*b*e^(10*d*x + 10*c) + 111*a*b^2*e^(10*d*x + 10*c) - 16*
b^3*e^(10*d*x + 10*c) + 256*a^3*e^(8*d*x + 8*c) - 64*a^2*b*e^(8*d*x + 8*c)
- 26*a*b^2*e^(8*d*x + 8*c) + 35*b^3*e^(8*d*x + 8*c) + 336*a^2*b*e^(6*d*x +
6*c) - 95*a*b^2*e^(6*d*x + 6*c) - 40*b^3*e^(6*d*x + 6*c) - 64*a^2*b*e^(4*d*
x + 4*c) + 54*a*b^2*e^(4*d*x + 4*c) + 25*b^3*e^(4*d*x + 4*c) - 19*a*b^2*e^(
2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + b^3)/((a^3*b - 2*a^2*b^2 +
 a*b^3)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6
*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2*d)
```

maple [C] time = 0.17, size = 2214, normalized size = 7.05

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -9/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+3/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
```

```
/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+77/16/d/(tanh(
1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b
*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^3-23/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-177/16/d/(tanh(1/2*d*x+
1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/
2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^5*a+131/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*ta
nh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+
a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/d/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)
^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^5*b
^2+109/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d
*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*a-367/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-9/d/
(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*
a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^7+109/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9*a-367/16/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)^9-9/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*
a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c
)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-177/16/d/(tanh(1/2*d
*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh
(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^11*a+131/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+
6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^
2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+1/d/(tanh(1/2*d*x+1/2*c)^
8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1
/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c
)^11*b^2+77/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-23/16/d/(tanh(1/2*d*x+1/2*c)^8*a
-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*
c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^1
3-9/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+
1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15+3/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15*b-3/128/d
/(a^2-2*a*b+b^2)/a*sum(((3*a-b)*_R^6+(-17*a+3*b)*_R^4+(17*a-3*b)*_R^2-3*a+b
)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=Ro
otOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*a*b^2*e^(14*d*x + 14*c) + 2*a*b^2 + b^3 - 3*(10*a*b^2*e^(12*c) - b^3
*e^(12*c))*e^(12*d*x) - (80*a^2*b*e^(10*c) - 111*a*b^2*e^(10*c) + 16*b^3*e^
(10*c))*e^(10*d*x) + (256*a^3*e^(8*c) - 64*a^2*b*e^(8*c) - 26*a*b^2*e^(8*c)
 + 35*b^3*e^(8*c))*e^(8*d*x) + (336*a^2*b*e^(6*c) - 95*a*b^2*e^(6*c) - 40*b
```

```
^3*e^(6*c))*e^(6*d*x) - (64*a^2*b*e^(4*c) - 54*a*b^2*e^(4*c) - 25*b^3*e^(4*
c))*e^(4*d*x) - (19*a*b^2*e^(2*c) + 8*b^3*e^(2*c))*e^(2*d*x))/(a^3*b^3*d -
2*a^2*b^4*d + a*b^5*d + (a^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*c) + a*b^5*
d*e^(16*c))*e^(16*d*x) - 8*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(14*c) + a*b
^5*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3*d*e^(12*c)
 + 22*a^2*b^4*d*e^(12*c) - 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b^2*d
*e^(10*c) - 39*a^3*b^3*d*e^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*b^5*d*e^(10
*c))*e^(10*d*x) + 2*(128*a^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c) + 355*a^3*
b^3*d*e^(8*c) - 166*a^2*b^4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8*d*x) + 8*(
16*a^4*b^2*d*e^(6*c) - 39*a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*c) - 7*a*b^
5*d*e^(6*c))*e^(6*d*x) - 4*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e^(4*c) + 22
*a^2*b^4*d*e^(4*c) - 7*a*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d*e^(2*c) -
2*a^2*b^4*d*e^(2*c) + a*b^5*d*e^(2*c))*e^(2*d*x)) + 1/16*integrate(-12*(2*(
4*a*e^(4*c) - b*e^(4*c))*e^(4*d*x) - a*e^(6*d*x + 6*c) - a*e^(2*d*x + 2*c))
/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^(8*c) - 2*a^2*b^2*e^(8*c) + a*b^3*e^
(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c) - 2*a^2*b^2*e^(6*c) + a*b^3*e^(6*c))*e^
(6*d*x) - 2*(8*a^4*e^(4*c) - 19*a^3*b*e^(4*c) + 14*a^2*b^2*e^(4*c) - 3*a*b^
3*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2*c) - 2*a^2*b^2*e^(2*c) + a*b^3*e^(2*c)
)*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^4}{\left(a-b\sinh(c+d\,x)^4\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^4/(a - b*sinh(c + d*x)^4)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.262 $\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=348

$$-\frac{\left(-14\sqrt{a}\sqrt{b}+12a+5b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(14\sqrt{a}\sqrt{b}+12a+5b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\cdots$$

```
[Out] -1/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(12*a+5*b-14*a^(
1/2)*b^(1/2))/a^(9/4)/d/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)+1/64*arctanh((a^(1/
2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(12*a+5*b+14*a^(1/2)*b^(1/2))/a^(9/4
)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(5/2)+1/8*b*tanh(d*x+c)*(a*(a+3*b)-(a^2+6*a*b
+b^2)*tanh(d*x+c)^2)/a/(a-b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^
2+1/32*tanh(d*x+c)*(2*a*(5*a^2-9*a*b-4*b^2)/(a-b)^3-5*(2*a^2+3*a*b-b^2)*tan
h(d*x+c)^2/(a-b)^2)/a^2/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A] time = 0.66, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 208}

$$\frac{\tanh(c+dx)\left(\frac{2a\left(5a^2-9ab-4b^2\right)}{(a-b)^3}-\frac{5\left(2a^2+3ab-b^2\right)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+\cdots\right.}{8ad(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c
 + d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) + ((12*
a + 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x
])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) + (b*Tanh[c +
 d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d
*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((
2*a*(5*a^2 - 9*a*b - 4*b^2))/(a - b)^3 - (5*(2*a^2 + 3*a*b - b^2)*Tanh[c +
d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d
*x]^4))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
```

```
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1-x^2\right)^4}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^2b^2(a+3b)}{(a-b)^3}\ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\tanh(c+dx)\left(\frac{2a(5a\ldots}{\ldots}\right.}{32a^2d\left(a-2a\tanh\ldots\right.}$$

$$= \frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\tanh(c+dx)\left(\frac{2a(5a\ldots}{\ldots}\right.}{32a^2d\left(a-2a\tanh\ldots\right.}$$

$$= -\frac{\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d} + \frac{\left(12a+14\sqrt{a}\sqrt{b}+5b\right)\tan\ldots}{64a^{9/4}\left(\sqrt{a}+\ldots\right.}$$

Mathematica [A] time = 5.00, size = 343, normalized size = 0.99

$$\frac{\frac{4\sinh(2(c+dx))\left(12a^2+b(5b-11a)\cosh(2(c+dx))+11ab-5b^2\right)}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}+\frac{\left(14\sqrt{a}\sqrt{b}+12a+5b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-1\right.}{}}{64a^2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (((Sqrt[a] + Sqrt[b])^2*(12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[((Sqrt[a]
- Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*S
qrt[b]]*Sqrt[b]) + ((Sqrt[a] - Sqrt[b])^2*(12*a + 14*Sqrt[a]*Sqrt[b] + 5*b)
*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(S
qrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (4*(12*a^2 + 11*a*b - 5*b^2 + b*(-11*a
+ 5*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d
*x)] - b*Cosh[4*(c + d*x)]) + (128*a*(a - b)*(2*a + b - b*Cosh[2*(c + d*x)]
)*Sinh[2*(c + d*x)])/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*
x)])^2)/(64*a^2*(a - b)^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 1.00, size = 449, normalized size = 1.29

$$-\frac{12\,a^2be^{(14\,dx+14\,c)}-11\,ab^2e^{(14\,dx+14\,c)}+5\,b^3e^{(14\,dx+14\,c)}-104\,a^2be^{(12\,dx+12\,c)}+85\,ab^2e^{(12\,dx+12\,c)}-35\,b^3e^{(12\,dx+12}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(12*a^2*b*e^(14*d*x + 14*c) - 11*a*b^2*e^(14*d*x + 14*c) + 5*b^3*e^(1
4*d*x + 14*c) - 104*a^2*b*e^(12*d*x + 12*c) + 85*a*b^2*e^(12*d*x + 12*c) -
35*b^3*e^(12*d*x + 12*c) - 320*a^3*e^(10*d*x + 10*c) + 652*a^2*b*e^(10*d*x
+ 10*c) - 407*a*b^2*e^(10*d*x + 10*c) + 105*b^3*e^(10*d*x + 10*c) + 1408*a^
3*e^(8*d*x + 8*c) - 1696*a^2*b*e^(8*d*x + 8*c) + 865*a*b^2*e^(8*d*x + 8*c)
- 175*b^3*e^(8*d*x + 8*c) + 320*a^3*e^(6*d*x + 6*c) + 756*a^2*b*e^(6*d*x +
6*c) - 849*a*b^2*e^(6*d*x + 6*c) + 175*b^3*e^(6*d*x + 6*c) - 248*a^2*b*e^(4
*d*x + 4*c) + 383*a*b^2*e^(4*d*x + 4*c) - 105*b^3*e^(4*d*x + 4*c) - 12*a^2*
b*e^(2*d*x + 2*c) - 77*a*b^2*e^(2*d*x + 2*c) + 35*b^3*e^(2*d*x + 2*c) + 11*
a*b^2 - 5*b^3)/((a^4 - 2*a^3*b + a^2*b^2)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x
 + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c)
+ b)^2*d)
```

maple [C] time = 0.19, size = 2670, normalized size = 7.67

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x)
```

[Out] -20/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*b^3+9/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(
1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*t
anh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b^2-20/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^9*b^3+9/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*
d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(
1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13*b^2-27/4/d
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2
)/a*tanh(1/2*d*x+1/2*c)^11*b^2-27/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^5*b^2+97/2/d/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b
+b^2)*tanh(1/2*d*x+1/2*c)^7+97/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+
1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*
d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-5/2/d/(tanh
(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*
b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^13-5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-1/4/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2
*c)*b-1/64/d/a^2/(a^2-2*a*b+b^2)*sum((a*(-5*a+2*b)*_R^6+(39*a^2-28*a*b+10*b
^2)*_R^4+(-39*a^2+28*a*b-10*b^2)*_R^2+5*a^2-2*a*b)/(_R^7*a-3*_R^5*a+3*_R^3*
a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-
16*b)*_Z^4-4*a*_Z^2+a))+5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c
)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1
/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-25/8/d/(tanh(1/2*d*x+1
/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2
*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x
+1/2*c)^3+45/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a+3/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-25/8
/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)
^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^7*a-1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-25/8/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2
*d*x+1/2*c)^9*a-1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+45/8/d/(tanh(1/2*d*x+1/2*c)^
8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1
/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
11*a+3/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+
1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11-25/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13+5/8/d/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2

```
)*tanh(1/2*d*x+1/2*c)^15-1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*
c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+
1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(11*a*b^2 - 5*b^3 + (12*a^2*b*e^(14*c) - 11*a*b^2*e^(14*c) + 5*b^3*e^
(14*c))*e^(14*d*x) - (104*a^2*b*e^(12*c) - 85*a*b^2*e^(12*c) + 35*b^3*e^(12
*c))*e^(12*d*x) - (320*a^3*e^(10*c) - 652*a^2*b*e^(10*c) + 407*a*b^2*e^(10*
c) - 105*b^3*e^(10*c))*e^(10*d*x) + (1408*a^3*e^(8*c) - 1696*a^2*b*e^(8*c)
+ 865*a*b^2*e^(8*c) - 175*b^3*e^(8*c))*e^(8*d*x) + (320*a^3*e^(6*c) + 756*a
^2*b*e^(6*c) - 849*a*b^2*e^(6*c) + 175*b^3*e^(6*c))*e^(6*d*x) - (248*a^2*b*
e^(4*c) - 383*a*b^2*e^(4*c) + 105*b^3*e^(4*c))*e^(4*d*x) - (12*a^2*b*e^(2*c
) + 77*a*b^2*e^(2*c) - 35*b^3*e^(2*c))*e^(2*d*x))/(a^4*b^2*d - 2*a^3*b^3*d
+ a^2*b^4*d + (a^4*b^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*
c))*e^(16*d*x) - 8*(a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e
^(14*c))*e^(14*d*x) - 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^
3*b^3*d*e^(12*c) - 7*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c
) - 39*a^4*b^2*d*e^(10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e
^(10*d*x) + 2*(128*a^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8
*c) - 166*a^3*b^3*d*e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b
*d*e^(6*c) - 39*a^4*b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6
*c))*e^(6*d*x) - 4*(8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d
*e^(4*c) - 7*a^2*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^
3*d*e^(2*c) + a^2*b^4*d*e^(2*c))*e^(2*d*x)) - 1/4*integrate(1/2*((12*a^2*e^
(6*c) - 11*a*b*e^(6*c) + 5*b^2*e^(6*c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 19*
a*b*e^(4*c) + 5*b^2*e^(4*c))*e^(4*d*x) + (12*a^2*e^(2*c) - 11*a*b*e^(2*c) +
 5*b^2*e^(2*c))*e^(2*d*x))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^4*b*e^(8*c) -
2*a^3*b^2*e^(8*c) + a^2*b^3*e^(8*c))*e^(8*d*x) - 4*(a^4*b*e^(6*c) - 2*a^3*b
^2*e^(6*c) + a^2*b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^5*e^(4*c) - 19*a^4*b*e^(4*
c) + 14*a^3*b^2*e^(4*c) - 3*a^2*b^3*e^(4*c))*e^(4*d*x) - 4*(a^4*b*e^(2*c) -
 2*a^3*b^2*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c+d\,x)^2}{\left(a-b\sinh(c+d\,x)^4\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(sinh(c + d*x)^2/(a - b*sinh(c + d*x)^4)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.263 $\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=320

$$\frac{\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{b}{32}$$

```
[Out] 1/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(32*a+21*b-50*a^(
1/2)*b^(1/2))/a^(11/4)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctanh((a^(1/2)+b^(1
/2))^(1/2)*tanh(d*x+c)/a^(1/4))*(32*a+21*b+50*a^(1/2)*b^(1/2))/a^(11/4)/d/(
a^(1/2)+b^(1/2))^(5/2)-1/8*b^2*tanh(d*x+c)*(3*a+b-4*(a+b)*tanh(d*x+c)^2)/a/
(a-b)^3/d/(a-2*a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)^2-1/32*b*tanh(d*x+c)*((
17*a^2-40*a*b+7*b^2)/(a-b)^3-(33*a-13*b)*tanh(d*x+c)^2/(a-b)^2)/a^2/d/(a-2*
a*tanh(d*x+c)^2+(a-b)*tanh(d*x+c)^4)
```

Rubi [A] time = 0.61, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3209, 1205, 1678, 1166, 208}

$$-\frac{b\tanh(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}-\frac{(33a-13b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[c + d*x]^4)^(-3),x]
```

```
[Out] ((32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c
 + d*x])/a^(1/4)])/(64*a^(11/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + ((32*a + 50*
Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(
1/4)])/(64*a^(11/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b^2*Tanh[c + d*x]*(3*a
+ b - 4*(a + b)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2
 + (a - b)*Tanh[c + d*x]^4)^2) - (b*Tanh[c + d*x]*((17*a^2 - 40*a*b + 7*b^2
)/(a - b)^3 - ((33*a - 13*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*
a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^5}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=-\frac{b^2\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}-\frac{\text{Subst}\left(\int -\frac{2ab(8}{}\right.}{}$$

$$=-\frac{b^2\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}-\frac{b\tanh(c+dx}{32a^2d\left(a-2a\right.}$$

$$=-\frac{b^2\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}-\frac{b\tanh(c+dx}{32a^2d\left(a-2a\right.}$$

$$=\frac{\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}+\frac{\left(32a+50\sqrt{a}\sqrt{b}+21b\right.}{64a^{11/4}\left(\right.}$$

Mathematica [A] time = 3.04, size = 333, normalized size = 1.04

$$\frac{\frac{64a^{3/2}b(a-b)(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{(-8a-4b\cosh(2(c+dx))+b\cosh(4(c+dx))+3b)^2}+\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-50\sqrt{a}\sqrt{b}+32a\right.}{\sqrt{}}}{64a^{5/2}d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-3),x]
```

```
[Out] (-(((Sqrt[a] + Sqrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[((Sqrt[
a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]
*Sqrt[b]]) + ((Sqrt[a] - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcT
anh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a
+ Sqrt[a]*Sqrt[b]] + (8*Sqrt[a]*b*(-19*a + 10*b + (6*a - 3*b)*Cosh[2*(c + d
*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c +
 d*x)]) + (64*a^(3/2)*(a - b)*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))
/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*a^(5/2)*
(a - b)^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.28, size = 391, normalized size = 1.22

$$\frac{7\,ab^2e^{(14\,dx+14\,c)} - 4\,b^3e^{(14\,dx+14\,c)} - 32\,a^2be^{(12\,dx+12\,c)} - 2\,ab^2e^{(12\,dx+12\,c)} + 7\,b^3e^{(12\,dx+12\,c)} - 16\,a^2be^{(10\,dx+10\,c)} + 3\,ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/8*(7*a*b^2*e^(14*d*x + 14*c) - 4*b^3*e^(14*d*x + 14*c) - 32*a^2*b*e^(12*d
*x + 12*c) - 2*a*b^2*e^(12*d*x + 12*c) + 7*b^3*e^(12*d*x + 12*c) - 16*a^2*b
*e^(10*d*x + 10*c) + 3*a*b^2*e^(10*d*x + 10*c) + 28*b^3*e^(10*d*x + 10*c) +
 768*a^3*e^(8*d*x + 8*c) - 960*a^2*b*e^(8*d*x + 8*c) + 498*a*b^2*e^(8*d*x +
 8*c) - 105*b^3*e^(8*d*x + 8*c) + 784*a^2*b*e^(6*d*x + 6*c) - 723*a*b^2*e^(
6*d*x + 6*c) + 140*b^3*e^(6*d*x + 6*c) - 160*a^2*b*e^(4*d*x + 4*c) + 266*a*
b^2*e^(4*d*x + 4*c) - 91*b^3*e^(4*d*x + 4*c) - 55*a*b^2*e^(2*d*x + 2*c) + 2
8*b^3*e^(2*d*x + 2*c) + 6*a*b^2 - 3*b^3)/((a^4 - 2*a^3*b + a^2*b^2)*(b*e^(8
*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4
*c) - 4*b*e^(2*d*x + 2*c) + b)^2*d)
```

maple [C] time = 0.19, size = 2290, normalized size = 7.16

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -17/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+
1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+11/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+149/16/d
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^3-95/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b^2-345/16/d/(ta
```

```
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)^5+427/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*
c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+
1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^5*b^2-7/d/(tanh(1/2*d
*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh
(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^5+213/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)
^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1111/16/d/(tanh(1/2*d
*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh
(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh
(1/2*d*x+1/2*c)^7+31/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*b^3+213/16/d/(tanh(1/2*d*
x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(
1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*
d*x+1/2*c)^9-1111/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+31/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^9*b^3-345/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+427/16/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/
2*c)^11*b^2-7/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2
/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+149/16/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^13-95/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2
/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13*b^2-17/16/d/(tanh(1/2*d*x+1/2*c)^
8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1
/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
15*b+11/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*
d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^
2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15-1/128/d/(a^2-2*a*b+b^2)/a^2*sum(
((32*a^2-47*a*b+21*b^2)*_R^6+(-96*a^2+85*a*b-31*b^2)*_R^4+(96*a^2-85*a*b+31
*b^2)*_R^2-32*a^2+47*a*b-21*b^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*l
n(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^
2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(6*a*b^2 - 3*b^3 + (7*a*b^2*e^(14*c) - 4*b^3*e^(14*c))*e^(14*d*x) - (32
*a^2*b*e^(12*c) + 2*a*b^2*e^(12*c) - 7*b^3*e^(12*c))*e^(12*d*x) - (16*a^2*b
*e^(10*c) - 3*a*b^2*e^(10*c) - 28*b^3*e^(10*c))*e^(10*d*x) + 3*(256*a^3*e^(
8*c) - 320*a^2*b*e^(8*c) + 166*a*b^2*e^(8*c) - 35*b^3*e^(8*c))*e^(8*d*x) +
(784*a^2*b*e^(6*c) - 723*a*b^2*e^(6*c) + 140*b^3*e^(6*c))*e^(6*d*x) - (160*
a^2*b*e^(4*c) - 266*a*b^2*e^(4*c) + 91*b^3*e^(4*c))*e^(4*d*x) - (55*a*b^2*e
^(2*c) - 28*b^3*e^(2*c))*e^(2*d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d +
(a^4*b^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*x)
```

```
 - 8*(a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^(14
*d*x) - 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^(12*
c) - 7*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^4*b^2
*d*e^(10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x) + 2*
(128*a^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 166*a^3*
b^3*d*e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(6*c) - 3
9*a^4*b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*e^(6*d*x)
 - 4*(8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(4*c) - 7*a
^2*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) +
a^2*b^4*d*e^(2*c))*e^(2*d*x)) + integrate(1/4*((7*a*b*e^(6*c) - 4*b^2*e^(6*
c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 40*a*b*e^(4*c) + 17*b^2*e^(4*c))*e^(4*d
*x) + (7*a*b*e^(2*c) - 4*b^2*e^(2*c))*e^(2*d*x))/(a^4*b - 2*a^3*b^2 + a^2*b
^3 + (a^4*b*e^(8*c) - 2*a^3*b^2*e^(8*c) + a^2*b^3*e^(8*c))*e^(8*d*x) - 4*(a
^4*b*e^(6*c) - 2*a^3*b^2*e^(6*c) + a^2*b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^5*e^
(4*c) - 19*a^4*b*e^(4*c) + 14*a^3*b^2*e^(4*c) - 3*a^2*b^3*e^(4*c))*e^(4*d*x
) - 4*(a^4*b*e^(2*c) - 2*a^3*b^2*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a - b \sinh\left(c + d\,x\right)^4\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sinh(c + d*x)^4)^3,x)
```

```
[Out] int(1/(a - b*sinh(c + d*x)^4)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.264 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=359

$$-\frac{3\sqrt{b}\left(-34\sqrt{a}\sqrt{b}+20a+15b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{3\sqrt{b}\left(34\sqrt{a}\sqrt{b}+20a+15b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\cdots}{\sqrt[4]{\cdots}}\right.}{64a^{13/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}$$

```
[Out] -coth(d*x+c)/a^3/d-3/64*arctanh((a^(1/2)-b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4)
)*b^(1/2)*(20*a+15*b-34*a^(1/2)*b^(1/2))/a^(13/4)/d/(a^(1/2)-b^(1/2))^(5/2)
+3/64*arctanh((a^(1/2)+b^(1/2))^(1/2)*tanh(d*x+c)/a^(1/4))*b^(1/2)*(20*a+15
*b+34*a^(1/2)*b^(1/2))/a^(13/4)/d/(a^(1/2)+b^(1/2))^(5/2)+1/8*b^2*tanh(d*x+
c)*(a*(a+3*b)-(a^2+6*a*b+b^2)*tanh(d*x+c)^2)/a^2/(a-b)^3/d/(a-2*a*tanh(d*x+
c)^2+(a-b)*tanh(d*x+c)^4)^2+1/32*b*tanh(d*x+c)*(2*a^2*(9*a-17*b)/(a-b)^3-(1
8*a^2+15*a*b-13*b^2)*tanh(d*x+c)^2/(a-b)^2)/a^3/d/(a-2*a*tanh(d*x+c)^2+(a-b
)*tanh(d*x+c)^4)
```

Rubi [A] time = 1.16, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3217, 1334, 1669, 1664, 1166, 208}

$$\frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2d(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}+\frac{b\tanh(c+dx)\left(\frac{2a^2(9a-17b)}{(a-b)^3}-\frac{\left(18a^2+15ab-13b^2\right)\tanh^2\cdots}{(a-b)^2}\right.}{32a^3d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+d\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt
[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + (
3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b
]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - Cot
h[c + d*x]/(a^3*d) + (b^2*Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*
Tanh[c + d*x]^2))/(8*a^2*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tan
h[c + d*x]^4)^2) + (b*Tanh[c + d*x]*((2*a^2*(9*a - 17*b))/(a - b)^3 - ((18*
a^2 + 15*a*b - 13*b^2)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^3*d*(a - 2*a*Tanh
[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1334

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
```

```
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^6}{x^2\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-16ab\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{b\tanh(c+dx)\cdots}{32a^3d\left(a-2a\cdots\right.}$$

$$= \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{b\tanh(c+dx)\cdots}{32a^3d\left(a-2a\cdots\right.}$$

$$= -\frac{\coth(c+dx)}{a^3d} + \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}$$

$$= -\frac{\coth(c+dx)}{a^3d} + \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}$$

$$= -\frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} + \frac{3\sqrt{b}\left(20a+34\cdots\right.}{\cdots}$$

Mathematica [A] time = 3.51, size = 357, normalized size = 0.99

$$\frac{\frac{4b\sinh(2(c+dx))\left(28a^2+b(13b-19a)\cosh(2(c+dx))+3ab-13b^2\right)}{(a-b)^2(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b)} + \frac{3\sqrt{b}\left(34\sqrt{a}\sqrt{b}+20a+15b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{3\sqrt{b}\left(-34\sqrt{a}\sqrt{b}+2\cdots\right.}{\left(\sqrt{\cdots}\right.}}{64a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[((Sqrt[a] - Sqrt[b])*
Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])^2*Sqrt[-a
+ Sqrt[a]*Sqrt[b]]) + (3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh
[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a]
+ Sqrt[b])^2*Sqrt[a + Sqrt[a]*Sqrt[b]]) - 64*Coth[c + d*x] + (4*b*(28*a^2 +
 3*a*b - 13*b^2 + b*(-19*a + 13*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((
a - b)^2*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (128*
a*b*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b)*(-8*a + 3*b
 - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2))/(64*a^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.51, size = 486, normalized size = 1.35

$$-\frac{\frac{28\,a^2b^2e^{(14\,dx+14\,c)}-35\,ab^3e^{(14\,dx+14\,c)}+13\,b^4e^{(14\,dx+14\,c)}-232\,a^2b^2e^{(12\,dx+12\,c)}+269\,ab^3e^{(12\,dx+12\,c)}-91\,b^4e^{(12\,dx+12\,c)}-576\,a^3be^{(10\,dx+10\,c)}+1372\,a^2b^2e^{(\cdots}}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*((28*a^2*b^2*e^(14*d*x + 14*c) - 35*a*b^3*e^(14*d*x + 14*c) + 13*b^4*
e^(14*d*x + 14*c) - 232*a^2*b^2*e^(12*d*x + 12*c) + 269*a*b^3*e^(12*d*x + 1
2*c) - 91*b^4*e^(12*d*x + 12*c) - 576*a^3*b*e^(10*d*x + 10*c) + 1372*a^2*b^
2*e^(10*d*x + 10*c) - 1039*a*b^3*e^(10*d*x + 10*c) + 273*b^4*e^(10*d*x + 10
*c) + 2432*a^3*b*e^(8*d*x + 8*c) - 3488*a^2*b^2*e^(8*d*x + 8*c) + 1913*a*b^
3*e^(8*d*x + 8*c) - 455*b^4*e^(8*d*x + 8*c) + 576*a^3*b*e^(6*d*x + 6*c) + 1
060*a^2*b^2*e^(6*d*x + 6*c) - 1689*a*b^3*e^(6*d*x + 6*c) + 455*b^4*e^(6*d*x
 + 6*c) - 376*a^2*b^2*e^(4*d*x + 4*c) + 679*a*b^3*e^(4*d*x + 4*c) - 273*b^4
*e^(4*d*x + 4*c) - 28*a^2*b^2*e^(2*d*x + 2*c) - 117*a*b^3*e^(2*d*x + 2*c) +
 91*b^4*e^(2*d*x + 2*c) + 19*a*b^3 - 13*b^4)/((a^5 - 2*a^4*b + a^3*b^2)*(b*
e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x
 + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2) + 32/(a^3*(e^(2*d*x + 2*c) - 1)))/d
```

maple [C] time = 0.25, size = 2747, normalized size = 7.65

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 153/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*b^3-2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(
1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*t
anh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b^2+153
/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9*b^3-2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13*b^2-7/2/d
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2
)/a*tanh(1/2*d*x+1/2*c)^11*b^2-7/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^5*b^2+25/4/d/(t
anh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-
16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+
b^2)*tanh(1/2*d*x+1/2*c)^7+25/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-52/d*b^4/a^3
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^7-52/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+17/4/d*b^3/a^2/
(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*
a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)
```

```
*tanh(1/2*d*x+1/2*c)^13+17/4/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*
d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(
1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-45/8/d/(tanh(
1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b
*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^13-45/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-1/2/d/a^3*tanh(1/2*d*x+
1/2*c)-1/2/d/a^3/tanh(1/2*d*x+1/2*c)+9/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(
1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*t
anh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+81/8/d/(t
anh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-
16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^5-45/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c
)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1
/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-45/8/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^9+81/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tan
h(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a
)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11-3/4/d/(tanh(1/2*d*x+1/2*c)^8*a
-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*
c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^15-3/64/d*b/a^3/(a^2-2*a*b+b^2)*sum((a*(-3*a+2*b)*_R^6+(49*a^2-72*a*b+30
*b^2)*_R^4+(-49*a^2+72*a*b-30*b^2)*_R^2+3*a^2-2*a*b)/(_R^7*a-3*_R^5*a+3*_R^
3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*
a-16*b)*_Z^4-4*a*_Z^2+a))+9/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15*b-3/4/d/(tanh(1/2*d
*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh
(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh
(1/2*d*x+1/2*c)-19/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-19/4/d/(tanh(1/2*d*x+
1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/
2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(
1/2*d*x+1/2*c)^11
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(32*a^2*b^2 - 83*a*b^3 + 45*b^4 + 3*(20*a^2*b^2*e^(16*c) - 33*a*b^3*e^
(16*c) + 15*b^4*e^(16*c))*e^(16*d*x) - 12*(43*a^2*b^2*e^(14*c) - 68*a*b^3*e
^(14*c) + 30*b^4*e^(14*c))*e^(14*d*x) - 4*(400*a^3*b*e^(12*c) - 1137*a^2*b^
2*e^(12*c) + 1031*a*b^3*e^(12*c) - 315*b^4*e^(12*c))*e^(12*d*x) + 12*(592*a
^3*b*e^(10*c) - 1237*a^2*b^2*e^(10*c) + 886*a*b^3*e^(10*c) - 210*b^4*e^(10*
c))*e^(10*d*x) + 2*(4096*a^4*e^(8*c) - 12192*a^3*b*e^(8*c) + 13634*a^2*b^2*
e^(8*c) - 7113*a*b^3*e^(8*c) + 1575*b^4*e^(8*c))*e^(8*d*x) + 4*(880*a^3*b*e
^(6*c) - 2855*a^2*b^2*e^(6*c) + 2512*a*b^3*e^(6*c) - 630*b^4*e^(6*c))*e^(6*
d*x) - 4*(256*a^3*b*e^(4*c) - 823*a^2*b^2*e^(4*c) + 903*a*b^3*e^(4*c) - 315
*b^4*e^(4*c))*e^(4*d*x) - 12*(19*a^2*b^2*e^(2*c) - 54*a*b^3*e^(2*c) + 30*b^
4*e^(2*c))*e^(2*d*x))/(a^5*b^2*d - 2*a^4*b^3*d + a^3*b^4*d - (a^5*b^2*d*e^(
18*c) - 2*a^4*b^3*d*e^(18*c) + a^3*b^4*d*e^(18*c))*e^(18*d*x) + 9*(a^5*b^2*
d*e^(16*c) - 2*a^4*b^3*d*e^(16*c) + a^3*b^4*d*e^(16*c))*e^(16*d*x) + 4*(8*a
^6*b*d*e^(14*c) - 25*a^5*b^2*d*e^(14*c) + 26*a^4*b^3*d*e^(14*c) - 9*a^3*b^4
*d*e^(14*c))*e^(14*d*x) - 4*(40*a^6*b*d*e^(12*c) - 101*a^5*b^2*d*e^(12*c) +
```

```
 82*a^4*b^3*d*e^(12*c) - 21*a^3*b^4*d*e^(12*c))*e^(12*d*x) - 2*(128*a^7*d*e
^(10*c) - 416*a^6*b*d*e^(10*c) + 511*a^5*b^2*d*e^(10*c) - 286*a^4*b^3*d*e^(
10*c) + 63*a^3*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a^7*d*e^(8*c) - 416*a^6*
b*d*e^(8*c) + 511*a^5*b^2*d*e^(8*c) - 286*a^4*b^3*d*e^(8*c) + 63*a^3*b^4*d*
e^(8*c))*e^(8*d*x) + 4*(40*a^6*b*d*e^(6*c) - 101*a^5*b^2*d*e^(6*c) + 82*a^4
*b^3*d*e^(6*c) - 21*a^3*b^4*d*e^(6*c))*e^(6*d*x) - 4*(8*a^6*b*d*e^(4*c) - 2
5*a^5*b^2*d*e^(4*c) + 26*a^4*b^3*d*e^(4*c) - 9*a^3*b^4*d*e^(4*c))*e^(4*d*x)
 - 9*(a^5*b^2*d*e^(2*c) - 2*a^4*b^3*d*e^(2*c) + a^3*b^4*d*e^(2*c))*e^(2*d*x
)) - 4*integrate(3/32*((20*a^2*b*e^(6*c) - 33*a*b^2*e^(6*c) + 15*b^3*e^(6*c
))*e^(6*d*x) - 2*(32*a^2*b*e^(4*c) - 41*a*b^2*e^(4*c) + 15*b^3*e^(4*c))*e^(
4*d*x) + (20*a^2*b*e^(2*c) - 33*a*b^2*e^(2*c) + 15*b^3*e^(2*c))*e^(2*d*x))/
(a^5*b - 2*a^4*b^2 + a^3*b^3 + (a^5*b*e^(8*c) - 2*a^4*b^2*e^(8*c) + a^3*b^3
*e^(8*c))*e^(8*d*x) - 4*(a^5*b*e^(6*c) - 2*a^4*b^2*e^(6*c) + a^3*b^3*e^(6*c
))*e^(6*d*x) - 2*(8*a^6*e^(4*c) - 19*a^5*b*e^(4*c) + 14*a^4*b^2*e^(4*c) - 3
*a^3*b^3*e^(4*c))*e^(4*d*x) - 4*(a^5*b*e^(2*c) - 2*a^4*b^2*e^(2*c) + a^3*b^
3*e^(2*c))*e^(2*d*x)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sinh\left(c+d\,x\right)^2\,{\left(a-b\,{\sinh\left(c+d\,x\right)}^4\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^3),x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a - b*sinh(c + d*x)^4)^3), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.265 $\int \frac{1}{1-\sinh^4(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

```
[Out] 1/4*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/2*tanh(x)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3209, 388, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sinh^4(x)}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{1-2x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{\tanh(x)}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.10, size = 24, normalized size = 0.96

$$\frac{1}{4}\left(\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)+2\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4
```

fricas [B] time = 0.61, size = 113, normalized size = 4.52

$$\frac{\left(\sqrt{2}\,\cosh(x)^2+2\,\sqrt{2}\,\cosh(x)\sinh(x)+\sqrt{2}\,\sinh(x)^2+\sqrt{2}\right)\log\left(-\frac{3\left(2\,\sqrt{2}-3\right)\cosh(x)^2-4\left(3\,\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\,\sqrt{2}\cdot\right.}{\cosh(x)^2+\sinh(x)^2-3}\right.}{8\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="fricas")
```

```
[Out] 1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + s
qrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(
x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 -
3)) - 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

giac [B] time = 0.12, size = 48, normalized size = 1.92

$$-\frac{1}{8}\,\sqrt{2}\,\log\left(\frac{\left|-4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}{\left|4\,\sqrt{2}+2\,e^{(2\,x)}-6\right|}\right)-\frac{1}{e^{(2\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - 1/(e^(2*x) + 1)
```

maple [B] time = 0.04, size = 55, normalized size = 2.20

$$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{4}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)}{4}+\frac{\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^4),x)
```

```
[Out] 1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+1/4*2^(1/2)*arctanh(1/4*
(2*tanh(1/2*x)+2)*2^(1/2))+tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

maxima [B] time = 0.45, size = 69, normalized size = 2.76

$$\frac{1}{8}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{8}\,\sqrt{2}\,\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)+\frac{1}{e^{(-2\,x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/8*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + 1/(e^(-2*x) + 1)
```

mupad [B] time = 0.16, size = 63, normalized size = 2.52

$$\frac{\sqrt{2}\,\ln\left(2\,e^{2\,x}+\frac{\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{8}\right)}{8}-\frac{\sqrt{2}\,\ln\left(2\,e^{2\,x}-\frac{\sqrt{2}\,\left(12\,e^{2\,x}-4\right)}{8}\right)}{8}-\frac{1}{e^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^4 - 1),x)
```

```
[Out] (2^(1/2)*log(2*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - (2^(1/2)*log(
2*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/8))/8 - 1/(exp(2*x) + 1)
```

sympy [B] time = 5.78, size = 908, normalized size = 36.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**4),x)
```

```
[Out] 3064704*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/
2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt
(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**
2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 3064704*log(tanh
(x/2) - 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt(2)*log(tanh(x/2) - 1 + sqrt(
2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(
2) + 17336584) + 3064704*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1225881
6*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 1733658
4) + 2167073*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(12258816*sq
rt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) +
 3064704*log(tanh(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 1733
6584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 2167073*sqrt(2)*log(tanh
(x/2) + 1 + sqrt(2))/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2) -
 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 1
2258816*sqrt(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2
)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt
(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(12258816*sq
rt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) -
 3064704*log(tanh(x/2) - sqrt(2) - 1)/(12258816*sqrt(2)*tanh(x/2)**2 + 1733
6584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh
(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584
*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt
(2) + 1)*tanh(x/2)**2/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**
2 + 12258816*sqrt(2) + 17336584) - 2167073*sqrt(2)*log(tanh(x/2) - sqrt(2)
+ 1)/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt
(2) + 17336584) - 3064704*log(tanh(x/2) - sqrt(2) + 1)/(12258816*sqrt(2)*ta
nh(x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584) + 1225881
6*sqrt(2)*tanh(x/2)/(12258816*sqrt(2)*tanh(x/2)**2 + 17336584*tanh(x/2)**2
+ 12258816*sqrt(2) + 17336584) + 17336584*tanh(x/2)/(12258816*sqrt(2)*tanh(
x/2)**2 + 17336584*tanh(x/2)**2 + 12258816*sqrt(2) + 17336584)
```

3.266 $\int \frac{1}{1+\sinh^4(x)}\,dx$

Optimal. Leaf size=176

$$-\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(2\tanh^2(x)-2\sqrt{1+\sqrt{2}}\,\tanh(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(\sqrt{2}\,\tanh^2(x)+\sqrt{2\left(1+\sqrt{2}\right)}\,\tanh\right.$$

```
[Out] -1/4*arctan(((1+2^(1/2))^(1/2)-2*tanh(x))/(2^(1/2)-1)^(1/2))/(1+2^(1/2))^(1
/2)+1/4*arctan(((1+2^(1/2))^(1/2)+2*tanh(x))/(2^(1/2)-1)^(1/2))/(1+2^(1/2))
^(1/2)-1/8*ln(2^(1/2)-2*(1+2^(1/2))^(1/2)*tanh(x)+2*tanh(x)^2)*(1+2^(1/2))^
(1/2)+1/8*ln(1+(2+2*2^(1/2))^(1/2)*tanh(x)+2^(1/2)*tanh(x)^2)*(1+2^(1/2))^(
1/2)
```

Rubi [A] time = 0.16, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3209, 1169, 634, 618, 204, 628}

$$-\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(2\tanh^2(x)-2\sqrt{1+\sqrt{2}}\,\tanh(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\,\log\left(\sqrt{2}\,\tanh^2(x)+\sqrt{2\left(1+\sqrt{2}\right)}\,\tanh\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[1 + Sqrt[2]] - 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[1 + Sqr
t[2]]) + ArcTan[(Sqrt[1 + Sqrt[2]] + 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt
[1 + Sqrt[2]]) - (Sqrt[1 + Sqrt[2]]*Log[Sqrt[2] - 2*Sqrt[1 + Sqrt[2]]*Tanh[
x] + 2*Tanh[x]^2])/8 + (Sqrt[1 + Sqrt[2]]*Log[1 + Sqrt[2*(1 + Sqrt[2])]*Tan
h[x] + Sqrt[2]*Tanh[x]^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{1+\sinh^4(x)}\,dx = \text{Subst}\left(\int \frac{1-x^2}{1-2x^2+2x^4}\,dx, x, \tanh(x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{2}}-\left(1+\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}-\sqrt{1+\sqrt{2}}\,x+x^2}\,dx, x, \tanh(x)\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{2}}+\left(1+\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}+\sqrt{1+\sqrt{2}}\,x+x^2}\,dx, x, \tanh(x)\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

$$= \frac{1}{8}\sqrt{3-2\sqrt{2}}\ \text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{2}}-\sqrt{1+\sqrt{2}}\,x+x^2}\,dx, x, \tanh(x)\right) + \frac{1}{8}\sqrt{3-2\sqrt{2}}\ \text{Subst}\left(\int\right.$$

$$= -\frac{1}{8}\sqrt{1+\sqrt{2}}\ \log\left(\sqrt{2}-2\sqrt{1+\sqrt{2}}\ \tanh(x)+2\tanh^2(x)\right) + \frac{1}{8}\sqrt{1+\sqrt{2}}\ \log\left(1+\sqrt{2(}\right.$$

$$= -\frac{1}{4}\sqrt{-1+\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{1+\sqrt{2}}-2\tanh(x)}{\sqrt{-1+\sqrt{2}}}\right) + \frac{1}{4}\sqrt{-1+\sqrt{2}}\ \tan^{-1}\left(\frac{\sqrt{1+\sqrt{2}}+2\tan}{\sqrt{-1+\sqrt{2}}}\right.$$

Mathematica [C] time = 0.08, size = 45, normalized size = 0.26

$$\frac{\tanh^{-1}\left(\sqrt{1-i}\ \tanh(x)\right)}{2\sqrt{1-i}} + \frac{\tanh^{-1}\left(\sqrt{1+i}\ \tanh(x)\right)}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 - I]*Tanh[x]]/(2*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Tanh[x]]
/(2*Sqrt[1 + I])
```

fricas [B] time = 1.23, size = 596, normalized size = 3.39

$$\frac{1}{16}\cdot 2^{\frac{1}{4}}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}\ \log\left(\left(2^{\frac{3}{4}}e^{(2x)}-2^{\frac{1}{4}}\left(3\sqrt{2}+4\right)\right)\sqrt{-2\sqrt{2}+4}+4\sqrt{2}+e^{(4x)}-2e^{(2x)}+5\right)-\frac{1}{16}\cdot 2^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^4),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log((2^(3/4)*e^(2*x) - 2^(1
/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x)
 + 5) - 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log(-(2^(3/4)*e^(2*
x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) -
2*e^(2*x) + 5) - 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(1/14*(sqrt(2)*(5*s
qrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) - 1/28*(2*sqrt(2)*(5*sqrt(2) + 6) - (2
^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sqrt(2) + 4) +
 16*sqrt(2) + 8)*sqrt(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*
sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 1/14*sqrt(2)*(3*sqrt(
2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*e^(2
*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(-2*sqrt(2)
+ 4) - 1/7*sqrt(2) + 3/7) - 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(-1/14*(
sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) + 1/28*(2*sqrt(2)*(5*sqrt(
2) + 6) + (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sq
rt(2) + 4) + 16*sqrt(2) + 8)*sqrt((2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4
))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) + 1/14*sqrt(
2)*(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2)
 + 6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(
-2*sqrt(2) + 4) + 1/7*sqrt(2) - 3/7)
```

giac [C] time = 0.20, size = 281, normalized size = 1.60

$$\left(\frac{1}{16}i + \frac{1}{16}\right)\sqrt{2\sqrt{2} - 2}\left(-\frac{i}{\sqrt{2} - 1} + 1\right)\log\left((20i + 10)\sqrt{2}e^{(2x)} + 10\sqrt{2}\sqrt{10\sqrt{2} + 14} - 50\sqrt{2} - (2i - 14)\sqrt{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^4),x, algorithm="giac")
```

```
[Out] (1/16*I + 1/16)*sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1)*log((20*I + 10)*
sqrt(2)*e^(2*x) + 10*sqrt(2)*sqrt(10*sqrt(2) + 14) - 50*sqrt(2) - (2*I - 14
)*sqrt(10*sqrt(2) + 14) + (28*I + 14)*e^(2*x) - 70) - (1/16*I + 1/16)*sqrt(
2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1)*log((20*I + 10)*sqrt(2)*e^(2*x) - 10*
sqrt(2)*sqrt(10*sqrt(2) + 14) - 50*sqrt(2) + (2*I - 14)*sqrt(10*sqrt(2) + 1
4) + (28*I + 14)*e^(2*x) - 70) + (1/16*I + 1/16)*sqrt(2*sqrt(2) + 2)*(-I/(s
qrt(2) + 1) + 1)*log(2*sqrt(2)*e^(2*x) + 2*sqrt(2)*sqrt(2*sqrt(2) - 2) - (4
*I + 2)*sqrt(2) + (2*I - 2)*sqrt(2*sqrt(2) - 2) - 2*e^(2*x) + 4*I + 2) - (1
/16*I + 1/16)*sqrt(2*sqrt(2) + 2)*(-I/(sqrt(2) + 1) + 1)*log(2*sqrt(2)*e^(2
*x) - 2*sqrt(2)*sqrt(2*sqrt(2) - 2) - (4*I + 2)*sqrt(2) - (2*I - 2)*sqrt(2*
sqrt(2) - 2) - 2*e^(2*x) + 4*I + 2)
```

maple [C] time = 0.05, size = 44, normalized size = 0.25

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4-2\_Z^2+1\right)}\_R\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-4\_R^3+4\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^4),x)
```

```
[Out] 1/4*sum(_R*ln(tanh(1/2*x)^2+(-4*_R^3+4*_R)*tanh(1/2*x)+1),_R=RootOf(2*_Z^4-
2*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh(x)^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sinh(x)^4 + 1), x)
```

mupad [B] time = 1.15, size = 205, normalized size = 1.16

$$-\frac{\sqrt{2}\ \sqrt{1-\mathrm{i}}\ \ln\left(\mathrm{e}^{2x}\ (436273152+91291648\mathrm{i})+\sqrt{2}\ \sqrt{1-\mathrm{i}}\ (-9830400+56623104\mathrm{i})+\sqrt{2}\ \sqrt{1-\mathrm{i}}\ \mathrm{e}^{2x}\ (-21}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^4 + 1),x)
```

```
[Out] (2^(1/2)*(1 - 1i)^(1/2)*log(exp(2*x)*(436273152 + 91291648i) + 2^(1/2)*(1 -
 1i)^(1/2)*(9830400 - 56623104i) + 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(2188902
40 + 149422080i) - (21168128 + 94306304i)))/8 - (2^(1/2)*(1 - 1i)^(1/2)*log
(exp(2*x)*(436273152 + 91291648i) - 2^(1/2)*(1 - 1i)^(1/2)*(9830400 - 56623
104i) - 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(218890240 + 149422080i) - (2116812
8 + 94306304i)))/8 - (2^(1/2)*(1 + 1i)^(1/2)*log(exp(2*x)*(436273152 - 9129
1648i) - 2^(1/2)*(1 + 1i)^(1/2)*(9830400 + 56623104i) - 2^(1/2)*(1 + 1i)^(1
/2)*exp(2*x)*(218890240 - 149422080i) - (21168128 - 94306304i)))/8 + (2^(1/
2)*(1 + 1i)^(1/2)*log(exp(2*x)*(436273152 - 91291648i) + 2^(1/2)*(1 + 1i)^(
1/2)*(9830400 + 56623104i) + 2^(1/2)*(1 + 1i)^(1/2)*exp(2*x)*(218890240 - 1
49422080i) - (21168128 - 94306304i)))/8
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**4),x)
```

```
[Out] Timed out
```

3.267 $\int \frac{1}{a+b\sinh^5(x)}\,dx$

Optimal. Leaf size=435

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}}+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{9/10}\left(\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}\right)}{\sqrt{\sqrt[5]{-1}\,b^{2/5}-(-1)^{4/5}a^{2/5}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{-1}\,b^{2/5}-(-1)^{4/5}a^{2/5}}}+\frac{2\sqrt[5]{-1}\tanh^{-1}\left(\frac{\sqrt[5]{-1}\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}+$$

```
[Out] -2/5*(-1)^(9/10)*arctanh((I*b^(1/5)-(-1)^(9/10)*a^(1/5)*tanh(1/2*x))/(-(-1)
^(4/5)*a^(2/5)-b^(2/5))^(1/2))/a^(4/5)/(-(-1)^(4/5)*a^(2/5)-b^(2/5))^(1/2)-
2/5*arctanh((b^(1/5)-a^(1/5)*tanh(1/2*x))/(a^(2/5)+b^(2/5))^(1/2))/a^(4/5)/
(a^(2/5)+b^(2/5))^(1/2)+2/5*(-1)^(1/5)*arctanh((b^(1/5)+(-1)^(1/5)*a^(1/5)*
tanh(1/2*x))/((-1)^(2/5)*a^(2/5)+b^(2/5))^(1/2))/a^(4/5)/((-1)^(2/5)*a^(2/5
)+b^(2/5))^(1/2)+2/5*(-1)^(9/10)*arctanh((-1)^(9/10)*((-1)^(1/5)*b^(1/5)+a^
(1/5)*tanh(1/2*x))/(-(-1)^(4/5)*a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2))/a^(4/5)/
(-(-1)^(4/5)*a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2)+2/5*(-1)^(9/10)*arctanh((-1)
^(3/10)*(b^(1/5)+(-1)^(3/5)*a^(1/5)*tanh(1/2*x))/(-(-1)^(4/5)*a^(2/5)+(-1)^
(3/5)*b^(2/5))^(1/2))/a^(4/5)/(-(-1)^(4/5)*a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2
)
```

Rubi [A] time = 0.97, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3213, 2660, 618, 206, 204}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}}+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{9/10}\left(\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}\right)}{\sqrt{\sqrt[5]{-1}\,b^{2/5}-(-1)^{4/5}a^{2/5}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{-1}\,b^{2/5}-(-1)^{4/5}a^{2/5}}}+\frac{2\sqrt[5]{-1}\tanh^{-1}\left(\frac{\sqrt[5]{-1}\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^5)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b^(1/5) - a^(1/5)*Tanh[x/2])/Sqrt[a^(2/5) + b^(2/5)]])/(5*a^(4
/5)*Sqrt[a^(2/5) + b^(2/5)]) + (2*(-1)^(9/10)*ArcTanh[((-1)^(9/10)*((-1)^(1
/5)*b^(1/5) + a^(1/5)*Tanh[x/2]))/Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(1/5)*b
^(2/5)]])/(5*a^(4/5)*Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(1/5)*b^(2/5)]) + (2
*(-1)^(1/5)*ArcTanh[(b^(1/5) + (-1)^(1/5)*a^(1/5)*Tanh[x/2])/Sqrt[(-1)^(2/5
)*a^(2/5) + b^(2/5)]])/(5*a^(4/5)*Sqrt[(-1)^(2/5)*a^(2/5) + b^(2/5)]) + (2*
(-1)^(9/10)*ArcTanh[((-1)^(3/10)*(b^(1/5) + (-1)^(3/5)*a^(1/5)*Tanh[x/2]))/
Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[-((-1)^(
4/5)*a^(2/5)) + (-1)^(3/5)*b^(2/5)]) - (2*(-1)^(9/10)*ArcTanh[(I*b^(1/5) -
(-1)^(9/10)*a^(1/5)*Tanh[x/2])/Sqrt[-((-1)^(4/5)*a^(2/5)) - b^(2/5)]])/(5*a
^(4/5)*Sqrt[-((-1)^(4/5)*a^(2/5)) - b^(2/5)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\sinh^5(x)}\,dx = \int \left(-\frac{(-1)^{9/10}}{5a^{4/5}\left(-(-1)^{9/10}\sqrt[5]{a}-i\sqrt[5]{b}\sinh(x)\right)}-\frac{(-1)^{9/10}}{5a^{4/5}\left(-(-1)^{9/10}\sqrt[5]{a}-\sqrt[10]{-1}\sqrt[5]{b}\sinh(x)\right)}-\ldots\right.$$

$$=-\frac{(-1)^{9/10}\int\frac{1}{-(-1)^{9/10}\sqrt[5]{a}-i\sqrt[5]{b}\sinh(x)}\,dx}{5a^{4/5}}-\frac{(-1)^{9/10}\int\frac{1}{-(-1)^{9/10}\sqrt[5]{a}-\sqrt[10]{-1}\sqrt[5]{b}\sinh(x)}\,dx}{5a^{4/5}}-\ldots$$

$$=-\frac{\left(2(-1)^{9/10}\right)\mathrm{Subst}\left(\int\frac{1}{-(-1)^{9/10}\sqrt[5]{a}-2i\sqrt[5]{b}x+(-1)^{9/10}\sqrt[5]{a}x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\ldots$$

$$=\frac{\left(4(-1)^{9/10}\right)\mathrm{Subst}\left(\int\frac{1}{-4(-1)^{4/5}\left(a^{2/5}+b^{2/5}\right)-x^2}\,dx,x,-2(-1)^{9/10}\sqrt[5]{b}+2(-1)^{9/10}\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)\right)}{5a^{4/5}}\ldots$$

$$=-\frac{2(-1)^{7/10}\tan^{-1}\left(\frac{i\sqrt[5]{b}+(-1)^{7/10}\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}}-\frac{2(-1)^{9/10}\tanh^{-1}\ldots}{5a^{4/5}\sqrt{-\ldots}}$$

Mathematica [C] time = 0.33, size = 141, normalized size = 0.32

$$\frac{8}{5}\mathrm{RootSum}\left[\#1^{10}b-5\#1^8b+10\#1^6b+32\#1^5a-10\#1^4b+5\#1^2b-b\&,\frac{\#1^3x+2\#1^3\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\ldots\right.}{\#1^8b-4\#1^6b+6\#1^4b+1\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^5)^(-1),x]
```

```
[Out] (8*RootSum[-b + 5*b*#1^2 - 10*b*#1^4 + 32*a*#1^5 + 10*b*#1^6 - 5*b*#1^8 + b
*#1^10 & , (x*#1^3 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2
]*#1]*#1^3)/(b - 4*b*#1^2 + 16*a*#1^3 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8) & ])/
5
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \sinh(x)^5 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sinh(x)^5 + a), x)
```

maple [C] time = 0.06, size = 113, normalized size = 0.26

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{10}-5a\_Z^{8}+10a\_Z^{6}-32b\_Z^{5}-10a\_Z^{4}+5a\_Z^{2}-a\right)} \frac{\left(-\_R^{8}+4\_R^{6}-6\_R^{4}+4\_R^{2}-1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a-4\_R^{7}a+6\_R^{5}a-16\_R^{4}b-4\_R^{3}a+\_Ra}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^5),x)
```

```
[Out] 1/5*sum((-_R^8+4*_R^6-6*_R^4+4*_R^2-1)/(_R^9*a-4*_R^7*a+6*_R^5*a-16*_R^4*b-
4*_R^3*a+_R*a)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^10*a-5*_Z^8*a+10*_Z^6*a-32*_
Z^5*b-10*_Z^4*a+5*_Z^2*a-a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \sinh(x)^5 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^5),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(x)^5 + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x)^5),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \sinh^{5}{(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**5),x)
```

```
[Out] Integral(1/(a + b*sinh(x)**5), x)
```

3.268 $\int \frac{1}{a+b\sinh^6(x)}\,dx$

Optimal. Leaf size=175

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] 1/3*arctanh((a^(1/3)-b^(1/3))^(1/2)*tanh(x)/a^(1/6))/a^(5/6)/(a^(1/3)-b^(1/
3))^(1/2)+1/3*arctanh((a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)*tanh(x)/a^(1/6))/a
^(5/6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctanh((a^(1/3)-(-1)^(2/3)*b
^(1/3))^(1/2)*tanh(x)/a^(1/6))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[a^(1/3) - b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3)
- b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tanh[x])/a^(1/6)]
/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) - (
-1)^(2/3)*b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b
^(1/3)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^6(x)}\,dx = \frac{\int \frac{1}{1+\frac{\sqrt[3]{b}\sinh^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{(-1)^{2/3}\sqrt[3]{b}\sinh^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tanh(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tanh(x)\right)}{3a} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Mathematica [C] time = 0.18, size = 134, normalized size = 0.77

$$\frac{16}{3}\text{RootSum}\left[\#1^6b-6\#1^5b+15\#1^4b+64\#1^3a-20\#1^3b+15\#1^2b-6\#1b+b\&, \frac{\#1^2x+\#1^2\log(-\#1\sinh(x)+\#}{\#1^5b-5\#1^4b+10\#1^3b+3}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^6)^(-1),x]
```

```
[Out] (16*RootSum[b - 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*
b*#1^5 + b*#1^6 & , (x*#1^2 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]
*#1]*#1^2)/(-b + 5*b*#1 + 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^3 - 5*b*#1^4 + b*
#1^5) & ])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="giac")
```

```
[Out] 0
```

maple [C] time = 0.06, size = 128, normalized size = 0.73

$$\frac{\left(\sum_{\_R=\text{RootOf}\left(a\_Z^{12}-6a\_Z^{10}+15a\_Z^8+(-20a+64b)\_Z^6+15a\_Z^4-6a\_Z^2+a\right)} \frac{\left(-\_R^{10}+5\_R^8-10\_R^6+10\_R^4-5\_R^2+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^{11}a-5\_R^9a+10\_R^7a-10\_R^5a+32\_R^5b+5\_R^3a-\_Ra}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^6),x)
```

```
[Out] 1/6*sum((-_R^10+5*_R^8-10*_R^6+10*_R^4-5*_R^2+1)/(_R^11*a-5*_R^9*a+10*_R^7*
a-10*_R^5*a+32*_R^5*b+5*_R^3*a-_R*a)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^12-6
*a*_Z^10+15*a*_Z^8+(-20*a+64*b)*_Z^6+15*a*_Z^4-6*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sinh(x)^6 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(x)^6 + a), x)
```

mupad [B] time = 58.56, size = 857, normalized size = 4.90

$$\sum_{k=1}^{6} \ln\left(\mathrm{root}\left(46656\,a^5\,b\,d^6-46656\,a^6\,d^6+3888\,a^4\,d^4-108\,a^2\,d^2+1,d,k\right)\left(\mathrm{root}\left(46656\,a^5\,b\,d^6-46656\,a^6\,d^6\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x)^6),x)
```

```
[Out] symsum(log(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*a^2*d^
2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*a^2
*d^2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 108*
a^2*d^2 + 1, d, k)*(root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*d^4 - 1
08*a^2*d^2 + 1, d, k)*((1459166279268040704*(327680*a^7*exp(2*x) + 298496*a
^6*b - 65536*a^7 + 158*a^2*b^5 - 91315*a^3*b^4 + 348176*a^4*b^3 - 489952*a^
5*b^2 - 196*a^2*b^5*exp(2*x) + 274019*a^3*b^4*exp(2*x) - 1132876*a^4*b^3*ex
p(2*x) + 1770440*a^5*b^2*exp(2*x) - 1239040*a^6*b*exp(2*x)))/(b^10*(a - b)^
3) + (17509995351216488448*root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 3888*a^4*
d^4 - 108*a^2*d^2 + 1, d, k)*(262144*a^7*exp(2*x) + 203520*a^6*b - 65536*a^
7 - 453*a^3*b^4 + 72022*a^4*b^3 - 209472*a^5*b^2 + 630*a^3*b^4*exp(2*x) - 2
54512*a^4*b^3*exp(2*x) + 767508*a^5*b^2*exp(2*x) - 775680*a^6*b*exp(2*x)))/
(b^10*(a - b)^2)) - (486388759756013568*(655360*a^5*exp(2*x) + 9*a*b^4 + 37
0176*a^4*b - 196608*a^5 - 24408*a^2*b^3 - 149088*a^3*b^2 + 63676*a^2*b^3*ex
p(2*x) + 526248*a^3*b^2*exp(2*x) - 10*a*b^4*exp(2*x) - 1245184*a^4*b*exp(2*
x)))/(b^10*(a - b)^2)) - (40532396646334464*(655360*a^5*exp(2*x) + b^5*exp(
2*x) + 24677*a*b^4 + 773120*a^4*b - 262144*a^5 - b^5 + 198071*a^2*b^3 - 733
696*a^3*b^2 - 477713*a^2*b^3*exp(2*x) + 1770640*a^3*b^2*exp(2*x) - 53861*a*
b^4*exp(2*x) - 1894400*a^4*b*exp(2*x)))/(b^10*(a - b)^3)) + (13510798882111
488*(655360*a^3*exp(2*x) - 11382*b^3*exp(2*x) - 144416*a*b^2 + 269056*a^2*b
 - 131072*a^3 + 6459*b^3 + 677524*a*b^2*exp(2*x) - 1321472*a^2*b*exp(2*x)))
/(b^10*(a - b)^2)) - (1125899906842624*(851968*a^4*exp(2*x) + 6006*b^4*exp(
2*x) + 211497*a*b^3 + 597504*a^3*b - 196608*a^4 - 3840*b^4 - 608544*a^2*b^2
 + 2562504*a^2*b^2*exp(2*x) - 864565*a*b^3*exp(2*x) - 2555904*a^3*b*exp(2*x
)))/(b^10*(a - b)^2*(a*b - a^2)))*root(46656*a^5*b*d^6 - 46656*a^6*d^6 + 38
88*a^4*d^4 - 108*a^2*d^2 + 1, d, k), k, 1, 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sinh^{6}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**6),x)
```

```
[Out] Integral(1/(a + b*sinh(x)**6), x)
```

3.269 $\int \frac{1}{a+b\sinh^8(x)}\,dx$

Optimal. Leaf size=245

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\,\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\,\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

```
[Out] -1/4*arctanh(((-a)^(1/4)-b^(1/4))^(1/2)*tanh(x)/(-a)^(1/8))/(-a)^(7/8)/((-a
)^(1/4)-b^(1/4))^(1/2)-1/4*arctanh(((-a)^(1/4)-I*b^(1/4))^(1/2)*tanh(x)/(-a
)^(1/8))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)-1/4*arctanh(((-a)^(1/4)+I*
b^(1/4))^(1/2)*tanh(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1/2)-
1/4*arctanh(((-a)^(1/4)+b^(1/4))^(1/2)*tanh(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)
^(1/4)+b^(1/4))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3211, 3181, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\,\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\,\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\,\tanh(x)}{(-a)^{5/8}}\right)}{4(-a)^{3/8}\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^8)^(-1),x]
```

```
[Out] -ArcTanh[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Tanh[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*S
qrt[(-a)^(1/4) - I*b^(1/4)]) - ArcTanh[(Sqrt[(-a)^(1/4) + I*b^(1/4)]*Tanh[x
])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*b^(1/4)]) - ArcTanh[(Sqrt[
(-a)^(1/4) + b^(1/4)]*Tanh[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) +
b^(1/4)]) - ArcTanh[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*Tanh[x])/(-a)^(5/8)]/(4*(
-a)^(3/8)*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\sinh^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \tanh(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \tanh(x)\right)}{4a} + \frac{\text{Subst}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

Mathematica [C] time = 0.26, size = 160, normalized size = 0.65

$$16\text{RootSum}\left[\#1^8b - 8\#1^7b + 28\#1^6b - 56\#1^5b + 256\#1^4a + 70\#1^4b - 56\#1^3b + 28\#1^2b - 8\#1b + b\&, \frac{\#}{\#1^7b - }\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^8)^(-1),x]
```

```
[Out] 16*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 - 56
*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh[x
] + Cosh[x]*#1 - Sinh[x]*#1]*#1^3)/(-b + 7*b*#1 - 21*b*#1^2 + 128*a*#1^3 +
35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.80, size = 1, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="giac")
```

```
[Out] 0
```

maple [C] time = 0.06, size = 162, normalized size = 0.66

$$\frac{\left(\sum_{\_R=\text{RootOf}\left(a\_Z^{16}-8a\_Z^{14}+28a\_Z^{12}-56a\_Z^{10}+(70a+256b)\_Z^8-56a\_Z^6+28a\_Z^4-8a\_Z^2+a\right)} \frac{\left(-\_R^{14}+7\_R^{12}-21\_R^{10}+35\_R^8-35\_R^6+21\_\right.}{\_R^{15}a-7\_R^{13}a+21\_R^{11}a-35\_R^9a+35\_R^7a+1}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^8),x)
```

```
[Out] 1/8*sum((-_R^14+7*_R^12-21*_R^10+35*_R^8-35*_R^6+21*_R^4-7*_R^2+1)/(_R^15*a
-7*_R^13*a+21*_R^11*a-35*_R^9*a+35*_R^7*a+128*_R^7*b-21*_R^5*a+7*_R^3*a-_R*
a)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^16-8*a*_Z^14+28*a*_Z^12-56*a*_Z^10+(70
*a+256*b)*_Z^8-56*a*_Z^6+28*a*_Z^4-8*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sinh(x)^8 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(x)^8 + a), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(x)^8),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sinh^8(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**8),x)
```

```
[Out] Integral(1/(a + b*sinh(x)**8), x)
```

3.270 $\int \frac{1}{1+\sinh^5(x)}\,dx$

Optimal. Leaf size=242

$$-\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{7/10}\left(\sqrt[5]{-1}\tanh\left(\frac{x}{2}\right)+1\right)}{\sqrt{-(-1)^{2/5}\left(1+(-1)^{2/5}\right)}}\right)}{5\sqrt{-(-1)^{2/5}\left(1+(-1)^{2/5}\right)}}-\frac{2(-1)^{4/5}\tanh^{-1}\left(\frac{1-(-1)^{4/5}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}}-\frac{2(}{}$$

```
[Out] -1/5*arctanh(1/2*(1-tanh(1/2*x))*2^(1/2))*2^(1/2)-2/5*(-1)^(3/5)*arctan((1+
(-1)^(3/5)*tanh(1/2*x))/(-1+(-1)^(1/5))^(1/2))/(-1+(-1)^(1/5))^(1/2)+2/5*(-
1)^(9/10)*arctanh((-1)^(7/10)*(1+(-1)^(1/5)*tanh(1/2*x))/(-(-1)^(2/5)*(1+(-
1)^(2/5)))^(1/2))/(-(-1)^(2/5)*(1+(-1)^(2/5)))^(1/2)-2/5*(-1)^(4/5)*arctanh
((1-(-1)^(4/5)*tanh(1/2*x))/(1-(-1)^(3/5))^(1/2))/(1-(-1)^(3/5))^(1/2)+2/5*
(-1)^(9/10)*arctan((I-(-1)^(9/10)*tanh(1/2*x))/(1+(-1)^(4/5))^(1/2))/(1+(-1
)^(4/5))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3213, 2660, 618, 204, 206, 617}

$$-\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{7/10}\left(\sqrt[5]{-1}\tanh\left(\frac{x}{2}\right)+1\right)}{\sqrt{-(-1)^{2/5}\left(1+(-1)^{2/5}\right)}}\right)}{5\sqrt{-(-1)^{2/5}\left(1+(-1)^{2/5}\right)}}-\frac{2(-1)^{4/5}\tanh^{-1}\left(\frac{1-(-1)^{4/5}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}}-\frac{2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^5)^(-1),x]
```

```
[Out] (-2*(-1)^(3/5)*ArcTan[(1 + (-1)^(3/5)*Tanh[x/2])/Sqrt[-1 + (-1)^(1/5)]])/(5
*Sqrt[-1 + (-1)^(1/5)]) + (2*(-1)^(9/10)*ArcTan[(I - (-1)^(9/10)*Tanh[x/2])
/Sqrt[1 + (-1)^(4/5)]])/(5*Sqrt[1 + (-1)^(4/5)]) - (Sqrt[2]*ArcTanh[(1 - Ta
nh[x/2])/Sqrt[2]])/5 + (2*(-1)^(9/10)*ArcTanh[((-1)^(7/10)*(1 + (-1)^(1/5)*
Tanh[x/2]))/Sqrt[-((-1)^(2/5)*(1 + (-1)^(2/5)))]])/(5*Sqrt[-((-1)^(2/5)*(1
+ (-1)^(2/5)))]) - (2*(-1)^(4/5)*ArcTanh[(1 - (-1)^(4/5)*Tanh[x/2])/Sqrt[1
- (-1)^(3/5)]])/(5*Sqrt[1 - (-1)^(3/5)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{1+\sinh^5(x)}\,dx = \int\left(-\frac{(-1)^{9/10}}{5\left(-(-1)^{9/10}-i\sinh(x)\right)}-\frac{(-1)^{9/10}}{5\left(-(-1)^{9/10}-\sqrt[10]{-1}\,\sinh(x)\right)}-\frac{(-1)^{9/10}}{5\left(-(-1)^{9/10}+(-1)^{3/10}\sin\right.}\right.$$

$$= -\left(\frac{1}{5}(-1)^{9/10}\int\frac{1}{-(-1)^{9/10}-i\sinh(x)}\,dx\right)-\frac{1}{5}(-1)^{9/10}\int\frac{1}{-(-1)^{9/10}-\sqrt[10]{-1}\,\sinh(x)}\,dx-\frac{1}{5}(-$$

$$= -\left(\frac{1}{5}\left(2(-1)^{9/10}\right)\text{Subst}\left(\int\frac{1}{-(-1)^{9/10}-2ix+(-1)^{9/10}x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\right)-\frac{1}{5}\left(2(-1)^{9/10}\right)\text{Su}$$

$$= -\left(\frac{2}{5}\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,1-\tanh\left(\frac{x}{2}\right)\right)\right)+\frac{1}{5}\left(4(-1)^{9/10}\right)\text{Subst}\left(\int\frac{1}{4(-1)^{3/5}\left(1-\sqrt[5]{-1}\right)\cdots}\right.$$

$$= \frac{2(-1)^{9/10}\tan^{-1}\left(\frac{i-(-1)^{9/10}\tanh\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{4/5}}}\right)}{5\sqrt{1+(-1)^{4/5}}}-\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{2\sqrt[10]{-1}\tanh^{-1}\left(\frac{i-\sqrt[10]{-1}\,\text{ta}\cdots}{\sqrt{-1+\sqrt[5]{\cdots}}}\right)}{5\sqrt{-1+\sqrt[5]{-1}}}$$

Mathematica [C] time = 1.03, size = 439, normalized size = 1.81

$$\frac{1}{10}\left(2\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)-\text{RootSum}\left[\#1^8-2\#1^7-2\#1^5+14\#1^4+2\#1^3+2\#1+1\&,\frac{\#1^6x+2\#1^6\log\left(-\cdots\right.}{\cdots}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^5)^(-1),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(-1 + Tanh[x/2])/Sqrt[2]] - RootSum[1 + 2*#1 + 2*#1^3 +
14*#1^4 - 2*#1^5 - 2*#1^7 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + C
osh[x/2]*#1 - Sinh[x/2]*#1] - 4*x*#1 - 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[
x/2]*#1 - Sinh[x/2]*#1]*#1 - 9*x*#1^2 - 18*Log[-Cosh[x/2] - Sinh[x/2] + Cos
h[x/2]*#1 - Sinh[x/2]*#1]*#1^2 - 24*x*#1^3 - 48*Log[-Cosh[x/2] - Sinh[x/2]
+ Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^3 + 9*x*#1^4 + 18*Log[-Cosh[x/2] - Sinh[x
/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 - 4*x*#1^5 - 8*Log[-Cosh[x/2] - Sin
h[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Si
nh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(1 + 3*#1^2 + 28*#1^3 - 5*#1^4
 - 7*#1^6 + 4*#1^7) & ])/10
```

fricas [B] time = 1.24, size = 3507, normalized size = 14.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^5),x, algorithm="fricas")
```

```
[Out] -1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5)
 + 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt
(5) + 3)*arctan(1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x - 7*sqrt(5) + 15)*sqrt(
2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 1/80*sqrt(2)*(sqrt(2)*((11*sqrt(5) - 25)
*e^x + 4*sqrt(5) - 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((3*sqrt(5
) - 5)*e^x + 7*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(5) + 3) + 1/128
00*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 40*
sqrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2
*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) + sqrt(2*sqrt(2)*(2*s
qrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(7*sqrt(5) - 15)*
sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5)
+ 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)*sqrt(2*sqrt(5)
+ 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5
) + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqrt(-20*sqrt(2)*sq
rt(sqrt(5) + 3)*(sqrt(5) - 3) + 40*(sqrt(5) - 1)*e^x - 2*(sqrt(2)*((2*sqrt(
5) - 5)*e^x - 3*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) - 5)*e^x - 3*sq
rt(5) + 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 2
0)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2*sqrt(2)*(2*sqrt
(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*sqrt(5) - 7)*e^x
 + 8*sqrt(5) - 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) -
11)*e^x + 2*sqrt(5) - 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(s
qrt(2)*((7*sqrt(5) - 17)*e^x + 8*sqrt(5) - 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqr
t(5) + 3) + 2*((5*sqrt(5) - 11)*e^x + 5*sqrt(5) - 9)*sqrt(2*sqrt(5) + 5))*(
8*sqrt(5) + 24)^(1/4)) + 1/20*(2*(4*sqrt(5) - 5)*e^x - sqrt(5) + 5)*sqrt(2*
sqrt(5) + 5)) - 1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) +
 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5)
 + 5)*sqrt(sqrt(5) + 3)*arctan(-1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x - 7*sqr
t(5) + 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) - 1/80*sqrt(2)*(sqrt(2)*((
11*sqrt(5) - 25)*e^x + 4*sqrt(5) - 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3
) + 2*((3*sqrt(5) - 5)*e^x + 7*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt
(5) + 3) - 1/12800*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sq
rt(5) + 3) + 40*sqrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(
sqrt(5) + 3) + 2*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) - sqr
t(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(
7*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)
*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)
*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5)
 + 5))*(8*sqrt(5) + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqr
t(-20*sqrt(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) + 40*(sqrt(5) - 1)*e^x + 2*(s
qrt(2)*((2*sqrt(5) - 5)*e^x - 3*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5)
 - 5)*e^x - 3*sqrt(5) + 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3)
 - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2
*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*
sqrt(5) - 7)*e^x + 8*sqrt(5) - 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) +
2*((5*sqrt(5) - 11)*e^x + 2*sqrt(5) - 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) +
24)^(3/4) + 4*(sqrt(2)*((7*sqrt(5) - 17)*e^x + 8*sqrt(5) - 18)*sqrt(2*sqrt(
5) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) - 11)*e^x + 5*sqrt(5) - 9)*sqrt(2
*sqrt(5) + 5))*(8*sqrt(5) + 24)^(1/4)) - 1/20*(2*(4*sqrt(5) - 5)*e^x - sqrt
(5) + 5)*sqrt(2*sqrt(5) + 5)) + 1/400*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5)
 + 24) + 4*sqrt(5) + 20)*(3*sqrt(5) + 5)*sqrt(-2*sqrt(5) + 5)*(-8*sqrt(5) +
 24)^(3/4)*arctan(-1/1280*((4*(5*sqrt(5) + 11)*e^x + ((3*sqrt(5) + 7)*e^x +
 8*sqrt(5) + 18)*sqrt(-8*sqrt(5) + 24) + 8*sqrt(5) + 16)*(-8*sqrt(5) + 24)^
(3/4) + 4*(4*(5*sqrt(5) + 11)*e^x + ((7*sqrt(5) + 17)*e^x + 8*sqrt(5) + 18)
*sqrt(-8*sqrt(5) + 24) + 20*sqrt(5) + 36)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2
*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5)
+ 1/25600*((((7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8
```

```
*sqrt(5) + 24)^(3/4) + 4*((17*sqrt(5) + 35)*sqrt(-8*sqrt(5) + 24) + 44*sqrt
(5) + 100)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) +
 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) + 20*(((5*sqrt(5) + 11)*sqrt(-8
*sqrt(5) + 24) + 4*sqrt(5) + 12)*sqrt(-8*sqrt(5) + 24) + 4*(5*sqrt(5) + 11)
*sqrt(-8*sqrt(5) + 24) + 32*sqrt(5) + 128)*sqrt(-2*sqrt(5) + 5))*sqrt(-40*(
sqrt(5) + 1)*e^x + (4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5)
 - 5)*sqrt(-8*sqrt(5) + 24) - 6*sqrt(5) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*
sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 10*(sqrt(5) + 3)*
sqrt(-8*sqrt(5) + 24) + 80*e^(2*x) + 80) - 1/320*(32*(4*sqrt(5) + 5)*e^x +
4*((11*sqrt(5) + 25)*e^x - 7*sqrt(5) - 15)*sqrt(-8*sqrt(5) + 24) + (4*(3*sq
rt(5) + 5)*e^x + ((11*sqrt(5) + 25)*e^x + 4*sqrt(5) + 10)*sqrt(-8*sqrt(5) +
 24) + 28*sqrt(5) + 60)*sqrt(-8*sqrt(5) + 24) - 16*sqrt(5) - 80)*sqrt(-2*sq
rt(5) + 5)) + 1/400*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5)
 + 20)*(3*sqrt(5) + 5)*sqrt(-2*sqrt(5) + 5)*(-8*sqrt(5) + 24)^(3/4)*arctan(
-1/1280*((4*(5*sqrt(5) + 11)*e^x + ((3*sqrt(5) + 7)*e^x + 8*sqrt(5) + 18)*s
qrt(-8*sqrt(5) + 24) + 8*sqrt(5) + 16)*(-8*sqrt(5) + 24)^(3/4) + 4*(4*(5*sq
rt(5) + 11)*e^x + ((7*sqrt(5) + 17)*e^x + 8*sqrt(5) + 18)*sqrt(-8*sqrt(5) +
 24) + 20*sqrt(5) + 36)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt
(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) + 1/25600*((((7*sq
rt(5) + 15)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*sqrt(5) + 24)^(3/
4) + 4*((17*sqrt(5) + 35)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*sqr
t(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) +
 20)*sqrt(-2*sqrt(5) + 5) - 20*(((5*sqrt(5) + 11)*sqrt(-8*sqrt(5) + 24) + 4
*sqrt(5) + 12)*sqrt(-8*sqrt(5) + 24) + 4*(5*sqrt(5) + 11)*sqrt(-8*sqrt(5) +
 24) + 32*sqrt(5) + 128)*sqrt(-2*sqrt(5) + 5))*sqrt(-40*(sqrt(5) + 1)*e^x -
 (4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5) - 5)*sqrt(-8*sqrt(
5) + 24) - 6*sqrt(5) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*
sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 10*(sqrt(5) + 3)*sqrt(-8*sqrt(5) +
24) + 80*e^(2*x) + 80) + 1/320*(32*(4*sqrt(5) + 5)*e^x + 4*((11*sqrt(5) + 2
5)*e^x - 7*sqrt(5) - 15)*sqrt(-8*sqrt(5) + 24) + (4*(3*sqrt(5) + 5)*e^x + (
(11*sqrt(5) + 25)*e^x + 4*sqrt(5) + 10)*sqrt(-8*sqrt(5) + 24) + 28*sqrt(5)
+ 60)*sqrt(-8*sqrt(5) + 24) - 16*sqrt(5) - 80)*sqrt(-2*sqrt(5) + 5)) + 1/80
0*(sqrt(2)*(3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 8*sqrt(5))*sqrt(2*sqrt(2)*(2
*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*lo
g(-4*sqrt(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) + 8*(sqrt(5) - 1)*e^x + 2/5*(s
qrt(2)*((2*sqrt(5) - 5)*e^x - 3*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5)
 - 5)*e^x - 3*sqrt(5) + 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3)
 - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 16*e^(2*x) + 16) - 1/800*(sqrt(
2)*(3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 8*sqrt(5))*sqrt(2*sqrt(2)*(2*sqrt(5)
 - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*log(-4*sqr
t(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) + 8*(sqrt(5) - 1)*e^x - 2/5*(sqrt(2)*(
(2*sqrt(5) - 5)*e^x - 3*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) - 5)*e^
x - 3*sqrt(5) + 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqr
t(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 16*e^(2*x) + 16) + 1/1600*((3*sqrt(5) +
 5)*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(
5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4)*log(-8*(sqrt(5) + 1)*e^x
 + 1/5*(4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5) - 5)*sqrt(-8
*sqrt(5) + 24) - 6*sqrt(5) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24
) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(
5) + 24) + 16*e^(2*x) + 16) - 1/1600*((3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24)
 + 16*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20
)*(-8*sqrt(5) + 24)^(1/4)*log(-8*(sqrt(5) + 1)*e^x - 1/5*(4*(sqrt(5) + 5)*e
^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5) - 5)*sqrt(-8*sqrt(5) + 24) - 6*sqrt(5
) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*s
qrt(5) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(5) + 24) + 16*e^(2*x) + 1
6) + 1/10*sqrt(2)*log(-(2*(sqrt(2) - 1)*e^x + 2*sqrt(2) - e^(2*x) - 3)/(e^(
2*x) + 2*e^x - 1))
```

giac [B] time = 2.86, size = 5246, normalized size = 21.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^5),x, algorithm="giac")
```

```
[Out] -8/25*5^(3/4)*sqrt(-1/32*sqrt(5) + 5/64)*arctan(2*(478931007287593595120 0*5
^(3/4)*sqrt(-2*sqrt(5) + 5) - 179974555429306222868768 0*sqrt(5)*sqrt(-2*sqr
t(5) + 5) - 3259140419799022448132 89*5^(3/4) - 1052060654860084919056 0*5^(1
/4)*sqrt(-2*sqrt(5) + 5) + 2650333406778869800551 83*sqrt(5) + 4025305730691
667696322880*sqrt(-2*sqrt(5) + 5) + 7288552456584503439489 19*5^(1/4) - 1637
333558120632636*e^x - 592460559708252630357201)/(920275442749632131440 6*5^(
3/4)*sqrt(-2*sqrt(5) + 5) + 1038239983143393667165790*sqrt(5)*sqrt(-2*sqrt(
5) + 5) + 186591807316241026405751*5^(3/4) - 207682196953203925502 10*5^(1/4
)*sqrt(-2*sqrt(5) + 5) + 576155331981489353033147*sqrt(5) - 232237011952592
5506249090*sqrt(-2*sqrt(5) + 5) - 417362544266571988465273*5^(1/4) - 128878
458038145102867 2113)) + 8/25*5^(3/4)*sqrt(-1/32*sqrt(5) + 5/64)*arctan(2*(4
315023771046590689440*5^(3/4)*sqrt(-2*sqrt(5) + 5) + 165124224190524729732 4
4480*sqrt(5)*sqrt(-2*sqrt(5) + 5) + 299155918195015663509604 1*5^(3/4) - 114
1548896112805999856 0*5^(1/4)*sqrt(-2*sqrt(5) + 5) + 476470546695231758102 79
9*sqrt(5) - 369310363594993781632412 80*sqrt(-2*sqrt(5) + 5) - 6690300251147
369625285239*5^(1/4) - 1637333558120632636*e^x - 1067630744269504182665681)
/(537689066142690749994*5^(3/4)*sqrt(-2*sqrt(5) + 5) + 45162997328032147105
966190*sqrt(5)*sqrt(-2*sqrt(5) + 5) + 818625762218671097515875 7*5^(3/4) - 5
838120100393683185390*5^(1/4)*sqrt(-2*sqrt(5) + 5) + 603739022767920301057 0
79*sqrt(5) - 1010088867986392440600019 70*sqrt(-2*sqrt(5) + 5) - 18307539608
818658210592747*5^(1/4) - 1355343042548851351155477)) - 1/10*sqrt(sqrt(5) +
 2)*log((302427386195713850867712*sqrt(5)*(2*sqrt(5) + 5)^3 + 1728156492546
93629067264*(2*sqrt(5) + 5)^(7/2) + 226820539646785388150784*sqrt(5)*(2*sqr
t(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 151213693097856925433856*(2*sqrt(5) + 5
)^3*sqrt(sqrt(5) + 2) + 708814186396204337971 20*sqrt(5)*(2*sqrt(5) + 5)^2*(
sqrt(5) + 2) + 567051349116963470376 96*(2*sqrt(5) + 5)^(5/2)*(sqrt(5) + 2)
+ 118135697732700722995 20*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2)
 + 118135697732700722995 20*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2) + 11075221
66244069278080*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 147669622165875903
7440*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 553761083122034639 04*sqrt(5)*s
qrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 110752216624406927808*(2*sqrt(5) +
 5)*(sqrt(5) + 2)^(5/2) + 1153668923170905498*sqrt(5)*(sqrt(5) + 2)^3 + 461
4675692683621992*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 82404923083636107*(s
qrt(5) + 2)^(7/2) - 622619531678741564620800*sqrt(5)*(2*sqrt(5) + 5)^(5/2)
- 415079687785827709747200*(2*sqrt(5) + 5)^3 - 389137207299213477888000*sqr
t(5)*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 311309765839370782310400*(2*sqrt
(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) - 97284301824803369472000*sqrt(5)*(2*sqrt(
5) + 5)^(3/2)*(sqrt(5) + 2) - 97284301824803369472000*(2*sqrt(5) + 5)^2*(sq
rt(5) + 2) - 12160537728100421184000*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^
(3/2) - 16214050304133894912000*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2) -
 760033608006276324000*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 152006
7216012552648000*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 19000840200156908100*sqr
t(5)*(sqrt(5) + 2)^(5/2) - 76003360800627632400*sqrt(2*sqrt(5) + 5)*(sqrt(5
) + 2)^(5/2) - 1583403350013075675*(sqrt(5) + 2)^3 - 346430300390652274610 1
760*sqrt(5)*(2*sqrt(5) + 5)^2 - 201537393763593356971212 8*(2*sqrt(5) + 5)^(
5/2) - 173215150195326137305088 0*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5)
 + 2) - 125960871102245848107008 0*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 324
778406616236507447040*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 3149021777556
14620267520*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 270648672180197089539 20*s
qrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 393627722194518275334 40*(2
*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 845777100563115904810*sqrt(5)*(sqrt(5)
+ 2)^2 - 2460173263715739220840*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 61504
```

331592893480521*(sqrt(5) + 2)^(5/2) + 3959703717250098693214208*sqrt(5)*(2*
sqrt(5) + 5)^(3/2) + 2662579692919387100254208*(2*sqrt(5) + 5)^2 + 14848888
93968787009955328*sqrt(5)*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 1331289846459
693550127104*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) + 18561111174609837624
4416*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) + 249616846211192540648832*(
2*sqrt(5) + 5)*(sqrt(5) + 2) + 77337963227540990101 84*sqrt(5)*(sqrt(5) + 2)
^(3/2) + 20801403850932711720736*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) +
6500438703416472412 73*(sqrt(5) + 2)^2 + 1099194045690938228328 2816*sqrt(5)*
(2*sqrt(5) + 5) + 856705374208110320622 0288*(2*sqrt(5) + 5)^(3/2) + 2747985
11422734557082070 4*sqrt(5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 32126451
53280413702332608*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 17174906963920909817 6
294*sqrt(5)*(sqrt(5) + 2) + 40158064416005171279 1576*sqrt(2*sqrt(5) + 5)*(s
qrt(5) + 2) + 16732526840002154699649*(sqrt(5) + 2)^(3/2) - 255726977589952
5489493536*sqrt(5)*sqrt(2*sqrt(5) + 5) - 3196587219874406861866 92*sqrt(5)*s
qrt(sqrt(5) + 2) + 398427752115625714426 72*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5)
 + 2) - 3308326863346966249269767*sqrt(5) - 4580301686563984868886360*sqrt(
2*sqrt(5) + 5) - 572537710820498108610795*sqrt(sqrt(5) + 2) + 2850824269841
065226382633)^2 + 64*(2432282250124078193049 6*sqrt(5)*(2*sqrt(5) + 5)^3 + 1
38987557149947325317 12*(2*sqrt(5) + 5)^(7/2) + 1824211687593058644787 2*sqrt
(5)*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 12161411250620390965248*(2*sq
rt(5) + 5)^3*sqrt(sqrt(5) + 2) + 570066152372830826496 0*sqrt(5)*(2*sqrt(5)
 + 5)^2*(sqrt(5) + 2) + 4560529218982646611968*(2*sqrt(5) + 5)^(5/2)*(sqrt(5
) + 2) + 950110253954718044160*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^
(3/2) + 950110253954718044160*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2) + 89072
836308254816640*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 11876378174433975
5520*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 4453641815412740832*sqrt(5)*sq
rt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 8907283630825481664*(2*sqrt(5) + 5)
*(sqrt(5) + 2)^(5/2) + 927842044877654 34*sqrt(5)*(sqrt(5) + 2)^3 + 37113681
7951061736*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 6627443177697531*(sqrt(5)
 + 2)^(7/2) - 13726081827177108602880*sqrt(5)*(2*sqrt(5) + 5)^(5/2) - 915072
1218118072401920*(2*sqrt(5) + 5)^3 - 8578801141985692876800*sqrt(5)*(2*sqrt
(5) + 5)^2*sqrt(sqrt(5) + 2) - 6863040913588554301440*(2*sqrt(5) + 5)^(5/2)
*sqrt(sqrt(5) + 2) - 2144700285496423219200*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(
sqrt(5) + 2) - 2144700285496423219200*(2*sqrt(5) + 5)^2*(sqrt(5) + 2) - 268
087535687052902400*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 3574500475
82737203200*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2) - 1675547098044080640
0*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 33510941960881612800*(2*sqr
t(5) + 5)*(sqrt(5) + 2)^2 - 418886774511020160*sqrt(5)*(sqrt(5) + 2)^(5/2)
 - 1675547098044080640*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) - 34907231209
251680*(sqrt(5) + 2)^3 - 3231678023348352407552 00*sqrt(5)*(2*sqrt(5) + 5)^2
 - 1977271856147662377779 20*(2*sqrt(5) + 5)^(5/2) - 16158390116741762037760
0*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) - 123579491009228898611 20
0*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 30296981468890803820800*sqrt(5)*(2*
sqrt(5) + 5)*(sqrt(5) + 2) - 30894872752307224652800*(2*sqrt(5) + 5)^(3/2)*
(sqrt(5) + 2) - 2524748455740900318400*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5)
 + 2)^(3/2) - 3861859094038403081600*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) -
78898389241903134950*sqrt(5)*(sqrt(5) + 2)^2 - 241366193377400192600*sqrt(2
sqrt(5) + 5)(sqrt(5) + 2)^2 - 6034154834435004815*(sqrt(5) + 2)^(5/2) - 1
00890523270644033265664*sqrt(5)*(2*sqrt(5) + 5)^(3/2) - 1294865270772630095
21664*(2*sqrt(5) + 5)^2 - 37833946226491512474624*sqrt(5)*(2*sqrt(5) + 5)*s
qrt(sqrt(5) + 2) - 64743263538631504760832*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(
5) + 2) - 4729243278311439059328*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)
 - 12139361913493407142656*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 19705180326297662
7472*sqrt(5)*(sqrt(5) + 2)^(3/2) - 1011613492791117261888*sqrt(2*sqrt(5) +
5)*(sqrt(5) + 2)^(3/2) - 31612921649722414434*(sqrt(5) + 2)^2 + 97605666773
8889843134336*sqrt(5)*(2*sqrt(5) + 5) + 7374596129883352417428 48*(2*sqrt(5)
 + 5)^(3/2) + 244014166934722460783584*sqrt(5)*sqrt(2*sqrt(5) + 5)*sqrt(sqr
t(5) + 2) + 276547354870625715653568*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 15
2508854334201537989 74*sqrt(5)*(sqrt(5) + 2) + 34568419358828214456696*sqrt(

```
2*sqrt(5) + 5)*(sqrt(5) + 2) + 1440350806617842269029*(sqrt(5) + 2)^(3/2) +
 865777074090951821677952*sqrt(5)*sqrt(2*sqrt(5) + 5) + 1082221342613689777
09744*sqrt(5)*sqrt(sqrt(5) + 2) + 403147498761313336459456*sqrt(2*sqrt(5) +
 5)*sqrt(sqrt(5) + 2) + 339901475433043622828423 4*sqrt(5) + 147978369874753
0204584760*sqrt(2*sqrt(5) + 5) + 184972962343441275573095*sqrt(sqrt(5) + 2)
 - 1312912080621749387731 04*e^x + 870069461703628226688110 2)^2) + 1/10*sqrt
(sqrt(5) + 2)*log((2967777257833108576665 60*sqrt(5)*(2*sqrt(5) + 5)^3 + 169
58727187617763295232 0*(2*sqrt(5) + 5)^(7/2) + 222583294337483143249920*sqrt
(5)*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 148388862891655428833280*(2*s
qrt(5) + 5)^3*sqrt(sqrt(5) + 2) + 69557279480463482265600*sqrt(5)*(2*sqrt(5
) + 5)^2*(sqrt(5) + 2) + 55645823584370785812480*(2*sqrt(5) + 5)^(5/2)*(sqr
t(5) + 2) + 11592879913410580377600*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5)
+ 2)^(3/2) + 11592879913410580377600*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2)
+ 1086832491882241910400*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 14491099
89176322547200*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 54341624594112095520
*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 108683249188224191040*(2
*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 1132117179044001990*sqrt(5)*(sqrt(5) +
2)^3 + 4528468716176007960*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 8086551278
8857285*(sqrt(5) + 2)^(7/2) - 562345414061023649464320*sqrt(5)*(2*sqrt(5) +
 5)^(5/2) - 374896942707349099642880*(2*sqrt(5) + 5)^3 - 351465883788139780
915200*sqrt(5)*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 281172707030511824732 1
60*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) - 87866470947034945228800*sqrt(5
)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 87866470947034945228800*(2*sqrt(5)
+ 5)^2*(sqrt(5) + 2) - 10983308868379368153600*sqrt(5)*(2*sqrt(5) + 5)*(sqr
t(5) + 2)^(3/2) - 14644411824505824204800*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) +
2)^(3/2) - 686456804273710509600*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^
2 - 1372913608547421019200*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 17161420106842
762740*sqrt(5)*(sqrt(5) + 2)^(5/2) - 68645680427371050960*sqrt(2*sqrt(5) +
5)*(sqrt(5) + 2)^(5/2) - 1430118342236896895*(sqrt(5) + 2)^3 - 352131158943
7476455997440*sqrt(5)*(2*sqrt(5) + 5)^2 - 2075104957091704020631552*(2*sqrt
(5) + 5)^(5/2) - 1760655794718738227998720*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*sq
rt(sqrt(5) + 2) - 1296940598182315012894720*(2*sqrt(5) + 5)^2*sqrt(sqrt(5)
+ 2) - 330122961509763417749760*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 324
235149545578753223680*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 275102467924802
84812480*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 4052939369319734
4152960*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 859695212265008900390*sqrt(5)
*(sqrt(5) + 2)^2 - 2533087105824834009560*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)
^2 - 633271776456208502 39*(sqrt(5) + 2)^(5/2) + 342706658437651377679974 4*s
qrt(5)*(2*sqrt(5) + 5)^(3/2) + 2255513638416047840415744*(2*sqrt(5) + 5)^2
+ 1285149969141192666299904*sqrt(5)*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 112
7756819208023920207872*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) + 1606437461
42649083287488*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) + 2114544036015044
85038976*(2*sqrt(5) + 5)*(sqrt(5) + 2) + 6693489422610378470312*sqrt(5)*(sq
rt(5) + 2)^(3/2) + 17621200300125373753248*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2
)^(3/2) + 550662509378917929789*(sqrt(5) + 2)^2 + 115193815540469058484814 0
8*sqrt(5)*(2*sqrt(5) + 5) + 918117929197579822791014 4*(2*sqrt(5) + 5)^(3/2)
 + 287984538851172646212035 2*sqrt(5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2)
+ 344294223449092433546630 4*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 17999033678
1982903882522*sqrt(5)*(sqrt(5) + 2) + 430367779311365541933288*sqrt(2*sqrt(
5) + 5)*(sqrt(5) + 2) + 17931990804640230913887*(sqrt(5) + 2)^(3/2) - 14939
8518691580642197238 4*sqrt(5)*sqrt(2*sqrt(5) + 5) - 18674814836447580274654 8
*sqrt(5)*sqrt(sqrt(5) + 2) + 397187773282286465287728*sqrt(2*sqrt(5) + 5)*s
qrt(sqrt(5) + 2) - 590811650205465011212999*sqrt(5) - 478425688960650950842
0584*sqrt(2*sqrt(5) + 5) - 598032111200813688552573*sqrt(sqrt(5) + 2) + 961
326558324007707256106 9)^2 + 64*(280947366475268436787 20*sqrt(5)*(2*sqrt(5)
+ 5)^3 + 160541352271581963878 40*(2*sqrt(5) + 5)^(7/2) + 210710524856451327
59040*sqrt(5)*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 1404736832376342183
9360*(2*sqrt(5) + 5)^3*sqrt(sqrt(5) + 2) + 6584703901764103987200*sqrt(5)*(
2*sqrt(5) + 5)^2*(sqrt(5) + 2) + 5267763121411283189760*(2*sqrt(5) + 5)^(5/
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2)*(sqrt(5) + 2) + 1097450650294017331200*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sq
rt(5) + 2)^(3/2) + 1097450650294017331200*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(
3/2) + 102885998465064124800*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 1371
81331286752166400*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 51442999232532062
40*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 10288599846506412480*(
2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 107172915067775130*sqrt(5)*(sqrt(5) +
2)^3 + 428691660271100520*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 76552082191
26795*(sqrt(5) + 2)^(7/2) - 20755836954363830992896*sqrt(5)*(2*sqrt(5) + 5)
^(5/2) - 13837224636242553995264*(2*sqrt(5) + 5)^3 - 1297239809647739437056
0*sqrt(5)*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 10377918477181915496448*(2*
sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) - 3243099524119348592640*sqrt(5)*(2*sq
rt(5) + 5)^(3/2)*(sqrt(5) + 2) - 3243099524119348592640*(2*sqrt(5) + 5)^2*(
sqrt(5) + 2) - 405387440514918574080*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^
(3/2) - 540516587353224765440*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2) - 2
5336715032182410880*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 506734300
64364821760*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 633417875804560272*sqrt(5)*(s
qrt(5) + 2)^(5/2) - 2533671503218241088*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(
5/2) - 52784822983713356*(sqrt(5) + 2)^3 - 363528280045460787978240*sqrt(5)
*(2*sqrt(5) + 5)^2 - 220585782417551521185792*(2*sqrt(5) + 5)^(5/2) - 18176
4140022730393989120*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) - 13786
6114010969700741120*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 34080776254261948
872960*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 34466528502742425185280*(2*s
qrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 2840064687855162406080*sqrt(5)*sqrt(2*sqr
t(5) + 5)*(sqrt(5) + 2)^(3/2) - 4308316062842803148160*(2*sqrt(5) + 5)*(sqr
t(5) + 2)^(3/2) - 88752021495473825190*sqrt(5)*(sqrt(5) + 2)^2 - 2692697539
27675196760*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 6731743848191879919*(sqrt
(5) + 2)^(5/2) - 70364981699301709291520*sqrt(5)*(2*sqrt(5) + 5)^(3/2) - 11
3606308687559690526720*(2*sqrt(5) + 5)^2 - 26386868137238140984320*sqrt(5)*
(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) - 56803154343779845263360*(2*sqrt(5) + 5)
^(3/2)*sqrt(sqrt(5) + 2) - 3298358517154767623040*sqrt(5)*sqrt(2*sqrt(5) +
5)*(sqrt(5) + 2) - 10650591439458720986880*(2*sqrt(5) + 5)*(sqrt(5) + 2) -
137431604881448650960*sqrt(5)*(sqrt(5) + 2)^(3/2) - 887549286621560082240*s
qrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 27735915206923752570*(sqrt(5) + 2)
^2 + 1084940669680612606048128*sqrt(5)*(2*sqrt(5) + 5) + 811441742157208365
935104*(2*sqrt(5) + 5)^(3/2) + 271235167420153151512032*sqrt(5)*sqrt(2*sqrt
(5) + 5)*sqrt(sqrt(5) + 2) + 304290653308953137225664*(2*sqrt(5) + 5)*sqrt(
sqrt(5) + 2) + 16952197963759571969502*sqrt(5)*(sqrt(5) + 2) + 380363316636
19142153208*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) + 1584847152650797589717*(sqr
t(5) + 2)^(3/2) + 916330240481116591230464*sqrt(5)*sqrt(2*sqrt(5) + 5) + 11
4541280060139573903808*sqrt(5)*sqrt(sqrt(5) + 2) + 438878646396292635288832
*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 3690050822522820384588494*sqrt(5)
+ 1591445182365082778211384*sqrt(2*sqrt(5) + 5) + 198930647795635347276423*
sqrt(sqrt(5) + 2) + 131291208062174938773104*e^x + 924005503530164856340594
2)^2) + 1/10*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*e^x + 2)/(sqrt(2) + e^x + 1
)) - 1/10*5^(1/4)*log(6400*(9202754427496321314406*5^(3/4)*sqrt(-2*sqrt(5)
+ 5) + 1038239983143393667165790*sqrt(5)*sqrt(-2*sqrt(5) + 5) + 18659180731
6241026405751*5^(3/4) - 20768219695320392550210*5^(1/4)*sqrt(-2*sqrt(5) + 5
) + 576155331981489353033147*sqrt(5) - 2322370119525925506249090*sqrt(-2*sq
rt(5) + 5) - 417362544266571988465273*5^(1/4) - 1288784580381451028672113)^
2 + 25600*(4789310072875935951200*5^(3/4)*sqrt(-2*sqrt(5) + 5) - 1799745554
293062228687680*sqrt(5)*sqrt(-2*sqrt(5) + 5) - 325914041979902244813289*5^(
3/4) - 10520606548600849190560*5^(1/4)*sqrt(-2*sqrt(5) + 5) + 2650333406778
86980055183*sqrt(5) + 4025305730691667696322880*sqrt(-2*sqrt(5) + 5) + 7288
55245658450343948919*5^(1/4) - 1637333558120632636*e^x - 592460559708252630
357201)^2) + 1/10*5^(1/4)*log(25600*(4315023771046590689440*5^(3/4)*sqrt(-2
*sqrt(5) + 5) + 16512422419052472973244480*sqrt(5)*sqrt(-2*sqrt(5) + 5) + 2
991559181950156635096041*5^(3/4) - 11415488961128059998560*5^(1/4)*sqrt(-2*
sqrt(5) + 5) + 476470546695231758102799*sqrt(5) - 3693103635949937816324128
0*sqrt(-2*sqrt(5) + 5) - 6690300251147369625285239*5^(1/4) - 16373335581206

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32636*e^x - 1067630744269504182665681)^2 + 6400*(537689066142690749994*5^(3
/4)*sqrt(-2*sqrt(5) + 5) + 451629973280321471059661 90*sqrt(5)*sqrt(-2*sqrt(
5) + 5) + 818625762218671097515875 7*5^(3/4) - 583812010039368318539 0*5^(1/4
)*sqrt(-2*sqrt(5) + 5) + 603739022767920301057079*sqrt(5) - 101008886798639
244060001970*sqrt(-2*sqrt(5) + 5) - 1830753960881865821059274 7*5^(1/4) - 13
553430425488513511554 77)^2) - 1/5*sqrt(2*sqrt(5) + 5)*arctan(-(110641272*(2
*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^7 - 475726088*(2*sqrt(5
) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^6 - 10105915139*(2*sqrt(5) + s
qrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^5 + 16180495104*(2*sqrt(5) + sqrt(2
*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^4 + 284235586966*(2*sqrt(5) + sqrt(2*sqr
t(5) + 5) + sqrt(sqrt(5) + 2))^3 - 13398309260*(2*sqrt(5) + sqrt(2*sqrt(5)
+ 5) + sqrt(sqrt(5) + 2))^2 - 4747850205816*sqrt(5) - 2373925102908*sqrt(2*
sqrt(5) + 5) - 2373925102908*sqrt(sqrt(5) + 2) - 759635933456*e^x - 1242609
575248)/(256556994*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^7
- 892031217*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^6 - 25195
966133*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^5 + 2895270815
8*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^4 + 709750301398*(2
*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^3 + 80692042496*(2*sqrt
(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^2 - 11068354399432*sqrt(5) -
 5534177199716*sqrt(2*sqrt(5) + 5) - 5534177199716*sqrt(sqrt(5) + 2) - 3881
375121088))/sqrt(sqrt(5) + 2) + 1/5*sqrt(2*sqrt(5) + 5)*arctan(-(83633448*(
2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^7 - 442112756*(2*sqrt(
5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^6 - 7188799155*(2*sqrt(5) + s
qrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^5 + 18979817940*(2*sqrt(5) + sqrt(2
*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^4 + 194564340278*(2*sqrt(5) + sqrt(2*sqr
t(5) + 5) + sqrt(sqrt(5) + 2))^3 - 178069044908*(2*sqrt(5) + sqrt(2*sqrt(5)
 + 5) + sqrt(sqrt(5) + 2))^2 - 2862929298552*sqrt(5) - 1431464649276*sqrt(2
*sqrt(5) + 5) - 1431464649276*sqrt(sqrt(5) + 2) + 759635933456*e^x - 101449
315520)/(82684590*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^7 -
 41690029*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^6 - 1005292
8883*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^5 - 3266507166*(
2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^4 + 302724737258*(2*sq
rt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^3 + 148206122616*(2*sqrt(5
) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^2 - 4842785241848*sqrt(5) - 24
21392620924*sqrt(2*sqrt(5) + 5) - 2421392620924*sqrt(sqrt(5) + 2) - 5110771
00176))/sqrt(sqrt(5) + 2)
```

maple [C] time = 0.07, size = 124, normalized size = 0.51

$$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+2\_Z^7+2\_Z^5+14\_Z^4-2\_Z^3-2\_Z+1\right)}\frac{\left(-2\_R^6-3\_R^5+2\_R^4+2\_R^3-2\_R^2-3\_R+2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{4\_R^7+7\_R^6+5\_R^4+28\_R^3-3\_R^2-1}\right)}{5}+\frac{\sqrt{2}\ \mathrm{arctanh}\left((2\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^5),x)
```

```
[Out] 2/5*sum((-2*_R^6-3*_R^5+2*_R^4+2*_R^3-2*_R^2-3*_R+2)/(4*_R^7+7*_R^6+5*_R^4+
28*_R^3-3*_R^2-1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^8+2*_Z^7+2*_Z^5+14*_Z^4-2
*_Z^3-2*_Z+1))+1/5*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{10}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{x}-1}{\sqrt{2}+e^{x}+1}\right)-\int\frac{2\left(e^{(7x)}-4e^{(6x)}+9e^{(5x)}-24e^{(4x)}-9e^{(3x)}-4e^{(2x)}-e^{x}\right)}{5\left(e^{(8x)}-2e^{(7x)}-2e^{(5x)}+14e^{(4x)}+2e^{(3x)}+2e^{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^5),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(2)*log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) - integrate(2/5*
(e^(7*x) - 4*e^(6*x) + 9*e^(5*x) - 24*e^(4*x) - 9*e^(3*x) - 4*e^(2*x) - e^x
)/(e^(8*x) - 2*e^(7*x) - 2*e^(5*x) + 14*e^(4*x) + 2*e^(3*x) + 2*e^x + 1), x
)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^5 + 1),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sinh(x)+1)\left(\sinh^4(x)-\sinh^3(x)+\sinh^2(x)-\sinh(x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**5),x)
```

```
[Out] Integral(1/((sinh(x) + 1)*(sinh(x)**4 - sinh(x)**3 + sinh(x)**2 - sinh(x) +
 1)), x)
```

3.271 $\int \frac{1}{1+\sinh^6(x)} \, dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\ \tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\ \tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tanh(x)}{3}$$

```
[Out] 1/3*arctanh((1+(-1)^(1/3))^(1/2)*tanh(x))/(1+(-1)^(1/3))^(1/2)+1/3*arctanh(
(1-(-1)^(2/3))^(1/2)*tanh(x))/(1-(-1)^(2/3))^(1/2)+1/3*tanh(x)
```

Rubi [A] time = 0.14, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3211, 3181, 206, 3175, 3767, 8}

$$\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\ \tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\ \tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tanh(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 + (-1)^(1/3)]*Tanh[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTanh[Sq
rt[1 - (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tanh[x]/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{1+\sinh^6(x)}\,dx = \frac{1}{3}\int \frac{1}{1+\sinh^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\sinh^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1+(-1)^{2/3}\sinh^2(x)}\,dx$$

$$= \frac{1}{3}\int \operatorname{sech}^2(x)\,dx + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-\left(1+\sqrt[3]{-1}\right)x^2}\,dx, x, \tanh(x)\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-\left(1-(-\right.}\right.$$

$$= \frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}} + \frac{1}{3}i\operatorname{Subst}(\int 1\,dx, x, -i\,\mathrm{ta}$$

$$= \frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}} + \frac{\tanh(x)}{3}$$

Mathematica [C] time = 0.22, size = 87, normalized size = 1.23

$$\frac{1}{18}\left(6\tanh(x) + \sqrt[4]{-3}\left(\left(-3-i\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\sqrt[4]{-3}\left(1+i\sqrt{3}\right)\tanh(x)\right) - \left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}\left(3+i\sqrt{3}\right)\mathrm{tanh}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^6)^(-1),x]
```

```
[Out] ((-3)^(1/4)*((-3 - I*Sqrt[3])*ArcTan[((-3)^(1/4)*(1 + I*Sqrt[3])*Tanh[x])/2
] - (3*I + Sqrt[3])*ArcTan[((-1/3)^(1/4)*(3 + I*Sqrt[3])*Tanh[x])/2]) + 6*T
anh[x])/18
```

fricas [B] time = 1.08, size = 692, normalized size = 9.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^6),x, algorithm="fricas")
```

```
[Out] -1/144*(4*(12^(1/4)*sqrt(6)*e^(2*x) + 12^(1/4)*sqrt(6))*sqrt(-4*sqrt(3) + 8
)*arctan((sqrt(3) + 2)*e^(2*x) + 1/216*sqrt(-6*(12^(1/4)*sqrt(6)*(sqrt(3) +
 3)*e^(2*x) - 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*
sqrt(3) + 36*e^(4*x) - 144*e^(2*x) + 252)*((12^(3/4)*sqrt(6)*(sqrt(3) + 3)
+ 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*sqrt(-4*sqrt(3) + 8) - 36*sqrt(3) - 72)
 - 2/3*sqrt(3)*(2*sqrt(3) - 3) - 1/36*(12^(3/4)*sqrt(6)*(sqrt(3) - 3) + (12
^(3/4)*sqrt(6)*(sqrt(3) + 3) + 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*e^(2*x) +
3*12^(1/4)*sqrt(6)*(sqrt(3) - 3))*sqrt(-4*sqrt(3) + 8) - 2*sqrt(3) + 4) + 4
*(12^(1/4)*sqrt(6)*e^(2*x) + 12^(1/4)*sqrt(6))*sqrt(-4*sqrt(3) + 8)*arctan(
-(sqrt(3) + 2)*e^(2*x) + 1/216*sqrt(6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(2*
x) - 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3) +
 36*e^(4*x) - 144*e^(2*x) + 252)*((12^(3/4)*sqrt(6)*(sqrt(3) + 3) + 3*12^(1
/4)*sqrt(6)*(sqrt(3) + 3))*sqrt(-4*sqrt(3) + 8) + 36*sqrt(3) + 72) + 2/3*sq
rt(3)*(2*sqrt(3) - 3) - 1/36*(12^(3/4)*sqrt(6)*(sqrt(3) - 3) + (12^(3/4)*sq
rt(6)*(sqrt(3) + 3) + 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*e^(2*x) + 3*12^(1/4
)*sqrt(6)*(sqrt(3) - 3))*sqrt(-4*sqrt(3) + 8) + 2*sqrt(3) - 4) - (12^(1/4)*
sqrt(6)*(sqrt(3) + 2)*e^(2*x) + 12^(1/4)*sqrt(6)*(sqrt(3) + 2))*sqrt(-4*sqr
t(3) + 8)*log(6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(2*x) - 12^(1/4)*sqrt(6)*
```

```
(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3) + 36*e^(4*x) - 144*e^(2
*x) + 252) + (12^(1/4)*sqrt(6)*(sqrt(3) + 2)*e^(2*x) + 12^(1/4)*sqrt(6)*(sq
rt(3) + 2))*sqrt(-4*sqrt(3) + 8)*log(-6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(
2*x) - 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3)
 + 36*e^(4*x) - 144*e^(2*x) + 252) + 96)/(e^(2*x) + 1)
```

giac [A] time = 0.12, size = 10, normalized size = 0.14

$$-\frac{2}{3\left(e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^6),x, algorithm="giac")
[Out] -2/3/(e^(2*x) + 1)
```

maple [C] time = 0.06, size = 61, normalized size = 0.86

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^4-3\_Z^2+1\right)}\_R\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-6\_R^3+6\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)\right)}{6}+\frac{2\tanh\left(\frac{x}{2}\right)}{3\left(\tanh^2\left(\frac{x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^6),x)
[Out] 1/6*sum(_R*ln(tanh(1/2*x)^2+(-6*_R^3+6*_R)*tanh(1/2*x)+1),_R=RootOf(3*_Z^4-
3*_Z^2+1))+2/3*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{3\left(e^{(2x)}+1\right)}-\int\frac{4\left(e^{(6x)}-10\,e^{(4x)}+e^{(2x)}\right)}{3\left(e^{(8x)}-8\,e^{(6x)}+30\,e^{(4x)}-8\,e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^6),x, algorithm="maxima")
[Out] -2/3/(e^(2*x) + 1) - integrate(4/3*(e^(6*x) - 10*e^(4*x) + e^(2*x))/(e^(8*x
) - 8*e^(6*x) + 30*e^(4*x) - 8*e^(2*x) + 1), x)
```

mupad [B] time = 4.21, size = 325, normalized size = 4.58

$$-\ln\left(\frac{1061158912\,\mathrm{e}^{2x}}{27}+\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1\mathrm{i}}{216}}\left(\frac{548405248}{27}+\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1\mathrm{i}}{216}}\left(\frac{3870294016}{9}+\sqrt{\frac{1}{72}-\frac{\sqrt{3}\,1\mathrm{i}}{216}}\left(19788\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^6 + 1),x)
[Out] log((1061158912*exp(2*x))/27 - (1/72 - (3^(1/2)*1i)/216)^(1/2)*((1/72 - (3^
(1/2)*1i)/216)^(1/2)*((21515730944*exp(2*x))/9 + (1/72 - (3^(1/2)*1i)/216)^
(1/2)*(19788726272*exp(2*x) - 2864709632) - 3870294016/9) - (2539651072*exp
(2*x))/9 + 548405248/27) - 351797248/81)*(1/72 - (3^(1/2)*1i)/216)^(1/2) -
log((1061158912*exp(2*x))/27 + ((3^(1/2)*1i)/216 + 1/72)^(1/2)*(((3^(1/2)*1
i)/216 + 1/72)^(1/2)*(((3^(1/2)*1i)/216 + 1/72)^(1/2)*(19788726272*exp(2*x)
 - 2864709632) - (21515730944*exp(2*x))/9 + 3870294016/9) - (2539651072*exp
(2*x))/9 + 548405248/27) - 351797248/81)*((3^(1/2)*1i)/216 + 1/72)^(1/2) -
```

```
log((1061158912*exp(2*x))/27 + (1/72 - (3^(1/2)*1i)/216)^(1/2)*((1/72 - (3^
(1/2)*1i)/216)^(1/2)*((1/72 - (3^(1/2)*1i)/216)^(1/2)*(19788726272*exp(2*x)
 - 2864709632) - (21515730944*exp(2*x))/9 + 3870294016/9) - (2539651072*exp
(2*x))/9 + 548405248/27) - 351797248/81)*(1/72 - (3^(1/2)*1i)/216)^(1/2) +
log((1061158912*exp(2*x))/27 - ((3^(1/2)*1i)/216 + 1/72)^(1/2)*(((3^(1/2)*1
i)/216 + 1/72)^(1/2)*((21515730944*exp(2*x))/9 + ((3^(1/2)*1i)/216 + 1/72)^
(1/2)*(19788726272*exp(2*x) - 2864709632) - 3870294016/9) - (2539651072*exp
(2*x))/9 + 548405248/27) - 351797248/81)*((3^(1/2)*1i)/216 + 1/72)^(1/2) -
2/(3*(exp(2*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**6),x)
```

```
[Out] Timed out
```

3.272 $\int \frac{1}{1+\sinh^8(x)}\,dx$

Optimal. Leaf size=129

$$\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{3/4}}\,\tanh(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\tanh^{-1}\left(\sqrt{1+\right.}{4\sqrt{1-}$$

```
[Out] 1/4*arctanh((1-(-1)^(1/4))^(1/2)*tanh(x))/(1-(-1)^(1/4))^(1/2)+1/4*arctanh(
(1+(-1)^(1/4))^(1/2)*tanh(x))/(1+(-1)^(1/4))^(1/2)+1/4*arctanh((1-(-1)^(3/4
))^(1/2)*tanh(x))/(1-(-1)^(3/4))^(1/2)+1/4*arctanh((1+(-1)^(3/4))^(1/2)*tan
h(x))/(1+(-1)^(3/4))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{3/4}}\,\tanh(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\tanh^{-1}\left(\sqrt{1+\right.}{4\sqrt{1-}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^8)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 - (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTanh[Sq
rt[1 + (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTanh[Sqrt[1 - (-1
)^(3/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTanh[Sqrt[1 + (-1)^(3/4)]*T
anh[x]]/(4*Sqrt[1 + (-1)^(3/4)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\sinh^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\,\sinh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt[4]{-1}\,\sinh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sinh^2(x)}\,dx + \frac{1}{4}$$

$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\left(1-\sqrt[4]{-1}\right)x^2}\,dx, x, \tanh(x)\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\left(1+\sqrt[4]{-1}\right)x^2}\,dx, x, \tanh(x\right.$$

$$= \frac{\tanh^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\tanh(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1-(-1)^{3/4}}\,\tan\right.}{4\sqrt{1-(-1)^{3/4}}}$$

Mathematica [C] time = 0.12, size = 127, normalized size = 0.98

$$16\text{RootSum}\left[\#1^8-8\#1^7+28\#1^6-56\#1^5+326\#1^4-56\#1^3+28\#1^2-8\#1+1\&, \frac{\#1^3x+\#1^3\log(-\#1\sinh(x)+}{\#1^7-7\#1^6+21\#1^5-35\#1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^8)^(-1),x]
```

```
[Out] 16*RootSum[1 - 8*#1 + 28*#1^2 - 56*#1^3 + 326*#1^4 - 56*#1^5 + 28*#1^6 - 8*
#1^7 + #1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]*#1]
*#1^3)/(-1 + 7*#1 - 21*#1^2 + 163*#1^3 - 35*#1^4 + 21*#1^5 - 7*#1^6 + #1^7)
 & ]
```

fricas [B] time = 2.76, size = 3773, normalized size = 29.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^8),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2)
 + 4)^(3/4)*sqrt(2*sqrt(2) + 3)*(sqrt(2) - 1)*arctan(1/31*(2*(13*sqrt(2) -
20)*e^(2*x) + 23*sqrt(2) - 33)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 1/
496*(32*(10*sqrt(2) - 13)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + (((355*
sqrt(2) - 508)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 6*(59*sqrt(2) - 86
)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(3/4) + 4*((82*sqrt(2) - 119)*sqrt(2
*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + (85*sqrt(2) - 126)*sqrt(2*sqrt(2) + 3))
*(2*sqrt(2) + 4)^(1/4))*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt
(2) + 8) + 4*((76*sqrt(2) - 105)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) +
2*(53*sqrt(2) - 72)*sqrt(2*sqrt(2) + 3))*sqrt(2*sqrt(2) + 4) + 16*(23*sqrt(
2) - 33)*sqrt(2*sqrt(2) + 3))*sqrt(4*(sqrt(2) - 1)*e^(2*x) - (2*(sqrt(2) -
1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) - 5*sqrt(2) + 6)*sqrt(2*sqrt(2) + 4) -
6*sqrt(2) + 6)*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*
(2*sqrt(2) + 4)^(1/4) - 4*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2) - 4*sqrt(2) + 2
*e^(4*x) + 10) + 1/248*((((254*sqrt(2) - 355)*e^(2*x) - 102*sqrt(2) + 145)*
sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*(3*(43*sqrt(2) - 59)*e^(2*x) -
23*sqrt(2) + 33)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt
(2) - 164)*e^(2*x) - 39*sqrt(2) + 60)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) +
3) + 2*((63*sqrt(2) - 85)*e^(2*x) - 17*sqrt(2) + 19)*sqrt(2*sqrt(2) + 3))*(
2*sqrt(2) + 4)^(1/4))*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2
) + 8) + 1/124*(((105*sqrt(2) - 152)*e^(2*x) + 13*sqrt(2) - 20)*sqrt(2*sqrt
(2) + 4)*sqrt(2*sqrt(2) + 3) + 4*((36*sqrt(2) - 53)*e^(2*x) - 23*sqrt(2) +
33)*sqrt(2*sqrt(2) + 3))*sqrt(2*sqrt(2) + 4) + 1/31*((33*sqrt(2) - 46)*e^(2
*x) - 3*sqrt(2) + 7)*sqrt(2*sqrt(2) + 3)) + 1/16*sqrt(2*sqrt(2*sqrt(2) + 4)
*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(3/4)*sqrt(2*sqrt(2) + 3)
*(sqrt(2) - 1)*arctan(-1/31*(2*(13*sqrt(2) - 20)*e^(2*x) + 23*sqrt(2) - 33)
```

```
*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) - 1/496*(32*(10*sqrt(2) - 13)*sqrt
(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) - (((355*sqrt(2) - 508)*sqrt(2*sqrt(2)
+ 4)*sqrt(2*sqrt(2) + 3) + 6*(59*sqrt(2) - 86)*sqrt(2*sqrt(2) + 3))*(2*sqrt
(2) + 4)^(3/4) + 4*((82*sqrt(2) - 119)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) +
 3) + (85*sqrt(2) - 126)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(1/4))*sqrt(2
*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8) + 4*((76*sqrt(2) - 10
5)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*(53*sqrt(2) - 72)*sqrt(2*sqr
t(2) + 3))*sqrt(2*sqrt(2) + 4) + 16*(23*sqrt(2) - 33)*sqrt(2*sqrt(2) + 3))*
sqrt(4*(sqrt(2) - 1)*e^(2*x) + (2*(sqrt(2) - 1)*e^(2*x) + ((sqrt(2) - 2)*e^
(2*x) - 5*sqrt(2) + 6)*sqrt(2*sqrt(2) + 4) - 6*sqrt(2) + 6)*sqrt(2*sqrt(2*s
qrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(1/4) - 4*sqrt
(2*sqrt(2) + 4)*(sqrt(2) - 2) - 4*sqrt(2) + 2*e^(4*x) + 10) + 1/248*((((254
*sqrt(2) - 355)*e^(2*x) - 102*sqrt(2) + 145)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqr
t(2) + 3) + 2*(3*(43*sqrt(2) - 59)*e^(2*x) - 23*sqrt(2) + 33)*sqrt(2*sqrt(2
) + 3))*(2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) - 164)*e^(2*x) - 39*sqrt(2
) + 60)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*((63*sqrt(2) - 85)*e^(2
*x) - 17*sqrt(2) + 19)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(1/4))*sqrt(2*s
qrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8) - 1/124*(((105*sqrt(2)
- 152)*e^(2*x) + 13*sqrt(2) - 20)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) +
 4*((36*sqrt(2) - 53)*e^(2*x) - 23*sqrt(2) + 33)*sqrt(2*sqrt(2) + 3))*sqrt(
2*sqrt(2) + 4) - 1/31*((33*sqrt(2) - 46)*e^(2*x) - 3*sqrt(2) + 7)*sqrt(2*sq
rt(2) + 3)) - 1/16*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2)
 + 8)*(sqrt(2) + 1)*(-2*sqrt(2) + 4)^(3/4)*sqrt(-2*sqrt(2) + 3)*arctan(1/31
*(2*(13*sqrt(2) + 20)*e^(2*x) + 23*sqrt(2) + 33)*sqrt(-2*sqrt(2) + 4)*sqrt(
-2*sqrt(2) + 3) - 1/496*(32*(10*sqrt(2) + 13)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*
sqrt(2) + 3) - (((355*sqrt(2) + 508)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) +
 3) + 6*(59*sqrt(2) + 86)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(3/4) + 4*
((82*sqrt(2) + 119)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + (85*sqrt(2)
 + 126)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(1/4))*sqrt(-2*(2*sqrt(2) +
3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) + 4*((76*sqrt(2) + 105)*sqrt(-2*sq
rt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*(53*sqrt(2) + 72)*sqrt(-2*sqrt(2) + 3))
*sqrt(-2*sqrt(2) + 4) + 16*(23*sqrt(2) + 33)*sqrt(-2*sqrt(2) + 3))*sqrt(-4*
(sqrt(2) + 1)*e^(2*x) - (2*(sqrt(2) + 1)*e^(2*x) + ((sqrt(2) + 2)*e^(2*x) -
 5*sqrt(2) - 6)*sqrt(-2*sqrt(2) + 4) - 6*sqrt(2) - 6)*sqrt(-2*(2*sqrt(2) +
3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*(-2*sqrt(2) + 4)^(1/4) + 4*(sqrt(2
) + 2)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 2*e^(4*x) + 10) - 1/248*((((254*s
qrt(2) + 355)*e^(2*x) - 102*sqrt(2) - 145)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqr
t(2) + 3) + 2*(3*(43*sqrt(2) + 59)*e^(2*x) - 23*sqrt(2) - 33)*sqrt(-2*sqrt(
2) + 3))*(-2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) + 164)*e^(2*x) - 39*sqrt
(2) - 60)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*((63*sqrt(2) + 85)*
e^(2*x) - 17*sqrt(2) - 19)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(1/4))*sq
rt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) + 1/124*(((105*
sqrt(2) + 152)*e^(2*x) + 13*sqrt(2) + 20)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt
(2) + 3) + 4*((36*sqrt(2) + 53)*e^(2*x) - 23*sqrt(2) - 33)*sqrt(-2*sqrt(2)
+ 3))*sqrt(-2*sqrt(2) + 4) + 1/31*((33*sqrt(2) + 46)*e^(2*x) - 3*sqrt(2) -
7)*sqrt(-2*sqrt(2) + 3)) - 1/16*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4
) + 4*sqrt(2) + 8)*(sqrt(2) + 1)*(-2*sqrt(2) + 4)^(3/4)*sqrt(-2*sqrt(2) + 3
)*arctan(-1/31*(2*(13*sqrt(2) + 20)*e^(2*x) + 23*sqrt(2) + 33)*sqrt(-2*sqrt
(2) + 4)*sqrt(-2*sqrt(2) + 3) + 1/496*(32*(10*sqrt(2) + 13)*sqrt(-2*sqrt(2)
 + 4)*sqrt(-2*sqrt(2) + 3) + (((355*sqrt(2) + 508)*sqrt(-2*sqrt(2) + 4)*sqr
t(-2*sqrt(2) + 3) + 6*(59*sqrt(2) + 86)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) +
 4)^(3/4) + 4*((82*sqrt(2) + 119)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3)
 + (85*sqrt(2) + 126)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(1/4))*sqrt(-2
*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) + 4*((76*sqrt(2) + 1
05)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*(53*sqrt(2) + 72)*sqrt(-2
*sqrt(2) + 3))*sqrt(-2*sqrt(2) + 4) + 16*(23*sqrt(2) + 33)*sqrt(-2*sqrt(2)
+ 3))*sqrt(-4*(sqrt(2) + 1)*e^(2*x) + (2*(sqrt(2) + 1)*e^(2*x) + ((sqrt(2)
+ 2)*e^(2*x) - 5*sqrt(2) - 6)*sqrt(-2*sqrt(2) + 4) - 6*sqrt(2) - 6)*sqrt(-2
*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*(-2*sqrt(2) + 4)^(1/
```

```
4) + 4*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 2*e^(4*x) + 10) - 1
/248*((((254*sqrt(2) + 355)*e^(2*x) - 102*sqrt(2) - 145)*sqrt(-2*sqrt(2) +
4)*sqrt(-2*sqrt(2) + 3) + 2*(3*(43*sqrt(2) + 59)*e^(2*x) - 23*sqrt(2) - 33)
*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) + 164)*e^(
2*x) - 39*sqrt(2) - 60)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*((63*
sqrt(2) + 85)*e^(2*x) - 17*sqrt(2) - 19)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2)
+ 4)^(1/4))*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) -
 1/124*(((105*sqrt(2) + 152)*e^(2*x) + 13*sqrt(2) + 20)*sqrt(-2*sqrt(2) + 4
)*sqrt(-2*sqrt(2) + 3) + 4*((36*sqrt(2) + 53)*e^(2*x) - 23*sqrt(2) - 33)*sq
rt(-2*sqrt(2) + 3))*sqrt(-2*sqrt(2) + 4) - 1/31*((33*sqrt(2) + 46)*e^(2*x)
- 3*sqrt(2) - 7)*sqrt(-2*sqrt(2) + 3)) + 1/64*sqrt(-2*(2*sqrt(2) + 3)*sqrt(
-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*((sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) + 2*sq
rt(2))*(-2*sqrt(2) + 4)^(1/4)*log(-2*(sqrt(2) + 1)*e^(2*x) + 1/2*(2*(sqrt(2
) + 1)*e^(2*x) + ((sqrt(2) + 2)*e^(2*x) - 5*sqrt(2) - 6)*sqrt(-2*sqrt(2) +
4) - 6*sqrt(2) - 6)*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2
) + 8)*(-2*sqrt(2) + 4)^(1/4) + 2*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + 2*sq
rt(2) + e^(4*x) + 5) - 1/64*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) +
4*sqrt(2) + 8)*((sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) + 2*sqrt(2))*(-2*sqrt(2)
 + 4)^(1/4)*log(-2*(sqrt(2) + 1)*e^(2*x) - 1/2*(2*(sqrt(2) + 1)*e^(2*x) + (
(sqrt(2) + 2)*e^(2*x) - 5*sqrt(2) - 6)*sqrt(-2*sqrt(2) + 4) - 6*sqrt(2) - 6
)*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*(-2*sqrt(2)
 + 4)^(1/4) + 2*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + 2*sqrt(2) + e^(4*x) +
5) + 1/64*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(sqrt
(2*sqrt(2) + 4)*(sqrt(2) - 1) + 2*sqrt(2))*(2*sqrt(2) + 4)^(1/4)*log(2*(sqr
t(2) - 1)*e^(2*x) + 1/2*(2*(sqrt(2) - 1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) -
 5*sqrt(2) + 6)*sqrt(2*sqrt(2) + 4) - 6*sqrt(2) + 6)*sqrt(2*sqrt(2*sqrt(2)
+ 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(1/4) - 2*sqrt(2*sqrt
(2) + 4)*(sqrt(2) - 2) - 2*sqrt(2) + e^(4*x) + 5) - 1/64*sqrt(2*sqrt(2*sqrt
(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1
) + 2*sqrt(2))*(2*sqrt(2) + 4)^(1/4)*log(2*(sqrt(2) - 1)*e^(2*x) - 1/2*(2*(
sqrt(2) - 1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) - 5*sqrt(2) + 6)*sqrt(2*sqrt(
2) + 4) - 6*sqrt(2) + 6)*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqr
t(2) + 8)*(2*sqrt(2) + 4)^(1/4) - 2*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2) - 2*s
qrt(2) + e^(4*x) + 5)
```

giac [A] time = 0.14, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^8),x, algorithm="giac")
```

```
[Out] 0
```

maple [C] time = 0.07, size = 64, normalized size = 0.50

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^{8}-4\_Z^{6}+6\_Z^{4}-4\_Z^{2}+1\right)}\_R\ln\left(\tanh^{2}\left(\frac{x}{2}\right)+\left(-4\_R^{7}+8\_R^{5}-12\_R^{3}+8\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^8),x)
```

```
[Out] 1/8*sum(_R*ln(tanh(1/2*x)^2+(-4*_R^7+8*_R^5-12*_R^3+8*_R)*tanh(1/2*x)+1),_R
=RootOf(2*_Z^8-4*_Z^6+6*_Z^4-4*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh(x)^{8}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^8),x, algorithm="maxima")
[Out] integrate(1/(sinh(x)^8 + 1), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(x)^8 + 1),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sinh^8(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**8),x)
[Out] Integral(1/(sinh(x)**8 + 1), x)
```

3.273 $\int \frac{1}{1-\sinh^5(x)}\,dx$

Optimal. Leaf size=228

$$-\frac{2\tanh^{-1}\left(\frac{(-1)^{3/5}-\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}+\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)+\frac{2\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}}-\frac{2\sqrt[10]{-1}\tanh^{-1}\left(\frac{(-1)^{3/10}((-1)^{4/5}}{\sqrt{\sqrt[5]{-1}+}}\right)}{5\sqrt{\sqrt[5]{-1}+(-1)^{3/5}}}$$

```
[Out] 1/5*arctanh(1/2*(1+tanh(1/2*x))*2^(1/2))*2^(1/2)-2/5*(-1)^(1/10)*arctan((I+
(-1)^(1/10)*tanh(1/2*x))/(1-(-1)^(1/5))^(1/2))/(1-(-1)^(1/5))^(1/2)-2/5*arc
tanh(((-1)^(3/5)-tanh(1/2*x))/(1-(-1)^(1/5))^(1/2))/(1-(-1)^(1/5))^(1/2)+2/
5*arctanh(((-1)^(4/5)+tanh(1/2*x))/(1-(-1)^(3/5))^(1/2))/(1-(-1)^(3/5))^(1/
2)-2/5*(-1)^(1/10)*arctanh((-1)^(3/10)*(1+(-1)^(4/5)*tanh(1/2*x))/((-1)^(1/
5)+(-1)^(3/5))^(1/2))/((-1)^(1/5)+(-1)^(3/5))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3213, 2660, 618, 204, 617, 206}

$$-\frac{2\tanh^{-1}\left(\frac{(-1)^{3/5}-\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}+\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)+\frac{2\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}}-\frac{2\sqrt[10]{-1}\tanh^{-1}\left(\frac{(-1)^{3/10}((-1)^{4/5}}{\sqrt{\sqrt[5]{-1}+}}\right)}{5\sqrt{\sqrt[5]{-1}+(-1)^{3/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^5)^(-1),x]
```

```
[Out] (-2*(-1)^(1/10)*ArcTan[(I + (-1)^(1/10)*Tanh[x/2])/Sqrt[1 - (-1)^(1/5)]])/(
5*Sqrt[1 - (-1)^(1/5)]) - (2*ArcTanh[((-1)^(3/5) - Tanh[x/2])/Sqrt[1 - (-1)
^(1/5)]])/(5*Sqrt[1 - (-1)^(1/5)]) + (Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[
2]])/5 + (2*ArcTanh[((-1)^(4/5) + Tanh[x/2])/Sqrt[1 - (-1)^(3/5)]])/(5*Sqrt
[1 - (-1)^(3/5)]) - (2*(-1)^(1/10)*ArcTanh[((-1)^(3/10)*(1 + (-1)^(4/5)*Tan
h[x/2]))/Sqrt[(-1)^(1/5) + (-1)^(3/5)]])/(5*Sqrt[(-1)^(1/5) + (-1)^(3/5)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1-\sinh^5(x)}\,dx &= \int \left(\frac{\sqrt[10]{-1}}{5\left(\sqrt[10]{-1}-i\sinh(x)\right)}+\frac{\sqrt[10]{-1}}{5\left(\sqrt[10]{-1}-\sqrt[10]{-1}\sinh(x)\right)}+\frac{\sqrt[10]{-1}}{5\left(\sqrt[10]{-1}+(-1)^{3/10}\sinh(x)\right)}+\right.\\
&= \frac{1}{5}\sqrt[10]{-1}\int \frac{1}{\sqrt[10]{-1}-i\sinh(x)}\,dx+\frac{1}{5}\sqrt[10]{-1}\int \frac{1}{\sqrt[10]{-1}-\sqrt[10]{-1}\sinh(x)}\,dx+\frac{1}{5}\sqrt[10]{-1}\int \frac{}{\sqrt[10]{-}}\\
&= \frac{1}{5}\left(2\sqrt[10]{-1}\right)\text{Subst}\left(\int \frac{1}{\sqrt[10]{-1}-2ix-\sqrt[10]{-1}\,x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{5}\left(2\sqrt[10]{-1}\right)\text{Subst}\left(\int \frac{}{}\right.\\
&= \frac{2}{5}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,1+\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{5}\left(4\sqrt[10]{-1}\right)\text{Subst}\left(\int \frac{1}{-4\left(1-\sqrt[5]{-1}\right)-x^2}\,dx,\right.\\
&= -\frac{2\sqrt[10]{-1}\tan^{-1}\left(\frac{i+\sqrt[10]{-1}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}-\frac{2\tanh^{-1}\left(\frac{(-1)^{3/5}-\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}+\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right.
\end{aligned}$$

Mathematica [C] time = 0.95, size = 437, normalized size = 1.92

$$\frac{1}{10}\left(\text{RootSum}\left[\#1^8+2\#1^7+2\#1^5+14\#1^4-2\#1^3-2\#1+1\&,\frac{\#1^6x+2\#1^6\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)\right.}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^5)^(-1),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[2]] + RootSum[1 - 2*#1 - 2*#1^3 + 1
4*#1^4 + 2*#1^5 + 2*#1^7 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Co
sh[x/2]*#1 - Sinh[x/2]*#1] + 4*x*#1 + 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x
/2]*#1 - Sinh[x/2]*#1]*#1 - 9*x*#1^2 - 18*Log[-Cosh[x/2] - Sinh[x/2] + Cosh
[x/2]*#1 - Sinh[x/2]*#1]*#1^2 + 24*x*#1^3 + 48*Log[-Cosh[x/2] - Sinh[x/2] +
 Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^3 + 9*x*#1^4 + 18*Log[-Cosh[x/2] - Sinh[x/
2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + 4*x*#1^5 + 8*Log[-Cosh[x/2] - Sinh
[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Sin
h[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-1 - 3*#1^2 + 28*#1^3 + 5*#1^4
 + 7*#1^6 + 4*#1^7) & ])/10
```

fricas [B] time = 2.65, size = 3500, normalized size = 15.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^5),x, algorithm="fricas")
```

```
[Out] 1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5)
+ 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(
5) + 3)*arctan(1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x + 7*sqrt(5) - 15)*sqrt(2
*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 1/80*sqrt(2)*(sqrt(2)*((11*sqrt(5) - 25)*
e^x - 4*sqrt(5) + 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((3*sqrt(5)
 - 5)*e^x - 7*sqrt(5) + 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(5) + 3) + 1/1280
0*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 40*s
qrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*
sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) + sqrt(2*sqrt(2)*(2*sq
rt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(7*sqrt(5) - 15)*s
qrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5) +
 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)*sqrt(2*sqrt(5) +
 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5)
 + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqrt(-20*sqrt(2)*sqr
t(sqrt(5) + 3)*(sqrt(5) - 3) - 40*(sqrt(5) - 1)*e^x + 2*(sqrt(2)*((2*sqrt(5
) - 5)*e^x + 3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) - 5)*e^x + 3*sqr
t(5) - 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20
)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2*sqrt(2)*(2*sqrt(
5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*sqrt(5) - 7)*e^x
- 8*sqrt(5) + 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) - 1
1)*e^x - 2*sqrt(5) + 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sq
rt(2)*((7*sqrt(5) - 17)*e^x - 8*sqrt(5) + 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt
(5) + 3) + 2*((5*sqrt(5) - 11)*e^x - 5*sqrt(5) + 9)*sqrt(2*sqrt(5) + 5))*(8
*sqrt(5) + 24)^(1/4)) + 1/20*(2*(4*sqrt(5) - 5)*e^x + sqrt(5) - 5)*sqrt(2*s
qrt(5) + 5)) + 1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) +
3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5)
+ 5)*sqrt(sqrt(5) + 3)*arctan(-1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x + 7*sqrt
(5) - 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) - 1/80*sqrt(2)*(sqrt(2)*((1
1*sqrt(5) - 25)*e^x - 4*sqrt(5) + 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3)
 + 2*((3*sqrt(5) - 5)*e^x - 7*sqrt(5) + 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(
5) + 3) - 1/12800*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqr
t(5) + 3) + 40*sqrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(s
qrt(5) + 3) + 2*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) - sqrt
(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(7
*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*
sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)*
sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5)
+ 5))*(8*sqrt(5) + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqrt
(-20*sqrt(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) - 40*(sqrt(5) - 1)*e^x - 2*(sq
rt(2)*((2*sqrt(5) - 5)*e^x + 3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5)
- 5)*e^x + 3*sqrt(5) - 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3)
- 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2*
sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*s
qrt(5) - 7)*e^x - 8*sqrt(5) + 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2
*((5*sqrt(5) - 11)*e^x - 2*sqrt(5) + 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 2
4)^(3/4) + 4*(sqrt(2)*((7*sqrt(5) - 17)*e^x - 8*sqrt(5) + 18)*sqrt(2*sqrt(5
) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) - 11)*e^x - 5*sqrt(5) + 9)*sqrt(2*
sqrt(5) + 5))*(8*sqrt(5) + 24)^(1/4)) - 1/20*(2*(4*sqrt(5) - 5)*e^x + sqrt(
5) - 5)*sqrt(2*sqrt(5) + 5)) - 1/400*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5)
+ 24) + 4*sqrt(5) + 20)*(3*sqrt(5) + 5)*sqrt(-2*sqrt(5) + 5)*(-8*sqrt(5) +
24)^(3/4)*arctan(-1/1280*((4*(5*sqrt(5) + 11)*e^x + ((3*sqrt(5) + 7)*e^x -
8*sqrt(5) - 18)*sqrt(-8*sqrt(5) + 24) - 8*sqrt(5) - 16)*(-8*sqrt(5) + 24)^(
3/4) + 4*(4*(5*sqrt(5) + 11)*e^x + ((7*sqrt(5) + 17)*e^x - 8*sqrt(5) - 18)*
sqrt(-8*sqrt(5) + 24) - 20*sqrt(5) - 36)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*
sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) +
 1/25600*((((7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*
```

```
sqrt(5) + 24)^(3/4) + 4*((17*sqrt(5) + 35)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(
5) + 100)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) +
24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) - 20*(((5*sqrt(5) + 11)*sqrt(-8*
sqrt(5) + 24) + 4*sqrt(5) + 12)*sqrt(-8*sqrt(5) + 24) + 4*(5*sqrt(5) + 11)*
sqrt(-8*sqrt(5) + 24) + 32*sqrt(5) + 128)*sqrt(-2*sqrt(5) + 5))*sqrt(40*(sq
rt(5) + 1)*e^x + (4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x + 3*sqrt(5) +
5)*sqrt(-8*sqrt(5) + 24) + 6*sqrt(5) + 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sq
rt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 10*(sqrt(5) + 3)*sq
rt(-8*sqrt(5) + 24) + 80*e^(2*x) + 80) + 1/320*(32*(4*sqrt(5) + 5)*e^x + 4*
((11*sqrt(5) + 25)*e^x + 7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + (4*(3*sqrt
(5) + 5)*e^x + ((11*sqrt(5) + 25)*e^x - 4*sqrt(5) - 10)*sqrt(-8*sqrt(5) + 2
4) - 28*sqrt(5) - 60)*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5) + 80)*sqrt(-2*sqrt
(5) + 5)) - 1/400*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) +
 20)*(3*sqrt(5) + 5)*sqrt(-2*sqrt(5) + 5)*(-8*sqrt(5) + 24)^(3/4)*arctan(-1
/1280*((4*(5*sqrt(5) + 11)*e^x + ((3*sqrt(5) + 7)*e^x - 8*sqrt(5) - 18)*sqr
t(-8*sqrt(5) + 24) - 8*sqrt(5) - 16)*(-8*sqrt(5) + 24)^(3/4) + 4*(4*(5*sqrt
(5) + 11)*e^x + ((7*sqrt(5) + 17)*e^x - 8*sqrt(5) - 18)*sqrt(-8*sqrt(5) + 2
4) - 20*sqrt(5) - 36)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-
8*sqrt(5) + 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) + 1/25600*((((7*sqrt
(5) + 15)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*sqrt(5) + 24)^(3/4)
 + 4*((17*sqrt(5) + 35)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*sqrt(
5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 2
0)*sqrt(-2*sqrt(5) + 5) + 20*(((5*sqrt(5) + 11)*sqrt(-8*sqrt(5) + 24) + 4*s
qrt(5) + 12)*sqrt(-8*sqrt(5) + 24) + 4*(5*sqrt(5) + 11)*sqrt(-8*sqrt(5) + 2
4) + 32*sqrt(5) + 128)*sqrt(-2*sqrt(5) + 5))*sqrt(40*(sqrt(5) + 1)*e^x - (4
*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x + 3*sqrt(5) + 5)*sqrt(-8*sqrt(5)
+ 24) + 6*sqrt(5) + 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqr
t(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 10*(sqrt(5) + 3)*sqrt(-8*sqrt(5) + 24)
 + 80*e^(2*x) + 80) - 1/320*(32*(4*sqrt(5) + 5)*e^x + 4*((11*sqrt(5) + 25)*
e^x + 7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + (4*(3*sqrt(5) + 5)*e^x + ((11
*sqrt(5) + 25)*e^x - 4*sqrt(5) - 10)*sqrt(-8*sqrt(5) + 24) - 28*sqrt(5) - 6
0)*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5) + 80)*sqrt(-2*sqrt(5) + 5)) - 1/800*(
sqrt(2)*(3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 8*sqrt(5))*sqrt(2*sqrt(2)*(2*sq
rt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*log(-
4*sqrt(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) - 8*(sqrt(5) - 1)*e^x + 2/5*(sqrt
(2)*((2*sqrt(5) - 5)*e^x + 3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) -
5)*e^x + 3*sqrt(5) - 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) -
4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 16*e^(2*x) + 16) + 1/800*(sqrt(2)*
(3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 8*sqrt(5))*sqrt(2*sqrt(2)*(2*sqrt(5) -
5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*log(-4*sqrt(2
)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) - 8*(sqrt(5) - 1)*e^x - 2/5*(sqrt(2)*((2*
sqrt(5) - 5)*e^x + 3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) - 5)*e^x +
 3*sqrt(5) - 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5
) + 20)*(8*sqrt(5) + 24)^(1/4) + 16*e^(2*x) + 16) - 1/1600*((3*sqrt(5) + 5)
*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5)
+ 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4)*log(8*(sqrt(5) + 1)*e^x + 1
/5*(4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x + 3*sqrt(5) + 5)*sqrt(-8*sqr
t(5) + 24) + 6*sqrt(5) + 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) +
4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(5) +
 24) + 16*e^(2*x) + 16) + 1/1600*((3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 1
6*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-
8*sqrt(5) + 24)^(1/4)*log(8*(sqrt(5) + 1)*e^x - 1/5*(4*(sqrt(5) + 5)*e^x +
((2*sqrt(5) + 5)*e^x + 3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 6*sqrt(5) + 1
0)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5
) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(5) + 24) + 16*e^(2*x) + 16) +
1/10*sqrt(2)*log((2*(sqrt(2) - 1)*e^x - 2*sqrt(2) + e^(2*x) + 3)/(e^(2*x) -
 2*e^x - 1))
```

giac [B] time = 2.95, size = 5248, normalized size = 23.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^5),x, algorithm="giac")
```

```
[Out] 8/25*5^(3/4)*sqrt(-1/32*sqrt(5) + 5/64)*arctan(-2*(4789310072875935951200*5
^(3/4)*sqrt(-2*sqrt(5) + 5) - 1799745554293062228687680*sqrt(5)*sqrt(-2*sqr
t(5) + 5) - 325914041979902244813289*5^(3/4) - 10520606548600849190560*5^(1
/4)*sqrt(-2*sqrt(5) + 5) + 265033340677886980055183*sqrt(5) + 4025305730691
667696322880*sqrt(-2*sqrt(5) + 5) + 728855245658450343948919*5^(1/4) + 1637
333558120632636*e^x - 592460559708252630357201)/(9202754427496321314406*5^(
3/4)*sqrt(-2*sqrt(5) + 5) + 1038239983143393667165790*sqrt(5)*sqrt(-2*sqrt(
5) + 5) + 186591807316241026405751*5^(3/4) - 20768219695320392550210*5^(1/4
)*sqrt(-2*sqrt(5) + 5) + 576155331981489353033147*sqrt(5) - 232237011952592
5506249090*sqrt(-2*sqrt(5) + 5) - 417362544266571988465273*5^(1/4) - 128878
4580381451028672113)) - 8/25*5^(3/4)*sqrt(-1/32*sqrt(5) + 5/64)*arctan(-2*(
4315023771046590689440*5^(3/4)*sqrt(-2*sqrt(5) + 5) + 165124224190524729732
44480*sqrt(5)*sqrt(-2*sqrt(5) + 5) + 2991559181950156635096041*5^(3/4) - 11
415488961128059998560*5^(1/4)*sqrt(-2*sqrt(5) + 5) + 4764705466952317581027
99*sqrt(5) - 36931036359499378163241280*sqrt(-2*sqrt(5) + 5) - 669030025114
7369625285239*5^(1/4) + 1637333558120632636*e^x - 1067630744269504182665681
)/(537689066142690749994*5^(3/4)*sqrt(-2*sqrt(5) + 5) + 4516299732803214710
5966190*sqrt(5)*sqrt(-2*sqrt(5) + 5) + 8186257622186710975158757*5^(3/4) -
5838120100393683185390*5^(1/4)*sqrt(-2*sqrt(5) + 5) + 603739022767920301057
079*sqrt(5) - 101008886798639244060001970*sqrt(-2*sqrt(5) + 5) - 1830753960
8818658210592747*5^(1/4) - 1355343042548851351155477)) - 1/10*sqrt(sqrt(5)
+ 2)*log((302427386195713850867712*sqrt(5)*(2*sqrt(5) + 5)^3 + 172815649254
693629067264*(2*sqrt(5) + 5)^(7/2) + 226820539646785388150784*sqrt(5)*(2*sq
rt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 151213693097856925433856*(2*sqrt(5) +
5)^3*sqrt(sqrt(5) + 2) + 70881418639620433797120*sqrt(5)*(2*sqrt(5) + 5)^2*
(sqrt(5) + 2) + 56705134911696347037696*(2*sqrt(5) + 5)^(5/2)*(sqrt(5) + 2)
 + 11813569773270072299520*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2
) + 11813569773270072299520*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2) + 1107522
166244069278080*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 14766962216587590
37440*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 55376108312203463904*sqrt(5)*
sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 110752216624406927808*(2*sqrt(5)
+ 5)*(sqrt(5) + 2)^(5/2) + 1153668923170905498*sqrt(5)*(sqrt(5) + 2)^3 + 46
14675692683621992*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 82404923083636107*(
sqrt(5) + 2)^(7/2) - 622619531678741564620800*sqrt(5)*(2*sqrt(5) + 5)^(5/2)
 - 415079687785827709747200*(2*sqrt(5) + 5)^3 - 389137207299213477888000*sq
rt(5)*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 311309765839370782310400*(2*sqr
t(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) - 97284301824803369472000*sqrt(5)*(2*sqrt
(5) + 5)^(3/2)*(sqrt(5) + 2) - 97284301824803369472000*(2*sqrt(5) + 5)^2*(s
qrt(5) + 2) - 12160537728100421184000*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)
^(3/2) - 16214050304133894912000*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2)
- 760033608006276324000*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 15200
67216012552648000*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 19000840200156908100*sq
rt(5)*(sqrt(5) + 2)^(5/2) - 76003360800627632400*sqrt(2*sqrt(5) + 5)*(sqrt(
5) + 2)^(5/2) - 1583403350013075675*(sqrt(5) + 2)^3 - 346430300390652274610
1760*sqrt(5)*(2*sqrt(5) + 5)^2 - 2015373937635933569712128*(2*sqrt(5) + 5)^
(5/2) - 1732151501953261373050880*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5
) + 2) - 1259608711022458481070080*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 32
4778406616236507447040*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 314902177755
614620267520*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 27064867218019708953920*
sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 39362772219451827533440*(
2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 845777100563115904810*sqrt(5)*(sqrt(5)
 + 2)^2 - 2460173263715739220840*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 6150
```

4331592893480521*(sqrt(5) + 2)^(5/2) + 3959703717250098693214208*sqrt(5)*(2*sqrt(5) + 5)^(3/2) + 2662579692919387100254208*(2*sqrt(5) + 5)^2 + 1484888893968787009955328*sqrt(5)*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 1331289846459693550127104*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) + 185611111746098376244416*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) + 249616846211192540648832*(2*sqrt(5) + 5)*(sqrt(5) + 2) + 7733796322754099010184*sqrt(5)*(sqrt(5) + 2)^(3/2) + 20801403850932711720736*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) + 650043870341647241273*(sqrt(5) + 2)^2 + 10991940456909382283282816*sqrt(5)*(2*sqrt(5) + 5) + 8567053742081103206220288*(2*sqrt(5) + 5)^(3/2) + 2747985114227345570820704*sqrt(5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 3212645153280413702332608*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 171749069639209098176294*sqrt(5)*(sqrt(5) + 2) + 401580644160051712791576*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) + 16732526840002154699649*(sqrt(5) + 2)^(3/2) - 2557269775899525489493536*sqrt(5)*sqrt(2*sqrt(5) + 5) - 319658721987440686186692*sqrt(5)*sqrt(sqrt(5) + 2) + 39842775211562571442672*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) - 3308326863346966249269767*sqrt(5) - 4580301686563984868886360*sqrt(2*sqrt(5) + 5) - 572537710820498108610795*sqrt(sqrt(5) + 2) + 2850824269841065226382633)^2 + 64*(24322822501240781930496*sqrt(5)*(2*sqrt(5) + 5)^3 + 13898755714994732531712*(2*sqrt(5) + 5)^(7/2) + 18242116875930586447872*sqrt(5)*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 12161411250620390965248*(2*sqrt(5) + 5)^3*sqrt(sqrt(5) + 2) + 5700661523728308264960*sqrt(5)*(2*sqrt(5) + 5)^2*(sqrt(5) + 2) + 4560529218982646611968*(2*sqrt(5) + 5)^(5/2)*(sqrt(5) + 2) + 950110253954718044160*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2) + 950110253954718044160*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2) + 89072836308254816640*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 118763781744339755520*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 4453641815412740832*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 8907283630825481664*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 92784204487765434*sqrt(5)*(sqrt(5) + 2)^3 + 371136817951061736*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 6627443177697531*(sqrt(5) + 2)^(7/2) - 13726081827177108602880*sqrt(5)*(2*sqrt(5) + 5)^(5/2) - 9150721218118072401920*(2*sqrt(5) + 5)^3 - 8578801141985692876800*sqrt(5)*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 6863040913588554301440*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) - 2144700285496423219200*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 2144700285496423219200*(2*sqrt(5) + 5)^2*(sqrt(5) + 2) - 268087535687052902400*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 357450047582737203200*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2) - 16755470980440806400*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 33510941960881612800*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 418886774511020160*sqrt(5)*(sqrt(5) + 2)^(5/2) - 1675547098044080640*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) - 34907231209251680*(sqrt(5) + 2)^3 - 323167802334835240755200*sqrt(5)*(2*sqrt(5) + 5)^2 - 197727185614766237777920*(2*sqrt(5) + 5)^(5/2) - 161583901167417620377600*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) - 123579491009228898611200*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 30296981468890803820800*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 30894872752307224652800*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 2524748455740900318400*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 3861859094038403081600*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 78898389241903134950*sqrt(5)*(sqrt(5) + 2)^2 - 241366193377400192600*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 6034154834435004815*(sqrt(5) + 2)^(5/2) - 100890523270644033265664*sqrt(5)*(2*sqrt(5) + 5)^(3/2) - 129486527077263009521664*(2*sqrt(5) + 5)^2 - 37833946226491512474624*sqrt(5)*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) - 64743263538631504760832*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) - 4729243278311439059328*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) - 12139361913493407142656*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 197051803262976627472*sqrt(5)*(sqrt(5) + 2)^(3/2) - 1011613492791117261888*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 31612921649722414434*(sqrt(5) + 2)^2 + 976056667738889843134336*sqrt(5)*(2*sqrt(5) + 5) + 737459612988335241742848*(2*sqrt(5) + 5)^(3/2) + 244014166934722460783584*sqrt(5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 276547354870625715653568*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 15250885433420153798974*sqrt(5)*(sqrt(5) + 2) + 34568419358828214456696*sqrt

(2*sqrt(5) + 5)*(sqrt(5) + 2) + 1440350806617842269029*(sqrt(5) + 2)^(3/2)
+ 865777074090951821677952*sqrt(5)*sqrt(2*sqrt(5) + 5) + 108222134261368977
709744*sqrt(5)*sqrt(sqrt(5) + 2) + 403147498761313336459456*sqrt(2*sqrt(5)
+ 5)*sqrt(sqrt(5) + 2) + 339901475433043622828 4234*sqrt(5) + 14797836987475
30204584760*sqrt(2*sqrt(5) + 5) + 1849729623434412755730 95*sqrt(sqrt(5) + 2
) + 131291208062174938773104*e^x + 870069461703628226688 1102)^2) + 1/10*sqr
t(sqrt(5) + 2)*log((29677772578331085766656 0*sqrt(5)*(2*sqrt(5) + 5)^3 + 16
958727187617763295232 0*(2*sqrt(5) + 5)^(7/2) + 222583294337483143249920*sqr
t(5)*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 148388862891655428833280*(2*
sqrt(5) + 5)^3*sqrt(sqrt(5) + 2) + 695572794804634822656 00*sqrt(5)*(2*sqrt(
5) + 5)^2*(sqrt(5) + 2) + 55645823584370785812480*(2*sqrt(5) + 5)^(5/2)*(sq
rt(5) + 2) + 11592879913410580377600*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5)
 + 2)^(3/2) + 11592879913410580377600*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2)
 + 1086832491882241910400*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 1449109
989176322547200*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 5434162459411209552
0*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 108683249188224191040*(
2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 1132117179044001990*sqrt(5)*(sqrt(5) +
 2)^3 + 4528468716176007960*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 808655127
88857285*(sqrt(5) + 2)^(7/2) - 562345414061023649464320*sqrt(5)*(2*sqrt(5)
+ 5)^(5/2) - 374896942707349099642880*(2*sqrt(5) + 5)^3 - 35146588378813978
0915200*sqrt(5)*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 281172707030511824732
160*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) - 87866470947034945228800*sqrt(
5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 87866470947034945228800*(2*sqrt(5)
 + 5)^2*(sqrt(5) + 2) - 10983308868379368153600*sqrt(5)*(2*sqrt(5) + 5)*(sq
rt(5) + 2)^(3/2) - 14644411824505824204800*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) +
 2)^(3/2) - 686456804273710509600*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)
^2 - 1372913608547421019200*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 1716142010684
2762740*sqrt(5)*(sqrt(5) + 2)^(5/2) - 68645680427371050960*sqrt(2*sqrt(5) +
 5)*(sqrt(5) + 2)^(5/2) - 1430118342236896895*(sqrt(5) + 2)^3 - 35213115894
37476455997440*sqrt(5)*(2*sqrt(5) + 5)^2 - 2075104957091704020631552*(2*sqr
t(5) + 5)^(5/2) - 1760655794718738227998720*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*s
qrt(sqrt(5) + 2) - 1296940598182315012894720*(2*sqrt(5) + 5)^2*sqrt(sqrt(5)
 + 2) - 330122961509763417749760*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 32
4235149545578753223680*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 27510246792480
284812480*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 405293936931973
44152960*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 859695212265008900390*sqrt(5
)*(sqrt(5) + 2)^2 - 2533087105824834009560*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2
)^2 - 63327177645620850239*(sqrt(5) + 2)^(5/2) + 3427066584376513776799744*
sqrt(5)*(2*sqrt(5) + 5)^(3/2) + 2255513638416047840415744*(2*sqrt(5) + 5)^2
 + 1285149969141192666299904*sqrt(5)*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 11
27756819208023920207872*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) + 160643746
142649083287488*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) + 211454403601504
485038976*(2*sqrt(5) + 5)*(sqrt(5) + 2) + 6693489422610378470312*sqrt(5)*(s
qrt(5) + 2)^(3/2) + 17621200300125373753248*sqrt(2*sqrt(5) + 5)*(sqrt(5) +
2)^(3/2) + 550662509378917929789*(sqrt(5) + 2)^2 + 115193815540469058484814
08*sqrt(5)*(2*sqrt(5) + 5) + 9181179291975798227910144*(2*sqrt(5) + 5)^(3/2
) + 2879845388511726462120352*sqrt(5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2)
 + 3442942234490924335466304*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 1799903367
81982903882522*sqrt(5)*(sqrt(5) + 2) + 430367779311365541933288*sqrt(2*sqrt
(5) + 5)*(sqrt(5) + 2) + 17931990804640230913887*(sqrt(5) + 2)^(3/2) - 1493
9851869158064219723 84*sqrt(5)*sqrt(2*sqrt(5) + 5) - 186748148364475802746 54
8*sqrt(5)*sqrt(sqrt(5) + 2) + 3971877328228646528772 8*sqrt(2*sqrt(5) + 5)*
sqrt(sqrt(5) + 2) - 5908116502054650112129 99*sqrt(5) - 47842568896065095084
20584*sqrt(2*sqrt(5) + 5) - 5980321112008136885525 73*sqrt(sqrt(5) + 2) + 96
13265583240077072561069)^2 + 64*(2809473664752684367872 0*sqrt(5)*(2*sqrt(5)
 + 5)^3 + 16054135227158196387840*(2*sqrt(5) + 5)^(7/2) + 21071052485645132
759040*sqrt(5)*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) + 140473683237634218
39360*(2*sqrt(5) + 5)^3*sqrt(sqrt(5) + 2) + 6584703901764103987200*sqrt(5)*
(2*sqrt(5) + 5)^2*(sqrt(5) + 2) + 5267763121411283189760*(2*sqrt(5) + 5)^(5

/2)*(sqrt(5) + 2) + 1097450650294017331200*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2) + 1097450650294017331200*(2*sqrt(5) + 5)^2*(sqrt(5) + 2)^(3/2) + 102885998465064124800*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 + 137181331286752166400*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^2 + 5144299923253206240*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 10288599846506412480*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) + 107172915067775130*sqrt(5)*(sqrt(5) + 2)^3 + 428691660271100520*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^3 + 7655208219126795*(sqrt(5) + 2)^(7/2) - 20755836954363830992896*sqrt(5)*(2*sqrt(5) + 5)^(5/2) - 13837224636242553995264*(2*sqrt(5) + 5)^3 - 12972398096477394370560*sqrt(5)*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 10377918477181915496448*(2*sqrt(5) + 5)^(5/2)*sqrt(sqrt(5) + 2) - 3243099524119348592640*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 3243099524119348592640*(2*sqrt(5) + 5)^2*(sqrt(5) + 2) - 405387440514918574080*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 540516587353224765440*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2)^(3/2) - 25336715032182410880*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 50673430064364821760*(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 633417875804560272*sqrt(5)*(sqrt(5) + 2)^(5/2) - 2533671503218241088*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(5/2) - 52784822983713356*(sqrt(5) + 2)^3 - 363528280045460787978240*sqrt(5)*(2*sqrt(5) + 5)^2 - 220585782417551521185792*(2*sqrt(5) + 5)^(5/2) - 181764140022730393989120*sqrt(5)*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) - 137866114010969700741120*(2*sqrt(5) + 5)^2*sqrt(sqrt(5) + 2) - 34080776254261948872960*sqrt(5)*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 34466528502742425185280*(2*sqrt(5) + 5)^(3/2)*(sqrt(5) + 2) - 2840064687855162406080*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 4308316062842803148160*(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 88752021495473825190*sqrt(5)*(sqrt(5) + 2)^2 - 269269753927675196760*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^2 - 6731743848191879919*(sqrt(5) + 2)^(5/2) - 70364981699301709291520*sqrt(5)*(2*sqrt(5) + 5)^(3/2) - 113606308687559690526720*(2*sqrt(5) + 5)^2 - 26386868137238140984320*sqrt(5)*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) - 56803154343779845263360*(2*sqrt(5) + 5)^(3/2)*sqrt(sqrt(5) + 2) - 3298358517154767623040*sqrt(5)*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) - 10650591439458720986880*(2*sqrt(5) + 5)*(sqrt(5) + 2) - 137431604881448650960*sqrt(5)*(sqrt(5) + 2)^(3/2) - 887549286621560082240*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2)^(3/2) - 27735915206923752570*(sqrt(5) + 2)^2 + 1084940669680612606048128*sqrt(5)*(2*sqrt(5) + 5) + 811441742157208365935104*(2*sqrt(5) + 5)^(3/2) + 271235167420153151512032*sqrt(5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 304290653308953137225664*(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 16952197963759571969502*sqrt(5)*(sqrt(5) + 2) + 38036331663619142153208*sqrt(2*sqrt(5) + 5)*(sqrt(5) + 2) + 1584847152650797589717*(sqrt(5) + 2)^(3/2) + 916330240481116591230464*sqrt(5)*sqrt(2*sqrt(5) + 5) + 114541280060139573903808*sqrt(5)*sqrt(sqrt(5) + 2) + 438878646396292635288832*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 2) + 3690050822522820384588494*sqrt(5) + 1591445182365082778211384*sqrt(2*sqrt(5) + 5) + 198930647795635347276423*sqrt(sqrt(5) + 2) - 131291208062174938773104*e^x + 9240055035301648563405942)^2) - 1/10*sqrt(2)*log(abs(-2*sqrt(2) + 2*e^x - 2)/abs(2*sqrt(2) + 2*e^x - 2)) - 1/10*5^(1/4)*log(6400*(9202754427496321314406*5^(3/4)*sqrt(-2*sqrt(5) + 5) + 1038239983143393667165790*sqrt(5)*sqrt(-2*sqrt(5) + 5) + 186591807316241026405751*5^(3/4) - 20768219695320392550210*5^(1/4)*sqrt(-2*sqrt(5) + 5) + 576155331981489353033147*sqrt(5) - 2322370119525925506249090*sqrt(-2*sqrt(5) + 5) - 417362544266571988465273*5^(1/4) - 1288784580381451028672113)^2 + 25600*(4789310072875935951200*5^(3/4)*sqrt(-2*sqrt(5) + 5) - 1799745554293062228687680*sqrt(5)*sqrt(-2*sqrt(5) + 5) - 325914041979902244813289*5^(3/4) - 10520606548600849190560*5^(1/4)*sqrt(-2*sqrt(5) + 5) + 265033340677886980055183*sqrt(5) + 4025305730691667696322880*sqrt(-2*sqrt(5) + 5) + 728855245658450343948919*5^(1/4) + 1637333558120632636*e^x - 592460559708252630357201)^2) + 1/10*5^(1/4)*log(25600*(4315023771046590689440*5^(3/4)*sqrt(-2*sqrt(5) + 5) + 16512422419052472973244480*sqrt(5)*sqrt(-2*sqrt(5) + 5) + 2991559181950156635096041*5^(3/4) - 11415488961128059998560*5^(1/4)*sqrt(-2*sqrt(5) + 5) + 476470546695231758102799*sqrt(5) - 36931036359499378163241280*sqrt(-2*sqrt(5) + 5) - 6690300251147369625285239*5^(1/4) + 1637333558

```
120632636*e^x - 1067630744269504182665681)^2 + 6400*(5376890661426907499994*
5^(3/4)*sqrt(-2*sqrt(5) + 5) + 4516299732803214710596619O*sqrt(5)*sqrt(-2*s
qrt(5) + 5) + 8186257622186710975158757*5^(3/4) - 583812010039368318539O*5^
(1/4)*sqrt(-2*sqrt(5) + 5) + 603739022767920301057079*sqrt(5) - 10100888679
863924406000197O*sqrt(-2*sqrt(5) + 5) - 18307539608818658210592747*5^(1/4)
- 1355343042548851351155477)^2) + 1/5*sqrt(2*sqrt(5) + 5)*arctan((110641272
*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^7 - 475726088*(2*sqr
t(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^6 - 10105915139*(2*sqrt(5)
+ sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^5 + 16180495104*(2*sqrt(5) + sqr
t(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^4 + 284235586966*(2*sqrt(5) + sqrt(2*
sqrt(5) + 5) + sqrt(sqrt(5) + 2))^3 - 13398309260*(2*sqrt(5) + sqrt(2*sqrt(
5) + 5) + sqrt(sqrt(5) + 2))^2 - 4747850205816*sqrt(5) - 2373925102908*sqrt
(2*sqrt(5) + 5) - 2373925102908*sqrt(sqrt(5) + 2) + 759635933456*e^x - 1242
609575248)/(256556994*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))
^7 - 892031217*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^6 - 25
195966133*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^5 + 2895270
8158*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^4 + 709750301398
*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^3 + 80692042496*(2*s
qrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^2 - 11068354399432*sqrt(5
) - 5534177199716*sqrt(2*sqrt(5) + 5) - 5534177199716*sqrt(sqrt(5) + 2) - 3
881375121088))/sqrt(sqrt(5) + 2) - 1/5*sqrt(2*sqrt(5) + 5)*arctan((83633448
*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^7 - 442112756*(2*sqr
t(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^6 - 7188799155*(2*sqrt(5) +
 sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^5 + 18979817940*(2*sqrt(5) + sqrt
(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^4 + 194564340278*(2*sqrt(5) + sqrt(2*s
qrt(5) + 5) + sqrt(sqrt(5) + 2))^3 - 178069044908*(2*sqrt(5) + sqrt(2*sqrt(
5) + 5) + sqrt(sqrt(5) + 2))^2 - 2862929298552*sqrt(5) - 1431464649276*sqrt
(2*sqrt(5) + 5) - 1431464649276*sqrt(sqrt(5) + 2) - 759635933456*e^x - 1014
49315520)/(82684590*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^7
 - 41690029*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^6 - 10052
928883*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^5 - 3266507166
*(2*sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^4 + 302724737258*(2*
sqrt(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^3 + 148206122616*(2*sqrt
(5) + sqrt(2*sqrt(5) + 5) + sqrt(sqrt(5) + 2))^2 - 4842785241848*sqrt(5) -
2421392620924*sqrt(2*sqrt(5) + 5) - 2421392620924*sqrt(sqrt(5) + 2) - 51107
7100176))/sqrt(sqrt(5) + 2)
```

maple [C] time = 0.07, size = 124, normalized size = 0.54

$$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-2\_Z^7-2\_Z^5+14\_Z^4+2\_Z^3+2\_Z+1\right)}\frac{\left(-2\_R^6+3\_R^5+2\_R^4-2\_R^3-2\_R^2+3\_R+2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{4\_R^7-7\_R^6-5\_R^4+28\_R^3+3\_R^2+1}\right)}{5}+\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{(2\tan}{}\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^5),x)
```

```
[Out] 2/5*sum((-2*_R^6+3*_R^5+2*_R^4-2*_R^3-2*_R^2+3*_R+2)/(4*_R^7-7*_R^6-5*_R^4+
28*_R^3+3*_R^2+1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^8-2*_Z^7-2*_Z^5+14*_Z^4+2
*_Z^3+2*_Z+1))+1/5*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{10}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{x}+1}{\sqrt{2}+e^{x}-1}\right)+\int\frac{2\left(e^{(7x)}+4e^{(6x)}+9e^{(5x)}+24e^{(4x)}-9e^{(3x)}+4e^{(2x)}-e^{x}\right)}{5\left(e^{(8x)}+2e^{(7x)}+2e^{(5x)}+14e^{(4x)}-2e^{(3x)}-2e^{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^5),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + integrate(2/5
*(e^(7*x) + 4*e^(6*x) + 9*e^(5*x) + 24*e^(4*x) - 9*e^(3*x) + 4*e^(2*x) - e^
x)/(e^(8*x) + 2*e^(7*x) + 2*e^(5*x) + 14*e^(4*x) - 2*e^(3*x) - 2*e^x + 1),
x)
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^5 - 1),x)
```

```
[Out] \text{Hanged}
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sinh^5(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**5),x)
```

```
[Out] -Integral(1/(sinh(x)**5 - 1), x)
```

3.274 $\int \frac{1}{1-\sinh^6(x)}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

```
[Out] 1/6*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/3*arctanh((1-(-1)^(1/3))^(1/2)*tanh(
x))/(1-(-1)^(1/3))^(1/2)+1/3*arctanh((1+(-1)^(2/3))^(1/2)*tanh(x))/(1+(-1)^
(2/3))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(3*Sqrt[2]) + ArcTanh[Sqrt[1 - (-1)^(1/3)]*Tanh[x]
]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTanh[Sqrt[1 + (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[
1 + (-1)^(2/3)])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sinh^6(x)}\,dx &= \frac{1}{3}\int \frac{1}{1-\sinh^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1+\sqrt[3]{-1}\sinh^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sinh^2(x)}\,dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\tanh(x)\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\left(1-\sqrt[3]{-1}\right)x^2}\,dx,x,\tanh(x)\right)+\frac{1}{3}\text{Su}\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1+(-1)^{2/3}}}\end{aligned}$$

Mathematica [C] time = 0.45, size = 70, normalized size = 0.84

$$\frac{1}{6}\left(\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)+i\sqrt{3}\left(\tan^{-1}\left(\frac{1-2i\tanh(x)}{\sqrt{3}}\right)-\tan^{-1}\left(\frac{1+2i\tanh(x)}{\sqrt{3}}\right)\right)-\tan^{-1}(\text{csch}(x)\text{sech}(x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^6)^(-1),x]
```

```
[Out] (-ArcTan[Csch[x]*Sech[x]] + I*Sqrt[3]*(ArcTan[(1 - (2*I)*Tanh[x])/Sqrt[3]]
- ArcTan[(1 + (2*I)*Tanh[x])/Sqrt[3]]) + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]])/
6
```

fricas [B] time = 0.95, size = 155, normalized size = 1.87

$$-\frac{1}{12}\sqrt{3}\log\left(16\sqrt{3}+4e^{(4x)}+28\right)+\frac{1}{12}\sqrt{3}\log\left(-16\sqrt{3}+4e^{(4x)}+28\right)+\frac{1}{12}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2}-3\right)e^{(2x)}-12\sqrt{}}{e^{(4x)}-6e^{(2x)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^6),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*log(16*sqrt(3) + 4*e^(4*x) + 28) + 1/12*sqrt(3)*log(-16*sqrt(
3) + 4*e^(4*x) + 28) + 1/12*sqrt(2)*log((2*(2*sqrt(2) - 3)*e^(2*x) - 12*sqr
t(2) + e^(4*x) + 17)/(e^(4*x) - 6*e^(2*x) + 1)) - 1/3*arctan(-(sqrt(3) + 2)
*e^(2*x) + 1/2*(sqrt(3) + 2)*sqrt(-16*sqrt(3) + 4*e^(4*x) + 28)) + 1/3*arct
an(-(sqrt(3) - 2)*e^(2*x) + sqrt(4*sqrt(3) + e^(4*x) + 7)*(sqrt(3) - 2))
```

giac [B] time = 0.16, size = 143, normalized size = 1.72

$$-\frac{1}{36}\left(\left(2\sqrt{3}-3\right)e^{(4x)}+2\sqrt{3}-3\right)\arctan\left(\frac{e^{(2x)}}{\sqrt{3}+2}\right)+\frac{1}{36}\left(\left(2\sqrt{3}+3\right)e^{(4x)}+2\sqrt{3}+3\right)\arctan\left(-\frac{e^{(2x)}}{\sqrt{3}-2}\right)-\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^6),x, algorithm="giac")
```

```
[Out] -1/36*((2*sqrt(3) - 3)*e^(4*x) + 2*sqrt(3) - 3)*arctan(e^(2*x)/(sqrt(3) + 2
)) + 1/36*((2*sqrt(3) + 3)*e^(4*x) + 2*sqrt(3) + 3)*arctan(-e^(2*x)/(sqrt(3
) - 2)) - 1/12*sqrt(3)*log((sqrt(3) + 2)^2 + e^(4*x)) + 1/12*sqrt(3)*log((s
qrt(3) - 2)^2 + e^(4*x)) - 1/12*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)
/abs(4*sqrt(2) + 2*e^(2*x) - 6))
```

maple [C] time = 0.06, size = 160, normalized size = 1.93

$$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^3+2\_Z^2-2\_Z+1\right)}\frac{\left(-\_R^2-\_R+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{2\_R^3+3\_R^2+2\_R-1}\right)}{3}+\frac{\sqrt{2}\ \text{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{6}+\frac{\sqrt{2}\ \text{arctanh}\left(\frac{(2\tan}{}\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^6),x)
```

```
[Out] 1/3*sum((-_R^2-_R+1)/(2*_R^3+3*_R^2+2*_R-1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z
^4+2*_Z^3+2*_Z^2-2*_Z+1))+1/6*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2)
)+1/6*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))+1/3*sum((-_R^2+_R+1)/(
2*_R^3-3*_R^2+2*_R+1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^4-2*_Z^3+2*_Z^2+2*_Z+
1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{x}+1}{\sqrt{2}+e^{x}-1}\right)+\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{x}-1}{\sqrt{2}+e^{x}+1}\right)+\int\frac{e^{(3x)}+4e^{(2x)}-e^{x}}{3\left(e^{(4x)}+2e^{(3x)}+2e^{(2x)}-2e^{x}+1\right)}\,dx-\int\frac{}{3\left(e^{(4x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^6),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + 1/12*sqrt(2)*
log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) + integrate(1/3*(e^(3*x) + 4*
e^(2*x) - e^x)/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x) - integrat
e(1/3*(e^(3*x) - 4*e^(2*x) - e^x)/(e^(4*x) - 2*e^(3*x) + 2*e^(2*x) + 2*e^x
+ 1), x)
```

mupad [B] time = 2.71, size = 285, normalized size = 3.43

$$\frac{\operatorname{atan}\left(\frac{14009449395540459520\,e^{2x}-955607545932677120\,\sqrt{3}+8088359377641144320\,\sqrt{3}\,e^{2x}-1655160823988879360}{6177144285775790080\,e^{2x}+2167269359741829120\,\sqrt{3}+3566375915854233600\,\sqrt{3}\,e^{2x}+3753820658157486080}\right)}{6}-\frac{\sqrt{3}\,\ln\left(\left(61771442\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^6 - 1),x)
```

```
[Out] (log(exp(2*x)*(14009449395540459520 + 6177144285775790080i) + 3^(1/2)*(9556
07545932677120 - 2167269359741829120i) - 3^(1/2)*exp(2*x)*(8088359377641144
320 + 3566375915854233600i) - (1655160823988879360 - 3753820658157486080i))
*1i)/12 - (log(exp(2*x)*(14009449395540459520 - 6177144285775790080i) + 3^(
1/2)*(955607545932677120 + 2167269359741829120i) - 3^(1/2)*exp(2*x)*(808835
9377641144320 - 3566375915854233600i) - (1655160823988879360 + 375382065815
7486080i))*1i)/12 + atan((14009449395540459520*exp(2*x) - 95560754593267712
0*3^(1/2) + 8088359377641144320*3^(1/2)*exp(2*x) - 1655160823988879360)/(61
77144285775790080*exp(2*x) + 2167269359741829120*3^(1/2) + 3566375915854233
600*3^(1/2)*exp(2*x) + 3753820658157486080))/6 - (3^(1/2)*log((617714428577
5790080*exp(2*x) - 2167269359741829120*3^(1/2) - 3566375915854233600*3^(1/2
)*exp(2*x) + 3753820658157486080)^2 + (14009449395540459520*exp(2*x) + 9556
07545932677120*3^(1/2) - 8088359377641144320*3^(1/2)*exp(2*x) - 16551608239
88879360)^2))/12 + (3^(1/2)*log((6177144285775790080*exp(2*x) + 21672693597
41829120*3^(1/2) + 3566375915854233600*3^(1/2)*exp(2*x) + 37538206581574860
80)^2 + (14009449395540459520*exp(2*x) - 955607545932677120*3^(1/2) + 80883
59377641144320*3^(1/2)*exp(2*x) - 1655160823988879360)^2))/12 + (2^(1/2)*lo
g(17674880313941032960*exp(2*x) - 2144322552070144000*2^(1/2) + 12498027726
650736640*2^(1/2)*exp(2*x) - 3032530035220152320))/12 - (2^(1/2)*log(176748
80313941032960*exp(2*x) + 2144322552070144000*2^(1/2) - 1249802772665073664
0*2^(1/2)*exp(2*x) - 3032530035220152320))/12
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**6),x)
```

```
[Out] Timed out
```

3.275 $\int \frac{1}{1-\sinh^8(x)}\,dx$

Optimal. Leaf size=69

$$\frac{\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\tanh(x)}{4}$$

```
[Out] 1/4*arctanh((1-I)^(1/2)*tanh(x))/(1-I)^(1/2)+1/4*arctanh((1+I)^(1/2)*tanh(x
))/(1+I)^(1/2)+1/8*arctanh(2^(1/2)*tanh(x))*2^(1/2)+1/4*tanh(x)
```

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3211, 3181, 206, 3175, 3767, 8}

$$\frac{\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\tanh(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^8)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 - I]*Tanh[x]]/(4*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Tanh[x]]
/(4*Sqrt[1 + I]) + ArcTanh[Sqrt[2]*Tanh[x]]/(4*Sqrt[2]) + Tanh[x]/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{1-\sinh^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sinh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-i\sinh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+i\sinh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sinh^2(x)}$$
$$= \frac{1}{4}\int \text{sech}^2(x)\,dx + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-(1+i)x^2}\,dx, x, \text{ta}\right.$$
$$= \frac{\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{4\sqrt{1-i}} + \frac{\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{4\sqrt{1+i}} + \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}} + \frac{1}{4}i\,\text{Subst}(\int$$
$$= \frac{\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{4\sqrt{1-i}} + \frac{\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{4\sqrt{1+i}} + \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}} + \frac{\tanh(x)}{4}$$

Mathematica [A] time = 0.50, size = 64, normalized size = 0.93

$$\frac{1}{8}\left(\frac{2\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{\sqrt{1-i}} + \frac{2\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{\sqrt{1+i}} + \sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right) + 2\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^8)^(-1),x]
```

```
[Out] ((2*ArcTanh[Sqrt[1 - I]*Tanh[x]])/Sqrt[1 - I] + (2*ArcTanh[Sqrt[1 + I]*Tanh
[x]])/Sqrt[1 + I] + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/8
```

fricas [B] time = 1.55, size = 708, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^8),x, algorithm="fricas")
```

```
[Out] -1/32*(4*(2^(1/4)*e^(2*x) + 2^(1/4))*sqrt(-2*sqrt(2) + 4)*arctan(1/14*(sqrt
(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) - 1/28*(2*sqrt(2)*(5*sqrt(2) +
 6) - (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sqrt(2
) + 4) + 16*sqrt(2) + 8)*sqrt(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*
sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 1/14*sqrt(2)*
(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) +
6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(-2*
sqrt(2) + 4) - 1/7*sqrt(2) + 3/7) + 4*(2^(1/4)*e^(2*x) + 2^(1/4))*sqrt(-2*s
qrt(2) + 4)*arctan(-1/14*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x)
+ 1/28*(2*sqrt(2)*(5*sqrt(2) + 6) + (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(
5*sqrt(2) + 6))*sqrt(-2*sqrt(2) + 4) + 16*sqrt(2) + 8)*sqrt((2^(3/4)*e^(2*x
) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2
*e^(2*x) + 5) + 1/14*sqrt(2)*(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) +
11) + 2*2^(1/4)*(5*sqrt(2) + 6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1
/4)*(3*sqrt(2) - 2))*sqrt(-2*sqrt(2) + 4) + 1/7*sqrt(2) - 3/7) - (2^(1/4)*(
sqrt(2) + 1)*e^(2*x) + 2^(1/4)*(sqrt(2) + 1))*sqrt(-2*sqrt(2) + 4)*log((2^(
3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) +
e^(4*x) - 2*e^(2*x) + 5) + (2^(1/4)*(sqrt(2) + 1)*e^(2*x) + 2^(1/4)*(sqrt(2
) + 1))*sqrt(-2*sqrt(2) + 4)*log(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4
))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 2*(sqrt(2)
*e^(2*x) + sqrt(2))*log((2*(2*sqrt(2) - 3)*e^(2*x) - 12*sqrt(2) + e^(4*x) +
 17)/(e^(4*x) - 6*e^(2*x) + 1)) + 16)/(e^(2*x) + 1)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^8),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.05, size = 99, normalized size = 1.43

$$\frac{\sqrt{2}\ \operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)}{8}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4-2\_Z^2+1\right)}\_R\ln\left(\tanh^2\left(\frac{x}{2}\right)+\left(-4\_R^3+4\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)\right)}{8}+\frac{\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^8),x)
```

```
[Out] 1/8*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+1/8*sum(_R*ln(tanh(1/2*x
)^2+(-4*_R^3+4*_R)*tanh(1/2*x)+1),_R=RootOf(2*_Z^4-2*_Z^2+1))+1/8*2^(1/2)*a
rctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))+1/2*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x+1}{\sqrt{2}+e^x-1}\right)+\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x-1}{\sqrt{2}+e^x+1}\right)-\frac{1}{2\left(e^{(2x)}+1\right)}+8\int\frac{e^{(4x)}}{e^{(8x)}-4e^{(6x)}+22e^{(4x)}-4e^{(2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^8),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + 1/16*sqrt(2)*
log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) - 1/2/(e^(2*x) + 1) + 8*integ
rate(e^(4*x)/(e^(8*x) - 4*e^(6*x) + 22*e^(4*x) - 4*e^(2*x) + 1), x)
```

mupad [B] time = 4.81, size = 273, normalized size = 3.96

$$\frac{\sqrt{2}\ \ln\left(582732658686033920\,e^{2x}-70697326355677184\sqrt{2}+412054214575915008\sqrt{2}\ e^{2x}-99981117754\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sinh(x)^8 - 1),x)
```

```
[Out] (2^(1/2)*log(582732658686033920*exp(2*x) - 70697326355677184*2^(1/2) + 4120
54214575915008*2^(1/2)*exp(2*x) - 99981117754441728))/16 - (2^(1/2)*log(582
732658686033920*exp(2*x) + 70697326355677184*2^(1/2) - 412054214575915008*2
^(1/2)*exp(2*x) - 99981117754441728))/16 - 1/(2*(exp(2*x) + 1)) - (2^(1/2)*
(1 - 1i)^(1/2)*log(exp(2*x)*(155613434002538496 + 429723297714798592i) - 2^
(1/2)*(1 - 1i)^(1/2)*(54684829282729984 - 21956972328779776i) + 2^(1/2)*(1
- 1i)^(1/2)*exp(2*x)*(12296353929494528 - 271474128182050816i) + (708364832
96067584 - 69311013991743488i)))/16 + (2^(1/2)*(1 - 1i)^(1/2)*log(exp(2*x)*
(155613434002538496 + 429723297714798592i) + 2^(1/2)*(1 - 1i)^(1/2)*(546848
29282729984 - 21956972328779776i) - 2^(1/2)*(1 - 1i)^(1/2)*exp(2*x)*(122963
53929494528 - 271474128182050816i) + (70836483296067584 - 69311013991743488
i)))/16 - (2^(1/2)*(1 + 1i)^(1/2)*log(exp(2*x)*(155613434002538496 - 429723
297714798592i) - 2^(1/2)*(1 + 1i)^(1/2)*(54684829282729984 + 21956972328779
776i) + 2^(1/2)*(1 + 1i)^(1/2)*exp(2*x)*(12296353929494528 + 27147412818205
0816i) + (70836483296067584 + 69311013991743488i)))/16 + (2^(1/2)*(1 + 1i)^
(1/2)*log(exp(2*x)*(155613434002538496 - 429723297714798592i) + 2^(1/2)*(1
+ 1i)^(1/2)*(54684829282729984 + 21956972328779776i) - 2^(1/2)*(1 + 1i)^(1/
```

```
2)*exp(2*x)*(12296353929494528 + 27147412818205 0816i) + (70836483296067584
+ 69311013991743488i)))/16
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**8),x)
```

```
[Out] Timed out
```

3.276 $\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)}{a}$$

[Out] sinh(x)/a+1/3*sinh(x)^3/a

Rubi [A] time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3175, 2633}

$$\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^5/(a + a*Sinh[x]^2),x]

[Out] Sinh[x]/a + Sinh[x]^3/(3*a)

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx&=\frac{\int \cosh^3(x)\,dx}{a}\\&=\frac{i\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(x)\right)}{a}\\&=\frac{\sinh(x)}{a}+\frac{\sinh^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{\frac{3\sinh(x)}{4}+\frac{1}{12}\sinh(3x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^5/(a + a*Sinh[x]^2),x]

[Out] ((3*Sinh[x])/4 + Sinh[3*x]/12)/a

fricas [A] time = 2.57, size = 20, normalized size = 1.11

$$\frac{\sinh(x)^3+3\left(\cosh(x)^2+3\right)\sinh(x)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(x)^3 + 3*(cosh(x)^2 + 3)*sinh(x))/a
```

giac [A] time = 2.02, size = 29, normalized size = 1.61

$$-\frac{\left(9\, e^{(2\,x)}+1\right)e^{(-3\,x)}-e^{(3\,x)}-9\, e^{x}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)/a
```

maple [B] time = 0.05, size = 67, normalized size = 3.72

$$\frac{-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a+a*sinh(x)^2),x)
```

```
[Out] 2/a*(-1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2-1/2/(tanh(1/2*x)-1)-1/6/(
tanh(1/2*x)+1)^3+1/4/(tanh(1/2*x)+1)^2-1/2/(tanh(1/2*x)+1))
```

maxima [B] time = 0.33, size = 34, normalized size = 1.89

$$\frac{\left(9\, e^{(-2\,x)}+1\right)e^{(3\,x)}}{24\,a}-\frac{9\, e^{(-x)}+e^{(-3\,x)}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/24*(9*e^(-2*x) + 1)*e^(3*x)/a - 1/24*(9*e^(-x) + e^(-3*x))/a
```

mupad [B] time = 1.36, size = 35, normalized size = 1.94

$$\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{\mathrm{e}^{-3\,x}}{24\,a}-\frac{3\,\mathrm{e}^{-x}}{8\,a}+\frac{3\,\mathrm{e}^{x}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a + a*sinh(x)^2),x)
```

```
[Out] exp(3*x)/(24*a) - exp(-3*x)/(24*a) - (3*exp(-x))/(8*a) + (3*exp(x))/(8*a)
```

sympy [B] time = 6.09, size = 124, normalized size = 6.89

$$-\frac{6\tanh^{5}\left(\frac{x}{2}\right)}{3a\tanh^{6}\left(\frac{x}{2}\right)-9a\tanh^{4}\left(\frac{x}{2}\right)+9a\tanh^{2}\left(\frac{x}{2}\right)-3a}+\frac{4\tanh^{3}\left(\frac{x}{2}\right)}{3a\tanh^{6}\left(\frac{x}{2}\right)-9a\tanh^{4}\left(\frac{x}{2}\right)+9a\tanh^{2}\left(\frac{x}{2}\right)-3a}-\frac{}{3a\tanh^{6}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(a+a*sinh(x)**2),x)
```

```
[Out] -6*tanh(x/2)**5/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)**2 - 3
*a) + 4*tanh(x/2)**3/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)**
2 - 3*a) - 6*tanh(x/2)/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)
**2 - 3*a)
```

3.277 $\int \frac{\cosh^4(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a}+\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] 1/2*x/a+1/2*cosh(x)*sinh(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3175, 2635, 8}

$$\frac{x}{2a}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + a*Sinh[x]^2),x]
```

```
[Out] x/(2*a) + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \cosh^2(x)\,dx}{a}\\ &= \frac{\cosh(x)\sinh(x)}{2a}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{\cosh(x)\sinh(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{\frac{x}{2}+\frac{1}{4}\sinh(2x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + a*Sinh[x]^2),x]
```

```
[Out] (x/2 + Sinh[2*x]/4)/a
```

fricas [A] time = 0.74, size = 12, normalized size = 0.60

$$\frac{\cosh(x)\sinh(x) + x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="fricas")
[Out] 1/2*(cosh(x)*sinh(x) + x)/a
```

giac [A] time = 0.13, size = 28, normalized size = 1.40

$$-\frac{\left(2e^{(2x)}+1\right)e^{(-2x)} - 4x - e^{(2x)}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="giac")
[Out] -1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))/a
```

maple [B] time = 0.04, size = 78, normalized size = 3.90

$$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right.}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+a*sinh(x)^2),x)
[Out] 1/2/a/(tanh(1/2*x)-1)^2+1/2/a/(tanh(1/2*x)-1)-1/2/a*ln(tanh(1/2*x)-1)-1/2/a
/(tanh(1/2*x)+1)^2+1/2/a/(tanh(1/2*x)+1)+1/2/a*ln(tanh(1/2*x)+1)
```

maxima [A] time = 0.32, size = 25, normalized size = 1.25

$$\frac{x}{2a} + \frac{e^{(2x)}}{8a} - \frac{e^{(-2x)}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="maxima")
[Out] 1/2*x/a + 1/8*e^(2*x)/a - 1/8*e^(-2*x)/a
```

mupad [B] time = 1.33, size = 25, normalized size = 1.25

$$\frac{e^{2x}}{8a} - \frac{e^{-2x}}{8a} + \frac{x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a + a*sinh(x)^2),x)
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) + x/(2*a)
```

sympy [B] time = 3.69, size = 153, normalized size = 7.65

$$\frac{x\tanh^4\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}-\frac{2x\tanh^2\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}+\frac{x}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}+\frac{}{2a\tanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+a*sinh(x)**2),x)
```

```
[Out] x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 2*x*tanh(x/2)*
*2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + x/(2*a*tanh(x/2)**4 - 4*a*
tanh(x/2)**2 + 2*a) + 2*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 +
 2*a) + 2*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

3.278 $\int \frac{\cosh^3(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=6

$$\frac{\sinh(x)}{a}$$

[Out] sinh(x)/a

Rubi [A] time = 0.04, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3175, 2637}

$$\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^3/(a + a*Sinh[x]^2),x]

[Out] Sinh[x]/a

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \cosh(x)\,dx}{a}\\ &= \frac{\sinh(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^3/(a + a*Sinh[x]^2),x]

[Out] Sinh[x]/a

fricas [A] time = 1.51, size = 6, normalized size = 1.00

$$\frac{\sinh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="fricas")

[Out] sinh(x)/a

giac [B] time = 0.12, size = 14, normalized size = 2.33

$$-\frac{e^{(-x)} - e^x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="giac")
[Out] -1/2*(e^(-x) - e^x)/a
```

maple [A] time = 0.03, size = 7, normalized size = 1.17

$$\frac{\sinh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+a*sinh(x)^2),x)
[Out] sinh(x)/a
```

maxima [B] time = 0.33, size = 17, normalized size = 2.83

$$-\frac{e^{(-x)}}{2a} + \frac{e^x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="maxima")
[Out] -1/2*e^(-x)/a + 1/2*e^x/a
```

mupad [B] time = 1.29, size = 6, normalized size = 1.00

$$\frac{\sinh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a + a*sinh(x)^2),x)
[Out] sinh(x)/a
```

sympy [B] time = 2.13, size = 17, normalized size = 2.83

$$-\frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right) - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+a*sinh(x)**2),x)
[Out] -2*tanh(x/2)/(a*tanh(x/2)**2 - a)
```

3.279 $\int \frac{\cosh^2(x)}{a+a\sinh^2(x)} dx$

Optimal. Leaf size=5

$$\frac{x}{a}$$

[Out] x/a

Rubi [A] time = 0.04, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3175, 8}

$$\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^2/(a + a*Sinh[x]^2),x]

[Out] x/a

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(x)}{a+a\sinh^2(x)} dx &= \frac{\int 1 \, dx}{a} \\ &= \frac{x}{a} \end{aligned}$$

Mathematica [A] time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^2/(a + a*Sinh[x]^2),x]

[Out] x/a

fricas [A] time = 0.81, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="fricas")

[Out] x/a

giac [A] time = 0.11, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="giac")
[Out] x/a
```

maple [C] time = 0.04, size = 11, normalized size = 2.20

$$\frac{2\,\mathrm{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+a*sinh(x)^2),x)
[Out] 2/a*arctanh(tanh(1/2*x))
```

maxima [A] time = 0.33, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="maxima")
[Out] x/a
```

mupad [B] time = 1.26, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a + a*sinh(x)^2),x)
[Out] x/a
```

sympy [A] time = 1.17, size = 2, normalized size = 0.40

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+a*sinh(x)**2),x)
[Out] x/a
```

3.280 $\int \frac{\cosh(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=7

$$\frac{\tan^{-1}(\sinh(x))}{a}$$

```
[Out] arctan(sinh(x))/a
```

Rubi [A] time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3175, 3770}

$$\frac{\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] ArcTan[Sinh[x]]/a
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \operatorname{sech}(x)\,dx}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.71

$$\frac{2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] (2*ArcTan[Tanh[x/2]])/a
```

fricas [A] time = 0.78, size = 11, normalized size = 1.57

$$\frac{2\,\arctan\left(\cosh(x)+\sinh(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(x) + sinh(x))/a
```

giac [A] time = 0.13, size = 8, normalized size = 1.14

$$\frac{2 \arctan(e^x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="giac")
[Out] 2*arctan(e^x)/a
```

maple [A] time = 0.02, size = 8, normalized size = 1.14

$$\frac{\arctan(\sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*sinh(x)^2),x)
[Out] arctan(sinh(x))/a
```

maxima [A] time = 0.45, size = 10, normalized size = 1.43

$$-\frac{2 \arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="maxima")
[Out] -2*arctan(e^(-x))/a
```

mupad [B] time = 0.07, size = 7, normalized size = 1.00

$$\frac{\operatorname{atan}(\sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a + a*sinh(x)^2),x)
[Out] atan(sinh(x))/a
```

sympy [A] time = 0.20, size = 5, normalized size = 0.71

$$\frac{\operatorname{atan}(\sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)**2),x)
[Out] atan(sinh(x))/a
```

3.281 $\int \frac{\operatorname{sech}(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

```
[Out] 1/2*arctan(sinh(x))/a+1/2*sech(x)*tanh(x)/a
```

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3175, 3768, 3770}

$$\frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] ArcTan[Sinh[x]]/(2*a) + (Sech[x]*Tanh[x])/(2*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \operatorname{sech}^3(x)\,dx}{a}\\ &= \frac{\operatorname{sech}(x)\tanh(x)}{2a}+\frac{\int \operatorname{sech}(x)\,dx}{2a}\\ &= \frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 0.91

$$\frac{\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\tanh(x)\operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] (ArcTan[Tanh[x/2]] + (Sech[x]*Tanh[x])/2)/a
```

fricas [B] time = 1.36, size = 151, normalized size = 6.86

$$\frac{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 1\right)\right.}{a\cosh(x)^4 + 4a\cosh(x)\sinh(x)^3 + a\sinh(x)^4 + 2a\cosh(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh(
x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)
^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)^2 - 1)*si
nh(x) - cosh(x))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*c
osh(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*si
nh(x) + a)
```

giac [B] time = 0.14, size = 52, normalized size = 2.36

$$\frac{\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)}{4a} - \frac{e^{(-x)} - e^{x}}{\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a - (e^(-x) - e^x)/(((e^(-x)
- e^x)^2 + 4)*a)
```

maple [B] time = 0.05, size = 50, normalized size = 2.27

$$-\frac{\tanh^3\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2} + \frac{\tanh\left(\frac{x}{2}\right)}{a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^2} + \frac{\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+a*sinh(x)^2),x)
```

```
[Out] -1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3+1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)+
1/a*arctan(tanh(1/2*x))
```

maxima [B] time = 0.43, size = 40, normalized size = 1.82

$$\frac{e^{(-x)} - e^{(-3x)}}{2ae^{(-2x)} + ae^{(-4x)} + a} - \frac{\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] (e^(-x) - e^(-3*x))/(2*a*e^(-2*x) + a*e^(-4*x) + a) - arctan(e^(-x))/a
```

mupad [B] time = 1.30, size = 54, normalized size = 2.45

$$\frac{\operatorname{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2e^x}{a\left(2e^{2x} + e^{4x} + 1\right)} + \frac{e^x}{a\left(e^{2x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)*(a + a*sinh(x)^2)),x)
```

```
[Out] atan((exp(x)*(a^2)^(1/2))/a)/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(
4*x) + 1)) + exp(x)/(a*(exp(2*x) + 1))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}(x)}{\sinh^2(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)**2),x)
```

```
[Out] Integral(sech(x)/(sinh(x)**2 + 1), x)/a
```

3.282 $\int \frac{\operatorname{sech}^3(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=35

$$\frac{3\tan^{-1}(\sinh(x))}{8a}+\frac{\tanh(x)\operatorname{sech}^3(x)}{4a}+\frac{3\tanh(x)\operatorname{sech}(x)}{8a}$$

```
[Out] 3/8*arctan(sinh(x))/a+3/8*sech(x)*tanh(x)/a+1/4*sech(x)^3*tanh(x)/a
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3175, 3768, 3770}

$$\frac{3\tan^{-1}(\sinh(x))}{8a}+\frac{\tanh(x)\operatorname{sech}^3(x)}{4a}+\frac{3\tanh(x)\operatorname{sech}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + a*Sinh[x]^2),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(8*a) + (3*Sech[x]*Tanh[x])/(8*a) + (Sech[x]^3*Tanh[x])
/(4*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \operatorname{sech}^5(x)\,dx}{a}\\
&= \frac{\operatorname{sech}^3(x)\tanh(x)}{4a}+\frac{3\int \operatorname{sech}^3(x)\,dx}{4a}\\
&= \frac{3\operatorname{sech}(x)\tanh(x)}{8a}+\frac{\operatorname{sech}^3(x)\tanh(x)}{4a}+\frac{3\int \operatorname{sech}(x)\,dx}{8a}\\
&= \frac{3\tan^{-1}(\sinh(x))}{8a}+\frac{3\operatorname{sech}(x)\tanh(x)}{8a}+\frac{\operatorname{sech}^3(x)\tanh(x)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 0.97

$$\frac{\frac{3}{4}\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{4}\tanh(x)\operatorname{sech}^3(x)+\frac{3}{8}\tanh(x)\operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + a*Sinh[x]^2),x]
```

```
[Out] ((3*ArcTan[Tanh[x/2]])/4 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4)/a
```

fricas [B] time = 1.55, size = 488, normalized size = 13.94

$$\frac{3\cosh(x)^7 + 21\cosh(x)\sinh(x)^6 + 3\sinh(x)^7 + \left(63\cosh(x)^2 + 11\right)\sinh(x)^5 + 11\cosh(x)^5 + 5\left(21\cosh(x)^3 + \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(x)^2 + 11)
*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*sinh(x)^4 + (105*
cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 11*cosh(x)^3 + (63*cosh(x)^5 +
110*cosh(x)^3 - 33*cosh(x))*sinh(x)^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7
+ sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3
+ 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*
cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cos
h(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(
x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + s
inh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*sinh(x) - 3*cosh
(x))/(a*cosh(x)^8 + 8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 + 4*a*cosh(x)^6 + 4
*(7*a*cosh(x)^2 + a)*sinh(x)^6 + 8*(7*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^5
+ 6*a*cosh(x)^4 + 2*(35*a*cosh(x)^4 + 30*a*cosh(x)^2 + 3*a)*sinh(x)^4 + 8*(
7*a*cosh(x)^5 + 10*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 4*a*cosh(x)^2 + 4
*(7*a*cosh(x)^6 + 15*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*sinh(x)^2 + 8*(a*cosh
(x)^7 + 3*a*cosh(x)^5 + 3*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

giac [B] time = 0.12, size = 67, normalized size = 1.91

$$\frac{3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)}{16a} - \frac{3\left(e^{(-x)} - e^{x}\right)^3 + 20e^{(-x)} - 20e^{x}}{4\left(\left(e^{(-x)} - e^{x}\right)^2 + 4\right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] 3/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a - 1/4*(3*(e^(-x) - e^x)^3
+ 20*e^(-x) - 20*e^x)/(((e^(-x) - e^x)^2 + 4)^2*a)
```

maple [B] time = 0.06, size = 94, normalized size = 2.69

$$-\frac{5\left(\tanh^7\left(\frac{x}{2}\right)\right)}{4a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}+\frac{3\left(\tanh^5\left(\frac{x}{2}\right)\right)}{4a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}-\frac{3\left(\tanh^3\left(\frac{x}{2}\right)\right)}{4a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}+\frac{5\tanh\left(\frac{x}{2}\right)}{4a\left(\tanh^2\left(\frac{x}{2}\right)+1\right)^4}+\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+a*sinh(x)^2),x)
```

```
[Out] -5/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7+3/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2
*x)^5-3/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3+5/4/a/(tanh(1/2*x)^2+1)^4*tan
h(1/2*x)+3/4/a*arctan(tanh(1/2*x))
```

maxima [B] time = 0.41, size = 69, normalized size = 1.97

$$\frac{3e^{(-x)} + 11e^{(-3x)} - 11e^{(-5x)} - 3e^{(-7x)}}{4\left(4ae^{(-2x)} + 6ae^{(-4x)} + 4ae^{(-6x)} + ae^{(-8x)} + a\right)} - \frac{3\arctan\left(e^{(-x)}\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*a*e^(-2*x) + 6*a
*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) - 3/4*arctan(e^(-x))/a
```

mupad [B] time = 1.29, size = 118, normalized size = 3.37

$$\frac{3\,\mathrm{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{4\,\sqrt{a^2}}-\frac{4\,e^{3\,x}}{a\,\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}-\frac{2\,e^x}{a\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)}+\frac{e^x}{2\,a\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}+\frac{}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(x)^3*(a + a*sinh(x)^2)),x)
```

```
[Out] (3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2)) - (4*exp(3*x))/(a*(4*exp(2
*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) - (2*exp(x))/(a*(3*exp(2*x)
+ 3*exp(4*x) + exp(6*x) + 1)) + exp(x)/(2*a*(2*exp(2*x) + exp(4*x) + 1)) +
(3*exp(x))/(4*a*(exp(2*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{sech}^{3}{\left(x \right)}}{\sinh^{2}{\left(x \right)} + 1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+a*sinh(x)**2),x)
```

```
[Out] Integral(sech(x)**3/(sinh(x)**2 + 1), x)/a
```

3.283 $\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=89

$$\frac{(6a-b)\sinh(c+dx)\cosh^3(c+dx)}{24d}+\frac{(6a-b)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x(6a-b)+\frac{b\sinh(c+dx)\cosh^5(c+dx)}{6d}$$

```
[Out] 1/16*(6*a-b)*x+1/16*(6*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/24*(6*a-b)*cosh(d*x
+c)^3*sinh(d*x+c)/d+1/6*b*cosh(d*x+c)^5*sinh(d*x+c)/d
```

Rubi [A] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3191, 385, 199, 206}

$$\frac{(6a-b)\sinh(c+dx)\cosh^3(c+dx)}{24d}+\frac{(6a-b)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x(6a-b)+\frac{b\sinh(c+dx)\cosh^5(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((6*a - b)*x)/16 + ((6*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) + ((6*a -
 b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*Cosh[c + d*x]^5*Sinh[c + d*x
])/(6*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{(1-x^2)^4}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d} + \frac{(6a-b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)^3}dx, x, \text{ta}\right.}{6d}$$

$$= \frac{(6a-b)\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b\cosh^5(c+dx)\sinh(c+d}{6d}$$

$$= \frac{(6a-b)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(6a-b)\cosh^3(c+dx)\sinh}{24d}$$

$$= \frac{1}{16}(6a-b)x + \frac{(6a-b)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(6a-b)\cosh}{}$$

Mathematica [A] time = 0.18, size = 63, normalized size = 0.71

$$\frac{(48a-3b)\sinh(2(c+dx)) + 3(2a+b)\sinh(4(c+dx)) + 72ac + 72adx + b\sinh(6(c+dx)) - 12bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (72*a*c + 72*a*d*x - 12*b*d*x + (48*a - 3*b)*Sinh[2*(c + d*x)] + 3*(2*a + b
)*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.70, size = 117, normalized size = 1.31

$$\frac{3\,b\cosh(dx+c)\sinh(dx+c)^5 + 2\left(5\,b\cosh(dx+c)^3 + 3\,(2\,a+b)\cosh(dx+c)\right)\sinh(dx+c)^3 + 6\,(6\,a-b)d}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 + 3*(2*a +
 b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(6*a - b)*d*x + 3*(b*cosh(d*x + c)^5
 + 2*(2*a + b)*cosh(d*x + c)^3 + (16*a - b)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.16, size = 121, normalized size = 1.36

$$\frac{1}{16}(6\,a-b)x + \frac{be^{(6\,dx+6\,c)}}{384\,d} + \frac{(2\,a+b)e^{(4\,dx+4\,c)}}{128\,d} + \frac{(16\,a-b)e^{(2\,dx+2\,c)}}{128\,d} - \frac{(16\,a-b)e^{(-2\,dx-2\,c)}}{128\,d} - \frac{(2\,a+b)e^{(-4\,dx-4\,c)}}{128\,d} - \frac{be^{(-}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*a - b)*x + 1/384*b*e^(6*d*x + 6*c)/d + 1/128*(2*a + b)*e^(4*d*x + 4
*c)/d + 1/128*(16*a - b)*e^(2*d*x + 2*c)/d - 1/128*(16*a - b)*e^(-2*d*x - 2
*c)/d - 1/128*(2*a + b)*e^(-4*d*x - 4*c)/d - 1/384*b*e^(-6*d*x - 6*c)/d
```

maple [A] time = 0.14, size = 95, normalized size = 1.07

$$\frac{b\left(\frac{\sinh(dx+c)\left(\cosh^5(dx+c)\right)}{6} - \frac{\left(\frac{\left(\cosh^3(dx+c)\right)}{4} + \frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)}{6} - \frac{dx}{16} - \frac{c}{16}\right) + a\left(\left(\frac{\left(\cosh^3(dx+c)\right)}{4} + \frac{3\cosh(dx+c)}{8}\right)\sinh(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/6*sinh(d*x+c)*cosh(d*x+c)^5-1/6*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c
))*sinh(d*x+c)-1/16*d*x-1/16*c)+a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh
(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.33, size = 152, normalized size = 1.71

$$\frac{1}{64}a\left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + \frac{1}{384}b\left(\frac{\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + 1\right)e^{(6dx+6c)}}{d} -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/384*b*((3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c
) + 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d + (3*e^(-2*d*x - 2*c) - 3*e^(-4*d
*x - 4*c) - e^(-6*d*x - 6*c))/d)
```

mupad [B] time = 1.43, size = 76, normalized size = 0.85

$$\frac{12\,a\sinh\left(2\,c+2\,d\,x\right)+\frac{3\,a\sinh(4\,c+4\,d\,x)}{2}-\frac{3\,b\sinh(2\,c+2\,d\,x)}{4}+\frac{3\,b\sinh(4\,c+4\,d\,x)}{4}+\frac{b\sinh(6\,c+6\,d\,x)}{4}+18\,a\,d\,x-3\,b\,d\,x}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (12*a*sinh(2*c + 2*d*x) + (3*a*sinh(4*c + 4*d*x))/2 - (3*b*sinh(2*c + 2*d*x
))/4 + (3*b*sinh(4*c + 4*d*x))/4 + (b*sinh(6*c + 6*d*x))/4 + 18*a*d*x - 3*b
*d*x)/(48*d)
```

sympy [A] time = 3.22, size = 250, normalized size = 2.81

$$\begin{cases} \frac{3ax\sinh^4(c+dx)}{8} - \frac{3ax\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3ax\cosh^4(c+dx)}{8} - \frac{3a\sinh^3(c+dx)\cosh(c+dx)}{8d} + \frac{5a\sinh(c+dx)\cosh^3(c+dx)}{8d} + \frac{bx\sinh}{1} \\ x\left(a+b\sinh^2(c)\right)\cosh^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 - 3*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
+ 5*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b*x*sinh(c + d*x)**6/16 - 3*b*
x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 3*b*x*sinh(c + d*x)**2*cosh(c + d*
x)**4/16 - b*x*cosh(c + d*x)**6/16 - b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d
) + b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + b*sinh(c + d*x)*cosh(c + d*
x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**4, True))
```

3.284 $\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=46

$$\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^5(c+dx)}{5d}$$

```
[Out] a*sinh(d*x+c)/d+1/3*(a+b)*sinh(d*x+c)^3/d+1/5*b*sinh(d*x+c)^5/d
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3190, 373}

$$\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^5)
/(5*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+bx^2\right)\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2+bx^4\right)\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{b\sinh^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 48, normalized size = 1.04

$$\frac{\sinh(c+dx)(4(5a+2b)\cosh(2(c+dx))+100a+3b\cosh(4(c+dx))-11b)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((100*a - 11*b + 4*(5*a + 2*b)*Cosh[2*(c + d*x)] + 3*b*Cosh[4*(c + d*x)])*S
inh[c + d*x])/(120*d)
```

fricas [A] time = 2.51, size = 82, normalized size = 1.78

$$\frac{3\,b\sinh\left(dx+c\right)^{5}+5\left(6\,b\cosh\left(dx+c\right)^{2}+4\,a+b\right)\sinh\left(dx+c\right)^{3}+15\left(b\cosh\left(dx+c\right)^{4}+\left(4\,a+b\right)\cosh\left(dx+c\right)\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*sinh(d*x + c)^5 + 5*(6*b*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c
)^3 + 15*(b*cosh(d*x + c)^4 + (4*a + b)*cosh(d*x + c)^2 + 12*a - 2*b)*sinh(
d*x + c))/d
```

giac [B] time = 0.13, size = 108, normalized size = 2.35

$$\frac{be^{(5\,dx+5\,c)}}{160\,d}+\frac{(4\,a+b)e^{(3\,dx+3\,c)}}{96\,d}+\frac{(6\,a-b)e^{(dx+c)}}{16\,d}-\frac{(6\,a-b)e^{(-dx-c)}}{16\,d}-\frac{(4\,a+b)e^{(-3\,dx-3\,c)}}{96\,d}-\frac{be^{(-5\,dx-5\,c)}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/160*b*e^(5*d*x + 5*c)/d + 1/96*(4*a + b)*e^(3*d*x + 3*c)/d + 1/16*(6*a -
b)*e^(d*x + c)/d - 1/16*(6*a - b)*e^(-d*x - c)/d - 1/96*(4*a + b)*e^(-3*d*x
 - 3*c)/d - 1/160*b*e^(-5*d*x - 5*c)/d
```

maple [A] time = 0.13, size = 65, normalized size = 1.41

$$\frac{b\left(\frac{\sinh(dx+c)\left(\cosh^{4}(dx+c)\right)}{5}-\frac{\left(\frac{2}{3}+\frac{\left(\cosh^{2}(dx+c)\right)}{3}\right)\sinh(dx+c)}{5}\right)+a\left(\frac{2}{3}+\frac{\left(\cosh^{2}(dx+c)\right)}{3}\right)\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/5*sinh(d*x+c)*cosh(d*x+c)^4-1/5*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+
c))+a*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))
```

maxima [B] time = 0.34, size = 136, normalized size = 2.96

$$\frac{1}{480}\,b\left(\frac{\left(5\,e^{(-2\,dx-2\,c)}-30\,e^{(-4\,dx-4\,c)}+3\right)e^{(5\,dx+5\,c)}}{d}+\frac{30\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}-3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*b*((5*e^(-2*d*x - 2*c) - 30*e^(-4*d*x - 4*c) + 3)*e^(5*d*x + 5*c)/d +
 (30*e^(-d*x - c) - 5*e^(-3*d*x - 3*c) - 3*e^(-5*d*x - 5*c))/d) + 1/24*a*(e
^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 1.34, size = 48, normalized size = 1.04

$$\frac{15\,a\sinh\left(c+d\,x\right)+5\,a\sinh\left(c+d\,x\right)^{3}+5\,b\sinh\left(c+d\,x\right)^{3}+3\,b\sinh\left(c+d\,x\right)^{5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (15*a*sinh(c + d*x) + 5*a*sinh(c + d*x)^3 + 5*b*sinh(c + d*x)^3 + 3*b*sinh(
c + d*x)^5)/(15*d)
```

sympy [A] time = 1.61, size = 85, normalized size = 1.85

$$\begin{cases} -\frac{2a\sinh^3(c+dx)}{3d} + \frac{a\sinh(c+dx)\cosh^2(c+dx)}{d} - \frac{2b\sinh^5(c+dx)}{15d} + \frac{b\sinh^3(c+dx)\cosh^2(c+dx)}{3d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^2(c)\right)\cosh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((-2*a*sinh(c + d*x)**3/(3*d) + a*sinh(c + d*x)*cosh(c + d*x)**2/d
 - 2*b*sinh(c + d*x)**5/(15*d) + b*sinh(c + d*x)**3*cosh(c + d*x)**2/(3*d),
 Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**3, True))
```

3.285 $\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(4a-b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(4a-b)+\frac{b\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

```
[Out] 1/8*(4*a-b)*x+1/8*(4*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*b*cosh(d*x+c)^3*sin
h(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3191, 385, 199, 206}

$$\frac{(4a-b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(4a-b)+\frac{b\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((4*a - b)*x)/8 + ((4*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c
 + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx = \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{(4a-b)\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2}\,dx, x, \text{ta}\right.}{4d}$$

$$= \frac{(4a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b\cosh^3(c+dx)\sinh(c+dx}{4d}$$

$$= \frac{1}{8}(4a-b)x + \frac{(4a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b\cosh^3(c+d}{4}$$

Mathematica [A] time = 0.07, size = 43, normalized size = 0.70

$$\frac{8a\sinh(2(c+dx)) + 16ac + 16adx + b\sinh(4(c+dx)) - 4bdx}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (16*a*c + 16*a*d*x - 4*b*d*x + 8*a*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)])
/(32*d)
```

fricas [A] time = 1.06, size = 59, normalized size = 0.97

$$\frac{b\cosh(dx+c)\sinh(dx+c)^3 + (4a-b)dx + \left(b\cosh(dx+c)^3 + 4a\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a - b)*d*x + (b*cosh(d*x + c)^3 +
 4*a*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.14, size = 71, normalized size = 1.16

$$\frac{1}{8}(4a-b)x + \frac{be^{(4dx+4c)}}{64d} + \frac{ae^{(2dx+2c)}}{8d} - \frac{ae^{(-2dx-2c)}}{8d} - \frac{be^{(-4dx-4c)}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*a - b)*x + 1/64*b*e^(4*d*x + 4*c)/d + 1/8*a*e^(2*d*x + 2*c)/d - 1/8*
a*e^(-2*d*x - 2*c)/d - 1/64*b*e^(-4*d*x - 4*c)/d
```

maple [A] time = 0.07, size = 70, normalized size = 1.15

$$\frac{b\left(\frac{\sinh(dx+c)\left(\cosh^3(dx+c)\right)}{4} - \frac{\cosh(dx+c)\sinh(dx+c)}{8} - \frac{dx}{8} - \frac{c}{8}\right) + a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
```

[Out] 1/d*(b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+a*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c))

maxima [A] time = 0.35, size = 76, normalized size = 1.25

$$\frac{1}{8}\,a\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{64}\,b\left(\frac{8\,(dx+c)}{d}-\frac{e^{(4\,dx+4\,c)}}{d}+\frac{e^{(-4\,dx-4\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/64*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)

mupad [B] time = 0.10, size = 38, normalized size = 0.62

$$\frac{a\,x}{2}-\frac{b\,x}{8}+\frac{\frac{a\sinh(2\,c+2\,d\,x)}{4}+\frac{b\sinh(4\,c+4\,d\,x)}{32}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2),x)

[Out] (a*x)/2 - (b*x)/8 + ((a*sinh(2*c + 2*d*x))/4 + (b*sinh(4*c + 4*d*x))/32)/d

sympy [A] time = 0.92, size = 150, normalized size = 2.46

$$\begin{cases} -\frac{ax\sinh^2(c+dx)}{2}+\frac{ax\cosh^2(c+dx)}{2}+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx\sinh^4(c+dx)}{8}+\frac{bx\sinh^2(c+dx)\cosh^2(c+dx)}{4}-\frac{bx\cosh^4(c+dx)}{8}+\frac{b\sinh \\ x\left(a+b\sinh^2(c)\right)\cosh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)

[Out] Piecewise((-a*x*sinh(c + d*x)**2/2 + a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x)*cosh(c + d*x)/(2*d) - b*x*sinh(c + d*x)**4/8 + b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - b*x*cosh(c + d*x)**4/8 + b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**2, True))

3.286 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

```
[Out] a*sinh(d*x+c)/d+1/3*b*sinh(d*x+c)^3/d
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3190}

$$\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Sinh[c + d*x])/d + (b*Sinh[c + d*x]^3)/(3*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.39

$$\frac{a\sinh(c)\cosh(dx)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/d + (b*Sinh[c + d*x]^3)/(3*
d)
```

fricas [A] time = 2.02, size = 41, normalized size = 1.46

$$\frac{b\sinh\left(dx+c\right)^3+3\left(b\cosh\left(dx+c\right)^2+4\,a-b\right)\sinh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(b*sinh(d*x + c)^3 + 3*(b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))/d
```

giac [**B**] time = 0.14, size = 70, normalized size = 2.50

$$\frac{be^{(3\,dx+3\,c)}}{24\,d}+\frac{(4\,a-b)e^{(dx+c)}}{8\,d}-\frac{(4\,a-b)e^{(-dx-c)}}{8\,d}-\frac{be^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*b*e^(3*d*x + 3*c)/d + 1/8*(4*a - b)*e^(d*x + c)/d - 1/8*(4*a - b)*e^(-
d*x - c)/d - 1/24*b*e^(-3*d*x - 3*c)/d
```

maple [**A**] time = 0.03, size = 25, normalized size = 0.89

$$\frac{\frac{b\left(\sinh^3(dx+c)\right)}{3}+a\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*b*sinh(d*x+c)^3+a*sinh(d*x+c))
```

maxima [**A**] time = 0.33, size = 26, normalized size = 0.93

$$\frac{b\sinh\left(dx+c\right)^3}{3\,d}+\frac{a\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*b*sinh(d*x + c)^3/d + a*sinh(d*x + c)/d
```

mupad [**B**] time = 0.09, size = 25, normalized size = 0.89

$$\frac{\sinh\left(c+d\,x\right)\,\left(b\,\sinh\left(c+d\,x\right)^2+3\,a\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (sinh(c + d*x)*(3*a + b*sinh(c + d*x)^2))/(3*d)
```

sympy [**A**] time = 0.42, size = 36, normalized size = 1.29

$$\begin{cases}\frac{a\sinh\left(c+dx\right)}{d}+\frac{b\sinh^3\left(c+dx\right)}{3d} & \text{for}\ d\neq 0\\ x\left(a+b\sinh^2\left(c\right)\right)\cosh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*sinh(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a +
 b*sinh(c)**2)*cosh(c), True))
```

3.287 $\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{(a-b)\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}$$

```
[Out] (a-b)*arctan(sinh(d*x+c))/d+b*sinh(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3190, 388, 203}

$$\frac{(a-b)\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - b)*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c + d*x])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{b\sinh(c+dx)}{d}+\frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{(a-b)\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.32

$$\frac{a\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}-\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/d - (b*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c + d*x
])/d
```

fricas [B] time = 0.58, size = 101, normalized size = 3.61

$$\frac{b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}+4\left(\left(a-b\right)\cosh\left(dx+c\right)+\left(a-b\right)\sinh\left(dx+c\right)\right)\ldots}{2\left(d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^
2 + 4*((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*arctan(cosh(d*x + c)
+ sinh(d*x + c)) - b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

giac [A] time = 0.14, size = 40, normalized size = 1.43

$$\frac{4\left(a-b\right)\arctan\left(e^{\left(dx+c\right)}\right)+be^{\left(dx+c\right)}-be^{\left(-dx-c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(4*(a - b)*arctan(e^(d*x + c)) + b*e^(d*x + c) - b*e^(-d*x - c))/d
```

maple [A] time = 0.06, size = 39, normalized size = 1.39

$$\frac{b\sinh\left(dx+c\right)}{d}+\frac{2a\arctan\left(\mathrm{e}^{dx+c}\right)}{d}-\frac{2b\arctan\left(\mathrm{e}^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] b*sinh(d*x+c)/d+2/d*a*arctan(exp(d*x+c))-2/d*b*arctan(exp(d*x+c))
```

maxima [A] time = 0.45, size = 56, normalized size = 2.00

$$\frac{1}{2}\,b\left(\frac{4\,\arctan\left(e^{\left(-dx-c\right)}\right)}{d}+\frac{e^{\left(dx+c\right)}}{d}-\frac{e^{\left(-dx-c\right)}}{d}\right)+\frac{a\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a*arcta
n(sinh(d*x + c))/d
```

mupad [B] time = 1.78, size = 88, normalized size = 3.14

$$\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a\,\sqrt{d^2}-b\,\sqrt{d^2}\right)}{d\,\sqrt{a^2-2\,a\,b+b^2}}\right)\,\sqrt{a^2-2\,a\,b+b^2}}{\sqrt{d^2}}-\frac{b\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{b\,\mathrm{e}^{c+d\,x}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x),x)
```

```
[Out] (2*atan((exp(d*x)*exp(c)*(a*(d^2)^(1/2) - b*(d^2)^(1/2)))/(d*(a^2 - 2*a*b +
 b^2)^(1/2)))*(a^2 - 2*a*b + b^2)^(1/2))/(d^2)^(1/2) - (b*exp(- c - d*x))/(
2*d) + (b*exp(c + d*x))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^2\left(c + dx\right)\right)\operatorname{sech}\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x), x)
```

3.288 $\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=19

$$\frac{(a-b)\tanh(c+dx)}{d}+bx$$

```
[Out] b*x+(a-b)*tanh(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3191, 388, 206}

$$\frac{(a-b)\tanh(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] b*x + ((a - b)*Tanh[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a-(a-b)x^2}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{(a-b)\tanh(c+dx)}{d}+\frac{b\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= bx+\frac{(a-b)\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.89

$$\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^{-1}(\tanh(c+dx))}{d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
[Out] (b*ArcTanh[Tanh[c + d*x]])/d + (a*Tanh[c + d*x])/d - (b*Tanh[c + d*x])/d
```

fricas [B] time = 0.89, size = 41, normalized size = 2.16

$$\frac{(bdx - a + b)\cosh(dx + c) + (a - b)\sinh(dx + c)}{d\cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
[Out] ((b*d*x - a + b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

giac [A] time = 0.13, size = 32, normalized size = 1.68

$$\frac{(dx + c)b - \frac{2(a-b)}{e^{(2dx+2c)}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
[Out] ((d*x + c)*b - 2*(a - b)/(e^(2*d*x + 2*c) + 1))/d
```

maple [A] time = 0.15, size = 29, normalized size = 1.53

$$\frac{\tanh(dx + c)a + b(dx + c - \tanh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
[Out] 1/d*(tanh(d*x+c)*a+b*(d*x+c-tanh(d*x+c)))
```

maxima [B] time = 0.35, size = 47, normalized size = 2.47

$$b\left(x + \frac{c}{d} - \frac{2}{d\left(e^{(-2dx-2c)} + 1\right)}\right) + \frac{2a}{d\left(e^{(-2dx-2c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
[Out] b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 0.80, size = 27, normalized size = 1.42

$$b\,x - \frac{2\,(a - b)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^2,x)
[Out] b*x - (2*(a - b))/(d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^2(c + dx)\right)\operatorname{sech}^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**2, x)
```

3.289 $\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)dx$

Optimal. Leaf size=42

$$\frac{(a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

```
[Out] 1/2*(a+b)*arctan(sinh(d*x+c))/d+1/2*(a-b)*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 385, 203}

$$\frac{(a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a - b)*Sech[c + d*x]*Tanh[c + d*x
])/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{\left(1+x^2\right)^2}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh\right.}{2d}\\ &= \frac{(a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 1.69

$$\frac{a\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Sech
[c + d*x]*Tanh[c + d*x])/(2*d) - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

fricas [B] time = 1.66, size = 324, normalized size = 7.71

$$\frac{(a-b)\cosh(dx+c)^3 + 3(a-b)\cosh(dx+c)\sinh(dx+c)^2 + (a-b)\sinh(dx+c)^3 + \left((a+b)\cosh(dx+c)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] ((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b
)*sinh(d*x + c)^3 + ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3
 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*arctan(cosh(d*x + c) + sin
h(d*x + c)) - (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 - a + b)*s
inh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*si
nh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.15, size = 105, normalized size = 2.50

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)(a+b) + \frac{4\left(a\left(e^{(dx+c)} - e^{(-dx-c)}\right) - b\left(e^{(dx+c)} - e^{(-dx-c)}\right)\right)}{\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a + b) + 4*(a
*(e^(d*x + c) - e^(-d*x - c)) - b*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x +
c) - e^(-d*x - c))^2 + 4))/d
```

maple [B] time = 0.09, size = 82, normalized size = 1.95

$$\frac{a\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2d} + \frac{a\arctan\left(\mathrm{e}^{dx+c}\right)}{d} - \frac{b\sinh(dx+c)}{d\cosh(dx+c)^2} + \frac{b\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2d} + \frac{b\arctan\left(\mathrm{e}^{d\ldots}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/2/d*a*sech(d*x+c)*tanh(d*x+c)+1/d*a*arctan(exp(d*x+c))-1/d*b*sinh(d*x+c)/
cosh(d*x+c)^2+1/2/d*b*sech(d*x+c)*tanh(d*x+c)+1/d*b*arctan(exp(d*x+c))
```

maxima [B] time = 0.42, size = 136, normalized size = 3.24

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1\right)}\right) - a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a*(arctan(e^(-d*x - c))/d - (e^(-d*x
 - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

mupad [B] time = 0.12, size = 127, normalized size = 3.02

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{a^2+2\,a\,b+b^2}}\right)\,\sqrt{a^2+2\,a\,b+b^2}}{\sqrt{d^2}}+\frac{e^{c+d\,x}\,\left(a-b\right)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{2\,e^{c+d\,x}\,\left(a-b\right)}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^3,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(2*a*b + a^2 + b
^2)^(1/2)))*(2*a*b + a^2 + b^2)^(1/2))/(d^2)^(1/2) + (exp(c + d*x)*(a - b))
/(d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(a - b))/(d*(2*exp(2*c + 2*d*
x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^2\left(c+dx\right)\right)\operatorname{sech}^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**3, x)
```

3.290 $\int \text{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)dx$

Optimal. Leaf size=32

$$\frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}$$

```
[Out] a*tanh(d*x+c)/d-1/3*(a-b)*tanh(d*x+c)^3/d
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3191}

$$\frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)dx &= \frac{\text{Subst}\left(\int\left(a-(a-b)x^2\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.38

$$-\frac{a\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^3)/(3*d)
```

fricas [B] time = 1.46, size = 159, normalized size = 4.97

$$-\frac{4\left((a+2\,b)\cosh\left(dx+c\right)^2-2\,(a-b)\cosh\left(dx+c\right)\sinh\left(d\right.\right.}{3\left(d\cosh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)^2+2\left(3\,d\cosh\left(d\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4/3*((a + 2*b)*cosh(d*x + c)^2 - 2*(a - b)*cosh(d*x + c)*sinh(d*x + c) + (
a + 2*b)*sinh(d*x + c)^2 + 3*a)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh
```

```
(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c
)^2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d
*x + c) + 3*d)
```

giac [A] time = 0.16, size = 47, normalized size = 1.47

$$-\frac{2\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+2\,a+b\right)}{3\,d\left(e^{(2\,dx+2\,c)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 2*a + b)/(d*(e^(2*d*x + 2
*c) + 1)^3)
```

maple [B] time = 0.10, size = 65, normalized size = 2.03

$$\frac{a\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^{2}}{3}\right)\tanh\left(dx+c\right)+b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^{3}}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^{2}}{3}\right)\tanh(dx+c)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^
3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.33, size = 185, normalized size = 5.78

$$\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}+\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}\right)+\frac{2}{3}\,b\left(\frac{}{d\left(3\,e^{(-2\,d}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1))) + 2/3*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e
^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(
-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

mupad [B] time = 0.82, size = 47, normalized size = 1.47

$$-\frac{2\,\left(2\,a+b+6\,a\,\mathrm{e}^{2\,c+2\,d\,x}+3\,b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{3\,d\,{\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^4,x)
```

```
[Out] -(2*(2*a + b + 6*a*exp(2*c + 2*d*x) + 3*b*exp(4*c + 4*d*x)))/(3*d*(exp(2*c
+ 2*d*x) + 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^{2}\left(c+dx\right)\right)\mathrm{sech}^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**4, x)
```

3.291 $\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)dx$

Optimal. Leaf size=70

$$\frac{(3a+b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}+\frac{(3a+b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}$$

```
[Out] 1/8*(3*a+b)*arctan(sinh(d*x+c))/d+1/8*(3*a+b)*sech(d*x+c)*tanh(d*x+c)/d+1/4
*(a-b)*sech(d*x+c)^3*tanh(d*x+c)/d
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3190, 385, 199, 203}

$$\frac{(3a+b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}+\frac{(3a+b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*a + b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((3*a + b)*Sech[c + d*x]*Tanh[c +
 d*x])/(8*d) + ((a - b)*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^3}dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{(3a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^2}dx,\right.}{4d}$$

$$= \frac{(3a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{(a-b)\operatorname{sech}^3(c+dx)\tanh(c}{4d}$$

$$= \frac{(3a+b)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{(3a+b)\operatorname{sech}(c+dx)\tanh(c+dx}{8d}$$

Mathematica [A] time = 0.16, size = 60, normalized size = 0.86

$$\frac{(3a+b)\tan^{-1}(\sinh(c+dx)) + 2(a-b)\tanh(c+dx)\operatorname{sech}^3(c+dx) + (3a+b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*a + b)*ArcTan[Sinh[c + d*x]] + (3*a + b)*Sech[c + d*x]*Tanh[c + d*x] +
2*(a - b)*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)
```

fricas [B] time = 1.37, size = 1046, normalized size = 14.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a + b)*cosh(d*x + c)^7 + 7*(3*a + b)*cosh(d*x + c)*sinh(d*x + c)^6
+ (3*a + b)*sinh(d*x + c)^7 + (11*a - 7*b)*cosh(d*x + c)^5 + (21*(3*a + b)*
cosh(d*x + c)^2 + 11*a - 7*b)*sinh(d*x + c)^5 + 5*(7*(3*a + b)*cosh(d*x + c
)^3 + (11*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (11*a - 7*b)*cosh(d*x +
 c)^3 + (35*(3*a + b)*cosh(d*x + c)^4 + 10*(11*a - 7*b)*cosh(d*x + c)^2 - 1
1*a + 7*b)*sinh(d*x + c)^3 + (21*(3*a + b)*cosh(d*x + c)^5 + 10*(11*a - 7*b
)*cosh(d*x + c)^3 - 3*(11*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a +
 b)*cosh(d*x + c)^8 + 8*(3*a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + b)
*sinh(d*x + c)^8 + 4*(3*a + b)*cosh(d*x + c)^6 + 4*(7*(3*a + b)*cosh(d*x +
c)^2 + 3*a + b)*sinh(d*x + c)^6 + 8*(7*(3*a + b)*cosh(d*x + c)^3 + 3*(3*a +
 b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a + b)*cosh(d*x + c)^4 + 2*(35*(3
*a + b)*cosh(d*x + c)^4 + 30*(3*a + b)*cosh(d*x + c)^2 + 9*a + 3*b)*sinh(d*
x + c)^4 + 8*(7*(3*a + b)*cosh(d*x + c)^5 + 10*(3*a + b)*cosh(d*x + c)^3 +
3*(3*a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a + b)*cosh(d*x + c)^2 +
4*(7*(3*a + b)*cosh(d*x + c)^6 + 15*(3*a + b)*cosh(d*x + c)^4 + 9*(3*a + b)
*cosh(d*x + c)^2 + 3*a + b)*sinh(d*x + c)^2 + 8*((3*a + b)*cosh(d*x + c)^7
+ 3*(3*a + b)*cosh(d*x + c)^5 + 3*(3*a + b)*cosh(d*x + c)^3 + (3*a + b)*cos
h(d*x + c))*sinh(d*x + c) + 3*a + b)*arctan(cosh(d*x + c) + sinh(d*x + c))
- (3*a + b)*cosh(d*x + c) + (7*(3*a + b)*cosh(d*x + c)^6 + 5*(11*a - 7*b)*c
osh(d*x + c)^4 - 3*(11*a - 7*b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c))/(
d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 +
 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d
*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4
 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 +
8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
```

```
 + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)
^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*
cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.15, size = 153, normalized size = 2.19

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)(3a+b) + \frac{4\left(3a\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+20a\left(e^{(dx+c)}-e^{(-dx-c)}\right)-4b\left(e^{(dx+c)}\right.\right.}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a + b) + 4
*(3*a*(e^(d*x + c) - e^(-d*x - c))^3 + b*(e^(d*x + c) - e^(-d*x - c))^3 + 2
0*a*(e^(d*x + c) - e^(-d*x - c)) - 4*b*(e^(d*x + c) - e^(-d*x - c)))/((e^(d
*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

maple [A] time = 0.10, size = 124, normalized size = 1.77

$$\frac{a\tanh(dx+c)\operatorname{sech}(dx+c)^3}{4d}+\frac{3a\operatorname{sech}(dx+c)\tanh(dx+c)}{8d}+\frac{3a\arctan\left(e^{dx+c}\right)}{4d}-\frac{b\sinh(dx+c)}{3d\cosh(dx+c)^4}+\frac{b\tanh(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/4/d*a*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*a*sech(d*x+c)*tanh(d*x+c)+3/4/d*a*a
rctan(exp(d*x+c))-1/3/d*b*sinh(d*x+c)/cosh(d*x+c)^4+1/12/d*b*tanh(d*x+c)*se
ch(d*x+c)^3+1/8/d*b*sech(d*x+c)*tanh(d*x+c)+1/4/d*b*arctan(exp(d*x+c))
```

maxima [B] time = 0.43, size = 228, normalized size = 3.26

$$-\frac{1}{4}a\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3e^{(-dx-c)}+11e^{(-3dx-3c)}-11e^{(-5dx-5c)}-3e^{(-7dx-7c)}}{d\left(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1\right)}\right)-\frac{1}{4}b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) -
11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/4*b*(arctan(e
^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c) -
e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) + e^(-8*d*x - 8*c) + 1)))
```

mupad [B] time = 0.83, size = 280, normalized size = 4.00

$$\frac{\operatorname{atan}\left(\frac{e^{dx}e^{c}\left(3a\sqrt{d^2}+b\sqrt{d^2}\right)}{d\sqrt{9a^2+6ab+b^2}}\right)\sqrt{9a^2+6ab+b^2}}{4\sqrt{d^2}}-\frac{\frac{b\,e^{5c+5dx}}{d}+\frac{2e^{3c+3dx}(2a-b)}{d}+\frac{b\,e^{c+dx}}{d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}+\frac{e^{c+dx}\,(3a+b}{4d\left(e^{2c+2dx}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^5,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(3*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(6*a*b + 9*a^2
 + b^2)^(1/2)))*(6*a*b + 9*a^2 + b^2)^(1/2))/(4*(d^2)^(1/2)) - ((b*exp(5*c
```

```
+ 5*d*x))/d + (2*exp(3*c + 3*d*x)*(2*a - b))/d + (b*exp(c + d*x))/d)/(4*exp
(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x)
+ 1) + (exp(c + d*x)*(3*a + b))/(4*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x
)*(a - 3*b))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*exp(c +
 d*x)*(a - b))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*
x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^2\left(c + dx\right)\right)\operatorname{sech}^5\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**5, x)
```

3.292 $\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{(a-b)\tanh^5(c+dx)}{5d}-\frac{(2a-b)\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}$$

```
[Out] a*tanh(d*x+c)/d-1/3*(2*a-b)*tanh(d*x+c)^3/d+1/5*(a-b)*tanh(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3191, 373}

$$\frac{(a-b)\tanh^5(c+dx)}{5d}-\frac{(2a-b)\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - ((2*a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)*Tanh[c +
 d*x]^5)/(5*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a-(a-b)x^2\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a-(2a-b)x^2+(a-b)x^4\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a\tanh(c+dx)}{d}-\frac{(2a-b)\tanh^3(c+dx)}{3d}+\frac{(a-b)\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 102, normalized size = 1.89

$$\frac{a\tanh^5(c+dx)}{5d}-\frac{2a\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}+\frac{2b\tanh(c+dx)}{15d}-\frac{b\tanh(c+dx)\operatorname{sech}^4(c+dx)}{5d}+\frac{b\tanh(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (2*b*Tanh[c + d*x])/(15*d) + (b*Sech[c + d*x]^2*Tanh[
c + d*x])/(15*d) - (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - (2*a*Tanh[c +
d*x]^3)/(3*d) + (a*Tanh[c + d*x]^5)/(5*d)
```

fricas [B] time = 2.10, size = 343, normalized size = 6.35

$$-\frac{}{15\left(d\cosh\left(dx+c\right)^{7}+7\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{7}+5\,d\cosh\left(dx+c\right)^{5}+\left(21\,d\cosh\left(d\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/15*(2*(a + 4*b)*cosh(d*x + c)^3 + 6*(a + 4*b)*cosh(d*x + c)*sinh(d*x + c
)^2 - (2*a - 7*b)*sinh(d*x + c)^3 + 30*a*cosh(d*x + c) - (3*(2*a - 7*b)*cos
h(d*x + c)^2 - 10*a + 5*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x
 + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (21*d*cos
h(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*cosh(d*x
 + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 50*
d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 50*d*cos
h(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d*x + c) + (
7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5*d)*si
nh(d*x + c))
```

giac [A] time = 0.15, size = 83, normalized size = 1.54

$$-\frac{4\left(15\,be^{(6\,dx+6\,c)}+40\,ae^{(4\,dx+4\,c)}-5\,be^{(4\,dx+4\,c)}+20\,ae^{(2\,dx+2\,c)}+5\,be^{(2\,dx+2\,c)}+4\,a+b\right)}{15\,d\left(e^{(2\,dx+2\,c)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -4/15*(15*b*e^(6*d*x + 6*c) + 40*a*e^(4*d*x + 4*c) - 5*b*e^(4*d*x + 4*c) +
20*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 4*a + b)/(d*(e^(2*d*x + 2*c) +
 1)^5)
```

maple [A] time = 0.13, size = 85, normalized size = 1.57

$$\frac{a\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^{4}}{5}+\frac{4\mathrm{sech}(dx+c)^{2}}{15}\right)\tanh\left(dx+c\right)+b\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^{5}}+\frac{\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^{4}}{5}+\frac{4\mathrm{sech}(dx+c)^{2}}{15}\right)\tanh(dx+c)}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+b*(-1/4*sinh
(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(
d*x+c)))
```

maxima [B] time = 0.33, size = 486, normalized size = 9.00

$$\frac{16}{15}\,a\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1\right)}+\frac{}{d\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 16/15*a*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
```

```
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 4/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c
) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10
*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d
*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(
5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*
x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

mupad [B] time = 0.82, size = 298, normalized size = 5.52

$$-\frac{\frac{8\,b\,e^{2\,c+2\,d\,x}}{5\,d}+\frac{8\,b\,e^{6\,c+6\,d\,x}}{5\,d}+\frac{16\,e^{4\,c+4\,d\,x}\,(2\,a-b)}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}-\frac{\frac{2\,b}{5\,d}+\frac{6\,b\,e^{4\,c+4\,d\,x}}{5\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(2\,a-b)}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)/cosh(c + d*x)^6,x)
```

```
[Out] - ((8*b*exp(2*c + 2*d*x))/(5*d) + (8*b*exp(6*c + 6*d*x))/(5*d) + (16*exp(4*
c + 4*d*x)*(2*a - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10
*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*b)/(
5*d) + (6*b*exp(4*c + 4*d*x))/(5*d) + (8*exp(2*c + 2*d*x)*(2*a - b))/(5*d))
/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c +
8*d*x) + 1) - ((8*(2*a - b))/(15*d) + (4*b*exp(2*c + 2*d*x))/(5*d))/(3*exp(
2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*b)/(5*d*(2*e
xp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.293 $\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=159

$$\frac{\left(48a^2-16ab+3b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}+\frac{\left(48a^2-16ab+3b^2\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(48a^2\right.$$

```
[Out] 1/128*(48*a^2-16*a*b+3*b^2)*x+1/128*(48*a^2-16*a*b+3*b^2)*cosh(d*x+c)*sinh(
d*x+c)/d+1/192*(48*a^2-16*a*b+3*b^2)*cosh(d*x+c)^3*sinh(d*x+c)/d+1/48*(10*a
-3*b)*b*cosh(d*x+c)^5*sinh(d*x+c)/d+1/8*b*cosh(d*x+c)^7*sinh(d*x+c)*(a-(a-b
)*tanh(d*x+c)^2)/d
```

Rubi [A] time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 413, 385, 199, 206}

$$\frac{\left(48a^2-16ab+3b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}+\frac{\left(48a^2-16ab+3b^2\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(48a^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((48*a^2 - 16*a*b + 3*b^2)*x)/128 + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x
]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x]^3*Sinh[
c + d*x])/(192*d) + ((10*a - 3*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) +
 (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(8*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{8d} - \frac{\text{Subst}\Big(\ldots}{\ldots}$$

$$= \frac{(10a-3b)b\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b\cosh^7(c+dx)\sinh(c+\ldots}{\ldots}$$

$$= \frac{\left(48a^2-16ab+3b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{192d} + \frac{(10a-3b)b\cosh\ldots}{\ldots}$$

$$= \frac{\left(48a^2-16ab+3b^2\right)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{\left(48a^2-16ab+3\ldots\right.}{\ldots}$$

$$= \frac{1}{128}\left(48a^2-16ab+3b^2\right)x + \frac{\left(48a^2-16ab+3b^2\right)\cosh(c+dx)\sinh\ldots}{128d}$$

Mathematica [A] time = 0.32, size = 98, normalized size = 0.62

$$\frac{24\left(48a^2-16ab+3b^2\right)(c+dx)+24\left(4a^2+4ab-b^2\right)\sinh(4(c+dx))+32ab\sinh(6(c+dx))+96a(8a-b)\sinh\ldots}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (24*(48*a^2 - 16*a*b + 3*b^2)*(c + d*x) + 96*a*(8*a - b)*Sinh[2*(c + d*x)]
+ 24*(4*a^2 + 4*a*b - b^2)*Sinh[4*(c + d*x)] + 32*a*b*Sinh[6*(c + d*x)] + 3
*b^2*Sinh[8*(c + d*x)])/(3072*d)
```

fricas [A] time = 0.52, size = 212, normalized size = 1.33

$$\frac{3\,b^2\cosh(dx+c)\sinh(dx+c)^7+3\left(7\,b^2\cosh(dx+c)^3+8\,ab\cosh(dx+c)\right)\sinh(dx+c)^5+\left(21\,b^2\cosh(dx+\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 + 8*a
*b*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 + 80*a*b*cosh(d
*x + c)^3 + 12*(4*a^2 + 4*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(48
*a^2 - 16*a*b + 3*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 + 8*a*b*cosh(d*x + c)^5
 + 4*(4*a^2 + 4*a*b - b^2)*cosh(d*x + c)^3 + 8*(8*a^2 - a*b)*cosh(d*x + c))
*sinh(d*x + c))/d
```

giac [A] time = 0.16, size = 191, normalized size = 1.20

$$\frac{1}{128}\left(48\,a^2-16\,ab+3\,b^2\right)x+\frac{b^2e^{(8\,dx+8\,c)}}{2048\,d}+\frac{abe^{(6\,dx+6\,c)}}{192\,d}-\frac{abe^{(-6\,dx-6\,c)}}{192\,d}-\frac{b^2e^{(-8\,dx-8\,c)}}{2048\,d}+\frac{\left(4\,a^2+4\,ab-b^2\right)e^{(4\,dx+4\,c)}}{256\,d}+\frac{(8\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/128*(48*a^2 - 16*a*b + 3*b^2)*x + 1/2048*b^2*e^(8*d*x + 8*c)/d + 1/192*a*
b*e^(6*d*x + 6*c)/d - 1/192*a*b*e^(-6*d*x - 6*c)/d - 1/2048*b^2*e^(-8*d*x -
 8*c)/d + 1/256*(4*a^2 + 4*a*b - b^2)*e^(4*d*x + 4*c)/d + 1/64*(8*a^2 - a*b
)*e^(2*d*x + 2*c)/d - 1/64*(8*a^2 - a*b)*e^(-2*d*x - 2*c)/d - 1/256*(4*a^2
+ 4*a*b - b^2)*e^(-4*d*x - 4*c)/d
```

maple [A] time = 0.08, size = 172, normalized size = 1.08

$$b^2\left(\frac{(\sinh^3(dx+c))(\cosh^5(dx+c))}{8}-\frac{\sinh(dx+c)(\cosh^5(dx+c))}{16}+\frac{\left(\frac{(\cosh^3(dx+c))}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)}{16}+\frac{3dx}{128}+\frac{3c}{128}\right)+2ab\left(\frac{\sinh(dx+c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(1/8*sinh(d*x+c)^3*cosh(d*x+c)^5-1/16*sinh(d*x+c)*cosh(d*x+c)^5+1/
16*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/128*d*x+3/128*c)+2*a*b
*(1/6*sinh(d*x+c)*cosh(d*x+c)^5-1/6*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sin
h(d*x+c)-1/16*d*x-1/16*c)+a^2*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x
+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.34, size = 225, normalized size = 1.42

$$\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{2048}b^2\left(\frac{\left(8e^{(-4dx-4c)}-1\right)e^{(8dx+8c)}}{d}-\frac{48(dx}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/2048*b^2*((8*e^(-4*d*x - 4*c) - 1)*e^(8*d*x
+ 8*c)/d - 48*(d*x + c)/d - (8*e^(-4*d*x - 4*c) - e^(-8*d*x - 8*c))/d) + 1/
192*a*b*((3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + 1)*e^(6*d*x + 6*c)/d -
24*(d*x + c)/d + (3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c
))/d)
```

mupad [B] time = 0.36, size = 121, normalized size = 0.76

$$\frac{96a^2\sinh(2c+2dx)+12a^2\sinh(4c+4dx)-3b^2\sinh(4c+4dx)+\frac{3b^2\sinh(8c+8dx)}{8}-12ab\sinh(2c+2}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (96*a^2*sinh(2*c + 2*d*x) + 12*a^2*sinh(4*c + 4*d*x) - 3*b^2*sinh(4*c + 4*d
*x) + (3*b^2*sinh(8*c + 8*d*x))/8 - 12*a*b*sinh(2*c + 2*d*x) + 12*a*b*sinh(
4*c + 4*d*x) + 4*a*b*sinh(6*c + 6*d*x) + 144*a^2*d*x + 9*b^2*d*x - 48*a*b*d
*x)/(384*d)
```

sympy [A] time = 9.78, size = 481, normalized size = 3.03

$$\begin{cases}\frac{3a^2x\sinh^4(c+dx)}{8}-\frac{3a^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3a^2x\cosh^4(c+dx)}{8}-\frac{3a^2\sinh^3(c+dx)\cosh(c+dx)}{8d}+\frac{5a^2\sinh(c+dx)\cosh^3(c+dx)}{8d}+ \\ x\left(a+b\sinh^2(c)\right)^2\cosh^4(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*a**2*x*sinh(c + d*x)**4/8 - 3*a**2*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**2*x*cosh(c + d*x)**4/8 - 3*a**2*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) + 5*a**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + a*b*x*sinh(c +
d*x)**6/8 - 3*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 3*a*b*x*sinh(c +
d*x)**2*cosh(c + d*x)**4/8 - a*b*x*cosh(c + d*x)**6/8 - a*b*sinh(c + d*x)**
5*cosh(c + d*x)/(8*d) + a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + a*b*s
inh(c + d*x)*cosh(c + d*x)**5/(8*d) + 3*b**2*x*sinh(c + d*x)**8/128 - 3*b**
2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 9*b**2*x*sinh(c + d*x)**4*cosh(c
 + d*x)**4/64 - 3*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 3*b**2*x*co
sh(c + d*x)**8/128 - 3*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 11*b**
2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 11*b**2*sinh(c + d*x)**3*cosh
(c + d*x)**5/(128*d) - 3*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d,
 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**4, True))
```

3.294 $\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=74

$$\frac{a^2\sinh(c+dx)}{d}+\frac{b(2a+b)\sinh^5(c+dx)}{5d}+\frac{a(a+2b)\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^7(c+dx)}{7d}$$

```
[Out] a^2*sinh(d*x+c)/d+1/3*a*(a+2*b)*sinh(d*x+c)^3/d+1/5*b*(2*a+b)*sinh(d*x+c)^5
/d+1/7*b^2*sinh(d*x+c)^7/d
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3190, 373}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{b(2a+b)\sinh^5(c+dx)}{5d}+\frac{a(a+2b)\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x])/d + (a*(a + 2*b)*Sinh[c + d*x]^3)/(3*d) + (b*(2*a + b)*
Sinh[c + d*x]^5)/(5*d) + (b^2*Sinh[c + d*x]^7)/(7*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx &= \frac{\operatorname{Subst}\left(\int \left(1+x^2\right)\left(a+bx^2\right)^2 dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^2+a(a+2b)x^2+b(2a+b)x^4+b^2x^6\right) dx, x, \sinh(c\right.}{d}\\ &= \frac{a^2\sinh(c+dx)}{d}+\frac{a(a+2b)\sinh^3(c+dx)}{3d}+\frac{b(2a+b)\sinh^5(c+}{5d}\end{aligned}$$

Mathematica [A] time = 0.11, size = 64, normalized size = 0.86

$$\frac{105a^2\sinh(c+dx)+21b(2a+b)\sinh^5(c+dx)+35a(a+2b)\sinh^3(c+dx)+15b^2\sinh^7(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (105*a^2*Sinh[c + d*x] + 35*a*(a + 2*b)*Sinh[c + d*x]^3 + 21*b*(2*a + b)*Si
nh[c + d*x]^5 + 15*b^2*Sinh[c + d*x]^7)/(105*d)
```

fricas [B] time = 1.55, size = 188, normalized size = 2.54

$$\frac{15\,b^2\sinh\left(dx+c\right)^7+21\left(15\,b^2\cosh\left(dx+c\right)^2+8\,ab-b^2\right)\sinh\left(dx+c\right)^5+35\left(15\,b^2\cosh\left(dx+c\right)^4+6\left(8\,ab-b^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(15*b^2*sinh(d*x + c)^7 + 21*(15*b^2*cosh(d*x + c)^2 + 8*a*b - b^2)*
sinh(d*x + c)^5 + 35*(15*b^2*cosh(d*x + c)^4 + 6*(8*a*b - b^2)*cosh(d*x + c
)^2 + 16*a^2 + 8*a*b - 3*b^2)*sinh(d*x + c)^3 + 105*(b^2*cosh(d*x + c)^6 +
(8*a*b - b^2)*cosh(d*x + c)^4 + (16*a^2 + 8*a*b - 3*b^2)*cosh(d*x + c)^2 +
48*a^2 - 16*a*b + 3*b^2)*sinh(d*x + c))/d
```

giac [B] time = 0.15, size = 196, normalized size = 2.65

$$\frac{b^2e^{(7\,dx+7\,c)}}{896\,d}-\frac{b^2e^{(-7\,dx-7\,c)}}{896\,d}+\frac{\left(8\,ab-b^2\right)e^{(5\,dx+5\,c)}}{640\,d}+\frac{\left(16\,a^2+8\,ab-3\,b^2\right)e^{(3\,dx+3\,c)}}{384\,d}+\frac{\left(48\,a^2-16\,ab+3\,b^2\right)e^{(dx+c)}}{128\,d}-\frac{\left(48\,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/896*b^2*e^(7*d*x + 7*c)/d - 1/896*b^2*e^(-7*d*x - 7*c)/d + 1/640*(8*a*b -
 b^2)*e^(5*d*x + 5*c)/d + 1/384*(16*a^2 + 8*a*b - 3*b^2)*e^(3*d*x + 3*c)/d
+ 1/128*(48*a^2 - 16*a*b + 3*b^2)*e^(d*x + c)/d - 1/128*(48*a^2 - 16*a*b +
3*b^2)*e^(-d*x - c)/d - 1/384*(16*a^2 + 8*a*b - 3*b^2)*e^(-3*d*x - 3*c)/d -
 1/640*(8*a*b - b^2)*e^(-5*d*x - 5*c)/d
```

maple [A] time = 0.08, size = 128, normalized size = 1.73

$$\frac{b^2\left(\frac{\left(\sinh^3(dx+c)\right)\left(\cosh^4(dx+c)\right)}{7}-\frac{3\sinh(dx+c)\left(\cosh^4(dx+c)\right)}{35}+\frac{3\left(\frac{2}{3}+\frac{\left(\cosh^2(dx+c)\right)}{3}\right)\sinh(dx+c)}{35}\right)+2ab\left(\frac{\sinh(dx+c)\left(\cosh^4(dx+c)\right)}{5}-\frac{\left(\frac{2}{3}+\frac{\left(\cosh\right.}{}\right.}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(1/7*sinh(d*x+c)^3*cosh(d*x+c)^4-3/35*sinh(d*x+c)*cosh(d*x+c)^4+3/
35*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+2*a*b*(1/5*sinh(d*x+c)*cosh(d*x+c)^
4-1/5*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+a^2*(2/3+1/3*cosh(d*x+c)^2)*sinh
(d*x+c))
```

maxima [B] time = 0.33, size = 242, normalized size = 3.27

$$-\frac{1}{4480}\,b^2\left(\frac{\left(7\,e^{(-2\,dx-2\,c)}+35\,e^{(-4\,dx-4\,c)}-105\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{105\,e^{(-dx-c)}-35\,e^{(-3\,dx-3\,c)}-7\,e^{(-5\,dx-5\,c)}+}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4480*b^2*((7*e^(-2*d*x - 2*c) + 35*e^(-4*d*x - 4*c) - 105*e^(-6*d*x - 6*
c) - 5)*e^(7*d*x + 7*c)/d + (105*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) - 7*e^(
-5*d*x - 5*c) + 5*e^(-7*d*x - 7*c))/d) + 1/240*a*b*((5*e^(-2*d*x - 2*c) - 3
0*e^(-4*d*x - 4*c) + 3)*e^(5*d*x + 5*c)/d + (30*e^(-d*x - c) - 5*e^(-3*d*x
- 3*c) - 3*e^(-5*d*x - 5*c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x +
c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 0.96, size = 80, normalized size = 1.08

$$\frac{35\,a^2\sinh\left(c+d\,x\right)^3+105\,a^2\sinh\left(c+d\,x\right)+42\,a\,b\sinh\left(c+d\,x\right)^5+70\,a\,b\sinh\left(c+d\,x\right)^3+15\,b^2\sinh\left(c+d\,x\right)^7+21\,b^2\sinh\left(c+d\,x\right)^5}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (105*a^2*sinh(c + d*x) + 35*a^2*sinh(c + d*x)^3 + 21*b^2*sinh(c + d*x)^5 +
15*b^2*sinh(c + d*x)^7 + 70*a*b*sinh(c + d*x)^3 + 42*a*b*sinh(c + d*x)^5)/(
105*d)
```

sympy [A] time = 5.27, size = 136, normalized size = 1.84

$$\begin{cases} -\frac{2a^2\sinh^3(c+dx)}{3d}+\frac{a^2\sinh(c+dx)\cosh^2(c+dx)}{d}-\frac{4ab\sinh^5(c+dx)}{15d}+\frac{2ab\sinh^3(c+dx)\cosh^2(c+dx)}{3d}-\frac{2b^2\sinh^7(c+dx)}{35d}+\frac{b^2\sinh^5(c+dx)\cosh^2(c+dx)}{5d} & \text{for}\: d\neq 0 \\ x\left(a+b\sinh^2(c)\right)^2\cosh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-2*a**2*sinh(c + d*x)**3/(3*d) + a**2*sinh(c + d*x)*cosh(c + d*x
)**2/d - 4*a*b*sinh(c + d*x)**5/(15*d) + 2*a*b*sinh(c + d*x)**3*cosh(c + d*
x)**2/(3*d) - 2*b**2*sinh(c + d*x)**7/(35*d) + b**2*sinh(c + d*x)**5*cosh(c
 + d*x)**2/(5*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**3, True))
```

3.295 $\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=119

$$\frac{\left(8a^2-4ab+b^2\right)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x\left(8a^2-4ab+b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh^3(c+dx)}{24d}+\frac{b\sin}{}$$

```
[Out] 1/16*(8*a^2-4*a*b+b^2)*x+1/16*(8*a^2-4*a*b+b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1
/24*(8*a-3*b)*b*cosh(d*x+c)^3*sinh(d*x+c)/d+1/6*b*cosh(d*x+c)^5*sinh(d*x+c)
*(a-(a-b)*tanh(d*x+c)^2)/d
```

Rubi [A] time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 413, 385, 199, 206}

$$\frac{\left(8a^2-4ab+b^2\right)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x\left(8a^2-4ab+b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh^3(c+dx)}{24d}+\frac{b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 4*a*b + b^2)*x)/16 + ((8*a^2 - 4*a*b + b^2)*Cosh[c + d*x]*Sinh[c
+ d*x])/(16*d) + ((8*a - 3*b)*b*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*
Cosh[c + d*x]^5*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(6*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^4}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^5(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{6d} - \text{Subs}$$

$$= \frac{(8a-3b)b\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b\cosh^5(c+dx)\sinh(c}{}$$

$$= \frac{\left(8a^2-4ab+b^2\right)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(8a-3b)b\cosh^3(}{2}$$

$$= \frac{1}{16}\left(8a^2-4ab+b^2\right)x + \frac{\left(8a^2-4ab+b^2\right)\cosh(c+dx)\sinh(c+}{16d}$$

Mathematica [A] time = 0.29, size = 79, normalized size = 0.66

$$\frac{12\left(8a^2-4ab+b^2\right)(c+dx)+3\left(16a^2-b^2\right)\sinh(2(c+dx))+3b(4a-b)\sinh(4(c+dx))+b^2\sinh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (12*(8*a^2 - 4*a*b + b^2)*(c + d*x) + 3*(16*a^2 - b^2)*Sinh[2*(c + d*x)] +
3*(4*a - b)*b*Sinh[4*(c + d*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

fricas [A] time = 2.22, size = 143, normalized size = 1.20

$$\frac{3b^2\cosh(dx+c)\sinh(dx+c)^5+2\left(5b^2\cosh(dx+c)^3+3\left(4ab-b^2\right)\cosh(dx+c)\right)\sinh(dx+c)^3+6\left(8a^2\right.}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^2*cosh(d*x + c)^3 + 3*(4
*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 - 4*a*b + b^2)*d*x +
3*(b^2*cosh(d*x + c)^5 + 2*(4*a*b - b^2)*cosh(d*x + c)^3 + (16*a^2 - b^2)*c
osh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.16, size = 149, normalized size = 1.25

$$\frac{1}{16}\left(8a^2-4ab+b^2\right)x+\frac{b^2e^{(6dx+6c)}}{384d}-\frac{b^2e^{(-6dx-6c)}}{384d}+\frac{\left(4ab-b^2\right)e^{(4dx+4c)}}{128d}+\frac{\left(16a^2-b^2\right)e^{(2dx+2c)}}{128d}-\frac{\left(16a^2-b^2\right)e^{(-2}}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^2 - 4*a*b + b^2)*x + 1/384*b^2*e^(6*d*x + 6*c)/d - 1/384*b^2*e^(-
6*d*x - 6*c)/d + 1/128*(4*a*b - b^2)*e^(4*d*x + 4*c)/d + 1/128*(16*a^2 - b^
2)*e^(2*d*x + 2*c)/d - 1/128*(16*a^2 - b^2)*e^(-2*d*x - 2*c)/d - 1/128*(4*a
*b - b^2)*e^(-4*d*x - 4*c)/d
```

maple [A] time = 0.08, size = 134, normalized size = 1.13

$$\frac{b^2\left(\frac{(\sinh^3(dx+c))(\cosh^3(dx+c))}{6}-\frac{\sinh(dx+c)(\cosh^3(dx+c))}{8}+\frac{\cosh(dx+c)\sinh(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(\frac{\sinh(dx+c)(\cosh^3(dx+c))}{4}-\mathrm{co}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(1/6*sinh(d*x+c)^3*cosh(d*x+c)^3-1/8*sinh(d*x+c)*cosh(d*x+c)^3+1/1
6*cosh(d*x+c)*sinh(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(1/4*sinh(d*x+c)*cosh(d*x+
c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+a^2*(1/2*cosh(d*x+c)*sinh(d
*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.34, size = 171, normalized size = 1.44

$$\frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{384}b^2\left(\frac{\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}-\frac{24(dx+c)}{d}-\frac{3e^{(-2dx-2c)}+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/384*b^2*((3*e^(-
2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d -
 (3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d) - 1/32*a*b
*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

mupad [B] time = 0.22, size = 95, normalized size = 0.80

$$\frac{12a^2\sinh(2c+2dx)-\frac{3b^2\sinh(2c+2dx)}{4}-\frac{3b^2\sinh(4c+4dx)}{4}+\frac{b^2\sinh(6c+6dx)}{4}+3ab\sinh(4c+4dx)+24a^2dx+3}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (12*a^2*sinh(2*c + 2*d*x) - (3*b^2*sinh(2*c + 2*d*x))/4 - (3*b^2*sinh(4*c +
 4*d*x))/4 + (b^2*sinh(6*c + 6*d*x))/4 + 3*a*b*sinh(4*c + 4*d*x) + 24*a^2*d
*x + 3*b^2*d*x - 12*a*b*d*x)/(48*d)
```

sympy [A] time = 3.54, size = 314, normalized size = 2.64

$$\begin{cases}-\frac{a^2x\sinh^2(c+dx)}{2}+\frac{a^2x\cosh^2(c+dx)}{2}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{abx\sinh^4(c+dx)}{4}+\frac{abx\sinh^2(c+dx)\cosh^2(c+dx)}{2}-\frac{abx\cosh^4(c+dx)}{4} \\ x\left(a+b\sinh^2(c)\right)^2\cosh^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-a**2*x*sinh(c + d*x)**2/2 + a**2*x*cosh(c + d*x)**2/2 + a**2*si
nh(c + d*x)*cosh(c + d*x)/(2*d) - a*b*x*sinh(c + d*x)**4/4 + a*b*x*sinh(c +
 d*x)**2*cosh(c + d*x)**2/2 - a*b*x*cosh(c + d*x)**4/4 + a*b*sinh(c + d*x)*
```

```
*3*cosh(c + d*x)/(4*d) + a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) - b**2*x*
sinh(c + d*x)**6/16 + 3*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 - 3*b**
2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 + b**2*x*cosh(c + d*x)**6/16 + b**
2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + b**2*sinh(c + d*x)**3*cosh(c + d*
x)**3/(6*d) - b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a
+ b*sinh(c)**2)**2*cosh(c)**2, True))
```

3.296 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}$$

```
[Out] a^2*sinh(d*x+c)/d+2/3*a*b*sinh(d*x+c)^3/d+1/5*b^2*sinh(d*x+c)^5/d
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3190, 194}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x])/d + (2*a*b*Sinh[c + d*x]^3)/(3*d) + (b^2*Sinh[c + d*x]^
5)/(5*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)^2\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(a^2+2abx^2+b^2x^4\right)\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.90

$$\frac{a^2\sinh(c+dx)+\frac{2}{3}ab\sinh^3(c+dx)+\frac{1}{5}b^2\sinh^5(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x] + (2*a*b*Sinh[c + d*x]^3)/3 + (b^2*Sinh[c + d*x]^5)/5)/d
```

fricas [B] time = 0.96, size = 106, normalized size = 2.16

$$\frac{3\,b^2\sinh\left(dx+c\right)^5+5\left(6\,b^2\cosh\left(dx+c\right)^2+8\,ab-3\,b^2\right)\sinh\left(dx+c\right)^3+15\left(b^2\cosh\left(dx+c\right)^4+\left(8\,ab-3\,b^2\right)\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^2*sinh(d*x + c)^5 + 5*(6*b^2*cosh(d*x + c)^2 + 8*a*b - 3*b^2)*si
nh(d*x + c)^3 + 15*(b^2*cosh(d*x + c)^4 + (8*a*b - 3*b^2)*cosh(d*x + c)^2 +
 16*a^2 - 8*a*b + 2*b^2)*sinh(d*x + c))/d
```

giac [B] time = 0.15, size = 134, normalized size = 2.73

$$\frac{b^2e^{(5\,dx+5\,c)}}{160\,d}-\frac{b^2e^{(-5\,dx-5\,c)}}{160\,d}+\frac{\left(8\,ab-3\,b^2\right)e^{(3\,dx+3\,c)}}{96\,d}+\frac{\left(8\,a^2-4\,ab+b^2\right)e^{(dx+c)}}{16\,d}-\frac{\left(8\,a^2-4\,ab+b^2\right)e^{(-dx-c)}}{16\,d}-\frac{\left(8\,ab-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/160*b^2*e^(5*d*x + 5*c)/d - 1/160*b^2*e^(-5*d*x - 5*c)/d + 1/96*(8*a*b -
3*b^2)*e^(3*d*x + 3*c)/d + 1/16*(8*a^2 - 4*a*b + b^2)*e^(d*x + c)/d - 1/16*
(8*a^2 - 4*a*b + b^2)*e^(-d*x - c)/d - 1/96*(8*a*b - 3*b^2)*e^(-3*d*x - 3*c
)/d
```

maple [A] time = 0.04, size = 41, normalized size = 0.84

$$\frac{\frac{b^2\left(\sinh^5\left(dx+c\right)\right)}{5}+\frac{2a\left(\sinh^3\left(dx+c\right)\right)b}{3}+a^2\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/5*b^2*sinh(d*x+c)^5+2/3*a*sinh(d*x+c)^3*b+a^2*sinh(d*x+c))
```

maxima [A] time = 0.33, size = 45, normalized size = 0.92

$$\frac{b^2\sinh\left(dx+c\right)^5}{5\,d}+\frac{2\,ab\sinh\left(dx+c\right)^3}{3\,d}+\frac{a^2\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*sinh(d*x + c)^5/d + 2/3*a*b*sinh(d*x + c)^3/d + a^2*sinh(d*x + c)/d
```

mupad [B] time = 0.83, size = 42, normalized size = 0.86

$$\frac{\sinh\left(c+d\,x\right)\,\left(15\,a^2+10\,a\,b\,\sinh\left(c+d\,x\right)^2+3\,b^2\,\sinh\left(c+d\,x\right)^4\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] (sinh(c + d*x)*(15*a^2 + 3*b^2*sinh(c + d*x)^4 + 10*a*b*sinh(c + d*x)^2))/(
15*d)
```

sympy [A] time = 1.72, size = 58, normalized size = 1.18

$$\begin{cases} \frac{a^2 \sinh{(c+dx)}}{d} + \frac{2ab \sinh^3{(c+dx)}}{3d} + \frac{b^2 \sinh^5{(c+dx)}}{5d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^2{(c)}\right)^2 \cosh{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*sinh(c + d*x)**3/(3*d) + b**2*sinh(
c + d*x)**5/(5*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c), True))
```

3.297 $\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{b(2a-b)\sinh(c+dx)}{d}+\frac{(a-b)^2\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\sinh^3(c+dx)}{3d}$$

```
[Out] (a-b)^2*arctan(sinh(d*x+c))/d+(2*a-b)*b*sinh(d*x+c)/d+1/3*b^2*sinh(d*x+c)^3
/d
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 390, 203}

$$\frac{b(2a-b)\sinh(c+dx)}{d}+\frac{(a-b)^2\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)^2*ArcTan[Sinh[c + d*x]])/d + ((2*a - b)*b*Sinh[c + d*x])/d + (b^2*
Sinh[c + d*x]^3)/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{(2a-b)b\sinh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)}{3d}+\frac{(a-b)^2\operatorname{Subst}\left(\int \frac{1}{1+\ldots}\right.}{}\\ &= \frac{(a-b)^2\tan^{-1}(\sinh(c+dx))}{d}+\frac{(2a-b)b\sinh(c+dx)}{d}+\frac{b^2\sinh^3}{3\ldots}\end{aligned}$$

Mathematica [A] time = 0.29, size = 70, normalized size = 1.27

$$\frac{\sinh(c+dx)\left(b\left(6a+b\left(\sinh^2(c+dx)-3\right)\right)+\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (Sinh[c + d*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c
+ d*x]^2] + b*(6*a + b*(-3 + Sinh[c + d*x]^2))))/(3*d)
```

fricas [B] time = 0.91, size = 446, normalized size = 8.11

$$\frac{b^2\cosh(dx+c)^6+6\,b^2\cosh(dx+c)\sinh(dx+c)^5+b^2\sinh(dx+c)^6+3\left(8\,ab-5\,b^2\right)\cosh(dx+c)^4+3\left(5\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(
d*x + c)^6 + 3*(8*a*b - 5*b^2)*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 +
 8*a*b - 5*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + 3*(8*a*b - 5*b
^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(8*a*b - 5*b^2)*cosh(d*x + c)^2 + 3*
(5*b^2*cosh(d*x + c)^4 + 6*(8*a*b - 5*b^2)*cosh(d*x + c)^2 - 8*a*b + 5*b^2)
*sinh(d*x + c)^2 - b^2 + 48*((a^2 - 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 -
 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^3)*arctan(cosh(d
*x + c) + sinh(d*x + c)) + 6*(b^2*cosh(d*x + c)^5 + 2*(8*a*b - 5*b^2)*cosh(
d*x + c)^3 - (8*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)
^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2
+ d*sinh(d*x + c)^3)
```

giac [A] time = 0.16, size = 102, normalized size = 1.85

$$\frac{b^2e^{(3\,dx+3\,c)}+24\,abe^{(dx+c)}-15\,b^2e^{(dx+c)}+48\left(a^2-2\,ab+b^2\right)\arctan\left(e^{(dx+c)}\right)-\left(24\,abe^{(2\,dx+2\,c)}-15\,b^2e^{(2\,dx+2\,c)}+b\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*d*x + 3*c) + 24*a*b*e^(d*x + c) - 15*b^2*e^(d*x + c) + 48*(a
^2 - 2*a*b + b^2)*arctan(e^(d*x + c)) - (24*a*b*e^(2*d*x + 2*c) - 15*b^2*e^
(2*d*x + 2*c) + b^2)*e^(-3*d*x - 3*c))/d
```

maple [A] time = 0.09, size = 89, normalized size = 1.62

$$\frac{2a^2\arctan\left(e^{dx+c}\right)}{d}+\frac{2ab\sinh(dx+c)}{d}-\frac{4ab\arctan\left(e^{dx+c}\right)}{d}+\frac{b^2\left(\sinh^3(dx+c)\right)}{3d}-\frac{b^2\sinh(dx+c)}{d}+\frac{2b^2\arctan\left(e^{d}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 2/d*a^2*arctan(exp(d*x+c))+2/d*a*b*sinh(d*x+c)-4/d*a*b*arctan(exp(d*x+c))+1
/3*b^2*sinh(d*x+c)^3/d-b^2*sinh(d*x+c)/d+2/d*b^2*arctan(exp(d*x+c))
```

maxima [**B**] time = 0.67, size = 133, normalized size = 2.42

$$-\frac{1}{24}b^2\left(\frac{\left(15e^{(-2dx-2c)}-1\right)e^{(3dx+3c)}}{d}-\frac{15e^{(-dx-c)}-e^{(-3dx-3c)}}{d}+\frac{48\arctan\left(e^{(-dx-c)}\right)}{d}\right)+ab\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*((15*e^(-2*d*x - 2*c) - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) -
 e^(-3*d*x - 3*c))/d + 48*arctan(e^(-d*x - c))/d) + a*b*(4*arctan(e^(-d*x -
 c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a^2*arctan(sinh(d*x + c))/d
```

mupad [**B**] time = 0.17, size = 182, normalized size = 3.31

$$\frac{b^2\,e^{3\,c+3\,d\,x}}{24\,d}-\frac{b^2\,e^{-3\,c-3\,d\,x}}{24\,d}-\frac{e^{-c-d\,x}\,\left(8\,a\,b-5\,b^2\right)}{8\,d}+\frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(a^2\,\sqrt{d^2}+b^2\,\sqrt{d^2}-2\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}}\right)\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-}}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x),x)
```

```
[Out] (b^2*exp(3*c + 3*d*x))/(24*d) - (b^2*exp(- 3*c - 3*d*x))/(24*d) - (exp(- c
- d*x)*(8*a*b - 5*b^2))/(8*d) + (2*atan((exp(d*x)*exp(c)*(a^2*(d^2)^(1/2) +
 b^2*(d^2)^(1/2) - 2*a*b*(d^2)^(1/2)))/(d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
6*a^2*b^2)^(1/2)))*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)^(1/2))/(d^2)
^(1/2) + (b*exp(c + d*x)*(8*a - 5*b))/(8*d)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^2\left(c+dx\right)\right)^2\operatorname{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)**2*sech(c + d*x), x)
```

3.298 $\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{(a-b)^2\tanh(c+dx)}{d}+\frac{1}{2}bx(4a-3b)+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] 1/2*(4*a-3*b)*b*x+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d+(a-b)^2*tanh(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 390, 385, 206}

$$\frac{(a-b)^2\tanh(c+dx)}{d}+\frac{1}{2}bx(4a-3b)+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - 3*b)*b*x)/2 + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + ((a - b)^2*
Tanh[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left((a-b)^2+\frac{(2a-b)b-2(a-b)bx^2}{\left(1-x^2\right)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a-b)^2\tanh(c+dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{\left(1-x^2\right)^2}dx, x, \tanh(c+\right.}{d}$$
$$= \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)^2\tanh(c+dx)}{d} + \frac{((4a-3}{}$$
$$= \frac{1}{2}(4a-3b)bx + \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)^2\tanh(c-}{d}$$

Mathematica [A] time = 0.31, size = 50, normalized size = 0.94

$$\frac{2b(4a-3b)(c+dx)+4(a-b)^2\tanh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*(4*a - 3*b)*b*(c + d*x) + b^2*Sinh[2*(c + d*x)] + 4*(a - b)^2*Tanh[c + d
*x])/(4*d)
```

fricas [A] time = 1.42, size = 97, normalized size = 1.83

$$\frac{b^2\sinh(dx+c)^3+4\left(\left(4\,ab-3\,b^2\right)dx-2\,a^2+4\,ab-2\,b^2\right)\cosh(dx+c)+\left(3\,b^2\cosh(dx+c)^2+8\,a^2-16\,ab-\right.}{8\,d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*sinh(d*x + c)^3 + 4*((4*a*b - 3*b^2)*d*x - 2*a^2 + 4*a*b - 2*b^2)*
cosh(d*x + c) + (3*b^2*cosh(d*x + c)^2 + 8*a^2 - 16*a*b + 9*b^2)*sinh(d*x +
 c))/(d*cosh(d*x + c))
```

giac [B] time = 0.19, size = 131, normalized size = 2.47

$$\frac{b^2e^{(2\,dx+2\,c)}+4\left(4\,ab-3\,b^2\right)(dx+c)-\frac{4\,abe^{(4\,dx+4\,c)}-3\,b^2e^{(4\,dx+4\,c)}+16\,a^2e^{(2\,dx+2\,c)}-28\,abe^{(2\,dx+2\,c)}+14\,b^2e^{(2\,dx+2\,c)}+b^2}{e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(b^2*e^(2*d*x + 2*c) + 4*(4*a*b - 3*b^2)*(d*x + c) - (4*a*b*e^(4*d*x +
4*c) - 3*b^2*e^(4*d*x + 4*c) + 16*a^2*e^(2*d*x + 2*c) - 28*a*b*e^(2*d*x + 2
*c) + 14*b^2*e^(2*d*x + 2*c) + b^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

maple [A] time = 0.09, size = 71, normalized size = 1.34

$$\frac{a^2\tanh(dx+c)+2ab\,(dx+c-\tanh(dx+c))+b^2\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*tanh(d*x+c)+2*a*b*(d*x+c-tanh(d*x+c))+b^2*(1/2*sinh(d*x+c)^3/cosh(
d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

maxima [B] time = 0.56, size = 119, normalized size = 2.25

$$2\,ab\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)-\frac{1}{8}\,b^2\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}\right)}\right)+\frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) - 1/8*b^2*(12*(d*x + c)/d +
e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4
*d*x - 4*c)))) + 2*a^2/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 0.88, size = 75, normalized size = 1.42

$$\frac{b\,x\,\left(4\,a-3\,b\right)}{2}-\frac{b^2\,{\mathrm{e}}^{-2\,c-2\,d\,x}}{8\,d}+\frac{b^2\,{\mathrm{e}}^{2\,c+2\,d\,x}}{8\,d}-\frac{2\,\left(a^2-2\,a\,b+b^2\right)}{d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^2,x)
```

```
[Out] (b*x*(4*a - 3*b))/2 - (b^2*exp(- 2*c - 2*d*x))/(8*d) + (b^2*exp(2*c + 2*d*x
))/(8*d) - (2*(a^2 - 2*a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2{(c+dx)}\right)^2\operatorname{sech}^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

3.299 $\int \mathrm{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=64

$$\frac{(a+3b)(a-b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)^2\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}+\frac{b^2\sinh(c+dx)}{d}$$

```
[Out] 1/2*(a-b)*(a+3*b)*arctan(sinh(d*x+c))/d+b^2*sinh(d*x+c)/d+1/2*(a-b)^2*sech(
d*x+c)*tanh(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 390, 385, 203}

$$\frac{(a+3b)(a-b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)^2\tanh(c+dx)\mathrm{sech}(c+dx)}{2d}+\frac{b^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)*(a + 3*b)*ArcTan[Sinh[c + d*x]])/(2*d) + (b^2*Sinh[c + d*x])/d + (
(a - b)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(b^2+\frac{a^2-b^2+2(a-b)bx^2}{(1+x^2)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b^2\sinh(c+dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{(1+x^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b^2\sinh(c+dx)}{d} + \frac{(a-b)^2\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{((a-b)(a+$$

$$= \frac{(a-b)(a+3b)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{b^2\sinh(c+dx)}{d} + \frac{(a-b)^2\operatorname{se}$$

Mathematica [C] time = 6.84, size = 233, normalized size = 3.64

$$\frac{\operatorname{csch}^3(c+dx)\left(-64\sinh^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 {}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};-\sinh^2(c+dx)\right)+\frac{105\tanh^{-1}\left(\sqrt{-\sinh}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c + d*x]^3*(-64*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[
c + d*x]^2]*Sinh[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2 - 35*(375*a^2 + a*(37
*a + 689*b + 61*b*Cosh[2*(c + d*x)])*Sinh[c + d*x]^2 + 303*b^2*Sinh[c + d*x
]^4 + 61*b^2*Sinh[c + d*x]^6) + (105*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(125*a
^2 + 2*a*(27*a + 125*b)*Sinh[c + d*x]^2 + (9*a^2 + 124*a*b + 101*b^2)*Sinh[
c + d*x]^4 + 2*b*(a + 27*b)*Sinh[c + d*x]^6 + b^2*Sinh[c + d*x]^8))/Sqrt[-S
inh[c + d*x]^2]))/(1680*d)
```

fricas [B] time = 1.91, size = 759, normalized size = 11.86

$$\frac{b^2\cosh(dx+c)^6 + 6\,b^2\cosh(dx+c)\sinh(dx+c)^5 + b^2\sinh(dx+c)^6 + \left(2\,a^2-4\,ab+3\,b^2\right)\cosh(dx+c)^4 + \left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d
*x + c)^6 + (2*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c)^4 + (15*b^2*cosh(d*x + c)
^2 + 2*a^2 - 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + (2
*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (2*a^2 - 4*a*b + 3*b
^2)*cosh(d*x + c)^2 + (15*b^2*cosh(d*x + c)^4 + 6*(2*a^2 - 4*a*b + 3*b^2)*c
osh(d*x + c)^2 - 2*a^2 + 4*a*b - 3*b^2)*sinh(d*x + c)^2 - b^2 + 2*((a^2 + 2
*a*b - 3*b^2)*cosh(d*x + c)^5 + 5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)*sinh(
d*x + c)^4 + (a^2 + 2*a*b - 3*b^2)*sinh(d*x + c)^5 + 2*(a^2 + 2*a*b - 3*b^2
)*cosh(d*x + c)^3 + 2*(5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*
b - 3*b^2)*sinh(d*x + c)^3 + 2*(5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^3 + 3
*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^2 + 2*a*b - 3*b^
2)*cosh(d*x + c) + (5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*
```

```
b - 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b - 3*b^2)*sinh(d*x + c))*arctan(cos
h(d*x + c) + sinh(d*x + c)) + 2*(3*b^2*cosh(d*x + c)^5 + 2*(2*a^2 - 4*a*b +
 3*b^2)*cosh(d*x + c)^3 - (2*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x +
 c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x +
c)^5 + 2*d*cosh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 +
2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x +
c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

giac [B] time = 0.17, size = 163, normalized size = 2.55

$$\frac{2\,b^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) + \left(\pi + 2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(a^2 + 2\,ab - 3\,b^2\right) + \frac{4\left(a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) - 2\,ab\left(e^{(d\ldots}\right.\right.}{\left(e^{(dx+c)} - \ldots\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^2*(e^(d*x + c) - e^(-d*x - c)) + (pi + 2*arctan(1/2*(e^(2*d*x + 2*
c) - 1)*e^(-d*x - c)))*(a^2 + 2*a*b - 3*b^2) + 4*(a^2*(e^(d*x + c) - e^(-d*
x - c)) - 2*a*b*(e^(d*x + c) - e^(-d*x - c)) + b^2*(e^(d*x + c) - e^(-d*x -
 c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

maple [B] time = 0.10, size = 169, normalized size = 2.64

$$\frac{a^2\mathrm{sech}\,(dx+c)\tanh\,(dx+c)}{2d}+\frac{a^2\arctan\left(\mathrm{e}^{dx+c}\right)}{d}-\frac{2ab\sinh\,(dx+c)}{d\cosh\,(dx+c)^2}+\frac{ab\,\mathrm{sech}\,(dx+c)\tanh\,(dx+c)}{d}+\frac{2ab\,\mathrm{arct}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*a^2*sech(d*x+c)*tanh(d*x+c)+1/d*a^2*arctan(exp(d*x+c))-2/d*a*b*sinh(d
*x+c)/cosh(d*x+c)^2+1/d*a*b*sech(d*x+c)*tanh(d*x+c)+2/d*a*b*arctan(exp(d*x+
c))+1/d*b^2*sinh(d*x+c)^3/cosh(d*x+c)^2+3/d*b^2*sinh(d*x+c)/cosh(d*x+c)^2-3
/2/d*b^2*sech(d*x+c)*tanh(d*x+c)-3/d*b^2*arctan(exp(d*x+c))
```

maxima [B] time = 0.45, size = 234, normalized size = 3.66

$$\frac{1}{2}\,b^2\left(\frac{6\,\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + 1}{d\left(e^{(-dx-c)} + 2\,e^{(-3\,dx-3\,c)} + e^{(-5\,dx-5\,c)}\right)}\right) - 2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(-\ldots}}{d\left(2\,e^{(-2\ldots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5
*c)))) - 2*a*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/
(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a^2*(arctan(e^(-d*x - c)
)/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x
- 4*c) + 1)))
```

mupad [B] time = 0.88, size = 220, normalized size = 3.44

$$\frac{b^2\,\mathrm{e}^{c+d\,x}}{2\,d}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^2\,\sqrt{d^2}-3\,b^2\,\sqrt{d^2}+2\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{a^4+4\,a^3\,b-2\,a^2\,b^2-12\,a\,b^3+9\,b^4}}\right)\,\sqrt{a^4+4\,a^3\,b-2\,a^2\,b^2-12\,a\,b^3+9\,b^4}}{\sqrt{d^2}}-\frac{b^2\,\mathrm{e}^{-c-d\,x}}{2\,d}+\frac{\mathrm{e}^{c+d\,x}\,\left(a^2-\ldots\right.}{d\,\left(\mathrm{e}^{2\,c+2\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^3,x)
```

```
[Out] (b^2*exp(c + d*x))/(2*d) + (atan((exp(d*x)*exp(c)*(a^2*(d^2)^(1/2) - 3*b^2*
(d^2)^(1/2) + 2*a*b*(d^2)^(1/2)))/(d*(4*a^3*b - 12*a*b^3 + a^4 + 9*b^4 - 2*
a^2*b^2)^(1/2)))*(4*a^3*b - 12*a*b^3 + a^4 + 9*b^4 - 2*a^2*b^2)^(1/2))/(d^2
)^(1/2) - (b^2*exp(- c - d*x))/(2*d) + (exp(c + d*x)*(a^2 - 2*a*b + b^2))/(
d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(a^2 - 2*a*b + b^2))/(d*(2*exp(
2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^2\left(c + dx\right)\right)^2 \operatorname{sech}^3\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

3.300 $\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=47

$$\frac{\left(a^2-b^2\right)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}+b^2x$$

```
[Out] b^2*x+(a^2-b^2)*tanh(d*x+c)/d-1/3*(a-b)^2*tanh(d*x+c)^3/d
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3191, 390, 206}

$$\frac{\left(a^2-b^2\right)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] b^2*x + ((a^2 - b^2)*Tanh[c + d*x])/d - ((a - b)^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(a^2-b^2-(a-b)^2x^2+\frac{b^2}{1-x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\left(a^2-b^2\right)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{1-x^2}\right.}{}\\
&= b^2x+\frac{\left(a^2-b^2\right)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 57, normalized size = 1.21

$$\frac{(a-b)\tanh(c+dx)\mathrm{sech}^2(c+dx)((a+2b)\cosh(2(c+dx))+2a+b)+3b^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (3*b^2*(c + d*x) + (a - b)*(2*a + b + (a + 2*b)*Cosh[2*(c + d*x)])*Sech[c +
 d*x]^2*Tanh[c + d*x])/(3*d)
```

fricas [B] time = 1.74, size = 200, normalized size = 4.26

$$\frac{\left(3\,b^2dx-2\,a^2-2\,ab+4\,b^2\right)\cosh\left(dx+c\right)^3+3\left(3\,b^2dx-2\,a^2-2\,ab+4\,b^2\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+2\left(a^2+\right.}{3\left(d\cosh\left(dx+c\right)^3+3\,d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*b^2*d*x - 2
*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*b - 2*b^2)
*sinh(d*x + c)^3 + 3*(3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c) + 6*
((a^2 + a*b - 2*b^2)*cosh(d*x + c)^2 + a^2 - a*b)*sinh(d*x + c))/(d*cosh(d*
x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

giac [B] time = 0.19, size = 98, normalized size = 2.09

$$\frac{3\,(dx+c)b^2-\frac{4\left(3\,abe^{(4\,dx+4\,c)}-3\,b^2e^{(4\,dx+4\,c)}+3\,a^2e^{(2\,dx+2\,c)}-3\,b^2e^{(2\,dx+2\,c)}+a^2+ab-2\,b^2\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*b^2 - 4*(3*a*b*e^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 3
*a^2*e^(2*d*x + 2*c) - 3*b^2*e^(2*d*x + 2*c) + a^2 + a*b - 2*b^2)/(e^(2*d*x
 + 2*c) + 1)^3)/d
```

maple [B] time = 0.11, size = 96, normalized size = 2.04

$$\frac{a^2\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh\left(dx+c\right)+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\mathrm{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(dx+c-\tanh\left(dx+c\right)-\frac{(\mathrm{ta}}{}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/2*sinh(d*x+c)/cosh(d
*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(d*x+c-tanh(d*x+c)-1/3
*tanh(d*x+c)^3))
```

maxima [B] time = 0.41, size = 267, normalized size = 5.68

$$\frac{1}{3}\,b^2\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\right)}\right)+\frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) + 1))) + 4/3*a*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-
4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

mupad [B] time = 0.84, size = 194, normalized size = 4.13

$$b^2\,x-\frac{\frac{4\,\left(a\,b-b^2\right)}{3\,d}-\frac{8\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b-a^2\right)}{3\,d}+\frac{4\,{\mathrm{e}}^{4\,c+4\,d\,x}\,\left(a\,b-b^2\right)}{3\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}+\frac{\frac{4\,\left(a\,b-a^2\right)}{3\,d}-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a\,b-b^2\right)}{3\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}-\frac{4\,\left(a\,b-b^2\right)}{3\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^4,x)
```

```
[Out] b^2*x - ((4*(a*b - b^2))/(3*d) - (8*exp(2*c + 2*d*x)*(a*b - a^2))/(3*d) + (
4*exp(4*c + 4*d*x)*(a*b - b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) + 1) + ((4*(a*b - a^2))/(3*d) - (4*exp(2*c + 2*d*x)
*(a*b - b^2))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (4*(a*b
- b^2))/(3*d*(exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.301 $\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=96

$$\frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{3\left(a^2-b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}$$

```
[Out] 1/8*(3*a^2+2*a*b+3*b^2)*arctan(sinh(d*x+c))/d+3/8*(a^2-b^2)*sech(d*x+c)*tan
h(d*x+c)/d+1/4*(a-b)*sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 413, 385, 203}

$$\frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{3\left(a^2-b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (3*(a^2 - b^2)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) + ((a - b)*Sech[c + d*x]^3*(a + b*Sinh[c + d
*x]^2)*Tanh[c + d*x])/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-b)\operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\tanh(c+dx)}{4d} + \frac{\text{Subs}}{}$$

$$= \frac{3\left(a^2-b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{(a-b)\operatorname{sech}^3(c+dx)\left(a\right.}{}$$

$$= \frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{3\left(a^2-b^2\right)\operatorname{sech}(c+dx}{8d}$$

Mathematica [C] time = 5.87, size = 303, normalized size = 3.16

$$-\frac{\operatorname{csch}^3(c+dx)\left(128\sinh^6(c+dx)\left(7a^2+12ab\sinh^2(c+dx)+5b^2\sinh^4(c+dx)\right){}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};-\sinh\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -1/6720*(Csch[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1
, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2 + 128*H
ypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*
x]^6*(7*a^2 + 12*a*b*Sinh[c + d*x]^2 + 5*b^2*Sinh[c + d*x]^4) + 35*(3375*a^
2 + a*(657*a + 4643*b + 607*b*Cosh[2*(c + d*x)])*Sinh[c + d*x]^2 + 1947*b^2
*Sinh[c + d*x]^4 + 485*b^2*Sinh[c + d*x]^6) - (105*ArcTanh[Sqrt[-Sinh[c + d
*x]^2]]*(1125*a^2 + 2*a*(297*a + 875*b)*Sinh[c + d*x]^2 + (37*a^2 + 988*a*b
 + 649*b^2)*Sinh[c + d*x]^4 + 2*b*(11*a + 189*b)*Sinh[c + d*x]^6 + 9*b^2*Si
nh[c + d*x]^8))/Sqrt[-Sinh[c + d*x]^2]))/d
```

fricas [B] time = 0.75, size = 1472, normalized size = 15.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^2 + 2*a*b - 5*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 + 2*a*b - 5*b^2)*co
sh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 2*a*b - 5*b^2)*sinh(d*x + c)^7 + (11
*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 2*a*b - 5*b^2)*cosh(d
*x + c)^2 + 11*a^2 - 14*a*b + 3*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 2*a*b
- 5*b^2)*cosh(d*x + c)^3 + (11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*
x + c)^4 - (11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 + 2*a*b -
 5*b^2)*cosh(d*x + c)^4 + 10*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^2 - 11
*a^2 + 14*a*b - 3*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 + 2*a*b - 5*b^2)*cosh(d
*x + c)^5 + 10*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^3 - 3*(11*a^2 - 14*a
*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^2 + 2*a*b + 3*b^2)*cosh(
d*x + c)^8 + 8*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a
^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x +
c)^6 + 4*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b + 3*b^2
)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(3*a^2
 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2 + 2*a*b + 3*b^2
)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 30*(3*a
```

```
^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 9*a^2 + 6*a*b + 9*b^2)*sinh(d*x + c)^
4 + 8*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(3*a^2 + 2*a*b + 3*b^
2)*cosh(d*x + c)^3 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 2*a*b + 3*b^
2)*cosh(d*x + c)^6 + 15*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 9*(3*a^2
+ 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^2 +
 3*a^2 + 2*a*b + 3*b^2 + 8*((3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^7 + 3*(3*
a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x +
 c)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d
*x + c) + sinh(d*x + c)) - (3*a^2 + 2*a*b - 5*b^2)*cosh(d*x + c) + (7*(3*a^
2 + 2*a*b - 5*b^2)*cosh(d*x + c)^6 + 5*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x +
 c)^4 - 3*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^2 - 3*a^2 - 2*a*b + 5*b^2
)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d
*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d
*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6
*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*
sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 +
15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh
(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*
sinh(d*x + c) + d)
```

giac [B] time = 0.18, size = 218, normalized size = 2.27

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)\left(3a^2 + 2ab + 3b^2\right) + \frac{4\left(3a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 2ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 - 5b^2\left(e^{(dx+c)} - e^{(\cdot}\right.\right.}{\left(\left(e^{(d\cdot}\right.\right.}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a^2 + 2*a*
b + 3*b^2) + 4*(3*a^2*(e^(d*x + c) - e^(-d*x - c))^3 + 2*a*b*(e^(d*x + c) -
 e^(-d*x - c))^3 - 5*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 20*a^2*(e^(d*x +
c) - e^(-d*x - c)) - 8*a*b*(e^(d*x + c) - e^(-d*x - c)) - 12*b^2*(e^(d*x +
c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

maple [B] time = 0.12, size = 237, normalized size = 2.47

$$\frac{a^2\tanh(dx+c)\operatorname{sech}(dx+c)^3}{4d} + \frac{3a^2\operatorname{sech}(dx+c)\tanh(dx+c)}{8d} + \frac{3a^2\arctan\left(e^{dx+c}\right)}{4d} - \frac{2ab\sinh(dx+c)}{3d\cosh(dx+c)^4} + \frac{ab\tanh}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/4/d*a^2*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*a^2*sech(d*x+c)*tanh(d*x+c)+3/4/d
*a^2*arctan(exp(d*x+c))-2/3/d*a*b*sinh(d*x+c)/cosh(d*x+c)^4+1/6/d*a*b*tanh(
d*x+c)*sech(d*x+c)^3+1/4/d*a*b*sech(d*x+c)*tanh(d*x+c)+1/2/d*a*b*arctan(exp
(d*x+c))-1/d*b^2*sinh(d*x+c)^3/cosh(d*x+c)^4-1/d*b^2*sinh(d*x+c)/cosh(d*x+c
)^4+1/4/d*b^2*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*b^2*sech(d*x+c)*tanh(d*x+c)+3
/4/d*b^2*arctan(exp(d*x+c))
```

maxima [B] time = 0.50, size = 347, normalized size = 3.61

$$-\frac{1}{4}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{5e^{(-dx-c)} - 3e^{(-3dx-3c)} + 3e^{(-5dx-5c)} - 5e^{(-7dx-7c)}}{d\left(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1\right)}\right) - \frac{1}{4}a^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*x - 3*c) +
 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/4*a^2*(3*arct
an(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x -
 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4
*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/2*a*b*(arctan(e^(-d*x - c))
/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c) - e^(-7*d*x -
7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^
(-8*d*x - 8*c) + 1)))
```

mupad [B] time = 0.89, size = 327, normalized size = 3.41

$$\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(3\,a^2\,\sqrt{d^2}+3\,b^2\,\sqrt{d^2}+2\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{9\,a^4+12\,a^3\,b+22\,a^2\,b^2+12\,a\,b^3+9\,b^4}}\right)\,\sqrt{9\,a^4+12\,a^3\,b+22\,a^2\,b^2+12\,a\,b^3+9\,b^4}}{4\,\sqrt{d^2}}-\frac{6\,e^{c+d\,x}\,\left(a^2-2\,a\,b+\right.}{d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^5,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(3*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 2*a*b*(d^2)
^(1/2)))/(d*(12*a*b^3 + 12*a^3*b + 9*a^4 + 9*b^4 + 22*a^2*b^2)^(1/2)))*(12*
a*b^3 + 12*a^3*b + 9*a^4 + 9*b^4 + 22*a^2*b^2)^(1/2))/(4*(d^2)^(1/2)) - (6*
exp(c + d*x)*(a^2 - 2*a*b + b^2))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1)) + (4*exp(c + d*x)*(a^2 - 2*a*b + b^2))/(d*(4*ex
p(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x)
 + 1)) + (exp(c + d*x)*(a^2 - 10*a*b + 9*b^2))/(2*d*(2*exp(2*c + 2*d*x) + e
xp(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(2*a*b + 3*a^2 - 5*b^2))/(4*d*(exp(2*
c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.302 $\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=57

$$\frac{a^2\tanh(c+dx)}{d}+\frac{(a-b)^2\tanh^5(c+dx)}{5d}-\frac{2a(a-b)\tanh^3(c+dx)}{3d}$$

```
[Out] a^2*tanh(d*x+c)/d-2/3*a*(a-b)*tanh(d*x+c)^3/d+1/5*(a-b)^2*tanh(d*x+c)^5/d
```

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3191, 194}

$$\frac{a^2\tanh(c+dx)}{d}+\frac{(a-b)^2\tanh^5(c+dx)}{5d}-\frac{2a(a-b)\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d - (2*a*(a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)^2*Ta
nh[c + d*x]^5)/(5*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \left(a-(a-b)x^2\right)^2\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^2-2a(a-b)x^2+(a-b)^2x^4\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a^2\tanh(c+dx)}{d}-\frac{2a(a-b)\tanh^3(c+dx)}{3d}+\frac{(a-b)^2\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.39, size = 69, normalized size = 1.21

$$\frac{\tanh(c+dx)\left(2\left(2a^2+ab-3b^2\right)\operatorname{sech}^2(c+dx)+8a^2+3(a-b)^2\operatorname{sech}^4(c+dx)+4ab+3b^2\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 4*a*b + 3*b^2 + 2*(2*a^2 + a*b - 3*b^2)*Sech[c + d*x]^2 + 3*(a -
b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(15*d)
```

fricas [B] time = 0.59, size = 403, normalized size = 7.07

$$-\frac{4\left(\left(4\,a^{2}+2\,ab+9\,b^{2}\right)\cosh\left(dx+c\right)^{4}-8\left(2\,a^{2}+ab-3\,b^{2}\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\right.}{15\left(d\cosh\left(dx+c\right)^{6}+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+d\sinh\left(dx+c\right)^{6}+6\,d\cosh\left(dx+c\right)^{4}+3\left(5\,d\cosh\left(\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/15*((4*a^2 + 2*a*b + 9*b^2)*cosh(d*x + c)^4 - 8*(2*a^2 + a*b - 3*b^2)*co
sh(d*x + c)*sinh(d*x + c)^3 + (4*a^2 + 2*a*b + 9*b^2)*sinh(d*x + c)^4 + 20*
(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(4*a^2 + 2*a*b + 9*b^2)*cosh(d*x + c)^
2 + 10*a^2 + 20*a*b)*sinh(d*x + c)^2 + 40*a^2 - 10*a*b + 15*b^2 - 8*((2*a^2
 + a*b - 3*b^2)*cosh(d*x + c)^3 + 5*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c
))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)
^6 + 6*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 +
4*(5*d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x
 + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c
)^2 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sin
h(d*x + c) + 10*d)
```

giac [B] time = 0.18, size = 128, normalized size = 2.25

$$-\frac{2\left(15\,b^{2}e^{\left(8\,dx+8\,c\right)}+60\,abe^{\left(6\,dx+6\,c\right)}+80\,a^{2}e^{\left(4\,dx+4\,c\right)}-20\,abe^{\left(4\,dx+4\,c\right)}+30\,b^{2}e^{\left(4\,dx+4\,c\right)}+40\,a^{2}e^{\left(2\,dx+2\,c\right)}+20\,abe^{\left(2\right.}\right.}{15\,d\left(e^{\left(2\,dx+2\,c\right)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*b^2*e^(8*d*x + 8*c) + 60*a*b*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x +
4*c) - 20*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 40*a^2*e^(2*d*x +
2*c) + 20*a*b*e^(2*d*x + 2*c) + 8*a^2 + 4*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c)
+ 1)^5)
```

maple [B] time = 0.12, size = 158, normalized size = 2.77

$$\frac{a^{2}\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^{4}}{5}+\frac{4\mathrm{sech}(dx+c)^{2}}{15}\right)\tanh\left(dx+c\right)+2ab\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^{5}}+\frac{\left(\frac{8}{15}+\frac{\mathrm{sech}(dx+c)^{4}}{5}+\frac{4\mathrm{sech}(dx+c)^{2}}{15}\right)\tanh(dx+c)}{4}\right)+b^{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/
4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)
*tanh(d*x+c))+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*
x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.34, size = 698, normalized size = 12.25

$$\frac{16}{15}\,a^{2}\left(\frac{5\,e^{\left(-2\,dx-2\,c\right)}}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}+10\,e^{\left(-4\,dx-4\,c\right)}+10\,e^{\left(-6\,dx-6\,c\right)}+5\,e^{\left(-8\,dx-8\,c\right)}+e^{\left(-10\,dx-10\,c\right)}+1\right)}+\frac{}{d\left(5\,e^{\left(-2\,dx-2\,c\right)}+10\,e^{\left(-4\right.}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 16/15*a^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*
e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*
x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) + 1))) + 8/15*a*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2/5*b^2*(10*e^(-4*d*x - 4*c)/(d*
(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d
*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

mupad [**B**] time = 0.86, size = 464, normalized size = 8.14

$$-\frac{\frac{2\left(8a^2-8ab+3b^2\right)}{15d}+\frac{2b^2e^{4c+4dx}}{5d}+\frac{4be^{2c+2dx}(2a-b)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{\frac{2b^2}{5d}+\frac{2b^2e^{8c+8dx}}{5d}+\frac{4e^{4c+4dx}\left(8a^2-8ab+3b^2\right)}{5d}+\frac{8be^{2c+2dx}(2a-b)}{5d}+\frac{8be^{6}}{}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^6,x)
```

```
[Out] - ((2*(8*a^2 - 8*a*b + 3*b^2))/(15*d) + (2*b^2*exp(4*c + 4*d*x))/(5*d) + (4
*b*exp(2*c + 2*d*x)*(2*a - b))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d
*x) + exp(6*c + 6*d*x) + 1) - ((2*b^2)/(5*d) + (2*b^2*exp(8*c + 8*d*x))/(5*
d) + (4*exp(4*c + 4*d*x)*(8*a^2 - 8*a*b + 3*b^2))/(5*d) + (8*b*exp(2*c + 2*
d*x)*(2*a - b))/(5*d) + (8*b*exp(6*c + 6*d*x)*(2*a - b))/(5*d))/(5*exp(2*c
+ 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) +
 exp(10*c + 10*d*x) + 1) - ((2*b*(2*a - b))/(5*d) + (2*b^2*exp(2*c + 2*d*x)
)/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*b*(2*a - b))/(5*
d) + (2*b^2*exp(6*c + 6*d*x))/(5*d) + (2*exp(2*c + 2*d*x)*(8*a^2 - 8*a*b +
3*b^2))/(5*d) + (6*b*exp(4*c + 4*d*x)*(2*a - b))/(5*d))/(4*exp(2*c + 2*d*x)
 + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*b^2
)/(5*d*(exp(2*c + 2*d*x) + 1))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.303 $\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=131

$$\frac{\left(5a^2+2ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{\left(5a^2+2ab+b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{16d}+\frac{(a-b)(5a+3b)\tanh(c+}{24d}$$

```
[Out] 1/16*(5*a^2+2*a*b+b^2)*arctan(sinh(d*x+c))/d+1/16*(5*a^2+2*a*b+b^2)*sech(d*
x+c)*tanh(d*x+c)/d+1/24*(a-b)*(5*a+3*b)*sech(d*x+c)^3*tanh(d*x+c)/d+1/6*(a-
b)*sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)*tanh(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 413, 385, 199, 203}

$$\frac{\left(5a^2+2ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{\left(5a^2+2ab+b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{16d}+\frac{(a-b)(5a+3b)\tanh(c+}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((5*a^2 + 2*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((5*a^2 + 2*a*b + b^
2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + ((a - b)*(5*a + 3*b)*Sech[c + d*x]
^3*Tanh[c + d*x])/(24*d) + ((a - b)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)
*Tanh[c + d*x])/(6*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-b)\operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{(a-b)(5a+3b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} + \frac{(a-b)\operatorname{sech}^5(c+dx)}{}$$

$$= \frac{\left(5a^2+2ab+b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} + \frac{(a-b)(5a+3b)\operatorname{sech}}{2}$$

$$= \frac{\left(5a^2+2ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{\left(5a^2+2ab+b^2\right)\operatorname{sech}(c+}{16d}$$

Mathematica [C] time = 10.23, size = 715, normalized size = 5.46

$$\frac{\operatorname{csch}^3(c+dx)\left(32\left(-\sinh^2(c+dx)\right)^{3/2}\sinh^4(c+dx)\left(5a^2+9ab\sinh^2(c+dx)+4b^2\sinh^4(c+dx)\right){}_5F_4\left(\frac{3}{2},2,2,2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c + d*x]^3*(65625*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]] + 36855*a^2*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 + 91875*a*b*ArcTanh[Sqrt[-Sinh
[c + d*x]^2]]*Sinh[c + d*x]^2 + 1680*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Si
nh[c + d*x]^4 + 54180*a*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 +
 32970*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 + 1365*a*b*ArcTa
nh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 + 19845*b^2*ArcTanh[Sqrt[-Sinh[c
 + d*x]^2]]*Sinh[c + d*x]^6 + 525*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[
c + d*x]^8 - 65625*a^2*Sqrt[-Sinh[c + d*x]^2] + 14980*a^2*(-Sinh[c + d*x]^2
)^(3/2) + 91875*a*b*(-Sinh[c + d*x]^2)^(3/2) + 8855*b^2*Sinh[c + d*x]^4*(-S
inh[c + d*x]^2)^(3/2) + 16*a^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1,
1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2) +
 32*a*b*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c
+ d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2) + 16*b^2*HypergeometricP
FQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]
^8*(-Sinh[c + d*x]^2)^(3/2) - 23555*a*b*(-Sinh[c + d*x]^2)^(5/2) - 32970*b^
2*(-Sinh[c + d*x]^2)^(5/2) + 32*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2)*(5*a^2
 + 9*a*b*Sinh[c + d*x]^2 + 4*b^2*Sinh[c + d*x]^4) + 4*HypergeometricPFQ[{3/
2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]
^2)^(3/2)*(155*a^2 + 242*a*b*Sinh[c + d*x]^2 + 95*b^2*Sinh[c + d*x]^4)))/(2
520*d*Sqrt[-Sinh[c + d*x]^2])
```

fricas [B] time = 2.18, size = 2824, normalized size = 21.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 + 33*(5*a^2 + 2*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^10 + 3*(5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^11 +
(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(5*a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + 85*a^2 + 34*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(5*a^2 + 2
*a*b + b^2)*cosh(d*x + c)^3 + (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c))*sin
h(d*x + c)^8 + 6*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*(5*a^2
 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c
)^2 + 33*a^2 - 38*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(5*a^2 + 2*a*b + b
^2)*cosh(d*x + c)^5 + 2*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^3 + (33*a^
2 - 38*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(33*a^2 - 38*a*b +
13*b^2)*cosh(d*x + c)^5 + 6*(231*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 21
*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(33*a^2 - 38*a*b + 13*b^2)
*cosh(d*x + c)^2 - 33*a^2 + 38*a*b - 13*b^2)*sinh(d*x + c)^5 + 6*(165*(5*a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 21*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x +
 c)^5 + 35*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(33*a^2 - 38*a*b
+ 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (85*a^2 + 34*a*b - 47*b^2)*cosh(
d*x + c)^3 + (495*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(85*a^2 + 34*a
*b - 47*b^2)*cosh(d*x + c)^6 + 210*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)
^4 - 60*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^2 - 85*a^2 - 34*a*b + 47*b
^2)*sinh(d*x + c)^3 + 3*(55*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 12*(85*
a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^7 + 42*(33*a^2 - 38*a*b + 13*b^2)*cosh
(d*x + c)^5 - 20*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^3 - (85*a^2 + 34*
a*b - 47*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((5*a^2 + 2*a*b + b^2)*cos
h(d*x + c)^12 + 12*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (
5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 6*(5*a^2 + 2*a*b + b^2)*cosh(d*x +
c)^10 + 6*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*
sinh(d*x + c)^10 + 20*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(5*a^2
+ 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a^2 + 2*a*b + b^2)*co
sh(d*x + c)^8 + 15*(33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 18*(5*a^2 +
2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 24*(3
3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 30*(5*a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^3 + 5*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(5*a^
2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(5*a^2 + 2*a*b + b^2)*cosh(d*x +
c)^6 + 315*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 105*(5*a^2 + 2*a*b + b^2
)*cosh(d*x + c)^2 + 25*a^2 + 10*a*b + 5*b^2)*sinh(d*x + c)^6 + 24*(33*(5*a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 63*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5
 + 35*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(5*a^2 + 2*a*b + b^2)*cosh(
d*x + c))*sinh(d*x + c)^5 + 15*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(
33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(5*a^2 + 2*a*b + b^2)*cosh(d*
x + c)^6 + 70*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 20*(5*a^2 + 2*a*b + b
^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(5*a^2
+ 2*a*b + b^2)*cosh(d*x + c)^9 + 36*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 +
 42*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 20*(5*a^2 + 2*a*b + b^2)*cosh(d
*x + c)^3 + 3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(5*a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 6*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x +
c)^10 + 45*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 70*(5*a^2 + 2*a*b + b^2)
*cosh(d*x + c)^6 + 50*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(5*a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + 5*a^2
+ 2*a*b + b^2 + 12*((5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 + 5*(5*a^2 + 2*a
*b + b^2)*cosh(d*x + c)^9 + 10*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 10*(
5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 5*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c
)^3 + (5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x +
```

```
 c) + sinh(d*x + c)) - 3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c) + 3*(11*(5*a^2
 + 2*a*b + b^2)*cosh(d*x + c)^10 + 3*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x +
c)^8 + 14*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^6 - 10*(33*a^2 - 38*a*b
+ 13*b^2)*cosh(d*x + c)^4 - (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^2 - 5*
a^2 - 2*a*b - b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*
sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh
(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x
 + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 +
18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d
*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^
6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^
2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5
+ 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x
 + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x +
 c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^
9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x +
 c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4
 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d
*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d
*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

giac [B] time = 0.22, size = 291, normalized size = 2.22

$$3\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(5\,a^2 + 2\,ab + b^2\right) + \frac{4\left(15\,a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 6\,ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 3\,b^2\left(e^{(dx+c)} - e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(5*a^2 + 2*
a*b + b^2) + 4*(15*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 6*a*b*(e^(d*x + c)
- e^(-d*x - c))^5 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 160*a^2*(e^(d*x
+ c) - e^(-d*x - c))^3 + 64*a*b*(e^(d*x + c) - e^(-d*x - c))^3 - 32*b^2*(e^
(d*x + c) - e^(-d*x - c))^3 + 528*a^2*(e^(d*x + c) - e^(-d*x - c)) - 96*a*b
*(e^(d*x + c) - e^(-d*x - c)) - 48*b^2*(e^(d*x + c) - e^(-d*x - c)))/((e^(d
*x + c) - e^(-d*x - c))^2 + 4)^3)/d
```

maple [B] time = 0.18, size = 302, normalized size = 2.31

$$\frac{a^2\tanh\left(dx+c\right)\operatorname{sech}\left(dx+c\right)^5}{6d} + \frac{5a^2\tanh\left(dx+c\right)\operatorname{sech}\left(dx+c\right)^3}{24d} + \frac{5a^2\operatorname{sech}\left(dx+c\right)\tanh\left(dx+c\right)}{16d} + \frac{5a^2\arctan\left(e\right.}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/6/d*a^2*tanh(d*x+c)*sech(d*x+c)^5+5/24/d*a^2*tanh(d*x+c)*sech(d*x+c)^3+5/
16/d*a^2*sech(d*x+c)*tanh(d*x+c)+5/8/d*a^2*arctan(exp(d*x+c))-2/5/d*a*b*sin
h(d*x+c)/cosh(d*x+c)^6+1/15/d*a*b*tanh(d*x+c)*sech(d*x+c)^5+1/12/d*a*b*tanh
(d*x+c)*sech(d*x+c)^3+1/8/d*a*b*sech(d*x+c)*tanh(d*x+c)+1/4/d*a*b*arctan(ex
p(d*x+c))-1/3/d*b^2*sinh(d*x+c)^3/cosh(d*x+c)^6-1/5/d*b^2*sinh(d*x+c)/cosh(
d*x+c)^6+1/30/d*b^2*tanh(d*x+c)*sech(d*x+c)^5+1/24/d*b^2*tanh(d*x+c)*sech(d
*x+c)^3+1/16/d*b^2*sech(d*x+c)*tanh(d*x+c)+1/8/d*b^2*arctan(exp(d*x+c))
```

maxima [B] time = 0.54, size = 483, normalized size = 3.69

$$-\frac{1}{24}\,a^2\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{15\,e^{(-dx-c)} + 85\,e^{(-3\,dx-3\,c)} + 198\,e^{(-5\,dx-5\,c)} - 198\,e^{(-7\,dx-7\,c)} - 85\,e^{(-9\,dx-9\,c)} - 15\,e^{(}}{d\left(6\,e^{(-2\,dx-2\,c)} + 15\,e^{(-4\,dx-4\,c)} + 20\,e^{(-6\,dx-6\,c)} + 15\,e^{(-8\,dx-8\,c)} + 6\,e^{(-10\,dx-10\,c)} + e^{(-12}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*a^2*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*
c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15
*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 1/12*a*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 17*e^(-
3*d*x - 3*c) - 114*e^(-5*d*x - 5*c) + 114*e^(-7*d*x - 7*c) - 17*e^(-9*d*x -
 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c)
+ 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12
*d*x - 12*c) + 1))) - 1/24*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c)
- 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c) + 47*e^(-
9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x
- 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) +
 e^(-12*d*x - 12*c) + 1)))
```

mupad [B] time = 0.92, size = 582, normalized size = 4.44

$$\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(5\,a^2\,\sqrt{d^2}+b^2\,\sqrt{d^2}+2\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{25\,a^4+20\,a^3\,b+14\,a^2\,b^2+4\,a\,b^3+b^4}}\right)\,\sqrt{25\,a^4+20\,a^3\,b+14\,a^2\,b^2+4\,a\,b^3+b^4}}{8\,\sqrt{d^2}}-\frac{\frac{2\,b^2\,e^{c+d\,x}}{3\,d}+\frac{2\,b^2\,e^{9\,c+9\,d\,x}}{3\,d}+\frac{4\,e^{5\,c+5\,d}\cdots}{\cdots}}{6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^2/cosh(c + d*x)^7,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*a^2*(d^2)^(1/2) + b^2*(d^2)^(1/2) + 2*a*b*(d^2)^(
1/2)))/(d*(4*a*b^3 + 20*a^3*b + 25*a^4 + b^4 + 14*a^2*b^2)^(1/2)))*(4*a*b^3
 + 20*a^3*b + 25*a^4 + b^4 + 14*a^2*b^2)^(1/2))/(8*(d^2)^(1/2)) - ((2*b^2*e
xp(c + d*x))/(3*d) + (2*b^2*exp(9*c + 9*d*x))/(3*d) + (4*exp(5*c + 5*d*x)*(
8*a^2 - 8*a*b + 3*b^2))/(3*d) + (8*b*exp(3*c + 3*d*x)*(2*a - b))/(3*d) + (8
*b*exp(7*c + 7*d*x)*(2*a - b))/(3*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*
d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + e
xp(12*c + 12*d*x) + 1) - (2*exp(c + d*x)*(11*a^2 - 26*a*b + 15*b^2))/(3*d*(
4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*
d*x) + 1)) + (exp(c + d*x)*(2*a*b + 5*a^2 + b^2))/(8*d*(exp(2*c + 2*d*x) +
1)) + (16*exp(c + d*x)*(a^2 - 2*a*b + b^2))/(3*d*(5*exp(2*c + 2*d*x) + 10*e
xp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*
d*x) + 1)) + (exp(c + d*x)*(a^2 - 22*a*b + 21*b^2))/(3*d*(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(2*a*b + 5*
a^2 - 23*b^2))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.304 $\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=238

$$\frac{b\left(44a^2-28ab+5b^2\right)\sinh(c+dx)\cosh^5(c+dx)}{160d}+\frac{(4a-b)\left(8a^2-2ab+b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{128d}+\frac{3(4a-}{}$$

```
[Out] 3/256*(4*a-b)*(8*a^2-2*a*b+b^2)*x+3/256*(4*a-b)*(8*a^2-2*a*b+b^2)*cosh(d*x+
c)*sinh(d*x+c)/d+1/128*(4*a-b)*(8*a^2-2*a*b+b^2)*cosh(d*x+c)^3*sinh(d*x+c)/
d+1/160*b*(44*a^2-28*a*b+5*b^2)*cosh(d*x+c)^5*sinh(d*x+c)/d+1/10*b*cosh(d*x
+c)^9*sinh(d*x+c)*(a-(a-b)*tanh(d*x+c)^2)^2/d+1/80*b*cosh(d*x+c)^7*sinh(d*x
+c)*(a*(10*a-b)-5*(a-b)*(2*a-b)*tanh(d*x+c)^2)/d
```

Rubi [A] time = 0.33, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3191, 413, 526, 385, 199, 206}

$$\frac{b\left(44a^2-28ab+5b^2\right)\sinh(c+dx)\cosh^5(c+dx)}{160d}+\frac{(4a-b)\left(8a^2-2ab+b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{128d}+\frac{3(4a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*(4*a - b)*(8*a^2 - 2*a*b + b^2)*x)/256 + (3*(4*a - b)*(8*a^2 - 2*a*b + b
^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + ((4*a - b)*(8*a^2 - 2*a*b + b^2)
*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*d) + (b*(44*a^2 - 28*a*b + 5*b^2)*Cosh
[c + d*x]^5*Sinh[c + d*x])/(160*d) + (b*Cosh[c + d*x]^9*Sinh[c + d*x]*(a -
(a - b)*Tanh[c + d*x]^2)^2)/(10*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a*(1
0*a - b) - 5*(a - b)*(2*a - b)*Tanh[c + d*x]^2))/(80*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^6}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^9(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{10d} - \frac{\text{Sub}}{}$$

$$= \frac{b\cosh^9(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{10d} + \frac{b\,\text{co}}{}$$

$$= \frac{b\left(44a^2-28ab+5b^2\right)\cosh^5(c+dx)\sinh(c+dx)}{160d} + \frac{b\cosh^9(c+}{}$$

$$= \frac{(4a-b)\left(8a^2-2ab+b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{128d} + \frac{b\left(44a^2\right.}{}$$

$$= \frac{3(4a-b)\left(8a^2-2ab+b^2\right)\cosh(c+dx)\sinh(c+dx)}{256d} + \frac{(4a-b)}{}$$

$$= \frac{3}{256}(4a-b)\left(8a^2-2ab+b^2\right)x + \frac{3(4a-b)\left(8a^2-2ab+b^2\right)\cos}{256d}$$

Mathematica [A] time = 0.53, size = 144, normalized size = 0.61

$$\frac{120(4a-b)\left(8a^2-2ab+b^2\right)(c+dx) - 10b\left(b^2-16a^2\right)\sinh(6(c+dx)) + 20\left(128a^3-24a^2b+b^3\right)\sinh(2(c+}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (120*(4*a - b)*(8*a^2 - 2*a*b + b^2)*(c + d*x) + 20*(128*a^3 - 24*a^2*b + b
^3)*Sinh[2*(c + d*x)] + 40*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*Sinh[4*(c + d
```

```
*x)] - 10*b*(-16*a^2 + b^2)*Sinh[6*(c + d*x)] + 5*(6*a - b)*b^2*Sinh[8*(c +
 d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)
```

fricas [A] time = 0.73, size = 376, normalized size = 1.58

$$\frac{5\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^9+10\left(6\,b^3\cosh\left(dx+c\right)^3+\left(6\,ab^2-b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^7+\left(126\,b^3\,\mathrm{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2560*(5*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 10*(6*b^3*cosh(d*x + c)^3 + (
6*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (126*b^3*cosh(d*x + c)^5 +
70*(6*a*b^2 - b^3)*cosh(d*x + c)^3 + 15*(16*a^2*b - b^3)*cosh(d*x + c))*sin
h(d*x + c)^5 + 10*(6*b^3*cosh(d*x + c)^7 + 7*(6*a*b^2 - b^3)*cosh(d*x + c)^
5 + 5*(16*a^2*b - b^3)*cosh(d*x + c)^3 + 4*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^
3)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(32*a^3 - 16*a^2*b + 6*a*b^2 - b^3)*
d*x + 5*(b^3*cosh(d*x + c)^9 + 2*(6*a*b^2 - b^3)*cosh(d*x + c)^7 + 3*(16*a^
2*b - b^3)*cosh(d*x + c)^5 + 8*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*cosh(d*x
+ c)^3 + 2*(128*a^3 - 24*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

giac [A] time = 0.19, size = 293, normalized size = 1.23

$$\frac{b^3e^{(10\,dx+10\,c)}}{10240\,d}-\frac{b^3e^{(-10\,dx-10\,c)}}{10240\,d}+\frac{3}{256}\left(32\,a^3-16\,a^2b+6\,ab^2-b^3\right)x+\frac{\left(6\,ab^2-b^3\right)e^{(8\,dx+8\,c)}}{4096\,d}+\frac{\left(16\,a^2b-b^3\right)e^{(6\,dx+6\,c)}}{2048\,d}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/10240*b^3*e^(10*d*x + 10*c)/d - 1/10240*b^3*e^(-10*d*x - 10*c)/d + 3/256*
(32*a^3 - 16*a^2*b + 6*a*b^2 - b^3)*x + 1/4096*(6*a*b^2 - b^3)*e^(8*d*x + 8
*c)/d + 1/2048*(16*a^2*b - b^3)*e^(6*d*x + 6*c)/d + 1/512*(8*a^3 + 12*a^2*b
 - 6*a*b^2 + b^3)*e^(4*d*x + 4*c)/d + 1/1024*(128*a^3 - 24*a^2*b + b^3)*e^(
2*d*x + 2*c)/d - 1/1024*(128*a^3 - 24*a^2*b + b^3)*e^(-2*d*x - 2*c)/d - 1/5
12*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*e^(-4*d*x - 4*c)/d - 1/2048*(16*a^2*b
 - b^3)*e^(-6*d*x - 6*c)/d - 1/4096*(6*a*b^2 - b^3)*e^(-8*d*x - 8*c)/d
```

maple [A] time = 0.08, size = 267, normalized size = 1.12

$$\frac{b^3\left(\frac{\left(\sinh^5(dx+c)\right)\left(\cosh^5(dx+c)\right)}{10}-\frac{\left(\sinh^3(dx+c)\right)\left(\cosh^5(dx+c)\right)}{16}+\frac{\sinh(dx+c)\left(\cosh^5(dx+c)\right)}{32}-\frac{\left(\frac{\left(\cosh^3(dx+c)\right)}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)}{32}-\frac{3dx}{256}\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*(1/10*sinh(d*x+c)^5*cosh(d*x+c)^5-1/16*sinh(d*x+c)^3*cosh(d*x+c)^5
+1/32*sinh(d*x+c)*cosh(d*x+c)^5-1/32*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*si
nh(d*x+c)-3/256*d*x-3/256*c)+3*a*b^2*(1/8*sinh(d*x+c)^3*cosh(d*x+c)^5-1/16*
sinh(d*x+c)*cosh(d*x+c)^5+1/16*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x
+c)+3/128*d*x+3/128*c)+3*a^2*b*(1/6*sinh(d*x+c)*cosh(d*x+c)^5-1/6*(1/4*cosh
(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)-1/16*d*x-1/16*c)+a^3*((1/4*cosh(d*x+
c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c))
```

maxima [A] time = 0.45, size = 363, normalized size = 1.53

$$\frac{1}{64}\,a^3\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{20480}\,b^3\left(\frac{\left(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}-40\,e^{(-6\cdots}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) - 40*e^(-6*d*x - 6*c) - 20*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 240*(d*x + c)/d + (20*e^(-2*d*x - 2*c) + 40*e^(-4*d*x - 4*c) - 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 3/2048*a*b^2*((8*e^(-4*d*x - 4*c) - 1)*e^(8*d*x + 8*c)/d - 48*(d*x + c)/d - (8*e^(-4*d*x - 4*c) - e^(-8*d*x - 8*c))/d) + 1/128*a^2*b*((3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d + (3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d)

mupad [B] time = 0.58, size = 209, normalized size = 0.88

$$\frac{320\,a^3\sinh\left(2\,c+2\,d\,x\right)+40\,a^3\sinh\left(4\,c+4\,d\,x\right)+\frac{5\,b^3\sinh(2\,c+2\,d\,x)}{2}+5\,b^3\sinh\left(4\,c+4\,d\,x\right)-\frac{5\,b^3\sinh(6\,c+6\,d\,x)}{4}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3,x)

[Out] (320*a^3*sinh(2*c + 2*d*x) + 40*a^3*sinh(4*c + 4*d*x) + (5*b^3*sinh(2*c + 2*d*x))/2 + 5*b^3*sinh(4*c + 4*d*x) - (5*b^3*sinh(6*c + 6*d*x))/4 - (5*b^3*sinh(8*c + 8*d*x))/8 + (b^3*sinh(10*c + 10*d*x))/4 - 60*a^2*b*sinh(2*c + 2*d*x) - 30*a*b^2*sinh(4*c + 4*d*x) + 60*a^2*b*sinh(4*c + 4*d*x) + 20*a^2*b*sinh(6*c + 6*d*x) + (15*a*b^2*sinh(8*c + 8*d*x))/4 + 480*a^3*d*x - 15*b^3*d*x + 90*a*b^2*d*x - 240*a^2*b*d*x)/(1280*d)

sympy [A] time = 24.56, size = 774, normalized size = 3.25

$$\begin{cases} \frac{3a^3x\sinh^4(c+dx)}{8}-\frac{3a^3x\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3a^3x\cosh^4(c+dx)}{8}-\frac{3a^3\sinh^3(c+dx)\cosh(c+dx)}{8d}+\frac{5a^3\sinh(c+dx)\cosh^3(c+dx)}{8d}+ \\ x\left(a+b\sinh^2(c)\right)^3\cosh^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Piecewise((3*a**3*x*sinh(c + d*x)**4/8 - 3*a**3*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 3*a**3*x*cosh(c + d*x)**4/8 - 3*a**3*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 5*a**3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 3*a**2*b*x*sinh(c + d*x)**6/16 - 9*a**2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 9*a**2*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 3*a**2*b*x*cosh(c + d*x)**6/16 - 3*a**2*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + a**2*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 3*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + 9*a*b**2*x*sinh(c + d*x)**8/128 - 9*a*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 27*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 9*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 9*a*b**2*x*cosh(c + d*x)**8/128 - 9*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 33*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 33*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(128*d) - 9*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 3*b**3*x*sinh(c + d*x)**10/256 - 15*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 15*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 15*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)**6/128 + 15*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 3*b**3*x*cosh(c + d*x)**10/256 - 3*b**3*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) + 7*b**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + b**3*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 7*b**3*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 3*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cosh(c)**4, True))

3.305 $\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=98

$$\frac{a^3\sinh(c+dx)}{d}+\frac{a^2(a+3b)\sinh^3(c+dx)}{3d}+\frac{b^2(3a+b)\sinh^7(c+dx)}{7d}+\frac{3ab(a+b)\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^9(c+dx)}{9d}$$

```
[Out] a^3*sinh(d*x+c)/d+1/3*a^2*(a+3*b)*sinh(d*x+c)^3/d+3/5*a*b*(a+b)*sinh(d*x+c)
^5/d+1/7*b^2*(3*a+b)*sinh(d*x+c)^7/d+1/9*b^3*sinh(d*x+c)^9/d
```

Rubi [A] time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3190, 373}

$$\frac{a^2(a+3b)\sinh^3(c+dx)}{3d}+\frac{a^3\sinh(c+dx)}{d}+\frac{b^2(3a+b)\sinh^7(c+dx)}{7d}+\frac{3ab(a+b)\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Sinh[c + d*x])/d + (a^2*(a + 3*b)*Sinh[c + d*x]^3)/(3*d) + (3*a*b*(a +
 b)*Sinh[c + d*x]^5)/(5*d) + (b^2*(3*a + b)*Sinh[c + d*x]^7)/(7*d) + (b^3*S
inh[c + d*x]^9)/(9*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^3\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^3+a^2(a+3b)x^2+3ab(a+b)x^4+b^2(3a+b)x^6+b^3x^8\right)\right.}{d}\\ &= \frac{a^3\sinh(c+dx)}{d}+\frac{a^2(a+3b)\sinh^3(c+dx)}{3d}+\frac{3ab(a+b)\sinh^5(c+\right.}{5d}\end{aligned}$$

Mathematica [A] time = 0.24, size = 83, normalized size = 0.85

$$\frac{315a^3\sinh(c+dx)+105a^2(a+3b)\sinh^3(c+dx)+45b^2(3a+b)\sinh^7(c+dx)+189ab(a+b)\sinh^5(c+dx)+3}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (315*a^3*Sinh[c + d*x] + 105*a^2*(a + 3*b)*Sinh[c + d*x]^3 + 189*a*b*(a + b
)*Sinh[c + d*x]^5 + 45*b^2*(3*a + b)*Sinh[c + d*x]^7 + 35*b^3*Sinh[c + d*x]
^9)/(315*d)
```

fricas [B] time = 0.58, size = 324, normalized size = 3.31

$$\frac{35\,b^3\sinh\left(dx+c\right)^9+45\left(28\,b^3\cosh\left(dx+c\right)^2+12\,ab^2-3\,b^3\right)\sinh\left(dx+c\right)^7+63\left(70\,b^3\cosh\left(dx+c\right)^4+48\,a\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^3*sinh(d*x + c)^9 + 45*(28*b^3*cosh(d*x + c)^2 + 12*a*b^2 - 3
*b^3)*sinh(d*x + c)^7 + 63*(70*b^3*cosh(d*x + c)^4 + 48*a^2*b - 12*a*b^2 +
45*(4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 105*(28*b^3*cosh(d*x
+ c)^6 + 45*(4*a*b^2 - b^3)*cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b - 36*a*b^2
+ 8*b^3 + 72*(4*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 315*(b^3*
cosh(d*x + c)^8 + 3*(4*a*b^2 - b^3)*cosh(d*x + c)^6 + 12*(4*a^2*b - a*b^2)*
cosh(d*x + c)^4 + 192*a^3 - 96*a^2*b + 36*a*b^2 - 6*b^3 + 4*(16*a^3 + 12*a^
2*b - 9*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/d
```

giac [B] time = 0.19, size = 286, normalized size = 2.92

$$\frac{b^3e^{(9\,dx+9\,c)}}{4608\,d}-\frac{b^3e^{(-9\,dx-9\,c)}}{4608\,d}+\frac{3\left(4\,ab^2-b^3\right)e^{(7\,dx+7\,c)}}{3584\,d}+\frac{3\left(4\,a^2b-ab^2\right)e^{(5\,dx+5\,c)}}{640\,d}+\frac{\left(16\,a^3+12\,a^2b-9\,ab^2+2\,b^3\right)e^{(3\,d\cdots}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4608*b^3*e^(9*d*x + 9*c)/d - 1/4608*b^3*e^(-9*d*x - 9*c)/d + 3/3584*(4*a*
b^2 - b^3)*e^(7*d*x + 7*c)/d + 3/640*(4*a^2*b - a*b^2)*e^(5*d*x + 5*c)/d +
1/384*(16*a^3 + 12*a^2*b - 9*a*b^2 + 2*b^3)*e^(3*d*x + 3*c)/d + 3/256*(32*a
^3 - 16*a^2*b + 6*a*b^2 - b^3)*e^(d*x + c)/d - 3/256*(32*a^3 - 16*a^2*b + 6
*a*b^2 - b^3)*e^(-d*x - c)/d - 1/384*(16*a^3 + 12*a^2*b - 9*a*b^2 + 2*b^3)*
e^(-3*d*x - 3*c)/d - 3/640*(4*a^2*b - a*b^2)*e^(-5*d*x - 5*c)/d - 3/3584*(4
*a*b^2 - b^3)*e^(-7*d*x - 7*c)/d
```

maple [B] time = 0.09, size = 209, normalized size = 2.13

$$\frac{b^3\left(\frac{\left(\sinh^5(dx+c)\right)\left(\cosh^4(dx+c)\right)}{9}-\frac{5\left(\sinh^3(dx+c)\right)\left(\cosh^4(dx+c)\right)}{63}+\frac{\sinh(dx+c)\left(\cosh^4(dx+c)\right)}{21}-\frac{\left(\frac{2}{3}+\frac{\left(\cosh^2(dx+c)\right)}{3}\right)\sinh(dx+c)}{21}\right)+3a\,b^2\left(\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*(1/9*sinh(d*x+c)^5*cosh(d*x+c)^4-5/63*sinh(d*x+c)^3*cosh(d*x+c)^4+
1/21*sinh(d*x+c)*cosh(d*x+c)^4-1/21*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+3*
a*b^2*(1/7*sinh(d*x+c)^3*cosh(d*x+c)^4-3/35*sinh(d*x+c)*cosh(d*x+c)^4+3/35*
(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+3*a^2*b*(1/5*sinh(d*x+c)*cosh(d*x+c)^4
-1/5*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+a^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(
d*x+c))
```

maxima [B] time = 0.41, size = 349, normalized size = 3.56

$$-\frac{1}{32256}\,b^3\left(\frac{\left(27\,e^{(-2\,dx-2\,c)}-168\,e^{(-6\,dx-6\,c)}+378\,e^{(-8\,dx-8\,c)}-7\right)e^{(9\,dx+9\,c)}}{d}-\frac{378\,e^{(-dx-c)}-168\,e^{(-3\,dx-3\,c)}+27\,e^{(-\cdots}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/32256*b^3*((27*e^(-2*d*x - 2*c) - 168*e^(-6*d*x - 6*c) + 378*e^(-8*d*x -
 8*c) - 7)*e^(9*d*x + 9*c)/d - (378*e^(-d*x - c) - 168*e^(-3*d*x - 3*c) + 2
7*e^(-7*d*x - 7*c) - 7*e^(-9*d*x - 9*c))/d) - 3/4480*a*b^2*((7*e^(-2*d*x -
2*c) + 35*e^(-4*d*x - 4*c) - 105*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d +
(105*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) - 7*e^(-5*d*x - 5*c) + 5*e^(-7*d*x
- 7*c))/d) + 1/160*a^2*b*((5*e^(-2*d*x - 2*c) - 30*e^(-4*d*x - 4*c) + 3)*e^
(5*d*x + 5*c)/d + (30*e^(-d*x - c) - 5*e^(-3*d*x - 3*c) - 3*e^(-5*d*x - 5*c
))/d) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d -
e^(-3*d*x - 3*c)/d)
```

mupad [B] time = 0.29, size = 112, normalized size = 1.14

$$\frac{105\,a^3\sinh\left(c+d\,x\right)^3+315\,a^3\sinh\left(c+d\,x\right)+189\,a^2\,b\sinh\left(c+d\,x\right)^5+315\,a^2\,b\sinh\left(c+d\,x\right)^3+135\,a\,b^2\sinh\left(c\right.}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (315*a^3*sinh(c + d*x) + 105*a^3*sinh(c + d*x)^3 + 45*b^3*sinh(c + d*x)^7 +
 35*b^3*sinh(c + d*x)^9 + 315*a^2*b*sinh(c + d*x)^3 + 189*a*b^2*sinh(c + d*
x)^5 + 189*a^2*b*sinh(c + d*x)^5 + 135*a*b^2*sinh(c + d*x)^7)/(315*d)
```

sympy [A] time = 13.84, size = 182, normalized size = 1.86

$$\begin{cases} -\frac{2a^3\sinh^3(c+dx)}{3d}+\frac{a^3\sinh(c+dx)\cosh^2(c+dx)}{d}-\frac{2a^2b\sinh^5(c+dx)}{5d}+\frac{a^2b\sinh^3(c+dx)\cosh^2(c+dx)}{d}-\frac{6ab^2\sinh^7(c+dx)}{35d}+\frac{3ab^2\sinh^5(c+d\ldots}{5\ldots} \\ x\left(a+b\sinh^2(c)\right)^3\cosh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((-2*a**3*sinh(c + d*x)**3/(3*d) + a**3*sinh(c + d*x)*cosh(c + d*x
)**2/d - 2*a**2*b*sinh(c + d*x)**5/(5*d) + a**2*b*sinh(c + d*x)**3*cosh(c +
 d*x)**2/d - 6*a*b**2*sinh(c + d*x)**7/(35*d) + 3*a*b**2*sinh(c + d*x)**5*c
osh(c + d*x)**2/(5*d) - 2*b**3*sinh(c + d*x)**9/(63*d) + b**3*sinh(c + d*x)
**7*cosh(c + d*x)**2/(7*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cosh(c)**3,
 True))
```

3.306 $\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=203

$$\frac{b\left(88a^2-68ab+15b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}+\frac{\left(64a^3-48a^2b+24ab^2-5b^3\right)\sinh(c+dx)\cosh(c+dx)}{128d}$$

```
[Out] 1/128*(64*a^3-48*a^2*b+24*a*b^2-5*b^3)*x+1/128*(64*a^3-48*a^2*b+24*a*b^2-5*
b^3)*cosh(d*x+c)*sinh(d*x+c)/d+1/192*b*(88*a^2-68*a*b+15*b^2)*cosh(d*x+c)^3
*sinh(d*x+c)/d+1/8*b*cosh(d*x+c)^7*sinh(d*x+c)*(a-(a-b)*tanh(d*x+c)^2)^2/d+
1/48*b*cosh(d*x+c)^5*sinh(d*x+c)*(a*(8*a-b)-(8*a-5*b)*(a-b)*tanh(d*x+c)^2)/
d
```

Rubi [A] time = 0.27, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3191, 413, 526, 385, 199, 206}

$$\frac{b\left(88a^2-68ab+15b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}+\frac{\left(-48a^2b+64a^3+24ab^2-5b^3\right)\sinh(c+dx)\cosh(c+dx}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*x)/128 + ((64*a^3 - 48*a^2*b + 24*a
*b^2 - 5*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (b*(88*a^2 - 68*a*b +
15*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) + (b*Cosh[c + d*x]^7*Sinh[c
+ d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(8*d) + (b*Cosh[c + d*x]^5*Sinh[c +
 d*x]*(a*(8*a - b) - (8*a - 5*b)*(a - b)*Tanh[c + d*x]^2))/(48*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{8d} - \frac{\text{Subst}}{}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{8d} + \frac{b\cosh}{}$$

$$= \frac{b\left(88a^2-68ab+15b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{192d} + \frac{b\cosh^7(c+}{}$$

$$= \frac{\left(64a^3-48a^2b+24ab^2-5b^3\right)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b\left(88a^2\right.}{}$$

$$= \frac{1}{128}\left(64a^3-48a^2b+24ab^2-5b^3\right)x + \frac{\left(64a^3-48a^2b+24ab^2-5b\right.}{12}$$

Mathematica [A] time = 0.34, size = 120, normalized size = 0.59

$$\frac{48\left(16a^3-3ab^2+b^3\right)\sinh(2(c+dx))+24b\left(12a^2-6ab+b^2\right)\sinh(4(c+dx))+24\left(64a^3-48a^2b+24ab^2-5b^3\right.}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (24*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*(c + d*x) + 48*(16*a^3 - 3*a*b^2
 + b^3)*Sinh[2*(c + d*x)] + 24*b*(12*a^2 - 6*a*b + b^2)*Sinh[4*(c + d*x)] +
 16*(3*a - b)*b^2*Sinh[6*(c + d*x)] + 3*b^3*Sinh[8*(c + d*x)])/(3072*d)
```

fricas [A] time = 0.55, size = 257, normalized size = 1.27

$$\frac{3\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+3\left(7\,b^3\cosh\left(dx+c\right)^3+4\left(3\,ab^2-b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+\left(21\,b^3\,\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^3*cosh(d*x + c)^3 + 4*(
3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^3*cosh(d*x + c)^5 + 4
0*(3*a*b^2 - b^3)*cosh(d*x + c)^3 + 12*(12*a^2*b - 6*a*b^2 + b^3)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 3*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*d*x + 3*(b
^3*cosh(d*x + c)^7 + 4*(3*a*b^2 - b^3)*cosh(d*x + c)^5 + 4*(12*a^2*b - 6*a*
b^2 + b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 3*a*b^2 + b^3)*cosh(d*x + c))*sinh
(d*x + c))/d
```

giac [A] time = 0.18, size = 231, normalized size = 1.14

$$\frac{b^3e^{(8\,dx+8\,c)}}{2048\,d}-\frac{b^3e^{(-8\,dx-8\,c)}}{2048\,d}+\frac{1}{128}\left(64\,a^3-48\,a^2b+24\,ab^2-5\,b^3\right)x+\frac{\left(3\,ab^2-b^3\right)e^{(6\,dx+6\,c)}}{384\,d}+\frac{\left(12\,a^2b-6\,ab^2+b^3\right)e}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2048*b^3*e^(8*d*x + 8*c)/d - 1/2048*b^3*e^(-8*d*x - 8*c)/d + 1/128*(64*a^
3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*x + 1/384*(3*a*b^2 - b^3)*e^(6*d*x + 6*c)/
d + 1/256*(12*a^2*b - 6*a*b^2 + b^3)*e^(4*d*x + 4*c)/d + 1/128*(16*a^3 - 3*
a*b^2 + b^3)*e^(2*d*x + 2*c)/d - 1/128*(16*a^3 - 3*a*b^2 + b^3)*e^(-2*d*x -
 2*c)/d - 1/256*(12*a^2*b - 6*a*b^2 + b^3)*e^(-4*d*x - 4*c)/d - 1/384*(3*a*
b^2 - b^3)*e^(-6*d*x - 6*c)/d
```

maple [A] time = 0.08, size = 216, normalized size = 1.06

$$\frac{b^3\left(\frac{\left(\sinh^5(dx+c)\right)\left(\cosh^3(dx+c)\right)}{8}-\frac{5\left(\sinh^3(dx+c)\right)\left(\cosh^3(dx+c)\right)}{48}+\frac{5\sinh(dx+c)\left(\cosh^3(dx+c)\right)}{64}-\frac{5\cosh(dx+c)\sinh(dx+c)}{128}-\frac{5dx}{128}-\frac{5c}{128}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*(1/8*sinh(d*x+c)^5*cosh(d*x+c)^3-5/48*sinh(d*x+c)^3*cosh(d*x+c)^3+
5/64*sinh(d*x+c)*cosh(d*x+c)^3-5/128*cosh(d*x+c)*sinh(d*x+c)-5/128*d*x-5/12
8*c)+3*a*b^2*(1/6*sinh(d*x+c)^3*cosh(d*x+c)^3-1/8*sinh(d*x+c)*cosh(d*x+c)^3
+1/16*cosh(d*x+c)*sinh(d*x+c)+1/16*d*x+1/16*c)+3*a^2*b*(1/4*sinh(d*x+c)*cos
h(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+a^3*(1/2*cosh(d*x+c)*
sinh(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.40, size = 287, normalized size = 1.41

$$\frac{1}{8}\,a^3\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{6144}\,b^3\left(\frac{\left(16\,e^{(-2\,dx-2\,c)}-24\,e^{(-4\,dx-4\,c)}-48\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}+\frac{240\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/6144*b^3*((16*e^
(-2*d*x - 2*c) - 24*e^(-4*d*x - 4*c) - 48*e^(-6*d*x - 6*c) - 3)*e^(8*d*x +
8*c)/d + 240*(d*x + c)/d + (48*e^(-2*d*x - 2*c) + 24*e^(-4*d*x - 4*c) - 16*
e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c))/d) - 1/128*a*b^2*((3*e^(-2*d*x - 2*c
) + 3*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d - (3*e^(-2*d
*x - 2*c) + 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d) - 3/64*a^2*b*(8*(d*x
+ c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

mupad [B] time = 0.42, size = 166, normalized size = 0.82

$$\frac{96\,a^3\sinh\left(2\,c+2\,d\,x\right)+6\,b^3\sinh\left(2\,c+2\,d\,x\right)+3\,b^3\sinh\left(4\,c+4\,d\,x\right)-2\,b^3\sinh\left(6\,c+6\,d\,x\right)+\frac{3\,b^3\sinh(8\,c+8\,d\,x)}{8}-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (96*a^3*sinh(2*c + 2*d*x) + 6*b^3*sinh(2*c + 2*d*x) + 3*b^3*sinh(4*c + 4*d*
x) - 2*b^3*sinh(6*c + 6*d*x) + (3*b^3*sinh(8*c + 8*d*x))/8 - 18*a*b^2*sinh(
2*c + 2*d*x) - 18*a*b^2*sinh(4*c + 4*d*x) + 36*a^2*b*sinh(4*c + 4*d*x) + 6*
a*b^2*sinh(6*c + 6*d*x) + 192*a^3*d*x - 15*b^3*d*x + 72*a*b^2*d*x - 144*a^2
*b*d*x)/(384*d)
```

sympy [A] time = 9.88, size = 559, normalized size = 2.75

$$\begin{cases} -\frac{a^3x\sinh^2(c+dx)}{2}+\frac{a^3x\cosh^2(c+dx)}{2}+\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3a^2bx\sinh^4(c+dx)}{8}+\frac{3a^2bx\sinh^2(c+dx)\cosh^2(c+dx)}{4}-\frac{3a^2bx\cosh^4(\cdots}{8}\cdots \\ x\left(a+b\sinh^2(c)\right)^3\cosh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((-a**3*x*sinh(c + d*x)**2/2 + a**3*x*cosh(c + d*x)**2/2 + a**3*si
nh(c + d*x)*cosh(c + d*x)/(2*d) - 3*a**2*b*x*sinh(c + d*x)**4/8 + 3*a**2*b*
x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - 3*a**2*b*x*cosh(c + d*x)**4/8 + 3*a
**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 3*a**2*b*sinh(c + d*x)*cosh(c
+ d*x)**3/(8*d) - 3*a*b**2*x*sinh(c + d*x)**6/16 + 9*a*b**2*x*sinh(c + d*x)
**4*cosh(c + d*x)**2/16 - 9*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 +
 3*a*b**2*x*cosh(c + d*x)**6/16 + 3*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(
16*d) + a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) - 3*a*b**2*sinh(c +
d*x)*cosh(c + d*x)**5/(16*d) - 5*b**3*x*sinh(c + d*x)**8/128 + 5*b**3*x*sin
h(c + d*x)**6*cosh(c + d*x)**2/32 - 15*b**3*x*sinh(c + d*x)**4*cosh(c + d*x
)**4/64 + 5*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 - 5*b**3*x*cosh(c +
 d*x)**8/128 + 5*b**3*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 73*b**3*sinh
(c + d*x)**5*cosh(c + d*x)**3/(384*d) - 55*b**3*sinh(c + d*x)**3*cosh(c + d
*x)**5/(384*d) + 5*b**3*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)),
(x*(a + b*sinh(c)**2)**3*cosh(c)**2, True))
```

3.307 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=67

$$\frac{a^3\sinh(c+dx)}{d}+\frac{a^2b\sinh^3(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^7(c+dx)}{7d}$$

```
[Out] a^3*sinh(d*x+c)/d+a^2*b*sinh(d*x+c)^3/d+3/5*a*b^2*sinh(d*x+c)^5/d+1/7*b^3*s
inh(d*x+c)^7/d
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3190, 194}

$$\frac{a^2b\sinh^3(c+dx)}{d}+\frac{a^3\sinh(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Sinh[c + d*x])/d + (a^2*b*Sinh[c + d*x]^3)/d + (3*a*b^2*Sinh[c + d*x]^
5)/(5*d) + (b^3*Sinh[c + d*x]^7)/(7*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)^3\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a^3\sinh(c+dx)}{d}+\frac{a^2b\sinh^3(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\text{s}}{}\end{aligned}$$

Mathematica [A] time = 0.11, size = 59, normalized size = 0.88

$$\frac{a^3\sinh(c+dx)+a^2b\sinh^3(c+dx)+\frac{3}{5}ab^2\sinh^5(c+dx)+\frac{1}{7}b^3\sinh^7(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Sinh[c + d*x] + a^2*b*Sinh[c + d*x]^3 + (3*a*b^2*Sinh[c + d*x]^5)/5 +
(b^3*Sinh[c + d*x]^7)/7)/d
```

fricas [B] time = 1.40, size = 209, normalized size = 3.12

$$\frac{5\,b^3\sinh\left(dx+c\right)^7+7\left(15\,b^3\cosh\left(dx+c\right)^2+12\,ab^2-5\,b^3\right)\sinh\left(dx+c\right)^5+35\left(5\,b^3\cosh\left(dx+c\right)^4+16\,a^2b-12}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b^3*sinh(d*x + c)^7 + 7*(15*b^3*cosh(d*x + c)^2 + 12*a*b^2 - 5*b^
3)*sinh(d*x + c)^5 + 35*(5*b^3*cosh(d*x + c)^4 + 16*a^2*b - 12*a*b^2 + 3*b^
3 + 2*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 35*(b^3*cosh(d*
x + c)^6 + (12*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 64*a^3 - 48*a^2*b + 24*a*b^
2 - 5*b^3 + 3*(16*a^2*b - 12*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))
/d
```

giac [B] time = 0.19, size = 222, normalized size = 3.31

$$\frac{b^3e^{(7\,dx+7\,c)}}{896\,d}-\frac{b^3e^{(-7\,dx-7\,c)}}{896\,d}+\frac{\left(12\,ab^2-5\,b^3\right)e^{(5\,dx+5\,c)}}{640\,d}+\frac{\left(16\,a^2b-12\,ab^2+3\,b^3\right)e^{(3\,dx+3\,c)}}{128\,d}+\frac{\left(64\,a^3-48\,a^2b+24\,ab^2-\right.}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/896*b^3*e^(7*d*x + 7*c)/d - 1/896*b^3*e^(-7*d*x - 7*c)/d + 1/640*(12*a*b^
2 - 5*b^3)*e^(5*d*x + 5*c)/d + 1/128*(16*a^2*b - 12*a*b^2 + 3*b^3)*e^(3*d*x
 + 3*c)/d + 1/128*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*e^(d*x + c)/d - 1/
128*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*e^(-d*x - c)/d - 1/128*(16*a^2*b
 - 12*a*b^2 + 3*b^3)*e^(-3*d*x - 3*c)/d - 1/640*(12*a*b^2 - 5*b^3)*e^(-5*d*
x - 5*c)/d
```

maple [A] time = 0.04, size = 56, normalized size = 0.84

$$\frac{\frac{b^3\left(\sinh^7\left(dx+c\right)\right)}{7}+\frac{3a\,b^2\left(\sinh^5\left(dx+c\right)\right)}{5}+a^2b\left(\sinh^3\left(dx+c\right)\right)+a^3\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(1/7*b^3*sinh(d*x+c)^7+3/5*a*b^2*sinh(d*x+c)^5+a^2*b*sinh(d*x+c)^3+a^3*
sinh(d*x+c))
```

maxima [A] time = 0.30, size = 63, normalized size = 0.94

$$\frac{b^3\sinh\left(dx+c\right)^7}{7\,d}+\frac{3\,ab^2\sinh\left(dx+c\right)^5}{5\,d}+\frac{a^2b\sinh\left(dx+c\right)^3}{d}+\frac{a^3\sinh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*sinh(d*x + c)^7/d + 3/5*a*b^2*sinh(d*x + c)^5/d + a^2*b*sinh(d*x +
c)^3/d + a^3*sinh(d*x + c)/d
```

mupad [B] time = 0.15, size = 58, normalized size = 0.87

$$\frac{\sinh\left(c+d\,x\right)\,\left(35\,a^3+35\,a^2\,b\,\sinh\left(c+d\,x\right)^2+21\,a\,b^2\,\sinh\left(c+d\,x\right)^4+5\,b^3\,\sinh\left(c+d\,x\right)^6\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] (sinh(c + d*x)*(35*a^3 + 5*b^3*sinh(c + d*x)^6 + 35*a^2*b*sinh(c + d*x)^2 +
 21*a*b^2*sinh(c + d*x)^4))/(35*d)
```

sympy [A] time = 4.97, size = 75, normalized size = 1.12

$$\begin{cases} \frac{a^3 \sinh{(c+dx)}}{d} + \frac{a^2 b \sinh^3{(c+dx)}}{d} + \frac{3ab^2 \sinh^5{(c+dx)}}{5d} + \frac{b^3 \sinh^7{(c+dx)}}{7d} & \text{for } d \neq 0 \\ x \left(a + b \sinh^2{(c)}\right)^3 \cosh{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)/d + a**2*b*sinh(c + d*x)**3/d + 3*a*b**2*sinh
(c + d*x)**5/(5*d) + b**3*sinh(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a + b*sinh
(c)**2)**3*cosh(c), True))
```

3.308 $\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=86

$$\frac{b\left(3a^2-3ab+b^2\right)\sinh(c+dx)}{d}+\frac{b^2(3a-b)\sinh^3(c+dx)}{3d}+\frac{(a-b)^3\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^3\sinh^5(c+dx)}{5d}$$

```
[Out] (a-b)^3*arctan(sinh(d*x+c))/d+b*(3*a^2-3*a*b+b^2)*sinh(d*x+c)/d+1/3*(3*a-b)
*b^2*sinh(d*x+c)^3/d+1/5*b^3*sinh(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 390, 203}

$$\frac{b\left(3a^2-3ab+b^2\right)\sinh(c+dx)}{d}+\frac{b^2(3a-b)\sinh^3(c+dx)}{3d}+\frac{(a-b)^3\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^3\sinh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - b)^3*ArcTan[Sinh[c + d*x]])/d + (b*(3*a^2 - 3*a*b + b^2)*Sinh[c + d*x
])/d + ((3*a - b)*b^2*Sinh[c + d*x]^3)/(3*d) + (b^3*Sinh[c + d*x]^5)/(5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(b\left(3a^2-3ab+b^2\right)+(3a-b)b^2x^2+b^3x^4+\frac{(a-b)^3}{1+x^2}\right)dx,x,s\right.}{d}\\ &= \frac{b\left(3a^2-3ab+b^2\right)\sinh(c+dx)}{d}+\frac{(3a-b)b^2\sinh^3(c+dx)}{3d}+\frac{b^3\sin}{}\\ &= \frac{(a-b)^3\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\left(3a^2-3ab+b^2\right)\sinh(c+dx)}{d}+\frac{(3}{}\end{aligned}$$

Mathematica [A] time = 0.55, size = 100, normalized size = 1.16

$$\frac{\sinh(c+dx)\left(b\left(45a^2+15ab\left(\sinh^2(c+dx)-3\right)+b^2\left(3\sinh^4(c+dx)-5\sinh^2(c+dx)+15\right)\right)+\frac{15(a-b)^3\tanh^{}}{\sqrt{-\sinh}}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Sinh[c + d*x]*((15*(a - b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c
 + d*x]^2] + b*(45*a^2 + 15*a*b*(-3 + Sinh[c + d*x]^2) + b^2*(15 - 5*Sinh[c
 + d*x]^2 + 3*Sinh[c + d*x]^4))))/(15*d)
```

fricas [B] time = 1.47, size = 1114, normalized size = 12.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^10 + 30*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^
3*sinh(d*x + c)^10 + 5*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^8 + 5*(27*b^3*cosh(
d*x + c)^2 + 12*a*b^2 - 7*b^3)*sinh(d*x + c)^8 + 40*(9*b^3*cosh(d*x + c)^3
+ (12*a*b^2 - 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(24*a^2*b - 30*a*b
^2 + 11*b^3)*cosh(d*x + c)^6 + 10*(63*b^3*cosh(d*x + c)^4 + 72*a^2*b - 90*a
*b^2 + 33*b^3 + 14*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*
(189*b^3*cosh(d*x + c)^5 + 70*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^3 + 45*(24*a
^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 30*(24*a^2*b - 3
0*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 10*(63*b^3*cosh(d*x + c)^6 + 35*(12*a*b
^2 - 7*b^3)*cosh(d*x + c)^4 - 72*a^2*b + 90*a*b^2 - 33*b^3 + 45*(24*a^2*b -
 30*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(9*b^3*cosh(d*x +
 c)^7 + 7*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^5 + 15*(24*a^2*b - 30*a*b^2 + 11
*b^3)*cosh(d*x + c)^3 - 3*(24*a^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c))*sin
h(d*x + c)^3 - 3*b^3 - 5*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^2 + 5*(27*b^3*cos
h(d*x + c)^8 + 28*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^6 + 90*(24*a^2*b - 30*a*
b^2 + 11*b^3)*cosh(d*x + c)^4 - 12*a*b^2 + 7*b^3 - 36*(24*a^2*b - 30*a*b^2
+ 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 960*((a^3 - 3*a^2*b + 3*a*b^2
- b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^4*
sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^3*sinh(d*x
 + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh(d*x + c)^
3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (a^3
- 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*arctan(cosh(d*x + c) + sinh(d*x
 + c)) + 10*(3*b^3*cosh(d*x + c)^9 + 4*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^7 +
 18*(24*a^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c)^5 - 12*(24*a^2*b - 30*a*b^
2 + 11*b^3)*cosh(d*x + c)^3 - (12*a*b^2 - 7*b^3)*cosh(d*x + c))*sinh(d*x +
c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x
+ c)^3*sinh(d*x + c)^2 + 10*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*
x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5)
```

giac [B] time = 0.21, size = 204, normalized size = 2.37

$$3\,b^3e^{(5\,dx+5\,c)}+60\,ab^2e^{(3\,dx+3\,c)}-35\,b^3e^{(3\,dx+3\,c)}+720\,a^2be^{(dx+c)}-900\,ab^2e^{(dx+c)}+330\,b^3e^{(dx+c)}+960\left(a^3-3\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^3*e^(5*d*x + 5*c) + 60*a*b^2*e^(3*d*x + 3*c) - 35*b^3*e^(3*d*x +
 3*c) + 720*a^2*b*e^(d*x + c) - 900*a*b^2*e^(d*x + c) + 330*b^3*e^(d*x + c)
 + 960*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*arctan(e^(d*x + c)) - (720*a^2*b*e^(
4*d*x + 4*c) - 900*a*b^2*e^(4*d*x + 4*c) + 330*b^3*e^(4*d*x + 4*c) + 60*a*b
^2*e^(2*d*x + 2*c) - 35*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-5*d*x - 5*c))/d
```

maple [A] time = 0.10, size = 155, normalized size = 1.80

$$\frac{2a^3\arctan\left(e^{dx+c}\right)}{d}+\frac{3a^2b\sinh\left(dx+c\right)}{d}-\frac{6a^2b\arctan\left(e^{dx+c}\right)}{d}+\frac{a\,b^2\left(\sinh^3\left(dx+c\right)\right)}{d}-\frac{3a\,b^2\sinh\left(dx+c\right)}{d}+\frac{6a\,b^2\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 2/d*a^3*arctan(exp(d*x+c))+3/d*a^2*b*sinh(d*x+c)-6/d*a^2*b*arctan(exp(d*x+c
))+1/d*a*b^2*sinh(d*x+c)^3-3/d*a*b^2*sinh(d*x+c)+6/d*a*b^2*arctan(exp(d*x+c
))+1/5*b^3*sinh(d*x+c)^5/d-1/3*b^3*sinh(d*x+c)^3/d+b^3*sinh(d*x+c)/d-2/d*b^
3*arctan(exp(d*x+c))
```

maxima [B] time = 0.44, size = 233, normalized size = 2.71

$$-\frac{1}{480}b^3\left(\frac{\left(35\,e^{(-2\,dx-2\,c)}-330\,e^{(-4\,dx-4\,c)}-3\right)e^{(5\,dx+5\,c)}}{d}+\frac{330\,e^{(-dx-c)}-35\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}}{d}-\frac{960\,\arctan\left(e\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/480*b^3*((35*e^(-2*d*x - 2*c) - 330*e^(-4*d*x - 4*c) - 3)*e^(5*d*x + 5*c
)/d + (330*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/d - 960
*arctan(e^(-d*x - c))/d) - 1/8*a*b^2*((15*e^(-2*d*x - 2*c) - 1)*e^(3*d*x +
3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arctan(e^(-d*x - c))/d
) + 3/2*a^2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) +
 a^3*arctan(sinh(d*x + c))/d
```

mupad [B] time = 1.04, size = 294, normalized size = 3.42

$$\frac{e^{c+d\,x}\,\left(24\,a^2\,b-30\,a\,b^2+11\,b^3\right)}{16\,d}-\frac{e^{-c-d\,x}\,\left(24\,a^2\,b-30\,a\,b^2+11\,b^3\right)}{16\,d}+\frac{2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^c\,\left(a^3\,\sqrt{d^2}-b^3\,\sqrt{d^2}+3\,a\,b^2\,\sqrt{d^2}-3\,a^2\,b\,\sqrt{d}\right.}{d\,\sqrt{a^6-6\,a^5\,b+15\,a^4\,b^2-20\,a^3\,b^3+15\,a^2\,b^4-6\,a\,b^5}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x),x)
```

```
[Out] (exp(c + d*x)*(24*a^2*b - 30*a*b^2 + 11*b^3))/(16*d) - (exp(- c - d*x)*(24*
a^2*b - 30*a*b^2 + 11*b^3))/(16*d) + (2*atan((exp(d*x)*exp(c)*(a^3*(d^2)^(1
/2) - b^3*(d^2)^(1/2) + 3*a*b^2*(d^2)^(1/2) - 3*a^2*b*(d^2)^(1/2)))/(d*(a^6
 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2)))*
(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2
))/(d^2)^(1/2) - (b^3*exp(- 5*c - 5*d*x))/(160*d) + (b^3*exp(5*c + 5*d*x))/
(160*d) - (b^2*exp(- 3*c - 3*d*x)*(12*a - 7*b))/(96*d) + (b^2*exp(3*c + 3*d
*x)*(12*a - 7*b))/(96*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sinh^2\left(c+dx\right)\right)^3\operatorname{sech}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)**3*sech(c + d*x), x)
```

3.309 $\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=92

$$\frac{3}{8}bx\left(8a^2-12ab+5b^2\right)+\frac{3b^2(4a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{(a-b)^3\tanh(c+dx)}{d}+\frac{b^3\sinh(c+dx)\cosh}{4d}$$

```
[Out] 3/8*b*(8*a^2-12*a*b+5*b^2)*x+3/8*(4*a-3*b)*b^2*cosh(d*x+c)*sinh(d*x+c)/d+1/
4*b^3*cosh(d*x+c)^3*sinh(d*x+c)/d+(a-b)^3*tanh(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3191, 390, 1157, 385, 206}

$$\frac{3}{8}bx\left(8a^2-12ab+5b^2\right)+\frac{3b^2(4a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{(a-b)^3\tanh(c+dx)}{d}+\frac{b^3\sinh(c+dx)\cosh}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*b*(8*a^2 - 12*a*b + 5*b^2)*x)/8 + (3*(4*a - 3*b)*b^2*Cosh[c + d*x]*Sinh[
c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + ((a - b)^3*Ta
nh[c + d*x])/d
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
```

```
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\operatorname{Subst}\left(\int\left((a-b)^3+\frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{\left(1-x^2\right)^3}\right)dx,x,\tan\right.}{d}$$

$$=\frac{(a-b)^3\tanh(c+dx)}{d}+\frac{\operatorname{Subst}\left(\int\frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2b}{\left(1-x^2\right)^3}\right.}{d}$$

$$=\frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{(a-b)^3\tanh(c+dx)}{d}-\frac{\operatorname{Subst}\left(\int\right.}{}$$

$$=\frac{3(4a-3b)b^2\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^3\cosh^3(c+dx)\sinh(c}{4d}$$

$$=\frac{3}{8}b\left(8a^2-12ab+5b^2\right)x+\frac{3(4a-3b)b^2\cosh(c+dx)\sinh(c+dx)}{8d}$$

Mathematica [A] time = 0.53, size = 78, normalized size = 0.85

$$\frac{12b\left(8a^2-12ab+5b^2\right)(c+dx)+8b^2(3a-2b)\sinh(2(c+dx))+32(a-b)^3\tanh(c+dx)+b^3\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (12*b*(8*a^2 - 12*a*b + 5*b^2)*(c + d*x) + 8*(3*a - 2*b)*b^2*Sinh[2*(c + d*
x)] + b^3*Sinh[4*(c + d*x)] + 32*(a - b)^3*Tanh[c + d*x])/(32*d)
```

fricas [B] time = 1.18, size = 178, normalized size = 1.93

$$\frac{b^3\sinh(dx+c)^5+\left(10\,b^3\cosh(dx+c)^2+24\,ab^2-15\,b^3\right)\sinh(dx+c)^3-8\left(8\,a^3-24\,a^2b+24\,ab^2-8\,b^3-3\left(8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(b^3*sinh(d*x + c)^5 + (10*b^3*cosh(d*x + c)^2 + 24*a*b^2 - 15*b^3)*si
nh(d*x + c)^3 - 8*(8*a^3 - 24*a^2*b + 24*a*b^2 - 8*b^3 - 3*(8*a^2*b - 12*a*
b^2 + 5*b^3)*d*x)*cosh(d*x + c) + (5*b^3*cosh(d*x + c)^4 + 64*a^3 - 192*a^2
*b + 216*a*b^2 - 80*b^3 + 9*(8*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
))/(d*cosh(d*x + c))
```

giac [B] time = 0.21, size = 197, normalized size = 2.14

$$\frac{b^3e^{(4\,dx+4\,c)}+24\,ab^2e^{(2\,dx+2\,c)}-16\,b^3e^{(2\,dx+2\,c)}+24\left(8\,a^2b-12\,ab^2+5\,b^3\right)(dx+c)-\left(144\,a^2be^{(4\,dx+4\,c)}-216\,ab^2e^{(4}\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 16*b^3*e^(2*d*x + 2*
c) + 24*(8*a^2*b - 12*a*b^2 + 5*b^3)*(d*x + c) - (144*a^2*b*e^(4*d*x + 4*c)
 - 216*a*b^2*e^(4*d*x + 4*c) + 90*b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*x +
 2*c) - 16*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c) - 128*(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)/(e^(2*d*x + 2*c) + 1))/d
```

maple [A] time = 0.09, size = 131, normalized size = 1.42

$$\frac{a^3 \tanh(dx+c) + 3a^2 b\,(dx+c-\tanh(dx+c)) + 3a\,b^2\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)} - \frac{3dx}{2} - \frac{3c}{2} + \frac{3\tanh(dx+c)}{2}\right) + b^3\left(\frac{\sinh^5(dx+c)}{4\cosh(dx+c)}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*tanh(d*x+c)+3*a^2*b*(d*x+c-tanh(d*x+c))+3*a*b^2*(1/2*sinh(d*x+c)^3
/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^3*(1/4*sinh(d*x+c)^5/cosh(d*x
+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+15/8*c-15/8*tanh(d*x+c)))
```

maxima [B] time = 0.35, size = 215, normalized size = 2.34

$$3\,a^2 b\left(x + \frac{c}{d} - \frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right) + \frac{1}{64}\,b^3\left(\frac{120\,(dx+c)}{d} + \frac{16\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)}}{d} - \frac{15\,e^{(-2\,dx-2\,c)} + 144\,e^{(-4\,dx-}}{d\left(e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3*a^2*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 1/64*b^3*(120*(d*x + c)/
d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144
*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)))) - 3/8*a*b
^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(
-2*d*x - 2*c) + e^(-4*d*x - 4*c)))) + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

mupad [B] time = 0.98, size = 141, normalized size = 1.53

$$\frac{b^3\,\mathrm{e}^{4\,c+4\,d\,x}}{64\,d} - \frac{b^3\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,d} - \frac{2\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} + \frac{3\,b\,x\,\left(8\,a^2-12\,a\,b+5\,b^2\right)}{8} - \frac{b^2\,\mathrm{e}^{-2\,c-2\,d\,x}\,(3\,a-2\,b)}{8\,d} + \frac{b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^2,x)
```

```
[Out] (b^3*exp(4*c + 4*d*x))/(64*d) - (b^3*exp(- 4*c - 4*d*x))/(64*d) - (2*(3*a*b
^2 - 3*a^2*b + a^3 - b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (3*b*x*(8*a^2 - 12*
a*b + 5*b^2))/8 - (b^2*exp(- 2*c - 2*d*x)*(3*a - 2*b))/(8*d) + (b^2*exp(2*c
 + 2*d*x)*(3*a - 2*b))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.310 $\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=91

$$\frac{b^2(3a-2b)\sinh(c+dx)}{d}+\frac{(a+5b)(a-b)^2\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)^3\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}+\frac{b^3\sinh^3(c+$$

```
[Out] 1/2*(a-b)^2*(a+5*b)*arctan(sinh(d*x+c))/d+(3*a-2*b)*b^2*sinh(d*x+c)/d+1/3*b
^3*sinh(d*x+c)^3/d+1/2*(a-b)^3*sech(d*x+c)*tanh(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 390, 385, 203}

$$\frac{b^2(3a-2b)\sinh(c+dx)}{d}+\frac{(a+5b)(a-b)^2\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)^3\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}+\frac{b^3\sinh^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - b)^2*(a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((3*a - 2*b)*b^2*Sinh[c
 + d*x])/d + (b^3*Sinh[c + d*x]^3)/(3*d) + ((a - b)^3*Sech[c + d*x]*Tanh[c
+ d*x])/(2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{\left(1+x^2\right)^2}dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left((3a-2b)b^2+b^3x^2+\frac{(a-b)^2(a+2b)+3(a-b)^2bx^2}{\left(1+x^2\right)^2}\right)dx, x, \sinh\right.}{d}$$

$$= \frac{(3a-2b)b^2\sinh(c+dx)}{d}+\frac{b^3\sinh^3(c+dx)}{3d}+\frac{\text{Subst}\left(\int \frac{(a-b)^2(a-}{(}\right.}{}$$

$$= \frac{(3a-2b)b^2\sinh(c+dx)}{d}+\frac{b^3\sinh^3(c+dx)}{3d}+\frac{(a-b)^3\operatorname{sech}(c+}{2}$$

$$= \frac{(a-b)^2(a+5b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(3a-2b)b^2\sinh(c+dx)}{d}$$

Mathematica [C] time = 7.99, size = 399, normalized size = 4.38

$$\frac{\operatorname{csch}^5(c+dx)\left(-256\sinh^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 {}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)+21\left(a^3\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}
, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3 + 21*(3*a^2*b
*Sinh[c + d*x]^2*(36015 + 16120*Sinh[c + d*x]^2 + 753*Sinh[c + d*x]^4) + b^
3*Sinh[c + d*x]^6*(32415 + 17320*Sinh[c + d*x]^2 + 753*Sinh[c + d*x]^4) + 3
*a*b^2*Sinh[c + d*x]^4*(36015 + 18280*Sinh[c + d*x]^2 + 753*Sinh[c + d*x]^4
) + a^3*(36015 + 16120*Sinh[c + d*x]^2 + 1473*Sinh[c + d*x]^4)) - (315*ArcT
anh[Sqrt[-Sinh[c + d*x]^2]]*(a^3*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c
+ d*x]^4 - 47*Sinh[c + d*x]^6) + 3*a^2*b*Sinh[c + d*x]^2*(2401 + 1875*Sinh[
c + d*x]^2 + 195*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + b^3*Sinh[c + d*x]^6*(
2161 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*
b^2*Sinh[c + d*x]^4*(2401 + 2019*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Si
nh[c + d*x]^6)))/Sqrt[-Sinh[c + d*x]^2]))/(30240*d)
```

fricas [B] time = 0.67, size = 1679, normalized size = 18.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sin
h(d*x + c)^10 + (36*a*b^2 - 25*b^3)*cosh(d*x + c)^8 + (45*b^3*cosh(d*x + c)
^2 + 36*a*b^2 - 25*b^3)*sinh(d*x + c)^8 + 8*(15*b^3*cosh(d*x + c)^3 + (36*a
*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(12*a^3 - 36*a^2*b + 54*a
*b^2 - 25*b^3)*cosh(d*x + c)^6 + 2*(105*b^3*cosh(d*x + c)^4 + 12*a^3 - 36*a
^2*b + 54*a*b^2 - 25*b^3 + 14*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^6 + 4*(63*b^3*cosh(d*x + c)^5 + 14*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^
3 + 3*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^
5 - 2*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)^4 + 2*(105*b^3*
```

```
cosh(d*x + c)^6 + 35*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^4 - 12*a^3 + 36*a^2*
b - 54*a*b^2 + 25*b^3 + 15*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 8*(15*b^3*cosh(d*x + c)^7 + 7*(36*a*b^2 - 25*b^3
)*cosh(d*x + c)^5 + 5*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)
^3 - (12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
 - b^3 - (36*a*b^2 - 25*b^3)*cosh(d*x + c)^2 + (45*b^3*cosh(d*x + c)^8 + 28
*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^6 + 30*(12*a^3 - 36*a^2*b + 54*a*b^2 - 2
5*b^3)*cosh(d*x + c)^4 - 36*a*b^2 + 25*b^3 - 12*(12*a^3 - 36*a^2*b + 54*a*b
^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*((a^3 + 3*a^2*b - 9*a*b^
2 + 5*b^3)*cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x +
 c)*sinh(d*x + c)^6 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh(d*x + c)^7 + 2
*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + (2*a^3 + 6*a^2*b - 18*
a*b^2 + 10*b^3 + 21*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^5 + 5*(7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 2*(a
^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + (a^3 + 3*a
^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (35*(a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3 + 20*(a^3 + 3*a^2*b
- 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(a^3 + 3*a^2*b -
9*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cos
h(d*x + c)^3 + 3*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^2 + (7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(a^3 + 3
*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 3*(a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c))
+ 2*(5*b^3*cosh(d*x + c)^9 + 4*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^7 + 6*(12*
a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)^5 - 4*(12*a^3 - 36*a^2*b
+ 54*a*b^2 - 25*b^3)*cosh(d*x + c)^3 - (36*a*b^2 - 25*b^3)*cosh(d*x + c))*s
inh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*si
nh(d*x + c)^7 + 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x
 + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*c
osh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^3 + (21*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))
*sinh(d*x + c)^2 + (7*d*cosh(d*x + c)^6 + 10*d*cosh(d*x + c)^4 + 3*d*cosh(d
*x + c)^2)*sinh(d*x + c))
```

giac [B] time = 0.24, size = 247, normalized size = 2.71

$$\frac{b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 36\,ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) - 24\,b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 6\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-c}\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 36*a*b^2*(e^(d*x + c) - e^(-d*x
- c)) - 24*b^3*(e^(d*x + c) - e^(-d*x - c)) + 6*(pi + 2*arctan(1/2*(e^(2*d*
x + 2*c) - 1)*e^(-d*x - c)))*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3) + 24*(a^3*(e
^(d*x + c) - e^(-d*x - c)) - 3*a^2*b*(e^(d*x + c) - e^(-d*x - c)) + 3*a*b^2
*(e^(d*x + c) - e^(-d*x - c)) - b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x
+ c) - e^(-d*x - c))^2 + 4))/d
```

maple [B] time = 0.12, size = 286, normalized size = 3.14

$$\frac{a^3\text{sech}\,(dx+c)\tanh\,(dx+c)}{2d} + \frac{a^3\arctan\left(\mathrm{e}^{dx+c}\right)}{d} - \frac{3a^2b\sinh\,(dx+c)}{d\cosh\,(dx+c)^2} + \frac{3a^2b\,\text{sech}\,(dx+c)\tanh\,(dx+c)}{2d} + \frac{3a^2b\,\text{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d*a^3*sech(d*x+c)*tanh(d*x+c)+1/d*a^3*arctan(exp(d*x+c))-3/d*a^2*b*sinh
(d*x+c)/cosh(d*x+c)^2+3/2/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/d*a^2*b*arctan(
exp(d*x+c))+3/d*a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^2+9/d*a*b^2*sinh(d*x+c)/cos
h(d*x+c)^2-9/2/d*a*b^2*sech(d*x+c)*tanh(d*x+c)-9/d*a*b^2*arctan(exp(d*x+c))
+1/3/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^2-5/3/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^
2-5/d*b^3*sinh(d*x+c)/cosh(d*x+c)^2+5/2/d*b^3*sech(d*x+c)*tanh(d*x+c)+5/d*b
^3*arctan(exp(d*x+c))
```

maxima [B] time = 0.56, size = 357, normalized size = 3.92

$$\frac{1}{24}\,b^3\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)}\right)}\right)+\frac{3}{2}\,ab^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))
/d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(
d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 3/2*a*b^2*
(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*
x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) -
 3*a^2*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*
e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d -
(e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)
 + 1)))
```

mupad [B] time = 2.28, size = 308, normalized size = 3.38

$$\frac{b^3\,e^{3\,c+3\,d\,x}}{24\,d}-\frac{b^3\,e^{-3\,c-3\,d\,x}}{24\,d}+\frac{3\,b^2\,e^{c+d\,x}\,(4\,a-3\,b)}{8\,d}+\frac{e^{c+d\,x}\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{2\,e^{c+d\,x}\,\left(a^3-3\,a^2\,b+3\,a\,b^2\right.}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^3,x)
```

```
[Out] (b^3*exp(3*c + 3*d*x))/(24*d) - (b^3*exp(- 3*c - 3*d*x))/(24*d) + (log(- (a
 - b)^2*(a + 5*b)*1i - exp(d*x)*exp(c)*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))*(
a - b)^2*(a + 5*b)*1i)/(2*d) - (log((a - b)^2*(a + 5*b)*1i - exp(d*x)*exp(c
)*(3*a^2*b - 9*a*b^2 + a^3 + 5*b^3))*(a - b)^2*(a + 5*b)*1i)/(2*d) + (3*b^2
*exp(c + d*x)*(4*a - 3*b))/(8*d) + (exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^3 -
 b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^
3 - b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (3*b^2*exp(- c
- d*x)*(4*a - 3*b))/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.311 $\int \text{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}b^2x(6a-5b)-\frac{(a-b)^3\tanh^3(c+dx)}{3d}+\frac{(a-b)^2(a+2b)\tanh(c+dx)}{d}+\frac{b^3\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] 1/2*(6*a-5*b)*b^2*x+1/2*b^3*cosh(d*x+c)*sinh(d*x+c)/d+(a-b)^2*(a+2*b)*tanh(
d*x+c)/d-1/3*(a-b)^3*tanh(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 390, 385, 206}

$$\frac{1}{2}b^2x(6a-5b)-\frac{(a-b)^3\tanh^3(c+dx)}{3d}+\frac{(a-b)^2(a+2b)\tanh(c+dx)}{d}+\frac{b^3\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((6*a - 5*b)*b^2*x)/2 + (b^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + ((a - b)^
2*(a + 2*b)*Tanh[c + d*x])/d - ((a - b)^3*Tanh[c + d*x]^3)/(3*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^2}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left((a-b)^2(a+2b)-(a-b)^3x^2+\frac{(3a-2b)b^2-3(a-b)b^2x^2}{\left(1-x^2\right)^2}\right)dx,\right.}{d}$$

$$= \frac{(a-b)^2(a+2b)\tanh(c+dx)}{d} - \frac{(a-b)^3\tanh^3(c+dx)}{3d} + \frac{\operatorname{Subst}}{}$$

$$= \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)^2(a+2b)\tanh(c+dx)}{d} -$$

$$= \frac{1}{2}(6a-5b)b^2x + \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)^2(a+2b}{d}$$

Mathematica [A] time = 0.76, size = 84, normalized size = 1.02

$$\frac{6b^2(6a-5b)(c+dx)+2(a-b)^2\tanh(c+dx)\operatorname{sech}^2(c+dx)((2a+7b)\cosh(2(c+dx))+4a+5b)+3b^3\sinh(2}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (6*(6*a - 5*b)*b^2*(c + d*x) + 3*b^3*Sinh[2*(c + d*x)] + 2*(a - b)^2*(4*a +
 5*b + (2*a + 7*b)*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*Tanh[c + d*x])/(12*d)
```

fricas [B] time = 0.74, size = 321, normalized size = 3.91

$$\frac{3\,b^3\sinh(dx+c)^5-4\left(4\,a^3+6\,a^2b-24\,ab^2+14\,b^3-3\left(6\,ab^2-5\,b^3\right)dx\right)\cosh(dx+c)^3-12\left(4\,a^3+6\,a^2b-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^3*sinh(d*x + c)^5 - 4*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(6
*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)^3 - 12*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*
b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*b^3*cosh
(d*x + c)^2 + 16*a^3 + 24*a^2*b - 96*a*b^2 + 65*b^3)*sinh(d*x + c)^3 - 12*(
4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c
) + 3*(5*b^3*cosh(d*x + c)^4 + 16*a^3 - 24*a^2*b + 10*b^3 + (16*a^3 + 24*a^
2*b - 96*a*b^2 + 65*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3
 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

giac [B] time = 0.23, size = 208, normalized size = 2.54

$$\frac{3\,b^3e^{(2\,dx+2\,c)}+12\left(6\,ab^2-5\,b^3\right)(dx+c)-3\left(12\,ab^2e^{(2\,dx+2\,c)}-10\,b^3e^{(2\,dx+2\,c)}+b^3\right)e^{(-2\,dx-2\,c)}-\frac{16\left(9\,a^2be^{(4\,dx+4\,c)}-18\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^3*e^(2*d*x + 2*c) + 12*(6*a*b^2 - 5*b^3)*(d*x + c) - 3*(12*a*b^2*
e^(2*d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + b^3)*e^(-2*d*x - 2*c) - 16*(9*a^
```

```
2*b*e^(4*d*x + 4*c) - 18*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 6*
a^3*e^(2*d*x + 2*c) - 18*a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 2
*a^3 + 3*a^2*b - 12*a*b^2 + 7*b^3)/(e^(2*d*x + 2*c) + 1)^3)/d
```

maple [A] time = 0.12, size = 148, normalized size = 1.80

$$\frac{a^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3a\,b^2\left(dx+c-\tanh(dx+c)-\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/2*sinh(d*x+c)/cosh
(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(d*x+c-tanh(d*x+
c)-1/3*tanh(d*x+c)^3)+b^3*(1/2*sinh(d*x+c)^5/cosh(d*x+c)^3-5/2*d*x-5/2*c+5/
2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

maxima [B] time = 0.38, size = 382, normalized size = 4.66

$$ab^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)-\frac{1}{24}b^3\left(\frac{60(dx+c)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx-2c}}{d\left(e^{(-2dx-2c)}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*
e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 1/24*b^3*
(60*(d*x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*
d*x - 4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
 4*c) + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) + 4/3*a^3*(3*e^(-2*d*x - 2
*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) +
1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2
*a^2*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-
6*d*x - 6*c) + 1)))
```

mupad [B] time = 0.16, size = 273, normalized size = 3.33

$$\frac{b^2x(6a-5b)}{2}-\frac{\frac{2(a^2b-2ab^2+b^3)}{d}+\frac{2e^{4c+4dx}(a^2b-2ab^2+b^3)}{d}+\frac{4e^{2c+2dx}(2a^3-3a^2b+b^3)}{3d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}-\frac{\frac{2(2a^3-3a^2b+b^3)}{3d}+\frac{2e^{2c+2dx}(a^2b-2ab}{d}}{2e^{2c+2dx}+e^{4c+4dx}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^4,x)
```

```
[Out] (b^2*x*(6*a - 5*b))/2 - ((2*(a^2*b - 2*a*b^2 + b^3))/d + (2*exp(4*c + 4*d*x
)*(a^2*b - 2*a*b^2 + b^3))/d + (4*exp(2*c + 2*d*x)*(2*a^3 - 3*a^2*b + b^3))
/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) -
((2*(2*a^3 - 3*a^2*b + b^3))/(3*d) + (2*exp(2*c + 2*d*x)*(a^2*b - 2*a*b^2 +
 b^3))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (b^3*exp(- 2*c - 2*
d*x))/(8*d) + (b^3*exp(2*c + 2*d*x))/(8*d) - (2*(a^2*b - 2*a*b^2 + b^3))/(d
*(exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.312 $\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{3(a-b)\left((a+b)^2+4b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(a-b)^3\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}+\frac{3(a-b)^2(a+3b)\tanh(c+dx)}{8d}$$

```
[Out] 3/8*(a-b)*(4*b^2+(a+b)^2)*arctan(sinh(d*x+c))/d+b^3*sinh(d*x+c)/d+3/8*(a-b)
^2*(a+3*b)*sech(d*x+c)*tanh(d*x+c)/d+1/4*(a-b)^3*sech(d*x+c)^3*tanh(d*x+c)/
d
```

Rubi [A] time = 0.13, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 390, 1157, 385, 203}

$$\frac{3(a-b)\left((a+b)^2+4b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(a-b)^3\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}+\frac{3(a-b)^2(a+3b)\tanh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a - b)*(4*b^2 + (a + b)^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (b^3*Sinh[c +
 d*x])/d + (3*(a - b)^2*(a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + ((a
- b)^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{(1+x^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b^3+\frac{a^3-b^3+3b(a^2-b^2)x^2+3(a-b)b^2x^4}{(1+x^2)^3}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{b^3\sinh(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{a^3-b^3+3b(a^2-b^2)x^2+3(a-b)b^2x^4}{(1+x^2)^3}\,dx, x, \sinh\right.}{d}$$

$$= \frac{b^3\sinh(c+dx)}{d} + \frac{(a-b)^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{b^3\sinh(c+dx)}{d} + \frac{3(a-b)^2(a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} +$$

$$= \frac{3(a-b)\left(4b^2+(a+b)^2\right)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{b^3\sinh(c+dx)}{d}$$

Mathematica [C] time = 10.07, size = 472, normalized size = 4.58

$$-\frac{\operatorname{csch}^5(c+dx)\left(256\sinh^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 {}_6F_5\left(\frac{3}{2},2,2,2,2,2;1,1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)+3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -1/60480*(Csch[c + d*x]^5*(256*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1,
1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3
 + 384*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]
^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^2*(7*a + 5*b*Sinh[c + d*x]^2) -
 21*(15*a*b^2*Sinh[c + d*x]^4*(36015 + 21529*Sinh[c + d*x]^2 + 1128*Sinh[c
+ d*x]^4) + 9*a^2*b*Sinh[c + d*x]^2*(72030 + 41615*Sinh[c + d*x]^2 + 2131*S
inh[c + d*x]^4) + b^3*Sinh[c + d*x]^6*(149460 + 90805*Sinh[c + d*x]^2 + 488
7*Sinh[c + d*x]^4) + a^3*(252105 + 140965*Sinh[c + d*x]^2 + 8226*Sinh[c + d
*x]^4)) + (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(a^3*(16807 + 15000*Sinh[c +
 d*x]^2 + 2187*Sinh[c + d*x]^4 - 62*Sinh[c + d*x]^6) + 9*a^2*b*Sinh[c + d*x
]^2*(4802 + 4375*Sinh[c + d*x]^2 + 640*Sinh[c + d*x]^4 + 3*Sinh[c + d*x]^6)
 + b^3*Sinh[c + d*x]^6*(9964 + 9375*Sinh[c + d*x]^2 + 1458*Sinh[c + d*x]^4
+ 7*Sinh[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]^4*(12005 + 11178*Sinh[c + d*x]
^2 + 1701*Sinh[c + d*x]^4 + 8*Sinh[c + d*x]^6)))/Sqrt[-Sinh[c + d*x]^2]))/d
```

fricas [B] time = 1.03, size = 2245, normalized size = 21.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/4*(2*b^3*cosh(d*x + c)^10 + 20*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 2*b^3*
sinh(d*x + c)^10 + 3*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 3*(3
0*b^3*cosh(d*x + c)^2 + a^3 + a^2*b - 5*a*b^2 + 5*b^3)*sinh(d*x + c)^8 + 24
*(10*b^3*cosh(d*x + c)^3 + (a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c))*s
inh(d*x + c)^7 + (11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + (4
20*b^3*cosh(d*x + c)^4 + 11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3 + 84*(a^3 + a^
2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 6*(84*b^3*cosh(d*
x + c)^5 + 28*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (11*a^3 - 2
1*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (11*a^3 - 21*a^
2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + (420*b^3*cosh(d*x + c)^6 + 210*(a^
3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - 11*a^3 + 21*a^2*b - 9*a*b^2
- 5*b^3 + 15*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(60*b^3*cosh(d*x + c)^7 + 42*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*c
osh(d*x + c)^5 + 5*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 -
(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*b^
3 - 3*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 3*(30*b^3*cosh(d*x
+ c)^8 + 28*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 5*(11*a^3 - 2
1*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - a^3 - a^2*b + 5*a*b^2 - 5*b^3
- 2*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 3*((a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^9 + 9*(a^3 + a^2*b + 3*a
*b^2 - 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + (a^3 + a^2*b + 3*a*b^2 - 5*b^
3)*sinh(d*x + c)^9 + 4*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 4*
(a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 9*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^7 + 28*(3*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x +
 c)^3 + (a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*
(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 6*(21*(a^3 + a^2*b + 3*a*
b^2 - 5*b^3)*cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 14*(a^3 + a^
2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(a^3 + a^2*
b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 70*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*c
osh(d*x + c)^3 + 15*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^4 + 4*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 4*(21*(a^3 +
a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 35*(a^3 + a^2*b + 3*a*b^2 - 5*b^
3)*cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 15*(a^3 + a^2*b + 3*a*
b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(3*(a^3 + a^2*b + 3*a*b^
2 - 5*b^3)*cosh(d*x + c)^7 + 7*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c
)^5 + 5*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b + 3*
a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^3 + a^2*b + 3*a*b^2 - 5*
b^3)*cosh(d*x + c) + (9*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^8 + 2
8*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 30*(a^3 + a^2*b + 3*a*b
^2 - 5*b^3)*cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 12*(a^3 + a^2
*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c)
+ sinh(d*x + c)) + 2*(10*b^3*cosh(d*x + c)^9 + 12*(a^3 + a^2*b - 5*a*b^2 +
5*b^3)*cosh(d*x + c)^7 + 3*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x +
 c)^5 - 2*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - 3*(a^3 +
a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^9 +
 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 4*d*cosh(d*x + c)^
7 + 4*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x + c)^3 +
 d*cosh(d*x + c))*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cosh(d*x
+ c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x + c)^
5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*d*cosh(d
*x + c)^3 + 4*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5
 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c)
 + (9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 12*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

giac [B] time = 0.23, size = 301, normalized size = 2.92

$$8\,b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 3\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(a^3 + a^2b + 3\,ab^2 - 5\,b^3\right) + \frac{4\left(3\,a^3\left(e^{(dx+c)} - e^{(-d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(8*b^3*(e^(d*x + c) - e^(-d*x - c)) + 3*(pi + 2*arctan(1/2*(e^(2*d*x +
 2*c) - 1)*e^(-d*x - c)))*(a^3 + a^2*b + 3*a*b^2 - 5*b^3) + 4*(3*a^3*(e^(d*
x + c) - e^(-d*x - c))^3 + 3*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 - 15*a*b^
2*(e^(d*x + c) - e^(-d*x - c))^3 + 9*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 2
0*a^3*(e^(d*x + c) - e^(-d*x - c)) - 12*a^2*b*(e^(d*x + c) - e^(-d*x - c))
- 36*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 28*b^3*(e^(d*x + c) - e^(-d*x - c
)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

maple [B] time = 0.14, size = 376, normalized size = 3.65

$$\frac{a^3\tanh(dx+c)\,\mathrm{sech}(dx+c)^3}{4d} + \frac{3a^3\mathrm{sech}(dx+c)\tanh(dx+c)}{8d} + \frac{3a^3\arctan\left(e^{dx+c}\right)}{4d} - \frac{a^2b\sinh(dx+c)}{d\cosh(dx+c)^4} + \frac{a^2b\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/4/d*a^3*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*a^3*sech(d*x+c)*tanh(d*x+c)+3/4/d
*a^3*arctan(exp(d*x+c))-1/d*a^2*b*sinh(d*x+c)/cosh(d*x+c)^4+1/4/d*a^2*b*tan
h(d*x+c)*sech(d*x+c)^3+3/8/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/4/d*a^2*b*arct
an(exp(d*x+c))-3/d*a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^4-3/d*a*b^2*sinh(d*x+c)/
cosh(d*x+c)^4+3/4/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^3+9/8/d*a*b^2*sech(d*x+c)
*tanh(d*x+c)+9/4/d*a*b^2*arctan(exp(d*x+c))+1/d*b^3*sinh(d*x+c)^5/cosh(d*x+
c)^4+5/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^4+5/d*b^3*sinh(d*x+c)/cosh(d*x+c)^4-
5/4/d*b^3*tanh(d*x+c)*sech(d*x+c)^3-15/8/d*b^3*sech(d*x+c)*tanh(d*x+c)-15/4
/d*b^3*arctan(exp(d*x+c))
```

maxima [B] time = 0.49, size = 489, normalized size = 4.75

$$\frac{1}{4}b^3\left(\frac{15\,\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{2\,e^{(-dx-c)}}{d} + \frac{17\,e^{(-2\,dx-2\,c)} + 13\,e^{(-4\,dx-4\,c)} + 7\,e^{(-6\,dx-6\,c)} - 7\,e^{(-8\,dx-8\,c)} + 2}{d\left(e^{(-dx-c)} + 4\,e^{(-3\,dx-3\,c)} + 6\,e^{(-5\,dx-5\,c)} + 4\,e^{(-7\,dx-7\,c)} + e^{(-9\,dx-9\,c)}\right)}\right) - \frac{3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*(15*arctan(e^(-d*x - c))/d - 2*e^(-d*x - c)/d + (17*e^(-2*d*x - 2*c
) + 13*e^(-4*d*x - 4*c) + 7*e^(-6*d*x - 6*c) - 7*e^(-8*d*x - 8*c) + 2)/(d*(
e^(-d*x - c) + 4*e^(-3*d*x - 3*c) + 6*e^(-5*d*x - 5*c) + 4*e^(-7*d*x - 7*c)
 + e^(-9*d*x - 9*c)))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x -
 c) - 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e
^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c
) + 1))) - 1/4*a^3*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d
*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c
) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3/4
*a^2*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(
-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*
c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1)))
```

mupad [B] time = 0.19, size = 430, normalized size = 4.17

$$\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a^3\,\sqrt{d^2}-5\,b^3\,\sqrt{d^2}+3\,a\,b^2\,\sqrt{d^2}+a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{a^6+2\,a^5\,b+7\,a^4\,b^2-4\,a^3\,b^3-a^2\,b^4-30\,a\,b^5+25\,b^6}}\right)\,\sqrt{a^6+2\,a^5\,b+7\,a^4\,b^2-4\,a^3\,b^3-a^2\,b^4-30\,a\,b^5+25\,b^6}}{4\,\sqrt{d^2}}+\frac{b^3\,\mathrm{e}^{c+d}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^5,x)
```

```
[Out] (3*atan((exp(d*x)*exp(c)*(a^3*(d^2)^(1/2) - 5*b^3*(d^2)^(1/2) + 3*a*b^2*(d^
2)^(1/2) + a^2*b*(d^2)^(1/2)))/(d*(2*a^5*b - 30*a*b^5 + a^6 + 25*b^6 - a^2*
b^4 - 4*a^3*b^3 + 7*a^4*b^2)^(1/2)))*(2*a^5*b - 30*a*b^5 + a^6 + 25*b^6 - a
^2*b^4 - 4*a^3*b^3 + 7*a^4*b^2)^(1/2))/(4*(d^2)^(1/2)) + (b^3*exp(c + d*x))
/(2*d) - (b^3*exp(- c - d*x))/(2*d) + (3*exp(c + d*x)*(a^2*b - 5*a*b^2 + a^
3 + 3*b^3))/(4*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(27*a*b^2 - 15*a^2
*b + a^3 - 13*b^3))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (6*
exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(d*(3*exp(2*c + 2*d*x) + 3*ex
p(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*exp(c + d*x)*(3*a*b^2 - 3*a^2*
b + a^3 - b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6
*d*x) + exp(8*c + 8*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.313 $\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=74

$$\frac{(a^3-b^3)\tanh(c+dx)}{d}+\frac{(a-b)^3\tanh^5(c+dx)}{5d}-\frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d}+b^3x$$

```
[Out] b^3*x+(a^3-b^3)*tanh(d*x+c)/d-1/3*(a-b)^2*(2*a+b)*tanh(d*x+c)^3/d+1/5*(a-b)
^3*tanh(d*x+c)^5/d
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3191, 390, 206}

$$\frac{(a^3-b^3)\tanh(c+dx)}{d}+\frac{(a-b)^3\tanh^5(c+dx)}{5d}-\frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] b^3*x + ((a^3 - b^3)*Tanh[c + d*x])/d - ((a - b)^2*(2*a + b)*Tanh[c + d*x]^
3)/(3*d) + ((a - b)^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a^3-b^3-(a-b)^2(2a+b)x^2+(a-b)^3x^4+\frac{b^3}{1-x^2}\right)dx, x\right.}{d}\\ &= \frac{(a^3-b^3)\tanh(c+dx)}{d}-\frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d}+\frac{(a-b}{}\\ &= b^3x+\frac{(a^3-b^3)\tanh(c+dx)}{d}-\frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d}+\end{aligned}$$

Mathematica [A] time = 0.77, size = 86, normalized size = 1.16

$$\frac{(a-b)\tanh(c+dx)\left(\left(4a^2+7ab-11b^2\right)\operatorname{sech}^2(c+dx)+8a^2+3(a-b)^2\operatorname{sech}^4(c+dx)+14ab+23b^2\right)+15b^3(c+\cdots}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (15*b^3*(c + d*x) + (a - b)*(8*a^2 + 14*a*b + 23*b^2 + (4*a^2 + 7*a*b - 11*
b^2)*Sech[c + d*x]^2 + 3*(a - b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(15*d)
```

fricas [B] time = 0.58, size = 530, normalized size = 7.16

$$\frac{\left(15\,b^3dx-8\,a^3-6\,a^2b-9\,ab^2+23\,b^3\right)\cosh\left(dx+c\right)^5+5\left(15\,b^3dx-8\,a^3-6\,a^2b-9\,ab^2+23\,b^3\right)\cosh\left(dx+c\right)\mathrm{s}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*((15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)^5 + 5
*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)*sinh(d*x +
 c)^4 + (8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*sinh(d*x + c)^5 + 5*(15*b^3*d*
x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)^3 + 5*(8*a^3 + 6*a^2*
b - 9*a*b^2 - 5*b^3 + 2*(8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 5*(2*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)
*cosh(d*x + c)^3 + 3*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh
(d*x + c))*sinh(d*x + c)^2 + 10*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 2
3*b^3)*cosh(d*x + c) + 5*((8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*cosh(d*x + c
)^4 + 16*a^3 - 24*a^2*b + 18*a*b^2 - 10*b^3 + 3*(8*a^3 + 6*a^2*b - 9*a*b^2
- 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x
+ c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*c
osh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

giac [B] time = 0.25, size = 213, normalized size = 2.88

$$\frac{15\,(dx+c)b^3-\frac{2\left(45\,ab^2e^{(8\,dx+8\,c)}-45\,b^3e^{(8\,dx+8\,c)}+90\,a^2be^{(6\,dx+6\,c)}-90\,b^3e^{(6\,dx+6\,c)}+80\,a^3e^{(4\,dx+4\,c)}-30\,a^2be^{(4\,dx+4\,c)}+90\,ab^2e^{(4\,dx+4\,c)}-140\,b^3e^{(4\,dx\cdots}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(d*x + c)*b^3 - 2*(45*a*b^2*e^(8*d*x + 8*c) - 45*b^3*e^(8*d*x + 8*
c) + 90*a^2*b*e^(6*d*x + 6*c) - 90*b^3*e^(6*d*x + 6*c) + 80*a^3*e^(4*d*x +
4*c) - 30*a^2*b*e^(4*d*x + 4*c) + 90*a*b^2*e^(4*d*x + 4*c) - 140*b^3*e^(4*d
*x + 4*c) + 40*a^3*e^(2*d*x + 2*c) + 30*a^2*b*e^(2*d*x + 2*c) - 70*b^3*e^(2
*d*x + 2*c) + 8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)/(e^(2*d*x + 2*c) + 1)^5)/
d
```

maple [B] time = 0.12, size = 199, normalized size = 2.69

$$\frac{a^3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh\left(dx+c\right)+3a^2b\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+3a\,b^2\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-
1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^
2)*tanh(d*x+c))+3*a*b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/c
osh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b
^3*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3-1/5*tanh(d*x+c)^5))
```

maxima [B] time = 0.39, size = 824, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + 16/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/5*a^2*b*(5*e^(-2*d*x - 2*c)/(
d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8
*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x
 - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 6/5*a*b^2*(10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x - 8*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1)))
```

mupad [B] time = 0.84, size = 563, normalized size = 7.61

$$b^3\,x-\frac{\frac{2\left(8\,a^3-12\,a^2\,b+9\,a\,b^2-5\,b^3\right)}{15\,d}-\frac{12\,{\mathrm{e}}^{2\,c+2\,d\,x}\left(a\,b^2-a^2\,b\right)}{5\,d}+\frac{6\,{\mathrm{e}}^{4\,c+4\,d\,x}\left(a\,b^2-b^3\right)}{5\,d}}{3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1}+\frac{\frac{6\left(a\,b^2-a^2\,b\right)}{5\,d}-\frac{6\,{\mathrm{e}}^{2\,c+2\,d\,x}\left(a\,b^2-b^3\right)}{5\,d}}{2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1}+\frac{\frac{6\left(a\,b^2-a^2\,b\right)}{5\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^6,x)
```

```
[Out] b^3*x - ((2*(9*a*b^2 - 12*a^2*b + 8*a^3 - 5*b^3))/(15*d) - (12*exp(2*c + 2*
d*x)*(a*b^2 - a^2*b))/(5*d) + (6*exp(4*c + 4*d*x)*(a*b^2 - b^3))/(5*d))/(3*
exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + ((6*(a*b^2
- a^2*b))/(5*d) - (6*exp(2*c + 2*d*x)*(a*b^2 - b^3))/(5*d))/(2*exp(2*c + 2*
d*x) + exp(4*c + 4*d*x) + 1) + ((6*(a*b^2 - a^2*b))/(5*d) + (18*exp(4*c + 4
*d*x)*(a*b^2 - a^2*b))/(5*d) - (2*exp(2*c + 2*d*x)*(9*a*b^2 - 12*a^2*b + 8*
a^3 - 5*b^3))/(5*d) - (6*exp(6*c + 6*d*x)*(a*b^2 - b^3))/(5*d))/(4*exp(2*c
+ 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)
- ((6*(a*b^2 - b^3))/(5*d) - (24*exp(2*c + 2*d*x)*(a*b^2 - a^2*b))/(5*d) -
(24*exp(6*c + 6*d*x)*(a*b^2 - a^2*b))/(5*d) + (4*exp(4*c + 4*d*x)*(9*a*b^2
- 12*a^2*b + 8*a^3 - 5*b^3))/(5*d) + (6*exp(8*c + 8*d*x)*(a*b^2 - b^3))/(5*
d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp
```

```
(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (6*(a*b^2 - b^3))/(5*d*(exp(2*c +
 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.314 $\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=154

$$\frac{(a+b)\left(5a^2-2ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{(a-b)\left(15a^2+14ab+15b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{48d}+\frac{5\left(a^2\right.}{}$$

```
[Out] 1/16*(a+b)*(5*a^2-2*a*b+5*b^2)*arctan(sinh(d*x+c))/d+1/48*(a-b)*(15*a^2+14*
a*b+15*b^2)*sech(d*x+c)*tanh(d*x+c)/d+5/24*(a^2-b^2)*sech(d*x+c)^3*(a+b*sin
h(d*x+c)^2)*tanh(d*x+c)/d+1/6*(a-b)*sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2*tan
h(d*x+c)/d
```

Rubi [A] time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 413, 526, 385, 203}

$$\frac{(a+b)\left(5a^2-2ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{(a-b)\left(15a^2+14ab+15b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{48d}+\frac{5\left(a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)*(5*a^2 - 2*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((a - b)*(
15*a^2 + 14*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*(a^2 - b
^2)*Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + ((a - b
)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{\left(1+x^2\right)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-b)\operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d} + \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{5\left(a^2-b^2\right)\operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\tanh(c+dx)}{24d} + \frac{(a-}{}$$

$$= \frac{(a-b)\left(15a^2+14ab+15b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d} + \frac{5\left(a^2-b\right.}{}$$

$$= \frac{(a+b)\left(5a^2-2ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{(a-b)\left(15a^2+14a\right.}{}$$

Mathematica [C] time = 14.01, size = 1192, normalized size = 7.74

$$\frac{\operatorname{csch}^5(c+dx)\left(1024b^3\,{}_7F_6\left(\frac{3}{2},2,2,2,2,2,2;1,1,1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right)\left(-\sinh^2(c+dx)\right)^{3/2}\sinh^{12}(c+dx)-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c + d*x]^5*(-117228825*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]] - 10926562
5*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 - 274542345*a^2*b*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 - 17069535*a^3*ArcTanh[Sqrt[-S
inh[c + d*x]^2]]*Sinh[c + d*x]^4 - 260465625*a^2*b*ArcTanh[Sqrt[-Sinh[c + d
*x]^2]]*Sinh[c + d*x]^4 - 215549775*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*S
inh[c + d*x]^4 + 142065*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6
 - 41427855*a^2*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 2071732
95*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 58009455*b^3*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 210735*a^2*b*ArcTanh[Sqrt[-S
inh[c + d*x]^2]]*Sinh[c + d*x]^8 - 33756345*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*
x]^2]]*Sinh[c + d*x]^8 - 56109375*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[
c + d*x]^8 - 174825*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10
- 9261945*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10 - 48825*b^3*
ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^12 + 117228825*a^3*Sqrt[-Sinh
[c + d*x]^2] + 4093425*a^3*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 1689515
10*a^2*b*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 215549775*a*b^2*Sinh[c +
d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 9514449*a^2*b*Sinh[c + d*x]^6*Sqrt[-Sinh[c
+ d*x]^2] + 135323370*a*b^2*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 580094
55*b^3*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 7808535*a*b^2*Sinh[c + d*x]
^8*Sqrt[-Sinh[c + d*x]^2] + 36772890*b^3*Sinh[c + d*x]^8*Sqrt[-Sinh[c + d*x
]^2] + 2160711*b^3*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*x]^2] - 70189350*a^3*(
```

```
-Sinh[c + d*x]^2)^(3/2) - 274542345*a^2*b*(-Sinh[c + d*x]^2)^(3/2) + 1024*a
^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[
c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2) + 3072*a^2*b*Hypergeom
etricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*
Sinh[c + d*x]^8*(-Sinh[c + d*x]^2)^(3/2) + 3072*a*b^2*HypergeometricPFQ[{3/
2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x
]^10*(-Sinh[c + d*x]^2)^(3/2) + 1024*b^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2
, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12*(-Sinh[c
 + d*x]^2)^(3/2) + 1536*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1
, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2)*(a + b*
Sinh[c + d*x]^2)^2*(9*a + 7*b*Sinh[c + d*x]^2) + 256*HypergeometricPFQ[{3/2
, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c
+ d*x]^2)^(3/2)*(295*a^3 + 741*a^2*b*Sinh[c + d*x]^2 + 621*a*b^2*Sinh[c + d
*x]^4 + 175*b^3*Sinh[c + d*x]^6)))/(725760*d*Sqrt[-Sinh[c + d*x]^2])
```

fricas [B] time = 0.71, size = 3675, normalized size = 23.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^11 + 33*(5*a^3 +
 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(5*a^3 + 3*
a^2*b + 3*a*b^2 - 11*b^3)*sinh(d*x + c)^11 + (85*a^3 + 51*a^2*b - 141*a*b^2
 + 5*b^3)*cosh(d*x + c)^9 + (85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3 + 165*(5
*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(55
*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^3 + (85*a^3 + 51*a^2*b
- 141*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 18*(11*a^3 - 19*a^2*b
 + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 -
11*b^3)*cosh(d*x + c)^4 + 11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3 + 2*(85*a^3
+ 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(
5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^5 + 2*(85*a^3 + 51*a^2*b
- 141*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*
b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 18*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*
b^3)*cosh(d*x + c)^5 + 18*(77*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x
 + c)^6 + 7*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - 11*a^
3 + 19*a^2*b - 13*a*b^2 + 5*b^3 + 21*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*
b^3)*cosh(d*x + c)^7 + 7*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x +
 c)^5 + 35*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - 5*(11*a
^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - (85*a^3
+ 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (495*(5*a^3 + 3*a^2*b + 3
*a*b^2 - 11*b^3)*cosh(d*x + c)^8 + 84*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^
3)*cosh(d*x + c)^6 + 630*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x +
c)^4 - 85*a^3 - 51*a^2*b + 141*a*b^2 - 5*b^3 - 180*(11*a^3 - 19*a^2*b + 13*
a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 3*(55*(5*a^3 + 3*a^2*b +
3*a*b^2 - 11*b^3)*cosh(d*x + c)^9 + 12*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b
^3)*cosh(d*x + c)^7 + 126*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x +
 c)^5 - 60*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - (85*a^3
 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((5*a^3
 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 12*(5*a^3 + 3*a^2*b + 3*a*
b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^3 + 3*a^2*b + 3*a*b^2 +
5*b^3)*sinh(d*x + c)^12 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x +
c)^10 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 11*(5*a^3 + 3*a^2*b + 3*a*b^
2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(5*a^3 + 3*a^2*b + 3*
a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh
(d*x + c))*sinh(d*x + c)^9 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*
x + c)^8 + 15*(33*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 5*a
^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 18*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cos
```

```
h(d*x + c)^2)*sinh(d*x + c)^8 + 24*(33*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*
cosh(d*x + c)^5 + 30*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^3 +
5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(
5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 4*(231*(5*a^3 + 3*a^2*
b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 315*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b
^3)*cosh(d*x + c)^4 + 25*a^3 + 15*a^2*b + 15*a*b^2 + 25*b^3 + 105*(5*a^3 +
3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(33*(5*a^3
 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 63*(5*a^3 + 3*a^2*b + 3*a*b
^2 + 5*b^3)*cosh(d*x + c)^5 + 35*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d
*x + c)^3 + 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 15*(33*(5*
a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 84*(5*a^3 + 3*a^2*b + 3*
a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 70*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cos
h(d*x + c)^4 + 5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 20*(5*a^3 + 3*a^2*b + 3*
a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 20*(11*(5*a^3 + 3*a^2*b +
 3*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 36*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*
cosh(d*x + c)^7 + 42*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^5 +
20*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(5*a^3 + 3*a^2*b
 + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a^3 + 3*a^2*b + 3*a*
b^2 + 5*b^3 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 6*(11
*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 45*(5*a^3 + 3*a^2*b
 + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 70*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3
)*cosh(d*x + c)^6 + 50*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^4
+ 5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3
)*cosh(d*x + c)^11 + 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^9
+ 10*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 10*(5*a^3 + 3*a^
2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b
^3)*cosh(d*x + c)^3 + (5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*si
nh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(5*a^3 + 3*a^2*b + 3
*a*b^2 - 11*b^3)*cosh(d*x + c) + 3*(11*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)
*cosh(d*x + c)^10 + 3*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)
^8 + 42*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^6 - 30*(11*a^3
 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^4 - 5*a^3 - 3*a^2*b - 3*a*b^2
 + 11*b^3 - (85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh
(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 +
15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)
*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*co
sh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c
)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6
+ 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 +
5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d
*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c
)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7
+ 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x +
c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 +
 d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*c
osh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x +
c))*sinh(d*x + c) + d)
```

giac [B] time = 0.24, size = 383, normalized size = 2.49

$$\frac{3\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(5\,a^3 + 3\,a^2b + 3\,ab^2 + 5\,b^3\right) + \frac{4\left(15\,a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 9\,a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(5*a^3 + 3*
a^2*b + 3*a*b^2 + 5*b^3) + 4*(15*a^3*(e^(d*x + c) - e^(-d*x - c))^5 + 9*a^2
*b*(e^(d*x + c) - e^(-d*x - c))^5 + 9*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5
- 33*b^3*(e^(d*x + c) - e^(-d*x - c))^5 + 160*a^3*(e^(d*x + c) - e^(-d*x -
c))^3 + 96*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 - 96*a*b^2*(e^(d*x + c) - e
^(-d*x - c))^3 - 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 528*a^3*(e^(d*x +
 c) - e^(-d*x - c)) - 144*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 144*a*b^2*(e
^(d*x + c) - e^(-d*x - c)) - 240*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x
 + c) - e^(-d*x - c))^2 + 4)^3)/d
```

maple [B] time = 0.14, size = 467, normalized size = 3.03

$$\frac{a^3\tanh(dx+c)\operatorname{sech}(dx+c)^5}{6d}+\frac{5a^3\tanh(dx+c)\operatorname{sech}(dx+c)^3}{24d}+\frac{5a^3\operatorname{sech}(dx+c)\tanh(dx+c)}{16d}+\frac{5a^3\arctan}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/6/d*a^3*tanh(d*x+c)*sech(d*x+c)^5+5/24/d*a^3*tanh(d*x+c)*sech(d*x+c)^3+5/
16/d*a^3*sech(d*x+c)*tanh(d*x+c)+5/8/d*a^3*arctan(exp(d*x+c))-3/5/d*a^2*b*s
inh(d*x+c)/cosh(d*x+c)^6+1/10/d*a^2*b*tanh(d*x+c)*sech(d*x+c)^5+1/8/d*a^2*b
*tanh(d*x+c)*sech(d*x+c)^3+3/16/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/8/d*a^2*b
*arctan(exp(d*x+c))-1/d*a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^6-3/5/d*a*b^2*sinh(
d*x+c)/cosh(d*x+c)^6+1/10/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^5+1/8/d*a*b^2*tan
h(d*x+c)*sech(d*x+c)^3+3/16/d*a*b^2*sech(d*x+c)*tanh(d*x+c)+3/8/d*a*b^2*arc
tan(exp(d*x+c))-1/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^6-5/3/d*b^3*sinh(d*x+c)^3
/cosh(d*x+c)^6-1/d*b^3*sinh(d*x+c)/cosh(d*x+c)^6+1/6/d*b^3*tanh(d*x+c)*sech
(d*x+c)^5+5/24/d*b^3*tanh(d*x+c)*sech(d*x+c)^3+5/16/d*b^3*sech(d*x+c)*tanh(
d*x+c)+5/8/d*b^3*arctan(exp(d*x+c))
```

maxima [B] time = 0.50, size = 646, normalized size = 4.19

$$-\frac{1}{24}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{33e^{(-dx-c)}-5e^{(-3dx-3c)}+90e^{(-5dx-5c)}-90e^{(-7dx-7c)}+5e^{(-9dx-9c)}-33e}{d\left(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*b^3*(15*arctan(e^(-d*x - c))/d + (33*e^(-d*x - c) - 5*e^(-3*d*x - 3*c
) + 90*e^(-5*d*x - 5*c) - 90*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) - 33*e^(
-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) - 1/24*a^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*
d*x - 3*c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9
*c) - 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) +
 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*
d*x - 12*c) + 1))) - 1/8*a^2*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c)
+ 17*e^(-3*d*x - 3*c) - 114*e^(-5*d*x - 5*c) + 114*e^(-7*d*x - 7*c) - 17*e^
(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*
x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
 + e^(-12*d*x - 12*c) + 1))) - 1/8*a*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(
-d*x - c) - 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c)
 + 47*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*
e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x
 - 10*c) + e^(-12*d*x - 12*c) + 1)))
```

mupad [B] time = 0.20, size = 601, normalized size = 3.90

$$\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(5\,a^3\,\sqrt{d^2}+5\,b^3\,\sqrt{d^2}+3\,a\,b^2\,\sqrt{d^2}+3\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{25\,a^6+30\,a^5\,b+39\,a^4\,b^2+68\,a^3\,b^3+39\,a^2\,b^4+30\,a\,b^5+25\,b^6}}\right)\sqrt{25\,a^6+30\,a^5\,b+39\,a^4\,b^2+68\,a^3\,b^3+39\,a^2\,b^4+30\,a\,b^5+}}{8\,\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^7,x)
```

```
[Out] (atan((exp(d*x)*exp(c)*(5*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 3*a*b^2*(d^
2)^(1/2) + 3*a^2*b*(d^2)^(1/2)))/(d*(30*a*b^5 + 30*a^5*b + 25*a^6 + 25*b^6
+ 39*a^2*b^4 + 68*a^3*b^3 + 39*a^4*b^2)^(1/2)))*(30*a*b^5 + 30*a^5*b + 25*a
^6 + 25*b^6 + 39*a^2*b^4 + 68*a^3*b^3 + 39*a^4*b^2)^(1/2))/(8*(d^2)^(1/2))
- (6*exp(c + d*x)*(13*a*b^2 - 11*a^2*b + 3*a^3 - 5*b^3))/(d*(4*exp(2*c + 2*
d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (
exp(c + d*x)*(111*a*b^2 - 57*a^2*b + a^3 - 55*b^3))/(3*d*(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*(3*a*b^2 +
3*a^2*b + 5*a^3 - 11*b^3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(3*
a^2*b - 93*a*b^2 + 5*a^3 + 85*b^3))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4
*d*x) + 1)) + (80*exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(3*d*(5*exp
(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d
*x) + exp(10*c + 10*d*x) + 1)) - (32*exp(c + d*x)*(3*a*b^2 - 3*a^2*b + a^3
- b^3))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x
) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.315 $\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=80

$$\frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-b)\tanh^3(c+dx)}{d}-\frac{(a-b)^3\tanh^7(c+dx)}{7d}+\frac{3a(a-b)^2\tanh^5(c+dx)}{5d}$$

```
[Out] a^3*tanh(d*x+c)/d-a^2*(a-b)*tanh(d*x+c)^3/d+3/5*a*(a-b)^2*tanh(d*x+c)^5/d-1
/7*(a-b)^3*tanh(d*x+c)^7/d
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3191, 194}

$$-\frac{a^2(a-b)\tanh^3(c+dx)}{d}+\frac{a^3\tanh(c+dx)}{d}-\frac{(a-b)^3\tanh^7(c+dx)}{7d}+\frac{3a(a-b)^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d - (a^2*(a - b)*Tanh[c + d*x]^3)/d + (3*a*(a - b)^2*Ta
nh[c + d*x]^5)/(5*d) - ((a - b)^3*Tanh[c + d*x]^7)/(7*d)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int \left(a-(a-b)x^2\right)^3\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(a^3-3a^2(a-b)x^2+3a(a-b)^2x^4-(a-b)^3x^6\right)dx,x,\text{t}\right.}{d}\\
&= \frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-b)\tanh^3(c+dx)}{d}+\frac{3a(a-b)^2\tanh^5(c}{5d}
\end{aligned}$$

Mathematica [B] time = 0.60, size = 163, normalized size = 2.04

$$\frac{\tanh(c+dx)\operatorname{sech}^6(c+dx)\left(16a^3\cosh(6(c+dx))+512a^3+8a^2b\cosh(6(c+dx))-304a^2b+\left(464a^3+232a^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((512*a^3 - 304*a^2*b + 192*a*b^2 - 50*b^3 + (464*a^3 + 232*a^2*b - 246*a*b
^2 + 75*b^3)*Cosh[2*(c + d*x)] + 2*(64*a^3 + 32*a^2*b + 24*a*b^2 - 15*b^3)*
```

```
Cosh[4*(c + d*x)] + 16*a^3*Cosh[6*(c + d*x)] + 8*a^2*b*Cosh[6*(c + d*x)] +
6*a*b^2*Cosh[6*(c + d*x)] + 5*b^3*Cosh[6*(c + d*x)])*Sech[c + d*x]^6*Tanh[c
 + d*x])/(1120*d)
```

fricas [B] time = 0.64, size = 814, normalized size = 10.18

$$-\frac{4\left(\left(8\,a^3+4\,a^2b+3\,ab^2+20\,b^3\right)\cosh\left(dx+c\right)^6-6\left(8\,a^3+4\,a^2b+3\,ab^2-15\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+\right.}{35\left(d\cosh\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/35*((8*a^3 + 4*a^2*b + 3*a*b^2 + 20*b^3)*cosh(d*x + c)^6 - 6*(8*a^3 + 4*
a^2*b + 3*a*b^2 - 15*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (8*a^3 + 4*a^2*b
+ 3*a*b^2 + 20*b^3)*sinh(d*x + c)^6 + 14*(4*a^3 + 2*a^2*b + 9*a*b^2)*cosh(d
*x + c)^4 + (56*a^3 + 28*a^2*b + 126*a*b^2 + 15*(8*a^3 + 4*a^2*b + 3*a*b^2
+ 20*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(8*a^3 + 4*a^2*b + 3*a*b^
2 - 15*b^3)*cosh(d*x + c)^3 + 28*(2*a^3 + a^2*b - 3*a*b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 + 280*a^3 - 140*a^2*b + 210*a*b^2 + 7*(24*a^3 + 52*a^2*b - 2
1*a*b^2 + 20*b^3)*cosh(d*x + c)^2 + (15*(8*a^3 + 4*a^2*b + 3*a*b^2 + 20*b^3
)*cosh(d*x + c)^4 + 168*a^3 + 364*a^2*b - 147*a*b^2 + 140*b^3 + 84*(4*a^3 +
 2*a^2*b + 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*(3*(8*a^3 + 4*a^2*
b + 3*a*b^2 - 15*b^3)*cosh(d*x + c)^5 + 56*(2*a^3 + a^2*b - 3*a*b^2)*cosh(d
*x + c)^3 + 7*(24*a^3 - 28*a^2*b + 9*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x
 + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x
+ c)^8 + 8*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^
6 + 4*(14*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cos
h(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 60*d*cosh(d*x + c)^2 + 14*d)*sinh(
d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 + 7*d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 56*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 30*d
*cosh(d*x + c)^4 + 42*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^2 + 4*(2*d*co
sh(d*x + c)^7 + 9*d*cosh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 + 7*d*cosh(d*x +
 c))*sinh(d*x + c) + 35*d)
```

giac [B] time = 0.24, size = 260, normalized size = 3.25

$$-\frac{2\left(35\,b^3e^{(12\,dx+12\,c)}+210\,ab^2e^{(10\,dx+10\,c)}+560\,a^2be^{(8\,dx+8\,c)}-210\,ab^2e^{(8\,dx+8\,c)}+175\,b^3e^{(8\,dx+8\,c)}+560\,a^3e^{(6\,dx+6\,c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*b^3*e^(12*d*x + 12*c) + 210*a*b^2*e^(10*d*x + 10*c) + 560*a^2*b*e
^(8*d*x + 8*c) - 210*a*b^2*e^(8*d*x + 8*c) + 175*b^3*e^(8*d*x + 8*c) + 560*
a^3*e^(6*d*x + 6*c) - 280*a^2*b*e^(6*d*x + 6*c) + 420*a*b^2*e^(6*d*x + 6*c)
 + 336*a^3*e^(4*d*x + 4*c) + 168*a^2*b*e^(4*d*x + 4*c) - 84*a*b^2*e^(4*d*x
+ 4*c) + 105*b^3*e^(4*d*x + 4*c) + 112*a^3*e^(2*d*x + 2*c) + 56*a^2*b*e^(2*
d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) + 16*a^3 + 8*a^2*b + 6*a*b^2 + 5*b^3)
/(d*(e^(2*d*x + 2*c) + 1)^7)
```

maple [B] time = 0.16, size = 289, normalized size = 3.61

$$\frac{a^3\left(\frac{16}{35}+\frac{\mathrm{sech}(dx+c)^6}{7}+\frac{6\mathrm{sech}(dx+c)^4}{35}+\frac{8\mathrm{sech}(dx+c)^2}{35}\right)\tanh\left(dx+c\right)+3a^2b\left(-\frac{\sinh(dx+c)}{6\cosh(dx+c)^7}+\frac{\left(\frac{16}{35}+\frac{\mathrm{sech}(dx+c)^6}{7}+\frac{6\mathrm{sech}(dx+c)^4}{35}+8\mathrm{sech}\right.}{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*ta
nh(d*x+c)+3*a^2*b*(-1/6*sinh(d*x+c)/cosh(d*x+c)^7+1/6*(16/35+1/7*sech(d*x+c
)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/4*sinh(
d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*
x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d
*x+c)^5/cosh(d*x+c)^7-5/8*sinh(d*x+c)^3/cosh(d*x+c)^7-5/16*sinh(d*x+c)/cosh
(d*x+c)^7+5/16*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)
^2)*tanh(d*x+c)))
```

maxima [B] time = 0.43, size = 1754, normalized size = 21.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 32/35*a^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*
e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 16/35*a^2*b*(7*e^
(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x
- 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)
 + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-6*d
*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 70*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d
*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c)
 + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
 + 12/35*a*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x -
4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) +
7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(d*(7
*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*
x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c
) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 3
5*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12
*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(
-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x -
 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/7*b^3*(21*e^(
-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x -
 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)
+ e^(-14*d*x - 14*c) + 1)) + 35*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x - 2*c) + 2
1*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*
d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 7*e^(-12*d*
```

```
x - 12*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 3
5*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12
*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
```

mupad [B] time = 0.87, size = 994, normalized size = 12.42

$$-\frac{\frac{2b^3}{7d}+\frac{8e^{6c+6dx}\left(16a^3-24a^2b+18ab^2-5b^3\right)}{7d}+\frac{2b^3e^{12c+12dx}}{7d}+\frac{6be^{4c+4dx}\left(16a^2-16ab+5b^2\right)}{7d}+\frac{6be^{8c+8dx}\left(16a^2-16ab+5b^2\right)}{7d}+\frac{12b^2}{}}{7e^{2c+2dx}+21e^{4c+4dx}+35e^{6c+6dx}+35e^{8c+8dx}+21e^{10c+10dx}+7e^{12c+12dx}+e^{14c+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(c + d*x)^2)^3/cosh(c + d*x)^8,x)
```

```
[Out] - ((2*b^3)/(7*d) + (8*exp(6*c + 6*d*x)*(18*a*b^2 - 24*a^2*b + 16*a^3 - 5*b^
3))/(7*d) + (2*b^3*exp(12*c + 12*d*x))/(7*d) + (6*b*exp(4*c + 4*d*x)*(16*a^
2 - 16*a*b + 5*b^2))/(7*d) + (6*b*exp(8*c + 8*d*x)*(16*a^2 - 16*a*b + 5*b^2
))/(7*d) + (12*b^2*exp(2*c + 2*d*x)*(2*a - b))/(7*d) + (12*b^2*exp(10*c + 1
0*d*x)*(2*a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp
(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c +
12*d*x) + exp(14*c + 14*d*x) + 1) - ((4*exp(4*c + 4*d*x)*(18*a*b^2 - 24*a^2
*b + 16*a^3 - 5*b^3))/(7*d) + (2*b^3*exp(10*c + 10*d*x))/(7*d) + (2*b^2*(2*
a - b))/(7*d) + (2*b*exp(2*c + 2*d*x)*(16*a^2 - 16*a*b + 5*b^2))/(7*d) + (4
*b*exp(6*c + 6*d*x)*(16*a^2 - 16*a*b + 5*b^2))/(7*d) + (10*b^2*exp(8*c + 8*
d*x)*(2*a - b))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6
*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*
x) + 1) - ((2*(18*a*b^2 - 24*a^2*b + 16*a^3 - 5*b^3))/(35*d) + (2*b^3*exp(6
*c + 6*d*x))/(7*d) + (6*b*exp(2*c + 2*d*x)*(16*a^2 - 16*a*b + 5*b^2))/(35*d
) + (6*b^2*exp(4*c + 4*d*x)*(2*a - b))/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4
*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*b^3*exp(2*c
+ 2*d*x))/(7*d) + (2*b^2*(2*a - b))/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c +
4*d*x) + 1) - ((2*b*(16*a^2 - 16*a*b + 5*b^2))/(35*d) + (8*exp(2*c + 2*d*x)
*(18*a*b^2 - 24*a^2*b + 16*a^3 - 5*b^3))/(35*d) + (2*b^3*exp(8*c + 8*d*x))/
(7*d) + (12*b*exp(4*c + 4*d*x)*(16*a^2 - 16*a*b + 5*b^2))/(35*d) + (8*b^2*e
xp(6*c + 6*d*x)*(2*a - b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x)
 + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2
*b*(16*a^2 - 16*a*b + 5*b^2))/(35*d) + (2*b^3*exp(4*c + 4*d*x))/(7*d) + (4*
b^2*exp(2*c + 2*d*x)*(2*a - b))/(7*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*
d*x) + exp(6*c + 6*d*x) + 1) - (2*b^3)/(7*d*(exp(2*c + 2*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**8*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.316 $\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d}-\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d}-\frac{(a-3b)\sinh^3(c+dx)}{3b^2d}+\frac{\sinh^5(c+dx)}{5bd}$$

```
[Out] (a^2-3*a*b+3*b^2)*sinh(d*x+c)/b^3/d-1/3*(a-3*b)*sinh(d*x+c)^3/b^2/d+1/5*sin
h(d*x+c)^5/b/d-(a-b)^3*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/b^(7/2)/d/a^(1/2
)
```

Rubi [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3190, 390, 205}

$$\frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d}-\frac{(a-3b)\sinh^3(c+dx)}{3b^2d}-\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d}+\frac{\sinh^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(((a - b)^3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)*d))
+ ((a^2 - 3*a*b + 3*b^2)*Sinh[c + d*x])/(b^3*d) - ((a - 3*b)*Sinh[c + d*x]^
3)/(3*b^2*d) + Sinh[c + d*x]^5/(5*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{b^3} - \frac{(a-3b)x^2}{b^2} + \frac{x^4}{b} + \frac{-a^3+3a^2b-3ab^2+b^3}{b^3(a+bx^2)}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d} - \frac{(a-3b)\sinh^3(c+dx)}{3b^2d} + \frac{\sinh^5(c+dx)}{5bd} - \frac{(a-b)^3\,\text{S}}{}$$
$$= -\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}d} + \frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d} - \frac{(a-3b)\sinh^3(c+}{3b^2d}$$

Mathematica [A] time = 0.49, size = 117, normalized size = 1.08

$$\frac{30\sqrt{b}\left(8a^2-22ab+19b^2\right)\sinh(c+dx) + 5b^{3/2}(9b-4a)\sinh(3(c+dx)) + \frac{3\left(\sqrt{a}\,b^{5/2}\sinh(5(c+dx))+80(a-b)^3\tan^{-1}\left(\frac{\sqrt{a}\,\text{csch}(}{\sqrt{b}}\right.\right.}{\sqrt{a}}}{240b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (30*Sqrt[b]*(8*a^2 - 22*a*b + 19*b^2)*Sinh[c + d*x] + 5*b^(3/2)*(-4*a + 9*b
)*Sinh[3*(c + d*x)] + (3*(80*(a - b)^3*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[
b]] + Sqrt[a]*b^(5/2)*Sinh[5*(c + d*x)]))/Sqrt[a])/(240*b^(7/2)*d)
```

fricas [B] time = 0.98, size = 3066, normalized size = 28.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/480*(3*a*b^3*cosh(d*x + c)^10 + 30*a*b^3*cosh(d*x + c)*sinh(d*x + c)^9 +
 3*a*b^3*sinh(d*x + c)^10 - 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^8 + 5*(27
*a*b^3*cosh(d*x + c)^2 - 4*a^2*b^2 + 9*a*b^3)*sinh(d*x + c)^8 + 40*(9*a*b^3
*cosh(d*x + c)^3 - (4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 3
0*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^6 + 10*(63*a*b^3*cosh(d*x
 + c)^4 + 24*a^3*b - 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 - 9*a*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*a*b^3*cosh(d*x + c)^5 - 70*(4*a^2*b^2
- 9*a*b^3)*cosh(d*x + c)^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 30*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^
4 + 10*(63*a*b^3*cosh(d*x + c)^6 - 35*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4
 - 24*a^3*b + 66*a^2*b^2 - 57*a*b^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 - 3*a*b^3 + 40*(9*a*b^3*cosh(d*x + c)^7 -
7*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^5 + 15*(8*a^3*b - 22*a^2*b^2 + 19*a*b
^3)*cosh(d*x + c)^3 - 3*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 + 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2 + 5*(27*a*b^3*cosh(
d*x + c)^8 - 28*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^6 + 90*(8*a^3*b - 22*a^
2*b^2 + 19*a*b^3)*cosh(d*x + c)^4 + 4*a^2*b^2 - 9*a*b^3 - 36*(8*a^3*b - 22*
a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 240*((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d
*x + c)^4*sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^
3*sinh(d*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh
(d*x + c)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)
```

```
^4 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*sqrt(-a*b)*log((b*cos
h(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2
*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2
 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(
d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*
x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x +
c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*
cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*
cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 10*(3*a*b^
3*cosh(d*x + c)^9 - 4*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^7 + 18*(8*a^3*b -
 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^5 - 12*(8*a^3*b - 22*a^2*b^2 + 19*a*b
^3)*cosh(d*x + c)^3 + (4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/(
a*b^4*d*cosh(d*x + c)^5 + 5*a*b^4*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*a*b^
4*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a*b^4*d*cosh(d*x + c)^2*sinh(d*x +
 c)^3 + 5*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a*b^4*d*sinh(d*x + c)^5),
 1/480*(3*a*b^3*cosh(d*x + c)^10 + 30*a*b^3*cosh(d*x + c)*sinh(d*x + c)^9 +
 3*a*b^3*sinh(d*x + c)^10 - 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^8 + 5*(27
*a*b^3*cosh(d*x + c)^2 - 4*a^2*b^2 + 9*a*b^3)*sinh(d*x + c)^8 + 40*(9*a*b^3
*cosh(d*x + c)^3 - (4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 3
0*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^6 + 10*(63*a*b^3*cosh(d*x
 + c)^4 + 24*a^3*b - 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 - 9*a*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*a*b^3*cosh(d*x + c)^5 - 70*(4*a^2*b^2
- 9*a*b^3)*cosh(d*x + c)^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 30*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^
4 + 10*(63*a*b^3*cosh(d*x + c)^6 - 35*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4
 - 24*a^3*b + 66*a^2*b^2 - 57*a*b^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 - 3*a*b^3 + 40*(9*a*b^3*cosh(d*x + c)^7 -
7*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^5 + 15*(8*a^3*b - 22*a^2*b^2 + 19*a*b
^3)*cosh(d*x + c)^3 - 3*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 + 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2 + 5*(27*a*b^3*cosh(
d*x + c)^8 - 28*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^6 + 90*(8*a^3*b - 22*a^
2*b^2 + 19*a*b^3)*cosh(d*x + c)^4 + 4*a^2*b^2 - 9*a*b^3 - 36*(8*a^3*b - 22*
a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d
*x + c)^4*sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^
3*sinh(d*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh
(d*x + c)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)
^4 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*sqrt(a*b)*arctan(1/2*
sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - 480*((a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^
4*sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^3*sinh(d
*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh(d*x + c
)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*sqrt(a*b)*arctan(1/2*(b*cosh(
d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a -
 b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b
)/(a*b)) + 10*(3*a*b^3*cosh(d*x + c)^9 - 4*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x +
 c)^7 + 18*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^5 - 12*(8*a^3*b
- 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^3 + (4*a^2*b^2 - 9*a*b^3)*cosh(d*x +
 c))*sinh(d*x + c))/(a*b^4*d*cosh(d*x + c)^5 + 5*a*b^4*d*cosh(d*x + c)^4*si
nh(d*x + c) + 10*a*b^4*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a*b^4*d*cosh(
d*x + c)^2*sinh(d*x + c)^3 + 5*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a*b^
4*d*sinh(d*x + c)^5)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[22,73]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[-10,75]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.The choice was done assuming [a,b]=[-1,84]Warning, need to choose a bran
ch for the root of a polynomial with parameters. This might be wrong.The ch
oice was done assuming [a,b]=[-91,-60]Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done assuming [a,b]=[-33,-40]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-18,-85]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[1,-81]Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.The choice was done assuming [a,b]=[
70,33]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[14,-81]Wa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming [a,b]=[39,-90]Warning, n
eed to choose a branch for the root of a polynomial with parameters. This m
ight be wrong.The choice was done assuming [a,b]=[77,26]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a,b]=[-97,-57]Warning, need to choose a
branch for the root of a polynomial with parameters. This might be wrong.Th
e choice was done assuming [a,b]=[-98,-45]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.The choic
e was done assuming [a,b]=[-76,76]Undef/Unsigned Inf encountered in limitEv
aluation time: 2.55Limit: Max order reached or unable to make series expans
ion Error: Bad Argument Value
```

maple [B] time = 0.14, size = 1656, normalized size = 15.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 3/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a+3/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a+1/2/d/b
^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-1/d/b^3/(tanh(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^
2/(tanh(1/2*d*x+1/2*c)+1)^2*a-1/d/b^3/(tanh(1/2*d*x+1/2*c)+1)*a^2+6/d*a^2/b
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+6/d*a^2/b/(-b*(a-b))^(1/2)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2))+1/3/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^3*a+1/3/d/b^
2/(tanh(1/2*d*x+1/2*c)+1)^3*a+3/d*a^2/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/5/
d/b/(tanh(1/2*d*x+1/2*c)-1)^5-1/5/d/b/(tanh(1/2*d*x+1/2*c)+1)^5-4/d/b^2*a^3
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/b^3*a^4/(-b*(a-b))^(1/2)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arc
```

```
tan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))+1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b
-3/d*a/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2))*b-4/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/d*a/b/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2))-4/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2))-3/d*a^2/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-4/d/b^2*a^3/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/b^3*a^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+
1/d/b^3*a^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^4
+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^4-11/8/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+11/8/
d/b/(tanh(1/2*d*x+1/2*c)+1)^2-5/4/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-5/4/d/b/(ta
nh(1/2*d*x+1/2*c)+1)^3-3/d/b/(tanh(1/2*d*x+1/2*c)-1)-3/d/b/(tanh(1/2*d*x+1/
2*c)+1)+1/d/b^3*a^4/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\,b^2e^{(10\,dx+10\,c)} - 3\,b^2 - 5\left(4\,abe^{(8\,c)} - 9\,b^2e^{(8\,c)}\right)e^{(8\,dx)} + 30\left(8\,a^2e^{(6\,c)} - 22\,abe^{(6\,c)} + 19\,b^2e^{(6\,c)}\right)e^{(6\,dx)} - 30\left(8\,a^2e^{(4\cdots\right.}{480\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(3*b^2*e^(10*d*x + 10*c) - 3*b^2 - 5*(4*a*b*e^(8*c) - 9*b^2*e^(8*c))*
e^(8*d*x) + 30*(8*a^2*e^(6*c) - 22*a*b*e^(6*c) + 19*b^2*e^(6*c))*e^(6*d*x)
- 30*(8*a^2*e^(4*c) - 22*a*b*e^(4*c) + 19*b^2*e^(4*c))*e^(4*d*x) + 5*(4*a*b
*e^(2*c) - 9*b^2*e^(2*c))*e^(2*d*x))*e^(-5*d*x - 5*c)/(b^3*d) - 1/128*integ
rate(256*((a^3*e^(3*c) - 3*a^2*b*e^(3*c) + 3*a*b^2*e^(3*c) - b^3*e^(3*c))*e
^(3*d*x) + (a^3*e^c - 3*a^2*b*e^c + 3*a*b^2*e^c - b^3*e^c)*e^(d*x))/(b^4*e^
(4*d*x + 4*c) + b^4 + 2*(2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.51, size = 954, normalized size = 8.83

$$\frac{e^{5\,c+5\,d\,x}}{160\,b\,d}-\frac{e^{-5\,c-5\,d\,x}}{160\,b\,d}-\frac{\left(2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,(a-b)^3\,\sqrt{a\,b^7\,d^2}}{2\,a\,b^3\,d\,\sqrt{(a-b)^6}}\right)+2\,\mathrm{atan}\left(\frac{a\,b^8\,e^{d\,x}\,e^{c}\left(\frac{4\left(12\,a^3\,b^5\,d\,\sqrt{a^6-6\,a^5\,b+15\,a^4\,b^2-20\,a^3\,b^3+15\,a^2\,b^4-6\,a\,b^5+b^6}-8\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^7/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(5*c + 5*d*x)/(160*b*d) - exp(- 5*c - 5*d*x)/(160*b*d) - ((2*atan((exp(d
*x)*exp(c)*(a - b)^3*(a*b^7*d^2)^(1/2))/(2*a*b^3*d*((a - b)^6)^(1/2))) + 2*
atan((a*b^8*exp(d*x)*exp(c)*((4*(12*a^3*b^5*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^
6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2) - 8*a^2*b^6*d*(a^6 - 6*a^5*
b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2) - 8*a^4*b^4
*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(
```

```
1/2) + 2*a^5*b^3*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3
 + 15*a^4*b^2)^(1/2) + 2*a*b^7*d*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^
4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/2)))/(a^2*b^15*d^2*(a - b)^3) - (2*(a^7*(a*
b^7*d^2)^(1/2) - b^7*(a*b^7*d^2)^(1/2) + 7*a*b^6*(a*b^7*d^2)^(1/2) - 7*a^6*
b*(a*b^7*d^2)^(1/2) - 21*a^2*b^5*(a*b^7*d^2)^(1/2) + 35*a^3*b^4*(a*b^7*d^2)
^(1/2) - 35*a^4*b^3*(a*b^7*d^2)^(1/2) + 21*a^5*b^2*(a*b^7*d^2)^(1/2)))/(a^2
*b^11*d*((a - b)^6)^(1/2)*(a*b^7*d^2)^(1/2)))*(a*b^7*d^2)^(1/2))/(4*a^4 - 1
6*a^3*b - 16*a*b^3 + 4*b^4 + 24*a^2*b^2) + (2*exp(3*c)*exp(3*d*x)*(a^7*(a*b
^7*d^2)^(1/2) - b^7*(a*b^7*d^2)^(1/2) + 7*a*b^6*(a*b^7*d^2)^(1/2) - 7*a^6*b
*(a*b^7*d^2)^(1/2) - 21*a^2*b^5*(a*b^7*d^2)^(1/2) + 35*a^3*b^4*(a*b^7*d^2)^
(1/2) - 35*a^4*b^3*(a*b^7*d^2)^(1/2) + 21*a^5*b^2*(a*b^7*d^2)^(1/2)))/(a*b^
3*d*((a - b)^6)^(1/2)*(4*a^4 - 16*a^3*b - 16*a*b^3 + 4*b^4 + 24*a^2*b^2))))
*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)^(1/
2))/(2*(a*b^7*d^2)^(1/2)) + (exp(c + d*x)*(8*a^2 - 22*a*b + 19*b^2))/(16*b^
3*d) - (exp(- c - d*x)*(8*a^2 - 22*a*b + 19*b^2))/(16*b^3*d) + (exp(- 3*c -
 3*d*x)*(4*a - 9*b))/(96*b^2*d) - (exp(3*c + 3*d*x)*(4*a - 9*b))/(96*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**7/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.317 $\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{x(8a^2-20ab+15b^2)}{8b^3}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3d}-\frac{(4a-7b)\sinh(c+dx)\cosh(c+dx)}{8b^2d}+\frac{\sinh(c+dx)}{4}$$

```
[Out] 1/8*(8*a^2-20*a*b+15*b^2)*x/b^3-1/8*(4*a-7*b)*cosh(d*x+c)*sinh(d*x+c)/b^2/d
+1/4*cosh(d*x+c)^3*sinh(d*x+c)/b/d-(a-b)^(5/2)*arctanh((a-b)^(1/2)*tanh(d*x
+c)/a^(1/2))/b^3/d/a^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3191, 414, 527, 522, 206, 208}

$$\frac{x(8a^2-20ab+15b^2)}{8b^3}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3d}-\frac{(4a-7b)\sinh(c+dx)\cosh(c+dx)}{8b^2d}+\frac{\sinh(c+dx)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((8*a^2 - 20*a*b + 15*b^2)*x)/(8*b^3) - ((a - b)^(5/2)*ArcTanh[(Sqrt[a - b]
*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^3*d) - ((4*a - 7*b)*Cosh[c + d*x]*Sinh
[c + d*x])/(8*b^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*b*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} + \frac{\text{Subst}\left(\int \frac{-a+4b-3(a-b)x^2}{(1-x^2)^2(a+(-a+b)x^2)}\,dx, x, \tanh(c+dx)\right)}{4bd}$$

$$= -\frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} + \frac{\text{Subst}\left(\int \frac{4a^2-}{}\right.}{}$$

$$= -\frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} - \frac{(a-b)^3\,\text{Subst}}{}$$

$$= \frac{(8a^2-20ab+15b^2)x}{8b^3} - \frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3d} - \frac{(4a-7b)\cosh(c+dx)}{8b^2d}$$

Mathematica [A] time = 0.34, size = 106, normalized size = 0.88

$$\frac{\sqrt{a}\left(4\left(8a^2-20ab+15b^2\right)(c+dx)-8b(a-2b)\sinh(2(c+dx))+b^2\sinh(4(c+dx))\right)-32(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-}}{}\right.}{32\sqrt{a}\,b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-32*(a - b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*(
4*(8*a^2 - 20*a*b + 15*b^2)*(c + d*x) - 8*(a - 2*b)*b*Sinh[2*(c + d*x)] + b
^2*Sinh[4*(c + d*x)]))/(32*Sqrt[a]*b^3*d)
```

fricas [B] time = 1.08, size = 1817, normalized size = 15.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh
(d*x + c)^8 + 8*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b - 2*
b^2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b + 4*b^2)*sinh(d*x +
 c)^6 + 8*(7*b^2*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x
```

```
+ c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x - 60*(
a*b - 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 +
4*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c) - 20*(a*b - 2*b^2)*cosh(d*x +
 c)^3)*sinh(d*x + c)^3 + 8*(a*b - 2*b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*
x + c)^6 + 12*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b - 2*b
^2)*cosh(d*x + c)^4 + 2*a*b - 4*b^2)*sinh(d*x + c)^2 + 32*((a^2 - 2*a*b + b
^2)*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^3*sinh(d*x + c) +
 6*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^2 - 2*a*b + b
^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^4)*sq
rt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d
*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cos
h(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x
 + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 -
 a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x +
 c))*sinh(d*x + c) + b)) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 - 20*a*b
 + 15*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d*x + c)^5 + 2*(a*b -
 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cosh
(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^3
*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4), 1/64*(b^2*cosh(d
*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 8*(
8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b - 2*b^2)*cosh(d*x + c
)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b + 4*b^2)*sinh(d*x + c)^6 + 8*(7*b^2*
cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^
2*cosh(d*x + c)^4 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x - 60*(a*b - 2*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 - 20*a*b
 + 15*b^2)*d*x*cosh(d*x + c) - 20*(a*b - 2*b^2)*cosh(d*x + c)^3)*sinh(d*x +
 c)^3 + 8*(a*b - 2*b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8*
a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b - 2*b^2)*cosh(d*x + c)
^4 + 2*a*b - 4*b^2)*sinh(d*x + c)^2 + 64*((a^2 - 2*a*b + b^2)*cosh(d*x + c)
^4 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^2 - 2*a*b +
 b^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^4)*sqrt(-(a - b)/a)*ar
ctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) - b^2 + 8*(b^2*cosh(d*x + c)^7
 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d
*x + c)^5 + 2*(a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x +
 c)^4 + 4*b^3*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^
4)]
```

giac [B] time = 4.14, size = 226, normalized size = 1.87

$$\frac{\frac{8\left(8a^2-20ab+15b^2\right)(dx+c)}{b^3}+\frac{be^{(4dx+4c)}-8ae^{(2dx+2c)}+16be^{(2dx+2c)}}{b^2}-\frac{\left(48a^2e^{(4dx+4c)}-120abe^{(4dx+4c)}+90b^2e^{(4dx+4c)}-8abe^{(2dx+2c)}+16b^2e^{(2\cdots}{b^3}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(8*(8*a^2 - 20*a*b + 15*b^2)*(d*x + c)/b^3 + (b*e^(4*d*x + 4*c) - 8*a*
e^(2*d*x + 2*c) + 16*b*e^(2*d*x + 2*c))/b^2 - (48*a^2*e^(4*d*x + 4*c) - 120
*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 8*a*b*e^(2*d*x + 2*c) + 16*
b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/b^3 - 64*(a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(
-a^2 + a*b)*b^3))/d
```

maple [B] time = 0.12, size = 1497, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2-
1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a+1/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2-1
/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a-
3/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*a^2/b/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)
+5/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+15/8/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1
5/8/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-3/d*a^2/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+
1/d/b^2*a^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2))+1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2))*b+3/d*a/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*ar
ctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/(-b*(a
-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b+3/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2))-3/d*a/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*a/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*a^2/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*
a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
)+1/d/b^2*a^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b^3*a^3/((
2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2))-1/d/b^3*a^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/4/d/
b/(tanh(1/2*d*x+1/2*c)-1)^4-1/4/d/b/(tanh(1/2*d*x+1/2*c)+1)^4+11/8/d/b/(tan
h(1/2*d*x+1/2*c)-1)^2-11/8/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(tanh(1/2*
d*x+1/2*c)-1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+9/8/d/b/(tanh(1/2*d*x+1/2
*c)-1)+9/8/d/b/(tanh(1/2*d*x+1/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 1.37, size = 264, normalized size = 2.18

$$\frac{x\left(8a^2-20ab+15b^2\right)}{8b^3}-\frac{e^{-4c-4dx}}{64bd}+\frac{e^{4c+4dx}}{64bd}+\frac{e^{-2c-2dx}\,(a-2b)}{8b^2d}-\frac{e^{2c+2dx}\,(a-2b)}{8b^2d}+\frac{\ln\left(\frac{4e^{2c+2dx}(a-b)^3}{b^4}-\frac{2(a-b)^{5/2}}{\cdots}\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] (x*(8*a^2 - 20*a*b + 15*b^2))/(8*b^3) - exp(- 4*c - 4*d*x)/(64*b*d) + exp(4
*c + 4*d*x)/(64*b*d) + (exp(- 2*c - 2*d*x)*(a - 2*b))/(8*b^2*d) - (exp(2*c
+ 2*d*x)*(a - 2*b))/(8*b^2*d) + (log((4*exp(2*c + 2*d*x)*(a - b)^3)/b^4 - (
2*(a - b)^(5/2)*(b + 2*a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a^(1/2)*b
^4))*(a - b)^(5/2))/(2*a^(1/2)*b^3*d) - (log((4*exp(2*c + 2*d*x)*(a - b)^3)
/b^4 + (2*(a - b)^(5/2)*(b + 2*a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a
^(1/2)*b^4))*(a - b)^(5/2))/(2*a^(1/2)*b^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.318 $\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

```
[Out] -(a-2*b)*sinh(d*x+c)/b^2/d+1/3*sinh(d*x+c)^3/b/d+(a-b)^2*arctan(sinh(d*x+c)
*b^(1/2)/a^(1/2))/b^(5/2)/d/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3190, 390, 205}

$$-\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}+\frac{\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - b)^2*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(a - 2*b)*Sinh[c + d*x])/(b^2*d) + Sinh[c + d*x]^3/(3*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a-2b}{b^2}+\frac{x^2}{b}+\frac{a^2-2ab+b^2}{b^2(a+bx^2)}\right) dx, x, \sinh(c+dx)\right)}{d}$$
$$= -\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{\sinh^3(c+dx)}{3bd}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+\right.}{b^2d}$$
$$= \frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}d}-\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

Mathematica [A] time = 0.25, size = 79, normalized size = 1.03

$$\frac{3\sqrt{b}\,(7b-4a)\sinh(c+dx)-\frac{12(a-b)^2\tan^{-1}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}}+b^{3/2}\sinh(3(c+dx))}{12b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((-12*(a - b)^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + 3*Sqrt[b
]*(-4*a + 7*b)*Sinh[c + d*x] + b^(3/2)*Sinh[3*(c + d*x)])/(12*b^(5/2)*d)
```

fricas [B] time = 0.54, size = 1490, normalized size = 19.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*(a*b^2*cosh(d*x + c)^6 + 6*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^
2*sinh(d*x + c)^6 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^4 + 3*(5*a*b^2*cosh
(d*x + c)^2 - 4*a^2*b + 7*a*b^2)*sinh(d*x + c)^4 + 4*(5*a*b^2*cosh(d*x + c)
^3 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a*b^2 + 3*(4*a^
2*b - 7*a*b^2)*cosh(d*x + c)^2 + 3*(5*a*b^2*cosh(d*x + c)^4 + 4*a^2*b - 7*a
*b^2 - 6*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 12*((a^2 -
2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d
*x + c) + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 - 2*a*
b + b^2)*sinh(d*x + c)^3)*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x
+ c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*
(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2
*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)
*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
- cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sin
h(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cos
h(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*
cosh(d*x + c))*sinh(d*x + c) + b)) + 6*(a*b^2*cosh(d*x + c)^5 - 2*(4*a^2*b
- 7*a*b^2)*cosh(d*x + c)^3 + (4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x +
c))/(a*b^3*d*cosh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*
a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + a*b^3*d*sinh(d*x + c)^3), 1/24*(a*b
^2*cosh(d*x + c)^6 + 6*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^2*sinh(d*x
 + c)^6 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^4 + 3*(5*a*b^2*cosh(d*x + c)^
2 - 4*a^2*b + 7*a*b^2)*sinh(d*x + c)^4 + 4*(5*a*b^2*cosh(d*x + c)^3 - 3*(4*
```

```
a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a*b^2 + 3*(4*a^2*b - 7*a*
b^2)*cosh(d*x + c)^2 + 3*(5*a*b^2*cosh(d*x + c)^4 + 4*a^2*b - 7*a*b^2 - 6*(
4*a^2*b - 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*((a^2 - 2*a*b + b^
2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c) +
3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*s
inh(d*x + c)^3)*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x +
c))/a) + 24*((a^2 - 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*co
sh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^2 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^3)*sqrt(a*b)*arctan(1/2*(b*cosh(
d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a -
 b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b
)/(a*b)) + 6*(a*b^2*cosh(d*x + c)^5 - 2*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^3
 + (4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x + c)
^3 + 3*a*b^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*b^3*d*cosh(d*x + c)*sinh
(d*x + c)^2 + a*b^3*d*sinh(d*x + c)^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[22,73]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[-10,75]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.The choice was done assuming [a,b]=[4,51]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.The cho
ice was done assuming [a,b]=[44,-86]Warning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming [a,b]=[34,-93]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b]=[80,-1]Warning, need to choose a branch for the root of a polyn
omial with parameters. This might be wrong.The choice was done assuming [a,
b]=[32,1]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[88,70]
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[-81,37]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-90,65]Undef/Unsigned I
nf encountered in limitEvaluation time: 2.06Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.11, size = 1148, normalized size = 14.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/3/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/d/b^
2/(tanh(1/2*d*x+1/2*c)-1)*a-2/d/b/(tanh(1/2*d*x+1/2*c)-1)-1/3/d/b/(tanh(1/2
*d*x+1/2*c)+1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/d/b^2/(tanh(1/2*d*x+1/
2*c)+1)*a-2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d/b^2*a^3/(-b*(a-b))^(1/2)/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2))+3/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2))-3/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a^2/b^2/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2))-2/d*a/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2))+1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b-1/d/b^
2*a^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*a^2/b/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2))-1/d*a^2/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tan
h(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d*a/b/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2))-1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/(-b*(a-b))^(1/2)/((
2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(3\left(4\,ae^{(4\,c)}-7\,be^{(4\,c)}\right)e^{(4\,dx)}-3\left(4\,ae^{(2\,c)}-7\,be^{(2\,c)}\right)e^{(2\,dx)}-be^{(6\,dx+6\,c)}+b\right)e^{(-3\,dx-3\,c)}}{24\,b^2d}+\frac{1}{32}\int\frac{64\left(\left(a^2e^{(3\,c)}-2\,ab\cdots\right.\right.}{b^3e^{(4\,dx}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(4*a*e^(4*c) - 7*b*e^(4*c))*e^(4*d*x) - 3*(4*a*e^(2*c) - 7*b*e^(2*
c))*e^(2*d*x) - b*e^(6*d*x + 6*c) + b)*e^(-3*d*x - 3*c)/(b^2*d) + 1/32*inte
grate(64*((a^2*e^(3*c) - 2*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (a^2*e^c
- 2*a*b*e^c + b^2*e^c)*e^(d*x))/(b^3*e^(4*d*x + 4*c) + b^3 + 2*(2*a*b^2*e^(
2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.28, size = 668, normalized size = 8.68

$$\left(2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,{\left(a-b\right)}^2\,\sqrt{a\,b^5\,d^2}}{2\,a\,b^2\,d\,\sqrt{{\left(a-b\right)}^4}}\right)-2\,\mathrm{atan}\left(\frac{a\,b^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\left(\frac{4\left(2\,a\,b^5\,d\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}-6\,a^2\,b^4\,d\,\sqrt{a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4}+6\,a^3\,b^3\,d\,\sqrt{a^4-\cdots}\right.}{a^2\,b^{11}\,d^2\,{\left(a-b\right)}^2}\cdots\right.}{\cdots}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] ((2*atan((exp(d*x)*exp(c)*(a - b)^2*(a*b^5*d^2)^(1/2))/(2*a*b^2*d*((a - b)^
4)^(1/2))) - 2*atan((a*b^6*exp(d*x)*exp(c)*((4*(2*a*b^5*d*(a^4 - 4*a^3*b -
4*a*b^3 + b^4 + 6*a^2*b^2)^(1/2) - 6*a^2*b^4*d*(a^4 - 4*a^3*b - 4*a*b^3 + b
^4 + 6*a^2*b^2)^(1/2) + 6*a^3*b^3*d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*
```

```
b^2)^(1/2) - 2*a^4*b^2*d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)^(1/2))
)/(a^2*b^11*d^2*(a - b)^2) + (2*(a^5*(a*b^5*d^2)^(1/2) - b^5*(a*b^5*d^2)^(1
/2) + 5*a*b^4*(a*b^5*d^2)^(1/2) - 5*a^4*b*(a*b^5*d^2)^(1/2) - 10*a^2*b^3*(a
*b^5*d^2)^(1/2) + 10*a^3*b^2*(a*b^5*d^2)^(1/2)))/(a^2*b^8*d*((a - b)^4)^(1/
2)*(a*b^5*d^2)^(1/2)))*(a*b^5*d^2)^(1/2))/(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*
b^3) - (2*exp(3*c)*exp(3*d*x)*(a^5*(a*b^5*d^2)^(1/2) - b^5*(a*b^5*d^2)^(1/2
) + 5*a*b^4*(a*b^5*d^2)^(1/2) - 5*a^4*b*(a*b^5*d^2)^(1/2) - 10*a^2*b^3*(a*b
^5*d^2)^(1/2) + 10*a^3*b^2*(a*b^5*d^2)^(1/2)))/(a*b^2*d*((a - b)^4)^(1/2)*(
12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))))*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*
a^2*b^2)^(1/2))/(2*(a*b^5*d^2)^(1/2)) - exp(- 3*c - 3*d*x)/(24*b*d) + exp(3
*c + 3*d*x)/(24*b*d) - (exp(c + d*x)*(4*a - 7*b))/(8*b^2*d) + (exp(- c - d*
x)*(4*a - 7*b))/(8*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
[Out] Timed out
```

3.319 $\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=81

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}-\frac{x(2a-3b)}{2b^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a-3*b)*x/b^2+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d+(a-b)^(3/2)*arctanh((a
-b)^(1/2)*tanh(d*x+c)/a^(1/2))/b^2/d/a^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 414, 522, 206, 208}

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d}-\frac{x(2a-3b)}{2b^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((2*a - 3*b)*x)/(2*b^2) + ((a - b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x
])/Sqrt[a]])/(Sqrt[a]*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
```

```
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\left(a-(a-b)x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{-a+2b+(-a+b)x^2}{(1-x^2)\left(a+(-a+b)x^2\right)}\,dx, x, \tanh(c+dx)\right)}{2bd}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2b^2d} + \frac{(a-b)^2}{}$$

$$= -\frac{(2a-3b)x}{2b^2} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

Mathematica [A] time = 0.16, size = 80, normalized size = 0.99

$$\frac{\sqrt{a}\,(b\sinh(2(c+dx)) - 2(2a-3b)(c+dx)) + 4(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{4\sqrt{a}\,b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (4*(a - b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*(-2
*(2*a - 3*b)*(c + d*x) + b*Sinh[2*(c + d*x)]))/(4*Sqrt[a]*b^2*d)
```

fricas [B] time = 0.58, size = 875, normalized size = 10.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - 3*b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x
 + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a - 3*b)*d*x - 3*b*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x + c)^2 + 2*(a - b)*cosh(
d*x + c)*sinh(d*x + c) + (a - b)*sinh(d*x + c)^2)*sqrt((a - b)/a)*log((b^2*
cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4
 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2
)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b -
 b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*
x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 - a*b)*sqrt((a - b)/a))/
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
) + 4*(2*(2*a - 3*b)*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*x + c) +
 b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*si
nh(d*x + c)^2), -1/8*(4*(2*a - 3*b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4
 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a - 3*b)
*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a - b)*cosh(d*x + c)^2 +
2*(a - b)*cosh(d*x + c)*sinh(d*x + c) + (a - b)*sinh(d*x + c)^2)*sqrt(-(a -
 b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
```

```
*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) + 4*(2*(2*a - 3*b)*d*
x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c
)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)]
```

giac [A] time = 2.93, size = 138, normalized size = 1.70

$$-\frac{\frac{4\,(dx+c)(2\,a-3\,b)}{b^2}-\frac{e^{(2\,dx+2\,c)}}{b}-\frac{\left(4\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-b\right)e^{(-2\,dx-2\,c)}}{b^2}-\frac{8\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(4*(d*x + c)*(2*a - 3*b)/b^2 - e^(2*d*x + 2*c)/b - (4*a*e^(2*d*x + 2*c
) - 6*b*e^(2*d*x + 2*c) - b)*e^(-2*d*x - 2*c)/b^2 - 8*(a^2 - 2*a*b + b^2)*a
rctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)
*b^2))/d
```

maple [B] time = 0.10, size = 993, normalized size = 12.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2
*ln(tanh(1/2*d*x+1/2*c)-1)-3/2/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/d/b/(tanh(
1/2*d*x+1/2*c)+1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)+3/2/d/b*ln(tanh(1/2*d*x
+1/2*c)+1)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d*a^2/b^2/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2))+2/d*a/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2))-1/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d*a/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2))*b+1/d*a^2/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(
a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-2/d*a/b/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))+1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*a^2/b/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b)
)^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+
a-2*b)*a)^(1/2))-1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 1.61, size = 300, normalized size = 3.70

$$\frac{e^{2c+2dx}}{8bd}-\frac{e^{-2c-2dx}}{8bd}-\frac{x\,(2a-3b)}{2b^2}-\frac{\ln\left(\frac{4(a-b)^3\left(2ab-b^2+8a^2e^{2c+2dx}+b^2e^{2c+2dx}-8abe^{2c+2dx}\right)}{ab^6}-\frac{8(a-b)^{7/2}\left(b+4ae^{2c+2dx}-2be^{2c+\cdots}\right.}{\sqrt{a}\,b^6}\right.}{2\sqrt{a}\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(2*c + 2*d*x)/(8*b*d) - exp(- 2*c - 2*d*x)/(8*b*d) - (x*(2*a - 3*b))/(2*
b^2) - (log((4*(a - b)^3*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*
c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a*b^6) - (8*(a - b)^(7/2)*(b + 4*a*e
xp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(1/2)*b^6))*(a - b)^(3/2))/(2*a
^(1/2)*b^2*d) + (log((4*(a - b)^3*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b
^2*exp(2*c + 2*d*x) - 8*a*b*exp(2*c + 2*d*x)))/(a*b^6) + (8*(a - b)^(7/2)*(
b + 4*a*exp(2*c + 2*d*x) - 2*b*exp(2*c + 2*d*x)))/(a^(1/2)*b^6))*(a - b)^(3
/2))/(2*a^(1/2)*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.320 $\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{\sinh(c+dx)}{bd}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}$$

```
[Out] sinh(d*x+c)/b/d-(a-b)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/b^(3/2)/d/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3190, 388, 205}

$$\frac{\sinh(c+dx)}{bd}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) +
Sinh[c + d*x]/(b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+bx^2}\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{\sinh(c+dx)}{bd}-\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sinh(c+dx)\right)}{bd}\\
&= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}d}+\frac{\sinh(c+dx)}{bd}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 0.96

$$\frac{\frac{\sinh(c+dx)}{b} - \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) + S
inh[c + d*x]/b)/d
```

fricas [B] time = 0.59, size = 659, normalized size = 12.67

$$\left[\frac{ab\cosh(dx+c)^2 + 2\,ab\cosh(dx+c)\sinh(dx+c) + ab\sinh(dx+c)^2 + \sqrt{-ab}\,((a-b)\cosh(dx+c) + (a-b)\,s}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*
x + c)^2 + sqrt(-a*b)*((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*log((
b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 -
 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x +
 c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(
cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*co
sh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d
*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
- b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) - a*b)/
(a*b^2*d*cosh(d*x + c) + a*b^2*d*sinh(d*x + c)), 1/2*(a*b*cosh(d*x + c)^2 +
 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 - 2*sqrt(a*b)*((a
- b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*arctan(1/2*sqrt(a*b)*(cosh(d*x
+ c) + sinh(d*x + c))/a) - 2*sqrt(a*b)*((a - b)*cosh(d*x + c) + (a - b)*sin
h(d*x + c))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)
^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4
*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) - a*b)/(a*b^2*d*cosh(d*x + c) + a*b
^2*d*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
```

```
This might be wrong.The choice was done assuming [a,b]=[-18,-81]Warning, ne
ed to choose a branch for the root of a polynomial with parameters. This mi
ght be wrong.The choice was done assuming [a,b]=[-10,75]Warning, need to ch
oose a branch for the root of a polynomial with parameters. This might be w
rong.The choice was done assuming [a,b]=[4,51]Warning, need to choose a bra
nch for the root of a polynomial with parameters. This might be wrong.The c
hoice was done assuming [a,b]=[44,-86]Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done assuming [a,b]=[34,-93]Warning, need to choose a branch for the root
 of a polynomial with parameters. This might be wrong.The choice was done a
ssuming [a,b]=[80,-1]Undef/Unsigned Inf encountered in limitEvaluation time
: 1.54Limit: Max order reached or unable to make series expansion Error: Ba
d Argument Value
```

maple [B] time = 0.10, size = 732, normalized size = 14.08

$$\frac{a^{2}\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{db\sqrt{-b\left(a-b\right)}\,\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}-\frac{a\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{db\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}-\frac{2a\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+\right.}}\right.}{d\sqrt{-b\left(a-b\right)}\,\sqrt{\left(2\sqrt{-b\left(a-b\right)}-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*a/b/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
-a+2*b)*a)^(1/2))-2/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a
^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a/b/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2))-2/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2))+1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
*b-1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/(
(2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2))*b-1/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d/b/(tanh(1/2*d*x+1/2*c)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}}{2\,bd}-\frac{1}{8}\int\frac{16\left(\left(ae^{(3\,c)}-be^{(3\,c)}\right)e^{(3\,dx)}+\left(ae^{c}-be^{c}\right)e^{(dx)}\right)}{b^{2}e^{(4\,dx+4\,c)}+b^{2}+2\left(2\,abe^{(2\,c)}-b^{2}e^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)/(b*d) - 1/8*integrate(16*((a*e^(3*c)
 - b*e^(3*c))*e^(3*d*x) + (a*e^c - b*e^c)*e^(d*x))/(b^2*e^(4*d*x + 4*c) + b
^2 + 2*(2*a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.14, size = 426, normalized size = 8.19

$$\frac{e^{c+d\,x}}{2\,b\,d}-\frac{\left(2\,\mathrm{atan}\left(\frac{a\,b^4\,e^{d\,x}\,e^{c}\left(\frac{4\left(2\,a\,b^3\,d\,\sqrt{a^2-2\,a\,b+b^2}+2\,a^3\,b\,d\,\sqrt{a^2-2\,a\,b+b^2}-4\,a^2\,b^2\,d\,\sqrt{a^2-2\,a\,b+b^2}\right)}{a^2\,b^7\,d^2\,(a-b)}-\frac{2\left(a^3\,\sqrt{a\,b^3\,d^2}-b^3\,\sqrt{a\,b^3\,d^2}+3\,a\,b^2\,\sqrt{a\,b^3\,d^2}-3\,a^2\,b\,\sqrt{a\,b^3\,d^2}\right.}{a^2\,b^5\,d\,\sqrt{(a-b)^2}\,\sqrt{a\,b^3\,d^2}}\right.}{4\,a^2-8\,a\,b+4\,b^2}\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] exp(c + d*x)/(2*b*d) - ((2*atan((a*b^4*exp(d*x)*exp(c)*((4*(2*a*b^3*d*(a^2
- 2*a*b + b^2)^(1/2) + 2*a^3*b*d*(a^2 - 2*a*b + b^2)^(1/2) - 4*a^2*b^2*d*(a
^2 - 2*a*b + b^2)^(1/2)))/(a^2*b^7*d^2*(a - b)) - (2*(a^3*(a*b^3*d^2)^(1/2)
 - b^3*(a*b^3*d^2)^(1/2) + 3*a*b^2*(a*b^3*d^2)^(1/2) - 3*a^2*b*(a*b^3*d^2)^
(1/2)))/(a^2*b^5*d*((a - b)^2)^(1/2)*(a*b^3*d^2)^(1/2)))*(a*b^3*d^2)^(1/2))
/(4*a^2 - 8*a*b + 4*b^2) + (2*exp(3*c)*exp(3*d*x)*(a^3*(a*b^3*d^2)^(1/2) -
b^3*(a*b^3*d^2)^(1/2) + 3*a*b^2*(a*b^3*d^2)^(1/2) - 3*a^2*b*(a*b^3*d^2)^(1/
2)))/(a*b*d*((a - b)^2)^(1/2)*(4*a^2 - 8*a*b + 4*b^2))) + 2*atan((exp(d*x)*
exp(c)*(a - b)*(a*b^3*d^2)^(1/2))/(2*a*b*d*((a - b)^2)^(1/2))))*(a^2 - 2*a*
b + b^2)^(1/2))/(2*(a*b^3*d^2)^(1/2)) - exp(- c - d*x)/(2*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.321 $\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{b}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

```
[Out] x/b-arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))*(a-b)^(1/2)/b/d/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 391, 206, 208}

$$\frac{x}{b}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] x/b - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b
*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{bd} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{bd}$$
$$= \frac{x}{b} - \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,bd}$$

Mathematica [A] time = 0.09, size = 50, normalized size = 1.00

$$\frac{-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} + c + dx}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (c + d*x - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[
a])/(b*d)
```

fricas [B] time = 0.49, size = 443, normalized size = 8.86

$$\left[\frac{2\,dx + \sqrt{\frac{a-b}{a}}\,\log\left(\frac{b^2\cosh(dx+c)^4 + 4\,b^2\cosh(dx+c)\sinh(dx+c)^3 + b^2\sinh(dx+c)^4 + 2\,(2\,ab-b^2)\cosh(dx+c)^2 + 2\,(3\,b^2\cosh(dx+c)^2 + 2\,ab-b^2)\sinh(\ldots}{b\cosh(dx+c)^4 + 4\,b\cosh(dx+c)\sinh(dx+c)^3 + b\sinh(dx+c)^4 + 2\,(2\,a-b)\cos\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c
)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) +
 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x
+ c)^2 + 2*a^2 - a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a
 - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/(b*d), (d*x + sqrt(-(a - b)/a)*ar
ctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)))/(b*d)]
```

giac [A] time = 1.57, size = 68, normalized size = 1.36

$$-\frac{\frac{(a-b)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,b} - \frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

[Out] -((a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(
-a^2 + a*b)*b) - (d*x + c)/b)/d

maple [B] time = 0.09, size = 577, normalized size = 11.54

$$-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{db}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{db}+\frac{a\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{db\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}+\frac{a\arctan\left(\frac{a\tanh(}{\sqrt{(2\sqrt{-b(a-}}}\right.}{d\sqrt{-b(a-b)}\sqrt{(2\sqrt{-b(a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)+1/d*a/b/((
2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2))-1/d*a/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)+a-2*b)*a)^(1/2))-1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/(-b*(a-b))^(1/2)/(
(2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2))*b+1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 0.48, size = 166, normalized size = 3.32

$$\frac{x}{b}+\frac{\ln\left(\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a-b\right)}{b^2}-\frac{2\,\sqrt{a-b}\,\left(b+2\,a\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b^2}\right)\,\sqrt{a-b}}{2\,\sqrt{a}\,b\,d}-\frac{\ln\left(\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a-b\right)}{b^2}+\frac{2\,\sqrt{a-b}\,\left(b+2\,a\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{\sqrt{a}\,b^2}\right)\,\sqrt{}}{2\,\sqrt{a}\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] x/b + (log((4*exp(2*c + 2*d*x)*(a - b))/b^2 - (2*(a - b)^(1/2)*(b + 2*a*exp
(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a^(1/2)*b^2))*(a - b)^(1/2))/(2*a^(1/
2)*b*d) - (log((4*exp(2*c + 2*d*x)*(a - b))/b^2 + (2*(a - b)^(1/2)*(b + 2*a
*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/(a^(1/2)*b^2))*(a - b)^(1/2))/(2*a
^(1/2)*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.322 $\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

```
[Out] arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3190, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

fricas [B] time = 0.59, size = 459, normalized size = 14.34

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a+b)\cosh(dx+c)^2+2\,\left(3\,b\cosh(dx+c)^2-2\,a-b\right)\sinh(dx+c)^2+4\,\left(b\cos\right.}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)\cosh(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3
 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c
))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + s
inh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqr
t(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sin
h(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a -
 b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(
d*x + c) + b))/(a*b*d), (sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + si
nh(d*x + c))/a) + sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x +
c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cos
h(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)))/(a*b*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.The choice was done assuming [a,b]=[-18,-81]Warning, ne
ed to choose a branch for the root of a polynomial with parameters. This mi
ght be wrong.The choice was done assuming [a,b]=[-10,75]Undef/Unsigned Inf
encountered in limitEvaluation time: 0.92Limit: Max order reached or unable
 to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.02, size = 24, normalized size = 0.75

$$\frac{\arctan\left(\frac{\sinh(dx+c)b}{\sqrt{ab}}\right)}{d\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d/(a*b)^(1/2)*arctan(sinh(d*x+c)*b/(a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{b\sinh(dx+c)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^2 + a), x)
```

mupad [B] time = 0.87, size = 23, normalized size = 0.72

$$\frac{\mathrm{atan}\left(\frac{b\,\sinh(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^2),x)
```

```
[Out] atan((b*sinh(c + d*x))/(a*b)^(1/2))/(d*(a*b)^(1/2))
```

sympy [A] time = 4.50, size = 128, normalized size = 4.00

$$\begin{cases} \frac{\tilde{\infty} x \cosh(c)}{\sinh^2(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\sinh(c+dx)}{ad} & \text{for } b = 0 \\ \frac{x \cosh(c)}{a+b\sinh^2(c)} & \text{for } d = 0 \\ -\frac{1}{bd\sinh(c+dx)} & \text{for } a = 0 \\ -\frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sinh(c+dx)\right)}{2\sqrt{a}bd\sqrt{\frac{1}{b}}} + \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sinh(c+dx)\right)}{2\sqrt{a}bd\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sinh
(c + d*x)/(a*d), Eq(b, 0)), (x*cosh(c)/(a + b*sinh(c)**2), Eq(d, 0)), (-1/(
b*d*sinh(c + d*x)), Eq(a, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x)
)/(2*sqrt(a)*b*d*sqrt(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))/(2
*sqrt(a)*b*d*sqrt(1/b)), True))
```

3.323 $\int \frac{\text{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)}$$

```
[Out] arctan(sinh(d*x+c))/(a-b)/d-arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/(a-
b)/d/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3190, 391, 203, 205}

$$\frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)*d) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[c + d*x])
/Sqrt[a]])/(Sqrt[a]*(a - b)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{(a-b)d} \\
&= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)d} - \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)d}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 54, normalized size = 0.92

$$
\frac{\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}} + 2\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad-bd}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + 2*ArcTan[Tanh[
(c + d*x)/2]])/(a*d - b*d)
```

fricas [B] time = 0.89, size = 511, normalized size = 8.66

$$
\left[-\frac{\sqrt{-\frac{b}{a}}\,\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a+b)\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2-2\,a-b\right)\sinh(dx+c)^2+4\,(b}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)}\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^
3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^
2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x +
c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^
2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*
x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
+ c))*sinh(d*x + c) + b)) - 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/((a -
b)*d), -(sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) +
sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2
 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a
 - b)*sinh(d*x + c))*sqrt(b/a)/b) - 2*arctan(cosh(d*x + c) + sinh(d*x + c))
)/((a - b)*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Undef/Unsigned Inf encountered in limitLimit: Max order reache
d or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.13, size = 481, normalized size = 8.15

$$\frac{ba\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\left(a-b\right)\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}-\frac{b\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\left(a-b\right)\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}-\frac{b^{2}\arctan\left(-\right.}{d\left(a-b\right)\sqrt{-b\left(a-b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*b/(a-b)*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(
a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*b/(a-b)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2))-1/d*b^2/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)
*a)^(1/2))+1/d*b/(a-b)*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d
*b/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^2/(a-b)/(-b*(a-b))^(1/2)/((2*(
-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)+a-2*b)*a)^(1/2))+2/d/(a-b)*arctan(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\arctan\left(e^{(dx+c)}\right)}{ad-bd}-2\int\frac{be^{(3\,dx+3\,c)}+be^{(dx+c)}}{ab-b^{2}+\left(abe^{(4\,c)}-b^{2}e^{(4\,c)}\right)e^{(4\,dx)}+2\left(2\,a^{2}e^{(2\,c)}-3\,abe^{(2\,c)}+b^{2}e^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(d*x + c))/(a*d - b*d) - 2*integrate((b*e^(3*d*x + 3*c) + b*e^(d
*x + c))/(a*b - b^2 + (a*b*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^2*e^(2
*c) - 3*a*b*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 1.36, size = 648, normalized size = 10.98

$$\frac{2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\left(16\,a^2\,\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}+b^2\,\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}-8\,a\,b\,\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}\right)}{16\,d\,a^3-24\,d\,a^2\,b+9\,d\,a\,b^2-d\,b^3}\right)}{\sqrt{a^2\,d^2-2\,a\,b\,d^2+b^2\,d^2}}-\frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{b}\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}\,\sqrt{a\,d^2\,{\left(a-b\right)}^2}}{2\,a\,d\,\left(a-b\right)}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (2*atan((exp(d*x)*exp(c)*(16*a^2*(a^2*d^2 + b^2*d^2 - 2*a*b*d^2)^(1/2) + b^
2*(a^2*d^2 + b^2*d^2 - 2*a*b*d^2)^(1/2) - 8*a*b*(a^2*d^2 + b^2*d^2 - 2*a*b*
d^2)^(1/2)))/(16*a^3*d - b^3*d + 9*a*b^2*d - 24*a^2*b*d)))/(a^2*d^2 + b^2*d
^2 - 2*a*b*d^2)^(1/2) - (b^(1/2)*(2*atan((b^(1/2)*exp(d*x)*exp(c)*(a*d^2*(a
```

```
 - b)^2)^(1/2))/(2*a*d*(a - b))) - 2*atan(((a^3*b^(5/2)*(a^3*d^2 + a*b^2*d^
2 - 2*a^2*b*d^2)^(1/2) - a^2*b^(7/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1
/2))*(exp(d*x)*exp(c)*((64*(8*a^3*d + 2*a*b^2*d - 10*a^2*b*d))/(a*b^3*(a*b
- a^2)*(a*d^2*(a - b)^2)^(1/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2)) +
 (32*(b^(3/2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2) - 4*a*b^(1/2)*(a^3*
d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2)))/(a^2*b^(5/2)*d*(a - b)*(a*b - a^2)*(
a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2))) - (32*exp(3*c)*exp(3*d*x)*(b^(3/
2)*(a^3*d^2 + a*b^2*d^2 - 2*a^2*b*d^2)^(1/2) - 4*a*b^(1/2)*(a^3*d^2 + a*b^2
*d^2 - 2*a^2*b*d^2)^(1/2)))/(a^2*b^(5/2)*d*(a - b)*(a*b - a^2)*(a^3*d^2 + a
*b^2*d^2 - 2*a^2*b*d^2)^(1/2))))/(256*a - 64*b))))/(2*(a^3*d^2 + a*b^2*d^2
- 2*a^2*b*d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sinh(c + d*x)**2), x)
```

3.324 $\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh(c+dx)}{d(a-b)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{3/2}}$$

```
[Out] -b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(3/2)/d/a^(1/2)+tanh(d*x+
c)/(a-b)/d
```

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3191, 388, 208}

$$\frac{\tanh(c+dx)}{d(a-b)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d
)) + Tanh[c + d*x]/((a - b)*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{a-(a-b)x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\tanh(c+dx)}{(a-b)d}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx,x,\tanh(c+dx)\right)}{(a-b)d}\\
&= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{3/2}d}+\frac{\tanh(c+dx)}{(a-b)d}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 60, normalized size = 1.00

$$\frac{\tanh(c+dx)}{d(a-b)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d
)) + Tanh[c + d*x]/((a - b)*d)
```

fricas [B] time = 0.59, size = 709, normalized size = 11.82

$$\left[-\frac{\left(b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}+b\right)\sqrt{a^{2}-ab}\,\log\left(\frac{b^{2}\cosh(dx+c)^{4}+4\,b^{2}\cosh}{\cdots}\right.}{2\left(\left(a^{3}-2\,a^{2}b+ab^{2}\right)d\cosh\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x +
c)^2 + b)*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*si
nh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(
3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2
+ 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(
b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2
*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x +
c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
 + c))*sinh(d*x + c) + b)) + 4*a^2 - 4*a*b)/((a^3 - 2*a^2*b + a*b^2)*d*cosh
(d*x + c)^2 + 2*(a^3 - 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^
3 - 2*a^2*b + a*b^2)*d*sinh(d*x + c)^2 + (a^3 - 2*a^2*b + a*b^2)*d), ((b*co
sh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sq
rt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
+ c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) - 2*a^2 +
 2*a*b)/((a^3 - 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + 2*(a^3 - 2*a^2*b + a*b
^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - 2*a^2*b + a*b^2)*d*sinh(d*x + c)
^2 + (a^3 - 2*a^2*b + a*b^2)*d)]
```

giac [A] time = 0.72, size = 80, normalized size = 1.33

$$-\frac{\frac{b\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\sqrt{-a^{2}+ab}\,(a-b)}+\frac{2}{(a-b)\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(b*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
 a*b)*(a - b)) + 2/((a - b)*(e^(2*d*x + 2*c) + 1)))/d
```

maple [B] time = 0.12, size = 335, normalized size = 5.58

$$\frac{b\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\,(a-b)\sqrt{\left(2\sqrt{-b\,(a-b)}\,-a+2b\right)a}}+\frac{b^{2}\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\,(a-b)\sqrt{-b\,(a-b)}\,\sqrt{\left(2\sqrt{-b\,(a-b)}\,-a+2b\right)a}}-\frac{b\,\mathrm{arctanh}\left(\cdots\right.}{d\,(a-b)\sqrt{\left(2\sqrt{-b\,(\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*b/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b^2/(a-b)/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2))-1/d*b/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^2
/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d/(a-b)*tanh(1/2*d*
x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 1.47, size = 265, normalized size = 4.42

$$\frac{b\ln\left(\frac{4\left(2ab-b^2+8a^2e^{2c+2dx}+b^2e^{2c+2dx}-8abe^{2c+2dx}\right)}{a(a-b)^3}-\frac{8b+32ae^{2c+2dx}-16be^{2c+2dx}}{\sqrt{a}\,(a-b)^{5/2}}\right)}{2\sqrt{a}\,d\,(a-b)^{3/2}}-\frac{b\ln\left(\frac{8b+32ae^{2c+2dx}-16be^{2c+2dx}}{\sqrt{a}\,(a-b)^{5/2}}+\frac{4\left(2a\right.}{}\right.}{2\sqrt{a}\,d\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (b*log((4*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x) - 8*
a*b*exp(2*c + 2*d*x)))/(a*(a - b)^3) - (8*b + 32*a*exp(2*c + 2*d*x) - 16*b*
exp(2*c + 2*d*x))/(a^(1/2)*(a - b)^(5/2))))/(2*a^(1/2)*d*(a - b)^(3/2)) - (
b*log((8*b + 32*a*exp(2*c + 2*d*x) - 16*b*exp(2*c + 2*d*x))/(a^(1/2)*(a - b
)^(5/2)) + (4*(2*a*b - b^2 + 8*a^2*exp(2*c + 2*d*x) + b^2*exp(2*c + 2*d*x)
- 8*a*b*exp(2*c + 2*d*x)))/(a*(a - b)^3)))/(2*a^(1/2)*d*(a - b)^(3/2)) - 2/
((exp(2*c + 2*d*x) + 1)*(a*d - b*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sinh(c + d*x)**2), x)
```

3.325 $\int \frac{\text{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^2}+\frac{(a-3b)\tan^{-1}(\sinh(c+dx))}{2d(a-b)^2}+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d(a-b)}$$

```
[Out] 1/2*(a-3*b)*arctan(sinh(d*x+c))/(a-b)^2/d+b^(3/2)*arctan(sinh(d*x+c)*b^(1/2
)/a^(1/2))/(a-b)^2/d/a^(1/2)+1/2*sech(d*x+c)*tanh(d*x+c)/(a-b)/d
```

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 414, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^2}+\frac{(a-3b)\tan^{-1}(\sinh(c+dx))}{2d(a-b)^2}+\frac{\tanh(c+dx)\text{sech}(c+dx)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - 3*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^2*d) + (b^(3/2)*ArcTan[(Sqrt[b
]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^2*d) + (Sech[c + d*x]*Tanh[c +
d*x])/(2*(a - b)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
```

```
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+bx^2\right)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-bx^2}{\left(1+x^2\right)\left(a+bx^2\right)}\,dx, x, \sinh(c+dx)\right)}{2(a-b)d}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d} + \frac{(a-3b)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2(a-b)^2d} + \frac{b^2\operatorname{Subst}}{}$$

$$= \frac{(a-3b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^2d} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^2d} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx}{2(a-b)d}$$

Mathematica [A] time = 0.24, size = 91, normalized size = 0.99

$$\frac{-2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right) + 2\sqrt{a}\,(a-3b)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \sqrt{a}\,(a-b)\tanh(c+dx)\operatorname{sech}(c+dx)}{2\sqrt{a}\,d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-2*b^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*Sqrt[a]*(a - 3*b)*A
rcTan[Tanh[(c + d*x)/2]] + Sqrt[a]*(a - b)*Sech[c + d*x]*Tanh[c + d*x])/(2*
Sqrt[a]*(a - b)^2*d)
```

fricas [B] time = 0.71, size = 1644, normalized size = 17.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a - b)*cosh(d*x + c)^3 + 6*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 +
 2*(a - b)*sinh(d*x + c)^3 + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*
x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x +
c) + b)*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)
^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d
*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x +
c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
 + c))*sinh(d*x + c) + b)) + 2*((a - 3*b)*cosh(d*x + c)^4 + 4*(a - 3*b)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a - 3*b)*sinh(d*x + c)^4 + 2*(a - 3*b)*cosh(d
*x + c)^2 + 2*(3*(a - 3*b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c)^2 + 4*(
(a - 3*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + a - 3*
b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a - b)*cosh(d*x + c) + 2*(3*(
a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c))/((a^2 - 2*a*b + b^2)*d*cosh(
```

```
d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 -
 2*a*b + b^2)*d*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^2 +
 2*(3*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*d)*sinh(d
*x + c)^2 + (a^2 - 2*a*b + b^2)*d + 4*((a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^
3 + (a^2 - 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*x + c)), ((a - b)*cosh(d*x
+ c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3
+ (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^
4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(
b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(b/a)*arctan(1/
2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) + (b*cosh(d*x + c)^4 + 4*b*cos
h(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x
 + c))*sinh(d*x + c) + b)*sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cos
h(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) +
(3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) + ((a - 3*b)*co
sh(d*x + c)^4 + 4*(a - 3*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - 3*b)*sinh(
d*x + c)^4 + 2*(a - 3*b)*cosh(d*x + c)^2 + 2*(3*(a - 3*b)*cosh(d*x + c)^2 +
 a - 3*b)*sinh(d*x + c)^2 + 4*((a - 3*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d
*x + c))*sinh(d*x + c) + a - 3*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (
a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c))/(
(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*d*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*d*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b
 + b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a
^2 - 2*a*b + b^2)*d)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*d + 4*((a^2 - 2*
a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*
x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Unde
f/Unsigned Inf encountered in limitEvaluation time: 0.52Limit: Max order re
ached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.15, size = 662, normalized size = 7.20

$$-\frac{b^{2} a \arctan\left(\frac{a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\left(a-b\right)^{2}\sqrt{-b\left(a-b\right)}\,\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}+\frac{b^{2} \arctan\left(\frac{a \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\left(a-b\right)^{2}\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}+\frac{b^{3}\,\mathrm{ar}}{d\left(a-b\right)^{2}\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d*b^2/(a-b)^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*ar
ctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b^2/(a
-b)^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
```

```
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b^3/(a-b)^2/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2))-1/d*b^2/(a-b)^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/
2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2))-1/d*b^2/(a-b)^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^3/(a-b)^2/(
-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(a-b)^2/(tanh(1/2*d*x+1/2
*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*a+1/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*
tanh(1/2*d*x+1/2*c)^3*b+1/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*
x+1/2*c)*a-1/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*b+1/
d/(a-b)^2*arctan(tanh(1/2*d*x+1/2*c))*a-3/d/(a-b)^2*arctan(tanh(1/2*d*x+1/2
*c))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ae^{c} - 3\,be^{c})\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{a^{2}d - 2\,abd + b^{2}d} + \frac{e^{(3\,dx+3\,c)} - e^{(dx+c)}}{ad - bd + \left(ade^{(4\,c)} - bde^{(4\,c)}\right)e^{(4\,dx)} + 2\left(ade^{(2\,c)} - bde^{(2\,c)}\right)e^{(2\,dx)}} + 8\int \frac{}{4\left(a^{2}b - 2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (a*e^c - 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^2*d - 2*a*b*d + b^2*d) + (e
^(3*d*x + 3*c) - e^(d*x + c))/(a*d - b*d + (a*d*e^(4*c) - b*d*e^(4*c))*e^(4
*d*x) + 2*(a*d*e^(2*c) - b*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/4*(b^2*e^(
3*d*x + 3*c) + b^2*e^(d*x + c))/(a^2*b - 2*a*b^2 + b^3 + (a^2*b*e^(4*c) - 2
*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^3*e^(2*c) - 5*a^2*b*e^(2*c
) + 4*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 6.14, size = 2797, normalized size = 30.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] exp(c + d*x)/((exp(2*c + 2*d*x) + 1)*(a*d - b*d)) - (2*exp(c + d*x))/((a*d
- b*d)*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + ((2*atan((b^2*exp(d*x
)*exp(c)*(a*d^2*(a - b)^4)^(1/2))/(2*a*d*(a - b)^2*(b^3)^(1/2))) - 2*atan((
exp(d*x)*exp(c)*((64*(20*a^3*d*(b^3)^(5/2) - 232*a^6*d*(b^3)^(3/2) + 2*a^9*
d*(b^3)^(1/2) + 2*a*b^5*d*(b^3)^(3/2) + 10*a^5*b*d*(b^3)^(3/2) - 22*a^8*b*d
*(b^3)^(1/2) - 10*a^2*b^4*d*(b^3)^(3/2) - 20*a^4*b^2*d*(b^3)^(3/2) - 18*a^2
*b^7*d*(b^3)^(1/2) + 102*a^3*b^6*d*(b^3)^(1/2) - 242*a^4*b^5*d*(b^3)^(1/2)
+ 310*a^5*b^4*d*(b^3)^(1/2) + 98*a^7*b^2*d*(b^3)^(1/2)))/(a*b^4*(a - b)^5*(
a*b - a^2)*(a^2 - 2*a*b + b^2)*(a*d^2*(a - b)^4)^(1/2)*(3*a*b^2 - 3*a^2*b +
 a^3 - b^3)*(9*a*b^2 - 6*a^2*b + a^3 - b^3)*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*
d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)) - (32*(b^8*(a^5*d^2 + a*b^4*d^2
 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 36*a^2*b^6*(a^5*d^2
 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) - 47*a^3*
b^5*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/
2) + 30*a^4*b^4*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*
b^2*d^2)^(1/2) - 9*a^5*b^3*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d
^2 + 6*a^3*b^2*d^2)^(1/2) + a^6*b^2*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*
a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) - 12*a*b^7*(a^5*d^2 + a*b^4*d^2 - 4*a^4*
b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))/(a^2*b^2*d*(a - b)^7*(a*b -
a^2)*(b^3)^(1/2)*(a^2 - 2*a*b + b^2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(9*a*b
^2 - 6*a^2*b + a^3 - b^3)*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^
2 + 6*a^3*b^2*d^2)^(1/2))) + (32*exp(3*c)*exp(3*d*x)*(b^8*(a^5*d^2 + a*b^4*
d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 36*a^2*b^6*(a^5*
```

```
d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) - 47*a
^3*b^5*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^
(1/2) + 30*a^4*b^4*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a
^3*b^2*d^2)^(1/2) - 9*a^5*b^3*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^
3*d^2 + 6*a^3*b^2*d^2)^(1/2) + a^6*b^2*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 -
 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) - 12*a*b^7*(a^5*d^2 + a*b^4*d^2 - 4*a
^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))/(a^2*b^2*d*(a - b)^7*(a*b
 - a^2)*(b^3)^(1/2)*(a^2 - 2*a*b + b^2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(9*
a*b^2 - 6*a^2*b + a^3 - b^3)*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3
*d^2 + 6*a^3*b^2*d^2)^(1/2)))*((a^2*b^10*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2
 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/64 - (a^3*b^9*(a^5*d^2 + a*b^4*d^2
 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/8 + (7*a^4*b^8*(a^5*
d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/16 -
(7*a^5*b^7*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d
^2)^(1/2))/8 + (35*a^6*b^6*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d
^2 + 6*a^3*b^2*d^2)^(1/2))/32 - (7*a^7*b^5*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d
^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/8 + (7*a^8*b^4*(a^5*d^2 + a*b^4*
d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/16 - (a^9*b^3*(a^
5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/8 +
 (a^10*b^2*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2*b^3*d^2 + 6*a^3*b^2*d
^2)^(1/2))/64)))*(b^3)^(1/2))/(2*(a^5*d^2 + a*b^4*d^2 - 4*a^4*b*d^2 - 4*a^2
*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)) + (atan((exp(d*x)*exp(c)*(a^7*(a^4*d^2 + b
^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) - 3*b^7*(a^4*d^2
+ b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) + 55*a*b^6*(a^
4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) - 15*a^6
*b*(a^4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2) -
297*a^2*b^5*(a^4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)
^(1/2) + 423*a^3*b^4*(a^4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2
*b^2*d^2)^(1/2) - 272*a^4*b^3*(a^4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^
2 + 6*a^2*b^2*d^2)^(1/2) + 90*a^5*b^2*(a^4*d^2 + b^4*d^2 - 4*a*b^3*d^2 - 4*
a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)))/(a^8*d*(a^2 - 6*a*b + 9*b^2)^(1/2) + b^8
*d*(a^2 - 6*a*b + 9*b^2)^(1/2) + 130*a^2*b^6*d*(a^2 - 6*a*b + 9*b^2)^(1/2)
- 314*a^3*b^5*d*(a^2 - 6*a*b + 9*b^2)^(1/2) + 367*a^4*b^4*d*(a^2 - 6*a*b +
9*b^2)^(1/2) - 230*a^5*b^3*d*(a^2 - 6*a*b + 9*b^2)^(1/2) + 79*a^6*b^2*d*(a^
2 - 6*a*b + 9*b^2)^(1/2) - 20*a*b^7*d*(a^2 - 6*a*b + 9*b^2)^(1/2) - 14*a^7*
b*d*(a^2 - 6*a*b + 9*b^2)^(1/2)))*(a^2 - 6*a*b + 9*b^2)^(1/2))/(a^4*d^2 + b
^4*d^2 - 4*a*b^3*d^2 - 4*a^3*b*d^2 + 6*a^2*b^2*d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)**2), x)
```

3.326 $\int \frac{\mathrm{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{5/2}}-\frac{\tanh^3(c+dx)}{3d(a-b)}+\frac{(a-2b)\tanh(c+dx)}{d(a-b)^2}$$

```
[Out] b^2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(5/2)/d/a^(1/2)+(a-2*b)*
tanh(d*x+c)/(a-b)^2/d-1/3*tanh(d*x+c)^3/(a-b)/d
```

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3191, 390, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{5/2}}-\frac{\tanh^3(c+dx)}{3d(a-b)}+\frac{(a-2b)\tanh(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)*d
) + ((a - 2*b)*Tanh[c + d*x])/((a - b)^2*d) - Tanh[c + d*x]^3/(3*(a - b)*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a-2b}{(a-b)^2} - \frac{x^2}{a-b} + \frac{b^2}{(a-b)^2(a-(a-b)x^2)}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a-2b)\tanh(c+dx)}{(a-b)^2 d} - \frac{\tanh^3(c+dx)}{3(a-b)d} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{(a-b)^2 d}$$
$$= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{5/2}d} + \frac{(a-2b)\tanh(c+dx)}{(a-b)^2 d} - \frac{\tanh^3(c+dx)}{3(a-b)d}$$

Mathematica [A] time = 0.54, size = 84, normalized size = 0.95

$$\frac{\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a-b)^{5/2}} + \frac{\tanh(c+dx)\left((a-b)\operatorname{sech}^2(c+dx)+2a-5b\right)}{(a-b)^2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2
)) + ((2*a - 5*b + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b)^2)/(3*d)
```

fricas [B] time = 0.57, size = 2444, normalized size = 27.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b - a*b^2)*sinh(d*x + c)^4 - 8*a^3 + 28*a^2*b -
20*a*b^2 - 24*(a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2 - 24*(a^3 - 3*a^2*b
 + 2*a*b^2 - 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^2*co
sh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 +
 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^4 +
3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sin
h(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 + 6*b^2*cosh(d*x + c)^2 + b^2)*sinh
(d*x + c)^2 + b^2 + 6*(b^2*cosh(d*x + c)^5 + 2*b^2*cosh(d*x + c)^3 + b^2*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^
2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cos
h(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*
x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 +
2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
(2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 48*((a^2*b - a*b^2)*cosh(d*x
 + c)^3 - (a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 - 3
*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^6 + 6*(a^4 - 3*a^3*b + 3*a^2*b^
2 - a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a
*b^3)*d*sinh(d*x + c)^6 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x
```

```
+ c)^4 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + (a^4
 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^4 + 3*(a^4 - 3*a^3*b + 3*a^
2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)
*d*cosh(d*x + c)^3 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c))
*sinh(d*x + c)^3 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)
^4 + 6*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + (a^4 - 3*a^3
*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a
*b^3)*d + 6*((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^5 + 2*(a^4
 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^3 + (a^4 - 3*a^3*b + 3*a^2*
b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b - a*b^2)*cosh(d
*x + c)^4 + 24*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b - a
*b^2)*sinh(d*x + c)^4 - 4*a^3 + 14*a^2*b - 10*a*b^2 - 12*(a^3 - 3*a^2*b + 2
*a*b^2)*cosh(d*x + c)^2 - 12*(a^3 - 3*a^2*b + 2*a*b^2 - 3*(a^2*b - a*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x +
 c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 + 3*b^2*cosh(d*x + c)^4 + 3*(5*b^
2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2
*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x
 + c)^4 + 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 6*(b^2*cosh(
d*x + c)^5 + 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt
(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 24*((a^2*
b - a*b^2)*cosh(d*x + c)^3 - (a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(
d*x + c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^6 + 6*(a^4 -
 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - 3*a^
3*b + 3*a^2*b^2 - a*b^3)*d*sinh(d*x + c)^6 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 -
 a*b^3)*d*cosh(d*x + c)^4 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh
(d*x + c)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^4 + 3*(a
^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 - 3*a^3*b +
 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^3 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^
3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b
^3)*d*cosh(d*x + c)^4 + 6*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x +
c)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^4 - 3*a^
3*b + 3*a^2*b^2 - a*b^3)*d + 6*((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(
d*x + c)^5 + 2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^3 + (a^4
 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [A] time = 0.68, size = 138, normalized size = 1.57

$$\frac{\frac{3\,b^{2}\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\left(a^{2}-2\,ab+b^{2}\right)\sqrt{-a^{2}+ab}}+\frac{2\,\left(3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+12\,be^{(2\,dx+2\,c)}-2\,a+5\,b\right)}{\left(a^{2}-2\,ab+b^{2}\right)\left(e^{(2\,dx+2\,c)}+1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^2
 - 2*a*b + b^2)*sqrt(-a^2 + a*b)) + 2*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x +
 2*c) + 12*b*e^(2*d*x + 2*c) - 2*a + 5*b)/((a^2 - 2*a*b + b^2)*(e^(2*d*x +
2*c) + 1)^3))/d
```

maple [B] time = 0.15, size = 535, normalized size = 6.08

$$-\frac{b^{2}\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\left(a-b\right)^{2}\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}-\frac{b^{3}\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{-b(a-b)}-a+2b\right)a}}\right)}{d\left(a-b\right)^{2}\sqrt{-b\left(a-b\right)}\sqrt{\left(2\sqrt{-b\left(a-b\right)}-a+2b\right)a}}+\frac{b^{2}\operatorname{arctanh}\left(\cdots\right)}{d\left(a-b\right)^{2}\sqrt{\left(2\sqrt{-b\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d*b^2/(a-b)^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*b^3/(a-b)^2/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b^2/(a-b)^2/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/
2))-1/d*b^3/(a-b)^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d/(a-b
)^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*a-4/d/(a-b)^2/(tanh(1
/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*b+4/3/d/(a-b)^2/(tanh(1/2*d*x+1/
2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3*a-16/3/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1
)^3*tanh(1/2*d*x+1/2*c)^3*b+2/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/
2*d*x+1/2*c)*a-4/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)*
b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 2.46, size = 710, normalized size = 8.07

$$\frac{8}{3\,\left(a\,d-b\,d\right)\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}-\frac{4}{\left(a\,d-b\,d\right)\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}+\frac{\mathrm{atan}\left(\left({\mathrm{e}}^{2\,c}\,{\mathrm{e}}^{2\,d\,x}\,\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] 8/(3*(a*d - b*d)*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x
) + 1)) - 4/((a*d - b*d)*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (at
an((exp(2*c)*exp(2*d*x)*(4/(d*(a - b)^2*(b^4)^(1/2)*(a^2 - 2*a*b + b^2)) +
((2*a - b)*(2*a^3*d*(b^4)^(1/2) - b^3*d*(b^4)^(1/2) + 4*a*b^2*d*(b^4)^(1/2)
 - 5*a^2*b*d*(b^4)^(1/2)))/(b^4*(a^2 - 2*a*b + b^2)*(-a*d^2*(a - b)^5)^(1/2
)*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*
a^4*b^2*d^2)^(1/2))) + ((2*a - b)*(b^3*d*(b^4)^(1/2) - 2*a*b^2*d*(b^4)^(1/2
) + a^2*b*d*(b^4)^(1/2)))/(b^4*(a^2 - 2*a*b + b^2)*(-a*d^2*(a - b)^5)^(1/2)
*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*a
^4*b^2*d^2)^(1/2)))*((b^3*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^
2 + 10*a^3*b^3*d^2 - 10*a^4*b^2*d^2)^(1/2))/2 - a*b^2*(a*b^5*d^2 - a^6*d^2
+ 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*a^4*b^2*d^2)^(1/2) + (a
^2*b*(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2 - 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 -
10*a^4*b^2*d^2)^(1/2))/2))*(b^4)^(1/2))/(a*b^5*d^2 - a^6*d^2 + 5*a^5*b*d^2
- 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 - 10*a^4*b^2*d^2)^(1/2) + (2*b)/((exp(2*c
+ 2*d*x) + 1)*(a - b)*(a*d - b*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{4}{\left(c + d x \right)}}{a + b \sinh^{2}{\left(c + d x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**4/(a + b*sinh(c + d*x)**2), x)
```

3.327 $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{\left(3a^2-10ab+15b^2\right)\tan^{-1}(\sinh(c+dx))}{8d(a-b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^3}+\frac{\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d(a-b)}+\frac{(3a-7b)\tan}{}$$

```
[Out] 1/8*(3*a^2-10*a*b+15*b^2)*arctan(sinh(d*x+c))/(a-b)^3/d-b^(5/2)*arctan(sinh
(d*x+c)*b^(1/2)/a^(1/2))/(a-b)^3/d/a^(1/2)+1/8*(3*a-7*b)*sech(d*x+c)*tanh(d
*x+c)/(a-b)^2/d+1/4*sech(d*x+c)^3*tanh(d*x+c)/(a-b)/d
```

Rubi [A] time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 414, 527, 522, 203, 205}

$$\frac{\left(3a^2-10ab+15b^2\right)\tan^{-1}(\sinh(c+dx))}{8d(a-b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^3}+\frac{\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d(a-b)}+\frac{(3a-7b)\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 10*a*b + 15*b^2)*ArcTan[Sinh[c + d*x]])/(8*(a - b)^3*d) - (b^(5/2
)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^3*d) + ((3*a -
7*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*(a - b)^2*d) + (Sech[c + d*x]^3*Tanh[c
 + d*x])/(4*(a - b)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^3(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} - \frac{\operatorname{Subst}\left(\int \frac{-3a+4b-3bx^2}{(1+x^2)^2(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{4(a-b)d}$$

$$= \frac{(3a-7b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)^2d} + \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} + \frac{\operatorname{Subst}\left(\int \frac{3a^2-7a}{(}\right.}{}$$

$$= \frac{(3a-7b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)^2d} + \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} - \frac{b^3\operatorname{Subst}\left(\int \frac{1}{a+b}\right.}{(}$$

$$= \frac{(3a^2-10ab+15b^2)\tan^{-1}(\sinh(c+dx))}{8(a-b)^3d} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^3d} + \frac{(3a-7b)\operatorname{sech}}{8}$$

Mathematica [A] time = 0.55, size = 139, normalized size = 1.01

$$\frac{2\sqrt{a}\left(3a^2-10ab+15b^2\right)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{a}\left(3a^2-10ab+7b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)+8b^{5/2}\operatorname{ta}}{8\sqrt{a}\,d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (8*b^(5/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*Sqrt[a]*(3*a^2 - 10*
a*b + 15*b^2)*ArcTan[Tanh[(c + d*x)/2]] + Sqrt[a]*(3*a^2 - 10*a*b + 7*b^2)*
Sech[c + d*x]*Tanh[c + d*x] + 2*Sqrt[a]*(a - b)^2*Sech[c + d*x]^3*Tanh[c +
d*x])/(8*Sqrt[a]*(a - b)^3*d)
```

fricas [B] time = 0.64, size = 5500, normalized size = 39.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 - 10*a*b + 7*b^2)
*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 - 10*a*b + 7*b^2)*sinh(d*x + c)^7 +
```

```
(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 - 10*a*b + 7*b^2)*
cosh(d*x + c)^2 + 11*a^2 - 26*a*b + 15*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 -
 10*a*b + 7*b^2)*cosh(d*x + c)^3 + (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^4 - (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 + (35*(3*a^2
 - 10*a*b + 7*b^2)*cosh(d*x + c)^4 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x
 + c)^2 - 11*a^2 + 26*a*b - 15*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 - 10*a*b +
 7*b^2)*cosh(d*x + c)^5 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 - 3
*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^2*cosh(d*
x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*b^
2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 6*b^2
*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x
 + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 30*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(
d*x + c)^4 + 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d*x + c)^5 + 10*b^2*cosh
(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^2*cosh(d*x + c)
^6 + 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2
+ b^2 + 8*(b^2*cosh(d*x + c)^7 + 3*b^2*cosh(d*x + c)^5 + 3*b^2*cosh(d*x + c
)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d
*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d
*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 +
 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) +
(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh
(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + ((3*a^2 - 10*a
*b + 15*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)*si
nh(d*x + c)^7 + (3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^8 + 4*(3*a^2 - 10*a
*b + 15*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)
^2 + 3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - 10*a*b + 15*b
^2)*cosh(d*x + c)^3 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 6*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 - 10*a*b
 + 15*b^2)*cosh(d*x + c)^4 + 30*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 +
 9*a^2 - 30*a*b + 45*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)*
cosh(d*x + c)^5 + 10*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 -
 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^2 - 10*a*b + 15*b
^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^6 + 15*(
3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^4 + 9*(3*a^2 - 10*a*b + 15*b^2)*cosh
(d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^2 + 3*a^2 - 10*a*b + 1
5*b^2 + 8*((3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^7 + 3*(3*a^2 - 10*a*b +
15*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + (3*
a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) +
 sinh(d*x + c)) - (3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c) + (7*(3*a^2 - 10*a
*b + 7*b^2)*cosh(d*x + c)^6 + 5*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^4
- 3*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^2 - 3*a^2 + 10*a*b - 7*b^2)*si
nh(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*d*sinh(d*x + c)^8 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos
h(d*x + c)^6 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*sinh(d*x + c)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*d*cosh(d*x + c)^4 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*
x + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(35*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 30*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*d)*sinh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2
+ 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^5 + 10*(a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos
h(d*x + c)^6 + 15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 9*(a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 -
```

```
b^3)*d)*sinh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 8*((a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*d*cosh(d*x + c)^5 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 +
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/4*((3*a
^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x
+ c)*sinh(d*x + c)^6 + (3*a^2 - 10*a*b + 7*b^2)*sinh(d*x + c)^7 + (11*a^2 -
 26*a*b + 15*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x +
 c)^2 + 11*a^2 - 26*a*b + 15*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 - 10*a*b +
7*b^2)*cosh(d*x + c)^3 + (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^4 - (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 - 10*a*b
+ 7*b^2)*cosh(d*x + c)^4 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^2 -
11*a^2 + 26*a*b - 15*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 - 10*a*b + 7*b^2)*co
sh(d*x + c)^5 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 - 3*(11*a^2 -
 26*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(b^2*cosh(d*x + c)^8 +
 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*b^2*cosh(d*x
 + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 6*b^2*cosh(d*x
+ c)^4 + 8*(7*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^5 +
2*(35*b^2*cosh(d*x + c)^4 + 30*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^4
 + 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d*x + c)^5 + 10*b^2*cosh(d*x + c)^
3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^2*cosh(d*x + c)^6 + 15*b^
2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 8*
(b^2*cosh(d*x + c)^7 + 3*b^2*cosh(d*x + c)^5 + 3*b^2*cosh(d*x + c)^3 + b^2*
cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cosh(d*x + c)
 + sinh(d*x + c))) - 4*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x
+ c)^7 + b^2*sinh(d*x + c)^8 + 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x +
c)^2 + b^2)*sinh(d*x + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c
)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 30
*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^2 + 8*(
7*b^2*cosh(d*x + c)^5 + 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(7*b^2*cosh(d*x + c)^6 + 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh
(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 + 3*b^2*c
osh(d*x + c)^5 + 3*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*
sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2
 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a
 - b)*sinh(d*x + c))*sqrt(b/a)/b) + ((3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c
)^8 + 8*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 -
10*a*b + 15*b^2)*sinh(d*x + c)^8 + 4*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c
)^6 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 3*a^2 - 10*a*b + 15*
b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + 3*(
3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2 - 10*a*b
 + 15*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^
4 + 30*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 9*a^2 - 30*a*b + 45*b^2)
*sinh(d*x + c)^4 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^5 + 10*(3*a
^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 4*(
7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^6 + 15*(3*a^2 - 10*a*b + 15*b^2)*
cosh(d*x + c)^4 + 9*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 3*a^2 - 10*
a*b + 15*b^2)*sinh(d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2 + 8*((3*a^2 - 10*a*
b + 15*b^2)*cosh(d*x + c)^7 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^5 +
 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + (3*a^2 - 10*a*b + 15*b^2)*co
sh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (3*a^2
- 10*a*b + 7*b^2)*cosh(d*x + c) + (7*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)
^6 + 5*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^4 - 3*(11*a^2 - 26*a*b + 15
*b^2)*cosh(d*x + c)^2 - 3*a^2 + 10*a*b - 7*b^2)*sinh(d*x + c))/((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*
d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sinh(d*
x + c)^8 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^3
- 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*d)*sinh(d*x + c)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^
```

```
4 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 + 3*(a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*
cosh(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*sinh(d*x + c)^4 + 4*
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d*cosh(d*x + c)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh
(d*x + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^6 + 15*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*d*cosh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*sinh(d*x + c)^2
+ (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*
cosh(d*x + c)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^5 + 3*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2
 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[31,78]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[85,31]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[46
,18]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[-27,57]Unde
f/Unsigned Inf encountered in limitEvaluation time: 0.63Limit: Max order re
ached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.15, size = 1023, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*b^3/(a-b)^3*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*b^3/(a-
b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*b^4/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2))+1/d*b^3/(a-b)^3*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2
)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)
*a)^(1/2))+1/d*b^3/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^4/(a-b)^3/(-
b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-5/4/d/(a-b)^3/(tanh(1/2*d*x+1/
2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)^7*a^2+7/2/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+
1)^4*tanh(1/2*d*x+1/2*c)^7*a*b-9/4/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*ta
nh(1/2*d*x+1/2*c)^7*b^2+3/4/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*
d*x+1/2*c)^5*a^2-1/2/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2
*c)^5*a*b-1/4/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)^5*b
^2-3/4/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)^3*a^2+1/2/
d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)^3*a*b+1/4/d/(a-b)
```

```
^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)^3*b^2+5/4/d/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)*a^2-7/2/d/(a-b)^3/(tanh(1/2*d*x
+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)*a*b+9/4/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2
+1)^4*tanh(1/2*d*x+1/2*c)*b^2+3/4/d/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))*a^2
-5/2/d/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))*a*b+15/4/d/(a-b)^3*arctan(tanh(1
/2*d*x+1/2*c))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(3\,a^2e^c - 10\,abe^c + 15\,b^2e^c\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{4\left(a^3d - 3\,a^2bd + 3\,ab^2d - b^3d\right)} + \frac{\left(3\,ae^{(7\,c)} - 7\,be^{(7\,c)}\right)e^{(7}}{4\left(a^2d - 2\,abd + b^2d + \left(a^2de^{(8\,c)} - 2\,abde^{(8\,c)} + b^2de^{(8\,c)}\right)e^{(8\,dx)} + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*a^2*e^c - 10*a*b*e^c + 15*b^2*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^3*d
 - 3*a^2*b*d + 3*a*b^2*d - b^3*d) + 1/4*((3*a*e^(7*c) - 7*b*e^(7*c))*e^(7*d
*x) + (11*a*e^(5*c) - 15*b*e^(5*c))*e^(5*d*x) - (11*a*e^(3*c) - 15*b*e^(3*c
))*e^(3*d*x) - (3*a*e^c - 7*b*e^c)*e^(d*x))/(a^2*d - 2*a*b*d + b^2*d + (a^2
*d*e^(8*c) - 2*a*b*d*e^(8*c) + b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^2*d*e^(6*c)
- 2*a*b*d*e^(6*c) + b^2*d*e^(6*c))*e^(6*d*x) + 6*(a^2*d*e^(4*c) - 2*a*b*d*e
^(4*c) + b^2*d*e^(4*c))*e^(4*d*x) + 4*(a^2*d*e^(2*c) - 2*a*b*d*e^(2*c) + b^
2*d*e^(2*c))*e^(2*d*x)) - 32*integrate(1/16*(b^3*e^(3*d*x + 3*c) + b^3*e^(d
*x + c))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4 + (a^3*b*e^(4*c) - 3*a^2*b^2*e^
(4*c) + 3*a*b^3*e^(4*c) - b^4*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 7*a^3
*b*e^(2*c) + 9*a^2*b^2*e^(2*c) - 5*a*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)),
 x)
```

mupad [B] time = 12.05, size = 6237, normalized size = 45.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^5*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] (4*exp(c + d*x))/((a*d - b*d)*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*
exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (6*exp(c + d*x))/((a*d - b*d)*(
3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (atan((e
xp(d*x)*exp(c)*(243*a^12*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 1
5*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 3840*b^12*(a^6*d^2
 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 +
15*a^4*b^2*d^2)^(1/2) - 110560*a*b^11*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*
a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) - 4050*
a^11*b*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20
*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 976143*a^2*b^10*(a^6*d^2 + b^6*d^2 -
 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d
^2)^(1/2) - 2740050*a^3*b^9*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2
+ 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 4252775*a^4*b^8
*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b
^3*d^2 + 15*a^4*b^2*d^2)^(1/2) - 4316760*a^5*b^7*(a^6*d^2 + b^6*d^2 - 6*a*b
^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1
/2) + 3087390*a^6*b^6*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a
^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) - 1608364*a^7*b^5*(a^6*
d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2
 + 15*a^4*b^2*d^2)^(1/2) + 615750*a^8*b^4*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2
- 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) - 1
71000*a^9*b^3*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d
^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2) + 33075*a^10*b^2*(a^6*d^2 + b^6
*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4
```

*b^2*d^2)^(1/2)))/(81*a^13*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 256*b^13*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 82593*a^2*b^11*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) + 343611*a^3*b^10*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 788535*a^4*b^9*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) + 1157013*a^5*b^8*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 1173354*a^6*b^7*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) + 857934*a^7*b^6*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 461358*a^8*b^5*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) + 182890*a^9*b^4*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 52581*a^10*b^3*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) + 10503*a^11*b^2*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) + 7968*a*b^12*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2) - 1323*a^12*b*d*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2)))*(9*a^4 - 60*a^3*b - 300*a*b^3 + 225*b^4 + 190*a^2*b^2)^(1/2))/(4*(a^6*d^2 + b^6*d^2 - 6*a*b^5*d^2 - 6*a^5*b*d^2 + 15*a^2*b^4*d^2 - 20*a^3*b^3*d^2 + 15*a^4*b^2*d^2)^(1/2)) - ((2*atan((b^3*exp(d*x)*exp(c)*(a*d^2*(a - b)^6)^(1/2))/(2*a*d*(a - b)^3*(b^5)^(1/2))) - 2*atan((exp(d*x)*exp(c)*((4*(4032*a^5*d*(b^5)^(5/2) - 74990*a^10*d*(b^5)^(3/2) + 18*a^15*d*(b^5)^(1/2) + 32*a*b^9*d*(b^5)^(3/2) + 288*a^9*b*d*(b^5)^(3/2) - 282*a^14*b*d*(b^5)^(1/2) - 288*a^2*b^8*d*(b^5)^(3/2) + 1152*a^3*b^7*d*(b^5)^(3/2) - 2688*a^4*b^6*d*(b^5)^(3/2) - 4032*a^6*b^4*d*(b^5)^(3/2) + 2688*a^7*b^3*d*(b^5)^(3/2) - 1152*a^8*b^2*d*(b^5)^(3/2) - 450*a^2*b^13*d*(b^5)^(1/2) + 4650*a^3*b^12*d*(b^5)^(1/2) - 21980*a^4*b^11*d*(b^5)^(1/2) + 62940*a^5*b^10*d*(b^5)^(1/2) - 121878*a^6*b^9*d*(b^5)^(1/2) + 168702*a^7*b^8*d*(b^5)^(1/2) - 172008*a^8*b^7*d*(b^5)^(1/2) + 131112*a^9*b^6*d*(b^5)^(1/2) + 31878*a^11*b^4*d*(b^5)^(1/2) - 9852*a^12*b^3*d*(b^5)^(1/2) + 2108*a^13*b^2*d*(b^5)^(1/2)))/(a*b^4*(a - b)^7*(a*b - a^2)*(a*d^2*(a - b)^6)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(225*a*b^4 - 60*a^4*b + 9*a^5 - 16*b^5 - 300*a^2*b^3 + 190*a^3*b^2)*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)) + (2*(16*b^14*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 321*a*b^13*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) + 1890*a^2*b^12*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 5685*a^3*b^11*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) + 10440*a^4*b^10*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 12690*a^5*b^9*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) + 10620*a^6*b^8*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 6210*a^7*b^7*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) + 2520*a^8*b^6*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 685*a^9*b^5*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) + 114*a^10*b^4*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 9*a^11*b^3*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)))/(a^2*b*d*(a - b)^10*(a*b - a^2)*(b^5)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)*(225*a*b^4 - 60*a^4*b + 9*a^5 - 16*b^5 - 300*a^2*b^3 + 190*a^3*b^2)*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2))) - (2*exp(3*c)*exp(3*d*x)*(16*b^14*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*

```
b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 321*a*b^13*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b
*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1
/2) + 1890*a^2*b^12*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15
*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 5685*a^3*b^11*(a^7*
d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3
*d^2 + 15*a^5*b^2*d^2)^(1/2) + 10440*a^4*b^10*(a^7*d^2 + a*b^6*d^2 - 6*a^6*
b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(
1/2) - 12690*a^5*b^9*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 1
5*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) + 10620*a^6*b^8*(a^7
*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^
3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 6210*a^7*b^7*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b
*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1
/2) + 2520*a^8*b^6*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*
a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) - 685*a^9*b^5*(a^7*d^2
 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^
2 + 15*a^5*b^2*d^2)^(1/2) + 114*a^10*b^4*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2
 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)
- 9*a^11*b^3*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^
4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)))/(a^2*b*d*(a - b)^10*(a*b -
 a^2)*(b^5)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a^4 - 4*a^3*b - 4*a*b^3
+ b^4 + 6*a^2*b^2)*(225*a*b^4 - 60*a^4*b + 9*a^5 - 16*b^5 - 300*a^2*b^3 + 1
90*a^3*b^2)*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4
*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)*(a^6 - 6*a^5*b - 6*a*b^5 + b^
6 + 15*a^2*b^4 - 20*a^3*b^3 + 15*a^4*b^2)))*((a^17*b*(a^7*d^2 + a*b^6*d^2 -
 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2
*d^2)^(1/2))/4 - (a^2*b^16*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d
^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/4 + (15*a^3*b
^15*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 2
0*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/4 - (105*a^4*b^14*(a^7*d^2 + a*b^6*d
^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5
*b^2*d^2)^(1/2))/4 + (455*a^5*b^13*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a
^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/4 - (
1365*a^6*b^12*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b
^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/4 + (3003*a^7*b^11*(a^7*d^
2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d
^2 + 15*a^5*b^2*d^2)^(1/2))/4 - (5005*a^8*b^10*(a^7*d^2 + a*b^6*d^2 - 6*a^6
*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^
(1/2))/4 + (6435*a^9*b^9*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2
 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/4 - (6435*a^10*
b^8*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 2
0*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/4 + (5005*a^11*b^7*(a^7*d^2 + a*b^6*
d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^
5*b^2*d^2)^(1/2))/4 - (3003*a^12*b^6*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6
*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/4 +
 (1365*a^13*b^5*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3
*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/4 - (455*a^14*b^4*(a^7*d
^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*
d^2 + 15*a^5*b^2*d^2)^(1/2))/4 + (105*a^15*b^3*(a^7*d^2 + a*b^6*d^2 - 6*a^6
*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^
(1/2))/4 - (15*a^16*b^2*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2
+ 15*a^3*b^4*d^2 - 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/4)))*(b^5)^(1/2)
)/(2*(a^7*d^2 + a*b^6*d^2 - 6*a^6*b*d^2 - 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 -
20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)) + (exp(c + d*x)*(a + 3*b))/(2*(a -
b)*(a*d - b*d)*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (exp(c + d*x)
*(3*a^2 - 10*a*b + 7*b^2))/(4*(exp(2*c + 2*d*x) + 1)*(a - b)^2*(a*d - b*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.328 $\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{(a^2-3ab+3b^2)\tanh(c+dx)}{d(a-b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{7/2}}+\frac{\tanh^5(c+dx)}{5d(a-b)}-\frac{(2a-3b)\tanh^3(c+dx)}{3d(a-b)^2}$$

```
[Out] -b^3*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/(a-b)^(7/2)/d/a^(1/2)+(a^2-3*
a*b+3*b^2)*tanh(d*x+c)/(a-b)^3/d-1/3*(2*a-3*b)*tanh(d*x+c)^3/(a-b)^2/d+1/5*
tanh(d*x+c)^5/(a-b)/d
```

Rubi [A] time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3191, 390, 208}

$$\frac{(a^2-3ab+3b^2)\tanh(c+dx)}{d(a-b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d(a-b)^{7/2}}+\frac{\tanh^5(c+dx)}{5d(a-b)}-\frac{(2a-3b)\tanh^3(c+dx)}{3d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/2)
*d)) + ((a^2 - 3*a*b + 3*b^2)*Tanh[c + d*x])/((a - b)^3*d) - ((2*a - 3*b)*T
anh[c + d*x]^3)/(3*(a - b)^2*d) + Tanh[c + d*x]^5/(5*(a - b)*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{(a-b)^3} - \frac{(2a-3b)x^2}{(a-b)^2} + \frac{x^4}{a-b} - \frac{b^3}{(a-b)^3(a-(a-b)x^2)}\right) dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3 d} - \frac{(2a-3b)\tanh^3(c+dx)}{3(a-b)^2 d} + \frac{\tanh^5(c+dx)}{5(a-b)d} - \frac{b^3 \mathrm{S}}{}$$

$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}d} + \frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3 d} - \frac{(2a-3b)\tanh^3(c}{3(a-b)^2 d}$$

Mathematica [A] time = 0.93, size = 119, normalized size = 0.94

$$\frac{\frac{\tanh(c+dx)\left((4a^2-13ab+9b^2)\operatorname{sech}^2(c+dx)+8a^2+3(a-b)^2\operatorname{sech}^4(c+dx)-26ab+33b^2\right)}{(a-b)^3} - \frac{15b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((-15*b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7
/2)) + ((8*a^2 - 26*a*b + 33*b^2 + (4*a^2 - 13*a*b + 9*b^2)*Sech[c + d*x]^2
 + 3*(a - b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(a - b)^3)/(15*d)
```

fricas [B] time = 0.83, size = 6046, normalized size = 47.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(60*(a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 480*(a^2*b^2 - a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^7 + 60*(a^2*b^2 - a*b^3)*sinh(d*x + c)^8 - 120*(a^3*b
 - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^6 - 120*(a^3*b - 4*a^2*b^2 + 3*a*b^3
- 14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 240*(14*(a^2*b^2
- a*b^3)*cosh(d*x + c)^3 - 3*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c))*s
inh(d*x + c)^5 + 40*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c
)^4 + 40*(105*(a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*a^4 - 31*a^3*b + 47*a^2
*b^2 - 24*a*b^3 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 32*a^4 - 136*a^3*b + 236*a^2*b^2 - 132*a*b^3 + 160*(21*(a^2*b^2
- a*b^3)*cosh(d*x + c)^5 - 15*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3
 + (8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^
3 + 40*(4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3)*cosh(d*x + c)^2 + 40*(42*
(a^2*b^2 - a*b^3)*cosh(d*x + c)^6 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d
*x + c)^4 + 4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3 + 6*(8*a^4 - 31*a^3*b
+ 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 15*(b^3*cosh(d*
x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sinh(d*x + c)^10 + 5
*b^3*cosh(d*x + c)^8 + 10*b^3*cosh(d*x + c)^6 + 5*(9*b^3*cosh(d*x + c)^2 +
b^3)*sinh(d*x + c)^8 + 40*(3*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(
d*x + c)^7 + 10*b^3*cosh(d*x + c)^4 + 10*(21*b^3*cosh(d*x + c)^4 + 14*b^3*c
osh(d*x + c)^2 + b^3)*sinh(d*x + c)^6 + 4*(63*b^3*cosh(d*x + c)^5 + 70*b^3*
cosh(d*x + c)^3 + 15*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 5*b^3*cosh(d*x +
```

c)^2 + 10*(21*b^3*cosh(d*x + c)^6 + 35*b^3*cosh(d*x + c)^4 + 15*b^3*cosh(d*
x + c)^2 + b^3)*sinh(d*x + c)^4 + 40*(3*b^3*cosh(d*x + c)^7 + 7*b^3*cosh(d*
x + c)^5 + 5*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3
 + 5*(9*b^3*cosh(d*x + c)^8 + 28*b^3*cosh(d*x + c)^6 + 30*b^3*cosh(d*x + c)
^4 + 12*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 10*(b^3*cosh(d*x + c)^
9 + 4*b^3*cosh(d*x + c)^7 + 6*b^3*cosh(d*x + c)^5 + 4*b^3*cosh(d*x + c)^3 +
 b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4
 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b
^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)
^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x
+ c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*
cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x +
c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x +
c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 80*(6*(a^2*b^2 - a*b^
3)*cosh(d*x + c)^7 - 9*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5 + 2*(8
*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 17*a^3*
b + 28*a^2*b^2 - 15*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 - 4*a^4*b +
6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^10 + 10*(a^5 - 4*a^4*b + 6*a
^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 - 4*a^4*
b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^10 + 5*(a^5 - 4*a^4*b +
6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^8 + 5*(9*(a^5 - 4*a^4*b + 6*
a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2
 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^8 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 -
4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4
*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^
3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*(a^5 - 4*a^4*b + 6*a^3
*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 14*(a^5 - 4*a^4*b + 6*a^3*b^2
 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^
2*b^3 + a*b^4)*d)*sinh(d*x + c)^6 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b
^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^
3 + a*b^4)*d*cosh(d*x + c)^5 + 70*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 +
a*b^4)*d*cosh(d*x + c)^3 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^
4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4
*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 35*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2
*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^
4)*d)*sinh(d*x + c)^4 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d
*cosh(d*x + c)^2 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*
cosh(d*x + c)^7 + 7*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(
d*x + c)^5 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x +
 c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 5*(9*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh
(d*x + c)^8 + 28*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x
 + c)^6 + 30*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c
)^4 + 12*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2
+ (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^2 + (a^5
 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d + 10*((a^5 - 4*a^4*b + 6*a^3*
b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^9 + 4*(a^5 - 4*a^4*b + 6*a^3*b^2 -
 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^7 + 6*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^
2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 4*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^
4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/15*(30*(a^2*b^2 - a*b^3)*cosh(d*x +
c)^8 + 240*(a^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + 30*(a^2*b^2 -
a*b^3)*sinh(d*x + c)^8 - 60*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^6 -
 60*(a^3*b - 4*a^2*b^2 + 3*a*b^3 - 14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^6 + 120*(14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^3 - 3*(a^3*b - 4*a^
2*b^2 + 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 20*(8*a^4 - 31*a^3*b + 47
*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^4 + 20*(105*(a^2*b^2 - a*b^3)*cosh(d*x +

c)^4 + 8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*a^4 - 68*a^3*b + 118*a^2*b^2 - 66*a*b^3 + 80*(21*(a^2*b^2 - a*b^3)*cosh(d*x + c)^5 - 15*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 + (8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 20*(4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3)*cosh(d*x + c)^2 + 20*(42*(a^2*b^2 - a*b^3)*cosh(d*x + c)^6 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3 + 6*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 15*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sinh(d*x + c)^10 + 5*b^3*cosh(d*x + c)^8 + 10*b^3*cosh(d*x + c)^6 + 5*(9*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^8 + 40*(3*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c)^7 + 10*b^3*cosh(d*x + c)^4 + 10*(21*b^3*cosh(d*x + c)^4 + 14*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^6 + 4*(63*b^3*cosh(d*x + c)^5 + 70*b^3*cosh(d*x + c)^3 + 15*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 5*b^3*cosh(d*x + c)^2 + 10*(21*b^3*cosh(d*x + c)^6 + 35*b^3*cosh(d*x + c)^4 + 15*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^4 + 40*(3*b^3*cosh(d*x + c)^7 + 7*b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + 5*(9*b^3*cosh(d*x + c)^8 + 28*b^3*cosh(d*x + c)^6 + 30*b^3*cosh(d*x + c)^4 + 12*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 10*(b^3*cosh(d*x + c)^9 + 4*b^3*cosh(d*x + c)^7 + 6*b^3*cosh(d*x + c)^5 + 4*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 40*(6*(a^2*b^2 - a*b^3)*cosh(d*x + c)^7 - 9*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5 + 2*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^10 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^10 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^8 + 5*(9*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^8 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 14*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^6 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 70*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 35*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^4 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^7 + 7*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^8 + 28*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 30*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 12*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d + 10*((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^9 + 4*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^7 + 6*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 4*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a

```
^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
+ a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 0.70, size = 253, normalized size = 2.01

$$-\frac{\frac{15\,b^3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\sqrt{-a^2+ab}}+\frac{2\left(15\,b^2e^{(8\,dx+8\,c)}-30\,abe^{(6\,dx+6\,c)}+90\,b^2e^{(6\,dx+6\,c)}+80\,a^2e^{(4\,dx+4\,c)}-230\,abe^{(4\,dx+4\,c)}+240\,b^2e^{(4\,dx+4\,c)}+40\,a^2e^{(2\,dx+2}\right.}{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*b^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-a^2 + a*b)) + 2*(15*b^2*e^(8*d*x + 8*c
) - 30*a*b*e^(6*d*x + 6*c) + 90*b^2*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x + 4*c
) - 230*a*b*e^(4*d*x + 4*c) + 240*b^2*e^(4*d*x + 4*c) + 40*a^2*e^(2*d*x + 2
*c) - 130*a*b*e^(2*d*x + 2*c) + 150*b^2*e^(2*d*x + 2*c) + 8*a^2 - 26*a*b +
33*b^2)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(e^(2*d*x + 2*c) + 1)^5))/d
```

maple [B] time = 0.17, size = 907, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*b^3/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b^4/(a-b)^3/(-b*(a-b))^(1/
2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*b^3/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
))+1/d*b^4/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*ar
ctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d/(a-b)
^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^9*a^2-6/d/(a-b)^3/(tanh(
1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^9*a*b+6/d/(a-b)^3/(tanh(1/2*d*x+1
/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^9*b^2+8/3/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2
+1)^5*tanh(1/2*d*x+1/2*c)^7*a^2-32/3/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*
tanh(1/2*d*x+1/2*c)^7*a*b+16/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2
*d*x+1/2*c)^7*b^2+116/15/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x
+1/2*c)^5*a^2-332/15/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2
*c)^5*a*b+132/5/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^5
*b^2+8/3/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^3*a^2-32
/3/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^3*a*b+16/d/(a-
b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^3*b^2+2/d/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)*a^2-6/d/(a-b)^3/(tanh(1/2*d*x+1
/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)*a*b+6/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^
5*tanh(1/2*d*x+1/2*c)*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [B] time = 2.89, size = 1152, normalized size = 9.14

$$\frac{16}{(a\,d-b\,d)\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}-\frac{32}{5\,(a\,d-b\,d)\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^6*(a + b*sinh(c + d*x)^2)),x)
```

```
[Out] 16/((a*d - b*d)*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*
x) + exp(8*c + 8*d*x) + 1)) - 32/(5*(a*d - b*d)*(5*exp(2*c + 2*d*x) + 10*ex
p(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d
*x) + 1)) + (atan((exp(2*c)*exp(2*d*x)*((4*b)/(d*(a - b)^3*(b^6)^(1/2)*(3*a
*b^2 - 3*a^2*b + a^3 - b^3)) + ((2*a - b)*(2*a^4*d*(b^6)^(1/2) + b^4*d*(b^6
)^(1/2) - 5*a*b^3*d*(b^6)^(1/2) - 7*a^3*b*d*(b^6)^(1/2) + 9*a^2*b^2*d*(b^6)
^(1/2)))/(b^5*(-a*d^2*(a - b)^7)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a*b
^7*d^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^
4*d^2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2))) - ((2*a - b)*(b^4*d*(b^6)^
(1/2) - 3*a*b^3*d*(b^6)^(1/2) - a^3*b*d*(b^6)^(1/2) + 3*a^2*b^2*d*(b^6)^(1/
2)))/(b^5*(-a*d^2*(a - b)^7)^(1/2)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(a*b^7*d
^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^
2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2)))*((b^4*(a*b^7*d^2 - a^8*d^2 + 7
*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d
^2 - 21*a^6*b^2*d^2)^(1/2))/2 + (3*a^2*b^2*(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d
^2 - 7*a^2*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21*
a^6*b^2*d^2)^(1/2))/2 - (3*a*b^3*(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2
*b^6*d^2 + 21*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^
2)^(1/2))/2 - (a^3*b*(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 2
1*a^3*b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2))/2)
)*(b^6)^(1/2))/(a*b^7*d^2 - a^8*d^2 + 7*a^7*b*d^2 - 7*a^2*b^6*d^2 + 21*a^3*
b^5*d^2 - 35*a^4*b^4*d^2 + 35*a^5*b^3*d^2 - 21*a^6*b^2*d^2)^(1/2) - (2*b^2)
/((exp(2*c + 2*d*x) + 1)*(a - b)^2*(a*d - b*d)) + (4*(a*b - b^2))/((a - b)^
2*(a*d - b*d)*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (8*(4*a - 3*b)
)/(3*(a - b)*(a*d - b*d)*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.329 $\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=158

$$\frac{(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}-\frac{x(4a-5b)}{2b^3}+\frac{(2a-b)(a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)}$$

```
[Out] -1/2*(4*a-5*b)*x/b^3+1/2*(a-b)^(3/2)*(4*a+b)*arctanh((a-b)^(1/2)*tanh(d*x+c
)/a^(1/2))/a^(3/2)/b^3/d+1/2*cosh(d*x+c)*sinh(d*x+c)/b/d/(a-(a-b)*tanh(d*x+
c)^2)+1/2*(a-b)*(2*a-b)*tanh(d*x+c)/a/b^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.26, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3191, 414, 527, 522, 206, 208}

$$\frac{(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(2a-b)(a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{x(4a-5b)}{2b^3}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((4*a - 5*b)*x)/(2*b^3) + ((a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b]*Ta
nh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2
*b*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((a - b)*(2*a - b)*Tanh[c + d*x])/(2*
a*b^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-a+2b-3(a-b)x^2}{(1-x^2)\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+d\right.}{2bd}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{(a-b)(2a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{(a-b)(2a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{(4a-5b}{}$$

$$= -\frac{(4a-5b)x}{2b^3} + \frac{(a-b)^{3/2}(4a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{\cosh(c+dx)\sin}{2bd\left(a-(a-b)\tan\right.}$$

Mathematica [A] time = 0.60, size = 118, normalized size = 0.75

$$\frac{\frac{2(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + 2(5b-4a)(c+dx) + \frac{2b(a-b)^2\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)} + b\sinh(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*(-4*a + 5*b)*(c + d*x) + (2*(a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b]
*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + b*Sinh[2*(c + d*x)] + (2*(a - b)^2*b*Si
nh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)]))))/(4*b^3*d)
```

fricas [B] time = 0.56, size = 3629, normalized size = 22.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(a*b^2*cosh(d*x + c)^8 + 8*a*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + a*b^2
*sinh(d*x + c)^8 + 2*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x
 + c)^6 + 2*(14*a*b^2*cosh(d*x + c)^2 + 2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*
b^2)*d*x)*sinh(d*x + c)^6 + 4*(14*a*b^2*cosh(d*x + c)^3 + 3*(2*a^2*b - a*b^
2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(2*a^3 -
5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^4
 + 2*(35*a*b^2*cosh(d*x + c)^4 - 8*a^3 + 20*a^2*b - 16*a*b^2 + 4*b^3 - 4*(8
*a^3 - 14*a^2*b + 5*a*b^2)*d*x + 15*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a*b^2*cosh(d*x + c)^5 + 5*(2
*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3 - 4*(2*a^3 - 5*
a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 - a*b^2 - 2*(6*a^2*b - 9*a*b^2 + 4*b^3 + 2*(4*a^2*b - 5*a*b^2
)*d*x)*cosh(d*x + c)^2 + 2*(14*a*b^2*cosh(d*x + c)^6 + 15*(2*a^2*b - a*b^2
- 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 - 6*a^2*b + 9*a*b^2 - 4*b^3 -
2*(4*a^2*b - 5*a*b^2)*d*x - 24*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 -
14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*((4*a^2*b - 3
*a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)*s
inh(d*x + c)^5 + (4*a^2*b - 3*a*b^2 - b^3)*sinh(d*x + c)^6 + 2*(8*a^3 - 10*
a^2*b + a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^3 - 20*a^2*b + 2*a*b^2 + 2*b^3
 + 15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*
a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(8*a^3 - 10*a^2*b + a*b^2 + b^3)
*cosh(d*x + c))*sinh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2
 + (15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^4 + 4*a^2*b - 3*a*b^2 - b^3
+ 12*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*
(3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 4*(8*a^3 - 10*a^2*b + a*b^2
+ b^3)*cosh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d
*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^
2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*
(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*
cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 +
 2*a^2 - a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b)) + 4*(2*a*b^2*cosh(d*x + c)^7 + 3*(2*a^2*b
- a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^5 - 8*(2*a^3 - 5*a^2*b +
 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3 - (6*a^2
*b - 9*a*b^2 + 4*b^3 + 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x +
 c))/(a*b^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a
*b^4*d*sinh(d*x + c)^6 + a*b^4*d*cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d*
cosh(d*x + c)^4 + (15*a*b^4*d*cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d)*si
nh(d*x + c)^4 + 4*(5*a*b^4*d*cosh(d*x + c)^3 + 2*(2*a^2*b^3 - a*b^4)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + (15*a*b^4*d*cosh(d*x + c)^4 + a*b^4*d + 12*(2*
a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*a*b^4*d*cosh(d*x
 + c)^5 + a*b^4*d*cosh(d*x + c) + 4*(2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3)*
sinh(d*x + c)), 1/8*(a*b^2*cosh(d*x + c)^8 + 8*a*b^2*cosh(d*x + c)*sinh(d*x
 + c)^7 + a*b^2*sinh(d*x + c)^8 + 2*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2
)*d*x)*cosh(d*x + c)^6 + 2*(14*a*b^2*cosh(d*x + c)^2 + 2*a^2*b - a*b^2 - 2*
(4*a^2*b - 5*a*b^2)*d*x)*sinh(d*x + c)^6 + 4*(14*a*b^2*cosh(d*x + c)^3 + 3*
(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^
5 - 8*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*
cosh(d*x + c)^4 + 2*(35*a*b^2*cosh(d*x + c)^4 - 8*a^3 + 20*a^2*b - 16*a*b^2
 + 4*b^3 - 4*(8*a^3 - 14*a^2*b + 5*a*b^2)*d*x + 15*(2*a^2*b - a*b^2 - 2*(4*
a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a*b^2*cosh(d*
x + c)^5 + 5*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3
- 4*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^3 - a*b^2 - 2*(6*a^2*b - 9*a*b^2 + 4*b^3 + 2*(4*
a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*a*b^2*cosh(d*x + c)^6 + 15*(2
*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 - 6*a^2*b + 9*a
```

```
*b^2 - 4*b^3 - 2*(4*a^2*b - 5*a*b^2)*d*x - 24*(2*a^3 - 5*a^2*b + 4*a*b^2 -
b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 -
4*((4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(4*a^2*b - 3*a*b^2 - b^3)*
cosh(d*x + c)*sinh(d*x + c)^5 + (4*a^2*b - 3*a*b^2 - b^3)*sinh(d*x + c)^6 +
 2*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^3 - 20*a^2*b +
2*a*b^2 + 2*b^3 + 15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 4*(5*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(8*a^3 - 10*a^2*b
 + a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*
cosh(d*x + c)^2 + (15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^4 + 4*a^2*b -
 3*a*b^2 - b^3 + 12*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 2*(3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 4*(8*a^3 - 10
*a^2*b + a*b^2 + b^3)*cosh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x
+ c))*sinh(d*x + c))*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)
/(a - b)) + 4*(2*a*b^2*cosh(d*x + c)^7 + 3*(2*a^2*b - a*b^2 - 2*(4*a^2*b -
5*a*b^2)*d*x)*cosh(d*x + c)^5 - 8*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3
 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3 - (6*a^2*b - 9*a*b^2 + 4*b^3 +
2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a*b^4*d*cosh(d*x
+ c)^6 + 6*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^4*d*sinh(d*x + c)^6
+ a*b^4*d*cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + (15*a
*b^4*d*cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^4 + 4*(5*a*
b^4*d*cosh(d*x + c)^3 + 2*(2*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c
)^3 + (15*a*b^4*d*cosh(d*x + c)^4 + a*b^4*d + 12*(2*a^2*b^3 - a*b^4)*d*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*a*b^4*d*cosh(d*x + c)^5 + a*b^4*d*cosh(
d*x + c) + 4*(2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3)*sinh(d*x + c))]
```

giac [B] time = 7.19, size = 305, normalized size = 1.93

$$-\frac{\frac{12\,(dx+c)(4\,a-5\,b)}{b^3}-\frac{3\,e^{(2\,dx+2\,c)}}{b^2}-\frac{12\left(4\,a^3-7\,a^2b+2\,ab^2+b^3\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,ab^3}-\frac{8\,a^2be^{(6\,dx+6\,c)}-10\,ab^2e^{(6\,dx+6\,c)}-16\,a^3e^{(4\,dx+4\,c)}+64\cdots}{(b\cdots}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(d*x + c)*(4*a - 5*b)/b^3 - 3*e^(2*d*x + 2*c)/b^2 - 12*(4*a^3 - 7
*a^2*b + 2*a*b^2 + b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2
+ a*b))/(sqrt(-a^2 + a*b)*a*b^3) - (8*a^2*b*e^(6*d*x + 6*c) - 10*a*b^2*e^(6
*d*x + 6*c) - 16*a^3*e^(4*d*x + 4*c) + 64*a^2*b*e^(4*d*x + 4*c) - 79*a*b^2*
e^(4*d*x + 4*c) + 24*b^3*e^(4*d*x + 4*c) - 28*a^2*b*e^(2*d*x + 2*c) + 44*a*
b^2*e^(2*d*x + 2*c) - 24*b^3*e^(2*d*x + 2*c) - 3*a*b^2)/((b*e^(6*d*x + 6*c)
 + 4*a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x + 4*c) + b*e^(2*d*x + 2*c))*a*b^3))/d
```

maple [B] time = 0.14, size = 1659, normalized size = 10.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2))-1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2))+1/2/d/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-2/d/b/(tanh(1/2*d*
x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/
```

```
2*d*x+1/2*c)^3-2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*t
anh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)-1/d/b/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2))+1/d/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)
*a-2/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3-5/2
/d/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+5/2/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+1/d/b^
2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^
2*b+a)*a*tanh(1/2*d*x+1/2*c)-2/d/b^3*a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d/b
^3*a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/(
(2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+7/2/d/b^2*a/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
))-7/2/d/b^2*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b^2/(tanh(1/2*d*x+1/2*c)^4
*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/
2*c)^3+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)+
1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2+7/2
/d/b*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+7/2/d/b*a/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2))*b-2/d/b^2*a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-2
/d/b^2*a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b)
)^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^6}{{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.330 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{\left(3a^2-2ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)}{b^2d}$$

```
[Out] -1/2*(3*a^2-2*a*b-b^2)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)/
d+sinh(d*x+c)/b^2/d+1/2*(a-b)^2*sinh(d*x+c)/a/b^2/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A] time = 0.14, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 390, 385, 205}

$$-\frac{\left(3a^2-2ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)
*b^(5/2)*d) + Sinh[c + d*x]/(b^2*d) + ((a - b)^2*Sinh[c + d*x])/(2*a*b^2*d*
(a + b*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a^2-b^2+2(a-b)bx^2}{b^2(a+bx^2)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\sinh(c+dx)}{b^2d} - \frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{b^2d}$$
$$= \frac{\sinh(c+dx)}{b^2d} + \frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)} - \frac{((a-b)(3a+b))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx\right.}{2ab^2d}$$
$$= -\frac{(a-b)(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{\sinh(c+dx)}{b^2d} + \frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx\right.}$$

Mathematica [A] time = 0.30, size = 106, normalized size = 1.02

$$\frac{-\frac{(-3a^2+2ab+b^2)\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{a^{3/2}} + \frac{2\sqrt{b}(a-b)^2\sinh(c+dx)}{a(2a+b\cosh(2(c+dx))-b)} + 2\sqrt{b}\sinh(c+dx)}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-(((-3*a^2 + 2*a*b + b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/a^(3/2)
) + 2*Sqrt[b]*Sinh[c + d*x] + (2*(a - b)^2*Sqrt[b]*Sinh[c + d*x])/(a*(2*a -
 b + b*Cosh[2*(c + d*x)])))/(2*b^(5/2)*d)
```

fricas [B] time = 0.75, size = 2739, normalized size = 26.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^2*b^2*cosh(d*x + c)^6 + 12*a^2*b^2*cosh(d*x + c)*sinh(d*x + c)^5
+ 2*a^2*b^2*sinh(d*x + c)^6 + 2*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x +
c)^4 + 2*(15*a^2*b^2*cosh(d*x + c)^2 + 6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*sinh(
d*x + c)^4 - 2*a^2*b^2 + 8*(5*a^2*b^2*cosh(d*x + c)^3 + (6*a^3*b - 7*a^2*b^
2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(6*a^3*b - 7*a^2*b^2 + 2*a*
b^3)*cosh(d*x + c)^2 + 2*(15*a^2*b^2*cosh(d*x + c)^4 - 6*a^3*b + 7*a^2*b^2
- 2*a*b^3 + 6*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + ((3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(3*a^2*b - 2*a*b^2 - b
^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (3*a^2*b - 2*a*b^2 - b^3)*sinh(d*x + c)
^5 + 2*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^3 + 2*(6*a^3 - 7*a^2*b + b^3 +
 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(3*a^2
*b - 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x +
c))*sinh(d*x + c)^2 + (3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c) + (5*(3*a^2*b
 - 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 3*a^2*b - 2*a*b^2 - b^3 + 6*(6*a^3 - 7*
a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a*b)*log((b*cosh(d*x + c
)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*c
osh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*c
```

```
osh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^
3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2
- 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*
b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x
+ c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x
+ c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(3*a^2*b^2*cosh(d
*x + c)^5 + 2*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^3 - (6*a^3*b -
7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^4*d*cosh(d*x + c)
^5 + 5*a^2*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*b^4*d*sinh(d*x + c)^5
+ a^2*b^4*d*cosh(d*x + c) + 2*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + 2*(
5*a^2*b^4*d*cosh(d*x + c)^2 + (2*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^3 + 2*
(5*a^2*b^4*d*cosh(d*x + c)^3 + 3*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sin
h(d*x + c)^2 + (5*a^2*b^4*d*cosh(d*x + c)^4 + a^2*b^4*d + 6*(2*a^3*b^3 - a^
2*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/2*(a^2*b^2*cosh(d*x + c)^6 + 6*
a^2*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*b^2*sinh(d*x + c)^6 + (6*a^3*b
- 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + (15*a^2*b^2*cosh(d*x + c)^2 + 6*a^
3*b - 7*a^2*b^2 + 2*a*b^3)*sinh(d*x + c)^4 - a^2*b^2 + 4*(5*a^2*b^2*cosh(d*
x + c)^3 + (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 -
 (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2 + (15*a^2*b^2*cosh(d*x + c
)^4 - 6*a^3*b + 7*a^2*b^2 - 2*a*b^3 + 6*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^5
+ 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (3*a^2*b - 2*
a*b^2 - b^3)*sinh(d*x + c)^5 + 2*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^3 +
2*(6*a^3 - 7*a^2*b + b^3 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^3 + 2*(5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(6*a^3 -
7*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (3*a^2*b - 2*a*b^2 - b^3)*c
osh(d*x + c) + (5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 3*a^2*b - 2*a
*b^2 - b^3 + 6*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt
(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - ((3*a^2*b -
 2*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)
*sinh(d*x + c)^4 + (3*a^2*b - 2*a*b^2 - b^3)*sinh(d*x + c)^5 + 2*(6*a^3 - 7
*a^2*b + b^3)*cosh(d*x + c)^3 + 2*(6*a^3 - 7*a^2*b + b^3 + 5*(3*a^2*b - 2*a
*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(3*a^2*b - 2*a*b^2 - b^
3)*cosh(d*x + c)^3 + 3*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)
^2 + (3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c) + (5*(3*a^2*b - 2*a*b^2 - b^3)
*cosh(d*x + c)^4 + 3*a^2*b - 2*a*b^2 - b^3 + 6*(6*a^3 - 7*a^2*b + b^3)*cosh
(d*x + c)^2)*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*c
osh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c)
+ (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) + 2*(3*a^
2*b^2*cosh(d*x + c)^5 + 2*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^3 -
 (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^4*d*c
osh(d*x + c)^5 + 5*a^2*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*b^4*d*sinh
(d*x + c)^5 + a^2*b^4*d*cosh(d*x + c) + 2*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x
+ c)^3 + 2*(5*a^2*b^4*d*cosh(d*x + c)^2 + (2*a^3*b^3 - a^2*b^4)*d)*sinh(d*x
 + c)^3 + 2*(5*a^2*b^4*d*cosh(d*x + c)^3 + 3*(2*a^3*b^3 - a^2*b^4)*d*cosh(d
*x + c))*sinh(d*x + c)^2 + (5*a^2*b^4*d*cosh(d*x + c)^4 + a^2*b^4*d + 6*(2*
a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
```

```
[a,b]=[66,-29]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[-
21,2]Warning, need to choose a branch for the root of a polynomial with par
ameters. This might be wrong.The choice was done assuming [a,b]=[15,2]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-92,94]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[44,-86]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.The choice was done assuming [a,b]=[-90,-5]Warning, need to choose a bra
nch for the root of a polynomial with parameters. This might be wrong.The c
hoice was done assuming [a,b]=[-94,-77]Warning, need to choose a branch for
 the root of a polynomial with parameters. This might be wrong.The choice w
as done assuming [a,b]=[36,-73]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[91,55]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[17,-27]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[2
4,-71]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.The choice was done assuming [a,b]=[-39,-6]Wa
rning, need to choose a branch for the root of a polynomial with parameters
. This might be wrong.The choice was done assuming [a,b]=[61,-49]Undef/Unsi
gned Inf encountered in limitEvaluation time: 2.51Limit: Max order reached
or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.12, size = 1403, normalized size = 13.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/b^2/(tanh(1/2*d*x+1/2*c)-1)-1/d/b^2/(tanh(1/2*d*x+1/2*c)+1)-1/d/b^2/(t
anh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+
a)*a*tanh(1/2*d*x+1/2*c)^3+2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d
*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh
(1/2*d*x+1/2*c)^3+1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)-2/d/b/(tanh(1/2*d*x+1/
2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*
x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)+3/2/d/b^2*a^2/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2))-3/2/d/b^2*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/2/d/b*a
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/2/d/b^2*a^2/(-b*(a-b))^(1/
2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/2/d/b^2*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-
5/2/d/b*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2))+1/2/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/b/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2))+1/2/d/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
```

```
1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/a/(-b*(a-b))^(1/2)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2))*b-1/2/d/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/a/(-b
*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1
/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{abe^{(6\,dx+6\,c)} - ab + \left(6\,a^2e^{(4\,c)} - 7\,abe^{(4\,c)} + 2\,b^2e^{(4\,c)}\right)e^{(4\,dx)} - \left(6\,a^2e^{(2\,c)} - 7\,abe^{(2\,c)} + 2\,b^2e^{(2\,c)}\right)e^{(2\,dx)}}{2\left(ab^3de^{(5\,dx+5\,c)} + ab^3de^{(dx+c)} + 2\left(2\,a^2b^2de^{(3\,c)} - ab^3de^{(3\,c)}\right)e^{(3\,dx)}\right)} - \frac{1}{32}\int \frac{32\left(\left(3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b*e^(6*d*x + 6*c) - a*b + (6*a^2*e^(4*c) - 7*a*b*e^(4*c) + 2*b^2*e^(
4*c))*e^(4*d*x) - (6*a^2*e^(2*c) - 7*a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*d*x)
)/(a*b^3*d*e^(5*d*x + 5*c) + a*b^3*d*e^(d*x + c) + 2*(2*a^2*b^2*d*e^(3*c) -
 a*b^3*d*e^(3*c))*e^(3*d*x)) - 1/32*integrate(32*((3*a^2*e^(3*c) - 2*a*b*e^
(3*c) - b^2*e^(3*c))*e^(3*d*x) + (3*a^2*e^c - 2*a*b*e^c - b^2*e^c)*e^(d*x))
/(a*b^3*e^(4*d*x + 4*c) + a*b^3 + 2*(2*a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(
2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(c+d\,x\right)}^5}{{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.331 $\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{x}{b^2}$$

```
[Out] x/b^2-1/2*(2*a+b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))*(a-b)^(1/2)/a^(3
/2)/b^2/d-1/2*(a-b)*tanh(d*x+c)/a/b/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.14, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 414, 522, 206, 208}

$$-\frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] x/b^2 - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(2*a^(3/2)*b^2*d) - ((a - b)*Tanh[c + d*x])/(2*a*b*d*(a - (a - b)*Tanh[c
+ d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
```

```
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a-b+(-a+b)x^2}{(1-x^2)\left(a+(-a+b)x^2\right)}\,dx, x, \tanh(c+\right.}{2abd}$$

$$= -\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{b^2d} - \frac{((a\cdots}{\cdots}$$

$$= \frac{x}{b^2} - \frac{\sqrt{a-b}\,(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx\right.}$$

Mathematica [A] time = 0.62, size = 108, normalized size = 1.08

$$\frac{-\frac{\left(2a^2-ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}} + \frac{b(b-a)\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)} + 2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) - ((2*a^2 - a*b - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqr
t[a]])/(a^(3/2)*Sqrt[a - b]) + (b*(-a + b)*Sinh[2*(c + d*x)])/(a*(2*a - b +
 b*Cosh[2*(c + d*x)])))/(2*b^2*d)
```

fricas [B] time = 0.50, size = 1527, normalized size = 15.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*d*x*cosh(d*x + c)^4 + 16*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 4*a*b*d*x*sinh(d*x + c)^4 + 4*a*b*d*x + 4*(2*(2*a^2 - a*b)*d*x + 2*a^2 -
3*a*b + b^2)*cosh(d*x + c)^2 + 4*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - a*
b)*d*x + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh(d*x + c
)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*
x + c)^4 + 2*(4*a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x +
c)^2 + 4*a^2 - b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d
*x + c)^3 + (4*a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a - b)/a)*log
((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x
+ c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
 - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2
*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*c
osh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 - a*b)*sqrt((a - b
)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
 c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sin
```

```
h(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c
) + b)) + 4*a*b - 4*b^2 + 8*(2*a*b*d*x*cosh(d*x + c)^3 + (2*(2*a^2 - a*b)*d
*x + 2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x +
 c)^4 + 4*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^3*d*sinh(d*x + c)^4 +
 a*b^3*d + 2*(2*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*a*b^3*d*cosh(d*x
+ c)^2 + (2*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + 4*(a*b^3*d*cosh(d*x + c)^
3 + (2*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*d*x*cos
h(d*x + c)^4 + 8*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*b*d*x*sinh(d*x
 + c)^4 + 2*a*b*d*x + 2*(2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a*b + b^2)*cosh(d*
x + c)^2 + 2*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a
*b + b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2
)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(4*a^2
- b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 4*a^2 - b^2)*
sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (4*a^2 -
 b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*s
qrt(-(a - b)/a)/(a - b)) + 2*a*b - 2*b^2 + 4*(2*a*b*d*x*cosh(d*x + c)^3 + (
2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(a
*b^3*d*cosh(d*x + c)^4 + 4*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^3*d*
sinh(d*x + c)^4 + a*b^3*d + 2*(2*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*
a*b^3*d*cosh(d*x + c)^2 + (2*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + 4*(a*b^3
*d*cosh(d*x + c)^3 + (2*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [A] time = 5.40, size = 178, normalized size = 1.78

$$\frac{\frac{2\,(dx+c)}{b^2}-\frac{\left(2\,a^2-ab-b^2\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,ab^2}+\frac{2\left(2\,a^2e^{(2\,dx+2\,c)}-3\,abe^{(2\,dx+2\,c)}+b^2e^{(2\,dx+2\,c)}+ab-b^2\right)}{\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)ab^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/b^2 - (2*a^2 - a*b - b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) +
2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a*b^2) + 2*(2*a^2*e^(2*d*x + 2
*c) - 3*a*b*e^(2*d*x + 2*c) + b^2*e^(2*d*x + 2*c) + a*b - b^2)/((b*e^(4*d*x
 + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)*a*b^2))/d
```

maple [B] time = 0.12, size = 1116, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b/
(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*
b+a)*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*
c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3-1/d/b/(tanh(1/2
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh
(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)+1/d/b^2*a/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)
*a)^(1/2))+1/d/b*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*ar
ctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/b^2*a/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*
a)^(1/2))-1/2/d/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/(-b*(a-b))^(1/2)/((2*(
```

```
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2))+1/2/d/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctan
h(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/((2*(-b*(a-b))^(1/2)-a+2*b)
*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
)-1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b+1/2/d/a/((2*(-b*(
a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a
)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))
*b
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+dx)^4}{\left(b\sinh(c+dx)^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.332 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2} dx$

Optimal. Leaf size=77

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*(a+b)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)/d-1/2*(a-b)*s
inh(d*x+c)/a/b/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3190, 385, 205}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) - (
(a - b)*Sinh[c + d*x])/(2*a*b*d*(a + b*Sinh[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{2abd}$$

$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} - \frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.35, size = 75, normalized size = 0.97

$$\frac{\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} - \frac{(a-b)\sinh(c+dx)}{2ab\left(a+b\sinh^2(c+dx)\right)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) - ((
a - b)*Sinh[c + d*x])/(2*a*b*(a + b*Sinh[c + d*x]^2)))/d
```

fricas [B] time = 0.56, size = 1615, normalized size = 20.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b - a*b^2)*sinh(d*x + c)^3 + ((a*b + b^2)*cosh(d*
x + c)^4 + 4*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b + b^2)*sinh(d
*x + c)^4 + 2*(2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*cosh(d
*x + c)^2 + 2*a^2 + a*b - b^2)*sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)
*cosh(d*x + c)^3 + (2*a^2 + a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-
a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*
x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*
sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x
+ c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)
^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)
/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
 + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b
)) - 4*(a^2*b - a*b^2)*cosh(d*x + c) - 4*(a^2*b - a*b^2 - 3*(a^2*b - a*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c))/(a^2*b^3*d*cosh(d*x + c)^4 + 4*a^2*b^3*d*c
osh(d*x + c)*sinh(d*x + c)^3 + a^2*b^3*d*sinh(d*x + c)^4 + a^2*b^3*d + 2*(2
*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^3*d*cosh(d*x + c)^2 + (2
*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + 4*(a^2*b^3*d*cosh(d*x + c)^3 + (2*
a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a^2*b - a*b^2)
*cosh(d*x + c)^3 + 6*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2
*b - a*b^2)*sinh(d*x + c)^3 - ((a*b + b^2)*cosh(d*x + c)^4 + 4*(a*b + b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 + a*
b - b^2)*cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + a*b -
```

```
 b^2)*sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(d*x + c)^3 + (2*a^2
 + a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*sqrt(a*b)*
(cosh(d*x + c) + sinh(d*x + c))/a) - ((a*b + b^2)*cosh(d*x + c)^4 + 4*(a*b
+ b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a
^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*cosh(d*x + c)^2 + 2*a^2
+ a*b - b^2)*sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(d*x + c)^3 +
 (2*a^2 + a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*(b*
cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (
4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqr
t(a*b)/(a*b)) - 2*(a^2*b - a*b^2)*cosh(d*x + c) - 2*(a^2*b - a*b^2 - 3*(a^2
*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^2*b^3*d*cosh(d*x + c)^4 + 4*
a^2*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b^3*d*sinh(d*x + c)^4 + a^2*b
^3*d + 2*(2*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^3*d*cosh(d*x
+ c)^2 + (2*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + 4*(a^2*b^3*d*cosh(d*x +
 c)^3 + (2*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[66,-29]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[-
21,2]Warning, need to choose a branch for the root of a polynomial with par
ameters. This might be wrong.The choice was done assuming [a,b]=[15,2]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-92,94]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[44,-86]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.The choice was done assuming [a,b]=[-27,-68]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.The
choice was done assuming [a,b]=[-70,50]Warning, need to choose a branch for
 the root of a polynomial with parameters. This might be wrong.The choice w
as done assuming [a,b]=[-63,-1]Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[91,-7]Undef/Unsigned Inf encountered in limitEvaluation tim
e: 1.8Limit: Max order reached or unable to make series expansion Error: Ba
d Argument Value
```

maple [B] time = 0.12, size = 808, normalized size = 10.49

$$\frac{\tanh^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)}-\frac{\tanh^{3}\left(\right.}{d\left(\left(\tanh^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^{2}\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*
```

```
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3-1/d/b/(ta
nh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)*tanh(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-1/2/d/b*a/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/
d/b*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/b/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2))+1/2/d/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/a/(-b*(a-b))^(1/
2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b-1/2/d/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2
/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(ae^{(3\,c)}-be^{(3\,c)}\right)e^{(3\,dx)}-\left(ae^{c}-be^{c}\right)e^{(dx)}}{ab^{2}de^{(4\,dx+4\,c)}+ab^{2}d+2\left(2\,a^{2}bde^{(2\,c)}-ab^{2}de^{(2\,c)}\right)e^{(2\,dx)}}+\frac{1}{8}\int\frac{8\left(\left(ae^{(3\,c)}+be^{(3\,c)}\right)e^{(3\,dx)}+\left(ae^{c}+be^{c}\right)e^{(dx)}\right)}{ab^{2}e^{(4\,dx+4\,c)}+ab^{2}+2\left(2\,a^{2}be^{(2\,c)}-ab^{2}e^{(2\,c)}\right)e^{(2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -((a*e^(3*c) - b*e^(3*c))*e^(3*d*x) - (a*e^c - b*e^c)*e^(d*x))/(a*b^2*d*e^(
4*d*x + 4*c) + a*b^2*d + 2*(2*a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x))
 + 1/8*integrate(8*((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) + (a*e^c + b*e^c)*e^(
d*x))/(a*b^2*e^(4*d*x + 4*c) + a*b^2 + 2*(2*a^2*b*e^(2*c) - a*b^2*e^(2*c))*
e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cosh\left(c+d\,x\right)^{3}}{{\left(b\,\sinh\left(c+d\,x\right)^{2}+a\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.333 $\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}}+\frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

```
[Out] 1/2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/d/(a-b)^(1/2)+1/2*tanh
(d*x+c)/a/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3191, 199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}}+\frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]*d) + Ta
nh[c + d*x]/(2*a*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}\,d} + \frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.21, size = 78, normalized size = 0.99

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a}\,\sinh(2(c+dx))}{2a+b\cosh(2(c+dx))-b}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[a - b] + (Sqrt[a]*Sinh[2
*(c + d*x)])/(2*a - b + b*Cosh[2*(c + d*x)]))/(2*a^(3/2)*d)
```

fricas [B] time = 0.68, size = 1421, normalized size = 17.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*
(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b
+ a*b^2)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(
d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b
^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x +
 c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^
2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)
^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b
^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(
d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
 - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/((a^3
*b^2 - a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)*s
inh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^4*b - 3*a^3
*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - a^2*b^3)*d*cosh(d*x + c
)^2 + (2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 - a^2*b
^3)*d + 4*((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b - 3*a^3*b^2 + a
^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b - 2*a*b^2 + 2*(2*a^3
 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x
+ c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c)^2 + (b^2*cos
h(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 +
2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*s
inh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c)
```

```
)*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh
(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^
2 - a*b)))/((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - a^2*b^3)*d
*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*sinh(d*x + c)^4 + 2*
(2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - a^2*b^3
)*d*cosh(d*x + c)^2 + (2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d)*sinh(d*x + c)^2 +
(a^3*b^2 - a^2*b^3)*d + 4*((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b
 - 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [A] time = 2.16, size = 126, normalized size = 1.59

$$\frac{\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a}-\frac{2\left(2\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b\right)}{\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)ab}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*a) - 2*(2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)/((b*e^(4*d*x +
4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)*a*b))/d
```

maple [B] time = 0.11, size = 404, normalized size = 5.11

$$\frac{\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)a}+\frac{\tanh\left(\right.}{d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2))*b+1/2/d/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b))^(1/2)/((2*(
-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)+a-2*b)*a)^(1/2))*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^2}{{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^2, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.334 $\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] 1/2*sinh(d*x+c)/a/d/(a+b*sinh(d*x+c)^2)+1/2*arctan(sinh(d*x+c)*b^(1/2)/a^(1
/2))/a^(3/2)/d/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d}+\frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Sinh[c + d*
x]/(2*a*d*(a + b*Sinh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{2ad}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\,d} + \frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.97

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{\sinh(c+dx)}{2a\left(a+b\sinh^2(c+dx)\right)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sinh[c + d*x
]/(2*a*(a + b*Sinh[c + d*x]^2)))/d
```

fricas [B] time = 0.71, size = 1320, normalized size = 20.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b*
sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(
3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*
a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-a*b)*log((b*cosh(d*x + c)^4
+ 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(
d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(
d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)
*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*co
sh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)
^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(3*a*b*cosh(d*x + c)^
2 - a*b)*sinh(d*x + c))/(a^2*b^2*d*cosh(d*x + c)^4 + 4*a^2*b^2*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + a^2*b^2*d*sinh(d*x + c)^4 + a^2*b^2*d + 2*(2*a^3*b -
a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^2*d*cosh(d*x + c)^2 + (2*a^3*b - a^
2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^2*b^2*d*cosh(d*x + c)^3 + (2*a^3*b - a^2*b
^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*cosh(d*x + c)^3 + 6*a*b*cos
h(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 - 2*a*b*cosh(d*x + c) +
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) + (b*cos
h(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2
```

```
*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(
a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*
sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)
*sinh(d*x + c))*sqrt(a*b)/(a*b)) + 2*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x
 + c))/(a^2*b^2*d*cosh(d*x + c)^4 + 4*a^2*b^2*d*cosh(d*x + c)*sinh(d*x + c)
^3 + a^2*b^2*d*sinh(d*x + c)^4 + a^2*b^2*d + 2*(2*a^3*b - a^2*b^2)*d*cosh(d
*x + c)^2 + 2*(3*a^2*b^2*d*cosh(d*x + c)^2 + (2*a^3*b - a^2*b^2)*d)*sinh(d*
x + c)^2 + 4*(a^2*b^2*d*cosh(d*x + c)^3 + (2*a^3*b - a^2*b^2)*d*cosh(d*x +
c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[66,-29]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[-
21,2]Warning, need to choose a branch for the root of a polynomial with par
ameters. This might be wrong.The choice was done assuming [a,b]=[15,2]Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-45,5]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[89,-20]Undef/Unsigned Inf enc
ountered in limitEvaluation time: 0.96Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

maple [A] time = 0.03, size = 57, normalized size = 0.86

$$\frac{\sinh \left(dx+c\right)}{2ad\left(a+b\left(\sinh^{2}\left(dx+c\right)\right)\right)}+\frac{\arctan\left(\frac{\sinh(dx+c)b}{\sqrt{ab}}\right)}{2da\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/2*sinh(d*x+c)/a/d/(a+b*sinh(d*x+c)^2)+1/2/d/a/(a*b)^(1/2)*arctan(sinh(d*x
+c)*b/(a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{(3\,dx+3\,c)}-e^{(dx+c)}}{abde^{(4\,dx+4\,c)}+abd+2\left(2\,a^{2}de^{(2\,c)}-abde^{(2\,c)}\right)e^{(2\,dx)}}+\frac{1}{2}\int\frac{2\left(e^{(3\,dx+3\,c)}+e^{(dx+c)}\right)}{abe^{(4\,dx+4\,c)}+ab+2\left(2\,a^{2}e^{(2\,c)}-abe^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) - e^(d*x + c))/(a*b*d*e^(4*d*x + 4*c) + a*b*d + 2*(2*a^2*d
*e^(2*c) - a*b*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(2*(e^(3*d*x + 3*c) + e
^(d*x + c))/(a*b*e^(4*d*x + 4*c) + a*b + 2*(2*a^2*e^(2*c) - a*b*e^(2*c))*e^
(2*d*x)), x)
```

mupad [B] time = 0.91, size = 54, normalized size = 0.82

$$\frac{\sinh(c+d\,x)}{2\,a\,\left(b\,d\sinh(c+d\,x)^2+a\,d\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sinh(c+d\,x)}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^2)^2,x)
```

```
[Out] sinh(c + d*x)/(2*a*(a*d + b*d*sinh(c + d*x)^2)) + atan((b^(1/2)*sinh(c + d*
x))/a^(1/2))/(2*a^(3/2)*b^(1/2)*d)
```

sympy [A] time = 22.64, size = 428, normalized size = 6.48

$$\begin{cases} \frac{\tilde{\infty}x\cosh(c)}{\sinh^4(c)} \\ \frac{\sinh(c+dx)}{a^2d} \\ -\frac{1}{3b^2d\sinh^3(c+dx)} \\ \frac{x\cosh(c)}{\left(a+b\sinh^2(c)\right)^2} \\ \frac{2i\sqrt{a}\,b\sqrt{\frac{1}{b}}\sinh(c+dx)}{4ia^{\frac{5}{2}}bd\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}}\sinh^2(c+dx)}+\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sinh(c+dx)\right)}{4ia^{\frac{5}{2}}bd\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}}\sinh^2(c+dx)}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sinh(c+dx)\right)}{4ia^{\frac{5}{2}}bd\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}}\sinh^2(c+dx)}+\frac{b\log\left(-i\sqrt{a}\sqrt{\cdots}\right.}{4ia^{\frac{5}{2}}bd\sqrt{\frac{1}{b}}+\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sinh
(c + d*x)/(a**2*d), Eq(b, 0)), (-1/(3*b**2*d*sinh(c + d*x)**3), Eq(a, 0)),
(x*cosh(c)/(a + b*sinh(c)**2)**2, Eq(d, 0)), (2*I*sqrt(a)*b*sqrt(1/b)*sinh(
c + d*x)/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqrt(1/b)*sinh(c
 + d*x)**2) + a*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))/(4*I*a**(5/2)*b*d
*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2) - a*log(I*sqrt
(a)*sqrt(1/b) + sinh(c + d*x))/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b
**2*d*sqrt(1/b)*sinh(c + d*x)**2) + b*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d
*x))*sinh(c + d*x)**2/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqr
t(1/b)*sinh(c + d*x)**2) - b*log(I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))*sinh(
c + d*x)**2/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqrt(1/b)*sin
h(c + d*x)**2), True))
```

3.335 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=106

$$-\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}-\frac{b\sinh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}+\frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)^2}$$

```
[Out] arctan(sinh(d*x+c))/(a-b)^2/d-1/2*b*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)^
2)-1/2*(3*a-b)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(a-b)^2/
d
```

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3190, 414, 522, 203, 205}

$$-\frac{\sqrt{b}\,(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}-\frac{b\sinh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}+\frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)^2*d) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Si
nh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^2*d) - (b*Sinh[c + d*x])/(2*a*(a
- b)*d*(a + b*Sinh[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
```

```
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{2a(a-b)d}$$

$$= -\frac{b\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{(a-b)^2 d} - \frac{((3a}{}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)^2 d} - \frac{(3a-b)\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2 d} - \frac{b\sinh(c+}{2a(a-b)d\left(a+b\sin\right.}$$

Mathematica [A] time = 0.41, size = 174, normalized size = 1.64

$$\frac{\cosh(2(c+dx))\left(4a^{3/2}b\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - b^{3/2}(b-3a)\tan^{-1}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)\right) + (2a-b)\left(4a^{3/2}\tan^{-1}\left(\text{t}\right.\right.}{2a^{3/2}d(a-b)^2(2a+b\cosh(2(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b)*(-(Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]]) +
 4*a^(3/2)*ArcTan[Tanh[(c + d*x)/2]]) + (-(b^(3/2)*(-3*a + b)*ArcTan[(Sqrt[
a]*Csch[c + d*x])/Sqrt[b]]) + 4*a^(3/2)*b*ArcTan[Tanh[(c + d*x)/2]])*Cosh[2
*(c + d*x)] - 2*Sqrt[a]*(a - b)*b*Sinh[c + d*x])/(2*a^(3/2)*(a - b)^2*d*(2*
a - b + b*Cosh[2*(c + d*x)]))
```

fricas [B] time = 0.65, size = 2143, normalized size = 20.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b - b^2)*cosh(d*x + c)^3 + 12*(a*b - b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a*b - b^2)*sinh(d*x + c)^3 + ((3*a*b - b^2)*cosh(d*x + c)^4 +
 4*(3*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - b^2)*sinh(d*x + c
)^4 + 2*(6*a^2 - 5*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - b^2)*cosh(d*x
 + c)^2 + 6*a^2 - 5*a*b + b^2)*sinh(d*x + c)^2 + 3*a*b - b^2 + 4*((3*a*b -
b^2)*cosh(d*x + c)^3 + (6*a^2 - 5*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*s
inh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a
 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sin
h(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*s
inh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))
*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2
```

```
*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*s
inh(d*x + c) + b)) - 8*(a*b*cosh(d*x + c)^4 + 4*a*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*b*sinh(d*x + c)^4 + 2*(2*a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*a*b*c
osh(d*x + c)^2 + 2*a^2 - a*b)*sinh(d*x + c)^2 + a*b + 4*(a*b*cosh(d*x + c)^
3 + (2*a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh
(d*x + c)) - 4*(a*b - b^2)*cosh(d*x + c) + 4*(3*(a*b - b^2)*cosh(d*x + c)^2
 - a*b + b^2)*sinh(d*x + c))/((a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4
 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b -
 2*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*
b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2
 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^3*b - 2*a^
2*b^2 + a*b^3)*d + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (2*a^
4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(
a*b - b^2)*cosh(d*x + c)^3 + 6*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^2 +
2*(a*b - b^2)*sinh(d*x + c)^3 + ((3*a*b - b^2)*cosh(d*x + c)^4 + 4*(3*a*b -
 b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - b^2)*sinh(d*x + c)^4 + 2*(6*
a^2 - 5*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - b^2)*cosh(d*x + c)^2 + 6
*a^2 - 5*a*b + b^2)*sinh(d*x + c)^2 + 3*a*b - b^2 + 4*((3*a*b - b^2)*cosh(d
*x + c)^3 + (6*a^2 - 5*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*a
rctan(1/2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) + ((3*a*b - b^2)*cosh(
d*x + c)^4 + 4*(3*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - b^2)*
sinh(d*x + c)^4 + 2*(6*a^2 - 5*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - b
^2)*cosh(d*x + c)^2 + 6*a^2 - 5*a*b + b^2)*sinh(d*x + c)^2 + 3*a*b - b^2 +
4*((3*a*b - b^2)*cosh(d*x + c)^3 + (6*a^2 - 5*a*b + b^2)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sin
h(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x
+ c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) - 4*(a*b*cosh(d*x + c)^4 + 4*
a*b*cosh(d*x + c)*sinh(d*x + c)^3 + a*b*sinh(d*x + c)^4 + 2*(2*a^2 - a*b)*c
osh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c)^2 + 2*a^2 - a*b)*sinh(d*x + c)^2 +
a*b + 4*(a*b*cosh(d*x + c)^3 + (2*a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*
arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a*b - b^2)*cosh(d*x + c) + 2*(3*
(a*b - b^2)*cosh(d*x + c)^2 - a*b + b^2)*sinh(d*x + c))/((a^3*b - 2*a^2*b^2
 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^3*b - 2*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^4
 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b - 2*a^2*b^2
 + a*b^3)*d*cosh(d*x + c)^2 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d)*sinh
(d*x + c)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*d + 4*((a^3*b - 2*a^2*b^2 + a*b^3
)*d*cosh(d*x + c)^3 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d*cosh(d*x + c)
)*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[66,-29]Undef/Unsigned Inf encountered in limitLimit: Max order reach
ed or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.15, size = 1080, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*b/(a-b)^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3-1/d*b^2/(a-b)^2/(tanh(1/2*d*x+1/2*
c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*
x+1/2*c)^3-1/d*b/(a-b)^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+1/d*b^2/(a-b)^2/(tanh(1/2
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*ta
nh(1/2*d*x+1/2*c)+3/2/d*b/(a-b)^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^
(1/2))-3/2/d*b/(a-b)^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-2/d*b^2/(a-b)^2/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/2/d*b/(a-b)^2*a/(-b*(a-b))^(1/2)/(
(2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(
a-b))^(1/2)+a-2*b)*a)^(1/2))+3/2/d*b/(a-b)^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-
2/d*b^2/(a-b)^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arcta
nh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d*b^2/(a
-b)^2/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/(
(2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d*b^3/(a-b)^2/a/(-b*(a-b))^(1/2)/(
(2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d*b^2/(a-b)^2/a/((2*(-b*(a-b))^(1/2)+a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
))+1/2/d*b^3/(a-b)^2/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d/(
a-b)^2*arctan(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{be^{(3\,dx+3\,c)} - be^{(dx+c)}}{a^2bd - ab^2d + \left(a^2bde^{(4\,c)} - ab^2de^{(4\,c)}\right)e^{(4\,dx)} + 2\left(2\,a^3de^{(2\,c)} - 3\,a^2bde^{(2\,c)} + ab^2de^{(2\,c)}\right)e^{(2\,dx)}} + \frac{2\,\arctan\left(e^{(dx+c)}\right)}{a^2d - 2\,abd + b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(4*c)
- a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(2*a^3*d*e^(2*c) - 3*a^2*b*d*e^(2*c) + a*b
^2*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(a^2*d - 2*a*b*d + b^2*d)
- 2*integrate(1/2*((3*a*b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (3*a*b*e^c - b
^2*e^c)*e^(d*x))/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^(4*c) - 2*a^2*b^2*e^
(4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 5*a^3*b*e^(2*c) + 4*a
^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out
```

3.336 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=114

$$-\frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}}+\frac{b^2\tanh(c+dx)}{2ad(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{d(a-b)^2}$$

```
[Out] -1/2*(4*a-b)*b*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(5/2)
/d+tanh(d*x+c)/(a-b)^2/d+1/2*b^2*tanh(d*x+c)/a/(a-b)^2/d/(a-(a-b)*tanh(d*x+
c)^2)
```

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 390, 385, 208}

$$-\frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}}+\frac{b^2\tanh(c+dx)}{2ad(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a -
 b)^(5/2)*d) + Tanh[c + d*x]/((a - b)^2*d) + (b^2*Tanh[c + d*x])/(2*a*(a -
b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2} - \frac{(2a-b)b-2(a-b)bx^2}{(a-b)^2\left(a+(-a+b)x^2\right)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{(a-b)^2d} - \frac{\text{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{(a-b)^2d}$$

$$= \frac{\tanh(c+dx)}{(a-b)^2d} + \frac{b^2\tanh(c+dx)}{2a(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{((4a-b)b)\,\text{Subst}\left(\int \frac{}{a+}\right.}{2a(}$$

$$= -\frac{(4a-b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d} + \frac{\tanh(c+dx)}{(a-b)^2d} + \frac{b^2\tanh(c+dx)}{2a(a-b)^2d\left(a-(a-b)\tan\right.}$$

Mathematica [A] time = 0.92, size = 105, normalized size = 0.92

$$\frac{\frac{\frac{b^2\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)}+2\tanh(c+dx)}{(a-b)^2} - \frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-(((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a -
 b)^(5/2))) + ((b^2*Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)]))
+ 2*Tanh[c + d*x])/(a - b)^2)/(2*d)
```

fricas [B] time = 0.61, size = 3147, normalized size = 27.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^4 + 16*(4*a^3*b - 5*a^
2*b^2 + a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(4*a^3*b - 5*a^2*b^2 + a*b
^3)*sinh(d*x + c)^4 + 8*a^3*b - 4*a^2*b^2 - 4*a*b^3 + 8*(4*a^4 - 5*a^3*b +
a^2*b^2)*cosh(d*x + c)^2 + 8*(4*a^4 - 5*a^3*b + a^2*b^2 + 3*(4*a^3*b - 5*a^
2*b^2 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a*b^2 - b^3)*cosh(d*x
 + c)^6 + 6*(4*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - b^3)
*sinh(d*x + c)^6 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^2*b -
 8*a*b^2 + b^3 + 15*(4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5
*(4*a*b^2 - b^3)*cosh(d*x + c)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*a*b^2 - b^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c
)^2 + (15*(4*a*b^2 - b^3)*cosh(d*x + c)^4 + 16*a^2*b - 8*a*b^2 + b^3 + 6*(1
6*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 -
 b^3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^3 + (16*
a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b
^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c
)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b -
```

```
b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*
b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh
(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*
a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*
((4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 5*a^3*b + a^2*b^2
)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*
d*cosh(d*x + c)^6 + 6*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*sinh(d*x
 + c)^6 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x
+ c)^4 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 +
(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^4 +
(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + 4
*(5*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + (4*a^6 -
13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^4 + 6*(4
*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (4*
a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^2 + (a^
5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d + 2*(3*(a^5*b - 3*a^4*b^2 + 3*a^3*
b^3 - a^2*b^4)*d*cosh(d*x + c)^5 + 2*(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3
*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*
b^3 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(4*a^3*b - 5*a^2*b^
2 + a*b^3)*cosh(d*x + c)^4 + 8*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)*
sinh(d*x + c)^3 + 2*(4*a^3*b - 5*a^2*b^2 + a*b^3)*sinh(d*x + c)^4 + 4*a^3*b
 - 2*a^2*b^2 - 2*a*b^3 + 4*(4*a^4 - 5*a^3*b + a^2*b^2)*cosh(d*x + c)^2 + 4*
(4*a^4 - 5*a^3*b + a^2*b^2 + 3*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 - ((4*a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(4*a*b^2 - b^3)*c
osh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - b^3)*sinh(d*x + c)^6 + (16*a^2*b
- 8*a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^2*b - 8*a*b^2 + b^3 + 15*(4*a*b^2
- b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*a*b^2 - b^3)*cosh(d*x + c
)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a*b^2 -
 b^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(4*a*b^2 - b^3)*cos
h(d*x + c)^4 + 16*a^2*b - 8*a*b^2 + b^3 + 6*(16*a^2*b - 8*a*b^2 + b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(16
*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d
*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2
*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a
*b)/(a^2 - a*b)) + 8*((4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + (4*a^
4 - 5*a^3*b + a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 3*a^4*b^2 +
3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^6 + 6*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 -
 a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3
- a^2*b^4)*d*sinh(d*x + c)^6 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 +
 a^2*b^4)*d*cosh(d*x + c)^4 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)
*d*cosh(d*x + c)^2 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*
d)*sinh(d*x + c)^4 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*
d*cosh(d*x + c)^2 + 4*(5*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d
*x + c)^3 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*c
osh(d*x + c)^4 + 6*(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*
cosh(d*x + c)^2 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)*
sinh(d*x + c)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d + 2*(3*(a^5*b
 - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^5 + 2*(4*a^6 - 13*a^5*b
 + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + (4*a^6 - 13*a^5*b
+ 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 0.82, size = 221, normalized size = 1.94

$$-\frac{\frac{\left(4\,ab-b^2\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^3-2\,a^2b+ab^2\right)\sqrt{-a^2+ab}}+\frac{2\left(4\,abe^{(4\,dx+4\,c)}-b^2e^{(4\,dx+4\,c)}+8\,a^2e^{(2\,dx+2\,c)}-2\,abe^{(2\,dx+2\,c)}+2\,ab+b^2\right)}{\left(a^3-2\,a^2b+ab^2\right)\left(be^{(6\,dx+6\,c)}+4\,ae^{(4\,dx+4\,c)}-be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((4*a*b - b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*
b))/((a^3 - 2*a^2*b + a*b^2)*sqrt(-a^2 + a*b)) + 2*(4*a*b*e^(4*d*x + 4*c) -
 b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 2*a*b*e^(2*d*x + 2*c) + 2*a*
b + b^2)/((a^3 - 2*a^2*b + a*b^2)*(b*e^(6*d*x + 6*c) + 4*a*e^(4*d*x + 4*c)
- b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)))/d
```

maple [B] time = 0.14, size = 798, normalized size = 7.00

$$\frac{b^2\left(\tanh^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,(a-b)^2\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)a}+\frac{\quad}{d\,(a-b)^2\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*b^2/(a-b)^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d*b^2/(a-b)^2/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/
2*d*x+1/2*c)+2/d*b/(a-b)^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d*b^2/(a-b)^2/(-b
*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-2/d*b/(a-b)^2/((2*(-b*(a-b))^(1/
2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2))+2/d*b^2/(a-b)^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-
1/2/d*b^2/(a-b)^2/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*
d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d*b^3/(a-b)^2/a/(-b*(a
-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d*b^2/(a-b)^2/a/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2))-1/2/d*b^3/(a-b)^2/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2))+2/d/(a-b)^2*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^2\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.337 $\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=157

$$\frac{b^{3/2}(5a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3}+\frac{b(a+b)\sinh(c+dx)}{2ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{(a-5b)\tan^{-1}(\sinh(c+dx))}{2d(a-b)^3}+\frac{\tanh(c+d\cdots}{2d(a-b)\left(a-\cdots\right.}$$

```
[Out] 1/2*(a-5*b)*arctan(sinh(d*x+c))/(a-b)^3/d+1/2*(5*a-b)*b^(3/2)*arctan(sinh(d
*x+c)*b^(1/2)/a^(1/2))/a^(3/2)/(a-b)^3/d+1/2*b*(a+b)*sinh(d*x+c)/a/(a-b)^2/
d/(a+b*sinh(d*x+c)^2)+1/2*sech(d*x+c)*tanh(d*x+c)/(a-b)/d/(a+b*sinh(d*x+c)^
2)
```

Rubi [A] time = 0.19, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}(5a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3}+\frac{b(a+b)\sinh(c+dx)}{2ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{(a-5b)\tan^{-1}(\sinh(c+dx))}{2d(a-b)^3}+\frac{\tanh(c+d\cdots}{2d(a-b)\left(a-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - 5*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^3*d) + ((5*a - b)*b^(3/2)*ArcT
an[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^3*d) + (b*(a + b)*S
inh[c + d*x])/(2*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)) + (Sech[c + d*x]*Ta
nh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{2(a-b)d}$$

$$= \frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\operatorname{Subst}\left(\right.}{}$$

$$= \frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{(a-5b)}{}$$

$$= \frac{(a-5b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^3d} + \frac{(5a-b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d} + \frac{b(a+}{2a(a-b)^2d}$$

Mathematica [A] time = 0.93, size = 230, normalized size = 1.46

$$\frac{(2a-b)\left(2a^{3/2}(a-5b)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + a^{3/2}(a-b)\tanh(c+dx)\operatorname{sech}(c+dx) + b^{3/2}(b-5a)\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*Sqrt[a]*(a - b)*b^2*Sinh[c + d*x] + (2*a - b)*(b^(3/2)*(-5*a + b)*ArcTan
[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*a^(3/2)*(a - 5*b)*ArcTan[Tanh[(c + d*
x)/2]] + a^(3/2)*(a - b)*Sech[c + d*x]*Tanh[c + d*x]) + b*Cosh[2*(c + d*x)]
*(b^(3/2)*(-5*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*a^(3/2)*(a
 - 5*b)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*(a - b)*Sech[c + d*x]*Tanh[c +
d*x]))/(2*a^(3/2)*(a - b)^3*d*(2*a - b + b*Cosh[2*(c + d*x)]))
```

fricas [B] time = 0.77, size = 6548, normalized size = 41.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b - b^3)*cosh(d*x + c)^7 + 28*(a^2*b - b^3)*cosh(d*x + c)*sinh
(d*x + c)^6 + 4*(a^2*b - b^3)*sinh(d*x + c)^7 + 4*(4*a^3 - 7*a^2*b + 4*a*b^
2 - b^3)*cosh(d*x + c)^5 + 4*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3 + 21*(a^2*b -
 b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b - b^3)*cosh(d*x + c)^
3 + (4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(4
*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 + 4*(35*(a^2*b - b^3)*cosh(
d*x + c)^4 - 4*a^3 + 7*a^2*b - 4*a*b^2 + b^3 + 10*(4*a^3 - 7*a^2*b + 4*a*b^
2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(a^2*b - b^3)*cosh(d*x +
c)^5 + 10*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^3 - 7*
a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((5*a*b^2 - b^3)*co
sh(d*x + c)^8 + 8*(5*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2
- b^3)*sinh(d*x + c)^8 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^6 + 4*(5*a^2*b -
 a*b^2 + 7*(5*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(5*a*b^2
 - b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - b^3)*co
sh(d*x + c)^4 + 20*a^2*b - 9*a*b^2 + b^3 + 30*(5*a^2*b - a*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b
- a*b^2)*cosh(d*x + c)^3 + (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c))*sinh(d
*x + c)^3 + 5*a*b^2 - b^3 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7*(5*a
*b^2 - b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - a*b^2)*cosh(d*x + c)^4 + 5*a^2*
b - a*b^2 + 3*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 8*((5*a*b^2 - b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c)^5 +
 (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^3 + (5*a^2*b - a*b^2)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*si
nh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*co
sh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)
*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*si
nh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2
- a)*sinh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*
sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*
cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a -
b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*((a^2*b - 5*a*b^2)*cosh(d*x + c)^
8 + 8*(a^2*b - 5*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - 5*a*b^2)*s
inh(d*x + c)^8 + 4*(a^3 - 5*a^2*b)*cosh(d*x + c)^6 + 4*(a^3 - 5*a^2*b + 7*(
a^2*b - 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b - 5*a*b^2)*
cosh(d*x + c)^3 + 3*(a^3 - 5*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(4*a
^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2*b - 5*a*b^2)*cosh(d*x
 + c)^4 + 4*a^3 - 21*a^2*b + 5*a*b^2 + 30*(a^3 - 5*a^2*b)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(7*(a^2*b - 5*a*b^2)*cosh(d*x + c)^5 + 10*(a^3 - 5*a^2*
b)*cosh(d*x + c)^3 + (4*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x +
 c)^3 + a^2*b - 5*a*b^2 + 4*(a^3 - 5*a^2*b)*cosh(d*x + c)^2 + 4*(7*(a^2*b -
 5*a*b^2)*cosh(d*x + c)^6 + 15*(a^3 - 5*a^2*b)*cosh(d*x + c)^4 + a^3 - 5*a^
2*b + 3*(4*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(
(a^2*b - 5*a*b^2)*cosh(d*x + c)^7 + 3*(a^3 - 5*a^2*b)*cosh(d*x + c)^5 + (4*
a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^3 + (a^3 - 5*a^2*b)*cosh(d*x + c))*
sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 4*(a^2*b - b^3)*cosh
(d*x + c) + 4*(7*(a^2*b - b^3)*cosh(d*x + c)^6 + 5*(4*a^3 - 7*a^2*b + 4*a*b
^2 - b^3)*cosh(d*x + c)^4 - a^2*b + b^3 - 3*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^
3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*d*cosh(d*x + c)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x +
 c)*sinh(d*x + c)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*sinh(d*x +
c)^8 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^
4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3
*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^6 + 2*(4*a^5 - 13*a^4*b + 15*a^3*b^2 -
 7*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d*cosh(d*x + c)^3 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cos
```

```
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*d*cosh(d*x + c)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x +
c)^2 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)
^4 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^4*
b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^5 + 10*(a^5 - 3*a^4*b +
3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7
*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b - 3*a^3*b^
2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 -
a^2*b^3)*d*cosh(d*x + c)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 +
 a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d)*sinh(d
*x + c)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d + 8*((a^4*b - 3*a^3*b
^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 -
a^2*b^3)*d*cosh(d*x + c)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a
*b^4)*d*cosh(d*x + c)^3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x
+ c))*sinh(d*x + c)), 1/2*(2*(a^2*b - b^3)*cosh(d*x + c)^7 + 14*(a^2*b - b^
3)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2*b - b^3)*sinh(d*x + c)^7 + 2*(4*a
^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(4*a^3 - 7*a^2*b + 4*a*b^
2 - b^3 + 21*(a^2*b - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(a^2*b
- b^3)*cosh(d*x + c)^3 + (4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*s
inh(d*x + c)^4 - 2*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(3
5*(a^2*b - b^3)*cosh(d*x + c)^4 - 4*a^3 + 7*a^2*b - 4*a*b^2 + b^3 + 10*(4*a
^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(a^2
*b - b^3)*cosh(d*x + c)^5 + 10*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x +
 c)^3 - 3*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^2
+ ((5*a*b^2 - b^3)*cosh(d*x + c)^8 + 8*(5*a*b^2 - b^3)*cosh(d*x + c)*sinh(d
*x + c)^7 + (5*a*b^2 - b^3)*sinh(d*x + c)^8 + 4*(5*a^2*b - a*b^2)*cosh(d*x
+ c)^6 + 4*(5*a^2*b - a*b^2 + 7*(5*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^6 + 8*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - a*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(
35*(5*a*b^2 - b^3)*cosh(d*x + c)^4 + 20*a^2*b - 9*a*b^2 + b^3 + 30*(5*a^2*b
 - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*b^2 - b^3)*cosh(d*x
+ c)^5 + 10*(5*a^2*b - a*b^2)*cosh(d*x + c)^3 + (20*a^2*b - 9*a*b^2 + b^3)*
cosh(d*x + c))*sinh(d*x + c)^3 + 5*a*b^2 - b^3 + 4*(5*a^2*b - a*b^2)*cosh(d
*x + c)^2 + 4*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - a*b^2)*cos
h(d*x + c)^4 + 5*a^2*b - a*b^2 + 3*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^2 + 8*((5*a*b^2 - b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - a*b
^2)*cosh(d*x + c)^5 + (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^3 + (5*a^2*b
 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cos
h(d*x + c) + sinh(d*x + c))) + ((5*a*b^2 - b^3)*cosh(d*x + c)^8 + 8*(5*a*b^
2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2 - b^3)*sinh(d*x + c)^8 +
4*(5*a^2*b - a*b^2)*cosh(d*x + c)^6 + 4*(5*a^2*b - a*b^2 + 7*(5*a*b^2 - b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^3 +
 3*(5*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(20*a^2*b - 9*a*b^2
 + b^3)*cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - b^3)*cosh(d*x + c)^4 + 20*a^2*b
- 9*a*b^2 + b^3 + 30*(5*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8
*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - a*b^2)*cosh(d*x + c)^3
+ (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a*b^2 - b^3
 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7*(5*a*b^2 - b^3)*cosh(d*x + c)
^6 + 15*(5*a^2*b - a*b^2)*cosh(d*x + c)^4 + 5*a^2*b - a*b^2 + 3*(20*a^2*b -
 9*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a*b^2 - b^3)*cosh(
d*x + c)^7 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c)^5 + (20*a^2*b - 9*a*b^2 + b^
3)*cosh(d*x + c)^3 + (5*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b
/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*s
inh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*
sinh(d*x + c))*sqrt(b/a)/b) + 2*((a^2*b - 5*a*b^2)*cosh(d*x + c)^8 + 8*(a^2
*b - 5*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - 5*a*b^2)*sinh(d*x +
c)^8 + 4*(a^3 - 5*a^2*b)*cosh(d*x + c)^6 + 4*(a^3 - 5*a^2*b + 7*(a^2*b - 5*
a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b - 5*a*b^2)*cosh(d*x +
 c)^3 + 3*(a^3 - 5*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(4*a^3 - 21*a^
```

```
2*b + 5*a*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2*b - 5*a*b^2)*cosh(d*x + c)^4 +
4*a^3 - 21*a^2*b + 5*a*b^2 + 30*(a^3 - 5*a^2*b)*cosh(d*x + c)^2)*sinh(d*x +
 c)^4 + 8*(7*(a^2*b - 5*a*b^2)*cosh(d*x + c)^5 + 10*(a^3 - 5*a^2*b)*cosh(d*
x + c)^3 + (4*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + a^
2*b - 5*a*b^2 + 4*(a^3 - 5*a^2*b)*cosh(d*x + c)^2 + 4*(7*(a^2*b - 5*a*b^2)*
cosh(d*x + c)^6 + 15*(a^3 - 5*a^2*b)*cosh(d*x + c)^4 + a^3 - 5*a^2*b + 3*(4
*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b - 5
*a*b^2)*cosh(d*x + c)^7 + 3*(a^3 - 5*a^2*b)*cosh(d*x + c)^5 + (4*a^3 - 21*a
^2*b + 5*a*b^2)*cosh(d*x + c)^3 + (a^3 - 5*a^2*b)*cosh(d*x + c))*sinh(d*x +
 c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a^2*b - b^3)*cosh(d*x + c)
+ 2*(7*(a^2*b - b^3)*cosh(d*x + c)^6 + 5*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*
cosh(d*x + c)^4 - a^2*b + b^3 - 3*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*
x + c)^2)*sinh(d*x + c))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*
x + c)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)*sinh(d
*x + c)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*sinh(d*x + c)^8 + 4*(
a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b - 3*a^
3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 -
 a^2*b^3)*d)*sinh(d*x + c)^6 + 2*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3
 + a*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*
d*cosh(d*x + c)^3 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(35*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*
x + c)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + (4*
a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^4 + 4*(a^
5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b - 3*a^3*
b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2
 - a^2*b^3)*d*cosh(d*x + c)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 +
 a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*
b^3 - a*b^4)*d*cosh(d*x + c)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d
*cosh(d*x + c)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d*
cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2
 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d + 8*((a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*d*cosh(d*x + c)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d
*cosh(d*x + c)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d*co
sh(d*x + c)^3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh
(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[6,-20]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[66,-29]Warning, need to choose a branch for the root of a polynomial
 with parameters. This might be wrong.The choice was done assuming [a,b]=[-
21,2]Warning, need to choose a branch for the root of a polynomial with par
ameters. This might be wrong.The choice was done assuming [a,b]=[15,2]Undef
/Unsigned Inf encountered in limitEvaluation time: 0.74Limit: Max order rea
ched or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.18, size = 1265, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3+1/d*b^3/(a-b)^3/(tanh(1/2*d*x+1
/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2
*d*x+1/2*c)^3+1/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)-1/d*b^3/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
/a*tanh(1/2*d*x+1/2*c)-5/2/d*b^2/(a-b)^3*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2))+5/2/d*b^2/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/d*b^3/(a-b)^
3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/2/d*b^2/(a-b)^3*a/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-5/2/d*b^2/(a-b)^3/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2))+3/d*b^3/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a
)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))
-1/2/d*b^3/(a-b)^3/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d*b^4/(a-b)^3/a/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d*b^3/(a-b)^3/a/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2))-1/2/d*b^4/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2))-1/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*a+1/d
/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*b+1/d/(a-b)^3/(t
anh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*a-1/d/(a-b)^3/(tanh(1/2*d*x+1
/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*b+1/d/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))*
a-5/d/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ae^c - 5\,be^c)\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{a^3d - 3\,a^2bd + 3\,ab^2d - b^3d} + \frac{\left(abe^{(7\,c)} + b^2e^{(7\,c)}\right)e^{(7\,dx)} + (}{a^3bd - 2\,a^2b^2d + ab^3d + \left(a^3bde^{(8\,c)} - 2\,a^2b^2de^{(8\,c)} + ab^3de^{(8\,c)}\right)e^{(8\,dx)} + 4\,(a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (a*e^c - 5*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^3*d - 3*a^2*b*d + 3*a*b^2*d
 - b^3*d) + ((a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) - 3*a*b
*e^(5*c) + b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) - 3*a*b*e^(3*c) + b^2*e^
(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b*d - 2*a^2*b^2*d + a*
b^3*d + (a^3*b*d*e^(8*c) - 2*a^2*b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x)
 + 4*(a^4*d*e^(6*c) - 2*a^3*b*d*e^(6*c) + a^2*b^2*d*e^(6*c))*e^(6*d*x) + 2*
(4*a^4*d*e^(4*c) - 9*a^3*b*d*e^(4*c) + 6*a^2*b^2*d*e^(4*c) - a*b^3*d*e^(4*c
))*e^(4*d*x) + 4*(a^4*d*e^(2*c) - 2*a^3*b*d*e^(2*c) + a^2*b^2*d*e^(2*c))*e^
(2*d*x)) + 8*integrate(1/8*((5*a*b^2*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) + (5*
a*b^2*e^c - b^3*e^c)*e^(d*x))/(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4 + (a^4
*b*e^(4*c) - 3*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c) - a*b^4*e^(4*c))*e^(4*d*
x) + 2*(2*a^5*e^(2*c) - 7*a^4*b*e^(2*c) + 9*a^3*b^2*e^(2*c) - 5*a^2*b^3*e^(
2*c) + a*b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + d\,x)^3\,\left(b\sinh(c + d\,x)^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.338 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=143

$$\frac{b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}}-\frac{b^3\tanh(c+dx)}{2ad(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\tanh^3(c+dx)}{3d(a-b)^2}+\frac{(a-3b)\tanh(c+dx)}{d(a-b)^3}$$

```
[Out] 1/2*(6*a-b)*b^2*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a-b)^(7/2
)/d+(a-3*b)*tanh(d*x+c)/(a-b)^3/d-1/3*tanh(d*x+c)^3/(a-b)^2/d-1/2*b^3*tanh(
d*x+c)/a/(a-b)^3/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.21, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 390, 385, 208}

$$\frac{b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}}-\frac{b^3\tanh(c+dx)}{2ad(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\tanh^3(c+dx)}{3d(a-b)^2}+\frac{(a-3b)\tanh(c+dx)}{d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a
- b)^(7/2)*d) + ((a - 3*b)*Tanh[c + d*x])/((a - b)^3*d) - Tanh[c + d*x]^3/(
3*(a - b)^2*d) - (b^3*Tanh[c + d*x])/(2*a*(a - b)^3*d*(a - (a - b)*Tanh[c +
 d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{(a-b)^3} - \frac{x^2}{(a-b)^2} + \frac{(3a-b)b^2-3(a-b)b^2x^2}{(a-b)^3\left(a+(-a+b)x^2\right)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a-3b)\tanh(c+dx)}{(a-b)^3 d} - \frac{\tanh^3(c+dx)}{3(a-b)^2 d} + \frac{\text{Subst}\left(\int \frac{(3a-b)b^2-3(a-b)b^2x^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c\right.}{(a-b)^3 d}$$

$$= \frac{(a-3b)\tanh(c+dx)}{(a-b)^3 d} - \frac{\tanh^3(c+dx)}{3(a-b)^2 d} - \frac{b^3\tanh(c+dx)}{2a(a-b)^3 d\left(a-(a-b)\tanh^2(c+dx)\right)}$$

$$= \frac{(6a-b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}d} + \frac{(a-3b)\tanh(c+dx)}{(a-b)^3 d} - \frac{\tanh^3(c+dx)}{3(a-b)^2 d} - \frac{}{2a}$$

Mathematica [A] time = 1.98, size = 130, normalized size = 0.91

$$\frac{\frac{3b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{7/2}} + \frac{2\tanh(c+dx)\left((a-b)\operatorname{sech}^2(c+dx)+2(a-4b)\right) - \frac{3b^3\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)}}{(a-b)^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((3*(6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a
 - b)^(7/2)) + ((-3*b^3*Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)
])) + 2*(2*(a - 4*b) + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b)^3)/(
6*d)
```

fricas [B] time = 0.75, size = 7894, normalized size = 55.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^8 + 96*(6*a^3*b^2 -
 7*a^2*b^3 + a*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + 12*(6*a^3*b^2 - 7*a^2*b
^3 + a*b^4)*sinh(d*x + c)^8 + 24*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*co
sh(d*x + c)^6 + 24*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4 + 14*(6*a^3*b^2 -
 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 48*(14*(6*a^3*b^2 -
7*a^2*b^3 + a*b^4)*cosh(d*x + c)^3 + 3*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b
^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 16*a^4*b + 80*a^3*b^2 - 52*a^2*b^3 - 1
2*a*b^4 - 8*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^4
- 8*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3 - 105*(6*a^3*b^2 - 7*a^2*
b^3 + a*b^4)*cosh(d*x + c)^4 - 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*co
sh(d*x + c)^5 + 15*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^3
- (24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c))*sinh(d*x +
c)^3 - 8*(8*a^5 - 38*a^4*b + 25*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c
)^2 + 8*(42*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 - 8*a^5 + 38*a^
```

```
4*b - 25*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 + 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3
+ a*b^4)*cosh(d*x + c)^4 - 6*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((6*a*b^3 - b^4)*cosh(d*x + c)^10 + 1
0*(6*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (6*a*b^3 - b^4)*sinh(d*x
+ c)^10 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^8 + (24*a^2*b^2 + 2*a*
b^3 - b^4 + 45*(6*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(6*
a*b^3 - b^4)*cosh(d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c))*
sinh(d*x + c)^7 + 2*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^6 + 2*(105*
(6*a*b^3 - b^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 + b^4 + 14*(24*a^2*
b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - b^
4)*cosh(d*x + c)^5 + 14*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(3
6*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(36*a^2*b^2
- 12*a*b^3 + b^4)*cosh(d*x + c)^4 + 2*(105*(6*a*b^3 - b^4)*cosh(d*x + c)^6
+ 35*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 +
 b^4 + 15*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 +
6*a*b^3 - b^4 + 8*(15*(6*a*b^3 - b^4)*cosh(d*x + c)^7 + 7*(24*a^2*b^2 + 2*a
*b^3 - b^4)*cosh(d*x + c)^5 + 5*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)
^3 + (36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (24*a^2
*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^2 + (45*(6*a*b^3 - b^4)*cosh(d*x + c)^8
 + 28*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 30*(36*a^2*b^2 - 12*a*
b^3 + b^4)*cosh(d*x + c)^4 + 24*a^2*b^2 + 2*a*b^3 - b^4 + 12*(36*a^2*b^2 -
12*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(6*a*b^3 - b^4)*cos
h(d*x + c)^9 + 4*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^7 + 6*(36*a^2*b
^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^5 + 4*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh
(d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d
*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cos
h(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x +
 c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt
(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sin
h(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a -
 b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(
d*x + c) + b)) + 16*(6*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^7 + 9*
(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 - 2*(24*a^5 - 106*a
^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^3 - (8*a^5 - 38*a^4*b + 25*a^
3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b - 4*a^5*
b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^10 + 10*(a^6*b - 4*a
^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^9 +
 (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*sinh(d*x + c)^10 +
 (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^8 +
 (45*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^
2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c)
^8 + 2*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*
d*cosh(d*x + c)^6 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*
b^5)*d*cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*
b^5)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6*b - 4*a^5*b^2 + 6*a^4*b
^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 14*(4*a^7 - 15*a^6*b + 20*a^5
*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^2 + (6*a^7 - 25*a^6*b + 40*a^5
*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(6*a^7 - 2
5*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4
 + 4*(63*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x +
 c)^5 + 14*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*
x + c)^3 + 3*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2
*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6*b - 4*a^5*b^2 + 6*a^4*
b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 35*(4*a^7 - 15*a^6*b + 20*a^
5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^4 + 15*(6*a^7 - 25*a^6*b + 40
*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (6*a^7 -
25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)
```

```
^4 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)
^2 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x
 + c)^7 + 7*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d
*x + c)^5 + 5*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^
2*b^5)*d*cosh(d*x + c)^3 + (6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10
*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^6*b - 4*a^5*b
^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 28*(4*a^7 - 15*a^
6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^6 + 30*(6*a^7 - 25
*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4
+ 12*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*
cosh(d*x + c)^2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d)
*sinh(d*x + c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d
+ 2*(5*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c
)^9 + 4*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x +
 c)^7 + 6*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^
5)*d*cosh(d*x + c)^5 + 4*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a
^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a
^4*b^3 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/6*(6*(6*a^3*b^2 - 7*a^
2*b^3 + a*b^4)*cosh(d*x + c)^8 + 48*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*
x + c)*sinh(d*x + c)^7 + 6*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*sinh(d*x + c)^8
+ 12*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 + 12*(6*a^4*b
- a^3*b^2 - 6*a^2*b^3 + a*b^4 + 14*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^6 + 24*(14*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x
+ c)^3 + 3*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^5 - 8*a^4*b + 40*a^3*b^2 - 26*a^2*b^3 - 6*a*b^4 - 4*(24*a^5 - 106*a^4*
b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^4 - 4*(24*a^5 - 106*a^4*b + 95*a
^3*b^2 - 13*a^2*b^3 - 105*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 -
 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 16*(21*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 + 15*(6*a^4*b -
a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^3 - (24*a^5 - 106*a^4*b + 95*a^3
*b^2 - 13*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(8*a^5 - 38*a^4*b + 2
5*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2 + 4*(42*(6*a^3*b^2 - 7*a^2
*b^3 + a*b^4)*cosh(d*x + c)^6 - 8*a^5 + 38*a^4*b - 25*a^3*b^2 - 2*a^2*b^3 -
 3*a*b^4 + 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 - 6*(
24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - 3*((6*a*b^3 - b^4)*cosh(d*x + c)^10 + 10*(6*a*b^3 - b^4)*cosh(d*x + c
)*sinh(d*x + c)^9 + (6*a*b^3 - b^4)*sinh(d*x + c)^10 + (24*a^2*b^2 + 2*a*b^
3 - b^4)*cosh(d*x + c)^8 + (24*a^2*b^2 + 2*a*b^3 - b^4 + 45*(6*a*b^3 - b^4)
*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(6*a*b^3 - b^4)*cosh(d*x + c)^3 +
 (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(36*a^2*b^
2 - 12*a*b^3 + b^4)*cosh(d*x + c)^6 + 2*(105*(6*a*b^3 - b^4)*cosh(d*x + c)^
4 + 36*a^2*b^2 - 12*a*b^3 + b^4 + 14*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x
+ c)^2)*sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - b^4)*cosh(d*x + c)^5 + 14*(24*a^
2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(36*a^2*b^2 - 12*a*b^3 + b^4)*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c
)^4 + 2*(105*(6*a*b^3 - b^4)*cosh(d*x + c)^6 + 35*(24*a^2*b^2 + 2*a*b^3 - b
^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 + b^4 + 15*(36*a^2*b^2 - 12*a*b
^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 6*a*b^3 - b^4 + 8*(15*(6*a*b^3
 - b^4)*cosh(d*x + c)^7 + 7*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^5 +
5*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^3 + (36*a^2*b^2 - 12*a*b^3 +
b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x
 + c)^2 + (45*(6*a*b^3 - b^4)*cosh(d*x + c)^8 + 28*(24*a^2*b^2 + 2*a*b^3 -
b^4)*cosh(d*x + c)^6 + 30*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^4 + 2
4*a^2*b^2 + 2*a*b^3 - b^4 + 12*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 2*(5*(6*a*b^3 - b^4)*cosh(d*x + c)^9 + 4*(24*a^2*b^2 +
 2*a*b^3 - b^4)*cosh(d*x + c)^7 + 6*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x
+ c)^5 + 4*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^3 + (24*a^2*b^2 + 2*
a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*
cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a
```

```
 - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 8*(6*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*
cosh(d*x + c)^7 + 9*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^5
 - 2*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^3 - (8*a^
5 - 38*a^4*b + 25*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c))*sinh(d*x +
c))/((a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^
10 + 10*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x +
c)*sinh(d*x + c)^9 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*
d*sinh(d*x + c)^10 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)
*d*cosh(d*x + c)^8 + (45*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b
^5)*d*cosh(d*x + c)^2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b
^5)*d)*sinh(d*x + c)^8 + 2*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10
*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^
3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b + 20*a^5*b^2
 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6*b -
 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 14*(4*a^7
 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^2 + (6*a^7
 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d)*sinh(d*x +
 c)^6 + 2*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^
5)*d*cosh(d*x + c)^4 + 4*(63*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a
^2*b^5)*d*cosh(d*x + c)^5 + 14*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3
+ a^2*b^5)*d*cosh(d*x + c)^3 + 3*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^
3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6*b
- 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 35*(4*a^
7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^4 + 15*(6
*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*
x + c)^2 + (6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b
^5)*d)*sinh(d*x + c)^4 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*
b^5)*d*cosh(d*x + c)^2 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 +
 a^2*b^5)*d*cosh(d*x + c)^7 + 7*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3
 + a^2*b^5)*d*cosh(d*x + c)^5 + 5*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b
^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (6*a^7 - 25*a^6*b + 40*a^5*b
^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 +
(45*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8
 + 28*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c
)^6 + 30*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5
)*d*cosh(d*x + c)^4 + 12*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a
^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a
^4*b^3 + a^2*b^5)*d)*sinh(d*x + c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a
^3*b^4 + a^2*b^5)*d + 2*(5*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2
*b^5)*d*cosh(d*x + c)^9 + 4*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a
^2*b^5)*d*cosh(d*x + c)^7 + 6*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 +
 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + 4*(6*a^7 - 25*a^6*b + 40*a^5*b^2
 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b
 + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 0.81, size = 270, normalized size = 1.89

$$\frac{\frac{3\left(6ab^2-b^3\right)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\left(a^4-3a^3b+3a^2b^2-ab^3\right)\sqrt{-a^2+ab}}+\frac{6\left(2ab^2e^{(2dx+2c)}-b^3e^{(2dx+2c)}+b^3\right)}{\left(a^4-3a^3b+3a^2b^2-ab^3\right)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{(2dx+2c)}+b\right)}+\frac{8\left(3be^{(4dx+4c)}-3ae^{(2dx+2c)}+9be^{(\cdots}\right.}{\left(a^3-3a^2b+3ab^2-b^3\right)\left(e^{(2dx\cdots}\right.}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(6*a*b^2 - b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 +
 a*b))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(-a^2 + a*b)) + 6*(2*a*b^2*
e^(2*d*x + 2*c) - b^3*e^(2*d*x + 2*c) + b^3)/((a^4 - 3*a^3*b + 3*a^2*b^2 -
a*b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b))
```

```
+ 8*(3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) + 9*b*e^(2*d*x + 2*c) - a +
4*b)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(e^(2*d*x + 2*c) + 1)^3))/d
```

maple [B] time = 0.18, size = 998, normalized size = 6.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3-1/d*b^3/(a-b)^3/(tanh(1/2*d*x
+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1
/2*d*x+1/2*c)-3/d*b^2/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/d*b^3/(a-b)^3/
(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/d*b^2/(a-b)^3/((2*(-b*(a-b)
)^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+
a-2*b)*a)^(1/2))-3/d*b^3/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2))+1/2/d*b^3/(a-b)^3/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d*b^4/(a-b)^3/a/(
-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d*b^3/(a-b)^3/a/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)+a-2*b)*a)^(1/2))+1/2/d*b^4/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/
2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2))+2/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*a
-6/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*b+4/3/d/(a-b
)^3/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3*a-28/3/d/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3*b+2/d/(a-b)^3/(tanh(1/2*d*x+1
/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)*a-6/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^3*
tanh(1/2*d*x+1/2*c)*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^4\,{\left(b\,\mathrm{sinh}\left(c+d\,x\right)^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2),x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.339 $\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=160

$$-\frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\sqrt{a-b}\left(8a^2+4ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}-\frac{(a-b)\tanh(c+}{4abd\left(a-(a-b)\tanh^2\right.}$$

```
[Out] x/b^3-1/8*(8*a^2+4*a*b+3*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))*(a-b
)^(1/2)/a^(5/2)/b^3/d-1/4*(a-b)*tanh(d*x+c)/a/b/d/(a-(a-b)*tanh(d*x+c)^2)^2
-1/8*(a-b)*(4*a+3*b)*tanh(d*x+c)/a^2/b^2/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.24, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3191, 414, 527, 522, 206, 208}

$$-\frac{\sqrt{a-b}\left(8a^2+4ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}-\frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{(a-b)\tanh(c+}{4abd\left(a-(a-b)\tanh^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] x/b^3 - (Sqrt[a - b]*(8*a^2 + 4*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(5/2)*b^3*d) - ((a - b)*Tanh[c + d*x])/(4*a*b*d*(a - (
a - b)*Tanh[c + d*x]^2)^2) - ((a - b)*(4*a + 3*b)*Tanh[c + d*x])/(8*a^2*b^2
*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-a-3b-3(a-b)x^2}{(1-x^2)\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c\right.}{4abd}$$

$$= -\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Su}}{}$$

$$= -\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Su}}{}$$

$$= \frac{x}{b^3} - \frac{\sqrt{a-b}\left(8a^2+4ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} - \frac{(a-b)\tanh(}{4abd\left(a-(a-b)\tan\right.}$$

Mathematica [A] time = 1.42, size = 164, normalized size = 1.02

$$\frac{\frac{3b\left(-2a^2+ab+b^2\right)\sinh(2(c+dx))}{a^2(2a+b\cosh(2(c+dx))-b)} - \frac{\left(8a^3-4a^2b-ab^2-3b^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}} + \frac{4b(a-b)^2\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)^2} + 8(c+dx)}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - ((8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*ArcTanh[(Sqrt[a - b]*Tanh
[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]) + (4*(a - b)^2*b*Sinh[2*(c + d*x
)])/(a*(2*a - b + b*Cosh[2*(c + d*x)])^2) + (3*b*(-2*a^2 + a*b + b^2)*Sinh[
2*(c + d*x)])/(a^2*(2*a - b + b*Cosh[2*(c + d*x)])))/(8*b^3*d)
```

fricas [B] time = 0.57, size = 5511, normalized size = 34.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^2*b^2*d*x*cosh(d*x + c)^8 + 128*a^2*b^2*d*x*cosh(d*x + c)*sinh(
d*x + c)^7 + 16*a^2*b^2*d*x*sinh(d*x + c)^8 + 4*(16*a^3*b - 20*a^2*b^2 + a*
b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^6 + 4*(112*a^2*b^2*
d*x*cosh(d*x + c)^2 + 16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b -
 a^2*b^2)*d*x)*sinh(d*x + c)^6 + 16*a^2*b^2*d*x + 8*(112*a^2*b^2*d*x*cosh(d
*x + c)^3 + 3*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^
2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(48*a^4 - 72*a^3*b + 18*a^2*b^2
+ 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^4 +
 4*(280*a^2*b^2*d*x*cosh(d*x + c)^4 + 48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a
*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x + 15*(16*a^3*b - 20*a^2*
b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 24*a^2*b^2 - 12*a*b^3 - 12*b^4 + 16*(56*a^2*b^2*d*x*cosh(d*x + c)
^5 + 5*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)
*cosh(d*x + c)^3 + (48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(
8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(32*a^
3*b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x
+ c)^2 + 4*(112*a^2*b^2*d*x*cosh(d*x + c)^6 + 15*(16*a^3*b - 20*a^2*b^2 + a
*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^4 + 32*a^3*b - 28*
a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x + 6*(48*a^4 - 72*a^
3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x
+ c)^8 + 8*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8
*a^2*b^2 + 4*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4
)*cosh(d*x + c)^6 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 + 4*a*b^3
+ 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 + 4*a*b^3 + 3*b
^4)*cosh(d*x + c)^3 + 3*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x +
 c)^4 + 2*(35*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 64*a^4 - 32*a
^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4 + 30*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 + 4*a*b^3 + 3*b^4 + 8*(7*(8*a^2*b^
2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh
(d*x + c)^3 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*
(7*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b + 2*a*b^3 -
 3*b^4)*cosh(d*x + c)^4 + 16*a^3*b + 2*a*b^3 - 3*b^4 + 3*(64*a^4 - 32*a^3*b
 + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*
a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(16*a^3*b + 2*a*b^3 - 3*b^4)
*cosh(d*x + c)^5 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh
(d*x + c)^3 + (16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d
*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cos
h(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x
 + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 -
 a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x +
 c))*sinh(d*x + c) + b)) + 8*(16*a^2*b^2*d*x*cosh(d*x + c)^7 + 3*(16*a^3*b
- 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^5
+ 2*(48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b
 + 3*a^2*b^2)*d*x)*cosh(d*x + c)^3 + (32*a^3*b - 28*a^2*b^2 - 13*a*b^3 + 9*
b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^5*d*
cosh(d*x + c)^8 + 8*a^2*b^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*b^5*d*sin
h(d*x + c)^8 + a^2*b^5*d + 4*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7
*a^2*b^5*d*cosh(d*x + c)^2 + (2*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(
8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*a^2*b^5*d*cosh(
d*x + c)^3 + 3*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
35*a^2*b^5*d*cosh(d*x + c)^4 + 30*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 +
```

(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + 8*(7*a^2*b^5*d*cosh(d*x + c)^5 + 10*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^2*b^5*d*cosh(d*x + c)^6 + 15*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^2 + (2*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^2 + 8*(a^2*b^5*d*cosh(d*x + c)^7 + 3*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(8*a^2*b^2*d*x*cosh(d*x + c)^8 + 64*a^2*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 8*a^2*b^2*d*x*sinh(d*x + c)^8 + 2*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^6 + 2*(112*a^2*b^2*d*x*cosh(d*x + c)^2 + 16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*sinh(d*x + c)^6 + 8*a^2*b^2*d*x + 4*(112*a^2*b^2*d*x*cosh(d*x + c)^3 + 3*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^4 + 2*(280*a^2*b^2*d*x*cosh(d*x + c)^4 + 48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x + 15*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 12*a^2*b^2 - 6*a*b^3 - 6*b^4 + 8*(56*a^2*b^2*d*x*cosh(d*x + c)^5 + 5*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^3 + (48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(32*a^3*b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^2 + 2*(112*a^2*b^2*d*x*cosh(d*x + c)^6 + 15*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^4 + 32*a^3*b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x + 6*(48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^2*b^2 + 4*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4 + 30*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 + 4*a*b^3 + 3*b^4 + 8*(7*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 16*a^3*b + 2*a*b^3 - 3*b^4 + 3*(64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) + 4*(16*a^2*b^2*d*x*cosh(d*x + c)^7 + 3*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^5 + 2*(48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^3 + (32*a^3*b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^5*d*cosh(d*x + c)^8 + 8*a^2*b^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*b^5*d*sinh(d*x + c)^8 + a^2*b^5*d + 4*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*a^2*b^5*d*cosh(d*x + c)^2 + (2*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*a^2*b^5*d*cosh(d*x + c)^3 + 3*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*b^5*d*cosh(d*x + c)^4 +

```
30*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2
*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + 8*(7
*a^2*b^5*d*cosh(d*x + c)^5 + 10*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (
8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*
a^2*b^5*d*cosh(d*x + c)^6 + 15*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4 + 3*
(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^2 + (2*a^3*b^4 - a^2*b^
5)*d)*sinh(d*x + c)^2 + 8*(a^2*b^5*d*cosh(d*x + c)^7 + 3*(2*a^3*b^4 - a^2*b
^5)*d*cosh(d*x + c)^5 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)
^3 + (2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 8.40, size = 353, normalized size = 2.21

$$\frac{\frac{8\,(dx+c)}{b^3} - \frac{\left(8\,a^3-4\,a^2b-ab^2-3\,b^3\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a^2b^3} + \frac{2\left(16\,a^3be^{(6\,dx+6\,c)}-20\,a^2b^2e^{(6\,dx+6\,c)}+ab^3e^{(6\,dx+6\,c)}+3\,b^4e^{(6\,dx+6\,c)}+48\,a^4e^{(4\,dx+4\,c)}-72 \cdots}{\cdots}}{8 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/b^3 - (8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*arctan(1/2*(b*e^(2
*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a^2*b^3) + 2*(16
*a^3*b*e^(6*d*x + 6*c) - 20*a^2*b^2*e^(6*d*x + 6*c) + a*b^3*e^(6*d*x + 6*c)
 + 3*b^4*e^(6*d*x + 6*c) + 48*a^4*e^(4*d*x + 4*c) - 72*a^3*b*e^(4*d*x + 4*c
) + 18*a^2*b^2*e^(4*d*x + 4*c) + 15*a*b^3*e^(4*d*x + 4*c) - 9*b^4*e^(4*d*x
+ 4*c) + 32*a^3*b*e^(2*d*x + 2*c) - 28*a^2*b^2*e^(2*d*x + 2*c) - 13*a*b^3*e
^(2*d*x + 2*c) + 9*b^4*e^(2*d*x + 2*c) + 6*a^2*b^2 - 3*a*b^3 - 3*b^4)/((b*e
^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2*a^2*b^3))
/d
```

maple [B] time = 0.14, size = 2048, normalized size = 12.80

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -3/8/d/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d/a^2/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2))+7/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c
)^7+7/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2
*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)
^7-23/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d
*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-23/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-
2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c
)^3-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d
*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)-1/2/d/b^2/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
))+1/2/d/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2
*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*
c)-3/8/d*b/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d*b/a^2/(-
b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b^2*a/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
```

```
))^(1/2)-a+2*b)*a)^(1/2))+1/d/b^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2))+1/d/b^3*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/8/d/a/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2))-1/8/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2))-1/d/b^3*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b^2/(tanh(1/2*d*x+1/2*c)^
4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x
+1/2*c)^3+3/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3-1/d/b^2/(tanh(1/2*d*x+1/2*
c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*
d*x+1/2*c)-1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^7+1/d/b^2/(tanh(1/2*d*x+1/2
*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2
*d*x+1/2*c)^5-1/8/d/b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/8/d/b/a/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2))-1/2/d/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2
/d/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*b/(tanh(1/2*d*x+1/2
*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1
/2*d*x+1/2*c)^5-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/b^3*ln(tanh(1/2*d*x+1
/2*c)+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^6}{{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^6/(a + b*sinh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.340 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=133

$$\frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\sinh(c+dx)}{8d\left(a+b\sinh^2(c+dx)\right)}+\frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d}-\frac{(a-b)\sinh(c+dx)\cosh^2(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] 1/8*(3*a^2+2*a*b+3*b^2)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/b^(5/2)
/d-1/4*(a-b)*cosh(d*x+c)^2*sinh(d*x+c)/a/b/d/(a+b*sinh(d*x+c)^2)^2+3/8*(1/a
^2-1/b^2)*sinh(d*x+c)/d/(a+b*sinh(d*x+c)^2)
```

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 413, 385, 205}

$$\frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\sinh(c+dx)}{8d\left(a+b\sinh^2(c+dx)\right)}+\frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d}-\frac{(a-b)\sinh(c+dx)\cosh^2(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2
)*b^(5/2)*d) - ((a - b)*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh
[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Sinh[c + d*x])/(8*d*(a + b*Sinh[c +
d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+bx^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\cosh^2(c+dx)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+3b+(3a+b)x^2}{\left(a+bx^2\right)^2}\,dx, x, \sinh(c+d$$

$$= -\frac{(a-b)\cosh^2(c+dx)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3\left(a^2-b^2\right)\sinh(c+dx)}{8a^2b^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\left(3a^2+\right.}{}$$

$$= \frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} - \frac{(a-b)\cosh^2(c+dx)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} -$$

Mathematica [A] time = 0.37, size = 149, normalized size = 1.12

$$\frac{\frac{8a^{3/2}\sqrt{b}(a-b)^2\sinh(c+dx)}{(2a+b\cosh(2(c+dx))-b)^2} - \frac{2\sqrt{a}\sqrt{b}\left(5a^2-2ab-3b^2\right)\sinh(c+dx)}{2a+b\cosh(2(c+dx))-b} - \left(3a^2+2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)}{8a^{5/2}b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-((3*a^2 + 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]]) + (8*a^
(3/2)*(a - b)^2*Sqrt[b]*Sinh[c + d*x])/(2*a - b + b*Cosh[2*(c + d*x)])^2 -
(2*Sqrt[a]*Sqrt[b]*(5*a^2 - 2*a*b - 3*b^2)*Sinh[c + d*x])/(2*a - b + b*Cosh
[2*(c + d*x)]))/(8*a^(5/2)*b^(5/2)*d)
```

fricas [B] time = 0.69, size = 5844, normalized size = 43.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^7 + 28*(5*a^3*b^2
 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(5*a^3*b^2 - 2*a^
2*b^3 - 3*a*b^4)*sinh(d*x + c)^7 + 4*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9
*a*b^4)*cosh(d*x + c)^5 + 4*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4 +
21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*
(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (12*a^4*b - 7*a^3*b^
2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(12*a^4*b - 7*
a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 4*(12*a^4*b - 7*a^3*b^2 -
 14*a^2*b^3 + 9*a*b^4 - 35*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^
4 - 10*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^3 + 4*(21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 10*(
12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 3*(12*a^4*b
- 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^
2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*
cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + 2*a*b^3 + 3*b^4)*sinh(d*x + c)
^8 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(6*a^3*b +
 a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^
```

2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3
*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(
3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 - 8*a^3*b + 17*a^2*b^
2 - 18*a*b^3 + 9*b^4 + 30*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(3*a^2*b^2 + 2*a
*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*co
sh(d*x + c)^3 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x +
 c)^2 + 4*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(6*a^3*b +
a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 6*a^3*b + a^2*b^2 + 4*a*b^3 -
3*b^4 + 3*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(
6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 - 8*a^3*b +
17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (6*a^3*b + a^2*b^2 + 4*a*b
^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log((b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh
(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*c
osh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c
)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c
)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) - 4*(5*a^3*b^2 - 2*a^2*b
^3 - 3*a*b^4)*cosh(d*x + c) + 4*(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d
*x + c)^6 - 5*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4 + 5*(12*a^4*b - 7*a^3*b^2 - 14*
a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 - 3*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 +
 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^3*b^5*d*cosh(d*x + c)^8 + 8*a^
3*b^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*b^5*d*sinh(d*x + c)^8 + a^3*b^5
d + 4(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*a^3*b^5*d*cosh(d*x +
c)^2 + (2*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^3 - 8*a^4*b^4
+ 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*a^3*b^5*d*cosh(d*x + c)^3 + 3*(2*a^4*
b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^3*b^5*d*cosh(d*x
+ c)^4 + 30*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^5*b^3 - 8*a^4*b^
4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)
^2 + 8*(7*a^3*b^5*d*cosh(d*x + c)^5 + 10*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x +
 c)^3 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^
3 + 4*(7*a^3*b^5*d*cosh(d*x + c)^6 + 15*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x +
c)^4 + 3*(8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^4*b^4
 - a^3*b^5)*d)*sinh(d*x + c)^2 + 8*(a^3*b^5*d*cosh(d*x + c)^7 + 3*(2*a^4*b^
4 - a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh
(d*x + c)^3 + (2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(
2*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^7 + 14*(5*a^3*b^2 - 2*a^2
*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(5*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4)*sinh(d*x + c)^7 + 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*
cosh(d*x + c)^5 + 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4 + 21*(5*a^
3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(5*a^
3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (12*a^4*b - 7*a^3*b^2 - 14*a
^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(12*a^4*b - 7*a^3*b^2
- 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*
b^3 + 9*a*b^4 - 35*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 - 10*(
12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)
^3 + 2*(21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 10*(12*a^4*b
 - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 3*(12*a^4*b - 7*a^3*
b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - ((3*a^2*b^2 +
2*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x
 + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + 2*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(
6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(6*a^3*b + a^2*b^2
 + 4*a*b^3 - 3*b^4 + 7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(

```
d*x + c)^6 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(6*a^3*
b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 -
 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b^
2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a
*b^3 + 9*b^4 + 30*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*si
nh(d*x + c)^4 + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3
*b^4)*cosh(d*x + c)^5 + 10*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x +
 c)^3 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c))*s
inh(d*x + c)^3 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^2 +
4*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(6*a^3*b + a^2*b^2
+ 4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4 +
3*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 8*((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(6*a^3*b
+ a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 - 8*a^3*b + 17*a^2*b
^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b
^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x
+ c) + sinh(d*x + c))/a) - ((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^8 +
 8*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2
 + 2*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^
4)*cosh(d*x + c)^6 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(3*a^2*b^2
+ 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + 2*a
*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 +
 9*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)
^4 + 24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4 + 30*(6*a^3*b + a^2*b
^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^2*b^2 + 2*a*b^
3 + 3*b^4 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(6*a^3*
b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (24*a^4 - 8*a^3*b + 17*a^2
*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(6*a^3*b + a^2*
b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)
*cosh(d*x + c)^6 + 15*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^4
 + 6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4 + 3*(24*a^4 - 8*a^3*b + 17*a^2*b^2 -
 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 2*a*b
^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(
d*x + c)^5 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x +
c)^3 + (6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2
 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a
 - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) - 2*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)
*cosh(d*x + c) + 2*(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 - 5
*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4 + 5*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a
*b^4)*cosh(d*x + c)^4 - 3*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cos
h(d*x + c)^2)*sinh(d*x + c))/(a^3*b^5*d*cosh(d*x + c)^8 + 8*a^3*b^5*d*cosh(
d*x + c)*sinh(d*x + c)^7 + a^3*b^5*d*sinh(d*x + c)^8 + a^3*b^5*d + 4*(2*a^4
*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*a^3*b^5*d*cosh(d*x + c)^2 + (2*a^4
*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*
d*cosh(d*x + c)^4 + 8*(7*a^3*b^5*d*cosh(d*x + c)^3 + 3*(2*a^4*b^4 - a^3*b^5
)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^3*b^5*d*cosh(d*x + c)^4 + 30*(
2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5
)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*a^3
*b^5*d*cosh(d*x + c)^5 + 10*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^
5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*
b^5*d*cosh(d*x + c)^6 + 15*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(8*a
^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^4*b^4 - a^3*b^5)*d
)*sinh(d*x + c)^2 + 8*(a^3*b^5*d*cosh(d*x + c)^7 + 3*(2*a^4*b^4 - a^3*b^5)*
d*cosh(d*x + c)^5 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 +
 (2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-85,-18]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[33,-80]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-98,-18]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-57,-1
0]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.The choice was done assuming [a,b]=[-57,-3]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [a,b]=[-53,60]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[80,-1]schur row 3 -6.9034e-07
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[-51,-3]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[15,-93]Warning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.The choice was done assuming [a,b]=[-62,-3]Warning, need to choose
a branch for the root of a polynomial with parameters. This might be wrong.
The choice was done assuming [a,b]=[-64,-88]Warning, need to choose a branc
h for the root of a polynomial with parameters. This might be wrong.The cho
ice was done assuming [a,b]=[82,-14]Warning, need to choose a branch for th
e root of a polynomial with parameters. This might be wrong.The choice was
done assuming [a,b]=[42,-23]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b]=[90,-28]Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.The choice was done assuming [a
,b]=[-71,-39]Warning, need to choose a branch for the root of a polynomial
with parameters. This might be wrong.The choice was done assuming [a,b]=[15
,70]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.The choice was done assuming [a,b]=[6,80]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [a,b]=[-27,31]Undef/Unsigned
 Inf encountered in limitEvaluation time: 4.35Limit: Max order reached or u
nable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.12, size = 1884, normalized size = 14.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 3/8/d/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d/a^2/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/
2))-7/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3-5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)
```

```
^7+7/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+
1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+1/2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^
7+7/2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-7/2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3
-1/2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+
1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+3/8/d/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-
3/8/d/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)+
3/8/d*b/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b/a^2/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d/b^2*a/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2))-3/8/d/b^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+
a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a
)^(1/2))-1/8/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/8/d/a/(-b
*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+9/4/d/b^2/(tanh(1/2*d*x+1/2*c)^4
*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+
1/2*c)^3+3/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/
2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3-3/4/d/b^2/(tanh(1/2*d*x+1/2
*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2
*d*x+1/2*c)+3/4/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^7-9/4/d/b^2/(tanh(1/2*d*
x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tan
h(1/2*d*x+1/2*c)^5+1/4/d/b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/4/d/b/a/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))+1/8/d/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)
*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
)+1/8/d/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*b/(tanh(1/2*d*
x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*t
anh(1/2*d*x+1/2*c)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(5 a^2 b e^{(7c)} - 2 a b^2 e^{(7c)} - 3 b^3 e^{(7c)}\right) e^{(7dx)} + \left(12 a^3 e^{(5c)} - 7 a^2 b e^{(5c)} - 14 a b^2 e^{(5c)} + 9 b^3 e^{(5c)}\right) e^{(5dx)} - \left(12 a^3 e^{(3c)} -}{4 \left(a^2 b^4 d e^{(8dx+8c)} + a^2 b^4 d + 4 \left(2 a^3 b^3 d e^{(6c)} - a^2 b^4 d e^{(6c)}\right) e^{(6dx)} + 2 \left(8 a^4 b^2 d e^{(4c)} - 8 a^3 b^3 d e^{(4c)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((5*a^2*b*e^(7*c) - 2*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*e^(7*d*x) + (12*a
^3*e^(5*c) - 7*a^2*b*e^(5*c) - 14*a*b^2*e^(5*c) + 9*b^3*e^(5*c))*e^(5*d*x)
- (12*a^3*e^(3*c) - 7*a^2*b*e^(3*c) - 14*a*b^2*e^(3*c) + 9*b^3*e^(3*c))*e^(
3*d*x) - (5*a^2*b*e^c - 2*a*b^2*e^c - 3*b^3*e^c)*e^(d*x))/(a^2*b^4*d*e^(8*d
*x + 8*c) + a^2*b^4*d + 4*(2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*
x) + 2*(8*a^4*b^2*d*e^(4*c) - 8*a^3*b^3*d*e^(4*c) + 3*a^2*b^4*d*e^(4*c))*e^
(4*d*x) + 4*(2*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 1/32*int
egrate(8*((3*a^2*e^(3*c) + 2*a*b*e^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (3*a^
2*e^c + 2*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^2*b^3*e^(4*d*x + 4*c) + a^2*b^3
+ 2*(2*a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+dx)^5}{\left(b\sinh(c+dx)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.341 $\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3} dx$

Optimal. Leaf size=114

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a-b}}+\frac{3\tanh(c+dx)}{8a^2d\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

```
[Out] 3/8*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/d/(a-b)^(1/2)+1/4*tanh
(d*x+c)/a/d/(a-(a-b)*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/a^2/d/(a-(a-b)*tanh(d
*x+c)^2)
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3191, 199, 208}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a-b}}+\frac{3\tanh(c+dx)}{8a^2d\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a - b]*d)
+ Tanh[c + d*x]/(4*a*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])
/(8*a^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= \frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8a^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a-b}\,d} + \frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c}{8a^2d\left(a-(a-b)\right.}$$

Mathematica [A] time = 0.69, size = 102, normalized size = 0.89

$$\frac{\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a}\sinh(2(c+dx))((2a+3b)\cosh(2(c+dx))+8a-3b)}{(2a+b\cosh(2(c+dx))-b)^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b] + (Sqrt[a]*(8
*a - 3*b + (2*a + 3*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(2*a - b + b*C
osh[2*(c + d*x)])^2)/(8*a^(5/2)*d)
```

fricas [B] time = 0.59, size = 4486, normalized size = 39.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 24*
(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 +
 4*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^6 + 8*a^3*b^2
+ 4*a^2*b^3 - 12*a*b^4 + 4*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*
a*b^4)*cosh(d*x + c)^4 + 4*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*
a*b^4 + 15*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 16*(5*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x +
 c)^3 + (16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4)*cosh(d*
x + c)^2 + 4*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4 + 15*(8*a^4*b - 8*
a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 8*a^4*b - 26*a
^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(b^4*co
sh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^4*sinh(d*x + c)^8 +
 4*(2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(7*b^4*cosh(d*x + c)^2 + 2*a*b^3 - b
^4)*sinh(d*x + c)^6 + 8*(7*b^4*cosh(d*x + c)^3 + 3*(2*a*b^3 - b^4)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^4 +
2*(35*b^4*cosh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^3 + 3*b^4 + 30*(2*a*b^3 - b^4
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4*cosh(d*x + c)^5 + 10*(2
*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)
```

```
)*sinh(d*x + c)^3 + 4*(2*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*b^4*cosh(d*x +
 c)^6 + 15*(2*a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a*b^3 - b^4 + 3*(8*a^2*b^2 -
 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b^4*cosh(d*x + c)^7
 + 3*(2*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d
*x + c)^3 + (2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*l
og((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*
x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a
*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 +
(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b
*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 +
2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x +
c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b))
 + 8*(3*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 + 2*(16*
a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^3 + (8*a^4
*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^4*
b^4 - a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)*si
nh(d*x + c)^7 + (a^4*b^4 - a^3*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^5*b^3 - 3*a^
4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b^4 - a^3*b^5)*d*cosh(d*x +
c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^6*b^2
 - 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b^4 -
a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 30*(
2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^6*b^2 - 16*a^5*b^
3 + 11*a^4*b^4 - 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 3*a^4*b^4 +
 a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 +
10*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^6*b^2 - 16*a^
5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^
4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^4 + 3*(8*a^6*b^2 - 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*co
sh(d*x + c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^2 + (a^4
*b^4 - a^3*b^5)*d + 8*((a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^5*b^3
 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^6*b^2 - 16*a^5*b^3 + 11*a^
4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^3 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3
*a*b^4)*cosh(d*x + c)^6 + 12*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*co
sh(d*x + c)*sinh(d*x + c)^5 + 2*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)
*sinh(d*x + c)^6 + 4*a^3*b^2 + 2*a^2*b^3 - 6*a*b^4 + 2*(16*a^5 - 8*a^4*b -
26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^4 + 2*(16*a^5 - 8*a^4*b -
26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4 + 15*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3
*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(8*a^4*b - 8*a^3*b^2 - 3*a^
2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + (16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*
b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^4*b + 8*a^3*b^2 - 25
*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2 + 2*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 +
 9*a*b^4 + 15*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 +
 6*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 3*(b^4*cosh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x + c
)^7 + b^4*sinh(d*x + c)^8 + 4*(2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(7*b^4*co
sh(d*x + c)^2 + 2*a*b^3 - b^4)*sinh(d*x + c)^6 + 8*(7*b^4*cosh(d*x + c)^3 +
 3*(2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2*b^2 - 8*a*b^3
+ 3*b^4)*cosh(d*x + c)^4 + 2*(35*b^4*cosh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^3
+ 3*b^4 + 30*(2*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + b^4 + 8*(7*
b^4*cosh(d*x + c)^5 + 10*(2*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^2*b^2 - 8*a
*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b^3 - b^4)*cosh(d*x +
 c)^2 + 4*(7*b^4*cosh(d*x + c)^6 + 15*(2*a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a
*b^3 - b^4 + 3*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 8*(b^4*cosh(d*x + c)^7 + 3*(2*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^2*b^
2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a*b^3 - b^4)*cosh(d*x + c))*sinh(
d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
```

```
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b
)) + 4*(3*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 + 2*(
16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^3 + (8*
a^4*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a
^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)
*sinh(d*x + c)^7 + (a^4*b^4 - a^3*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^5*b^3 - 3
*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b^4 - a^3*b^5)*d*cosh(d*x
 + c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^6*b
^2 - 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b^4
 - a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(
d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3
0*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^6*b^2 - 16*a^5
*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 3*a^4*b^
4 + a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5
 + 10*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^6*b^2 - 16
*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*
(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5
)*d*cosh(d*x + c)^4 + 3*(8*a^6*b^2 - 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d
*cosh(d*x + c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^2 + (
a^4*b^4 - a^3*b^5)*d + 8*((a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^5*
b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^6*b^2 - 16*a^5*b^3 + 11
*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^3 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)
*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 5.49, size = 244, normalized size = 2.14

$$\frac{\frac{3\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}\,a^2} - \frac{2\left(8a^2be^{(6dx+6c)}-3b^3e^{(6dx+6c)}+16a^3e^{(4dx+4c)}+8a^2be^{(4dx+4c)}-18ab^2e^{(4dx+4c)}+9b^3e^{(4dx+4c)}+8a^2be^{(2dx+2c)}+16ab^2\cdots\right.}{\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{(2dx+2c)}+b\right)^2a^2b^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^
2 + a*b)*a^2) - 2*(8*a^2*b*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 16*a^3
*e^(4*d*x + 4*c) + 8*a^2*b*e^(4*d*x + 4*c) - 18*a*b^2*e^(4*d*x + 4*c) + 9*b
^3*e^(4*d*x + 4*c) + 8*a^2*b*e^(2*d*x + 2*c) + 16*a*b^2*e^(2*d*x + 2*c) - 9
*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + 3*b^3)/((b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x
+ 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2*a^2*b^2))/d
```

maple [B] time = 0.12, size = 664, normalized size = 5.82

$$\frac{5\left(\tanh^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+4\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+a\right)^2a}+\frac{3\left(\tan\cdots\right.}{4d\left(\left(\tanh^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2\left(\tanh^2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+3
/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+
3/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tan
```

```
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-3
/8/d/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d*b/a^2/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2))+3/8/d/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d*b/a^2/(-
b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^4}{{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^4/(a + b*sinh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.342 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=117

$$\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}+\frac{(a+3b)\sinh(c+dx)}{8a^2bd\left(a+b\sinh^2(c+dx)\right)}-\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] 1/8*(a+3*b)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)/d-1/4*(a-b)
*sinh(d*x+c)/a/b/d/(a+b*sinh(d*x+c)^2)^2+1/8*(a+3*b)*sinh(d*x+c)/a^2/b/d/(a
+b*sinh(d*x+c)^2)
```

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 385, 199, 205}

$$\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}+\frac{(a+3b)\sinh(c+dx)}{8a^2bd\left(a+b\sinh^2(c+dx)\right)}-\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a + 3*b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*d) -
 ((a - b)*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh[c + d*x]^2)^2) + ((a + 3*b)*S
inh[c + d*x])/(8*a^2*b*d*(a + b*Sinh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{4abd}$$

$$= -\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(a+3b)\sinh(c+dx)}{8a^2bd\left(a+b\sinh^2(c+dx)\right)} + \frac{(a+3b)\,\text{Subst}}{}$$

$$= \frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} - \frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(a+3b)\sinh}{8a^2bd\left(a+b\sin\right.}$$

Mathematica [A] time = 0.68, size = 114, normalized size = 0.97

$$\frac{(a+3b)\left(\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} + \frac{\sinh(c+dx)\left(5a+3b\sinh^2(c+dx)\right)}{8a^2\left(a+b\sinh^2(c+dx)\right)^2}\right) - \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-(Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2) + (a + 3*b)*((3*ArcTan[(Sqrt[b]
*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]) + (Sinh[c + d*x]*(5*a + 3*b*S
inh[c + d*x]^2))/(8*a^2*(a + b*Sinh[c + d*x]^2)^2)))/(3*b*d)
```

fricas [B] time = 0.79, size = 4907, normalized size = 41.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^7 + 28*(a^2*b^2 + 3*a*b^3)*cosh(
d*x + c)*sinh(d*x + c)^6 + 4*(a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^7 - 4*(4*a^3
*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^5 - 4*(4*a^3*b - 17*a^2*b^2 + 9*a*
b^3 - 21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*
b^2 + 3*a*b^3)*cosh(d*x + c)^3 - (4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^4 + 4*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^3
+ 4*(35*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 4*a^3*b - 17*a^2*b^2 + 9*a*b^
3 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
4*(21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*
b^3)*cosh(d*x + c)^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^2 - ((a*b^2 + 3*b^3)*cosh(d*x + c)^8 + 8*(a*b^2 + 3*b^3)*cosh(d
*x + c)*sinh(d*x + c)^7 + (a*b^2 + 3*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b + 5*
a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3 + 7*(a*b^2 +
3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b^3)*cosh(d*x + c
)^3 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a
^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 + 3*b^3)*c
osh(d*x + c)^4 + 8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3 + 30*(2*a^2*b + 5*a*b^
2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a*b^2 + 3*b^3)*cosh(d*x
```

```
 + c)^5 + 10*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^3 + (8*a^3 + 16*a^2*
b - 21*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 + 3*b^3 + 4*(2
*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 4*(7*(a*b^2 + 3*b^3)*cosh(d*x +
 c)^6 + 15*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^4 + 2*a^2*b + 5*a*b^2
- 3*b^3 + 3*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*((a*b^2 + 3*b^3)*cosh(d*x + c)^7 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3
)*cosh(d*x + c)^5 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^3 +
 (2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log((
b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 -
 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x +
 c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(
cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*co
sh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d
*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
- b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) - 4*(a^
2*b^2 + 3*a*b^3)*cosh(d*x + c) + 4*(7*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^6 -
 5*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^4 - a^2*b^2 - 3*a*b^3 + 3
*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^3*b^4*
d*cosh(d*x + c)^8 + 8*a^3*b^4*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*b^4*d*s
inh(d*x + c)^8 + a^3*b^4*d + 4*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^6 + 4*
(7*a^3*b^4*d*cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^6 + 2
*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*a^3*b^4*d*cos
h(d*x + c)^3 + 3*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(35*a^3*b^4*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2
 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^3 -
a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^4*d*cosh(d*x + c)^5 + 10*(2*a^4*b^3
 - a^3*b^4)*d*cosh(d*x + c)^3 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^4*d*cosh(d*x + c)^6 + 15*(2*a^4*b^3
- a^3*b^4)*d*cosh(d*x + c)^4 + 3*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh
(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + 8*(a^3*b^4*d*cosh(
d*x + c)^7 + 3*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^5 + (8*a^5*b^2 - 8*a^4
*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^3 + (2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)
)*sinh(d*x + c)), 1/8*(2*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^7 + 14*(a^2*b^2
+ 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2*b^2 + 3*a*b^3)*sinh(d*x +
 c)^7 - 2*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^5 - 2*(4*a^3*b - 1
7*a^2*b^2 + 9*a*b^3 - 21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^5 + 10*(7*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 - (4*a^3*b - 17*a^2*b^2 + 9*
a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*
cosh(d*x + c)^3 + 2*(35*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 4*a^3*b - 17*
a^2*b^2 + 9*a*b^3 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^3 + 2*(21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5 - 10*(4*a^3*b - 1
7*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*c
osh(d*x + c))*sinh(d*x + c)^2 + ((a*b^2 + 3*b^3)*cosh(d*x + c)^8 + 8*(a*b^2
 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 + 3*b^3)*sinh(d*x + c)^8 +
 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b
^3 + 7*(a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b
^3)*cosh(d*x + c)^3 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 2*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + 2*(35*(
a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3 + 30*(
2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a*b^2 +
 3*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^3 +
(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^
2 + 3*b^3 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 4*(7*(a*b^2 + 3
*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^4 + 2*
a^2*b + 5*a*b^2 - 3*b^3 + 3*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 + 3*b^3)*cosh(d*x + c)^7 + 3*(2*a^2*b +
 5*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*c
osh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c))*
```

```
sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) + ((a*b^2
 + 3*b^3)*cosh(d*x + c)^8 + 8*(a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^7
 + (a*b^2 + 3*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x
 + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3 + 7*(a*b^2 + 3*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 3*(2*a^2*b + 5*a*
b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 + 16*a^2*b - 21*a*b^
2 + 9*b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 8*a^3
+ 16*a^2*b - 21*a*b^2 + 9*b^3 + 30*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c
)^2)*sinh(d*x + c)^4 + 8*(7*(a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b +
 5*a*b^2 - 3*b^3)*cosh(d*x + c)^3 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + a*b^2 + 3*b^3 + 4*(2*a^2*b + 5*a*b^2 - 3*b^
3)*cosh(d*x + c)^2 + 4*(7*(a*b^2 + 3*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b + 5
*a*b^2 - 3*b^3)*cosh(d*x + c)^4 + 2*a^2*b + 5*a*b^2 - 3*b^3 + 3*(8*a^3 + 16
*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 + 3
*b^3)*cosh(d*x + c)^7 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + (8*
a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 - 3
*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3
 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d
*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) -
 2*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c) + 2*(7*(a^2*b^2 + 3*a*b^3)*cosh(d*x +
c)^6 - 5*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^4 - a^2*b^2 - 3*a*b
^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^
3*b^4*d*cosh(d*x + c)^8 + 8*a^3*b^4*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*b
^4*d*sinh(d*x + c)^8 + a^3*b^4*d + 4*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^
6 + 4*(7*a^3*b^4*d*cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)
^6 + 2*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*a^3*b^4
*d*cosh(d*x + c)^3 + 3*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(35*a^3*b^4*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x
+ c)^2 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(2*a^4*
b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^4*d*cosh(d*x + c)^5 + 10*(2*a
^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^3 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d
*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^4*d*cosh(d*x + c)^6 + 15*(2*a^
4*b^3 - a^3*b^4)*d*cosh(d*x + c)^4 + 3*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*
d*cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + 8*(a^3*b^4*d
*cosh(d*x + c)^7 + 3*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^5 + (8*a^5*b^2 -
 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^3 + (2*a^4*b^3 - a^3*b^4)*d*cosh(d*
x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-85,-18]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[33,-80]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-98,-18]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-57,-1
0]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.The choice was done assuming [a,b]=[-57,-3]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [a,b]=[-53,60]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[80,-1]schur row 3 -6.9034e-07
```

```
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[-51,-3]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-78,38]Warning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.The choice was done assuming [a,b]=[-75,-16]Warning, need to choose
 a branch for the root of a polynomial with parameters. This might be wrong
.The choice was done assuming [a,b]=[-58,-64]Warning, need to choose a bran
ch for the root of a polynomial with parameters. This might be wrong.The ch
oice was done assuming [a,b]=[-72,82]Warning, need to choose a branch for t
he root of a polynomial with parameters. This might be wrong.The choice was
 done assuming [a,b]=[27,42]Warning, need to choose a branch for the root o
f a polynomial with parameters. This might be wrong.The choice was done ass
uming [a,b]=[-64,90]Undef/Unsigned Inf encountered in limitEvaluation time:
 2.89Limit: Max order reached or unable to make series expansion Error: Bad
 Argument Value
```

maple [B] time = 0.13, size = 1348, normalized size = 11.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7-3
/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+11/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-3
/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5+3/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-
11/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+3/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^
3-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-1/8
/d/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/8/d/b/a/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2))-1/4/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/8/d/
b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*
d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/8/d/b/a/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2))-1/4/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d/
a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2))-3/8/d/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d*b/a^2/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(abe^{(7c)} + 3b^2e^{(7c)}\right)e^{(7dx)} - \left(4a^2e^{(5c)} - 17abe^{(5c)} + 9b^2e^{(5c)}\right)e^{(5dx)} + \left(4a^2e^{(3c)} - 17abe^{(3c)} + 9b^2e^{(3c)}\right)e}{4\left(a^2b^3de^{(8dx+8c)} + a^2b^3d + 4\left(2a^3b^2de^{(6c)} - a^2b^3de^{(6c)}\right)e^{(6dx)} + 2\left(8a^4bde^{(4c)} - 8a^3b^2de^{(4c)} + 3a^2b^3de^{(4c)}\right)e^{(4dx)}\cdot\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((a*b*e^(7*c) + 3*b^2*e^(7*c))*e^(7*d*x) - (4*a^2*e^(5*c) - 17*a*b*e^(5
*c) + 9*b^2*e^(5*c))*e^(5*d*x) + (4*a^2*e^(3*c) - 17*a*b*e^(3*c) + 9*b^2*e^
(3*c))*e^(3*d*x) - (a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^2*b^3*d*e^(8*d*x + 8*c
) + a^2*b^3*d + 4*(2*a^3*b^2*d*e^(6*c) - a^2*b^3*d*e^(6*c))*e^(6*d*x) + 2*(
8*a^4*b*d*e^(4*c) - 8*a^3*b^2*d*e^(4*c) + 3*a^2*b^3*d*e^(4*c))*e^(4*d*x) +
4*(2*a^3*b^2*d*e^(2*c) - a^2*b^3*d*e^(2*c))*e^(2*d*x)) + 1/8*integrate(2*((
a*e^(3*c) + 3*b*e^(3*c))*e^(3*d*x) + (a*e^c + 3*b*e^c)*e^(d*x))/(a^2*b^2*e^
(4*d*x + 4*c) + a^2*b^2 + 2*(2*a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh \left(c+d\,x\right)^3}{{\left(b\,\sinh \left(c+d\,x\right)^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.343 $\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3} dx$

Optimal. Leaf size=143

$$\frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{3/2}}+\frac{(4a-3b)\tanh(c+dx)}{8a^2d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{b\tanh(c+dx)}{4ad(a-b)\left(a-(a-b)\tanh^2(c+dx)\right.}$$

```
[Out] 1/8*(4*a-3*b)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a-b)^(3/2)/
d-1/4*b*tanh(d*x+c)/a/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)^2+1/8*(4*a-3*b)*tanh(
d*x+c)/a^2/(a-b)/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 385, 199, 208}

$$\frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{3/2}}+\frac{(4a-3b)\tanh(c+dx)}{8a^2d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{b\tanh(c+dx)}{4ad(a-b)\left(a-(a-b)\tanh^2(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a -
b)^(3/2)*d) - (b*Tanh[c + d*x])/(4*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2
)^2) + ((4*a - 3*b)*Tanh[c + d*x])/(8*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d
*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{(4a-3b)\,\text{Subst}\left(\int \frac{1}{\left(a+(-a+b)x^2\right)^2}\,dx,\right.}{4a(a-b)d}$$

$$= -\frac{b\tanh(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{(4a-3b)\tanh(c+dx)}{8a^2(a-b)d\left(a-(a-b)\tanh^2(c+\right.}$$

$$= \frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{3/2}d} - \frac{b\tanh(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} +$$

Mathematica [A] time = 0.92, size = 124, normalized size = 0.87

$$\frac{\frac{\sqrt{a}\sinh(2(c+dx))\left(8a^2+b(2a-3b)\cosh(2(c+dx))-12ab+3b^2\right)}{(a-b)(2a+b\cosh(2(c+dx))-b)^2}+\frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a - b)^(3/2) +
 (Sqrt[a]*(8*a^2 - 12*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*(c + d*x)])*Sinh[2
*(c + d*x)])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*a^(5/2)*d)
```

fricas [B] time = 0.84, size = 5183, normalized size = 36.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 24*(4*a^3*b^2
- 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(4*a^3*b^2 - 7*a^2
*b^3 + 3*a*b^4)*sinh(d*x + c)^6 - 8*a^3*b^2 + 20*a^2*b^3 - 12*a*b^4 - 4*(16
*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 - 4*(1
6*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^
2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(4*a^3*b^2 - 7*a^
2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 - (16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2
*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(16*a^4*b - 44*a^3*b^2 +
 37*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 - 4*(16*a^4*b - 44*a^3*b^2 + 37*a^2*
b^3 - 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 6*(1
6*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + ((4*a*b^3 - 3*b^4)*cosh(d*x + c)^8 + 8*(4*a*b^3 - 3*b^4)*cosh
(d*x + c)*sinh(d*x + c)^7 + (4*a*b^3 - 3*b^4)*sinh(d*x + c)^8 + 4*(8*a^2*b^
2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4 + 7
*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a*b^3 - 3*b^4
)*cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^4 + 2
*(35*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 32*a^3*b - 56*a^2*b^2 + 36*a*b^3 -
 9*b^4 + 30*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4
```

+ 4*a*b^3 - 3*b^4 + 8*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + 10*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 15*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 8*a^2*b^2 - 10*a*b^3 + 3*b^4 + 3*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*a*b^3 - 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^3 + (8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 8*(3*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 - 2*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - (16*a^4*b - 44*a^3*b^2 + 37*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 30*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + 10*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + (a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d + 8*((a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(2*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 12*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^6 - 4*a^3*b^2 + 10*a^2*b^3 - 6*a*b^4 - 2*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 - 2*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 - (16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(16*a^4*b - 44*a^3*b^2 + 37*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 - 2*(16*a^4*b - 44*a^3*b^2 + 37*a^2*b^3 - 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((4*a*b^3 - 3*b^4)*cosh(d*x + c)^8 + 8*(4*a*b^3 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a*b^3 - 3*b^4)*sinh(d*x + c)^8 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4 + 7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(

```
d*x + c)^4 + 2*(35*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 32*a^3*b - 56*a^2*b^
2 + 36*a*b^3 - 9*b^4 + 30*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*s
inh(d*x + c)^4 + 4*a*b^3 - 3*b^4 + 8*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 +
 10*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (32*a^3*b - 56*a^2*b^2
 + 36*a*b^3 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 10*a*b
^3 + 3*b^4)*cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 15*(
8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 8*a^2*b^2 - 10*a*b^3 + 3*b^
4 + 3*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 + 8*((4*a*b^3 - 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 10*a*b^3 + 3
*b^4)*cosh(d*x + c)^5 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x
 + c)^3 + (8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt
(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 4*(3*(4*a
^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 - 2*(16*a^5 - 56*a^4*b + 70*a
^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - (16*a^4*b - 44*a^3*b^2 + 3
7*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 - 2*a^4*b^4 +
a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x +
 c)*sinh(d*x + c)^7 + (a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4
*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^
5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 5*a^5*b^3 + 4
*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^7*b - 24*a^6*b^2 + 27*a^5*b
^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 - 2*a^4*b^4
+ a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b
^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5
)*d*cosh(d*x + c)^4 + 30*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*co
sh(d*x + c)^2 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5
)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*co
sh(d*x + c)^2 + 8*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + 10
*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^7*b
 - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2
*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^7*b
- 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*
a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + (a^5*b^3 -
2*a^4*b^4 + a^3*b^5)*d + 8*((a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)
^7 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (8
*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^
3 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x
 + c))]
```

giac [B] time = 3.74, size = 269, normalized size = 1.88

$$\frac{\frac{(4a-3b)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{(a^3-a^2b)\sqrt{-a^2+ab}}+\frac{2\left(4ab^2e^{(6dx+6c)}-3b^3e^{(6dx+6c)}-16a^3e^{(4dx+4c)}+40a^2be^{(4dx+4c)}-30ab^2e^{(4dx+4c)}+9b^3e^{(4dx+4c)}-16a^2be^{(2d\cdots}\right.}{(a^3b-a^2b^2)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{(2dx+2c)}+b\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((4*a - 3*b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))
/((a^3 - a^2*b)*sqrt(-a^2 + a*b)) + 2*(4*a*b^2*e^(6*d*x + 6*c) - 3*b^3*e^(6
*d*x + 6*c) - 16*a^3*e^(4*d*x + 4*c) + 40*a^2*b*e^(4*d*x + 4*c) - 30*a*b^2*
e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) - 16*a^2*b*e^(2*d*x + 2*c) + 28*a*b
^2*e^(2*d*x + 2*c) - 9*b^3*e^(2*d*x + 2*c) - 2*a*b^2 + 3*b^3)/((a^3*b - a^2
*b^2)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2
))/d
```

maple [B] time = 0.12, size = 1322, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)^2/(a-b)*tanh(1/2*d*x+1/2*c)^7-5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(1/2*d*x+1/
2*c)^7*b-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2/(a-b)*tanh(1/2*d*x+1/2*c)^5+13/4/d/(tanh(1/2*d*x+1/2*c)
^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(
1/2*d*x+1/2*c)^5*b-3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2/(a-b)*tanh(1/2*d*x+1/2*c)^5*b^2-1/d/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2/(a-b)*tanh(1/2*d*x+1/2*c)^3+13/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(1/2*d*x+1/2*c)^3*b
-3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/a^2/(a-b)*tanh(1/2*d*x+1/2*c)^3*b^2+1/d/(tanh(1/2*d*x+1/2*c)^4*
a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a-b)*tanh(1/2*d
*x+1/2*c)-5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(1/2*d*x+1/2*c)*b-1/2/d/a/(a-b)/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2))-1/2/d/a/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2))*b+1/2/d/a/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/(a-b)/(-b*(a-b)
)^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b+3/8/d/a^2/(a-b)*b/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2))+3/8/d/a^2/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b
^2-3/8/d/a^2/(a-b)*b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d/a^2/(a-b)/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^2}{{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b*sinh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.344 $\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=96

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] 1/4*sinh(d*x+c)/a/d/(a+b*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+b*sinh(d
*x+c)^2)+3/8*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/d/b^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 199, 205}

$$\frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d}+\frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d) + Sinh[c
+ d*x]/(4*a*d*(a + b*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/(8*a^2*d*(a +
b*Sinh[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{4ad}$$

$$= \frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx\right.}{8a^2}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\,d} + \frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+d\right.}$$

Mathematica [A] time = 0.18, size = 79, normalized size = 0.82

$$\frac{\frac{\sqrt{a}\sinh(c+dx)\left(5a+3b\sinh^2(c+dx)\right)}{\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/Sqrt[b] + (Sqrt[a]*Sinh[c + d*
x]*(5*a + 3*b*Sinh[c + d*x]^2))/(a + b*Sinh[c + d*x]^2)^2)/(8*a^(5/2)*d)
```

fricas [B] time = 0.56, size = 3934, normalized size = 40.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*a*b^2*cosh(d*x + c)^7 + 84*a*b^2*cosh(d*x + c)*sinh(d*x + c)^6 +
12*a*b^2*sinh(d*x + c)^7 + 4*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^5 + 4*(63*a
*b^2*cosh(d*x + c)^2 + 20*a^2*b - 9*a*b^2)*sinh(d*x + c)^5 + 20*(21*a*b^2*c
osh(d*x + c)^3 + (20*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - 12*a
*b^2*cosh(d*x + c) - 4*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 + 4*(105*a*b^2*
cosh(d*x + c)^4 - 20*a^2*b + 9*a*b^2 + 10*(20*a^2*b - 9*a*b^2)*cosh(d*x + c
)^2)*sinh(d*x + c)^3 + 4*(63*a*b^2*cosh(d*x + c)^5 + 10*(20*a^2*b - 9*a*b^2
)*cosh(d*x + c)^3 - 3*(20*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 -
 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*
x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a
*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*cosh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh
(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*
(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b - b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b
+ 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh
(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2
*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*c
osh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*s
inh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 + 3*(2*a*b - b^2)*cosh(d*x +
c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c
```

))*sinh(d*x + c))*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sin
h(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cos
h(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*
cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d
*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x +
c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
 + c))*sinh(d*x + c) + b)) + 4*(21*a*b^2*cosh(d*x + c)^6 + 5*(20*a^2*b - 9*
a*b^2)*cosh(d*x + c)^4 - 3*a*b^2 - 3*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c))/(a^3*b^3*d*cosh(d*x + c)^8 + 8*a^3*b^3*d*cosh(d*x + c)*sinh(
d*x + c)^7 + a^3*b^3*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x
 + c)^6 + a^3*b^3*d + 4*(7*a^3*b^3*d*cosh(d*x + c)^2 + (2*a^4*b^2 - a^3*b^3
)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^
4 + 8*(7*a^3*b^3*d*cosh(d*x + c)^3 + 3*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c
))*sinh(d*x + c)^5 + 2*(35*a^3*b^3*d*cosh(d*x + c)^4 + 30*(2*a^4*b^2 - a^3*
b^3)*d*cosh(d*x + c)^2 + (8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d)*sinh(d*x + c)
^4 + 4*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^3*d*cosh(d*x +
c)^5 + 10*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^3 + (8*a^5*b - 8*a^4*b^2 +
3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^3*d*cosh(d*x + c)^
6 + 15*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 8*a^4*b^2 + 3
*a^3*b^3)*d*cosh(d*x + c)^2 + (2*a^4*b^2 - a^3*b^3)*d)*sinh(d*x + c)^2 + 8*
(a^3*b^3*d*cosh(d*x + c)^7 + 3*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^5 + (8
*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b^2 - a^3*b^3)*d
*cosh(d*x + c))*sinh(d*x + c)), 1/8*(6*a*b^2*cosh(d*x + c)^7 + 42*a*b^2*cos
h(d*x + c)*sinh(d*x + c)^6 + 6*a*b^2*sinh(d*x + c)^7 + 2*(20*a^2*b - 9*a*b^
2)*cosh(d*x + c)^5 + 2*(63*a*b^2*cosh(d*x + c)^2 + 20*a^2*b - 9*a*b^2)*sinh
(d*x + c)^5 + 10*(21*a*b^2*cosh(d*x + c)^3 + (20*a^2*b - 9*a*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^4 - 6*a*b^2*cosh(d*x + c) - 2*(20*a^2*b - 9*a*b^2)*cosh
(d*x + c)^3 + 2*(105*a*b^2*cosh(d*x + c)^4 - 20*a^2*b + 9*a*b^2 + 10*(20*a^
2*b - 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*a*b^2*cosh(d*x + c)
^5 + 10*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 - 3*(20*a^2*b - 9*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*si
nh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x + c)^6 + 4*(
7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*cosh(d*x +
c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2 - 8*a*b +
3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b - b^2)*cosh(
d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c
)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x
+ c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(
d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 +
3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^3 +
 (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*sqrt(a*b)
*(cosh(d*x + c) + sinh(d*x + c))/a) + 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d
*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x +
c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*c
osh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2
- 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b -
b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*co
sh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2
*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*
b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x
 + c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*
x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2
*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))
*sqrt(a*b)/(a*b)) + 2*(21*a*b^2*cosh(d*x + c)^6 + 5*(20*a^2*b - 9*a*b^2)*co

```
sh(d*x + c)^4 - 3*a*b^2 - 3*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x
+ c))/(a^3*b^3*d*cosh(d*x + c)^8 + 8*a^3*b^3*d*cosh(d*x + c)*sinh(d*x + c)^
7 + a^3*b^3*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^6 +
 a^3*b^3*d + 4*(7*a^3*b^3*d*cosh(d*x + c)^2 + (2*a^4*b^2 - a^3*b^3)*d)*sinh
(d*x + c)^6 + 2*(8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^4 + 8*(7*
a^3*b^3*d*cosh(d*x + c)^3 + 3*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c))*sinh(d
*x + c)^5 + 2*(35*a^3*b^3*d*cosh(d*x + c)^4 + 30*(2*a^4*b^2 - a^3*b^3)*d*co
sh(d*x + c)^2 + (8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d)*sinh(d*x + c)^4 + 4*(2
*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^3*d*cosh(d*x + c)^5 + 10
*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^3 + (8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3
)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^3*d*cosh(d*x + c)^6 + 15*(2
*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)
*d*cosh(d*x + c)^2 + (2*a^4*b^2 - a^3*b^3)*d)*sinh(d*x + c)^2 + 8*(a^3*b^3*
d*cosh(d*x + c)^7 + 3*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^5 + (8*a^5*b -
8*a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b^2 - a^3*b^3)*d*cosh(d*x
 + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-85,-18]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[33,-80]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-98,-18]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-57,-1
0]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.The choice was done assuming [a,b]=[-57,-3]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [a,b]=[-53,60]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[80,-1]schur row 3 -6.9034e-07
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[-51,-3]Warning,
 need to choose a branch for the root of a polynomial with parameters. This
 might be wrong.The choice was done assuming [a,b]=[-64,-74]Warning, need t
o choose a branch for the root of a polynomial with parameters. This might
be wrong.The choice was done assuming [a,b]=[-56,-37]Undef/Unsigned Inf enc
ountered in limitEvaluation time: 2.01Limit: Max order reached or unable to
 make series expansion Error: Bad Argument Value
```

maple [A] time = 0.03, size = 85, normalized size = 0.89

$$\frac{\sinh\left(dx+c\right)}{4ad\left(a+b\left(\sinh^{2}\left(dx+c\right)\right)\right)^{2}}+\frac{3\sinh\left(dx+c\right)}{8a^{2}d\left(a+b\left(\sinh^{2}\left(dx+c\right)\right)\right)}+\frac{3\arctan\left(\frac{\sinh(dx+c)b}{\sqrt{ab}}\right)}{8d\,a^{2}\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/4*sinh(d*x+c)/a/d/(a+b*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+b*sinh(d
*x+c)^2)+3/8/d/a^2/(a*b)^(1/2)*arctan(sinh(d*x+c)*b/(a*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(20\,ae^{(5\,c)}-9\,be^{(5\,c)}\right)e^{(5\,dx)}-\left(20\,ae^{(3\,c)}-9\,be^{(3\,c)}\right)e^{(3\,dx)}+3\,be^{(7\,dx+7\,c)}-3\,be^{(dx+c)}}{4\left(a^2b^2de^{(8\,dx+8\,c)}+a^2b^2d+4\left(2\,a^3bde^{(6\,c)}-a^2b^2de^{(6\,c)}\right)e^{(6\,dx)}+2\left(8\,a^4de^{(4\,c)}-8\,a^3bde^{(4\,c)}+3\,a^2b^2de^{(4\,c)}\right)e^{(4\,dx)}+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((20*a*e^(5*c) - 9*b*e^(5*c))*e^(5*d*x) - (20*a*e^(3*c) - 9*b*e^(3*c))*
e^(3*d*x) + 3*b*e^(7*d*x + 7*c) - 3*b*e^(d*x + c))/(a^2*b^2*d*e^(8*d*x + 8*
c) + a^2*b^2*d + 4*(2*a^3*b*d*e^(6*c) - a^2*b^2*d*e^(6*c))*e^(6*d*x) + 2*(8
*a^4*d*e^(4*c) - 8*a^3*b*d*e^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x) + 4*(2*
a^3*b*d*e^(2*c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(3/2*(e^(3*d
*x + 3*c) + e^(d*x + c))/(a^2*b*e^(4*d*x + 4*c) + a^2*b + 2*(2*a^3*e^(2*c)
- a^2*b*e^(2*c))*e^(2*d*x)), x)
```

mupad [B] time = 0.94, size = 87, normalized size = 0.91

$$\frac{\frac{5\sinh(c+d\,x)}{8\,a}+\frac{3\,b\sinh(c+d\,x)^3}{8\,a^2}}{d\,a^2+2\,d\,a\,b\sinh\left(c+d\,x\right)^2+d\,b^2\sinh\left(c+d\,x\right)^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sinh(c+d\,x)}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^2)^3,x)
```

```
[Out] ((5*sinh(c + d*x))/(8*a) + (3*b*sinh(c + d*x)^3)/(8*a^2))/(a^2*d + b^2*d*si
nh(c + d*x)^4 + 2*a*b*d*sinh(c + d*x)^2) + (3*atan((b^(1/2)*sinh(c + d*x))/
a^(1/2)))/(8*a^(5/2)*b^(1/2)*d)
```

sympy [A] time = 89.11, size = 915, normalized size = 9.53

$$\begin{cases} \frac{\tilde{\infty} x\cosh\left(c\right)}{\sinh^6\left(c\right)} \\ \frac{\sinh\left(c+dx\right)}{a^3d} \\ -\frac{1}{5b^3d\sinh^5\left(c+dx\right)} \\ \frac{x\cosh\left(c\right)}{\left(a+b\sinh^2\left(c\right)\right)^3} \\ \frac{10ia^{\frac{3}{2}}b\sqrt{\frac{1}{b}}\sinh\left(c+dx\right)}{16ia^{\frac{9}{2}}bd\sqrt{\frac{1}{b}}+32ia^{\frac{7}{2}}b^2d\sqrt{\frac{1}{b}}\sinh^2\left(c+dx\right)+16ia^{\frac{5}{2}}b^3d\sqrt{\frac{1}{b}}\sinh^4\left(c+dx\right)}+\frac{6i\sqrt{a}b^2\sqrt{\frac{1}{b}}\sinh^3\left(c+dx\right)}{16ia^{\frac{9}{2}}bd\sqrt{\frac{1}{b}}+32ia^{\frac{7}{2}}b^2d\sqrt{\frac{1}{b}}\sinh^2\left(c+dx\right)+16ia^{\frac{5}{2}}b^3d\sqrt{\frac{1}{b}}\sinh^4\left(c+dx\right)}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sinh
(c + d*x)/(a**3*d), Eq(b, 0)), (-1/(5*b**3*d*sinh(c + d*x)**5), Eq(a, 0)),
(x*cosh(c)/(a + b*sinh(c)**2)**3, Eq(d, 0)), (10*I*a**(3/2)*b*sqrt(1/b)*sin
h(c + d*x)/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*si
nh(c + d*x)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**4) + 6*I*sqr
t(a)*b**2*sqrt(1/b)*sinh(c + d*x)**3/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a*
*(7/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*s
inh(c + d*x)**4) + 3*a**2*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))/(16*I*a
**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2 + 1
6*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**4) - 3*a**2*log(I*sqrt(a)*sqrt
```

```
(1/b) + sinh(c + d*x))/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*
sqrt(1/b)*sinh(c + d*x)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**
4) + 6*a*b*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))*sinh(c + d*x)**2/(16*I
*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2 +
 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**4) - 6*a*b*log(I*sqrt(a)*sqr
t(1/b) + sinh(c + d*x))*sinh(c + d*x)**2/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*
I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/
b)*sinh(c + d*x)**4) + 3*b**2*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))*sin
h(c + d*x)**4/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)
*sinh(c + d*x)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**4) - 3*b*
*2*log(I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))*sinh(c + d*x)**4/(16*I*a**(9/2)
*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2 + 16*I*a**
(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**4), True))
```

3.345 $\int \frac{\text{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=159

$$-\frac{b(7a-3b)\sinh(c+dx)}{8a^2d(a-b)^2\left(a+b\sinh^2(c+dx)\right)}-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}-\frac{b\sinh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+d\right.}$$

```
[Out] arctan(sinh(d*x+c))/(a-b)^3/d-1/4*b*sinh(d*x+c)/a/(a-b)/d/(a+b*sinh(d*x+c)^
2)^2-1/8*(7*a-3*b)*b*sinh(d*x+c)/a^2/(a-b)^2/d/(a+b*sinh(d*x+c)^2)-1/8*(15*
a^2-10*a*b+3*b^2)*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)/(a-b)
^3/d
```

Rubi [A] time = 0.18, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3190, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}-\frac{b(7a-3b)\sinh(c+dx)}{8a^2d(a-b)^2\left(a+b\sinh^2(c+dx)\right)}-\frac{b\sinh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)^3*d) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*Ar
cTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) - (b*Sinh[c
+ d*x])/(4*a*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2) - ((7*a - 3*b)*b*Sinh[c +
 d*x])/(8*a^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \sinh(c+dx\right.}{4a(a-b)d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{(7a-3b)b\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \text{Sul}$$

$$= -\frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{(7a-3b)b\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \text{Sul}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)^3d} - \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} - \frac{}{4a(a-}$$

Mathematica [B] time = 0.78, size = 321, normalized size = 2.02

$$(b-2a)^2\left(16a^{5/2}\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)\right)-2b\cosh(2(c+dx$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((-2*a + b)^2*(Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d
*x])/Sqrt[b]] + 16*a^(5/2)*ArcTan[Tanh[(c + d*x)/2]]) + (b^(5/2)*(15*a^2 -
10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 16*a^(5/2)*b^2*Ar
cTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)]^2 - 2*Sqrt[a]*b*(18*a^3 - 35*a^2
*b + 20*a*b^2 - 3*b^3)*Sinh[c + d*x] - 2*b*Cosh[2*(c + d*x)]*(-((2*a - b)*(
Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] +
 16*a^(5/2)*ArcTan[Tanh[(c + d*x)/2]])) + Sqrt[a]*b*(7*a^2 - 10*a*b + 3*b^2
)*Sinh[c + d*x]))/(8*a^(5/2)*(a - b)^3*d*(2*a - b + b*Cosh[2*(c + d*x)])^2)
```

fricas [B] time = 0.74, size = 8083, normalized size = 50.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 28*(7*a^2*b^2 -
10*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(7*a^2*b^2 - 10*a*b^3 +
 3*b^4)*sinh(d*x + c)^7 + 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh
(d*x + c)^5 + 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4 + 21*(7*a^2*b^2 -
 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(7*a^2*b^2 - 10
*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4
)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*
b^4)*cosh(d*x + c)^3 + 4*(35*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4
 - 36*a^3*b + 77*a^2*b^2 - 50*a*b^3 + 9*b^4 + 10*(36*a^3*b - 77*a^2*b^2 + 5
0*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(7*a^2*b^2 - 10*a
*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^
4)*cosh(d*x + c)^3 - 3*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^2 + ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^8 +
8*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^2*b
^2 - 10*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^
3 - 3*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 +
7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(
15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(30*a^3*b - 35*a^2*b^2 +
 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(120*a^4 - 200*a^3*b
+ 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b
^3 + 9*b^4 + 30*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 15*a^2*b^2 - 10*a*b^3 + 3*b^4 + 8*(7*(15*a^2*b^2 - 10*a*
b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4
)*cosh(d*x + c)^3 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^
4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 +
 15*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 30*a^3*b -
 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 3*(120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*
a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^2*b^2 - 10*a*b^3
 + 3*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*co
sh(d*x + c)^5 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh
(d*x + c)^3 + (30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
+ c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d
*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x
+ c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*si
nh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*
x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*
x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh
(d*x + c))*sinh(d*x + c) + b)) - 32*(a^2*b^2*cosh(d*x + c)^8 + 8*a^2*b^2*co
sh(d*x + c)*sinh(d*x + c)^7 + a^2*b^2*sinh(d*x + c)^8 + 4*(2*a^3*b - a^2*b^
2)*cosh(d*x + c)^6 + 4*(7*a^2*b^2*cosh(d*x + c)^2 + 2*a^3*b - a^2*b^2)*sinh
(d*x + c)^6 + 8*(7*a^2*b^2*cosh(d*x + c)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4 +
2*(35*a^2*b^2*cosh(d*x + c)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b -
 a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(d*
x + c)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^3 + (8*a^4 - 8*a^3*b + 3*a^
2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(d*x + c)
^2 + 4*(7*a^2*b^2*cosh(d*x + c)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^4
+ 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*sinh
```

```
(d*x + c)^2 + 8*(a^2*b^2*cosh(d*x + c)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(d*x +
 c)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + (2*a^3*b - a^2*b^2)
*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 4*(7
*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c) + 4*(7*(7*a^2*b^2 - 10*a*b^3 + 3
*b^4)*cosh(d*x + c)^6 + 5*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d
*x + c)^4 - 7*a^2*b^2 + 10*a*b^3 - 3*b^4 - 3*(36*a^3*b - 77*a^2*b^2 + 50*a*
b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*
b^4 - a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2
*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 -
a^2*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4
 + a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2
*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^
2*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 +
 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*
a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 -
 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^2 - 3
*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4 + 30*(2*a^6*b - 7*a^5*b^2
 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 + (8*a^7 - 32*a^6*b +
 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(
2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 +
8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + 10*(2*
a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + (8
*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5
)*d*cosh(d*x + c)^6 + 15*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2
*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 1
7*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3
 - 5*a^3*b^4 + a^2*b^5)*d)*sinh(d*x + c)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b
^4 - a^2*b^5)*d + 8*((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x
 + c)^7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(
d*x + c)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a
^2*b^5)*d*cosh(d*x + c)^3 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 +
a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(7*a^2*b^2 - 10*a*b^3 + 3
*b^4)*cosh(d*x + c)^7 + 14*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)*sin
h(d*x + c)^6 + 2*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*sinh(d*x + c)^7 + 2*(36*a^3
*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^5 + 2*(36*a^3*b - 77*a^2*
b^2 + 50*a*b^3 - 9*b^4 + 21*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 10*(7*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (
36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 -
2*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^3 + 2*(35*(7*a^2
*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4 - 36*a^3*b + 77*a^2*b^2 - 50*a*b^3
 + 9*b^4 + 10*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + 2*(21*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*
(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^3 - 3*(36*a^3*b -
77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((15*a^2*b^
2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*c
osh(d*x + c)*sinh(d*x + c)^7 + (15*a^2*b^2 - 10*a*b^3 + 3*b^4)*sinh(d*x + c
)^8 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(30*
a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh
(d*x + c)^3 + 3*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c))*s
inh(d*x + c)^5 + 2*(120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*c
osh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 12
0*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4 + 30*(30*a^3*b - 35*a^2*
b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 10*
a*b^3 + 3*b^4 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(
30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (120*a^4 - 200*
a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*
(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b
```

^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(30*a^3*b - 35*a^2*b^2 + 16*a*b
^3 - 3*b^4)*cosh(d*x + c)^4 + 30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 3*
(120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 8*((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(30*a
^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (120*a^4 - 200*a^3*
b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (30*a^3*b - 35*a^2*b^
2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*sq
rt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) + ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)
*cosh(d*x + c)^8 + 8*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x
 + c)^7 + (15*a^2*b^2 - 10*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 3
5*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 35*a^2*b^2 +
16*a*b^3 - 3*b^4 + 7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^6 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(30*a
^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(1
20*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(3
5*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 120*a^4 - 200*a^3*b + 1
49*a^2*b^2 - 54*a*b^3 + 9*b^4 + 30*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 10*a*b^3 + 3*b^4 + 8*(7*
(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 35*a^2*b^2
 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 -
 54*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 35*a^2*b^
2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4
)*cosh(d*x + c)^6 + 15*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x
+ c)^4 + 30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 3*(120*a^4 - 200*a^3*b
+ 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15
*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 35*a^2*b^2 + 1
6*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*
a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)
*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*
b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x +
c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) - 16*(a^2*
b^2*cosh(d*x + c)^8 + 8*a^2*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*b^2*sin
h(d*x + c)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^6 + 4*(7*a^2*b^2*cosh(d*
x + c)^2 + 2*a^3*b - a^2*b^2)*sinh(d*x + c)^6 + 8*(7*a^2*b^2*cosh(d*x + c)^
3 + 3*(2*a^3*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 8*a^3
*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*b^2*cosh(d*x + c)^4 + 8*a^4 - 8
*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(d*x + c)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(d*
x + c)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4
*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^2 + 4*(7*a^2*b^2*cosh(d*x + c)^6 + 15*(2
*a^3*b - a^2*b^2)*cosh(d*x + c)^4 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b
+ 3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^2*b^2*cosh(d*x + c)^7
+ 3*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cos
h(d*x + c)^3 + (2*a^3*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cos
h(d*x + c) + sinh(d*x + c)) - 2*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c
) + 2*(7*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 5*(36*a^3*b - 77*
a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^4 - 7*a^2*b^2 + 10*a*b^3 - 3*b^4
- 3*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x +
c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (
a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^6*b -
 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b - 7*a^
5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^7 - 32
*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^
4 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + 3*
(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(35*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*
x + c)^4 + 30*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cos

```
h(d*x + c)^2 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3
*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b
^4 + a^2*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*d*cosh(d*x + c)^5 + 10*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4
 + a^2*b^5)*d*cosh(d*x + c)^3 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3
 + 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^2
 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^6*b - 7*a^5
*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^7 - 32*a
^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^2
+ (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d)*sinh(d*x + c)^
2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d + 8*((a^5*b^2 - 3*a^4*b^3
 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*
b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b
^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^3 + (2*a^6*b - 7*
a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-85,-18]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[33,-80]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-98,-18]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-57,-1
0]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.The choice was done assuming [a,b]=[-57,-3]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [a,b]=[-53,60]Undef/Unsigned
 Inf encountered in limitEvaluation time: 1.16Limit: Max order reached or u
nable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.17, size = 2118, normalized size = 13.32

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 13/8/d*b^3/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d*b
^4/(a-b)^3/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctan
h(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+15/8/d*b/(a-b
)^3*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+15/8/d*b/(a-b)^3*a/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+13/8/d*b^3/(a-b)^3/a/(-b*(a-b))^
(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d*b^4/(a-b)^3/a^2/(-b*(a-b))^(1/2)/(
(2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2))+5/4/d*b^2/(a-b)^3/a/((2*(-b*(a-b))^(1/2)-a+2*b)
*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
```

```
)+3/d*b^4/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh
(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5-35/2/d*b^2/(a-b)^3/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2*tanh(1/2*d*x+1/2*c)^3+2/d/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))-27/4/d*b/(a
-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^5+27/4/d*b/(a-b)^3/(tanh(1/2*d*x+1/2*c)^
4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x
+1/2*c)^3-9/4/d*b/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)-3/d*b^4/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2/a^2*tanh(1/2*d*x+1/2*c)^3+5/4/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)
^7+55/4/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3-55/4/d*b^3/(a-b)^3/(ta
nh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)^2/a*tanh(1/2*d*x+1/2*c)^5-5/4/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-
3/8/d*b^3/(a-b)^3/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/4/d*b^2/(a-b)^3/a/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2))+3/8/d*b^3/(a-b)^3/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
))-25/8/d*b^2/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-25/8/d*
b^2/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+9/4/d*b/(a-b)^3/
(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*
b+a)^2*a*tanh(1/2*d*x+1/2*c)^7+7/2/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2
*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)
+35/2/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-7/2/d*b^2/(a-b)^3/(tanh(1/
2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*t
anh(1/2*d*x+1/2*c)^7-15/8/d*b/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+15/8/d*b
/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(7\,ab^2e^{(7\,c)}-3\,b^3e^{(7\,c)}\right)e^{(7\,dx)}+\left(36\,a^2be^{(5\,c}\right.}{4\left(a^4b^2d-2\,a^3b^3d+a^2b^4d+\left(a^4b^2de^{(8\,c)}-2\,a^3b^3de^{(8\,c)}+a^2b^4de^{(8\,c)}\right)e^{(8\,dx)}+4\left(2\,a^5bde^{(6\,c)}-5\,a^4b^2de^{(6\,c)}+4\,a^3b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((7*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*e^(7*d*x) + (36*a^2*b*e^(5*c) - 41*
a*b^2*e^(5*c) + 9*b^3*e^(5*c))*e^(5*d*x) - (36*a^2*b*e^(3*c) - 41*a*b^2*e^(
3*c) + 9*b^3*e^(3*c))*e^(3*d*x) - (7*a*b^2*e^c - 3*b^3*e^c)*e^(d*x))/(a^4*b
^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) - 2*a^3*b^3*d*e^(8*c) +
 a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(2*a^5*b*d*e^(6*c) - 5*a^4*b^2*d*e^(6*c)
+ 4*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(8*a^6*d*e^(4*c) -
 24*a^5*b*d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) - 14*a^3*b^3*d*e^(4*c) + 3*a^2*b
^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^5*b*d*e^(2*c) - 5*a^4*b^2*d*e^(2*c) + 4*a^
3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(a^
3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d) - 2*integrate(1/8*((15*a^2*b*e^(3*c) -
 10*a*b^2*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) + (15*a^2*b*e^c - 10*a*b^2*e^c
 + 3*b^3*e^c)*e^(d*x))/(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4 + (a^5*b*e^
(4*c) - 3*a^4*b^2*e^(4*c) + 3*a^3*b^3*e^(4*c) - a^2*b^4*e^(4*c))*e^(4*d*x)
+ 2*(2*a^6*e^(2*c) - 7*a^5*b*e^(2*c) + 9*a^4*b^2*e^(2*c) - 5*a^3*b^3*e^(2*c
) + a^2*b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh (c+d\,x)\,\left(b\sinh (c+d\,x)^2+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^3),x)
[Out] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
[Out] Timed out
```

3.346 $\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=172

$$\frac{3b^2(4a-b)\tanh(c+dx)}{8a^2d(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{3b\left(8a^2-4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{7/2}}-\frac{b^3\tanh(c+d}{4ad(a-b)^3\left(a-(a-b)\operatorname{ta}\right.}$$

```
[Out] -3/8*b*(8*a^2-4*a*b+b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(
a-b)^(7/2)/d+tanh(d*x+c)/(a-b)^3/d-1/4*b^3*tanh(d*x+c)/a/(a-b)^3/d/(a-(a-b)
*tanh(d*x+c)^2)^2+3/8*(4*a-b)*b^2*tanh(d*x+c)/a^2/(a-b)^3/d/(a-(a-b)*tanh(d
*x+c)^2)
```

Rubi [A] time = 0.26, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3191, 390, 1157, 385, 208}

$$\frac{3b^2(4a-b)\tanh(c+dx)}{8a^2d(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{3b\left(8a^2-4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{7/2}}-\frac{b^3\tanh(c+d}{4ad(a-b)^3\left(a-(a-b)\operatorname{ta}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(
8*a^(5/2)*(a - b)^(7/2)*d) + Tanh[c + d*x]/((a - b)^3*d) - (b^3*Tanh[c + d*
x])/(4*a*(a - b)^3*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*(4*a - b)*b^2*Ta
nh[c + d*x])/(8*a^2*(a - b)^3*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
```

```
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a-b)^3} - \frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{(a-b)^3\left(a+(-a+b)x^2\right)^3}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{\operatorname{Subst}\left(\int \frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{\left(a+(-a+b)x^2\right)^3}\,dx, x, \tanh(c+d\right.}{(a-b)^3d}$$

$$= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3\tanh(c+dx)}{4a(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{-3(2a-b)^2b+}{\left(a+(-a\right.}\right.}{}$$

$$= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3\tanh(c+dx)}{4a(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3(4a-b)b^2}{8a^2(a-b)^3d\left(a-(a\right.}$$

$$= -\frac{3b\left(8a^2-4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{7/2}d} + \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3}{4a(a-b)^3d\,(a}$$

Mathematica [A] time = 1.14, size = 165, normalized size = 0.96

$$\frac{\frac{b^2(10a-3b)\sinh(2(c+dx))}{a^2(a-b)^3(2a+b\cosh(2(c+dx))-b)} - \frac{3b\left(8a^2-4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^{7/2}} + \frac{4b^2\sinh(2(c+dx))}{a(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} + \frac{8\tanh(c+dx)}{(a-b)^3}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((-3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/
(a^(5/2)*(a - b)^(7/2)) + (4*b^2*Sinh[2*(c + d*x)])/(a*(a - b)^2*(2*a - b +
 b*Cosh[2*(c + d*x)])^2) + ((10*a - 3*b)*b^2*Sinh[2*(c + d*x)])/(a^2*(a - b
)^3*(2*a - b + b*Cosh[2*(c + d*x)])) + (8*Tanh[c + d*x])/(a - b)^3)/(8*d)
```

fricas [B] time = 0.83, size = 9442, normalized size = 54.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(12*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^8 + 9
6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)*sinh(d*x + c)^
7 + 12*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*sinh(d*x + c)^8 + 24*(2
4*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^6 + 24
*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5 + 14*(8*a^4*b^2 -
12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 32*a^4*b
^2 + 8*a^3*b^3 - 52*a^2*b^4 + 12*a*b^5 + 48*(14*(8*a^4*b^2 - 12*a^3*b^3 + 5
*a^2*b^4 - a*b^5)*cosh(d*x + c)^3 + 3*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 -
 8*a^2*b^4 + a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(64*a^6 - 88*a^5*b +
 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^4 + 8*(64*a^6 - 88*a^5*b +
 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4 + 105*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4
 - a*b^5)*cosh(d*x + c)^4 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*
b^4 + a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(8*a^4*b^2 - 12*a^3*
b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^5 + 15*(24*a^5*b - 44*a^4*b^2 + 27*a
^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^3 + (64*a^6 - 88*a^5*b + 28*a^4*b
^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(32*a^5*b - 16
*a^4*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5)*cosh(d*x + c)^2 + 8*(42*(8*a^
4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^6 + 32*a^5*b - 16*a^4
*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a
^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^4 + 6*(64*a^6 - 88*a^5*b + 28*a^4
*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((8*a^2*b^
3 - 4*a*b^4 + b^5)*cosh(d*x + c)^10 + 10*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d
*x + c)*sinh(d*x + c)^9 + (8*a^2*b^3 - 4*a*b^4 + b^5)*sinh(d*x + c)^10 + (6
4*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^8 + (64*a^3*b^2 -
56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 45*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c
)^2)*sinh(d*x + c)^8 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^3 +
(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^7
 + 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^6 +
 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5 + 105*(8*a^2*b^3 - 4
*a*b^4 + b^5)*cosh(d*x + c)^4 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*
b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(8*a^2*b^3 - 4*a*b^4 + b^5)*c
osh(d*x + c)^5 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x +
 c)^3 + 3*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c
))*sinh(d*x + c)^5 + 8*a^2*b^3 - 4*a*b^4 + b^5 + 2*(64*a^4*b - 64*a^3*b^2 +
 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 2*(105*(8*a^2*b^3 - 4*a*b^4
+ b^5)*cosh(d*x + c)^6 + 64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5
 + 35*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^4 + 15*(64
*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^7 + 7*(64*a^3*b^2
 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^5 + 5*(64*a^4*b - 64*a^3*b^
2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^4*b - 64*a^3*b^2 +
32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + (64*a^3*b^2 -
56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^2 + (45*(8*a^2*b^3 - 4*a*b^4 +
 b^5)*cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cos
h(d*x + c)^6 + 64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 30*(64*a^4*b -
64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 12*(64*a^4*b - 6
4*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
2*(5*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^9 + 4*(64*a^3*b^2 - 56*a^2*b
^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^7 + 6*(64*a^4*b - 64*a^3*b^2 + 32*a^2*
b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^5 + 4*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^
3 - 8*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 -
3*b^5)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)
^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b -
 b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x +
c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*
x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*
b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x
+ c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x
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+ c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*(6*(8*a^4*b^2 -
12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^7 + 9*(24*a^5*b - 44*a^4*b^2
+ 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^5 + 2*(64*a^6 - 88*a^5*b +
28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^3 + (32*a^5*b - 16*a^4*b^2
- 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b^
2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^10 + 10*(a
^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)*sinh(
d*x + c)^9 + (a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*sinh
(d*x + c)^10 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5
 - 3*a^3*b^6)*d*cosh(d*x + c)^8 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*
a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 -
 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d)*sinh(d*x + c)^8 + 2*(8*a^9 - 36*a^
8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*
x + c)^6 + 8*(15*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*
cosh(d*x + c)^3 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*
b^5 - 3*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7*b^2 - 4*a^6
*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 14*(8*a^8*b - 3
5*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x +
c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5
 + a^3*b^6)*d)*sinh(d*x + c)^6 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*
b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 4*(63*(a^7*b^2
- 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^5 + 14*(8*a^
8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh
(d*x + c)^3 + 3*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 -
8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7*b^2 - 4
*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^6 + 35*(8*a^8*b
 - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*
x + c)^4 + 15*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*
a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*
a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d)*sinh(d*x + c)^4 + (8*a^8*b -
 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x
+ c)^2 + 8*(15*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*co
sh(d*x + c)^7 + 7*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*
b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*
a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^3 + (8*a^9 - 36
*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b
^5 + a^3*b^6)*d*cosh(d*x + c)^8 + 28*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 5
0*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^6 + 30*(8*a^9 - 36*a^8*
b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x
+ c)^4 + 12*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^
4*b^5 + a^3*b^6)*d*cosh(d*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 5
0*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d)*sinh(d*x + c)^2 + (a^7*b^2 - 4*a^6*b
^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d + 2*(5*(a^7*b^2 - 4*a^6*b^3 + 6*a^5
*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^9 + 4*(8*a^8*b - 35*a^7*b^2 + 6
0*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^7 + 6*(8*a
^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)
*d*cosh(d*x + c)^5 + 4*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5
*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^3 + (8*a^8*b - 35*a^7*b^2 + 60*
a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x +
c)), -1/8*(6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^8 +
 48*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)*sinh(d*x + c
)^7 + 6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*sinh(d*x + c)^8 + 12*(
24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^6 + 1
2*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5 + 14*(8*a^4*b^2 -
 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*a^4*
b^2 + 4*a^3*b^3 - 26*a^2*b^4 + 6*a*b^5 + 24*(14*(8*a^4*b^2 - 12*a^3*b^3 + 5
*a^2*b^4 - a*b^5)*cosh(d*x + c)^3 + 3*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 -
 8*a^2*b^4 + a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(64*a^6 - 88*a^5*b +
```

28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^4 + 4*(64*a^6 - 88*a^5*b + 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4 + 105*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^4 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(21*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^5 + 15*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^3 + (64*a^6 - 88*a^5*b + 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(32*a^5*b - 16*a^4*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5)*cosh(d*x + c)^2 + 4*(42*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^6 + 32*a^5*b - 16*a^4*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^4 + 6*(64*a^6 - 88*a^5*b + 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^10 + 10*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^9 + (8*a^2*b^3 - 4*a*b^4 + b^5)*sinh(d*x + c)^10 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^8 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 45*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^6 + 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5 + 105*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^4 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^5 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^3 + 3*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*a^2*b^3 - 4*a*b^4 + b^5 + 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 2*(105*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^6 + 64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5 + 35*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^4 + 15*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^7 + 7*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^5 + 5*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^2 + (45*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^6 + 64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 30*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 12*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^9 + 4*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^7 + 6*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^5 + 4*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 8*(6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^7 + 9*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^5 + 2*(64*a^6 - 88*a^5*b + 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^3 + (32*a^5*b - 16*a^4*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^10 + 10*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*sinh(d*x + c)^10 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^8 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d)*sinh(d*x + c)^8 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^6 + 8*(15*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^3 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d

```
*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*
b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 14*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 -
50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^2 + (8*a^9 - 36*a^8*b
+ 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d)*sinh(d*x +
 c)^6 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*
b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 4*(63*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 -
 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^5 + 14*(8*a^8*b - 35*a^7*b^2 + 60*a^6
*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^3 + 3*(8*a^9 -
36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*
a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^6 + 35*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^
3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^4 + 15*(8*a^9 - 36
*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh
(d*x + c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*
a^4*b^5 + a^3*b^6)*d)*sinh(d*x + c)^4 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3
- 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^2 + 8*(15*(a^7*b^2 -
 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^7 + 7*(8*a^8*
b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d
*x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*
a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^3 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*
a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^
3 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x
+ c)^8 + 28*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 -
3*a^3*b^6)*d*cosh(d*x + c)^6 + 30*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b
^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 12*(8*a^9 - 36*a
^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d
*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 -
3*a^3*b^6)*d)*sinh(d*x + c)^2 + (a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^
5 + a^3*b^6)*d + 2*(5*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^
6)*d*cosh(d*x + c)^9 + 4*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 +
20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^7 + 6*(8*a^9 - 36*a^8*b + 65*a^7*b^
2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^5 + 4*(8
*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^
6)*d*cosh(d*x + c)^3 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20
*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

giac [B] time = 1.77, size = 367, normalized size = 2.13

$$-\frac{\frac{3\left(8a^2b-4ab^2+b^3\right)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\left(a^5-3a^4b+3a^3b^2-a^2b^3\right)\sqrt{-a^2+ab}}+\frac{2\left(16a^2b^2e^{(6dx+6c)}-12ab^3e^{(6dx+6c)}+3b^4e^{(6dx+6c)}+80a^3be^{(4dx+4c)}-104a^2b^2e^{(4dx+4c)}+54ab^3e\cdots\right.}{\left(a^5-3a^4b+3a^3b^2-a^2b^3\right)\left(be^{(4dx+4c)}+\cdots\right.}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(8*a^2*b - 4*a*b^2 + b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/
sqrt(-a^2 + a*b))/((a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sqrt(-a^2 + a*b))
+ 2*(16*a^2*b^2*e^(6*d*x + 6*c) - 12*a*b^3*e^(6*d*x + 6*c) + 3*b^4*e^(6*d*x
 + 6*c) + 80*a^3*b*e^(4*d*x + 4*c) - 104*a^2*b^2*e^(4*d*x + 4*c) + 54*a*b^3
*e^(4*d*x + 4*c) - 9*b^4*e^(4*d*x + 4*c) + 64*a^2*b^2*e^(2*d*x + 2*c) - 52*
a*b^3*e^(2*d*x + 2*c) + 9*b^4*e^(2*d*x + 2*c) + 10*a*b^3 - 3*b^4)/((a^5 - 3
*a^4*b + 3*a^3*b^2 - a^2*b^3)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*
b*e^(2*d*x + 2*c) + b)^2) + 16/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(e^(2*d*x +
 2*c) + 1)))/d
```

maple [B] time = 0.16, size = 1694, normalized size = 9.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -5/4/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+3/d*b^2/(a-b)^3/(tanh(1/2
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*ta
nh(1/2*d*x+1/2*c)^7-3/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x
+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+45/4/d*b^3
/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+
1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-3/d*b^4/(a-b)^3/(tanh(1/2*d*x+1/2*c
)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2
*d*x+1/2*c)^5-3/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+45/4/d*b^3/(a-b)
^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)
^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3-3/d*b^4/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-
2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1
/2*c)^3-5/4/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)+3/d*b^2/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2*tanh(1/2*d*x+1/2*c)+3/d*b/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/2/d*b^2
/(a-b)^3/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b^3/(a-b)^3/a^2/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2))+3/d*b^2/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
))-3/2/d*b^3/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*
b^4/(a-b)^3/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arcta
n(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/d*b/(a-b)^3
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/2/d*b^2/(a-b)^3/a/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(
1/2))-3/8/d*b^3/(a-b)^3/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*b^2/(a-b)^3/(
-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/2/d*b^3/(a-b)^3/a/(-b*(a-b)
)^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d*b^4/(a-b)^3/a^2/(-b*(a-b))^(1/2
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d/(a-b)^3*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d
*x+1/2*c)^2+1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(c+d\,x\right)^2\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.347 $\int \frac{\text{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=217

$$\frac{b(4a-b)(a+3b)\sinh(c+dx)}{8a^2d(a-b)^3\left(a+b\sinh^2(c+dx)\right)}+\frac{b^{3/2}\left(35a^2-14ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^4}+\frac{b(2a+b)\sinh(c+dx)}{4ad(a-b)^2\left(a+b\sinh^2(c+d\right.}$$

```
[Out] 1/2*(a-7*b)*arctan(sinh(d*x+c))/(a-b)^4/d+1/8*b^(3/2)*(35*a^2-14*a*b+3*b^2)
*arctan(sinh(d*x+c)*b^(1/2)/a^(1/2))/a^(5/2)/(a-b)^4/d+1/4*b*(2*a+b)*sinh(d
*x+c)/a/(a-b)^2/d/(a+b*sinh(d*x+c)^2)^2+1/8*(4*a-b)*b*(a+3*b)*sinh(d*x+c)/a
^2/(a-b)^3/d/(a+b*sinh(d*x+c)^2)+1/2*sech(d*x+c)*tanh(d*x+c)/(a-b)/d/(a+b*s
inh(d*x+c)^2)^2
```

Rubi [A] time = 0.30, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}\left(35a^2-14ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^4}+\frac{b(4a-b)(a+3b)\sinh(c+dx)}{8a^2d(a-b)^3\left(a+b\sinh^2(c+dx)\right)}+\frac{b(2a+b)\sinh(c+dx)}{4ad(a-b)^2\left(a+b\sinh^2(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - 7*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^4*d) + (b^(3/2)*(35*a^2 - 14*a
*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^4*d
) + (b*(2*a + b)*Sinh[c + d*x])/(4*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)^2)
 + ((4*a - b)*b*(a + 3*b)*Sinh[c + d*x])/(8*a^2*(a - b)^3*d*(a + b*Sinh[c +
 d*x]^2)) + (Sech[c + d*x]*Tanh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]
^2)^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^3}\,dx,x,\sinh(c+dx)\right)}{d}$$

$$=\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)^2}-\frac{\operatorname{Subst}\left(\int \frac{-a+2b-5bx^2}{(1+x^2)(a+bx^2)^3}\,dx,x,\sinh(c+dx)\right)}{2(a-b)d}$$

$$=\frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)^2}+\frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)^2}-\frac{\operatorname{Subst}}{}$$

$$=\frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(4a-b)b(a+3b)\sinh(c+dx)}{8a^2(a-b)^3d\left(a+b\sinh^2(c+dx)\right)}+\frac{\mathrm{s}}{2(a}$$

$$=\frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)^2}+\frac{(4a-b)b(a+3b)\sinh(c+dx)}{8a^2(a-b)^3d\left(a+b\sinh^2(c+dx)\right)}+\frac{\mathrm{s}}{2(a}$$

$$=\frac{(a-7b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^4d}+\frac{b^{3/2}\left(35a^2-14ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^4d}+$$

Mathematica [A] time = 1.88, size = 222, normalized size = 1.02

$$\frac{-\frac{3b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{a^{5/2}}+\frac{14b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{a^{3/2}}+\frac{2b^2(a-b)\sinh(c+dx)\left(26a^2+b(11a-3b)\cosh(2(c+dx))-21ab+3b^2\right)}{a^2(2a+b\cosh(2(c+dx))-b)^2}-\frac{35b^{3/2}\tan^{-1}}{}}{8d(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((-35*b^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + (14*b^(5/2
)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/a^(3/2) - (3*b^(7/2)*ArcTan[(Sqr
t[a]*Csch[c + d*x])/Sqrt[b]])/a^(5/2) + 8*a*ArcTan[Tanh[(c + d*x)/2]] - 56*
b*ArcTan[Tanh[(c + d*x)/2]] + (2*(a - b)*b^2*(26*a^2 - 21*a*b + 3*b^2 + (11
*a - 3*b)*b*Cosh[2*(c + d*x)])*Sinh[c + d*x])/(a^2*(2*a - b + b*Cosh[2*(c +
 d*x)])^2) + 4*(a - b)*Sech[c + d*x]*Tanh[c + d*x])/(8*(a - b)^4*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-85,-18]Warning, need to choose a branch for the root of a
polynomial with parameters. This might be wrong.The choice was done assumin
g [a,b]=[33,-80]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.The choice was done assuming [a,b]=
[-98,-18]Warning, need to choose a branch for the root of a polynomial with
 parameters. This might be wrong.The choice was done assuming [a,b]=[-57,-1
0]Warning, need to choose a branch for the root of a polynomial with parame
ters. This might be wrong.The choice was done assuming [a,b]=[-57,-3]Warnin
g, need to choose a branch for the root of a polynomial with parameters. Th
is might be wrong.The choice was done assuming [a,b]=[-53,60]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[80,-1]schur row 3 -6.9034e-07
Warning, need to choose a branch for the root of a polynomial with paramete
rs. This might be wrong.The choice was done assuming [a,b]=[-51,-3]Undef/Un
signed Inf encountered in limitEvaluation time: 1.66Limit: Max order reache
d or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.21, size = 2307, normalized size = 10.63

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -35/8/d*b^2/(a-b)^4*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-35/8/d*
b^2/(a-b)^4*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-17/8/d*b^4/(a-
b)^4/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b^5/(a-b)^4/a^2/
(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-17/8/d*b^4/(a-b)^4/a/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
```

```
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d*b^5/(a-b)^4/a^2/(-b*(a-b))^(1/
2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+35/8/d*b^2/(a-b)^4/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2))-35/8/d*b^2/(a-b)^4/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tan
h(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(a-b)^4/(tanh(1/
2*d*x+1/2*c)^2+1)^2*a*tanh(1/2*d*x+1/2*c)^3+1/d/(a-b)^4/(tanh(1/2*d*x+1/2*c
)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*b+1/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^2*a*
tanh(1/2*d*x+1/2*c)-1/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/
2*c)*b+9/2/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-49/2/d*b^3/(a-b)^4/(t
anh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+
a)^2*tanh(1/2*d*x+1/2*c)^5+49/2/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-
9/2/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh
(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+1/d/(a-b)^4*arctan(tanh(1/2*d*
x+1/2*c))*a-7/d/(a-b)^4*arctan(tanh(1/2*d*x+1/2*c))*b-7/4/d*b^3/(a-b)^4/a/(
(2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b^4/(a-b)^4/a^2/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2))+7/4/d*b^3/(a-b)^4/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-5/4/d*b^4/(a-b)^4/(ta
nh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)^2/a*tanh(1/2*d*x+1/2*c)^7+71/4/d*b^4/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)
^5-3/d*b^5/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5-71/4/d*b^4/(a-b)^4/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2/a*tanh(1/2*d*x+1/2*c)^3-3/8/d*b^4/(a-b)^4/a^2/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/
2))+49/8/d*b^3/(a-b)^4/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+49/8/d
*b^3/(a-b)^4/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(
a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*b^5/(a-b)^4
/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3+5/4/d*b^4/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*
a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1
/2*c)-13/4/d*b^2/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^7+39/4/d*b^2/(a-b)^4/
(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*
b+a)^2*a*tanh(1/2*d*x+1/2*c)^5-39/4/d*b^2/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-
2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2
*c)^3+13/4/d*b^2/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a
+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (a*e^c - 7*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^4*d - 4*a^3*b*d + 6*a^2*b^2
*d - 4*a*b^3*d + b^4*d) + 1/4*((4*a^2*b^2*e^(11*c) + 11*a*b^3*e^(11*c) - 3*
b^4*e^(11*c))*e^(11*d*x) + (32*a^3*b*e^(9*c) + 32*a^2*b^2*e^(9*c) - 31*a*b^
3*e^(9*c) + 3*b^4*e^(9*c))*e^(9*d*x) + 2*(32*a^4*e^(7*c) - 48*a^3*b*e^(7*c)
 + 46*a^2*b^2*e^(7*c) - 21*a*b^3*e^(7*c) + 3*b^4*e^(7*c))*e^(7*d*x) - 2*(32
*a^4*e^(5*c) - 48*a^3*b*e^(5*c) + 46*a^2*b^2*e^(5*c) - 21*a*b^3*e^(5*c) + 3
*b^4*e^(5*c))*e^(5*d*x) - (32*a^3*b*e^(3*c) + 32*a^2*b^2*e^(3*c) - 31*a*b^3
*e^(3*c) + 3*b^4*e^(3*c))*e^(3*d*x) - (4*a^2*b^2*e^c + 11*a*b^3*e^c - 3*b^4
```

```
*e^c)*e^(d*x))/(a^5*b^2*d - 3*a^4*b^3*d + 3*a^3*b^4*d - a^2*b^5*d + (a^5*b^
2*d*e^(12*c) - 3*a^4*b^3*d*e^(12*c) + 3*a^3*b^4*d*e^(12*c) - a^2*b^5*d*e^(1
2*c))*e^(12*d*x) + 2*(4*a^6*b*d*e^(10*c) - 13*a^5*b^2*d*e^(10*c) + 15*a^4*b
^3*d*e^(10*c) - 7*a^3*b^4*d*e^(10*c) + a^2*b^5*d*e^(10*c))*e^(10*d*x) + (16
*a^7*d*e^(8*c) - 48*a^6*b*d*e^(8*c) + 47*a^5*b^2*d*e^(8*c) - 13*a^4*b^3*d*e
^(8*c) - 3*a^3*b^4*d*e^(8*c) + a^2*b^5*d*e^(8*c))*e^(8*d*x) + 4*(8*a^7*d*e^
(6*c) - 28*a^6*b*d*e^(6*c) + 37*a^5*b^2*d*e^(6*c) - 23*a^4*b^3*d*e^(6*c) +
7*a^3*b^4*d*e^(6*c) - a^2*b^5*d*e^(6*c))*e^(6*d*x) + (16*a^7*d*e^(4*c) - 48
*a^6*b*d*e^(4*c) + 47*a^5*b^2*d*e^(4*c) - 13*a^4*b^3*d*e^(4*c) - 3*a^3*b^4*
d*e^(4*c) + a^2*b^5*d*e^(4*c))*e^(4*d*x) + 2*(4*a^6*b*d*e^(2*c) - 13*a^5*b^
2*d*e^(2*c) + 15*a^4*b^3*d*e^(2*c) - 7*a^3*b^4*d*e^(2*c) + a^2*b^5*d*e^(2*c
))*e^(2*d*x)) + 8*integrate(1/32*((35*a^2*b^2*e^(3*c) - 14*a*b^3*e^(3*c) +
3*b^4*e^(3*c))*e^(3*d*x) + (35*a^2*b^2*e^c - 14*a*b^3*e^c + 3*b^4*e^c)*e^(d
*x))/(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5 + (a^6*b*e^(4*c)
- 4*a^5*b^2*e^(4*c) + 6*a^4*b^3*e^(4*c) - 4*a^3*b^4*e^(4*c) + a^2*b^5*e^(4*
c))*e^(4*d*x) + 2*(2*a^7*e^(2*c) - 9*a^6*b*e^(2*c) + 16*a^5*b^2*e^(2*c) - 1
4*a^4*b^3*e^(2*c) + 6*a^3*b^4*e^(2*c) - a^2*b^5*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(c+d\,x\right)^3\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.348 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=203

$$-\frac{b^3(16a-3b)\tanh(c+dx)}{8a^2d(a-b)^4\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{b^2\left(48a^2-16ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{9/2}}+\frac{b^4\tan}{4ad(a-b)^4\left(a-(\right.}$$

```
[Out] 1/8*b^2*(48*a^2-16*a*b+3*b^2)*arctanh((a-b)^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5
/2)/(a-b)^(9/2)/d+(a-4*b)*tanh(d*x+c)/(a-b)^4/d-1/3*tanh(d*x+c)^3/(a-b)^3/d
+1/4*b^4*tanh(d*x+c)/a/(a-b)^4/d/(a-(a-b)*tanh(d*x+c)^2)^2-1/8*(16*a-3*b)*b
^3*tanh(d*x+c)/a^2/(a-b)^4/d/(a-(a-b)*tanh(d*x+c)^2)
```

Rubi [A] time = 0.34, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 390, 1157, 385, 208}

$$-\frac{b^3(16a-3b)\tanh(c+dx)}{8a^2d(a-b)^4\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{b^2\left(48a^2-16ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{9/2}}+\frac{b^4\tan}{4ad(a-b)^4\left(a-(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(8*a^(5/2)*(a - b)^(9/2)*d) + ((a - 4*b)*Tanh[c + d*x])/((a - b)^4*d) - T
anh[c + d*x]^3/(3*(a - b)^3*d) + (b^4*Tanh[c + d*x])/(4*a*(a - b)^4*d*(a -
(a - b)*Tanh[c + d*x]^2)^2) - ((16*a - 3*b)*b^3*Tanh[c + d*x])/(8*a^2*(a -
b)^4*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
```

```
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^4}{(a-(a-b)x^2)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a-4b}{(a-b)^4} - \frac{x^2}{(a-b)^3} + \frac{b^2(6a^2-4ab+b^2)-4(a-b)(3a-b)b^2x^2+6(a-b)^2b^2x^4}{(a-b)^4(a+(-a+b)x^2)^3}\right) dx, x, \tanh(c+\right.}{d}$$

$$= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4 d} - \frac{\tanh^3(c+dx)}{3(a-b)^3 d} + \frac{\operatorname{Subst}\left(\int \frac{b^2(6a^2-4ab+b^2)-4(a-b)(3a-b)b^2x^2+6(\right.}{(a+(-a+b)x^2)^3}}{(a-b)^4 d}$$

$$= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4 d} - \frac{\tanh^3(c+dx)}{3(a-b)^3 d} + \frac{b^4\tanh(c+dx)}{4a(a-b)^4 d\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

$$= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4 d} - \frac{\tanh^3(c+dx)}{3(a-b)^3 d} + \frac{b^4\tanh(c+dx)}{4a(a-b)^4 d\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

$$= \frac{b^2(48a^2-16ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{9/2}d} + \frac{(a-4b)\tanh(c+dx)}{(a-b)^4 d} - \frac{\tanh^3(}{3(a-}$$

Mathematica [A] time = 2.31, size = 169, normalized size = 0.83

$$\frac{\frac{\frac{3b^3\sinh(2(c+dx))\left(-32a^2+b(3b-14a)\cosh(2(c+dx))+24ab-3b^2\right)}{a^2(2a+b\cosh(2(c+dx))-b)^2}+8\tanh(c+dx)\left((a-b)\operatorname{sech}^2(c+dx)+2a-11b\right)}{(a-b)^4}+\frac{3b^2(48a^2-16ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^{9/2}}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[
a]])/(a^(5/2)*(a - b)^(9/2)) + ((3*b^3*(-32*a^2 + 24*a*b - 3*b^2 + b*(-14*a
 + 3*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a^2*(2*a - b + b*Cosh[2*(c +
 d*x)])^2) + 8*(2*a - 11*b + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b
)^4)/(24*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.78, size = 436, normalized size = 2.15

$$\frac{\frac{3\left(48\,a^{2}b^{2}-16\,ab^{3}+3\,b^{4}\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{\left(a^{6}-4\,a^{5}b+6\,a^{4}b^{2}-4\,a^{3}b^{3}+a^{2}b^{4}\right)\sqrt{-a^{2}+ab}}+\frac{6\left(24\,a^{2}b^{3}e^{(6\,dx+6\,c)}-16\,ab^{4}e^{(6\,dx+6\,c)}+3\,b^{5}e^{(6\,dx+6\,c)}+112\,a^{3}b^{2}e^{(4\,dx+4\,c)}-136\,a^{2}b^{3}e^{(4\,dx+4\,c)}+6\right.}{\left(a^{6}-4\,a^{5}b+6\,a^{4}b^{2}-4\,a^{3}b^{3}+a^{2}b^{4}\right)\left(be^{(4\right.}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(48*a^2*b^2 - 16*a*b^3 + 3*b^4)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a
 - b)/sqrt(-a^2 + a*b))/((a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*
sqrt(-a^2 + a*b)) + 6*(24*a^2*b^3*e^(6*d*x + 6*c) - 16*a*b^4*e^(6*d*x + 6*c
) + 3*b^5*e^(6*d*x + 6*c) + 112*a^3*b^2*e^(4*d*x + 4*c) - 136*a^2*b^3*e^(4*
d*x + 4*c) + 66*a*b^4*e^(4*d*x + 4*c) - 9*b^5*e^(4*d*x + 4*c) + 88*a^2*b^3*
e^(2*d*x + 2*c) - 64*a*b^4*e^(2*d*x + 2*c) + 9*b^5*e^(2*d*x + 2*c) + 14*a*b
^4 - 3*b^5)/((a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*(b*e^(4*d*x
+ 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2) + 16*(9*b*e^(4*d
*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) - 2*a + 11*b)/((a^4
- 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(e^(2*d*x + 2*c) + 1)^3))/d
```

maple [B] time = 0.20, size = 1892, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 2/d*b^4/(a-b)^4/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d*b^5/(
a-b)^4/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d*b^4/(a-b)^4/a/
(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d*b^5/(a-b)^4/a^2/(-b*(a
-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-6/d*b^2/(a-b)^4/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)
*a)^(1/2))+6/d*b^2/(a-b)^4/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-4/d*b^3/(a-b)^4/(t
anh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+
a)^2*tanh(1/2*d*x+1/2*c)^7+4/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+4/d
*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-4/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2
*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d
*x+1/2*c)+2/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*a-8
/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^3*b*tanh(1/2*d*x+1/2*c)^5+4/3/d/(a-b)^
4/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3*a-40/3/d/(a-b)^4/(tanh(
1/2*d*x+1/2*c)^2+1)^3*b*tanh(1/2*d*x+1/2*c)^3+2/d/(a-b)^4/(tanh(1/2*d*x+1/2
*c)^2+1)^3*tanh(1/2*d*x+1/2*c)*a-8/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^3*b*
tanh(1/2*d*x+1/2*c)+2/d*b^3/(a-b)^4/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d*b^
4/(a-b)^4/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-2/d*b^3/(a-b)^4/a/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2))+5/4/d*b^4/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1
```

```
/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7-61/4/d*b^4
/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+
1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+3/d*b^5/(a-b)^4/(tanh(1/2*d*x+1/2*c
)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2
*d*x+1/2*c)^5-61/4/d*b^4/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+3/8/d*b^4/(
a-b)^4/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-6/d*b^3/(a-b)^4/(-b*(a-b))^(1/2)/(
(2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2))-6/d*b^3/(a-b)^4/(-b*(a-b))^(1/2)/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2))+3/d*b^5/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2
*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3+5/4/d*b^4/
(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is b-a positive or negative?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(c+d\,x\right)^4\,{\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3),x)
```

```
[Out] int(1/(cosh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.349 $\int \frac{\cosh^2(x)}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=19

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-x$$

```
[Out] -x+arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3191, 391, 206}

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 - Sinh[x]^2),x]
```

```
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{1-\sinh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-2x^2\right)\left(1-x^2\right)}\,dx, x, \tanh(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right)\\ &= -x+\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)\end{aligned}$$

Mathematica [A] time = 0.08, size = 24, normalized size = 1.26

$$-2\left(\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 - Sinh[x]^2),x]
```

```
[Out] -2*(x/2 - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2])
```

fricas [B] time = 0.63, size = 70, normalized size = 3.68

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2-2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2-3}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3)) - x
```

giac [B] time = 0.14, size = 41, normalized size = 2.16

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - x
```

maple [B] time = 0.06, size = 54, normalized size = 2.84

$$\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1-sinh(x)^2),x)
```

```
[Out] ln(tanh(1/2*x)-1)+2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))-ln(tanh(1/
2*x)+1)+2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

maxima [B] time = 0.49, size = 64, normalized size = 3.37

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

mupad [B] time = 0.14, size = 56, normalized size = 2.95

$$\frac{\sqrt{2}\,\ln\left(8\,e^{2x}+\frac{\sqrt{2}\,\left(12\,e^{2x}-4\right)}{2}\right)}{2}-\frac{\sqrt{2}\,\ln\left(8\,e^{2x}-\frac{\sqrt{2}\,\left(12\,e^{2x}-4\right)}{2}\right)}{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)^2/(sinh(x)^2 - 1),x)
```

```
[Out] (2^(1/2)*log(8*exp(2*x) + (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - (2^(1/2)*log(
8*exp(2*x) - (2^(1/2)*(12*exp(2*x) - 4))/2))/2 - x
```

sympy [B] time = 6.53, size = 238, normalized size = 12.53

$$-\frac{1331714x}{941664\sqrt{2}+1331714}-\frac{941664\sqrt{2}\,x}{941664\sqrt{2}+1331714}+\frac{941664\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{941664\sqrt{2}+1331714}+\frac{665857\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)\right.}{941664\sqrt{2}+1331}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1-sinh(x)**2),x)
```

```
[Out] -1331714*x/(941664*sqrt(2) + 1331714) - 941664*sqrt(2)*x/(941664*sqrt(2) +
1331714) + 941664*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714) +
 665857*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))/(941664*sqrt(2) + 1331714) + 9
41664*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) + 665857*sqrt
(2)*log(tanh(x/2) + 1 + sqrt(2))/(941664*sqrt(2) + 1331714) - 665857*sqrt(2
)*log(tanh(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 941664*log(tanh
(x/2) - sqrt(2) - 1)/(941664*sqrt(2) + 1331714) - 665857*sqrt(2)*log(tanh(x
/2) - sqrt(2) + 1)/(941664*sqrt(2) + 1331714) - 941664*log(tanh(x/2) - sqrt
(2) + 1)/(941664*sqrt(2) + 1331714)
```

3.350 $\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=10

$$2\tanh^{-1}(\sinh(x))-\sinh(x)$$

```
[Out] 2*arctanh(sinh(x))-sinh(x)
```

Rubi [A] time = 0.04, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 388, 206}

$$2\tanh^{-1}(\sinh(x))-\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 - Sinh[x]^2),x]
```

```
[Out] 2*ArcTanh[Sinh[x]] - Sinh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{1-x^2}\,dx,x,\sinh(x)\right)\\ &= -\sinh(x)+2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sinh(x)\right)\\ &= 2\tanh^{-1}(\sinh(x))-\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.40

$$-2\left(\frac{\sinh(x)}{2}-\tanh^{-1}(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 - Sinh[x]^2),x]
```

```
[Out] -2*(-ArcTanh[Sinh[x]] + Sinh[x]/2)
```

fricas [B] time = 1.01, size = 71, normalized size = 7.10

$$-\frac{\cosh(x)^2 - 2(\cosh(x) + \sinh(x))\log\left(\frac{2(\sinh(x)+1)}{\cosh(x)-\sinh(x)}\right) + 2(\cosh(x) + \sinh(x))\log\left(\frac{2(\sinh(x)-1)}{\cosh(x)-\sinh(x)}\right) + 2\cosh(x}{2(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh(x)^2 - 2*(cosh(x) + sinh(x))*log(2*(sinh(x) + 1)/(cosh(x) - sinh
(x))) + 2*(cosh(x) + sinh(x))*log(2*(sinh(x) - 1)/(cosh(x) - sinh(x))) + 2*
cosh(x)*sinh(x) + sinh(x)^2 - 1)/(cosh(x) + sinh(x))
```

giac [B] time = 0.14, size = 37, normalized size = 3.70

$$\frac{1}{2}e^{(-x)} - \frac{1}{2}e^x + \log\left(\left|-e^{(-x)} + e^x + 2\right|\right) - \log\left(\left|-e^{(-x)} + e^x - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) - 1/2*e^x + log(abs(-e^(-x) + e^x + 2)) - log(abs(-e^(-x) + e^x
- 2))
```

maple [B] time = 0.05, size = 50, normalized size = 5.00

$$\ln\left(\tanh^2\left(\frac{x}{2}\right) - 2\tanh\left(\frac{x}{2}\right) - 1\right) + \frac{1}{\tanh\left(\frac{x}{2}\right) + 1} - \ln\left(\tanh^2\left(\frac{x}{2}\right) + 2\tanh\left(\frac{x}{2}\right) - 1\right) + \frac{1}{\tanh\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1-sinh(x)^2),x)
```

```
[Out] ln(tanh(1/2*x)^2-2*tanh(1/2*x)-1)+1/(tanh(1/2*x)+1)-ln(tanh(1/2*x)^2+2*tanh
(1/2*x)-1)+1/(tanh(1/2*x)-1)
```

maxima [B] time = 0.46, size = 39, normalized size = 3.90

$$\frac{1}{2}e^{(-x)} - \frac{1}{2}e^x - \log\left(2e^{(-x)} + e^{(-2x)} - 1\right) + \log\left(-2e^{(-x)} + e^{(-2x)} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-x) - 1/2*e^x - log(2*e^(-x) + e^(-2*x) - 1) + log(-2*e^(-x) + e^(-2
*x) - 1)
```

mupad [B] time = 0.06, size = 39, normalized size = 3.90

$$\frac{e^{-x}}{2} - \ln\left(32e^{2x} - 64e^x - 32\right) + \ln\left(32e^{2x} + 64e^x - 32\right) - \frac{e^x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)^3/(sinh(x)^2 - 1),x)
```

```
[Out] exp(-x)/2 - log(32*exp(2*x) - 64*exp(x) - 32) + log(32*exp(2*x) + 64*exp(x)
 - 32) - exp(x)/2
```

sympy [B] time = 1.61, size = 129, normalized size = 12.90

$$\frac{\log\left(\tanh^2\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)-1\right)\tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)-1}-\frac{\log\left(\tanh^2\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)-1\right)}{\tanh^2\left(\frac{x}{2}\right)-1}-\frac{\log\left(\tanh^2\left(\frac{x}{2}\right)+2\tanh\left(\frac{x}{2}\right)-1\right)}{\tanh^2\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1-sinh(x)**2),x)
```

```
[Out] log(tanh(x/2)**2 - 2*tanh(x/2) - 1)*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(t
anh(x/2)**2 - 2*tanh(x/2) - 1)/(tanh(x/2)**2 - 1) - log(tanh(x/2)**2 + 2*ta
nh(x/2) - 1)*tanh(x/2)**2/(tanh(x/2)**2 - 1) + log(tanh(x/2)**2 + 2*tanh(x/
2) - 1)/(tanh(x/2)**2 - 1) + 2*tanh(x/2)/(tanh(x/2)**2 - 1)
```

3.351 $\int \frac{\cosh^4(x)}{1-\sinh^2(x)} dx$

Optimal. Leaf size=30

$$-\frac{5x}{2}+2\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-\frac{1}{2}\sinh(x)\cosh(x)$$

```
[Out] -5/2*x-1/2*cosh(x)*sinh(x)+2*arctanh(2^(1/2)*tanh(x))*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3191, 414, 522, 206}

$$-\frac{5x}{2}+2\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(1 - Sinh[x]^2),x]
```

```
[Out] (-5*x)/2 + 2*Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] - (Cosh[x]*Sinh[x])/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1-2x^2)(1-x^2)^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}\cosh(x)\sinh(x) - \frac{1}{2}\text{Subst}\left(\int \frac{-3-2x^2}{(1-2x^2)(1-x^2)}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}\cosh(x)\sinh(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right) + 4\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(\right.$$
$$= -\frac{5x}{2} + 2\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right) - \frac{1}{2}\cosh(x)\sinh(x)$$

Mathematica [**A**] time = 0.07, size = 32, normalized size = 1.07

$$-2\left(\frac{5x}{4} + \frac{1}{8}\sinh(2x) - \sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(1 - Sinh[x]^2),x]
```

```
[Out] -2*((5*x)/4 - Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + Sinh[2*x]/8)
```

fricas [**B**] time = 0.60, size = 163, normalized size = 5.43

$$-\frac{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 20\,x\cosh(x)^2 + 2\left(3\cosh(x)^2 + 10\,x\right)\sinh(x)^2 - 8\left(\sqrt{2}\cosh(x)^2 + \right.}{8\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 20*x*cosh(x)^2 + 2*(3*c
osh(x)^2 + 10*x)*sinh(x)^2 - 8*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(
x) + sqrt(2)*sinh(x)^2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) -
4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^
2 + sinh(x)^2 - 3)) + 4*(cosh(x)^3 + 10*x*cosh(x))*sinh(x) - 1)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)
```

giac [**B**] time = 0.15, size = 61, normalized size = 2.03

$$\frac{1}{8}\left(10\,e^{(2\,x)} + 1\right)e^{(-2\,x)} - \sqrt{2}\log\left(\frac{\left|-4\sqrt{2} + 2\,e^{(2\,x)} - 6\right|}{\left|4\sqrt{2} + 2\,e^{(2\,x)} - 6\right|}\right) - \frac{5}{2}\,x - \frac{1}{8}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/8*(10*e^(2*x) + 1)*e^(-2*x) - sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)
/abs(4*sqrt(2) + 2*e^(2*x) - 6)) - 5/2*x - 1/8*e^(2*x)
```

maple [**B**] time = 0.06, size = 98, normalized size = 3.27

$$-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{5\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2} + 2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2\right)\sqrt{2}}{4}\right) + \frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(1-sinh(x)^2),x)
```

```
[Out] -1/2/(tanh(1/2*x)-1)^2-1/2/(tanh(1/2*x)-1)+5/2*ln(tanh(1/2*x)-1)+2*2^(1/2)*
arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+1/2/(tanh(1/2*x)+1)^2-1/2/(tanh(1/2*
x)+1)-5/2*ln(tanh(1/2*x)+1)+2*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2)
)
```

maxima [B] time = 0.61, size = 75, normalized size = 2.50

$$\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-\frac{5}{2}x-\frac{1}{8}e^{(2x)}+\frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - sqrt(2)*log(-
(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 5/2*x - 1/8*e^(2*x) + 1/8*
e^(-2*x)
```

mupad [B] time = 0.84, size = 66, normalized size = 2.20

$$\frac{e^{-2x}}{8}-\frac{5x}{2}-\frac{e^{2x}}{8}+\sqrt{2}\ln\left(16e^{2x}+\sqrt{2}\left(12e^{2x}-4\right)\right)-\sqrt{2}\ln\left(16e^{2x}-\sqrt{2}\left(12e^{2x}-4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cosh(x)^4/(sinh(x)^2 - 1),x)
```

```
[Out] exp(-2*x)/8 - (5*x)/2 - exp(2*x)/8 + 2^(1/2)*log(16*exp(2*x) + 2^(1/2)*(12*
exp(2*x) - 4)) - 2^(1/2)*log(16*exp(2*x) - 2^(1/2)*(12*exp(2*x) - 4))
```

sympy [B] time = 17.30, size = 2431, normalized size = 81.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(1-sinh(x)**2),x)
```

```
[Out] -2716698600*sqrt(2)*x*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sq
rt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)
**2 + 1536796802 + 1086679440*sqrt(2)) - 3841992005*x*tanh(x/2)**4/(1536796
802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**
2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) + 76
83984010*x*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(
x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 15367
96802 + 1086679440*sqrt(2)) + 5433397200*sqrt(2)*x*tanh(x/2)**2/(1536796802
*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 -
 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 27166
98600*sqrt(2)*x/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4
- 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 +
1086679440*sqrt(2)) - 3841992005*x/(1536796802*tanh(x/2)**4 + 1086679440*sq
rt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)
**2 + 1536796802 + 1086679440*sqrt(2)) + 2173358880*log(tanh(x/2) - 1 + sqr
t(2))*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)*
*4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802
 + 1086679440*sqrt(2)) + 1536796802*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*ta
nh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 307
3593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 10866
79440*sqrt(2)) - 4346717760*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(1536
796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2
```

```
)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) -
 3073593604*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))*tanh(x/2)**2/(1536796802*t
anh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2
173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) + 2173358
880*log(tanh(x/2) - 1 + sqrt(2))/(1536796802*tanh(x/2)**4 + 1086679440*sqrt
(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**
2 + 1536796802 + 1086679440*sqrt(2)) + 1536796802*sqrt(2)*log(tanh(x/2) - 1
 + sqrt(2))/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 30
73593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086
679440*sqrt(2)) + 2173358880*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**4/(153
6796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/
2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2))
+ 1536796802*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**4/(1536796802*
tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 -
2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 434671
7760*log(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 1
086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(
2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 3073593604*sqrt(2)*log
(tanh(x/2) + 1 + sqrt(2))*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 108667944
0*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(
x/2)**2 + 1536796802 + 1086679440*sqrt(2)) + 2173358880*log(tanh(x/2) + 1 +
 sqrt(2))/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073
593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 108667
9440*sqrt(2)) + 1536796802*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(1536796802
*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 -
 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 21733
58880*log(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**4/(1536796802*tanh(x/2)**4 +
1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt
(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 1536796802*sqrt(2)*lo
g(tanh(x/2) - sqrt(2) - 1)*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 10866794
40*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh
(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) + 4346717760*log(tanh(x/2) - sq
rt(2) - 1)*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(
x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 15367
96802 + 1086679440*sqrt(2)) + 3073593604*sqrt(2)*log(tanh(x/2) - sqrt(2) -
1)*tanh(x/2)**2/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4
- 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 +
1086679440*sqrt(2)) - 2173358880*log(tanh(x/2) - sqrt(2) - 1)/(1536796802*t
anh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2
173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 1536796
802*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(1536796802*tanh(x/2)**4 + 1086679
440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tan
h(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 2173358880*log(tanh(x/2) - s
qrt(2) + 1)*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh
(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536
796802 + 1086679440*sqrt(2)) - 1536796802*sqrt(2)*log(tanh(x/2) - sqrt(2) +
 1)*tanh(x/2)**4/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4
 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 +
 1086679440*sqrt(2)) + 4346717760*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**2
/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*ta
nh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt
(2)) + 3073593604*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)*tanh(x/2)**2/(153679
6802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)*
*2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 2
173358880*log(tanh(x/2) - sqrt(2) + 1)/(1536796802*tanh(x/2)**4 + 108667944
0*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(
x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 1536796802*sqrt(2)*log(tanh(x/
2) - sqrt(2) + 1)/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**
4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802
```

```
+ 1086679440*sqrt(2)) - 1536796802*tanh(x/2)**3/(1536796802*tanh(x/2)**4 +
1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt
(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 1086679440*sqrt(2)*ta
nh(x/2)**3/(1536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 307
3593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 10866
79440*sqrt(2)) - 1536796802*tanh(x/2)/(1536796802*tanh(x/2)**4 + 1086679440
*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(x/2)**2 - 2173358880*sqrt(2)*tanh(x
/2)**2 + 1536796802 + 1086679440*sqrt(2)) - 1086679440*sqrt(2)*tanh(x/2)/(1
536796802*tanh(x/2)**4 + 1086679440*sqrt(2)*tanh(x/2)**4 - 3073593604*tanh(
x/2)**2 - 2173358880*sqrt(2)*tanh(x/2)**2 + 1536796802 + 1086679440*sqrt(2)
)
```

3.352 $\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a(a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4bf}-\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf}$$

```
[Out] -1/8*a*(a-4*b)*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(3/
2)/f+1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/b/f-1/8*(a-4*b)*sinh(f*x+e)*
(a+b*sinh(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 388, 195, 217, 206}

$$-\frac{a(a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4bf}-\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(a*(a - 4*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(8*b^(3/2)*f) - ((a - 4*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*b
*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
```

```
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)\sqrt{a+bx^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4bf} - \frac{(a-4b)\,\text{Subst}\left(\int \sqrt{a+b\right.}{4b}$$

$$= -\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf} + \frac{\sinh(e+fx)\left(a+\right.}{}$$

$$= -\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf} + \frac{\sinh(e+fx)\left(a+\right.}{}$$

$$= -\frac{a(a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-4b)\sinh(e+fx)\sqrt{a+}}{8bf}$$

Mathematica [A] time = 0.69, size = 124, normalized size = 1.06

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(\sqrt{b}\sinh(e+fx)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,(a+b\cosh(2(e+fx))+3b)-\sqrt{a}\,(a-4b)\sinh^{-1}\left(\right.\right.}{8b^{3/2}f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(-(Sqrt[a]*(a - 4*b)*ArcSinh[(Sqrt[b]*Sinh[e +
 f*x])/Sqrt[a]]) + Sqrt[b]*(a + 3*b + b*Cosh[2*(e + f*x)])*Sinh[e + f*x]*Sq
rt[1 + (b*Sinh[e + f*x]^2)/a]))/(8*b^(3/2)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
])
```

fricas [B] time = 0.81, size = 3281, normalized size = 28.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(2*((a^2 - 4*a*b)*cosh(f*x + e)^4 + 4*(a^2 - 4*a*b)*cosh(f*x + e)^3*
sinh(f*x + e) + 6*(a^2 - 4*a*b)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 -
4*a*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 4*a*b)*sinh(f*x + e)^4)*sqrt(
b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3
)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 +
 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5
*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^
6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*
b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*
cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 1
```

```
4*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*
a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cos
h(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2
*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2
 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x +
 e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b
+ b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2
- 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*
(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*
sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)
*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*
sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 -
 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e
))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b -
 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(
f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^
3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(
f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x
 + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e
)^5 + sinh(f*x + e)^6)) + 2*((a^2 - 4*a*b)*cosh(f*x + e)^4 + 4*(a^2 - 4*a*b
)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 4*a*b)*cosh(f*x + e)^2*sinh(f*x
+ e)^2 + 4*(a^2 - 4*a*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 4*a*b)*sinh
(f*x + e)^4)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a
)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(2)*(b^
2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)
^6 + (2*a*b + 5*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 2*a*b + 5*
b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (2*a*b + 5*b^2)*cosh(f*x
+ e))*sinh(f*x + e)^3 - (2*a*b + 5*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x
+ e)^4 + 6*(2*a*b + 5*b^2)*cosh(f*x + e)^2 - 2*a*b - 5*b^2)*sinh(f*x + e)^2
 - b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(2*a*b + 5*b^2)*cosh(f*x + e)^3 - (2*
a*b + 5*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh
(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + s
inh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^4 + 4*b^2*f*cosh(f*x + e)^3*sinh(f*x
 + e) + 6*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b^2*f*cosh(f*x + e)*sin
h(f*x + e)^3 + b^2*f*sinh(f*x + e)^4), 1/64*(4*((a^2 - 4*a*b)*cosh(f*x + e)
^4 + 4*(a^2 - 4*a*b)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 4*a*b)*cosh(f
*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 4*a*b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a^2 - 4*a*b)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x +
 e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b
)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*co
sh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*s
inh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x
+ e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x +
 e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + 4*((a^2 - 4*a*b)*c
osh(f*x + e)^4 + 4*(a^2 - 4*a*b)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 4
*a*b)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 4*a*b)*cosh(f*x + e)*sinh(
f*x + e)^3 + (a^2 - 4*a*b)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*co
```

```
sh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)
^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)
^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*(b^2*cosh(f*x +
 e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (2*a*b
+ 5*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 2*a*b + 5*b^2)*sinh(f*
x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (2*a*b + 5*b^2)*cosh(f*x + e))*sinh(f
*x + e)^3 - (2*a*b + 5*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(
2*a*b + 5*b^2)*cosh(f*x + e)^2 - 2*a*b - 5*b^2)*sinh(f*x + e)^2 - b^2 + 2*(
3*b^2*cosh(f*x + e)^5 + 2*(2*a*b + 5*b^2)*cosh(f*x + e)^3 - (2*a*b + 5*b^2)
*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/(b^2*f*cosh(f*x + e)^4 + 4*b^2*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b^
2*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^3
 + b^2*f*sinh(f*x + e)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.14, size = 52, normalized size = 0.44

$$\frac{`int/indef0`\left(\frac{b\left(\cosh^4\left(fx+e\right)\right)+\left(a-b\right)\left(\cosh^2\left(fx+e\right)\right)}{\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(1/2
),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \cosh\left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(e+fx\right)^3 \sqrt{b \sinh\left(e+fx\right)^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.353 $\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}\,f}$$

```
[Out] 1/2*a*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*
sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 195, 217, 206}

$$\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (a*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[b]
*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \sinh(e+fx)\right)}{f}$$
$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sinh(e$$
$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sinh(e}{\sqrt{a+b\sinh}}$$
$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

Mathematica [A] time = 0.26, size = 96, normalized size = 1.33

$$\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}+\sqrt{b}\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)}{2\sqrt{b}f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2) + a^(3/2)*ArcSinh[(Sqrt[b]*S
inh[e + f*x])/Sqrt[a]]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(2*Sqrt[b]*f*Sqrt[a
 + b*Sinh[e + f*x]^2])
```

fricas [B] time = 0.69, size = 2419, normalized size = 33.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e
)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a
*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*
x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4
*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh
(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^
2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2
 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9
*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x +
 e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10
*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 +
6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x
+ e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b
+ 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^
2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*
cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^
2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 +
3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)
^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(
f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*
```

```
a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b
 + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5
- 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*si
nh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(
2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*
b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*
b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x +
e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^
3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*si
nh(f*x + e)^5 + sinh(f*x + e)^6)) + (a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*
sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cos
h(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3
*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh
(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b
*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*
f*cosh(f*x + e)^2 + 2*b*f*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2
), -1/8*(2*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*
x + e)^2)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh
(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cos
h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
 e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b
 - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*a*
b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2
)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2
)*cosh(f*x + e))*sinh(f*x + e))) + 2*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)
*sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^
2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
 e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(
3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*
a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*
b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^2 + 2*b*f*cosh(f*x + e)*si
nh(f*x + e) + b*f*sinh(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.03, size = 62, normalized size = 0.86

$$\frac{\sinh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}{2f}+\frac{a\ln\left(\sqrt{b}\,\sinh\left(fx+e\right)+\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\right)}{2f\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/2/f*a/b^(1/2)*ln(b^(1/2)*sinh
(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \cosh\left(fx+e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e), x)
```

mupad [B] time = 0.96, size = 61, normalized size = 0.85

$$\frac{\sinh\left(e+f\,x\right)\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{2\,f}+\frac{a\,\ln\left(\sqrt{b}\,\sinh\left(e+f\,x\right)+\sqrt{b\sinh\left(e+f\,x\right)^2+a}\right)}{2\,\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] (sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2))/(2*f) + (a*log(b^(1/2)*sinh(e
 + f*x) + (a + b*sinh(e + f*x)^2)^(1/2)))/(2*b^(1/2)*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh^{2}{\left(e + fx \right)}} \cosh{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*cosh(e + f*x), x)
```

3.354 $\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

```
[Out] arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+arc
tanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f} + \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx \ldots}}\right.}{}$$

$$= \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f} + \frac{b\operatorname{Subst}\left(\int \frac{1}{1-bx^2} \ldots\right.}{}$$

$$= \frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.75, size = 130, normalized size = 1.53

$$\frac{\frac{\sqrt{a}\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}{\sqrt{2a+b\cosh(2(e+fx))-b}} + \sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]] + (Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]]*S
qrt[(2*a - b + b*Cosh[2*(e + f*x)])/a])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
)/f
```

fricas [B] time = 0.81, size = 5139, normalized size = 60.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*
a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f
*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a
^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^
2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 +
9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 1
0*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 +
 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x
```

+ e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 2*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/f, 1/4*(4*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)

```
)*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^
2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2
)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2
 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b
 - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos
h(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*
b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sin
h(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f
*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x +
 e)^5 + sinh(f*x + e)^6)) + sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
 e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh
(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x +
e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)
))/f, -1/2*(sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*co
sh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a
*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*
a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b
^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b
^2)*cosh(f*x + e))*sinh(f*x + e))) + sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)
^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x
+ e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*
(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2
*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(-a + b)*log(((a - 2*b)*co
sh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(
f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2
 - 3*a + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh
(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh
(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 +
 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)
))/f, 1/2*(2*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)
^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)
^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x +
 e))*sinh(f*x + e) + b)) - sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2
 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqr
t(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*
x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f
*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^
2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3
 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) - sqrt(-b)*arctan(sqrt(2)
*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sq
rt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4
+ 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(
```

```
f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(
f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.13, size = 51, normalized size = 0.60

$$\frac{\text{`}int/indef0\text{`}\left(-\frac{-b\left(\sinh^{2}\left(fx+e\right)\right)-a}{\cosh\left(fx+e\right)^{2}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(-(-b*sinh(f*x+e)^2-a)/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),
sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\ \mathrm{sech}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}{\cosh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^{2}\left(e+fx\right)}\ \mathrm{sech}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x), x)
```

3.355 $\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f\sqrt{a-b}}+\frac{\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

```
[Out] 1/2*a*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/(a-b)^(1/
2)+1/2*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3190, 378, 377, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f\sqrt{a-b}}+\frac{\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (a*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt
[a - b]*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(2*f
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2f} + \frac{a\,\text{Subst}\left(\int \frac{}{(1}\right.}{}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2f} + \frac{a\,\text{Subst}\left(\int \frac{}{1-}\right.}{}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{a-b}\,f} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\ \text{ta}}{2f}$$

Mathematica [B] time = 1.56, size = 175, normalized size = 2.03

$$\frac{\sinh(e+fx)\left(\sqrt{2}\,a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\tanh^{-1}\left(\frac{\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)+\operatorname{sech}^2(e+fx)\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}\,(2a+b\,\text{cc}\right.}{4f\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sinh[e + f*x]*(Sqrt[2]*a*ArcTanh[Sqrt[-(((a - b)*Sinh[e + f*x]^2)/a)]/Sqrt
[1 + (b*Sinh[e + f*x]^2)/a]]*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a] + (2*a
 - b + b*Cosh[2*(e + f*x)])*Sech[e + f*x]^2*Sqrt[-(((a - b)*Sinh[e + f*x]^2
)/a)]))/(4*f*Sqrt[-(((a - b)*Sinh[e + f*x]^2)/a)]*Sqrt[a + b*Sinh[e + f*x]^
2])
```

fricas [B] time = 0.73, size = 1327, normalized size = 15.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh(f*x
+ e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2
+ 4*(a*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + a)*sqrt(-a + b)*l
og(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)
*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2
+ 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 -
(3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*co
sh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*s
inh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*si
nh(f*x + e) + 1)) - 2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x
 + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
```

```
f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)^4 + 4*(a - b)*f*cosh
(f*x + e)*sinh(f*x + e)^3 + (a - b)*f*sinh(f*x + e)^4 + 2*(a - b)*f*cosh(f*
x + e)^2 + 2*(3*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^2 + (a
 - b)*f + 4*((a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x + e))*sinh(f*x
+ e)), 1/2*((a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh
(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2 + a)*sinh(f*x +
e)^2 + 4*(a*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + a)*sqrt(a -
b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x
 + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b
)) + sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x +
e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)))/((a - b)*f*cosh(f*x + e)^4 + 4*(a - b)*f*cosh(f*x + e)*sinh(f*
x + e)^3 + (a - b)*f*sinh(f*x + e)^4 + 2*(a - b)*f*cosh(f*x + e)^2 + 2*(3*(
a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^2 + (a - b)*f + 4*((a -
 b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.14, size = 35, normalized size = 0.41

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}{\cosh\left(fx+e\right)^{4}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^{2}+a}\,\operatorname{sech}\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}{\cosh\left(e+f\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\ \mathrm{sech}^3\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**3, x)
```

3.356 $\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=151

$$\frac{a(3a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8f(a-b)^{3/2}}+\frac{\tanh(e+fx)\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f(a-b)}+\frac{(3a-4b)\tanh(e+fx)\operatorname{s}}{}$$

```
[Out] 1/8*a*(3*a-4*b)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/(
a-b)^(3/2)/f+1/4*sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)/(a-b)/
f+1/8*(3*a-4*b)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f
```

Rubi [A] time = 0.14, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 382, 378, 377, 203}

$$\frac{a(3a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8f(a-b)^{3/2}}+\frac{\tanh(e+fx)\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f(a-b)}+\frac{(3a-4b)\tanh(e+fx)\operatorname{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (a*(3*a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/(8*(a - b)^(3/2)*f) + ((3*a - 4*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2]*Tanh[e + f*x])/(8*(a - b)*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]
^2)^(3/2)*Tanh[e + f*x])/(4*(a - b)*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1+x^2\right)^3}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{4(a-b)f} + \frac{(3a-4b)\,\mathrm{S}}{}$$

$$= \frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{8(a-b)f} + \mathrm{sech}$$

$$= \frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{8(a-b)f} + \mathrm{sech}$$

$$= \frac{a(3a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8(a-b)^{3/2}f} + \frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+}}{8(a}$$

Mathematica [C] time = 11.68, size = 684, normalized size = 4.53

$$-\frac{\tanh(e+fx)\operatorname{sech}^3(e+fx)\left(\frac{b\sinh^2(e+fx)}{a}+1\right)\left(10b\sinh^2(e+fx)\sqrt{\frac{(a-b)\tanh^2(e+fx)\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a^2}}+1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/40*(Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)*Tanh[e + f*x]*(-15*a*Arc
Sin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]] - 10*b*ArcSin[Sqrt[((a - b)*Tanh[e +
 f*x]^2)/a]]*Sinh[e + f*x]^2 - 30*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f
*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) - 20*b*Sinh[e + f*x]^2*Sqrt[
(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^
(3/2) - 32*a*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)
^(5/2) - 32*b*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sin
h[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Ta
nh[e + f*x]^2)/a)^(5/2) + 32*a*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e
 + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*T
anh[e + f*x]^2)/a)^(7/2) + 32*b*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[
e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 15*a*Sqrt[((a - b)*Sech[e + f*x
]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2] + 10*b*Sinh[e + f*x]^2*Sq
rt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2]))
```

```
/(f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^
2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))
```

fricas [B] time = 0.98, size = 3727, normalized size = 24.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [-1/16*(((3*a^2 - 4*a*b)*cosh(f*x + e)^8 + 8*(3*a^2 - 4*a*b)*cosh(f*x + e)*
sinh(f*x + e)^7 + (3*a^2 - 4*a*b)*sinh(f*x + e)^8 + 4*(3*a^2 - 4*a*b)*cosh(
f*x + e)^6 + 4*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a^2 - 4*a*b)*sinh(f*x
 + e)^6 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x
 + e))*sinh(f*x + e)^5 + 6*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 2*(35*(3*a^2 -
 4*a*b)*cosh(f*x + e)^4 + 30*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 9*a^2 - 12*a
*b)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^5 + 10*(3*a^2 - 4*
a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 4
*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 4*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^6 + 1
5*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 9*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a
^2 - 4*a*b)*sinh(f*x + e)^2 + 3*a^2 - 4*a*b + 8*((3*a^2 - 4*a*b)*cosh(f*x +
 e)^7 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)^5 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)
^3 + (3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2
*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)
*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x
+ e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b
)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e
)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x +
e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e
) + 1)) - 2*sqrt(2)*((3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^6 + 6*(3*a^2 - 5
*a*b + 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b + 2*b^2)*sinh(
f*x + e)^6 + (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^4 + (15*(3*a^2 - 5*a*
b + 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 21*a*b + 10*b^2)*sinh(f*x + e)^4 + 4*
(5*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 21*a*b + 10*b^2)*cos
h(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^2 +
(15*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 21*a*b + 10*b^2)*
cosh(f*x + e)^2 - 11*a^2 + 21*a*b - 10*b^2)*sinh(f*x + e)^2 - 3*a^2 + 5*a*b
 - 2*b^2 + 2*(3*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^5 + 2*(11*a^2 - 21*a*
b + 10*b^2)*cosh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b
 + b^2)*f*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x
+ e)^7 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^8 + 4*(a^2 - 2*a*b + b^2)*f*co
sh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b +
 b^2)*f)*sinh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 8*(7*(
a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e
))*sinh(f*x + e)^5 + 2*(35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 30*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^2 + 3*(a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^4
 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 8*(7*(a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^5 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)^6 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 9*(a^2 - 2*a*b + b^2)*
f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b +
 b^2)*f + 8*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 3*(a^2 - 2*a*b + b^2)*
f*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b
+ b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((3*a^2 - 4*a*b)*cosh(f*x + e)
^8 + 8*(3*a^2 - 4*a*b)*cosh(f*x + e)*sinh(f*x + e)^7 + (3*a^2 - 4*a*b)*sinh
```

```
(f*x + e)^8 + 4*(3*a^2 - 4*a*b)*cosh(f*x + e)^6 + 4*(7*(3*a^2 - 4*a*b)*cosh
(f*x + e)^2 + 3*a^2 - 4*a*b)*sinh(f*x + e)^6 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*
x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(3*a^2 - 4*
a*b)*cosh(f*x + e)^4 + 2*(35*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 30*(3*a^2 -
4*a*b)*cosh(f*x + e)^2 + 9*a^2 - 12*a*b)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 4*
a*b)*cosh(f*x + e)^5 + 10*(3*a^2 - 4*a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*
b)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 4*(
7*(3*a^2 - 4*a*b)*cosh(f*x + e)^6 + 15*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 9*
(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a^2 - 4*a*b)*sinh(f*x + e)^2 + 3*a^2 -
4*a*b + 8*((3*a^2 - 4*a*b)*cosh(f*x + e)^7 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e
)^5 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)^3 + (3*a^2 - 4*a*b)*cosh(f*x + e))*si
nh(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)
^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)
^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x +
 e))*sinh(f*x + e) + b)) + sqrt(2)*((3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^6
 + 6*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b
 + 2*b^2)*sinh(f*x + e)^6 + (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^4 + (1
5*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 21*a*b + 10*b^2)*sinh(
f*x + e)^4 + 4*(5*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 21*a*
b + 10*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cos
h(f*x + e)^2 + (15*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 21
*a*b + 10*b^2)*cosh(f*x + e)^2 - 11*a^2 + 21*a*b - 10*b^2)*sinh(f*x + e)^2
- 3*a^2 + 5*a*b - 2*b^2 + 2*(3*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^5 + 2*
(11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cos
h(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
)/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)*sinh(f*x + e)^7 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^8 + 4*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 +
 (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^4 + 8*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)
*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2 - 2*a*b + b^2)*f*cosh(f*x +
e)^4 + 30*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 3*(a^2 - 2*a*b + b^2)*f)*
sinh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 8*(7*(a^2 - 2*a
*b + b^2)*f*cosh(f*x + e)^5 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e)^6 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 9*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 +
 (a^2 - 2*a*b + b^2)*f + 8*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 3*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3
+ (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.16, size = 35, normalized size = 0.23

$$\frac{`int/indef0`\left(\frac{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}{\cosh\left(fx+e\right)^{6}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
[Out] `int/indef0`(1/cosh(f*x+e)^6*(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,\operatorname{sech}\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\cosh\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^5,x)
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.357 $\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=301

$$-\frac{(2a^2-7ab-3b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f}+\frac{(2a^2-7ab-3b^2)\,\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)(a+b\sinh^2(e\cdot}{a}}}$$

```
[Out] 1/5*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/b/f-2/15*(a-3*b)*cosh
(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+1/15*(2*a^2-7*a*b-3*b^2)*
(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/15*(a-9*b)*(1/(1+sinh(
f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+
e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sech(
f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/15*(2*a^2-7*a*b-3*b^2)*(a+b*sinh(f*
x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Rubi [A] time = 0.30, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3192, 416, 528, 531, 418, 492, 411}

$$-\frac{(2a^2-7ab-3b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f}+\frac{(2a^2-7ab-3b^2)\,\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)(a+b\sinh^2(e\cdot}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-2*(a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*
b*f) + (Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*b*f)
+ ((2*a^2 - 7*a*b - 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e
 + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) - ((a - 9*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/
a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2
*(a + b*Sinh[e + f*x]^2))/a]) - ((2*a^2 - 7*a*b - 3*b^2)*Sqrt[a + b*Sinh[e
+ f*x]^2]*Tanh[e + f*x])/(15*b^2*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \left(1+x^2\right)^{3/2}\sqrt{a+bx^2}\,dx\right.}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5bf} + \frac{\left(\sqrt{\cosh^2(e\right.}{}$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \mathrm{co}$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \mathrm{co}$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \mathrm{co}$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \mathrm{co}$$

Mathematica [C] time = 1.41, size = 211, normalized size = 0.70

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(8a^2+4b(4a+3b)\cosh(2(e+fx))+32ab+3b^2\cosh(4(e+fx))-15b^2\right)-32ia\left(a^2-4\right.}{240b^2f\sqrt{2a+b\cos}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((16*I)*a*(2*a^2 - 7*a*b - 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticE[I*(e + f*x), b/a] - (32*I)*a*(a^2 - 4*a*b + 3*b^2)*Sqrt[(2*a - b +
 b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 + 3
2*a*b - 15*b^2 + 4*b*(4*a + 3*b)*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x)
])*Sinh[2*(e + f*x)])/(240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\cosh\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [A] time = 0.19, size = 521, normalized size = 1.73

$$3\sqrt{-\frac{b}{a}}\,b^2\sinh\left(fx+e\right)\left(\cosh^6\left(fx+e\right)\right)+4\sqrt{-\frac{b}{a}}\,ab\sinh\left(fx+e\right)\left(\cosh^4\left(fx+e\right)\right)+\left(\sqrt{-\frac{b}{a}}\,a^2+2\sqrt{-\frac{b}{a}}\,ab-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^6+4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-3*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+7*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,\cosh\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh\left(e+f\,x\right)^4\,\sqrt{b\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)

[Out] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

3.358 $\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=223

$$\frac{(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf}+\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{2\operatorname{sech}(e+fx)}{}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*(a+b)*(1/(1+sin
h(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*
x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f/(sec
h(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+2/3*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+
sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^
(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x
+e)^2)/a)^(1/2)+1/3*(a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b/f
```

Rubi [A] time = 0.20, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3192, 417, 531, 418, 492, 411}

$$\frac{(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf}+\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{2\operatorname{sech}(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((a + b)*
EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e +
 f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*El
lipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*
Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 417

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*(c + d*x^n)^q)/(n*(p + q) + 1), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \sqrt{1+x^2}\sqrt{a+bx^2}\,dx, x, \text{si}\right.}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cosh^2(e+f\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2a\sqrt{\cosh^2(e+\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{2F\left(\tan^{-1}(\sinh(\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{(a+b)E\left(\tan^{-1}(\right.}{}$$

Mathematica [C] time = 0.82, size = 168, normalized size = 0.75

$$\frac{b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)+2i\sqrt{2}\,a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right)-2i\sqrt{2}\,a(a+b)}{6bf\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE
[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e +
```

```
f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*S
inh[2*(e + f*x)])/(6*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\cosh\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.17, size = 339, normalized size = 1.52

$$\frac{\sqrt{-\frac{b}{a}}\,b\sinh\left(fx+e\right)\left(\cosh^{4}\left(fx+e\right)\right)+\left(\sqrt{-\frac{b}{a}}\,a-\sqrt{-\frac{b}{a}}\,b\right)\left(\cosh^{2}\left(fx+e\right)\right)\sinh\left(fx+e\right)+\sqrt{\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^
(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*b+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(b/a*cosh(f*x+e)
^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2
),(a/b)^(1/2))*b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\cosh\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\cosh\left(e+f\,x\right)^{2}\,\sqrt{b\,\sinh\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.359 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\left|\frac{b}{a}\right.\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(
1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\left|\frac{b}{a}\right.\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\\ &= -\frac{iE\left(ie+ifx\left|\frac{b}{a}\right.\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 69, normalized size = 1.15

$$-\frac{ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,E\left(i(e+fx)\left|\frac{b}{a}\right.\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a]
)/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.00, size = 140, normalized size = 2.33

$$\frac{\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\left(a\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right)-b\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}}, \sqrt{\right.\right.}{\sqrt{-\frac{b}{a}}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int((a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

3.360 $\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] (1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3192, 411}

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
+ f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1+x^2\right)^{3/2}}\,dx,x,\sinh(e+fx\right)}{f}\\ &= \frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}\end{aligned}$$

Mathematica [C] time = 0.49, size = 148, normalized size = 2.11

$$\frac{\sqrt{2}\tanh(e+fx)(2a+b\cosh(2(e+fx))-b)-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)+2ia\sqrt{\frac{2a+b\cosh(2(e+fx))}{a}}}{2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
] - (2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x),
b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*f*Sqrt[2*a
 - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\operatorname{sech}\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.18, size = 177, normalized size = 2.53

$$\frac{\sqrt{-\frac{b}{a}}\,b\left(\sinh^3\left(fx+e\right)\right)+b\sqrt{\frac{a+b\left(\sinh^2\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\operatorname{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\sqrt{\frac{a+}{}}}{\sqrt{-\frac{b}{a}}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sin\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] ((-1/a*b)^(1/2)*b*sinh(f*x+e)^3+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*((a+b*sinh(f
*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2
),(a/b)^(1/2))+(-1/a*b)^(1/2)*a*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+
b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh\left(fx+e\right)^2+a}\,\operatorname{sech}\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}{\cosh\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^2,x)
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^2\left(e+fx\right)}\,\operatorname{sech}^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**2, x)
```

3.361 $\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=206

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))\right.}{3f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] 1/3*(2*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x
+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*b*(1/(
1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+si
nh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-
b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*sech(f*x+e)^2*(a+b*sin
h(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.17, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3192, 412, 525, 418, 411}

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))\right.}{3f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])
/(3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1+x^2\right)^{5/2}}\,dx, x, \sinh(e+fx\right.}{f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3f} - \frac{\left(\sqrt{\cosh^2(e+f\right.}{}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3f} + \frac{\left((2a-b)\sqrt{\cosh\right.}{}$$

$$= \frac{(2a-b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 3.01, size = 204, normalized size = 0.99

$$\frac{\sqrt{2}\tanh(e+fx)\operatorname{sech}^2(e+fx)\left(\left(8a^2-4b^2\right)\cosh(2(e+fx))+(2a-b)(8a+b\cosh(4(e+fx))-5b)\right)-16ia(a-}{24f(a-b)\sqrt{2a+b\cosh(2(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((8*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] - (16*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] + Sqrt[2]*((8*a^2 - 4*b^2)*Cosh[2*(e + f*x)] + (2*a
 - b)*(8*a - 5*b + b*Cosh[4*(e + f*x)]))*Sech[e + f*x]^2*Tanh[e + f*x])/(24
*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\operatorname{sech}\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.25, size = 318, normalized size = 1.54

$$\frac{\left(2\sqrt{-\frac{b}{a}}\, ab-\sqrt{-\frac{b}{a}}\, b^{2}\right)\sinh\left(fx+e\right)\left(\cosh^{4}\left(fx+e\right)\right)+\left(2\sqrt{-\frac{b}{a}}\, a^{2}-2\sqrt{-\frac{b}{a}}\, ab\right)\left(\cosh^{2}\left(fx+e\right)\right)\sinh\left(fx+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+(2
*(-1/a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b)*cosh(f*x+e)^2*sinh(f*x+e)+((-1/a*
b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)+(b/a*cosh
(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(a*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))-2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+b*EllipticE(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2)/cosh(f*x+e)^3/(a-b)/(-1/
a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^{2}+a}\, \operatorname{sech}\left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b \sinh\left(e+f\,x\right)^{2}+a}}{\cosh\left(e+f\,x\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2)/cosh(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b \sinh^{2}\left(e+fx\right)}\, \operatorname{sech}^{4}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**4, x)
```

3.362 $\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=157

$$-\frac{a^2(a-6b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6bf}-\frac{(a-6b)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24bf}$$

```
[Out] -1/16*a^2*(a-6*b)*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/b^
(3/2)/f-1/24*(a-6*b)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/b/f+1/6*sinh(f*x
+e)*(a+b*sinh(f*x+e)^2)^(5/2)/b/f-1/16*a*(a-6*b)*sinh(f*x+e)*(a+b*sinh(f*x+
e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 388, 195, 217, 206}

$$-\frac{a^2(a-6b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6bf}-\frac{(a-6b)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(a^2*(a - 6*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/(16*b^(3/2)*f) - (a*(a - 6*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(16*b*f) - ((a - 6*b)*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*b*f
) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(5/2))/(6*b*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^{3/2}dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6bf} - \frac{(a-6b)\,\text{Subst}\left(\int\left(a+\right.\right.}{}$$

$$= -\frac{(a-6b)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24bf} + \frac{\sinh(e+fx}{}$$

$$= -\frac{a(a-6b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{16bf} - \frac{(a-6b)\sinh}{}$$

$$= -\frac{a(a-6b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{16bf} - \frac{(a-6b)\sinh}{}$$

$$= -\frac{a^2(a-6b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{a(a-6b)\sinh(e+fx)}{16}$$

Mathematica [A] time = 1.25, size = 149, normalized size = 0.95

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(\sqrt{b}\sinh(e+fx)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\left(2b(7a+6b)\sinh^2(e+fx)+3a(a+10b)+8b^2\sin\right.\right.}{48b^{3/2}f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(-3*a^(3/2)*(a - 6*b)*ArcSinh[(Sqrt[b]*Sinh[e
+ f*x])/Sqrt[a]] + Sqrt[b]*Sinh[e + f*x]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]*(3
*a*(a + 10*b) + 2*b*(7*a + 6*b)*Sinh[e + f*x]^2 + 8*b^2*Sinh[e + f*x]^4)))/
(48*b^(3/2)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

fricas [B] time = 1.03, size = 4603, normalized size = 29.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(6*((a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 6*(a^3 - 6*a^2*b)*cosh(f*x +
e)^5*sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 2
0*(a^3 - 6*a^2*b)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 - 6*a^2*b)*cosh
(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)*sinh(f*x + e)
^5 + (a^3 - 6*a^2*b)*sinh(f*x + e)^6)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)
```

*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*((a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 - 6*a^2*b)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 - 6*a^2*b)*sinh(f*x + e)^6)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(2)*(b^3*cosh(f*x + e)^10 + 10*b^3*cosh(f*x + e)*sinh(f*x + e)^9 + b^3*sinh(f*x + e)^10 + (7*a*b^2 + b^3)*cosh(f*x + e)^8 + (45*b^3*cosh(f*x + e)^2 + 7*a*b^2 + b^3)*sinh(f*x + e)^8 + 8*(15*b^3*cosh(f*x + e)^3 + (7*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^6 + (210*b^3*cosh(f*x + e)^4 + 6*a^2*b + 39*a*b^2 - 8*b^3 + 28*(7*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 28*(7*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^4 + (210*b^3*cosh(f*x + e)^6 + 70*(7*a*b^2 + b^3)*cosh(f*x + e)^4 - 6*a^2*b - 39*a*b^2 + 8*b^3 + 15*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(30*b^3*cosh(f*x + e)^7 + 14*(7*a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^3 - (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - b^3 - (7*a*b^2 + b^3)*cosh(f*x + e)^2 + (45*b^3*cosh(f*x + e)^8 + 28*(7*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^4 - 7*a*b^2 - b^3 - 6*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*(7*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^5 - 2*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^3 - (7*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a

```
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/
(b^2*f*cosh(f*x + e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 15*b^2*f*c
osh(f*x + e)^4*sinh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x + e)^3 +
 15*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*sinh(f*x
+ e)^5 + b^2*f*sinh(f*x + e)^6), 1/384*(12*((a^3 - 6*a^2*b)*cosh(f*x + e)^6
 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*cos
h(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 - 6*a^2*b)*cosh(f*x + e)^3*sinh(f*x
+ e)^3 + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 - 6*a^
2*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 - 6*a^2*b)*sinh(f*x + e)^6)*sqrt(
-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(
f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*
x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh
(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)
^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e)
)*sinh(f*x + e))) + 12*((a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 6*(a^3 - 6*a^2*b)
*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^4*sinh(f*
x + e)^2 + 20*(a^3 - 6*a^2*b)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 - 6
*a^2*b)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)*s
inh(f*x + e)^5 + (a^3 - 6*a^2*b)*sinh(f*x + e)^6)*sqrt(-b)*arctan(sqrt(2)*(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt
(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 +
4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*
x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*
x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*(b^3*cosh
(f*x + e)^10 + 10*b^3*cosh(f*x + e)*sinh(f*x + e)^9 + b^3*sinh(f*x + e)^10
+ (7*a*b^2 + b^3)*cosh(f*x + e)^8 + (45*b^3*cosh(f*x + e)^2 + 7*a*b^2 + b^3
)*sinh(f*x + e)^8 + 8*(15*b^3*cosh(f*x + e)^3 + (7*a*b^2 + b^3)*cosh(f*x +
e))*sinh(f*x + e)^7 + (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^6 + (210*b
^3*cosh(f*x + e)^4 + 6*a^2*b + 39*a*b^2 - 8*b^3 + 28*(7*a*b^2 + b^3)*cosh(f
*x + e)^2)*sinh(f*x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 28*(7*a*b^2 + b^3
)*cosh(f*x + e)^3 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e))*sinh(f*x
+ e)^5 - (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^4 + (210*b^3*cosh(f*x +
 e)^6 + 70*(7*a*b^2 + b^3)*cosh(f*x + e)^4 - 6*a^2*b - 39*a*b^2 + 8*b^3 + 1
5*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(30*b^3
*cosh(f*x + e)^7 + 14*(7*a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(6*a^2*b + 39*a*b
^2 - 8*b^3)*cosh(f*x + e)^3 - (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e))*s
inh(f*x + e)^3 - b^3 - (7*a*b^2 + b^3)*cosh(f*x + e)^2 + (45*b^3*cosh(f*x +
 e)^8 + 28*(7*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b + 39*a*b^2 - 8*b^3
)*cosh(f*x + e)^4 - 7*a*b^2 - b^3 - 6*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x
 + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*(7*a*b^2 + b^3)*cos
h(f*x + e)^7 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^5 - 2*(6*a^2*b
+ 39*a*b^2 - 8*b^3)*cosh(f*x + e)^3 - (7*a*b^2 + b^3)*cosh(f*x + e))*sinh(f
*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x
+ e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 15*b^2*f*cosh(f*x + e)^4*s
inh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*b^2*f*cosh(f
*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*f*s
inh(f*x + e)^6)]
```

giac **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.17, size = 77, normalized size = 0.49

$$\frac{\text{`}\,int/indef0\text{`}\left(\frac{b^2\left(\sinh^6\left(fx+e\right)\right)+\left(2ab+b^2\right)\left(\sinh^4\left(fx+e\right)\right)+\left(a^2+2ab\right)\left(\sinh^2\left(fx+e\right)\right)+a^2}{\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^6+(2*a*b+b^2)*sinh(f*x+e)^4+(a^2+2*a*b)*sinh(
f*x+e)^2+a^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(e+f\,x\right)^3\,{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.363 $\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=104

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}$$

```
[Out] 1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+3/8*a^2*arctanh(sinh(f*x+e)*b^(
1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/b^(1/2)+3/8*a*sinh(f*x+e)*(a+b*sinh(f*x+e
)^2)^(1/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*Sqr
t[b]*f) + (3*a*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*f) + (Sinh[e +
 f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f} + \frac{(3a)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,d\right.}{4f}$$

$$= \frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f} + \frac{\sinh(e+fx)\left(a+b\sinh^2\right.}{4f}$$

$$= \frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f} + \frac{\sinh(e+fx)\left(a+b\sinh^2\right.}{4f}$$

$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+}}{8f}$$

Mathematica [A] time = 0.49, size = 93, normalized size = 0.89

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}+5a\sinh(e+fx)+2b\sinh^3(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(5*a*Sinh[e + f*x] + 2*b*Sinh[e + f*x]^3 + (3*
a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*Sinh
[e + f*x]^2)/a])))/(8*f)
```

fricas [B] time = 1.04, size = 3161, normalized size = 30.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(6*(a^2*cosh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2
*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^
2*sinh(f*x + e)^4)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 +
8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2
+ b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^
6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*
x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 +
 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^
2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f
*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b
 - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*
b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(
a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a
^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 -
```

2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*(a^2*cosh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (10*a*b - 3*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*a*b - 3*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (10*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(10*a*b - 3*b^2)*cosh(f*x + e)^2 - 10*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 3*b^2)*cosh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + b*f*sinh(f*x + e)^4), -1/64*(12*(a^2*cosh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + 12*(a^2*cosh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (10*a*b - 3*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*a*b - 3*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (10*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(10*a*b - 3*b^2)*cosh(f*x + e)^2 - 10*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 3*b^2)*c

```
osh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cos
h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
 e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*
x + e)^3*sinh(f*x + e) + 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh
(f*x + e)*sinh(f*x + e)^3 + b*f*sinh(f*x + e)^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.03, size = 90, normalized size = 0.87

$$\frac{\sinh\left(fx+e\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{4f}+\frac{3a\sinh\left(fx+e\right)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}}{8f}+\frac{3a^2\ln\left(\sqrt{b}\ \sinh\left(fx+e\right)+\right.}{8f\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+3/8*a*sinh(f*x+e)*(a+b*sinh(f*x
+e)^2)^(1/2)/f+3/8/f*a^2/b^(1/2)*ln(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^2)
^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e), x)
```

mupad [B] time = 1.02, size = 60, normalized size = 0.58

$$\frac{\sinh\left(e+f\,x\right)\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,\sinh\left(e+f\,x\right)^2}{a}\right)}{f\,\left(\frac{b\,\sinh\left(e+f\,x\right)^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] (sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(
b*sinh(e + f*x)^2)/a))/(f*((b*sinh(e + f*x)^2)/a + 1)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.364 $\int \mathrm{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}$$

```
[Out] (a-b)^(3/2)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f+1/2
*(3*a-2*b)*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))*b^(1/2)/f
+1/2*b*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3190, 416, 523, 217, 206, 377, 203}

$$\frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\,(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Si
nh[e + f*x]^2]])/(2*f) + (b*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
```

```
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1+x^2}dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{\operatorname{Subst}\left(\int \frac{a(2a-b)+(3a-2b)bx^2}{\left(1+x^2\right)\sqrt{a+bx^2}} d\right.}{2f}$$

$$= \frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(a-b)^2\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+}}\right.}{f}$$

$$= \frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(a-b)^2\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\right.}{f}$$

$$= \frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f} + \frac{(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(}{\sqrt{a+b\sinh}}\right.}{2f}$$

Mathematica [A] time = 0.47, size = 142, normalized size = 1.14

$$\frac{b\sinh(e+fx)\sqrt{4a+2b\cosh(2(e+fx))-2b}+4(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)+2\sqrt{b}(3a-2b)\tanh^{-1}\left(\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*(a - b)^(3/2)*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Co
sh[2*(e + f*x)]]] + 2*(3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e +
 f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + b*Sqrt[4*a - 2*b + 2*b*Cosh[2
*(e + f*x)]]*Sinh[e + f*x])/(4*f)
```

fricas [B] time = 1.19, size = 6337, normalized size = 50.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x
+ e) + (3*a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*c
osh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 +
(a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^
3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b
^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*
cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f
*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a
*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*
b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*
b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x
+ e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3
+ 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2
 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2
- sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh
(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 -
2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a
^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^
3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*
cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b
 + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2
- 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4
*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 +
3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 +
 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(
cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh
(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(
f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 4*((a -
b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f
*x + e)^2)*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f
*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*
x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*
sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
- 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a
- 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b
)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*
(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x +
 e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + ((3*a - 2*b)*cosh(f*x + e)^2 +
 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*sinh(f*x + e)^2)*s
qrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(
f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e
)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*
cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(2)*(b*cosh(f*x + e)^
2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*
sinh(f*x + e) + f*sinh(f*x + e)^2), 1/8*(8*((a - b)*cosh(f*x + e)^2 + 2*(a
- b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2)*sqrt(a - b)*arc
tan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*co
```

sh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(
2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^
2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - (
(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (
3*a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x
+ e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b -
2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(
f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3
)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x
+ e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^
5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b
^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*
b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^
3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 +
(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a
*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)
^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3
+ 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 - sqrt(2
)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e
)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b +
b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a
*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a
^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x
+ e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*
cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3
*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sin
h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*
cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x
+ e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e
)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)
^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) - ((3*a - 2*b)*cos
h(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*sinh
(f*x + e)^2)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a
)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2)*(b*
cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*
cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x + e)^2), -1/8*(2*((3*a - 2*b)*cosh
(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*sinh(
f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*co
sh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a
*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*
a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b
^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b
^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*
a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*sinh(f*x + e)^2)*sqrt(-b
)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*

```
cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2
*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e
)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) +
 4*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a -
b)*sinh(f*x + e)^2)*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*
b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b
)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x +
e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
 + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e)
+ a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x +
e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(c
osh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - sqrt(2)*(b*cosh(f*x +
 e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*co
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x +
 e)*sinh(f*x + e) + f*sinh(f*x + e)^2), 1/8*(8*((a - b)*cosh(f*x + e)^2 + 2
*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2)*sqrt(a - b)
*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(
b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x +
 e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))
 - 2*((3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x +
e) + (3*a - 2*b)*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x
 + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 +
 b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*
cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)
*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*
x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x
 + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) - 2*((3*a - 2*b)*c
osh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*si
nh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
 b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))
*sinh(f*x + e) + b)) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(
f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x
+ e)^2)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.13, size = 63, normalized size = 0.50

$$\frac{\text{`}int/indef0\text{`}\left(\frac{b^2\left(\sinh^4\left(fx+e\right)\right)+2ab\left(\sinh^2\left(fx+e\right)\right)+a^2}{\cosh\left(fx+e\right)^2\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^2/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.365 $\int \mathrm{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=133

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{a-b}\,(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}+\frac{(a-b)\tanh(e+fx)\mathrm{sech}(e+fx)\sqrt{a-}}{2f}$$

```
[Out] b^(3/2)*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f+1/2*(a+2*b
)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))*(a-b)^(1/2)/f+1
/2*(a-b)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.15, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3190, 413, 523, 217, 206, 377, 203}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{a-b}\,(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}+\frac{(a-b)\tanh(e+fx)\mathrm{sech}(e+fx)\sqrt{a-}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e
 + f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Si
nh[e + f*x]^2]])/f + ((a - b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tan
h[e + f*x])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2f} + \frac{\operatorname{Subst}\left(\right.}{}$$

$$= \frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2f} + \frac{b^2\operatorname{Subs}}{}$$

$$= \frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2f} + \frac{b^2\operatorname{Subs}}{}$$

$$= \frac{\sqrt{a-b}\,(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+}{\sqrt{a+b\sinh^2(}}\right.}{f}$$

Mathematica [A] time = 0.79, size = 150, normalized size = 1.13

$$\frac{4b^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)+2\sqrt{a-b}\,(a+2b)\tan^{-1}\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)+(a-b)\tanh(e+fx)\operatorname{sech}(e+}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a -
b + b*Cosh[2*(e + f*x)]]] + 4*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f*x
])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + (a - b)*Sqrt[4*a - 2*b + 2*b*Cosh
[2*(e + f*x)]]*Sech[e + f*x]*Tanh[e + f*x])/(4*f)
```

fricas [B] time = 1.11, size = 7350, normalized size = 55.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/4*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a + 2*b)*cosh(f*x + e)^4 + 4*(a + 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + 2*b)*sinh(f*x + e)^4 + 2*(a + 2*b)*cosh(f*x + e)^2 + 2*(3*(a + 2*b)*cosh(f*x + e)^2 + a + 2*b)*sinh(f*x + e)^2 + 4*((a + 2*b)*cosh(f*x + e)^3 + (a + 2*b)*cosh(f*x + e))*sinh(f*x + e) + a + 2*b)*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x +

```
e)^4 + 4*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 + 2*f*cosh(f*
x + e)^2 + 2*(3*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)
^3 + f*cosh(f*x + e))*sinh(f*x + e) + f), 1/4*(2*((a + 2*b)*cosh(f*x + e)^4
 + 4*(a + 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + 2*b)*sinh(f*x + e)^4 +
2*(a + 2*b)*cosh(f*x + e)^2 + 2*(3*(a + 2*b)*cosh(f*x + e)^2 + a + 2*b)*sin
h(f*x + e)^2 + 4*((a + 2*b)*cosh(f*x + e)^3 + (a + 2*b)*cosh(f*x + e))*sinh
(f*x + e) + a + 2*b)*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(
f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(
f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*co
sh(f*x + e))*sinh(f*x + e) + b)) + (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x
+ e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(
f*x + e) + b)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^
2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3
)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2
*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e
)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a
^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b -
 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
 e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*c
osh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14
*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*
cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 -
 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b
- 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*
b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e
)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x
+ e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh
(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b
^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*
(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^
2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*
x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x
 + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a
*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^
3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*co
sh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(
f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*
x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + (b*cosh(f*x + e)^4 + 4*b*cosh(
f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b
*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x +
 e))*sinh(f*x + e) + b)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*
sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x
 + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sin
h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) +
 b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) +
2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e)
+ (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e
)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f
*x + e)^4 + 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)
```

```
^2 + 4*(f*cosh(f*x + e)^3 + f*cosh(f*x + e))*sinh(f*x + e) + f), -1/4*(2*(b
*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*co
sh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)
*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)
*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x +
e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(
a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a
*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e)))
+ 2*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e
)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4
*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(s
qrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 +
 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x +
 e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)
*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b
*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - ((a + 2*b
)*cosh(f*x + e)^4 + 4*(a + 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + 2*b)*s
inh(f*x + e)^4 + 2*(a + 2*b)*cosh(f*x + e)^2 + 2*(3*(a + 2*b)*cosh(f*x + e)
^2 + a + 2*b)*sinh(f*x + e)^2 + 4*((a + 2*b)*cosh(f*x + e)^3 + (a + 2*b)*co
sh(f*x + e))*sinh(f*x + e) + a + 2*b)*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x
+ e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e
)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a
+ 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x +
e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x +
e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh
(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*s
qrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (
a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f*x
+ e)^4 + 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^2
+ 4*(f*cosh(f*x + e)^3 + f*cosh(f*x + e))*sinh(f*x + e) + f), 1/2*(((a + 2*
b)*cosh(f*x + e)^4 + 4*(a + 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + 2*b)*
sinh(f*x + e)^4 + 2*(a + 2*b)*cosh(f*x + e)^2 + 2*(3*(a + 2*b)*cosh(f*x + e
)^2 + a + 2*b)*sinh(f*x + e)^2 + 4*((a + 2*b)*cosh(f*x + e)^3 + (a + 2*b)*c
osh(f*x + e))*sinh(f*x + e) + a + 2*b)*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x
 + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - (b*cosh(f*x + e)^4 + 4
*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2
+ 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*co
sh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x +
 e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b
)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*co
sh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*s
inh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x
+ e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x +
 e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) - (b*cosh(f*x + e)^4
```

```
 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 +
b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^
2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
 e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(
3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*
a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((a - b)*cosh(f*x + e)^
2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^4 + 4*
f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 + 2*f*cosh(f*x + e)^2 +
 2*(3*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 + f*cos
h(f*x + e))*sinh(f*x + e) + f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.16, size = 63, normalized size = 0.47

$$\frac{\text{`}int/indef0\text{`}\left(\frac{b^2\left(\sinh^4\left(fx+e\right)\right)+2ab\left(\sinh^2\left(fx+e\right)\right)+a^2}{\cosh\left(fx+e\right)^4\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^4/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^3, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.366 $\int \operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8f\sqrt{a-b}}+\frac{\tanh(e+fx)\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}+\frac{3a\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{}}{8f}$$

```
[Out] 3/8*a^2*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/(a-b)^(
1/2)+1/4*sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)/f+3/8*a*sech(f
*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.12, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3190, 378, 377, 203}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8f\sqrt{a-b}}+\frac{\tanh(e+fx)\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}+\frac{3a\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*
Sqrt[a - b]*f) + (3*a*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*
x])/(8*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(
4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^3}dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{4f} + \frac{(3a)\,\text{Su}}{}$$

$$= \frac{3a\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{8f} + \frac{\operatorname{sech}^3(e}{}$$

$$= \frac{3a\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{8f} + \frac{\operatorname{sech}^3(e}{}$$

$$= \frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{a-b}\,f} + \frac{3a\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e-}}{8f}$$

Mathematica [C] time = 0.13, size = 66, normalized size = 0.52

$$\frac{a^2\sinh(e+fx)\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{(a-b)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Hypergeometric2F1[1/2, 3, 3/2, -(((a - b)*Sinh[e + f*x]^2)/(a + b*Sinh
[e + f*x]^2))]*Sinh[e + f*x])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

fricas [B] time = 1.02, size = 3089, normalized size = 24.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(a^2*cosh(f*x + e)^8 + 8*a^2*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*
sinh(f*x + e)^8 + 4*a^2*cosh(f*x + e)^6 + 4*(7*a^2*cosh(f*x + e)^2 + a^2)*s
inh(f*x + e)^6 + 6*a^2*cosh(f*x + e)^4 + 8*(7*a^2*cosh(f*x + e)^3 + 3*a^2*c
osh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a^2*cosh(f*x + e)^4 + 30*a^2*cosh(f*x
 + e)^2 + 3*a^2)*sinh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^2 + 8*(7*a^2*cosh(f*
x + e)^5 + 10*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 +
4*(7*a^2*cosh(f*x + e)^6 + 15*a^2*cosh(f*x + e)^4 + 9*a^2*cosh(f*x + e)^2 +
 a^2)*sinh(f*x + e)^2 + a^2 + 8*(a^2*cosh(f*x + e)^7 + 3*a^2*cosh(f*x + e)^
5 + 3*a^2*cosh(f*x + e)^3 + a^2*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*
log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3
+ (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b
)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2
 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((
b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 -
 (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*c
```

```
osh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*
sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*s
inh(f*x + e) + 1)) - 2*sqrt(2)*((3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^6 + 6*(
3*a^2 - a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - a*b - 2*b^2)*
sinh(f*x + e)^6 + (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^4 + (15*(3*a^2 -
a*b - 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 17*a*b + 6*b^2)*sinh(f*x + e)^4 + 4
*(5*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 17*a*b + 6*b^2)*cosh(
f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^2 + (15
*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 17*a*b + 6*b^2)*cosh(f
*x + e)^2 - 11*a^2 + 17*a*b - 6*b^2)*sinh(f*x + e)^2 - 3*a^2 + a*b + 2*b^2
 + 2*(3*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^5 + 2*(11*a^2 - 17*a*b + 6*b^2)*
cosh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e))*sinh(f*x + e))*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)^8
 + 8*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a - b)*f*sinh(f*x + e)^8 +
4*(a - b)*f*cosh(f*x + e)^6 + 4*(7*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*s
inh(f*x + e)^6 + 6*(a - b)*f*cosh(f*x + e)^4 + 8*(7*(a - b)*f*cosh(f*x + e)
^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a - b)*f*cosh(f*x
+ e)^4 + 30*(a - b)*f*cosh(f*x + e)^2 + 3*(a - b)*f)*sinh(f*x + e)^4 + 4*(a
 - b)*f*cosh(f*x + e)^2 + 8*(7*(a - b)*f*cosh(f*x + e)^5 + 10*(a - b)*f*cos
h(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a - b)*f*
cosh(f*x + e)^6 + 15*(a - b)*f*cosh(f*x + e)^4 + 9*(a - b)*f*cosh(f*x + e)^
2 + (a - b)*f)*sinh(f*x + e)^2 + (a - b)*f + 8*((a - b)*f*cosh(f*x + e)^7 +
 3*(a - b)*f*cosh(f*x + e)^5 + 3*(a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh
(f*x + e))*sinh(f*x + e)), 1/8*(3*(a^2*cosh(f*x + e)^8 + 8*a^2*cosh(f*x + e
)*sinh(f*x + e)^7 + a^2*sinh(f*x + e)^8 + 4*a^2*cosh(f*x + e)^6 + 4*(7*a^2*
cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^6 + 6*a^2*cosh(f*x + e)^4 + 8*(7*a^2*c
osh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a^2*cosh(f*x
+ e)^4 + 30*a^2*cosh(f*x + e)^2 + 3*a^2)*sinh(f*x + e)^4 + 4*a^2*cosh(f*x +
 e)^2 + 8*(7*a^2*cosh(f*x + e)^5 + 10*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x
+ e))*sinh(f*x + e)^3 + 4*(7*a^2*cosh(f*x + e)^6 + 15*a^2*cosh(f*x + e)^4 +
 9*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^2 + a^2 + 8*(a^2*cosh(f*x + e)^
7 + 3*a^2*cosh(f*x + e)^5 + 3*a^2*cosh(f*x + e)^3 + a^2*cosh(f*x + e))*sinh
(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2
 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e
))*sinh(f*x + e) + b)) + sqrt(2)*((3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^6 + 6
*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - a*b - 2*b^2
)*sinh(f*x + e)^6 + (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^4 + (15*(3*a^2
- a*b - 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 17*a*b + 6*b^2)*sinh(f*x + e)^4 +
 4*(5*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 17*a*b + 6*b^2)*cos
h(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^2 + (
15*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 17*a*b + 6*b^2)*cosh
(f*x + e)^2 - 11*a^2 + 17*a*b - 6*b^2)*sinh(f*x + e)^2 - 3*a^2 + a*b + 2*b^
2 + 2*(3*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^5 + 2*(11*a^2 - 17*a*b + 6*b^2
)*cosh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e))*sinh(f*x + e))
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)
^8 + 8*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a - b)*f*sinh(f*x + e)^8
+ 4*(a - b)*f*cosh(f*x + e)^6 + 4*(7*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)
*sinh(f*x + e)^6 + 6*(a - b)*f*cosh(f*x + e)^4 + 8*(7*(a - b)*f*cosh(f*x +
e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a - b)*f*cosh(f*
x + e)^4 + 30*(a - b)*f*cosh(f*x + e)^2 + 3*(a - b)*f)*sinh(f*x + e)^4 + 4*
(a - b)*f*cosh(f*x + e)^2 + 8*(7*(a - b)*f*cosh(f*x + e)^5 + 10*(a - b)*f*c
osh(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a - b)*
f*cosh(f*x + e)^6 + 15*(a - b)*f*cosh(f*x + e)^4 + 9*(a - b)*f*cosh(f*x + e
```

```
)^2 + (a - b)*f)*sinh(f*x + e)^2 + (a - b)*f + 8*((a - b)*f*cosh(f*x + e)^7
 + 3*(a - b)*f*cosh(f*x + e)^5 + 3*(a - b)*f*cosh(f*x + e)^3 + (a - b)*f*co
sh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.18, size = 63, normalized size = 0.50

$$\frac{\text{`}int/indef0\text{`}\left(\frac{b^{2}\left(\sinh^{4}\left(fx+e\right)\right)+2ab\left(\sinh^{2}\left(fx+e\right)\right)+a^{2}}{\cosh\left(fx+e\right)^{6}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^6/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^5,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.367 $\int \operatorname{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=205

$$\frac{a^2(5a-6b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16f(a-b)^{3/2}}+\frac{\tanh(e+fx)\operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6f(a-b)}+\frac{(5a-6b)\tanh(e+fx)}{}$$

```
[Out] 1/16*a^2*(5*a-6*b)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)
)/(a-b)^(3/2)/f+1/24*(5*a-6*b)*sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2)*tanh
(f*x+e)/(a-b)/f+1/6*sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(5/2)*tanh(f*x+e)/(a-
b)/f+1/16*a*(5*a-6*b)*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-
b)/f
```

Rubi [A] time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3190, 382, 378, 377, 203}

$$\frac{a^2(5a-6b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16f(a-b)^{3/2}}+\frac{\tanh(e+fx)\operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6f(a-b)}+\frac{(5a-6b)\tanh(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*(5*a - 6*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x
]^2]])/(16*(a - b)^(3/2)*f) + (a*(5*a - 6*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2]*Tanh[e + f*x])/(16*(a - b)*f) + ((5*a - 6*b)*Sech[e + f*x]^3*(a
 + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(24*(a - b)*f) + (Sech[e + f*x]^
5*(a + b*Sinh[e + f*x]^2)^(5/2)*Tanh[e + f*x])/(6*(a - b)*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
```

```
[q, -1]) && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^4}dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}\tanh(e+fx)}{6(a-b)f} + \frac{(5a-6\ldots}{}$$

$$= \frac{(5a-6b)\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{24(a-b)f} \ldots$$

$$= \frac{a(5a-6b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{16(a-b)f} + \ldots$$

$$= \frac{a(5a-6b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{16(a-b)f} + \ldots$$

$$= \frac{a^2(5a-6b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16(a-b)^{3/2}f} + \frac{a(5a-6b)\operatorname{sech}(e+fx)\ldots}{}$$

Mathematica [C] time = 15.16, size = 959, normalized size = 4.68

$$a^2\operatorname{sech}^3(e+fx)\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^2\tanh(e+fx)\left(256b\,{}_2F_1\left(2,5;\frac{7}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sinh^2(e+fx)\sqrt{\frac{\operatorname{sech}^2(e+fx)(\ldots}{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)^2*Tanh[e + f*x]*(45*a*ArcS
in[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]] + 30*b*ArcSin[Sqrt[((a - b)*Tanh[e +
f*x]^2)/a]]*Sinh[e + f*x]^2 + 210*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f
*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) + 140*b*Sinh[e + f*x]^2*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)
^(3/2) - 120*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*
Tanh[e + f*x]^2)/a)^(5/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tan
h[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b
)*Tanh[e + f*x]^2)/a)^(5/2) - 80*b*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a
 + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) + 256*b*Hyper
```

```
geometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(
5/2) - 512*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)
^(7/2) - 512*b*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Si
nh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*T
anh[e + f*x]^2)/a)^(7/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh
[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)
*Tanh[e + f*x]^2)/a)^(9/2) + 256*b*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Ta
nh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(9/2) - 45*a*Sqrt[((a - b)*Sech[e +
f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2] - 30*b*Sinh[e + f*x]^2
*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2
]))/(240*f*(a + b*Sinh[e + f*x]^2)^(3/2)*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[
e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))
```

fricas [B] time = 2.71, size = 7633, normalized size = 37.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*((5*a^3 - 6*a^2*b)*cosh(f*x + e)^12 + 12*(5*a^3 - 6*a^2*b)*cosh(f
*x + e)*sinh(f*x + e)^11 + (5*a^3 - 6*a^2*b)*sinh(f*x + e)^12 + 6*(5*a^3 -
6*a^2*b)*cosh(f*x + e)^10 + 6*(5*a^3 - 6*a^2*b + 11*(5*a^3 - 6*a^2*b)*cosh(
f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 3
*(5*a^3 - 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^9 + 15*(5*a^3 - 6*a^2*b)*co
sh(f*x + e)^8 + 15*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b
+ 18*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 24*(33*(5*a^3 - 6
*a^2*b)*cosh(f*x + e)^5 + 30*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 5*(5*a^3 -
 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^7 + 20*(5*a^3 - 6*a^2*b)*cosh(f*x +
e)^6 + 4*(231*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 315*(5*a^3 - 6*a^2*b)*cos
h(f*x + e)^4 + 25*a^3 - 30*a^2*b + 105*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^2)*s
inh(f*x + e)^6 + 24*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 63*(5*a^3 - 6*a
^2*b)*cosh(f*x + e)^5 + 35*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 5*(5*a^3 - 6
*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 6*a^2*b)*cosh(f*x + e)
^4 + 15*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^8 + 84*(5*a^3 - 6*a^2*b)*cosh(f
*x + e)^6 + 70*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 20*(5*
a^3 - 6*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 20*(11*(5*a^3 - 6*a^2*b)*
cosh(f*x + e)^9 + 36*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 42*(5*a^3 - 6*a^2*
b)*cosh(f*x + e)^5 + 20*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 3*(5*a^3 - 6*a^
2*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 5*a^3 - 6*a^2*b + 6*(5*a^3 - 6*a^2*b)
*cosh(f*x + e)^2 + 6*(11*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^10 + 45*(5*a^3 - 6
*a^2*b)*cosh(f*x + e)^8 + 70*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 50*(5*a^3
- 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 15*(5*a^3 - 6*a^2*b)*cosh(f*
x + e)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 6*a^2*b)*cosh(f*x + e)^11 + 5*(5*a
^3 - 6*a^2*b)*cosh(f*x + e)^9 + 10*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 10*(
5*a^3 - 6*a^2*b)*cosh(f*x + e)^5 + 5*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + (5
*a^3 - 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2*b)*c
osh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh
(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^
2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cos
h(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sin
h(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2
+ 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1
)) - 2*sqrt(2)*((15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^10 + 10
```

*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)*sinh(f*x + e)^9 + (15*
a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*sinh(f*x + e)^10 + (85*a^3 - 133*a^2*b +
20*a*b^2 + 28*b^3)*cosh(f*x + e)^8 + (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^
3 + 45*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)
^8 + 8*(15*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^3 + (85*a^3
- 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(99*a^3
 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^6 + 2*(105*(15*a^3 - 23*a^
2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^4 + 99*a^3 - 247*a^2*b + 200*a*b^2 - 5
2*b^3 + 14*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^2)*sinh(f
x + e)^6 + 4(63*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^5 + 1
4*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^3 + 3*(99*a^3 - 24
7*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(99*a^3 -
247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^4 + 2*(105*(15*a^3 - 23*a^2*b
 + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^6 + 35*(85*a^3 - 133*a^2*b + 20*a*b^2 + 2
8*b^3)*cosh(f*x + e)^4 - 99*a^3 + 247*a^2*b - 200*a*b^2 + 52*b^3 + 15*(99*a
^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(
15*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^7 + 7*(85*a^3 - 133*
a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^5 + 5*(99*a^3 - 247*a^2*b + 200*a*
b^2 - 52*b^3)*cosh(f*x + e)^3 - (99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*c
osh(f*x + e))*sinh(f*x + e)^3 - 15*a^3 + 23*a^2*b - 4*a*b^2 - 4*b^3 - (85*a
^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^2 + (45*(15*a^3 - 23*a^2*
b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^8 + 28*(85*a^3 - 133*a^2*b + 20*a*b^2 +
28*b^3)*cosh(f*x + e)^6 + 30*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh
(f*x + e)^4 - 85*a^3 + 133*a^2*b - 20*a*b^2 - 28*b^3 - 12*(99*a^3 - 247*a^2
*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(15*a^3 -
23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^9 + 4*(85*a^3 - 133*a^2*b + 20*a*
b^2 + 28*b^3)*cosh(f*x + e)^7 + 6*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)
*cosh(f*x + e)^5 - 4*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e
)^3 - (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e))*sinh(f*x + e)
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*
cosh(f*x + e)^12 + 12*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^11
+ (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^12 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)^10 + 6*(11*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2
)*f)*sinh(f*x + e)^10 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 20*(11*(
a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e
))*sinh(f*x + e)^9 + 15*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 18*(a^2
 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^8
+ 20*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 24*(33*(a^2 - 2*a*b + b^2)*f*c
osh(f*x + e)^5 + 30*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 5*(a^2 - 2*a*b
+ b^2)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231*(a^2 - 2*a*b + b^2)*f*cosh
(f*x + e)^6 + 315*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 105*(a^2 - 2*a*b
+ b^2)*f*cosh(f*x + e)^2 + 5*(a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^6 + 15*(a
^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 24*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)^7 + 63*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 35*(a^2 - 2*a*b + b^2)
*f*cosh(f*x + e)^3 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5
 + 15*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 84*(a^2 - 2*a*b + b^2)*f*
cosh(f*x + e)^6 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 20*(a^2 - 2*a*
b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^4 + 6*(a^
2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 20*(11*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^9 + 36*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 42*(a^2 - 2*a*b + b^2)*
f*cosh(f*x + e)^5 + 20*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a
*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 6*(11*(a^2 - 2*a*b + b^2)*f*co
sh(f*x + e)^10 + 45*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 70*(a^2 - 2*a*b
 + b^2)*f*cosh(f*x + e)^6 + 50*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*(
a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)
^2 + (a^2 - 2*a*b + b^2)*f + 12*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^11 + 5
*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^9 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^7 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b^2)*f

```
*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/4
8*(3*((5*a^3 - 6*a^2*b)*cosh(f*x + e)^12 + 12*(5*a^3 - 6*a^2*b)*cosh(f*x +
e)*sinh(f*x + e)^11 + (5*a^3 - 6*a^2*b)*sinh(f*x + e)^12 + 6*(5*a^3 - 6*a^2
*b)*cosh(f*x + e)^10 + 6*(5*a^3 - 6*a^2*b + 11*(5*a^3 - 6*a^2*b)*cosh(f*x +
 e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 3*(5*a
^3 - 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^9 + 15*(5*a^3 - 6*a^2*b)*cosh(f*
x + e)^8 + 15*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 18*
(5*a^3 - 6*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 24*(33*(5*a^3 - 6*a^2*
b)*cosh(f*x + e)^5 + 30*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 5*(5*a^3 - 6*a^
2*b)*cosh(f*x + e))*sinh(f*x + e)^7 + 20*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6
+ 4*(231*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 315*(5*a^3 - 6*a^2*b)*cosh(f*x
 + e)^4 + 25*a^3 - 30*a^2*b + 105*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^2)*sinh(f
*x + e)^6 + 24*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 63*(5*a^3 - 6*a^2*b)
*cosh(f*x + e)^5 + 35*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 5*(5*a^3 - 6*a^2*
b)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 +
15*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^8 + 84*(5*a^3 - 6*a^2*b)*cosh(f*x +
e)^6 + 70*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 20*(5*a^3 -
 6*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 20*(11*(5*a^3 - 6*a^2*b)*cosh(
f*x + e)^9 + 36*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 42*(5*a^3 - 6*a^2*b)*co
sh(f*x + e)^5 + 20*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 3*(5*a^3 - 6*a^2*b)*
cosh(f*x + e))*sinh(f*x + e)^3 + 5*a^3 - 6*a^2*b + 6*(5*a^3 - 6*a^2*b)*cosh
(f*x + e)^2 + 6*(11*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^10 + 45*(5*a^3 - 6*a^2*
b)*cosh(f*x + e)^8 + 70*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 50*(5*a^3 - 6*a
^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 15*(5*a^3 - 6*a^2*b)*cosh(f*x + e
)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 6*a^2*b)*cosh(f*x + e)^11 + 5*(5*a^3 -
6*a^2*b)*cosh(f*x + e)^9 + 10*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 10*(5*a^3
 - 6*a^2*b)*cosh(f*x + e)^5 + 5*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + (5*a^3
- 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b
*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x +
 e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
 e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((15*a^3 - 2
3*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^10 + 10*(15*a^3 - 23*a^2*b + 4*a*b
^2 + 4*b^3)*cosh(f*x + e)*sinh(f*x + e)^9 + (15*a^3 - 23*a^2*b + 4*a*b^2 +
4*b^3)*sinh(f*x + e)^10 + (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x
 + e)^8 + (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3 + 45*(15*a^3 - 23*a^2*b +
 4*a*b^2 + 4*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(15*a^3 - 23*a^2
*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^3 + (85*a^3 - 133*a^2*b + 20*a*b^2 + 28
*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(99*a^3 - 247*a^2*b + 200*a*b^2 -
52*b^3)*cosh(f*x + e)^6 + 2*(105*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh
(f*x + e)^4 + 99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3 + 14*(85*a^3 - 133*a^
2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(15*a^3 -
 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^5 + 14*(85*a^3 - 133*a^2*b + 20*
a*b^2 + 28*b^3)*cosh(f*x + e)^3 + 3*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^
3)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*
b^3)*cosh(f*x + e)^4 + 2*(105*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*
x + e)^6 + 35*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^4 - 99
*a^3 + 247*a^2*b - 200*a*b^2 + 52*b^3 + 15*(99*a^3 - 247*a^2*b + 200*a*b^2
- 52*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(15*(15*a^3 - 23*a^2*b + 4*a
*b^2 + 4*b^3)*cosh(f*x + e)^7 + 7*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*
cosh(f*x + e)^5 + 5*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)
^3 - (99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e))*sinh(f*x + e)
^3 - 15*a^3 + 23*a^2*b - 4*a*b^2 - 4*b^3 - (85*a^3 - 133*a^2*b + 20*a*b^2 +
 28*b^3)*cosh(f*x + e)^2 + (45*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f
*x + e)^8 + 28*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^6 + 3
0*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^4 - 85*a^3 + 133*
a^2*b - 20*a*b^2 - 28*b^3 - 12*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*co
```

```
sh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)
*cosh(f*x + e)^9 + 4*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)
^7 + 6*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^5 - 4*(99*a^
3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^3 - (85*a^3 - 133*a^2*b +
 20*a*b^2 + 28*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^12 + 12*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^11 + (a^2 - 2*a*b + b^2)*f*sin
h(f*x + e)^12 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^10 + 6*(11*(a^2 - 2*a
*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^10 + 15*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 20*(11*(a^2 - 2*a*b + b^2)*f*cosh(f
*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^9 + 15*(33
*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 18*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^8 + 20*(a^2 - 2*a*b + b^2)*f*
cosh(f*x + e)^6 + 24*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 30*(a^2 -
2*a*b + b^2)*f*cosh(f*x + e)^3 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sin
h(f*x + e)^7 + 4*(231*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 315*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^4 + 105*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 +
5*(a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^6 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*
x + e)^4 + 24*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 63*(a^2 - 2*a*b +
 b^2)*f*cosh(f*x + e)^5 + 35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 5*(a^2
 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(33*(a^2 - 2*a*b + b^
2)*f*cosh(f*x + e)^8 + 84*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 70*(a^2 -
 2*a*b + b^2)*f*cosh(f*x + e)^4 + 20*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2
+ (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^4 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)^2 + 20*(11*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^9 + 36*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e)^7 + 42*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 20*(a^2
 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*
sinh(f*x + e)^3 + 6*(11*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^10 + 45*(a^2 -
2*a*b + b^2)*f*cosh(f*x + e)^8 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 +
 50*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f
*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f
+ 12*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^11 + 5*(a^2 - 2*a*b + b^2)*f*cosh
(f*x + e)^9 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 10*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2
*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.18, size = 63, normalized size = 0.31

$$\frac{\text{`}int/indef0\text{`}\left(\frac{b^{2}\left(\sinh^{4}\left(fx+e\right)\right)+2ab\left(\sinh^{2}\left(fx+e\right)\right)+a^{2}}{\cosh\left(fx+e\right)^{8}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^8/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^7,x)
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**7*(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.368 $\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=357

$$-\frac{2(a+b)\left(a^2-6ab+b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35b^2f}+\frac{\left(a^2+9ab-2b^2\right)\sinh(e+fx)\cosh(e+fx)\sqrt{a}}{35bf}$$

```
[Out] 1/35*(a^2+9*a*b-2*b^2)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/
f+2/35*(4*a-b)*cosh(f*x+e)^3*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/7*b*
cosh(f*x+e)^5*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+2/35*(a+b)*(a^2-6*a*b
+b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*
x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)
^(1/2)/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/35*(a^2-18*a*b+b
^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+
e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(
1/2)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/35*(a+b)*(a^2-6*a*b+
b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b^2/f
```

Rubi [A] time = 0.39, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3192, 416, 528, 531, 418, 492, 411}

$$-\frac{2(a+b)\left(a^2-6ab+b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35b^2f}+\frac{\left(a^2+9ab-2b^2\right)\sinh(e+fx)\cosh(e+fx)\sqrt{a}}{35bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a^2 + 9*a*b - 2*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]
^2])/(35*b*f) + (2*(4*a - b)*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]^5*Sinh[e + f*x]*Sqrt[a + b*Sinh[e +
f*x]^2])/(7*f) + (2*(a + b)*(a^2 - 6*a*b + b^2)*EllipticE[ArcTan[Sinh[e + f
*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 18*a*b + b^2)*Ellipti
cF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a + b)*
(a^2 - 6*a*b + b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(35*b^2*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int\left(1+x^2\right)^{3/2}\left(a+bx^2\right)\right.}{f}$$

$$= \frac{b\cosh^5(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{7f} + \frac{\left(\sqrt{\cosh\right.}{}$$

$$= \frac{2(4a-b)\cosh^3(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35f} +$$

$$= \frac{\left(a^2+9ab-2b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+f}}{35bf}$$

$$= \frac{\left(a^2+9ab-2b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+f}}{35bf}$$

$$= \frac{\left(a^2+9ab-2b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+f}}{35bf}$$

$$= \frac{\left(a^2+9ab-2b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+f}}{35bf}$$

Mathematica [C] time = 2.61, size = 256, normalized size = 0.72

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(32a^3+b\left(144a^2+192ab-37b^2\right)\cosh(2(e+fx))+400a^2b+2b^2(26a+b)\cosh(4(e+f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((128*I)*a*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*
x)])/a]*EllipticE[I*(e + f*x), b/a] - (64*I)*a*(2*a^3 - 11*a^2*b + 8*a*b^2
+ b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a]
+ Sqrt[2]*b*(32*a^3 + 400*a^2*b - 212*a*b^2 + 30*b^3 + b*(144*a^2 + 192*a*b
 - 37*b^2)*Cosh[2*(e + f*x)] + 2*b^2*(26*a + b)*Cosh[4*(e + f*x)] + 5*b^3*C
osh[6*(e + f*x)])*Sinh[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\cosh\left(fx+e\right)^4\sinh\left(fx+e\right)^2+a\cosh\left(fx+e\right)^4\right)\sqrt{b\sinh\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(f*x + e)^4*sinh(f*x + e)^2 + a*cosh(f*x + e)^4)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.20, size = 730, normalized size = 2.04

$$\frac{5\sqrt{-\frac{b}{a}}\, b^3 \sinh\left(fx+e\right)\left(\cosh^8\left(fx+e\right)\right)+\left(13\sqrt{-\frac{b}{a}}\, a\, b^2-7\sqrt{-\frac{b}{a}}\, b^3\right)\left(\cosh^6\left(fx+e\right)\right)\sinh\left(fx+e\right)+\left(9\sqrt{-\frac{b}{a}}\, a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/35*(5*(-1/a*b)^(1/2)*b^3*sinh(f*x+e)*cosh(f*x+e)^8+(13*(-1/a*b)^(1/2)*a*b
^2-7*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-1/a*b)^(1/2)*a^2*b-
(-1/a*b)^(1/2)*a*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+((-1/a*b)^(1/2)*a^3+8*(-1/a
*b)^(1/2)*a^2*b-11*(-1/a*b)^(1/2)*a*b^2+2*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^2
*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3+8*(b/a*cosh(f*x+e)^2+(a-b)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*a^2*b-11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+2*(b/a*cosh(f*x+e)^2+(a
-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*b^3-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ell
ipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3+10*(b/a*cosh(f*x+e)^2+(a
-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*a^2*b+10*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-2*(b/a*cosh(f*x+e)^
2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*b^3)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh\left(e+f\,x\right)^4\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.369 $\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{\left(3a^2+7ab-2b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf}-\frac{\left(3a^2+7ab-2b^2\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ta}\right.}{15bf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] 2/15*(3*a-b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/5*b*cosh
(f*x+e)^3*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/15*(3*a^2+7*a*b-2*b^2)*
(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/15*(9*a-b)*(1/(1+sinh(f*
x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)
^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+
e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/15*(3*a^2+7*a*b-2*b^2)*(a+b*sinh(f*x+e)
^2)^(1/2)*tanh(f*x+e)/b/f
```

Rubi [A] time = 0.29, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = $\{3192, 416, 528, 531, 418, 492, 411\}$

$$\frac{\left(3a^2+7ab-2b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf}-\frac{\left(3a^2+7ab-2b^2\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\text{ta}\right.}{15bf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(3*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f
) + (b*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) - (
(3*a^2 + 7*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e +
f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((9*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Se
ch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) + ((3*a^2 + 7*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/(15*b*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \sqrt{1+x^2}\,\left(a+bx^2\right)^{3/2}\,d\right.}{f}$$

$$= \frac{b\cosh^3(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{5f} + \frac{\left(\sqrt{\cosh^2(e\right.}{}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{b\,\mathrm{co}}{}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{b\,\mathrm{co}}{}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{b\,\mathrm{co}}{}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{b\,\mathrm{co}}{}$$

Mathematica [C] time = 1.38, size = 213, normalized size = 0.71

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(48a^2+4b(9a-2b)\cosh(2(e+fx))-28ab+3b^2\cosh(4(e+fx))+5b^2\right)+16ia\left(3a^2-2ab\right.}{240bf\sqrt{2a+b\cosh(2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-16*I)*a*(3*a^2 + 7*a*b - 2*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*
EllipticE[I*(e + f*x), b/a] + (16*I)*a*(3*a^2 - 2*a*b - b^2)*Sqrt[(2*a - b
+ b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(48*a^2 -
 28*a*b + 5*b^2 + 4*(9*a - 2*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x
)])*Sinh[2*(e + f*x)])/(240*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\cosh\left(fx+e\right)^2\sinh\left(fx+e\right)^2+a\cosh\left(fx+e\right)^2\right)\sqrt{b\sinh\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(f*x + e)^2*sinh(f*x + e)^2 + a*cosh(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type

maple [A] time = 0.19, size = 535, normalized size = 1.79

$$\frac{3\sqrt{-\frac{b}{a}}\, b^2 \sinh\left(fx+e\right)\left(\cosh^6\left(fx+e\right)\right)+\left(9\sqrt{-\frac{b}{a}}\, ab-5\sqrt{-\frac{b}{a}}\, b^2\right)\left(\cosh^4\left(fx+e\right)\right)\sinh\left(fx+e\right)+\left(6\sqrt{-\frac{b}{a}}\, a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^6+(9*(-1/a*b)^(1/2)*a*b-5*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(6*(-1/a*b)^(1/2)*a^2-8*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+6*a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-8*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+7*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh\left(e+fx\right)^2\left(b\sinh\left(e+fx\right)^2+a\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)

[Out] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

3.370 $\int \left(a + b\sinh^2(e + fx)\right)^{3/2} \, dx$

Optimal. Leaf size=174

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\text{s}\cdots}}{3f\sqrt{\cdots}}$$

```
[Out] 1/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2/3*I*(2*a-b)*(co
s(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*
(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+1/3*I*a*(a-b)*(cos(
I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1
+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\text{s}\cdots}}{3f\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{1}{3}\int \frac{a(3a-b)+2(2a-}{\sqrt{a+b\sinh}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{1}{3}(a(a-b))\int \frac{}{\sqrt{a+b\sinh}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2(2a-b)\sqrt{a+b\sinh^2}\right.}{3\sqrt{1}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{2i(2a-b)E\left(ie+ifx\,\middle|\,\frac{b}{a}\right)}{3f\sqrt{1+\frac{b}{}}}$$

Mathematica [A] time = 0.76, size = 169, normalized size = 0.97

$$\frac{b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)+2i\sqrt{2}\,a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-4i\sqrt{2}\,a(2a}{6f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.00, size = 416, normalized size = 2.39

$$\frac{\sqrt{-\frac{b}{a}}\, b^2 \sinh\left(fx+e\right)\left(\cosh^4\left(fx+e\right)\right)+\left(\sqrt{-\frac{b}{a}}\, ab-\sqrt{-\frac{b}{a}}\, b^2\right)\left(\cosh^2\left(fx+e\right)\right)\sinh\left(fx+e\right)+3a^2\sqrt{\frac{b\left(\cosh^2\left(fx+\right.\right.}{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a*b-(-1/a
*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^
2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b \sinh\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.371 $\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=210

$$-\frac{(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{(a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{b\operatorname{sech}(e+fx)\sqrt{a}}{}$$

```
[Out] (a-2*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^
2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+b*(1/(1+sinh(f*x+e)^
2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(
1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*
(a+b*sinh(f*x+e)^2)/a)^(1/2)-(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/
f+(a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.20, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3192, 413, 531, 418, 492, 411}

$$-\frac{(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{(a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{b\operatorname{sech}(e+fx)\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
+ (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sin
h[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a
- 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f + ((a - b)*Sqrt[a + b*S
inh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^{3/2}}dx, x, \sinh(e+$$

$$= \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{se}\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f} + \frac{\left(ab\sqrt{\cosh^2(e+fx)}\right.}{}$$

$$= \frac{bF\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

$$= \frac{(a-2b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 0.97, size = 160, normalized size = 0.76

$$\frac{2ia(a-2b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)+(a-b)\left(\sqrt{2}\tanh(e+fx)(2a+b\cosh(2(e+fx))-b)-2ia\sqrt{\frac{2a}{}}\right.}{2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] + (a - b)*((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ell
```

```
ipticF[I*(e + f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e +
 f*x]))/(2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\,\text{sech}\left(fx+e\right)^{2}\sinh\left(fx+e\right)^{2}+a\,\text{sech}\left(fx+e\right)^{2}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sech(f*x + e)^2*sinh(f*x + e)^2 + a*sech(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.17, size = 334, normalized size = 1.59

$$\frac{\sqrt{-\frac{b}{a}}\,ab\left(\sinh^{3}\left(fx+e\right)\right)-\sqrt{-\frac{b}{a}}\,b^{2}\left(\sinh^{3}\left(fx+e\right)\right)+2a\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] ((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^3-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+2*a*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*b-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1
/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-((a+b*sinh(f*x+e)
^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*a*b+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+(-1/a*b)^(1/2)*a^2*sinh(f*x+
e)-(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x
+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\text{sech}\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(b\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.372 $\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=193

$$\frac{(a-b)\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e\right.}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] 2/3*(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sin
h(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e
)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*b*(1/(1+sinh(f
*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e
)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x
+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(a-b)*sech(f*x+e)^2*(a+b*sinh(f*x+e)
^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.19, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3192, 413, 525, 418, 411}

$$\frac{(a-b)\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e\right.}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*S
inh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) +
((a - b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^{5/2}}dx, x, \sinh(e+\right.}{f}$$

$$= \frac{(a-b)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f} + \frac{\left(\sqrt{\cos\right.}{}$$

$$= \frac{(a-b)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f} - \frac{\left(ab\sqrt{\right.}{}$$

$$= \frac{2(a+b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 2.02, size = 197, normalized size = 1.02

$$\frac{\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\left(\left(4a^2+6ab-2b^2\right)\cosh(2(e+fx))+8a^2+b(a+b)\cosh(4(e+fx))-3ab+b^2\right)}{\sqrt{2}} - 2ia(2a+b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+\right.}{6f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((4*I)*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f
*x), b/a] - (2*I)*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellip
ticF[I*(e + f*x), b/a] + ((8*a^2 - 3*a*b + b^2 + (4*a^2 + 6*a*b - 2*b^2)*Co
sh[2*(e + f*x)] + b*(a + b)*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh[e + f*x
])/Sqrt[2])/(6*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\operatorname{sech}\left(fx+e\right)^4\sinh\left(fx+e\right)^2+a\operatorname{sech}\left(fx+e\right)^4\right)\sqrt{b\sinh\left(fx+e\right)^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sech(f*x + e)^4*sinh(f*x + e)^2 + a*sech(f*x + e)^4)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.19, size = 328, normalized size = 1.70

$$-\frac{\left(-2\sqrt{-\frac{b}{a}}\,ab-2\sqrt{-\frac{b}{a}}\,b^{2}\right)\sinh\left(fx+e\right)\left(\cosh^{4}\left(fx+e\right)\right)+\left(-2\sqrt{-\frac{b}{a}}\,a^{2}-\sqrt{-\frac{b}{a}}\,ab+3\sqrt{-\frac{b}{a}}\,b^{2}\right)\left(\cosh^{2}\left(fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*((-2*(-1/a*b)^(1/2)*a*b-2*(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^
4+(-2*(-1/a*b)^(1/2)*a^2-(-1/a*b)^(1/2)*a*b+3*(-1/a*b)^(1/2)*b^2)*cosh(f*x+
e)^2*sinh(f*x+e)+(-(-1/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b
^2)*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(
2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+2*b*EllipticE(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-a*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))-2*b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e
)^2)/cosh(f*x+e)^3/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}}{\cosh\left(e+f\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2)/cosh(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.373 $\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=79

$$\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f}$$

```
[Out] -1/2*(a-2*b)*arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(3/2)
/f+1/2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3190, 388, 217, 206}

$$\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(
2*b^(3/2)*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$
$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf} - \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+\right.}{2bf}$$
$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf} - \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sinh(e+fx}{\sqrt{a+b\sinh^2(e}}\right.}{2bf}$$
$$= -\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf}$$

Mathematica [A] time = 0.11, size = 77, normalized size = 0.97

$$\frac{\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2b} - \frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-1/2*((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2
]])/b^(3/2) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b))/f
```

fricas [B] time = 1.00, size = 2479, normalized size = 31.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((a - 2*b)*cosh(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)
 + (a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*
x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b
 - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos
h(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f
*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e
)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 +
 b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*
a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 +
b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3
 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3
*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b
^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt
(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x +
 e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2
*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*
(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f
```

```
*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2
)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*
b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b -
 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3
 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3
)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f
*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x +
 e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x +
e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a - 2*b)*cos
h(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh(f*x
 + e)^2)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*si
nh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x +
 e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(2)*(b*cosh
(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt
((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^2 + 2*b^
2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2), 1/8*(2*((a - 2*b)
*cosh(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh
(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*c
osh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(
a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3
*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*
b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*
b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((a - 2*b)*cosh(f*x + e)^2 + 2*(a -
 2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh(f*x + e)^2)*sqrt(-b)*arc
tan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(
f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a
 - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 +
 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt
(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^
2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x +
e)^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.12, size = 35, normalized size = 0.44

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\cosh^2(fx+e)}{\sqrt{a+b\left(\sinh^2(fx+e)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(fx+e\right)^{3}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(e+f\,x\right)^{3}}{\sqrt{b\,\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.374 $\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

```
[Out] arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3190, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}\,f}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)
```

fricas [B] time = 0.81, size = 1990, normalized size = 52.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*
a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f
*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a
^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^
2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 +
9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 1
0*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 +
 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x
 + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b
 + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b
^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)
*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a
^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 +
 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e
)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*
b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5
 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*s
inh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*
(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2
*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a
*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x +
 e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)
^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*s
inh(f*x + e)^5 + sinh(f*x + e)^6)) + sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*
(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*co
sh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*si
nh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2)))/(b*f), -1/2*(sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 +
 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(
-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x
```

```
+ e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x
 + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2
- 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 -
 (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + sqrt(-b)*arctan(sqrt(2)*(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt
(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 +
4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*
x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*
x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)))/(b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.03, size = 34, normalized size = 0.89

$$\frac{\ln\left(\sqrt{b}\ \sinh\left(fx+e\right)+\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\right)}{f\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*ln(b^(1/2)*sinh(f*x+e)+(a+b*sinh(f*x+e)^2)^(1/2))/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [B] time = 1.02, size = 33, normalized size = 0.87

$$\frac{\ln\left(\sqrt{b}\ \sinh\left(e+f\,x\right)+\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}\right)}{\sqrt{b}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*sinh(e + f*x) + (a + b*sinh(e + f*x)^2)^(1/2))/(b^(1/2)*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cosh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.375 $\int \frac{\text{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

```
[Out] arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3190, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a - b
]*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{a-b}\,f}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a - b
]*f)
```

fricas [B] time = 0.51, size = 598, normalized size = 13.00

$$\left[-\frac{\sqrt{-a+b}\,\log\left(\frac{(a-2b)\cosh(fx+e)^4+4(a-2b)\cosh(fx+e)\sinh(fx+e)^3+(a-2b)\sinh(fx+e)^4-2(3a-2b)\cosh(fx+e)^2+2\left(3(a-2b)\cosh(f\right.}{\cosh(fx+e)^4+4\cosh(fx+e)\,s}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x +
e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e
)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(
2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*
sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b
)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(co
sh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*co
sh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3
 + cosh(f*x + e))*sinh(f*x + e) + 1))/((a - b)*f), arctan(sqrt(2)*(cosh(f*x
 + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
```

```
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))/(sqrt(a - b)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.12, size = 35, normalized size = 0.76

$$\frac{\text{`}int/indef0\text{`}\left(\frac{1}{\cosh\left(fx+e\right)^{2}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cosh\left(e+f\,x\right)\,\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.376 $\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=97

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f(a-b)^{3/2}}+\frac{\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}$$

```
[Out] 1/2*(a-2*b)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)
^(3/2)/f+1/2*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3190, 382, 377, 203}

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f(a-b)^{3/2}}+\frac{\tanh(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(2*(a - b)^(3/2)*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e +
f*x])/(2*(a - b)*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2(a-b)f} + \frac{(a-2b)\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\right.}{2(a-b)f}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{2(a-b)f} + \frac{(a-2b)\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx\right.}{2(a-b)f}$$

$$= \frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{3/2}f} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+f}{2(a-b)f}$$

Mathematica [C] time = 9.37, size = 443, normalized size = 4.57

$$\frac{\tanh(e+fx)\operatorname{sech}^3(e+fx)\left(\frac{b\sinh^2(e+fx)}{a}+1\right)\left(-30b\sinh^2(e+fx)\sqrt{\frac{\tanh^2(e+fx)\operatorname{sech}^2(e+fx)\left(a^2+ab\left(\sinh^2(e+fx)-1\right)-b^2\sinh^2\right.}{a^2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)*Tanh[e + f*x]*(45*a*ArcSin[Sqr
t[((a - b)*Tanh[e + f*x]^2)/a]] + 30*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2
)/a]]*Sinh[e + f*x]^2 + 16*a*Hypergeometric2F1[2, 3, 7/2, ((a - b)*Tanh[e +
 f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tan
h[e + f*x]^2)/a)^(5/2) + 16*b*Hypergeometric2F1[2, 3, 7/2, ((a - b)*Tanh[e
+ f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) - 45*a*Sqrt[(Sech[e + f*x]^2*(a^2 -
 b^2*Sinh[e + f*x]^2 + a*b*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2] -
30*b*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[e + f*x]^2 + a*b
*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2]))/(30*a*f*Sqrt[a + b*Sinh[e
+ f*x]^2]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[
e + f*x]^2)/a)^(3/2))
```

fricas [B] time = 0.67, size = 1503, normalized size = 15.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)
^3 + (a - 2*b)*sinh(f*x + e)^4 + 2*(a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*
b)*cosh(f*x + e)^2 + a - 2*b)*sinh(f*x + e)^2 + 4*((a - 2*b)*cosh(f*x + e)^
3 + (a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)*sqrt(-a + b)*log(((a
- 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2
*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f
*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f
```

```
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a -
2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x
+ e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x
 + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
+ e) + 1)) - 2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*s
inh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*sinh(f*x
 + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*a*b + b^2
)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b
 + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*
f*cosh(f*x + e))*sinh(f*x + e)), 1/2*(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2
*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 + 2*(a - 2*b)
*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 + a - 2*b)*sinh(f*x + e)^
2 + 4*((a - 2*b)*cosh(f*x + e)^3 + (a - 2*b)*cosh(f*x + e))*sinh(f*x + e) +
 a - 2*b)*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
+ b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2
+ 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e)
)*sinh(f*x + e) + b)) + sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f
*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f
*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^
2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*
b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.15, size = 35, normalized size = 0.36

$$\frac{\text{`}int/indef0\text{`}\left(\frac{1}{\cosh\left(fx+e\right)^{4}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\mathrm{sech}\left(fx+e\right)^{3}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(e+f\,x\right)^3\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.377 $\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=241

$$-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f}+\frac{2(a-2b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+\right.}{3b^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b/f+2/3*(a-2*b)*(1/(1
+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sin
h(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/b^2/
f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(a-3*b)*(1/(1+sinh(f*x+e)
^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^
(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/b/f/(sech(f*x+
e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/3*(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tan
h(f*x+e)/b^2/f
```

Rubi [A] time = 0.22, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3192, 416, 531, 418, 492, 411}

$$-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f}+\frac{2(a-2b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+\right.}{3b^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a -
 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Si
nh[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
 - ((a - 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]
^2))/a]) - (2*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f
)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\left(2(a-2b)\sqrt{\cosh^2(e+fx)}\operatorname{sec}\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{(a-3b)F\left(\tan^{-1}(\sinh(e+fx)\right.}{3abf\sqrt{\frac{\operatorname{se}}{}}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{2(a-2b)E\left(\tan^{-1}(\sinh(e+f\right.}{3b^2f\sqrt{\frac{\operatorname{se}}{}}}$$

Mathematica [C] time = 0.86, size = 179, normalized size = 0.74

$$\frac{-2i\sqrt{2}\left(2a^2-5ab+3b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)+}{6b^2f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((4*I)*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elliptic
E[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b +
 b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[
2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*b^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e +
f*x)]])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.18, size = 344, normalized size = 1.43

$$\sqrt{-\frac{b}{a}}\, b\sinh\left(fx+e\right)\left(\cosh^{4}\left(fx+e\right)\right)+\left(\sqrt{-\frac{b}{a}}\, a-\sqrt{-\frac{b}{a}}\, b\right)\left(\cosh^{2}\left(fx+e\right)\right)\sinh\left(fx+e\right)+a\sqrt{\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^
(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+4*(b/a*cosh(f*
x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^
(1/2),(a/b)^(1/2))*b)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2
)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(e+f\,x\right)^4}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.378 $\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=177

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\operatorname{sech}(e$$

```
[Out] -(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/
(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2
)/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))^(1/
2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1
-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*
sinh(f*x+e)^2)/a)^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/b/f
```

Rubi [A] time = 0.15, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3192, 422, 418, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\operatorname{sech}(e$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])) + (El
lipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a +
 b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(b*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\,\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f} + \frac{\left(\sqrt{\cdots\right.}}{\cdots}$$

$$= \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{\sqrt{a+b\sinh^2\cdots}}{\cdots}$$

$$= -\frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{F\left(\tan^{-1}(\sin\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.24, size = 95, normalized size = 0.54

$$-\frac{i\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\left((b-a)F\left(i(e+fx)\left|\frac{b}{a}\right.\right)+aE\left(i(e+fx)\left|\frac{b}{a}\right.\right)\right)}{bf\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*(a*EllipticE[I*(e + f*x), b/a
] + (-a + b)*EllipticF[I*(e + f*x), b/a]))/(b*f*Sqrt[2*a - b + b*Cosh[2*(e
+ f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.12, size = 86, normalized size = 0.49

$$\frac{\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\;\mathrm{EllipticE}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\;\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(fx+e\right)^{2}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(e+f\,x\right)^{2}}{\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{2}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cosh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.379 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(
1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sinh^2(e+fx)}}\\ &= -\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 68, normalized size = 1.13

$$-\frac{i\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/
(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.00, size = 86, normalized size = 1.43

$$\frac{\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\operatorname{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\sinh\left(e+fx\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sinh^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.380 $\int \frac{\text{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=160

$$\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{b\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}\right.}{af(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh\right.}{a}}}$$

```
[Out] (1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(
1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)
/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-b*(1/(1+sinh(f*x+e)^2)
)^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/
2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)/f/(sech(f*x
+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3192, 414, 21, 422, 418, 492, 411}

$$\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{b\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}\right.}{af(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
+ f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
(b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(a*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{(a-b)f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \right.}{(-a+b)f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{(a-b)f} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \right.}{(-a+b)f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{(a-b)f} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \right.}{(-a+b)f}$$

$$= -\frac{bF\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{\left(\sqrt{\cosh^2(e\right.}{}$$

$$= \frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{bF\left(\tan^{-1}(\sin\right.}{}$$

Mathematica [C] time = 0.65, size = 159, normalized size = 0.99

$$\frac{\sqrt{2}\tanh(e+fx)(2a+b\cosh(2(e+fx))-b)-2i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)+2ia\sqrt{\frac{2a+b\cosh(2}{a}}}{2f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
] - (2*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e +
f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*(a -
 b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{sech}\left(fx+e\right)^{2}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.20, size = 133, normalized size = 0.83

$$-\frac{-\sqrt{-\frac{b}{a}}\,b\left(\sinh^{3}\left(fx+e\right)\right)+b\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\operatorname{EllipticE}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-\sqrt{-}}{\left(a-b\right)\sqrt{-\frac{b}{a}}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -(-(-1/a*b)^(1/2)*b*sinh(f*x+e)^3+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x
+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(-1/a*b)^(1/
2)*a*sinh(f*x+e))/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2
)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(fx+e\right)^{2}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(e+fx\right)^2\sqrt{b\sinh\left(e+fx\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)),x)
[Out] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(sech(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.381 $\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=219

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)} - \frac{b(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(\right.}{3af(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] 2/3*(a-2*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x
+e)^2)^(1/2)/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(a-3
*b)*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*
x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)
^(1/2)/a/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*sech(f*x
+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f
```

Rubi [A] time = 0.19, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3192, 414, 525, 418, 411}

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)} - \frac{b(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(\right.}{3af(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (2*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
 + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) - ((a - 3*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech
[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*(a - b)^2*f*Sqrt[(Sech[e + f*x]
^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]
^2]*Tanh[e + f*x])/(3*(a - b)*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{5/2}\sqrt{a+bx^2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+f\right.}{}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} + \frac{\left(2(a-2b)\sqrt{\cosh^2(e+fx)}\,\text{se}\right.}{}$$

$$= \frac{2(a-2b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{(a-}{}$$

Mathematica [C] time = 2.23, size = 219, normalized size = 1.00

$$\frac{-2i\left(2a^2-5ab+3b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right)+\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\left(\left(4a^2-6ab-2b^2\right)\cosh(2(e+fx))+8a^2+b(a-2b\right.}{\sqrt{2}}}{6f(a-b)^2\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((4*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] - (2*I)*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f
*x)])/a]*EllipticF[I*(e + f*x), b/a] + ((8*a^2 - 15*a*b + 4*b^2 + (4*a^2 -
6*a*b - 2*b^2)*Cosh[2*(e + f*x)] + (a - 2*b)*b*Cosh[4*(e + f*x)])*Sech[e +
f*x]^2*Tanh[e + f*x])/Sqrt[2])/(6*(a - b)^2*f*Sqrt[2*a - b + b*Cosh[2*(e +
f*x)]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\operatorname{sech}\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.28, size = 343, normalized size = 1.57

$$\frac{\sqrt{\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)\left(\cosh^2\left(fx+e\right)\right)}\left(2\left(\cosh^4\left(fx+e\right)\right)\sqrt{-\frac{b}{a}}\,b\,(a-2b)\sinh\left(fx+e\right)+\left(\cosh^2\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/cosh(f*x+e)^3/(-1/a*b)^(1/2)/
(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)/(a^2-2*a*b+b^2)*(2*cosh(f*x+e)^
4*(-1/a*b)^(1/2)*b*(a-2*b)*sinh(f*x+e)+cosh(f*x+e)^2*(-1/a*b)^(1/2)*(2*a^2-
5*a*b+3*b^2)*sinh(f*x+e)+(-1/a*b)^(1/2)*(a^2-2*a*b+b^2)*sinh(f*x+e)+(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(a*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))-2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+4*b*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^
(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(e+f\,x\right)^4\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(sech(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.382 $\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}f}-\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(3/2)/f-(a-b)*sinh
(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3190, 385, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}f}-\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(3/2)*f) -
((a - b)*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{bf}$$

$$= -\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{bf}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}f} - \frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A] time = 0.17, size = 89, normalized size = 1.16

$$\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1} + \sqrt{b}(b-a)\sinh(e+fx)}{ab^{3/2}f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(-a + b)*Sinh[e + f*x] + a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/S
qrt[a]]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a*b^(3/2)*f*Sqrt[a + b*Sinh[e + f
*x]^2])
```

fricas [B] time = 1.18, size = 3126, normalized size = 40.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh
(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 +
2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)
*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f
*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*
b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*co
sh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 +
b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x +
e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2
+ b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5
*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 +
 b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^
3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(
3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
 e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*
b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqr
```

```
t(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x
+ e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b
 + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 +
2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3
*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(
f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^
2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b
- 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^
3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^
3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(
f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x
+ e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x +
 e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + (a*b*cosh(f*x
 + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + 2*(2*
a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f
*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sin
h(f*x + e))*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e
)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)
*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*
x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - 4*sqrt(2)*((
a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a
*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
 + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)))/(a*b^3*f*cosh(f*x + e)^4 + 4*a*b^3*f*cosh(f*x + e)*sinh(f*x +
 e)^3 + a*b^3*f*sinh(f*x + e)^4 + a*b^3*f + 2*(2*a^2*b^2 - a*b^3)*f*cosh(f*
x + e)^2 + 2*(3*a*b^3*f*cosh(f*x + e)^2 + (2*a^2*b^2 - a*b^3)*f)*sinh(f*x +
 e)^2 + 4*(a*b^3*f*cosh(f*x + e)^3 + (2*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*s
inh(f*x + e)), -1/2*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x +
e)^3 + a*b*sinh(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cos
h(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3
+ (2*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*((a -
 b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(
f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(
(a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a*b - b^2)*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b -
b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b
^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + (a*b
*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^
4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 + 2*a^2 - a*
b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x
+ e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
+ b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x
 + e))*sinh(f*x + e) + b)) + 2*sqrt(2)*((a*b - b^2)*cosh(f*x + e)^2 + 2*(a*
b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a*b - b^2)*sinh(f*x + e)^2 - a*b +
b^2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*b^3*f*cosh(f*x +
e)^4 + 4*a*b^3*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*b^3*f*sinh(f*x + e)^4 +
a*b^3*f + 2*(2*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^2 + 2*(3*a*b^3*f*cosh(f*x +
```

```
e)^2 + (2*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 + 4*(a*b^3*f*cosh(f*x + e)^3
+ (2*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.12, size = 35, normalized size = 0.45

$$\frac{`int/indef0`\left(\frac{\cosh^2(fx+e)}{\left(a+b\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}}}, \sinh(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(fx+e)^3}{\left(b\sinh(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(e+fx)^3}{\left(b\sinh(e+fx)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.383 $\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3190, 191}

$$\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}\\ &= \frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.00

$$\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

fricas [B] time = 0.62, size = 245, normalized size = 8.45

$$\frac{\sqrt{2}\left(\cosh\left(fx+e\right)^{2}+2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right.\right.}{abf\cosh\left(fx+e\right)^{4}+4\,abf\cosh\left(fx+e\right)\sinh\left(fx+e\right)^{3}+abf\sinh\left(fx+e\right)^{4}+2\left(2\,a^{2}-ab\right)f\cosh\left(fx+e\right)^{2}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
- 1)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*f*cosh(f*x + e)^
4 + 4*a*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*f*sinh(f*x + e)^4 + 2*(2*a^
2 - a*b)*f*cosh(f*x + e)^2 + a*b*f + 2*(3*a*b*f*cosh(f*x + e)^2 + (2*a^2 -
a*b)*f)*sinh(f*x + e)^2 + 4*(a*b*f*cosh(f*x + e)^3 + (2*a^2 - a*b)*f*cosh(f
*x + e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.03, size = 28, normalized size = 0.97

$$\frac{\sinh\left(fx+e\right)}{af\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

maxima [B] time = 0.83, size = 236, normalized size = 8.14

$$\frac{b^{2}e^{\left(-6\,fx-6\,e\right)}+2\,ab-b^{2}+\left(8\,a^{2}-8\,ab+3\,b^{2}\right)e^{\left(-2\,fx-2\,e\right)}+3\left(2\,ab-b^{2}\right)e^{\left(-4\,fx-4\,e\right)}}{2\left(a^{2}-ab\right)\left(2\left(2\,a-b\right)e^{\left(-2\,fx-2\,e\right)}+be^{\left(-4\,fx-4\,e\right)}+b\right)^{\frac{3}{2}}f}-\frac{b^{2}+3\left(2\,ab-b^{2}\right)e^{\left(-2\,fx-2\,e\right)}+}{2\left(a^{2}-ab\right)\left(2\left(2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*e^(-6*f*x - 6*e) + 2*a*b - b^2 + (8*a^2 - 8*a*b + 3*b^2)*e^(-2*f*x
 - 2*e) + 3*(2*a*b - b^2)*e^(-4*f*x - 4*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(3/2)*f) - 1/2*(b^2 + 3*(2*a*b - b^2)
*e^(-2*f*x - 2*e) + (8*a^2 - 8*a*b + 3*b^2)*e^(-4*f*x - 4*e) + (2*a*b - b^2
)*e^(-6*f*x - 6*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*
x - 4*e) + b)^(3/2)*f)
```

mupad [B] time = 1.13, size = 191, normalized size = 6.59

$$-\frac{\mathrm{e}^{e+f\,x}\,\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}\,\left(\frac{2\cosh\left(e+f\,x\right)\,\mathrm{e}^{e+f\,x}\,\left(b\,\left(2\,a-b\right)-b\,\left(4\,a-2\,b\right)\right)}{f\,\left(a\,b^{2}-a^{2}\,b\right)}-\frac{2\,b^{2}\,\mathrm{e}^{e+f\,x}\,\sinh\left(e+f\,x\right)}{f\,\left(a\,b^{2}-a^{2}\,b\right)}+\frac{b\,\mathrm{e}^{2\,e+2\,f\,x}\,\left(4\,a-2\,b\right)}{f\,\left(a\,b^{2}-a^{2}\,b\right)}\right)}{4\,a\,\mathrm{e}^{2\,e+2\,f\,x}-2\,b\,\mathrm{e}^{2\,e+2\,f\,x}+2\,b\,\mathrm{e}^{2\,e+2\,f\,x}\cosh\left(2\,e+2\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] -(exp(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2)*((2*cosh(e + f*x)*exp(e + f*x)
*(b*(2*a - b) - b*(4*a - 2*b)))/(f*(a*b^2 - a^2*b)) - (2*b^2*exp(e + f*x)*s
inh(e + f*x))/(f*(a*b^2 - a^2*b)) + (b*exp(2*e + 2*f*x)*(4*a - 2*b))/(f*(a*
b^2 - a^2*b))))/(4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + 2*b*exp(2*e
+ 2*f*x)*cosh(2*e + 2*f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cosh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.384 $\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\sinh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)^(3/2)/f-b*s
inh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\sinh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(3
/2)*f) - (b*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx\right.}{(a-b)f}$$

$$= -\frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right.}{(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] time = 7.58, size = 315, normalized size = 3.71

$$\tanh(e+fx)\operatorname{sech}^7(e+fx)\sqrt{a+b\sinh^2(e+fx)}\left(4(a-b)^2\sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sech[e + f*x]^7*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]*(4*(a - b)^2*Hyp
ergeometric2F1[2, 2, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^4*(a +
 b*Sinh[e + f*x]^2)*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[e + f*x]^2 + a*b*
(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2] + 15*a*Cosh[e + f*x]^2*(3*a +
 2*b*Sinh[e + f*x]^2)*(-(ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*(a + b*S
inh[e + f*x]^2)) + a*Cosh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[
e + f*x]^2 + a*b*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2])))/(15*a^5*f
*(((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2)^(3
/2))
```

fricas [B] time = 0.77, size = 1717, normalized size = 20.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh
(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 +
2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)
*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4
+ 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2
*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*
sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
```

```
*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sin
h(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 +
sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x +
e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*
((a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) +
(a*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f
*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sin
h(f*x + e)^2)))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^4 + 4*(a^3*b -
 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^3*b - 2*a^2*b^2 +
a*b^3)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f*cosh(f
*x + e)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^2 + (2*a^4 - 5
*a^3*b + 4*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3
)*f + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^3 + (2*a^4 - 5*a^3*b +
 4*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e)), ((a*b*cosh(f*x + e)^4
+ 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + 2*(2*a^2 - a*
b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x + e)^
2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*
x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*si
nh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a
- b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh
(f*x + e) + b)) - sqrt(2)*((a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh
(f*x + e)*sinh(f*x + e) + (a*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*co
sh(f*x + e)^4 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)*sinh(f*x + e)
^3 + (a^3*b - 2*a^2*b^2 + a*b^3)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 5*a^3*b + 4
*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*f*co
sh(f*x + e)^2 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 +
(a^3*b - 2*a^2*b^2 + a*b^3)*f + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x +
 e)^3 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e
))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [C] time = 0.17, size = 101, normalized size = 1.19

$$\frac{\text{`}int/indef0\text{`}\left(\frac{-b\left(\sinh^2\left(fx+e\right)\right)-a}{\left(-b^2\left(\sinh^6\left(fx+e\right)\right)+\left(-2ab-b^2\right)\left(\sinh^4\left(fx+e\right)\right)+\left(-a^2-2ab\right)\left(\sinh^2\left(fx+e\right)\right)-a^2\right)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((-b*sinh(f*x+e)^2-a)/(-b^2*sinh(f*x+e)^6+(-2*a*b-b^2)*sinh(f*x
+e)^4+(-a^2-2*a*b)*sinh(f*x+e)^2-a^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e)
)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sech(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(e+f\,x\right)\,{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)),x)
[Out] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Integral(sech(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.385 $\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{b(a+2b)\sinh(e+fx)}{2af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f(a-b)^{5/2}}+\frac{\tanh(e+fx)\operatorname{sech}(e+fx)}{2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] 1/2*(a-4*b)*arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)
^(5/2)/f+1/2*b*(a+2*b)*sinh(f*x+e)/a/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/
2*sech(f*x+e)*tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3190, 414, 527, 12, 377, 203}

$$\frac{b(a+2b)\sinh(e+fx)}{2af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f(a-b)^{5/2}}+\frac{\tanh(e+fx)\operatorname{sech}(e+fx)}{2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(2*(a - b)^(5/2)*f) + (b*(a + 2*b)*Sinh[e + f*x])/(2*a*(a - b)^2*f*Sqrt[a
+ b*Sinh[e + f*x]^2]) + (Sech[e + f*x]*Tanh[e + f*x])/(2*(a - b)*f*Sqrt[a +
 b*Sinh[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-2bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx\right)}{2(a-b)f}$$

$$= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subs}}{}$$

$$= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a-4}{}$$

$$= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a-4}{}$$

$$= \frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{5/2}f} + \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sec}}{2(a-}$$

Mathematica [C] time = 5.21, size = 231, normalized size = 1.63

$$\frac{\tanh(e+fx)\operatorname{sech}^5(e+fx)\left(16(a-b)\sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^2 {}_3F_2\left(2,2,3;1,\frac{9}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sech[e + f*x]^5*(16*(a - b)*HypergeometricPFQ[{2, 2, 3}, {1, 9/2}, ((a - b
)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^2 + 16*(a - b
```

```
)*Hypergeometric2F1[2, 3, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2
*(4*a^2 + 7*a*b*Sinh[e + f*x]^2 + 3*b^2*Sinh[e + f*x]^4) + 7*a*Cosh[e + f*x
]^2*Hypergeometric2F1[1, 2, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*(15*a^2 + 20*
a*b*Sinh[e + f*x]^2 + 8*b^2*Sinh[e + f*x]^4))*Tanh[e + f*x]])/(105*a^4*f*Sqr
t[a + b*Sinh[e + f*x]^2])
```

fricas [B] time = 1.28, size = 4845, normalized size = 34.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2*b - 4*a*b^2)*cosh(f*x + e)^8 + 8*(a^2*b - 4*a*b^2)*cosh(f*x + e
)*sinh(f*x + e)^7 + (a^2*b - 4*a*b^2)*sinh(f*x + e)^8 + 4*(a^3 - 4*a^2*b)*c
osh(f*x + e)^6 + 4*(a^3 - 4*a^2*b + 7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^2)*si
nh(f*x + e)^6 + 8*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b)*
cosh(f*x + e))*sinh(f*x + e)^5 + 2*(4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x +
e)^4 + 2*(35*(a^2*b - 4*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 17*a^2*b + 4*a*b^2
 + 30*(a^3 - 4*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b - 4*a*
b^2)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b)*cosh(f*x + e)^3 + (4*a^3 - 17*a^2
*b + 4*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b - 4*a*b^2 + 4*(a^3 - 4
*a^2*b)*cosh(f*x + e)^2 + 4*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^6 + 15*(a^3
- 4*a^2*b)*cosh(f*x + e)^4 + a^3 - 4*a^2*b + 3*(4*a^3 - 17*a^2*b + 4*a*b^2)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b - 4*a*b^2)*cosh(f*x + e)^7 +
3*(a^3 - 4*a^2*b)*cosh(f*x + e)^5 + (4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x +
 e)^3 + (a^3 - 4*a^2*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a
- 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2
*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f
*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a -
2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x
+ e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x
 + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
+ e) + 1)) + 2*sqrt(2)*((a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 6*(a^2*b
+ a*b^2 - 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b + a*b^2 - 2*b^3)*si
nh(f*x + e)^6 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + (4*a^
3 - 7*a^2*b + 5*a*b^2 - 2*b^3 + 15*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^2)
*sinh(f*x + e)^4 + 4*(5*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (4*a^3 -
7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - a^2*b - a*b^2 +
 2*b^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + (15*(a^2*b +
 a*b^2 - 2*b^3)*cosh(f*x + e)^4 - 4*a^3 + 7*a^2*b - 5*a*b^2 + 2*b^3 + 6*(4*
a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a
^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^5 + 2*(4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^
3)*cosh(f*x + e)^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*f*sinh(f*x + e)^8 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f
*cosh(f*x + e)^6 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x
+ e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^6 + 2*(4*a^
5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^
4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^3 + 3*(a^5 - 3*a^4*b +
 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b
^2 - a^2*b^3)*f*cosh(f*x + e)^2 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^
3 + a*b^4)*f)*sinh(f*x + e)^4 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*c
```

osh(f*x + e)^2 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f + 8*((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e)), 1/2*(((a^2*b - 4*a*b^2)*cosh(f*x + e)^8 + 8*(a^2*b - 4*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 4*a*b^2)*sinh(f*x + e)^8 + 4*(a^3 - 4*a^2*b)*cosh(f*x + e)^6 + 4*(a^3 - 4*a^2*b + 7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x + e)^4 + 2*(35*(a^2*b - 4*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 17*a^2*b + 4*a*b^2 + 30*(a^3 - 4*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b)*cosh(f*x + e)^3 + (4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b - 4*a*b^2 + 4*(a^3 - 4*a^2*b)*cosh(f*x + e)^2 + 4*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b)*cosh(f*x + e)^4 + a^3 - 4*a^2*b + 3*(4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b - 4*a*b^2)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b)*cosh(f*x + e)^5 + (4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x + e)^3 + (a^3 - 4*a^2*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 6*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b + a*b^2 - 2*b^3)*sinh(f*x + e)^6 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3 + 15*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - a^2*b - a*b^2 + 2*b^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + (15*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^4 - 4*a^3 + 7*a^2*b - 5*a*b^2 + 2*b^3 + 6*(4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^5 + 2*(4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*sinh(f*x + e)^8 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^6 + 2*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^3 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^2 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f)*sinh(f*x + e)^4 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^2 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x +

```
e)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e
)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^2 + (a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f + 8*((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4
)*f*cosh(f*x + e)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x +
e)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^
3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.41Error: Bad Argument Typ
e
```

maple [C] time = 0.23, size = 95, normalized size = 0.67

$$\frac{\text{`}int/indef0\text{`}\left(-\frac{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,\left(\cosh^{2}\left(fx+e\right)\right)}{-b^{2}\left(\cosh^{10}\left(fx+e\right)\right)+\left(-2ab+2b^{2}\right)\left(\cosh^{8}\left(fx+e\right)\right)+\left(-a^{2}+2ab-b^{2}\right)\left(\cosh^{6}\left(fx+e\right)\right)},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2/(-b^2*cosh(f*x+e)^10+
(-2*a*b+2*b^2)*cosh(f*x+e)^8+(-a^2+2*a*b-b^2)*cosh(f*x+e)^6),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(e+f\,x\right)^{3}\,{\left(b\,\sinh\left(e+f\,x\right)^{2}+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{3}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sech(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.386 $\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(8a^2-13ab+3b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^3f}+\frac{\left(8a^2-13ab+3b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,I}{3ab^3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+\right.}{a}}}$$

```
[Out] -(a-b)*cosh(f*x+e)^3*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(4*a-3
*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/b^2/f+1/3*(8*a^2-13
*a*b+3*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x
+e)^2)^(1/2)/a/b^3/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/3*(2*a-3
*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+
e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(
1/2)/a/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-1/3*(8*a^2-13*a*b+
3*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/b^3/f
```

Rubi [A] time = 0.30, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3192, 413, 528, 531, 418, 492, 411}

$$-\frac{\left(8a^2-13ab+3b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^3f}+\frac{\left(8a^2-13ab+3b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,I}{3ab^3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a*b^2*f) + ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1
 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^3*f*Sqrt[(Sech[e
+ f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(2*a - 3*b)*EllipticF[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^2*f
*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 - 13*a*b + 3*
b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*b^3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\Big(}{ab}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a-}}{3ab^2f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a-}}{3ab^2f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a-}}{3ab^2f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a-}}{3ab^2f}$$

Mathematica [C] time = 1.11, size = 196, normalized size = 0.60

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(8a^2+ab\cosh(2(e+fx))-13ab+6b^2\right)-4ia\left(8a^2-17ab+9b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\Big(i(e}{12ab^3f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((4*I)*a*(8*a^2 - 13*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticE[I*(e + f*x), b/a] - (4*I)*a*(8*a^2 - 17*a*b + 9*b^2)*Sqrt[(2*a - b
 + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 -
 13*a*b + 6*b^2 + a*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(12*a*b^3*f*Sqr
t[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\cosh\left(fx+e\right)^6}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^6/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.18, size = 498, normalized size = 1.53

$$\frac{\sqrt{-\frac{b}{a}}\, ab \sinh\left(fx+e\right)\left(\cosh^4\left(fx+e\right)\right)+\left(4\sqrt{-\frac{b}{a}}\, a^2-7\sqrt{-\frac{b}{a}}\, ab+3\sqrt{-\frac{b}{a}}\, b^2\right)\left(\cosh^2\left(fx+e\right)\right)\sinh\left(fx+e\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*a*b*sinh(f*x+e)*cosh(f*x+e)^4+(4*(-1/a*b)^(1/2)*a^2-7*(
-1/a*b)^(1/2)*a*b+3*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+4*a^2*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))-7*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x
+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+3*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*b^2-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+13*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a*b-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/b^2/(-1/a*
b)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cosh\left(e+f\,x\right)}^6}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.387 $\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=244

$$\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{ab^2f}-\frac{(2a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+fx))|1\right.}{ab^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -(a-b)*cosh(f*x+e)*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-(2*a-b)*(1/(
1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+si
nh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/b
^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))^(1/2
)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-
b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/b/f/(sech(f*x+e)^2*(a+b
*sinh(f*x+e)^2)/a)^(1/2)+(2*a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/b^
2/f
```

Rubi [A] time = 0.22, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3192, 413, 531, 418, 492, 411}

$$\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{ab^2f}-\frac{(2a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+fx))|1\right.}{ab^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(a*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
 + b*Sinh[e + f*x]^2])/(a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)
)/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a*b^2*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}}{abj}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}}{bf}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathrm{sech}(e+}{abf\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\,\mathrm{si}\right.}{a}}}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(2a-b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathrm{s}}{ab^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)(}{}}}$$

Mathematica [C] time = 0.62, size = 155, normalized size = 0.64

$$\frac{(a-b)\left(-\sqrt{2}\,b\sinh(2(e+fx))+4ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)\right)-2ia(2a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,E\left(\right.}{2ab^2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (a - b)*((4*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ell
ipticF[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]))/(2*a*b^2*f*Sqrt[2*
a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\cosh\left(fx+e\right)^{4}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.19, size = 322, normalized size = 1.32

$$-\frac{\left(\sqrt{-\frac{b}{a}}\,a-\sqrt{-\frac{b}{a}}\,b\right)\left(\cosh^{2}\left(fx+e\right)\right)\sinh\left(fx+e\right)+a\sqrt{\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh
(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2
+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),
(a/b)^(1/2))*a+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/b/(-1/a*b)^(1/2)/a/cosh(f*
x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(e+f\,x\right)^4}{{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.388 $\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

```
[Out] cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)*Ell
ipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2
))/f/a^(1/2)/b^(1/2)/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(
f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3192, 411}

$$\frac{\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])
/(Sqrt[a]*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\left(a+bx^2\right)^{3/2}}\,dx,x,\sinh(e+fx)\right)}{f}\\ &= \frac{\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.32, size = 143, normalized size = 1.57

$$\frac{-i\sqrt{2}\,a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right)+i\sqrt{2}\,a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,E\left(i(e+fx)\middle|\frac{b}{a}\right)+b\sinh(2(e+fx))}{abf\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x),
 b/a] - I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e
+ f*x), b/a] + b*Sinh[2*(e + f*x)])/(a*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e +
 f*x)]])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\cosh\left(fx+e\right)^{2}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.18, size = 181, normalized size = 1.99

$$\frac{\sqrt{-\frac{b}{a}}\,\sinh\left(fx+e\right)\left(\cosh^{2}\left(fx+e\right)\right)+\sqrt{\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\right.}{\sqrt{-\frac{b}{a}}\,a\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] ((-1/a*b)^(1/2)*sinh(f*x+e)*cosh(f*x+e)^2+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(b
/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)
^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(e+f\,x\right)^2}{{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.389 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] -b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-I*(cos(I*e+I
*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*si
nh(f*x+e)^2)^(1/2)/a/(a-b)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3184, 21, 3178, 3177}

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{-a-b\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,d}{a(a-b)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.16, size = 100, normalized size = 0.87

$$\frac{-\sqrt{2}\,b\sinh(2(e+fx)) - 2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,E\left(i(e+fx)\middle|\frac{b}{a}\right)}{2af(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x
+ e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.00, size = 253, normalized size = 2.20

$$-\frac{\sqrt{-\frac{b}{a}}\,b\sinh\left(fx+e\right)\left(\cosh^2\left(fx+e\right)\right) - a\sqrt{\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -((-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^2-a*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/a/(a-
b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b \sinh\left(e+fx\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-3/2), x)
```

3.390 $\int \frac{\text{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=217

$$\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{b}\,(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\,f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{2b\text{sech}(e+fx)\sqrt{a+b\,\text{si}}}{af(a-b)^2}$$

```
[Out] (a+b)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/
2)*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b
)^(1/2))*b^(1/2)/(a-b)^2/f/a^(1/2)/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1
/2)/(a+b*sinh(f*x+e)^2)^(1/2)-2*b*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sec
h(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x
+e)^2)/a)^(1/2)+tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3192, 414, 525, 418, 411}

$$\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{b}\,(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\,f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{2b\text{sech}(e+fx)\sqrt{a+b\,\text{si}}}{af(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqr
t[a]], 1 - a/b])/(Sqrt[a]*(a - b)^2*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[
e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*b*EllipticF[ArcTan[Sinh[e +
f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)^2*f*S
qrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*
f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx,x,\sinh(e+fx)\right.}{f}$$

$$= \frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int -\right.}{(-a+b)f}$$

$$= \frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\right.}{(a-b)(-a+}$$

$$= \frac{\sqrt{b}\,(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\,(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{2bF\left(\tan^{-1}(\sinh(e\right.}{a(a}$$

Mathematica [C] time = 1.22, size = 178, normalized size = 0.82

$$\frac{\tanh(e+fx)\left(2a^2+b(a+b)\cosh(2(e+fx))-ab+b^2\right)-i\sqrt{2}\,a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)+i}{af(a-b)^2\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (I*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] - I*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticF[I*(e + f*x), b/a] + (2*a^2 - a*b + b^2 + b*(a + b)*Cosh[2*(e + f
*x)])*Tanh[e + f*x])/(a*(a - b)^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]]
)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\operatorname{sech}\left(fx+e\right)^2}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.21, size = 345, normalized size = 1.59

$$-\frac{-\sqrt{-\frac{b}{a}}\, ab\left(\sinh^3\left(fx+e\right)\right)-\sqrt{-\frac{b}{a}}\, b^2\left(\sinh^3\left(fx+e\right)\right)+a\sqrt{\frac{a+b\left(\sinh^2\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(-(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^3-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+a*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*b-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2
)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+((a+b*sinh(f*x+e)^2
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*a*b+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-(-1/a*b)^(1/2)*a^2*sinh(f*x+e)-(
-1/a*b)^(1/2)*b^2*sinh(f*x+e))/(a-b)^2/a/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(e+f\,x\right)^2\,{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(e + fx)}{\left(a + b\sinh^2(e + fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sech(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.391 $\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f}-\frac{(a-b)\sinh(e+fx)\cosh^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

```
[Out] arctanh(sinh(f*x+e)*b^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/b^(5/2)/f-1/3*(a-b)*
cosh(f*x+e)^2*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/3*(a-b)*(3*a+2*
b)*sinh(f*x+e)/a^2/b^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 413, 385, 217, 206}

$$-\frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f}-\frac{(a-b)\sinh(e+fx)\cosh^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(5/2)*f) -
((a - b)*Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3
/2)) - ((a - b)*(3*a + 2*b)*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sinh[e +
 f*x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+2b+3ax^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx\right)}{3abf}$$

$$= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \text{Subs}$$

$$= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \text{Subs}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(a-b)(3a-}{3a^2b^2f\sqrt{a}}$$

Mathematica [A] time = 0.89, size = 126, normalized size = 0.94

$$\frac{\frac{2\sqrt{2}(b-a)\sinh(e+fx)\left(3a^2+b(2a+b)\cosh(2(e+fx))+ab-b^2\right)}{3a^2b^2(2a+b\cosh(2(e+fx))-b)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{b^{5/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]
]]/b^(5/2) + (2*Sqrt[2]*(-a + b)*(3*a^2 + a*b - b^2 + b*(2*a + b)*Cosh[2*(e
 + f*x)])*Sinh[e + f*x])/(3*a^2*b^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2)))
/f
```

fricas [B] time = 2.14, size = 6774, normalized size = 50.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7
 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a
^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*
```

```
cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*
(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)
^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2)*
sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(f*x + e)^5 + 10*(2*a^3*b - a^
2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e))*sinh(
f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x
+ e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*
a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*(a^2*b^2*co
sh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b +
3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*
b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x
 + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*
a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(
f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2
*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2
+ 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*
a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x +
e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*
(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6
*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x +
 e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b +
 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2
 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*c
osh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2
 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3
*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^
4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f
*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b
+ 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 -
 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2
*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b
^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b
^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e
)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3
*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sin
h(f*x + e)^5 + sinh(f*x + e)^6)) + 3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*c
osh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b
^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sin
h(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*
x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 +
 2*(35*a^2*b^2*cosh(f*x + e)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b
- a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(f
*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a
^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e
)^2 + 4*(7*a^2*b^2*cosh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^4
 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*sin
h(f*x + e)^2 + 8*(a^2*b^2*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x
+ e)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2
)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f
*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*
cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*
x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
```

)^2)) - 8*sqrt(2)*((2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^6 + 6*(2*a^2*b^2
 - a*b^3 - b^4)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b^2 - a*b^3 - b^4)*s
inh(f*x + e)^6 + 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^4 + 3*
(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4 + 5*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x +
 e)^2)*sinh(f*x + e)^4 - 2*a^2*b^2 + a*b^3 + b^4 + 4*(5*(2*a^2*b^2 - a*b^3
 - b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e
))*sinh(f*x + e)^3 - 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^2
 + 3*(5*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^4 - 2*a^3*b + 2*a^2*b^2 + a*
b^3 - b^4 + 6*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x
 + e)^2 + 6*((2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^5 + 2*(2*a^3*b - 2*a^2
*b^2 - a*b^3 + b^4)*cosh(f*x + e)^3 - (2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*c
osh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
)))/(a^2*b^5*f*cosh(f*x + e)^8 + 8*a^2*b^5*f*cosh(f*x + e)*sinh(f*x + e)^7
 + a^2*b^5*f*sinh(f*x + e)^8 + a^2*b^5*f + 4*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*
x + e)^6 + 4*(7*a^2*b^5*f*cosh(f*x + e)^2 + (2*a^3*b^4 - a^2*b^5)*f)*sinh(f
x + e)^6 + 2(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x + e)^4 + 8*(7*
a^2*b^5*f*cosh(f*x + e)^3 + 3*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e))*sinh(f
x + e)^5 + 2(35*a^2*b^5*f*cosh(f*x + e)^4 + 30*(2*a^3*b^4 - a^2*b^5)*f*co
sh(f*x + e)^2 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f)*sinh(f*x + e)^4 + 4*
(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^2 + 8*(7*a^2*b^5*f*cosh(f*x + e)^5 +
10*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^3 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2
*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^2*b^5*f*cosh(f*x + e)^6 + 1
5*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^
2*b^5)*f*cosh(f*x + e)^2 + (2*a^3*b^4 - a^2*b^5)*f)*sinh(f*x + e)^2 + 8*(a^
2*b^5*f*cosh(f*x + e)^7 + 3*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^5 + (8*a^
4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x + e)^3 + (2*a^3*b^4 - a^2*b^5)*f*
cosh(f*x + e))*sinh(f*x + e)), -1/6*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2
*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2
*b^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*s
inh(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(
f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4
 + 2*(35*a^2*b^2*cosh(f*x + e)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*
b - a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh
(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3
*a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x +
 e)^2 + 4*(7*a^2*b^2*cosh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)
^4 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*s
inh(f*x + e)^2 + 8*(a^2*b^2*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*
x + e)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b
^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x
 + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 +
 b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*
cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)
*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*
x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x
 + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + 3*(a^2*b^2*cosh(
f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e
)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2
 + 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*
a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^
2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)^4 + 8*a^4 - 8*a^3*b +
3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b
^2 + 8*(7*a^2*b^2*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^3
 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b
 - a^2*b^2)*cosh(f*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x + e)^6 + 15*(2*a^3*b -
a^2*b^2)*cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b
^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*(a^2*b^2*cosh(f*x + e)^7 + 3*(2*a^

```
3*b - a^2*b^2)*cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e
)^3 + (2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-b)*arctan(sqr
t(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1
)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e
)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*c
osh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*c
osh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + 4*sqrt(2)*(
(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^6 + 6*(2*a^2*b^2 - a*b^3 - b^4)*cos
h(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b^2 - a*b^3 - b^4)*sinh(f*x + e)^6 + 3*
(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^4 + 3*(2*a^3*b - 2*a^2*b^
2 - a*b^3 + b^4 + 5*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e
)^4 - 2*a^2*b^2 + a*b^3 + b^4 + 4*(5*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e
)^3 + 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e))*sinh(f*x + e)^3
- 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^2 + 3*(5*(2*a^2*b^2 -
 a*b^3 - b^4)*cosh(f*x + e)^4 - 2*a^3*b + 2*a^2*b^2 + a*b^3 - b^4 + 6*(2*a^
3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((2*a^2
*b^2 - a*b^3 - b^4)*cosh(f*x + e)^5 + 2*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)
*cosh(f*x + e)^3 - (2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e))*sinh(
f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b^5*f*cosh
(f*x + e)^8 + 8*a^2*b^5*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^5*f*sinh(f*
x + e)^8 + a^2*b^5*f + 4*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^6 + 4*(7*a^2
*b^5*f*cosh(f*x + e)^2 + (2*a^3*b^4 - a^2*b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^
4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x + e)^4 + 8*(7*a^2*b^5*f*cosh(f*x
+ e)^3 + 3*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a
^2*b^5*f*cosh(f*x + e)^4 + 30*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^2 + (8*
a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b^4 - a^2*b^
5)*f*cosh(f*x + e)^2 + 8*(7*a^2*b^5*f*cosh(f*x + e)^5 + 10*(2*a^3*b^4 - a^2
*b^5)*f*cosh(f*x + e)^3 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x +
e))*sinh(f*x + e)^3 + 4*(7*a^2*b^5*f*cosh(f*x + e)^6 + 15*(2*a^3*b^4 - a^2*
b^5)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x +
 e)^2 + (2*a^3*b^4 - a^2*b^5)*f)*sinh(f*x + e)^2 + 8*(a^2*b^5*f*cosh(f*x +
e)^7 + 3*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^5 + (8*a^4*b^3 - 8*a^3*b^4 +
 3*a^2*b^5)*f*cosh(f*x + e)^3 + (2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e))*sinh
(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.5Error: Bad Argument Type
```

maple [C] time = 0.14, size = 65, normalized size = 0.49

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\cosh^4(fx+e)}{\left(b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2\right)\sqrt{a+b\left(\sinh^2(fx+e)\right)}}, \sinh(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^4/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(cosh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(e+f\,x\right)^5}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(cosh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

3.392 $\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh^2(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/3*cosh(f*x+e)^2*sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*sinh(f*x+e)
/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3190, 378, 191}

$$\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh^2(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*
Sinh[e + f*x])/(3*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh^2(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{3af}$$

$$= \frac{\cosh^2(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A] time = 0.10, size = 50, normalized size = 0.68

$$\frac{(a+2b)\sinh^3(e+fx)+3a\sinh(e+fx)}{3a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*a*Sinh[e + f*x] + (a + 2*b)*Sinh[e + f*x]^3)/(3*a^2*f*(a + b*Sinh[e + f*
x]^2)^(3/2))
```

fricas [B] time = 0.99, size = 945, normalized size = 12.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*((a + 2*b)*cosh(f*x + e)^6 + 6*(a + 2*b)*cosh(f*x + e)*sinh(f*x
 + e)^5 + (a + 2*b)*sinh(f*x + e)^6 + 3*(3*a - 2*b)*cosh(f*x + e)^4 + 3*(5*
(a + 2*b)*cosh(f*x + e)^2 + 3*a - 2*b)*sinh(f*x + e)^4 + 4*(5*(a + 2*b)*cos
h(f*x + e)^3 + 3*(3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(3*a - 2*b)
*cosh(f*x + e)^2 + 3*(5*(a + 2*b)*cosh(f*x + e)^4 + 6*(3*a - 2*b)*cosh(f*x
+ e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 6*((a + 2*b)*cosh(f*x + e)^5 + 2*(3*a
 - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) - a - 2*
b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a^2*b^2*f*cosh(f*x + e
)^8 + 8*a^2*b^2*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*f*sinh(f*x + e)^8
 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^6 + 4*(7*a^2*b^2*f*cosh(f*x + e)^2
 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)
*f*cosh(f*x + e)^4 + 8*(7*a^2*b^2*f*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)
*f*cosh(f*x + e))*sinh(f*x + e)^5 + a^2*b^2*f + 2*(35*a^2*b^2*f*cosh(f*x +
e)^4 + 30*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (8*a^4 - 8*a^3*b + 3*a^2*
b^2)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + 8*(7*a^
2*b^2*f*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (8*a^4
 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^2*b^2*f*c
osh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 3*(8*a^4 - 8*a^
3*b + 3*a^2*b^2)*f*cosh(f*x + e)^2 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^2
 + 8*(a^2*b^2*f*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^5 +
 (8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*f*co
sh(f*x + e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.43Error: Bad Argument Typ
e
```

maple [C] time = 0.14, size = 65, normalized size = 0.89

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\cosh^2(fx+e)}{\left(b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2\right)\sqrt{a+b\left(\sinh^2(fx+e)\right)}}, \sinh(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [B] time = 0.52, size = 927, normalized size = 12.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b^4*e^(-10*f*x - 10*e) - 4*a^3*b + 6*a^2*b^2 - b^4 - (16*a^4 - 32*a^
3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-2*f*x - 2*e) + 10*(2*a^3*b - 3*a^2*
b^2 + 3*a*b^3 - b^4)*e^(-4*f*x - 4*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-
6*f*x - 6*e) + 5*(2*a*b^3 - b^4)*e^(-8*f*x - 8*e))/((a^4 - 2*a^3*b + a^2*b^
2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) + 1/4*(
2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-2*f
*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-4*f*x
 - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) + 5*(4*
a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x - 10
*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x
 - 4*e) + b)^(5/2)*f) - 1/4*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*
e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x - 4*e
) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x - 6*e)
+ 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^2 - 2
*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e
^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) + 1/12*(b^4 + 5*(2*a*b^3
 - b^4)*e^(-2*f*x - 2*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-4*f*x - 4*e)
+ 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^(-6*f*x - 6*e) - (16*a^4 - 32*
a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-8*f*x - 8*e) - (4*a^3*b - 6*a^2*b
^2 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

mupad [B] time = 1.83, size = 144, normalized size = 1.97

$$\frac{2\,e^{e+f\,x}\,\left(e^{2\,e+2\,f\,x}-1\right)\,\sqrt{a+b\,{\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)}^2}\,\left(a+2\,b+10\,a\,e^{2\,e+2\,f\,x}+a\,e^{4\,e+4\,f\,x}-4\,b\,e^{2\,e+2\,f\,x}+2\,b\,e^{4\,e+4\,f\,x}\right)}{3\,a^2\,f\,{\left(b+4\,a\,e^{2\,e+2\,f\,x}-2\,b\,e^{2\,e+2\,f\,x}+b\,e^{4\,e+4\,f\,x}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] (2*exp(e + f*x)*(exp(2*e + 2*f*x) - 1)*(a + b*(exp(e + f*x)/2 - exp(- e - f
*x)/2)^2)^(1/2)*(a + 2*b + 10*a*exp(2*e + 2*f*x) + a*exp(4*e + 4*f*x) - 4*b
*exp(2*e + 2*f*x) + 2*b*exp(4*e + 4*f*x)))/(3*a^2*f*(b + 4*a*exp(2*e + 2*f*
x) - 2*b*exp(2*e + 2*f*x) + b*exp(4*e + 4*f*x))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.393 $\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/3*sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*sinh(f*x+e)/a^2/f/(a+b*si
nh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3190, 192, 191}

$$\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Sinh[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*Sinh[e + f*x])/(3*
a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx,x,\sinh(e+fx)\right)}{f}\\ &= \frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx,x,\sinh(e+fx)\right)}{3af}\\ &= \frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 47, normalized size = 0.72

$$\frac{\sinh(e+fx)\left(3a+2b\sinh^2(e+fx)\right)}{3a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sinh[e + f*x]*(3*a + 2*b*Sinh[e + f*x]^2))/(3*a^2*f*(a + b*Sinh[e + f*x]^2
)^(3/2))
```

fricas [B] time = 2.05, size = 912, normalized size = 14.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(2)*(b*cosh(f*x + e)^6 + 6*b*cosh(f*x + e)*sinh(f*x + e)^5 + b*sinh
(f*x + e)^6 + 3*(2*a - b)*cosh(f*x + e)^4 + 3*(5*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^4 + 4*(5*b*cosh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e))*si
nh(f*x + e)^3 - 3*(2*a - b)*cosh(f*x + e)^2 + 3*(5*b*cosh(f*x + e)^4 + 6*(2
*a - b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 6*(b*cosh(f*x + e)^5 +
 2*(2*a - b)*cosh(f*x + e)^3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) - b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a^2*b^2*f*cosh(f*x + e)^8
 + 8*a^2*b^2*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*f*sinh(f*x + e)^8 +
4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^6 + 4*(7*a^2*b^2*f*cosh(f*x + e)^2 +
(2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*
cosh(f*x + e)^4 + 8*(7*a^2*b^2*f*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*f*
cosh(f*x + e))*sinh(f*x + e)^5 + a^2*b^2*f + 2*(35*a^2*b^2*f*cosh(f*x + e)^
4 + 30*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (8*a^4 - 8*a^3*b + 3*a^2*b^2
)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + 8*(7*a^2*b
^2*f*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (8*a^4 -
8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^2*b^2*f*cosh
(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 3*(8*a^4 - 8*a^3*b
 + 3*a^2*b^2)*f*cosh(f*x + e)^2 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^2 +
8*(a^2*b^2*f*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^5 + (8
*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*f*cosh(
f*x + e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.46Error: Bad Argument Typ
e
```

maple [A] time = 0.02, size = 56, normalized size = 0.86

$$\frac{\frac{\sinh(fx+e)}{3a\left(a+b\left(\sinh^2(fx+e)\right)\right)^{\frac{3}{2}}}+\frac{2\sinh(fx+e)}{3a^2\sqrt{a+b\left(\sinh^2(fx+e)\right)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/f*(1/3*sinh(f*x+e)/a/(a+b*sinh(f*x+e)^2)^(3/2)+2/3/a^2*sinh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2))
```

maxima [B] time = 0.47, size = 485, normalized size = 7.46

$$\frac{2\,a^2b^2 - 2\,ab^3 + b^4 + 5\left(4\,a^3b - 6\,a^2b^2 + 4\,ab^3 - b^4\right)e^{\left(-2\,fx-2\,e\right)} + 2\left(24\,a^4 - 48\,a^3b + 49\,a^2b^2 - 25\,ab^3 + 5\,b^4\right)e^{\left(-4\,f\right.}}{3\left(a^4 - 2\,a^3b + a^2b^2\right)\left(2\,(2\,a - b)e^{\left(\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^
(-2*f*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-
4*f*x - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) +
5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x
 - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-
4*f*x - 4*e) + b)^(5/2)*f) - 1/3*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 +
b^4)*e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x
- 4*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x -
6*e) + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^
2 - 2*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a -
 b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

mupad [B] time = 1.49, size = 129, normalized size = 1.98

$$\frac{4\,\mathrm{e}^{e+f\,x}\,\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)\,\sqrt{a+b\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}\,\left(b+6\,a\,\mathrm{e}^{2\,e+2\,f\,x}-2\,b\,\mathrm{e}^{2\,e+2\,f\,x}+b\,\mathrm{e}^{4\,e+4\,f\,x}\right)}{3\,a^2\,f\,{\left(b+4\,a\,\mathrm{e}^{2\,e+2\,f\,x}-2\,b\,\mathrm{e}^{2\,e+2\,f\,x}+b\,\mathrm{e}^{4\,e+4\,f\,x}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] (4*exp(e + f*x)*(exp(2*e + 2*f*x) - 1)*(a + b*(exp(e + f*x)/2 - exp(- e - f
*x)/2)^2)^(1/2)*(b + 6*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + b*exp(4*
e + 4*f*x)))/(3*a^2*f*(b + 4*a*exp(2*e + 2*f*x) - 2*b*exp(2*e + 2*f*x) + b*
exp(4*e + 4*f*x))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.394 $\int \frac{\text{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{b(5a-2b)\sinh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{b\sinh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

```
[Out] arctan(sinh(f*x+e)*(a-b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2))/(a-b)^(5/2)/f-1/3
*b*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/3*(5*a-2*b)*b*sinh(f*x
+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3190, 414, 527, 12, 377, 203}

$$-\frac{b(5a-2b)\sinh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{b\sinh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(5
/2)*f) - (b*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) -
((5*a - 2*b)*b*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+f\right.}{3a(a-b)f}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \text{Su}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \text{Su}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \text{Su}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b}{3a^2(a-b)^2f\sqrt{a}}$$

Mathematica [C] time = 9.37, size = 1331, normalized size = 9.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sech[e + f*x]*Tanh[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]
] + (2100*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2)/a +
(840*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^4)/a^2 - (
3150*(a - b)*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Tanh[e + f*x]^2)/a -
```

```
(4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2*
Tanh[e + f*x]^2)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]
^2)/a]]*Sinh[e + f*x]^4*Tanh[e + f*x]^2)/a^3 + (1575*(a - b)^2*ArcSin[Sqrt[
((a - b)*Tanh[e + f*x]^2)/a]]*Tanh[e + f*x]^4)/a^2 + (2100*(a - b)^2*b*ArcS
in[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2*Tanh[e + f*x]^4)/a^3
+ (840*(a - b)^2*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x
]^4*Tanh[e + f*x]^4)/a^4 + 2100*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^
2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) + (2800*b*Sinh[e + f*x]^2*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(
3/2))/a + (1120*b^2*Sinh[e + f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f
*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))/a^2 + 96*Hypergeometric2F1[
2, 2, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
 + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 24*HypergeometricPFQ[{
2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a +
 b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + (168*b*Hyperg
eometric2F1[2, 2, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7
/2))/a + (48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x
]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(
((a - b)*Tanh[e + f*x]^2)/a)^(7/2))/a + (72*b^2*Hypergeometric2F1[2, 2, 9/2
, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2))/a^2 + (24*b^2*Hy
pergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e +
f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e +
 f*x]^2)/a)^(7/2))/a^2 - 1575*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2)*Tanh[e + f*x]^2)/a^2] - (2100*b*Sinh[e + f*x]^2*Sqrt[((a - b)*Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2])/a - (840*b^2*Sin
h[e + f*x]^4*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e +
 f*x]^2)/a^2])/a^2))/(315*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e +
f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(1 + (b*Sinh[e + f*x]^2)/a)*(((a - b)*Ta
nh[e + f*x]^2)/a)^(5/2))
```

fricas [B] time = 1.24, size = 5396, normalized size = 40.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7
 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a
^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*
cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*
(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)
^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2)*
sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(f*x + e)^5 + 10*(2*a^3*b - a^
2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e))*sinh(
f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x
+ e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*
a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*(a^2*b^2*co
sh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b +
3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)
*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^
2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)
*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sq
rt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*
cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh
(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh
```

(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^6 + 6*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)*sinh(f*x + e)^5 + (5*a^2*b^2 - 7*a*b^3 + 2*b^4)*sinh(f*x + e)^6 + 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^4 + 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4 + 5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 - 5*a^2*b^2 + 7*a*b^3 - 2*b^4 + 4*(5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^3 + 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^2 + 3*(5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^4 - 8*a^3*b + 17*a^2*b^2 - 11*a*b^3 + 2*b^4 + 6*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^5 + 2*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^3 - (8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^8 + 8*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^4 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^3 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^2 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f)*sinh(f*x + e)^4 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^2 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^5 + 10*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^3 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^6 + 15*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^4 + 3*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f)*sinh(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f + 8*((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^3 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e)), 1/3*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*(a^2*b^2*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)

```
^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)
^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*((5*a^2*b^2 - 7
*a*b^3 + 2*b^4)*cosh(f*x + e)^6 + 6*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x
+ e)*sinh(f*x + e)^5 + (5*a^2*b^2 - 7*a*b^3 + 2*b^4)*sinh(f*x + e)^6 + 3*(8
*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^4 + 3*(8*a^3*b - 17*a
^2*b^2 + 11*a*b^3 - 2*b^4 + 5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^2
)*sinh(f*x + e)^4 - 5*a^2*b^2 + 7*a*b^3 - 2*b^4 + 4*(5*(5*a^2*b^2 - 7*a*b^3
 + 2*b^4)*cosh(f*x + e)^3 + 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos
h(f*x + e))*sinh(f*x + e)^3 - 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*c
osh(f*x + e)^2 + 3*(5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^4 - 8*a^3
*b + 17*a^2*b^2 - 11*a*b^3 + 2*b^4 + 6*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2
*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*c
osh(f*x + e)^5 + 2*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^
3 - (8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e))*sinh(f*x + e))
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^5*b^2 - 3*a^4*b^3 +
3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^8 + 8*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
 - a^2*b^5)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*
b^4 - a^2*b^5)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a
^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
 - a^2*b^5)*f*cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^
4 + a^2*b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4
*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^4 + 8*(7*(a^5*b^2 - 3*a^4*b^
3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^3 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4
*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^5*b
^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 7*a
^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^2 + (8*a^7 - 32*a
^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f)*sinh(f*x + e)^4
 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e
)^2 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^5 +
10*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^
3 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f
*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*f*cosh(f*x + e)^6 + 15*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4
 + a^2*b^5)*f*cosh(f*x + e)^4 + 3*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b
^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a
^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f)*sinh(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3
*a^3*b^4 - a^2*b^5)*f + 8*((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*co
sh(f*x + e)^7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f
*cosh(f*x + e)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4
 - 3*a^2*b^5)*f*cosh(f*x + e)^3 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*
b^4 + a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.49Error: Bad Argument Typ
e
```

maple [C] time = 0.21, size = 169, normalized size = 1.26

$$\frac{`int/indef0`\left(\frac{-b^2\left(\sinh^4(fx+e)\right)-2ab\left(\sinh^2(fx+e)\right)-a^2}{\left(-b^4\left(\sinh^{10}(fx+e)\right)+\left(-4a\,b^3-b^4\right)\left(\sinh^8(fx+e)\right)+\left(-6a^2b^2-4a\,b^3\right)\left(\sinh^6(fx+e)\right)+\left(-4a^3b-6a^2b^2\right)\left(\sinh^4(fx+e)\right)+\left(-a^4-4a^3b\right)\left(\right.}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`((-b^2*sinh(f*x+e)^4-2*a*b*sinh(f*x+e)^2-a^2)/(-b^4*sinh(f*x+e)
^10+(-4*a*b^3-b^4)*sinh(f*x+e)^8+(-6*a^2*b^2-4*a*b^3)*sinh(f*x+e)^6+(-4*a^3
*b-6*a^2*b^2)*sinh(f*x+e)^4+(-a^4-4*a^3*b)*sinh(f*x+e)^2-a^4)/(a+b*sinh(f*x
+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh\left(e+f\,x\right)\,{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sech(e + f*x)/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.395 $\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=330

$$-\frac{2(a-b)(2a+b)\sinh(e+fx)\cosh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{(4a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+f\right.}{3a^2b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -1/3*(a-b)*cosh(f*x+e)^3*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(a
-b)*(2*a+b)*cosh(f*x+e)*sinh(f*x+e)/a^2/b^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3
*(8*a^2-3*a*b-2*b^2)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*El
lipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/a^2/b^3/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
+1/3*(4*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(
sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/a^2/b^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(8*
a^2-3*a*b-2*b^2)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^2/b^3/f
```

Rubi [A] time = 0.33, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3192, 413, 526, 531, 418, 492, 411}

$$\frac{\left(8a^2-3ab-2b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2b^3f}-\frac{2(a-b)(2a+b)\sinh(e+fx)\cosh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{(4a-b)\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*(a - b)*(2*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt
[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e
 + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b^3*f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((4*a - b)*EllipticF[A
rcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(
3*a^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((8*a^2 -
3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*b^3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^{5/2}}{(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subs}}{}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] time = 2.24, size = 206, normalized size = 0.62

$$\frac{\frac{1}{2}(a-b)\left(-2\sqrt{2}\,b\sinh(2(e+fx))\left(8a^2+b(5a+2b)\cosh(2(e+fx))+ab-2b^2\right)+4ia^2(8a+b)\left(\frac{2a+b\cosh(2(e+f}{a}\right.\right.}{6a^2b^3f(2a+b\cosh(2(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-2*I)*a^2*(8*a^2 - 3*a*b - 2*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/
2)*EllipticE[I*(e + f*x), b/a] + ((a - b)*((4*I)*a^2*(8*a + b)*((2*a - b +
b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - 2*Sqrt[2]*b*(8*
a^2 + a*b - 2*b^2 + b*(5*a + 2*b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]))/2)
/(6*a^2*b^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\cosh\left(fx+e\right)^6}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^6/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.46Error: Bad Argument Typ
e
```

maple [B] time = 0.22, size = 812, normalized size = 2.46

$$-\frac{\left(5\sqrt{-\frac{b}{a}}\,a^{2}b-3\sqrt{-\frac{b}{a}}\,a\,b^{2}-2\sqrt{-\frac{b}{a}}\,b^{3}\right)\sinh\left(fx+e\right)\left(\cosh^{4}\left(fx+e\right)\right)+\left(4\sqrt{-\frac{b}{a}}\,a^{3}-6\sqrt{-\frac{b}{a}}\,a^{2}b+2\sqrt{-\frac{b}{a}}\,b^{3}\right)\left(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((5*(-1/a*b)^(1/2)*a^2*b-3*(-1/a*b)^(1/2)*a*b^2-2*(-1/a*b)^(1/2)*b^3)*
sinh(f*x+e)*cosh(f*x+e)^4+(4*(-1/a*b)^(1/2)*a^3-6*(-1/a*b)^(1/2)*a^2*b+2*(-
1/a*b)^(1/2)*b^3)*cosh(f*x+e)^2*sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh
(f*x+e)^2+(a-b)/a)^(1/2)*b*(4*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a^2-2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*Elliptic
F(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-8*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a^2+3*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b+2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e
)^2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sin
h(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3-6*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a
^2*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a^3+11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a*b^2-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/a^2/(a+b*sinh(f*x+e)^2)^(3/2)/(
-1/a*b)^(1/2)/b^2/cosh(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(fx+e\right)^{6}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(e+f\,x\right)^{6}}{\left(b\sinh\left(e+f\,x\right)^{2}+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cosh(e + f*x)^6/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.396 $\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{2(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}b^{3/2}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1\right.}{3a^2bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -1/3*(a-b)*cosh(f*x+e)*sinh(f*x+e)/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)+2/3*(a+b
)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)*E
llipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1
/2))/a^(3/2)/b^(3/2)/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sin
h(f*x+e)^2)^(1/2)-1/3*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+
b*sinh(f*x+e)^2)^(1/2)/a^2/b/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3192, 413, 525, 418, 411}

$$\frac{2(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}b^{3/2}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1\right.}{3a^2bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) + (2*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqr
t[a]], 1 - a/b])/(3*a^(3/2)*b^(3/2)*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[
e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (EllipticF[ArcTan[Sinh[e + f*x]
], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b*f*Sqrt[(Sec
h[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}}{3a}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}}{3ab}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh}{\sqrt{a}}\right.\right.}{3a^{3/2}b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sin}}$$

Mathematica [C] time = 1.44, size = 178, normalized size = 0.80

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(a^2+b(a+b)\cosh(2(e+fx))+2ab-b^2\right)-ia^2(2a+b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}F\left(i(e+f\right.}{3a^2b^2f(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e
 + f*x), b/a] - I*a^2*(2*a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*E
llipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(a^2 + 2*a*b - b^2 + b*(a + b)*Cosh[
2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*b^2*f*(2*a - b + b*Cosh[2*(e + f*x)
])^(3/2))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\cosh\left(fx+e\right)^4}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.44Error: Bad Argument Typ
e
```

maple [B] time = 0.20, size = 597, normalized size = 2.68

$$\left(2\sqrt{-\frac{b}{a}}\, ab + 2\sqrt{-\frac{b}{a}}\, b^2\right)\sinh\left(fx+e\right)\left(\cosh^4\left(fx+e\right)\right) + \left(\sqrt{-\frac{b}{a}}\, a^2 + \sqrt{-\frac{b}{a}}\, ab - 2\sqrt{-\frac{b}{a}}\, b^2\right)\left(\cosh^2\left(fx+e\right)\right)\sinh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((2*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+
((-1/a*b)^(1/2)*a^2+(-1/a*b)^(1/2)*a*b-2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*
sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*El
lipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))-2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a-2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2+
a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+2*(b/a
*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/a^2/(a+b*sinh(f*x+e)^2)^(3/2)/(-1/a*b)^(1/2
)/b/cosh(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(e+f\,x\right)^4}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cosh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.397 $\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=228

$$\frac{(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}\sqrt{b}f(a-b)\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx\right.}{3a^2f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] 1/3*cosh(f*x+e)*sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(a-2*b)*cosh(
f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)*Elliptic
E(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(1/2))/a^
(3/2)/(a-b)/f/b^(1/2)/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)/(a+b*sinh
(f*x+e)^2)^(1/2)+1/3*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*El
lipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/
2)
```

Rubi [A] time = 0.20, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3192, 412, 525, 418, 411}

$$\frac{(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}\sqrt{b}f(a-b)\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx\right.}{3a^2f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((a -
 2*b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 -
a/b])/(3*a^(3/2)*(a - b)*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e +
 f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + (EllipticF[ArcTan[Sinh[e + f*x]],
1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)*f*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{1+}}\right.}{3af}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{1+}}\right.}{3a(a-b)f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right.\right.}{3a^{3/2}(a-b)\sqrt{b}\,f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2}}$$

Mathematica [C] time = 1.44, size = 193, normalized size = 0.85

$$\frac{-\sqrt{2}\,b\sinh(2(e+fx))\left(-4a^2-b(a-2b)\cosh(2(e+fx))+7ab-2b^2\right)-2ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}F\left(\right.}{6a^2bf(a-b)(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a - 2*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*
(e + f*x), b/a] - (2*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/
2)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*(-4*a^2 + 7*a*b - 2*b^2 - (a - 2
*b)*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*a^2*(a - b)*b*f*(2*a - b + b
*Cosh[2*(e + f*x)])^(3/2))
```

fricas [F] time = 2.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\cosh\left(fx+e\right)^2}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.41Error: Bad Argument Typ
e
```

maple [B] time = 0.22, size = 662, normalized size = 2.90

$$\sqrt{-\frac{b}{a}}\, ab \left(\sinh^5\left(fx+e\right)\right) - 2\sqrt{-\frac{b}{a}}\, b^2 \left(\sinh^5\left(fx+e\right)\right) + 2\sqrt{\frac{a+b\left(\sinh^2\left(fx+e\right)\right)}{a}}\, \sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \mathrm{EllipticF}\left(\sinh\left(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5-2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+2*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b
^2*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^2+2*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2*sinh(f*x+e)^2+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-2*(-1/
a*b)^(1/2)*a*b*sinh(f*x+e)^3-2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+2*a^2*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))-2*a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-((a+b*sinh(f*x+e)^2)
/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))*a^2+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e
)-3*(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/a^2/(a-b)/(a+b*sinh(f*x+e)^2)^(3/2)/(-1
/a*b)^(1/2)/cosh(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(e+f\,x\right)^2}{{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(cosh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

3.398 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(}{3af(a-b)\left(a+b\sinh^2(\right.}$$

```
[Out] -1/3*b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(2*a
-b)*b*cosh(f*x+e)*sinh(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-2/3*I
*(2*a-b)*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(
b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)^2/f/(1+b*sinh(f*x+e)^2/a)^(
1/2)+1/3*I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x)
,(b/a)^(1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/
2)
```

Rubi [A] time = 0.28, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(}{3af(a-b)\left(a+b\sinh^2(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] -(b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*S
qrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a
*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a+2b+b\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A] time = 1.30, size = 190, normalized size = 0.76

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(-5a^2+b(b-2a)\cosh(2(e+fx))+5ab-b^2\right)+ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}F\left(i(e+\right.}{3a^2f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*
EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b
)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cos
h[2*(e + f*x)])^(3/2))
```

fricas [F] time = 1.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*
x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.00, size = 406, normalized size = 1.62

$$\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(-\frac{\sinh\left(fx+e\right)\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}}{3ab(a-b)\left(\sinh^{2}\left(fx+e\right)+\frac{a}{b}\right)^{2}}-\frac{2b\left(\cosh^{2}\left(fx+e\right)\right)\sinh\left(fx-\right.}{3a^{2}(a-b)^{2}\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a/b/(a-b)*sinh(f*x+e)*((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+a/b)^2-2/3*b*cosh(f*x+e
)^2/a^2/(a-b)^2*sinh(f*x+e)*(2*a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/
2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2/3*b*(2*a-b)/a^2/(a-b)^2/(-1/a
*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*
x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))))/cosh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\sinh\left(e+fx\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(1/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Integral((a + b*sinh(e + f*x)**2)**(-5/2), x)
```

3.399 $\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{b(9a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{3a^2f(a-b)^3\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\sqrt{b}\left(3a^2+7ab-2b^2\right)\cosh(e+fx)\ldots}{3a^{3/2}f(a-b)^3\sqrt{a+b\sinh^2(\ldots}}$$

```
[Out] 1/3*b*(3*a+b)*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)
+1/3*(3*a^2+7*a*b-2*b^2)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*s
inh(f*x+e)^2/a)^(1/2)*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e
)^2/a)^(1/2),(1-a/b)^(1/2))*b^(1/2)/a^(3/2)/(a-b)^3/f/(a*cosh(f*x+e)^2/(a+b
*sinh(f*x+e)^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(9*a-b)*b*(1/(1+sinh(f
*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e
)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)^3
/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+tanh(f*x+e)/(a-b)/f/(a+b*sin
h(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.31, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3192, 414, 527, 525, 418, 411}

$$\frac{\sqrt{b}\left(3a^2+7ab-2b^2\right)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{b(9a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\ldots}{3a^2f(a-b)^3\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\ldots\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (b*(3*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*(a + b*Sinh[e +
f*x]^2)^(3/2)) + (Sqrt[b]*(3*a^2 + 7*a*b - 2*b^2)*Cosh[e + f*x]*EllipticE[A
rcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*a^(3/2)*(a - b)^3*f*Sq
rt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]
) - ((9*a - b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sq
rt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*
Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*f*(a + b*Sinh[e + f*x]^2)^(3
/2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx,x,\sinh(e+fx)\right.}{f}$$

$$= \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\right.}{(-a+b}$$

$$= \frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} +$$

$$= \frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} +$$

$$= \frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\sqrt{b}\left(3a^2+7ab-2b^2\right)\cosh(e+fx)E}{3a^{3/2}(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

Mathematica [C] time = 3.46, size = 260, normalized size = 0.89

$$\frac{-2ia^2\left(3a^2-2ab-b^2\right)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}F\left(i(e+fx)\middle|\frac{b}{a}\right)+2ia^2\left(3a^2+7ab-2b^2\right)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}E\left(i\right.}{6a^2f(a-b)^3(2a+b\cosh(2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(3*a^2 + 7*a*b - 2*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2
)*EllipticE[I*(e + f*x), b/a] - (2*I)*a^2*(3*a^2 - 2*a*b - b^2)*((2*a - b +
 b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + ((24*a^4 - 24*
a^3*b + 41*a^2*b^2 - 19*a*b^3 + 2*b^4 + 4*a*b*(6*a^2 + 5*a*b - 3*b^2)*Cosh[
2*(e + f*x)] + b^2*(3*a^2 + 7*a*b - 2*b^2)*Cosh[4*(e + f*x)])*Tanh[e + f*x]
)/Sqrt[2])/(6*a^2*(a - b)^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

fricas [F] time = 1.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\mathrm{sech}\left(fx+e\right)^2}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.47Error: Bad Argument Typ
e
```

maple [B] time = 0.24, size = 1002, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(-3*(-1/a*b)^(1/2)*a^2*b^2*sinh(f*x+e)^5-7*(-1/a*b)^(1/2)*a*b^3*sinh(f
*x+e)^5+2*(-1/a*b)^(1/2)*b^4*sinh(f*x+e)^5+6*((a+b*sinh(f*x+e)^2)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2
*b^2*sinh(f*x+e)^2-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*El
lipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^3*sinh(f*x+e)^2+2*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*b^4*sinh(f*x+e)^2+3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b^
2*sinh(f*x+e)^2+7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^3*sinh(f*x+e)^2-2*((a+b*si
nh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^
(1/2),(a/b)^(1/2))*b^4*sinh(f*x+e)^2-6*(-1/a*b)^(1/2)*a^3*b*sinh(f*x+e)^3-8
```

```
*(-1/a*b)^(1/2)*a^2*b^2*sinh(f*x+e)^3-4*(-1/a*b)^(1/2)*a*b^3*sinh(f*x+e)^3+
2*(-1/a*b)^(1/2)*b^4*sinh(f*x+e)^3+6*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*
x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3*b-8*((a
+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*a^2*b^2+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^3+3*((a+b*
sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*a^3*b+7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^
(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b^2-2*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))*a*b^3-3*(-1/a*b)^(1/2)*a^4*sinh(f*x+e)-8*(-1/a*b)^(1/2)*a
^2*b^2*sinh(f*x+e)+3*(-1/a*b)^(1/2)*a*b^3*sinh(f*x+e))/(-1/a*b)^(1/2)/(a+b*
sinh(f*x+e)^2)^(3/2)/a^2/(a-b)^3/cosh(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(e+f\,x\right)^2\,{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sech(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.400 $\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=117

$$\frac{d\sinh(e+fx)\cosh^2(e+fx)^{\frac{1-m}{2}}(d\cosh(e+fx))^{m-1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\right.}{f}$$

```
[Out] d*AppellF1(1/2,1/2-1/2*m,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(d*cosh(
f*x+e))^(-1+m)*(cosh(f*x+e)^2)^(1/2-1/2*m)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^
p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3193, 430, 429}

$$\frac{d\sinh(e+fx)\cosh^2(e+fx)^{\frac{1-m}{2}}(d\cosh(e+fx))^{m-1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2
)/a)]*(d*Cosh[e + f*x])^(-1 + m)*(Cosh[e + f*x]^2)^((1 - m)/2)*Sinh[e + f*x
]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3193

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(
ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Cos[e + f*x])^(2*FracPart[(m - 1)/2]))/(
f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/
2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f
, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(d(d\cosh(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cosh^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \left(1+\right.\right.}{f}$$

$$= \frac{\left(d(d\cosh(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cosh^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\left(a+b\sinh^2(e\right.\right.}{}$$

$$= \frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)(d\cosh(e}{}$$

Mathematica [F] time = 9.20, size = 0, normalized size = 0.00

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 2.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\left(d\cosh\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\left(d\cosh\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \left(d\cosh\left(fx+e\right)\right)^m\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\left(d\cosh\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\cosh\left(e+f\,x\right)\right)^m\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cosh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p,x)
[Out] int((d*cosh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))**m*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

3.401 $\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=214

$$\frac{\left(3a^2-2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\right.}{b^2f(2p+3)(2p+5)}$$

```
[Out] -(3*a-b*(7+2*p))*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1+p)/b^2/f/(4*p^2+16*p+15
)+cosh(f*x+e)^2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1+p)/b/f/(5+2*p)+(3*a^2-2*
a*b*(5+2*p)+b^2*(4*p^2+16*p+15))*hypergeom([1/2, -p],[3/2],-b*sinh(f*x+e)^2
/a)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p/b^2/f/(4*p^2+16*p+15)/((1+b*sinh(f*x+
e)^2/a)^p)
```

Rubi [A] time = 0.20, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 416, 388, 246, 245}

$$\frac{\left(3a^2-2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\right.}{b^2f(2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a - b*(7 + 2*p))*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b^2*
f*(3 + 2*p)*(5 + 2*p))) + (Cosh[e + f*x]^2*Sinh[e + f*x]*(a + b*Sinh[e + f*
x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 2*a*b*(5 + 2*p) + b^2*(15 + 16*p
 + 4*p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e
 + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Sinh[
e + f*x]^2)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
```

```
1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\frac{\operatorname{Subst}\left(\int\left(1+x^2\right)^2\left(a+bx^2\right)^p\,dx,x,\sinh(e+fx)\right)}{f}$$

$$=\frac{\cosh^2(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(5+2p)}+\frac{\operatorname{Subst}\left(\int\left(\right.\right.}{}$$

$$=-\frac{(3a-b(7+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}+\frac{\cosh^2(e}{}$$

$$=-\frac{(3a-b(7+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}+\frac{\cosh^2(e}{}$$

$$=-\frac{(3a-b(7+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}+\frac{\cosh^2(e}{}$$

Mathematica [F] time = 10.76, size = 0, normalized size = 0.00

$$\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \left(\cosh^5\left(fx+e\right)\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh\left(e+f\,x\right)^5\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p,x)
[Out] int(cosh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

3.402 $\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=125

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{bf(2p+3)}-\frac{(a-b(2p+3))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\ {}_2F}{bf(2p+3)}$$

```
[Out] sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1+p)/b/f/(3+2*p)-(a-b*(3+2*p))*hypergeom([
1/2, -p],[3/2],-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p/b/f/(3
+2*p)/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 388, 246, 245}

$$\frac{\left(1-\frac{a}{2bp+3b}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{f}+\frac{\sinh(e+fx)\left(a\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) + ((1 - a/(
3*b + 2*b*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sin
h[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+bx^2\right)^p dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(1-\frac{a}{3b+2bp}\right)\text{Subst}\left(\int \left(\right.\right.}{}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(\left(1-\frac{a}{3b+2bp}\right)\left(a+b\sin\right.\right.}{}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(1-\frac{a}{3b+2bp}\right){}_2F_1\left(\frac{1}{2},-p\right.}{}$$

Mathematica [A] time = 0.20, size = 120, normalized size = 0.96

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\left((b(2p+3)-a)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)+\left(a+b\,\mathrm{si}\right.\right.}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p*((-a + b*(3 + 2*p))*Hypergeometric
2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)] + (a + b*Sinh[e + f*x]^2)*(1 +
(b*Sinh[e + f*x]^2)/a)^p))/(b*f*(3 + 2*p)*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \left(\cosh^3\left(fx+e\right)\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sinh \left(fx+e\right)^2+a\right)^p \cosh \left(fx+e\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh \left(e+f\,x\right)^3\,{\left(b\,\sinh \left(e+f\,x\right)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p,x)
[Out] int(cosh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

3.403 $\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] hypergeom([1/2, -p],[3/2],-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*sinh(f*x+e)^
2)^p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 246, 245}

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a
 + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^p\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx^2}{a}\right)^p\right.}{f}$$

$$= \frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\right.}{f}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 1.00

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a
 + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

fricas [F] time = 1.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int\cosh\left(fx+e\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

mupad [B] time = 1.50, size = 64, normalized size = 0.96

$$\frac{\sinh\left(e+f\,x\right)\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,\sinh\left(e+f\,x\right)^2}{a}\right)}{f\,\left(\frac{b\,\sinh\left(e+f\,x\right)^2}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] (sinh(e + f*x)*(a + b*sinh(e + f*x)^2)^p*hypergeom([1/2, -p], 3/2, -(b*sinh
(e + f*x)^2)/a))/(f*((b*sinh(e + f*x)^2)/a + 1)^p)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.404 $\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*s
inh(f*x+e)^2)^p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 430, 429}

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh
[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{1+x^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\,dx\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\right.}{f}$$

Mathematica [F] time = 3.51, size = 0, normalized size = 0.00

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(fx+e\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b \sinh \left(e+f x\right)^2+a\right)^p}{\cosh \left(e+f x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x),x)
[Out] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
[Out] Timed out
```

3.405 $\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*sinh(f*x+e)*(a+b*s
inh(f*x+e)^2)^p/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3190, 430, 429}

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh
[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{(1+x^2)^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(1+x^2)^2}\,dx, x\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh\right.}{f}$$

Mathematica [F] time = 5.72, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(fx+e\right)^3\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b \sinh\left(e + f\, x\right)^2 + a\right)^p}{\cosh\left(e + f\, x\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^3,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.406 $\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,-3/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(a+b*sinh(f*x+e
)^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Si
nh[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^p\right.}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh}{}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}}{f}$$

Mathematica [F] time = 10.02, size = 0, normalized size = 0.00

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \left(\cosh^4\left(fx+e\right)\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(e+f\,x\right)^4\,{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(cosh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.407 $\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,-1/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(a+b*sinh(f*x+e
)^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Si
nh[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \sqrt{1+x^2}\,\left(a+bx^2\right)^p\,dx, x\right.}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+}{a}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,\left(a\right.}{f}$$

Mathematica [F] time = 10.60, size = 0, normalized size = 0.00

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \left(\cosh^2\left(fx+e\right)\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh\left(e+f\,x\right)^2\,{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int(cosh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.408 $\int \left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,1/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(a+b*sinh(f*x+e)
^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3185, 430, 429}

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3185

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f
*x]), Subst[Int[(a + b*ff^2*x^2)^p/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{\sqrt{1+x^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\mathrm{Sub}}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\sinh^2(e\right.}{f}$$

Mathematica [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 1.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^p,x)
[Out] int((a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b \sinh \left(e+f x\right)^2+a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.409 $\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e+fx)}\,\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sin}{}\right.}{f}$$

```
[Out] AppellF1(1/2,3/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(a+b*sinh(f*x+e)
^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e+fx)}\,\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sin}{}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{\left(1+x^2\right)^{3/2}}\,dx, x, \sinh(e+f\right.}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+}{a}\right.\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\,(a+}{f}$$

Mathematica [F] time = 4.43, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(fx+e\right)^2\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sinh\left(e+f\,x\right)^2+a\right)^p}{\cosh\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^2,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.410 $\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(\cdot}{a}\right.}{f}$$

```
[Out] AppellF1(1/2,5/2,-p,3/2,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a)*(a+b*sinh(f*x+e)
^2)^p*(cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(\cdot}{a}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 5/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{\left(1+x^2\right)^{5/2}}dx,x,\sinh(e\right.}{f}$$

$$=\frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh}{}\right.\right.}{f}$$

$$=\frac{F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(\right.}{f}$$

Mathematica [F] time = 8.40, size = 0, normalized size = 0.00

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
[Out] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \operatorname{sech}\left(fx+e\right)^4\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
[Out] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b \sinh\left(e+f\,x\right)^2+a\right)^p}{\cosh\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^4,x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^p/cosh(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.411 $\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=259

$$-\frac{2a\left(a^4+b^4\right)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d}+\frac{2\left(a^4+b^4\right)^2\sqrt{\sinh(c+dx)}}{b^9d}-\frac{a\left(a^4+2b^4\right)\sinh^2(c+dx)}{2b^6d}+\frac{2\left(a^4+2\right.}{}$$

```
[Out] -2*a*(a^4+b^4)^2*ln(a+b*sinh(d*x+c)^(1/2))/b^10/d-a^3*(a^4+2*b^4)*sinh(d*x+
c)/b^8/d+2/3*a^2*(a^4+2*b^4)*sinh(d*x+c)^(3/2)/b^7/d-1/2*a*(a^4+2*b^4)*sinh
(d*x+c)^2/b^6/d+2/5*(a^4+2*b^4)*sinh(d*x+c)^(5/2)/b^5/d-1/3*a^3*sinh(d*x+c)
^3/b^4/d+2/7*a^2*sinh(d*x+c)^(7/2)/b^3/d-1/4*a*sinh(d*x+c)^4/b^2/d+2/9*sinh
(d*x+c)^(9/2)/b/d+2*(a^4+b^4)^2*sinh(d*x+c)^(1/2)/b^9/d
```

Rubi [A] time = 0.30, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3223, 1890, 1620}

$$\frac{2a^2\sinh^{\frac{7}{2}}(c+dx)}{7b^3d}-\frac{a^3\sinh^3(c+dx)}{3b^4d}+\frac{2\left(a^4+2b^4\right)\sinh^{\frac{5}{2}}(c+dx)}{5b^5d}-\frac{a\left(a^4+2b^4\right)\sinh^2(c+dx)}{2b^6d}+\frac{2a^2\left(a^4+2b^4\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2*a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*(a^4 + b^
4)^2*Sqrt[Sinh[c + d*x]])/(b^9*d) - (a^3*(a^4 + 2*b^4)*Sinh[c + d*x])/(b^8*
d) + (2*a^2*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) - (a*(a^4 + 2*b^4)
*Sinh[c + d*x]^2)/(2*b^6*d) + (2*(a^4 + 2*b^4)*Sinh[c + d*x]^(5/2))/(5*b^5*
d) - (a^3*Sinh[c + d*x]^3)/(3*b^4*d) + (2*a^2*Sinh[c + d*x]^(7/2))/(7*b^3*d
) - (a*Sinh[c + d*x]^4)/(4*b^2*d) + (2*Sinh[c + d*x]^(9/2))/(9*b*d)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1890

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff, x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+b\sqrt{x}}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x(1+x^4)^2}{a+bx}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(a^4+b^4)^2}{b^9} - \frac{a^3(a^4+2b^4)x}{b^8} + \frac{a^2(a^4+2b^4)x^2}{b^7} - \frac{a(a^4+2b^4)x^3}{b^6} + \frac{(a^4+2b^4)x^4}{b^5} - \frac{a^3x^5}{b^4} + \frac{a^2x^6}{b^3}\right.\right.}{d}$$

$$= -\frac{2a\left(a^4+b^4\right)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d} + \frac{2\left(a^4+b^4\right)^2\sqrt{\sinh(c+dx)}}{b^9d} - \frac{a^3\left(a^4+\right.}{}$$

Mathematica [A] time = 0.46, size = 220, normalized size = 0.85

$$\frac{-630ab^4\left(a^4+2b^4\right)\sinh^2(c+dx) + 2520b\left(a^4+b^4\right)^2\sqrt{\sinh(c+dx)} - 2520a\left(a^4+b^4\right)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2520*a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]] + 2520*b*(a^4 + b^4)^
2*Sqrt[Sinh[c + d*x]] - 1260*a^3*b^2*(a^4 + 2*b^4)*Sinh[c + d*x] + 840*a^2*
b^3*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2) - 630*a*b^4*(a^4 + 2*b^4)*Sinh[c + d*
x]^2 + 504*b^5*(a^4 + 2*b^4)*Sinh[c + d*x]^(5/2) - 420*a^3*b^6*Sinh[c + d*x
]^3 + 360*a^2*b^7*Sinh[c + d*x]^(7/2) - 315*a*b^8*Sinh[c + d*x]^4 + 280*b^9
*Sinh[c + d*x]^(9/2))/(1260*b^10*d)
```

fricas [B] time = 3.91, size = 2595, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/20160*(315*a*b^8*cosh(d*x + c)^8 + 315*a*b^8*sinh(d*x + c)^8 + 840*a^3*b
^6*cosh(d*x + c)^7 - 840*a^3*b^6*cosh(d*x + c) + 315*a*b^8 + 840*(3*a*b^8*c
osh(d*x + c) + a^3*b^6)*sinh(d*x + c)^7 + 1260*(2*a^5*b^4 + 3*a*b^8)*cosh(d
*x + c)^6 + 420*(21*a*b^8*cosh(d*x + c)^2 + 14*a^3*b^6*cosh(d*x + c) + 6*a^
5*b^4 + 9*a*b^8)*sinh(d*x + c)^6 + 2520*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x +
c)^5 + 2520*(7*a*b^8*cosh(d*x + c)^3 + 7*a^3*b^6*cosh(d*x + c)^2 + 4*a^7*b^
2 + 7*a^3*b^6 + 3*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c))*sinh(d*x + c)^5 - 20
160*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8)*c)*cosh(d*x
+ c)^4 + 210*(105*a*b^8*cosh(d*x + c)^4 + 140*a^3*b^6*cosh(d*x + c)^3 - 96*
(a^9 + 2*a^5*b^4 + a*b^8)*d*x + 90*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^2 -
96*(a^9 + 2*a^5*b^4 + a*b^8)*c + 60*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c))*
sinh(d*x + c)^4 - 2520*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c)^3 + 840*(21*a*
b^8*cosh(d*x + c)^5 + 35*a^3*b^6*cosh(d*x + c)^4 - 12*a^7*b^2 - 21*a^3*b^6
+ 30*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^3 + 30*(4*a^7*b^2 + 7*a^3*b^6)*cos
h(d*x + c)^2 - 96*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8
)*c)*cosh(d*x + c))*sinh(d*x + c)^3 + 1260*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x +
 c)^2 + 1260*(7*a*b^8*cosh(d*x + c)^6 + 14*a^3*b^6*cosh(d*x + c)^5 + 2*a^5*
b^4 + 3*a*b^8 + 15*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^4 + 20*(4*a^7*b^2 +
7*a^3*b^6)*cosh(d*x + c)^3 - 96*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*a
```

```
^5*b^4 + a*b^8)*c)*cosh(d*x + c)^2 - 6*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c
))*sinh(d*x + c)^2 - 20160*((a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)^4 + 4*(
a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^9 + 2*a^5*b^4
 + a*b^8)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^9 + 2*a^5*b^4 + a*b^8)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a^9 + 2*a^5*b^4 + a*b^8)*sinh(d*x + c)^4)*log
((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2*cosh(d*x + c) - b^2 + 2
*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*cosh(d*x + c) + a*b*sinh(
d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 -
 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh(d*x + c))) +
20160*((a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)^4 + 4*(a^9 + 2*a^5*b^4 + a*b
^8)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*(a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a^9 + 2*a^5*b^4 + a*b^8)*sinh(d*x + c)^4)*log(2*(b^2*sinh(d*x + c)
 - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 840*(3*a*b^8*cosh(d*x + c)^7 + 7
*a^3*b^6*cosh(d*x + c)^6 - a^3*b^6 + 9*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^
5 + 15*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c)^4 - 96*((a^9 + 2*a^5*b^4 + a*b
^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8)*c)*cosh(d*x + c)^3 - 9*(4*a^7*b^2 + 7*a
^3*b^6)*cosh(d*x + c)^2 + 3*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c))*sinh(d*x +
 c) - 8*(35*b^9*cosh(d*x + c)^8 + 35*b^9*sinh(d*x + c)^8 + 90*a^2*b^7*cosh(
d*x + c)^7 - 90*a^2*b^7*cosh(d*x + c) + 35*b^9 + 10*(28*b^9*cosh(d*x + c) +
 9*a^2*b^7)*sinh(d*x + c)^7 + 28*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c)^6 + 14*
(70*b^9*cosh(d*x + c)^2 + 45*a^2*b^7*cosh(d*x + c) + 18*a^4*b^5 + 26*b^9)*s
inh(d*x + c)^6 + 30*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)^5 + 2*(980*b^9*
cosh(d*x + c)^3 + 945*a^2*b^7*cosh(d*x + c)^2 + 420*a^6*b^3 + 705*a^2*b^7 +
 84*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c))*sinh(d*x + c)^5 + 42*(120*a^8*b + 2
28*a^4*b^5 + 101*b^9)*cosh(d*x + c)^4 + 2*(1225*b^9*cosh(d*x + c)^4 + 1575*
a^2*b^7*cosh(d*x + c)^3 + 2520*a^8*b + 4788*a^4*b^5 + 2121*b^9 + 210*(9*a^4
*b^5 + 13*b^9)*cosh(d*x + c)^2 + 75*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)
)*sinh(d*x + c)^4 - 30*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)^3 + 2*(980*b
^9*cosh(d*x + c)^5 + 1575*a^2*b^7*cosh(d*x + c)^4 - 420*a^6*b^3 - 705*a^2*b
^7 + 280*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c)^3 + 150*(28*a^6*b^3 + 47*a^2*b^
7)*cosh(d*x + c)^2 + 84*(120*a^8*b + 228*a^4*b^5 + 101*b^9)*cosh(d*x + c))*
sinh(d*x + c)^3 + 28*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c)^2 + 2*(490*b^9*cosh
(d*x + c)^6 + 945*a^2*b^7*cosh(d*x + c)^5 + 126*a^4*b^5 + 182*b^9 + 210*(9*
a^4*b^5 + 13*b^9)*cosh(d*x + c)^4 + 150*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x
+ c)^3 + 126*(120*a^8*b + 228*a^4*b^5 + 101*b^9)*cosh(d*x + c)^2 - 45*(28*a
^6*b^3 + 47*a^2*b^7)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(140*b^9*cosh(d*x +
 c)^7 + 315*a^2*b^7*cosh(d*x + c)^6 - 45*a^2*b^7 + 84*(9*a^4*b^5 + 13*b^9)*
cosh(d*x + c)^5 + 75*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)^4 + 84*(120*a^
8*b + 228*a^4*b^5 + 101*b^9)*cosh(d*x + c)^3 - 45*(28*a^6*b^3 + 47*a^2*b^7)
*cosh(d*x + c)^2 + 28*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(sinh(d*x + c)))/(b^10*d*cosh(d*x + c)^4 + 4*b^10*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^10*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^10*d*cosh(d*x + c
)*sinh(d*x + c)^3 + b^10*d*sinh(d*x + c)^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)^5}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a), x)
```

maple [C] time = 0.22, size = 780, normalized size = 3.01

$$-\frac{7a}{8d\,b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{7a}{8d\,b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{9a}{8d\,b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{9a}{8d\,b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x)
```

```
[Out] -7/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a+7/8/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a-9/
8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-9/8/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a+a
^7/d/b^8/(tanh(1/2*d*x+1/2*c)-1)-1/2*a^5/d/b^6/(tanh(1/2*d*x+1/2*c)-1)+1/3*
a^3/d/b^4/(tanh(1/2*d*x+1/2*c)+1)^3-1/2*a^5/d/b^6/(tanh(1/2*d*x+1/2*c)+1)^2
-1/2*a^3/d/b^4/(tanh(1/2*d*x+1/2*c)+1)^2+1/3*a^3/d/b^4/(tanh(1/2*d*x+1/2*c)
-1)^3-1/2*a^5/d/b^6/(tanh(1/2*d*x+1/2*c)-1)^2+1/2*a^3/d/b^4/(tanh(1/2*d*x+1
/2*c)-1)^2+a^9/d/b^10*ln(tanh(1/2*d*x+1/2*c)-1)-a/d/b^2*ln(a^2*tanh(1/2*d*x
+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)-a^9/d/b^10*ln(a^2*tanh(1/2*d*x+1/2
*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)-2*a^5/d/b^6*ln(a^2*tanh(1/2*d*x+1/2*c)
^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+a^9/d/b^10*ln(tanh(1/2*d*x+1/2*c)+1)+2*a^
5/d/b^6*ln(tanh(1/2*d*x+1/2*c)+1)+2*a^5/d/b^6*ln(tanh(1/2*d*x+1/2*c)-1)-1/4
*a/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^4-1/4*a/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^4+a^
7/d/b^8/(tanh(1/2*d*x+1/2*c)+1)+1/2*a^5/d/b^6/(tanh(1/2*d*x+1/2*c)+1)+1/d*a
/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/d/b^
2/(tanh(1/2*d*x+1/2*c)-1)^3*a+2/d/b^4/(tanh(1/2*d*x+1/2*c)-1)*a^3+1/2/d/b^2
/(tanh(1/2*d*x+1/2*c)+1)^3*a+2/d/b^4/(tanh(1/2*d*x+1/2*c)+1)*a^3+`int/indef
0`(-cosh(d*x+c)^4*b*sinh(d*x+c)^(1/2)/(-b^2*sinh(d*x+c)+a^2),sinh(d*x+c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)^5}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(c+d\,x)^5}{a+b\,\sqrt{\sinh(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2)),x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Timed out
```

3.412 $\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=136

$$-\frac{2a\left(a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}+\frac{2\left(a^4+b^4\right)\sqrt{\sinh(c+dx)}}{b^5d}-\frac{a^3\sinh(c+dx)}{b^4d}+\frac{2a^2\sinh^{\frac{3}{2}}(c+dx)}{3b^3d}-\frac{a\,s}{}$$

[Out] -2*a*(a^4+b^4)*ln(a+b*sinh(d*x+c)^(1/2))/b^6/d-a^3*sinh(d*x+c)/b^4/d+2/3*a^2*sinh(d*x+c)^(3/2)/b^3/d-1/2*a*sinh(d*x+c)^2/b^2/d+2/5*sinh(d*x+c)^(5/2)/b/d+2*(a^4+b^4)*sinh(d*x+c)^(1/2)/b^5/d

Rubi [A] time = 0.16, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3223, 1890, 1620}

$$\frac{2a^2\sinh^{\frac{3}{2}}(c+dx)}{3b^3d}-\frac{a^3\sinh(c+dx)}{b^4d}+\frac{2\left(a^4+b^4\right)\sqrt{\sinh(c+dx)}}{b^5d}-\frac{2a\left(a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}-\frac{a\,\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2*a*(a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^6*d) + (2*(a^4 + b^4)*
Sqrt[Sinh[c + d*x]])/(b^5*d) - (a^3*Sinh[c + d*x])/(b^4*d) + (2*a^2*Sinh[c
+ d*x]^(3/2))/(3*b^3*d) - (a*Sinh[c + d*x]^2)/(2*b^2*d) + (2*Sinh[c + d*x]^
(5/2))/(5*b*d)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1890

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+b\sqrt{x}}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x(1+x^4)}{a+bx}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{a^4+b^4}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a(a^4+b^4)}{b^5(a+bx)}\right)dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= -\frac{2a\left(a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d} + \frac{2\left(a^4+b^4\right)\sqrt{\sinh(c+dx)}}{b^5d} - \frac{a^3\sinh(c+}{b^4d}$$

Mathematica [A] time = 0.15, size = 117, normalized size = 0.86

$$\frac{60b\left(a^4+b^4\right)\sqrt{\sinh(c+dx)} - 60a\left(a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right) - 30a^3b^2\sinh(c+dx) + 20a^2b^3\sinh^{\frac{3}{2}}(c-}{30b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-60*a*(a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]] + 60*b*(a^4 + b^4)*Sqrt[S
inh[c + d*x]] - 30*a^3*b^2*Sinh[c + d*x] + 20*a^2*b^3*Sinh[c + d*x]^(3/2) -
 15*a*b^4*Sinh[c + d*x]^2 + 12*b^5*Sinh[c + d*x]^(5/2))/(30*b^6*d)
```

fricas [B] time = 2.07, size = 879, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/120*(15*a*b^4*cosh(d*x + c)^4 + 15*a*b^4*sinh(d*x + c)^4 + 60*a^3*b^2*co
sh(d*x + c)^3 - 60*a^3*b^2*cosh(d*x + c) + 15*a*b^4 + 60*(a*b^4*cosh(d*x +
c) + a^3*b^2)*sinh(d*x + c)^3 - 120*((a^5 + a*b^4)*d*x + (a^5 + a*b^4)*c)*c
osh(d*x + c)^2 + 30*(3*a*b^4*cosh(d*x + c)^2 + 6*a^3*b^2*cosh(d*x + c) - 4*
(a^5 + a*b^4)*d*x - 4*(a^5 + a*b^4)*c)*sinh(d*x + c)^2 - 120*((a^5 + a*b^4)
*cosh(d*x + c)^2 + 2*(a^5 + a*b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^5 + a*b
^4)*sinh(d*x + c)^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2
*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*c
osh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2
 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) -
 a^2)*sinh(d*x + c))) + 120*((a^5 + a*b^4)*cosh(d*x + c)^2 + 2*(a^5 + a*b^4
)*cosh(d*x + c)*sinh(d*x + c) + (a^5 + a*b^4)*sinh(d*x + c)^2)*log(2*(b^2*s
inh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 60*(a*b^4*cosh(d*x +
 c)^3 + 3*a^3*b^2*cosh(d*x + c)^2 - a^3*b^2 - 4*((a^5 + a*b^4)*d*x + (a^5 +
 a*b^4)*c)*cosh(d*x + c))*sinh(d*x + c) - 4*(3*b^5*cosh(d*x + c)^4 + 3*b^5*
sinh(d*x + c)^4 + 10*a^2*b^3*cosh(d*x + c)^3 - 10*a^2*b^3*cosh(d*x + c) + 3
*b^5 + 2*(6*b^5*cosh(d*x + c) + 5*a^2*b^3)*sinh(d*x + c)^3 + 6*(10*a^4*b +
9*b^5)*cosh(d*x + c)^2 + 6*(3*b^5*cosh(d*x + c)^2 + 5*a^2*b^3*cosh(d*x + c)
 + 10*a^4*b + 9*b^5)*sinh(d*x + c)^2 + 2*(6*b^5*cosh(d*x + c)^3 + 15*a^2*b^
3*cosh(d*x + c)^2 - 5*a^2*b^3 + 6*(10*a^4*b + 9*b^5)*cosh(d*x + c))*sinh(d*
x + c))*sqrt(sinh(d*x + c)))/(b^6*d*cosh(d*x + c)^2 + 2*b^6*d*cosh(d*x + c)
*sinh(d*x + c) + b^6*d*sinh(d*x + c)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)^3}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a), x)
```

maple [C] time = 0.20, size = 359, normalized size = 2.64

$$-\frac{a}{2d\,b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{a^3}{d\,b^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{a}{2d\,b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a^5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x)
```

```
[Out] -1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a+1/d/b^4/(tanh(1/2*d*x+1/2*c)-1)*a^3-
1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a+a^5/d/b^6*ln(tanh(1/2*d*x+1/2*c)-1)+1/d
*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a+1/d/
b^4/(tanh(1/2*d*x+1/2*c)+1)*a^3+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a+a^5/d/b
^6*ln(tanh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-a^5/d/b^6*
ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)-a/d/b^2*ln(a^2*
tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+`int/indef0`(-cosh(d*x
+c)^2*b*sinh(d*x+c)^(1/2)/(-b^2*sinh(d*x+c)+a^2),sinh(d*x+c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)^3}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^3}{a+b\,\sqrt{\sinh(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2)),x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Timed out
```

3.413 $\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{\sinh(c+dx)}}{bd}-\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}$$

```
[Out] -2*a*ln(a+b*sinh(d*x+c)^(1/2))/b^2/d+2*sinh(d*x+c)^(1/2)/b/d
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3223, 190, 43}

$$\frac{2\sqrt{\sinh(c+dx)}}{bd}-\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2*a*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^2*d) + (2*Sqrt[Sinh[c + d*x]])/(b*
d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b\sqrt{x}}\,dx,x,\sinh(c+dx)\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \frac{x}{a+bx}\,dx,x,\sqrt{\sinh(c+dx)}\right)}{d}\\
&= \frac{2\,\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx,x,\sqrt{\sinh(c+dx)}\right)}{d}\\
&= -\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}+\frac{2\sqrt{\sinh(c+dx)}}{bd}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{\sinh(c+dx)}}{b}-\frac{a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (2*(-((a*Log[a + b*Sqrt[Sinh[c + d*x]]])/b^2) + Sqrt[Sinh[c + d*x]]/b))/d
```

fricas [B] time = 2.31, size = 225, normalized size = 5.23

$$\frac{adx + a\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2a^2\cosh(dx+c)-b^2+2\left(b^2\cosh(dx+c)+a^2\right)\sinh(dx+c)-4\left(ab\cosh(dx+c)+ab\sinh(dx+c)\right)\sqrt{\sinh}}{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2-2a^2\cosh(dx+c)-b^2+2\left(b^2\cosh(dx+c)-a^2\right)\sinh(dx+c)}\right.}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] (a*d*x + a*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2*cosh(d*x
+ c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*cosh(d*x +
c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2 + b^2*sin
h(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh
(d*x + c))) - a*log(2*(b^2*sinh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x +
 c))) + 2*b*sqrt(sinh(d*x + c)))/(b^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

maple [B] time = 0.03, size = 89, normalized size = 2.07

$$\frac{2\left(\sqrt{\sinh}\,(dx+c)\right)}{bd}+\frac{a\ln\left(b\left(\sqrt{\sinh}\,(dx+c)\right)-a\right)}{d\,b^2}-\frac{a\ln\left(a+b\left(\sqrt{\sinh}\,(dx+c)\right)\right)}{b^2d}-\frac{a\ln\left(b^2\sinh(dx+c)-a^2\right)}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x)
```

```
[Out] 2*sinh(d*x+c)^(1/2)/b/d+1/d/b^2*a*ln(b*sinh(d*x+c)^(1/2)-a)-a*ln(a+b*sinh(d
*x+c)^(1/2))/b^2/d-1/d*a*ln(b^2*sinh(d*x+c)-a^2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

mupad [B] time = 1.01, size = 39, normalized size = 0.91

$$\frac{2\sqrt{\sinh(c+d\,x)}}{b\,d}-\frac{2\,a\,\ln\left(a+b\,\sqrt{\sinh(c+d\,x)}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^(1/2)),x)
```

```
[Out] (2*sinh(c + d*x)^(1/2))/(b*d) - (2*a*log(a + b*sinh(c + d*x)^(1/2)))/(b^2*d
)
```

sympy [A] time = 1.82, size = 68, normalized size = 1.58

$$\begin{cases} \frac{x\cosh(c)}{a} & \text{for } b=0\wedge d=0 \\ \frac{\sinh(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\cosh(c)}{a+b\sqrt{\sinh(c)}} & \text{for } d=0 \\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)}{b^2d}+\frac{2\sqrt{\sinh(c+dx)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a*d), Eq(b, 0
)), (x*cosh(c)/(a + b*sqrt(sinh(c))), Eq(d, 0)), (-2*a*log(a/b + sqrt(sinh(
c + d*x)))/(b**2*d) + 2*sqrt(sinh(c + d*x))/(b*d), True))
```

3.414 $\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=286

$$-\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{d\left(a^4+b^4\right)}+\frac{ab^2\log(\cosh(c+dx))}{d\left(a^4+b^4\right)}+\frac{a^3\tan^{-1}(\sinh(c+dx))}{d\left(a^4+b^4\right)}-\frac{b\left(a^2+b^2\right)\log\left(\sinh(c+d\right.}{2\sqrt{2}\,d\,(}$$

```
[Out] a^3*arctan(sinh(d*x+c))/(a^4+b^4)/d+a*b^2*ln(cosh(d*x+c))/(a^4+b^4)/d-2*a*b
^2*ln(a+b*sinh(d*x+c)^(1/2))/(a^4+b^4)/d-1/2*b*(a^2-b^2)*arctan(-1+2^(1/2)*
sinh(d*x+c)^(1/2))/(a^4+b^4)/d*2^(1/2)-1/2*b*(a^2-b^2)*arctan(1+2^(1/2)*sin
h(d*x+c)^(1/2))/(a^4+b^4)/d*2^(1/2)-1/4*b*(a^2+b^2)*ln(1+sinh(d*x+c)-2^(1/2
)*sinh(d*x+c)^(1/2))/(a^4+b^4)/d*2^(1/2)+1/4*b*(a^2+b^2)*ln(1+sinh(d*x+c)+2
^(1/2)*sinh(d*x+c)^(1/2))/(a^4+b^4)/d*2^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3223, 6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$-\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{d\left(a^4+b^4\right)}-\frac{b\left(a^2+b^2\right)\log\left(\sinh(c+dx)-\sqrt{2}\sqrt{\sinh(c+dx)}+1\right)}{2\sqrt{2}\,d\left(a^4+b^4\right)}+\frac{b\left(a^2+b^2\right)\log\left(\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (b*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^4 + b^4
)*d) - (b*(a^2 - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^
4 + b^4)*d) + (a^3*ArcTan[Sinh[c + d*x]])/((a^4 + b^4)*d) + (a*b^2*Log[Cosh
[c + d*x]])/((a^4 + b^4)*d) - (2*a*b^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/((a^
4 + b^4)*d) - (b*(a^2 + b^2)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c +
 d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d) + (b*(a^2 + b^2)*Log[1 + Sqrt[2]*Sqrt[Sin
h[c + d*x]] + Sinh[c + d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)\left(1+x^2\right)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x}{(a+bx)\left(1+x^4\right)}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{ab^3}{\left(a^4+b^4\right)(a+bx)}+\frac{b^3+a^3x-a^2bx^2+ab^2x^3}{\left(a^4+b^4\right)\left(1+x^4\right)}\right)dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{2\,\text{Subst}\left(\int \frac{b^3+a^3x-a^2bx^2+ab^2x^3}{1+x^4}\,dx, x, \sqrt{\sinh(c+\right.}{\left(a^4+b^4\right)d}$$

$$= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{2\,\text{Subst}\left(\int \left(\frac{b^3-a^2bx^2}{1+x^4}+\frac{x\left(a^3+ab^2x^2\right)}{1+x^4}\right)dx, x, \sqrt{\text{si}\right.}{\left(a^4+b^4\right)d}$$

$$= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{2\,\text{Subst}\left(\int \frac{b^3-a^2bx^2}{1+x^4}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+$$

$$= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{\text{Subst}\left(\int \frac{a^3+ab^2x}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{\left(a^4+b^4\right)d}-\frac{\left(b\left(a\right.\right.}{}$$

$$= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{\left(a^4+b^4\right)d}+\frac{\left(ab^2\right.}{}$$

$$= \frac{a^3\tan^{-1}(\sinh(c+dx))}{\left(a^4+b^4\right)d}+\frac{ab^2\log(\cosh(c+dx))}{\left(a^4+b^4\right)d}-\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right.}{\left(a^4+b^4\right)d}$$

$$= \frac{b\left(a^2-b^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{\sqrt{2}\left(a^4+b^4\right)d}-\frac{b\left(a^2-b^2\right)\tan^{-1}\left(1+\sqrt{2}\sqrt{\sinh(c+\right.}{\sqrt{2}\left(a^4+b^4\right)d}$$

Mathematica [C] time = 0.25, size = 229, normalized size = 0.80

$$3\left(4a^3\tan^{-1}(\sinh(c+dx))-8ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)+4ab^2\log(\cosh(c+dx))-\sqrt{2}\,b^3\log\left(\sinh(c+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (3*(-2*Sqrt[2]*b^3*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]] + 2*Sqrt[2]*b^3*
ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]] + 4*a^3*ArcTan[Sinh[c + d*x]] + 4*a
*b^2*Log[Cosh[c + d*x]] - 8*a*b^2*Log[a + b*Sqrt[Sinh[c + d*x]]] - Sqrt[2]*
b^3*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]] + Sqrt[2]*b^3*Log[
1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]]) - 8*a^2*b*Hypergeometric2
F1[3/4, 1, 7/4, -Sinh[c + d*x]^2]*Sinh[c + d*x]^(3/2))/(12*(a^4 + b^4)*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

maple [C] time = 0.19, size = 206, normalized size = 0.72

$$-\frac{a\,b^2\ln\left(a^2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^2\right)}{d\left(a^4+b^4\right)}+\frac{a\,b^2\ln\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\left(a^4+b^4\right)}+\frac{2a^3\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d\left(a^4+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x)
[Out] -a/d*b^2/(a^4+b^4)*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a
^2)+a/d/(a^4+b^4)*b^2*ln(tanh(1/2*d*x+1/2*c)^2+1)+2*a^3/d/(a^4+b^4)*arctan(
tanh(1/2*d*x+1/2*c))+`int/indef0`(b*sinh(d*x+c)^(1/2)*(-b^2*sinh(d*x+c)+a^2
)/(2*a^2*b^2*sinh(d*x+c)*cosh(d*x+c)^2-b^4*cosh(d*x+c)^4+(-a^4+b^4)*cosh(d*
x+c)^2),sinh(d*x+c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh(c+d\,x)\,\left(a+b\,\sqrt{\sinh(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))),x)
[Out] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sqrt(sinh(c + d*x))), x)
```

3.415 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=270

$$\frac{2a\left(a^4+b^4\right)^2}{b^{10}d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2\left(a^4+b^4\right)\left(9a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d}-\frac{4a\left(3a^4+2b^4\right)\sinh^{\frac{3}{2}}(c+dx)}{3b^7d}+\frac{(5a$$

[Out] 2*(a^4+b^4)*(9*a^4+b^4)*ln(a+b*sinh(d*x+c)^(1/2))/b^10/d+a^2*(7*a^4+6*b^4)*sinh(d*x+c)/b^8/d-4/3*a*(3*a^4+2*b^4)*sinh(d*x+c)^(3/2)/b^7/d+1/2*(5*a^4+2*b^4)*sinh(d*x+c)^2/b^6/d-8/5*a^3*sinh(d*x+c)^(5/2)/b^5/d+a^2*sinh(d*x+c)^3/b^4/d-4/7*a*sinh(d*x+c)^(7/2)/b^3/d+1/4*sinh(d*x+c)^4/b^2/d-16*a^3*(a^4+b^4)*sinh(d*x+c)^(1/2)/b^9/d+2*a*(a^4+b^4)^2/b^10/d/(a+b*sinh(d*x+c)^(1/2))

Rubi [A] time = 0.32, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3223, 1890, 1620}

$$\frac{a^2\sinh^3(c+dx)}{b^4d}-\frac{8a^3\sinh^{\frac{5}{2}}(c+dx)}{5b^5d}+\frac{\left(5a^4+2b^4\right)\sinh^2(c+dx)}{2b^6d}-\frac{4a\left(3a^4+2b^4\right)\sinh^{\frac{3}{2}}(c+dx)}{3b^7d}+\frac{a^2\left(7a^4+6b^4\right)s}{b^8d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]])^2,x]

[Out] (2*(a^4 + b^4)*(9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*a*(a^4 + b^4)^2)/(b^10*d*(a + b*Sqrt[Sinh[c + d*x]])) - (16*a^3*(a^4 + b^4)*Sqrt[Sinh[c + d*x]])/(b^9*d) + (a^2*(7*a^4 + 6*b^4)*Sinh[c + d*x])/(b^8*d) - (4*a*(3*a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) + ((5*a^4 + 2*b^4)*Sinh[c + d*x]^2)/(2*b^6*d) - (8*a^3*Sinh[c + d*x]^(5/2))/(5*b^5*d) + (a^2*Sinh[c + d*x]^3)/(b^4*d) - (4*a*Sinh[c + d*x]^(7/2))/(7*b^3*d) + Sinh[c + d*x]^4/(4*b^2*d)

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1890

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b\sqrt{x})^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x(1+x^4)^2}{(a+bx)^2}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{8a^3(a^4+b^4)}{b^9} + \frac{a^2(7a^4+6b^4)x}{b^8} - \frac{2a(3a^4+2b^4)x^2}{b^7} + \frac{(5a^4+2b^4)x^3}{b^6} - \frac{4a^3x^4}{b^5} + \frac{3a^2}{b^4}\right.\right.}{d}$$

$$= \frac{2(a^4+b^4)(9a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d} + \frac{2a(a^4+b^4)^2}{b^{10}d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

Mathematica [A] time = 0.61, size = 288, normalized size = 1.07

$$\frac{-70ab^4(9a^4+10b^4)\sinh^2(c+dx) + 840b(a^4+b^4)\sqrt{\sinh(c+dx)}\left((9a^4+b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right) - 8a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (840*a*(a^4 + b^4)*(a^4 + b^4 + (9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]
]) + 840*b*(a^4 + b^4)*(-8*a^4 + (9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]
]])*Sqrt[Sinh[c + d*x]] - 420*a^3*b^2*(9*a^4 + 10*b^4)*Sinh[c + d*x] + 140*
a^2*b^3*(9*a^4 + 10*b^4)*Sinh[c + d*x]^(3/2) - 70*a*b^4*(9*a^4 + 10*b^4)*Si
nh[c + d*x]^2 + 42*b^5*(9*a^4 + 10*b^4)*Sinh[c + d*x]^(5/2) - 252*a^3*b^6*S
inh[c + d*x]^3 + 180*a^2*b^7*Sinh[c + d*x]^(7/2) - 135*a*b^8*Sinh[c + d*x]^
4 + 105*b^9*Sinh[c + d*x]^(9/2))/(420*b^10*d*(a + b*Sqrt[Sinh[c + d*x]]))
```

fricas [B] time = 2.13, size = 5181, normalized size = 19.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(105*b^10*cosh(d*x + c)^10 + 105*b^10*sinh(d*x + c)^10 + 630*a^2*b^8
*cosh(d*x + c)^9 + 630*a^2*b^8*cosh(d*x + c) - 105*b^10 + 210*(5*b^10*cosh(
d*x + c) + 3*a^2*b^8)*sinh(d*x + c)^9 + 105*(24*a^4*b^6 + 11*b^10)*cosh(d*x
 + c)^8 + 105*(45*b^10*cosh(d*x + c)^2 + 54*a^2*b^8*cosh(d*x + c) + 24*a^4*
b^6 + 11*b^10)*sinh(d*x + c)^8 + 840*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c
)^7 + 840*(15*b^10*cosh(d*x + c)^3 + 27*a^2*b^8*cosh(d*x + c)^2 + 18*a^6*b^
4 + 17*a^2*b^8 + (24*a^4*b^6 + 11*b^10)*cosh(d*x + c))*sinh(d*x + c)^7 - 42
0*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d
*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^6 + 210*(105*b^10*
cosh(d*x + c)^4 + 252*a^2*b^8*cosh(d*x + c)^3 - 224*a^8*b^2 - 188*a^4*b^6 -
 6*b^10 - 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 14*(24*a^4*b^6 + 11*b^10
)*cosh(d*x + c)^2 - 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c + 28*(18*a^6*b^4 +
 17*a^2*b^8)*cosh(d*x + c))*sinh(d*x + c)^6 - 1680*(16*a^10 + 60*a^6*b^4 +
37*a^2*b^8 - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4
 + a^2*b^8)*c)*cosh(d*x + c)^5 + 420*(63*b^10*cosh(d*x + c)^5 + 189*a^2*b^8
*cosh(d*x + c)^4 - 64*a^10 - 240*a^6*b^4 - 148*a^2*b^8 + 14*(24*a^4*b^6 + 1
1*b^10)*cosh(d*x + c)^3 + 32*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x + 42*(18*a
```

^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^2 + 32*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c
 - 6*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10
)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c))*sinh(d*x + c)^
5 + 420*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b
^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^4 + 210*(105
*b^10*cosh(d*x + c)^6 + 378*a^2*b^8*cosh(d*x + c)^5 + 224*a^8*b^2 + 188*a^4
*b^6 + 6*b^10 + 35*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^4 + 140*(18*a^6*b^4
 + 17*a^2*b^8)*cosh(d*x + c)^3 + 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x - 3
0*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d
x + 16(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^2 + 32*(9*a^8*b^2
+ 10*a^4*b^6 + b^10)*c - 40*(16*a^10 + 60*a^6*b^4 + 37*a^2*b^8 - 8*(9*a^10
+ 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c)*cosh(d*x
 + c))*sinh(d*x + c)^4 + 840*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^3 + 84
0*(15*b^10*cosh(d*x + c)^7 + 63*a^2*b^8*cosh(d*x + c)^6 + 18*a^6*b^4 + 17*a
^2*b^8 + 7*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^5 + 35*(18*a^6*b^4 + 17*a^2
*b^8)*cosh(d*x + c)^4 - 10*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b
^2 + 10*a^4*b^6 + b^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*
x + c)^3 - 20*(16*a^10 + 60*a^6*b^4 + 37*a^2*b^8 - 8*(9*a^10 + 10*a^6*b^4 +
 a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c)*cosh(d*x + c)^2 + 2*(1
12*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x +
 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c))*sinh(d*x + c)^3 - 105
*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^2 + 105*(45*b^10*cosh(d*x + c)^8 + 21
6*a^2*b^8*cosh(d*x + c)^7 - 24*a^4*b^6 - 11*b^10 + 28*(24*a^4*b^6 + 11*b^10
)*cosh(d*x + c)^6 + 168*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^5 - 60*(112
*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 1
6*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^4 - 160*(16*a^10 + 60*a^
6*b^4 + 37*a^2*b^8 - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10
*a^6*b^4 + a^2*b^8)*c)*cosh(d*x + c)^3 + 24*(112*a^8*b^2 + 94*a^4*b^6 + 3*b
^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 +
 b^10)*c)*cosh(d*x + c)^2 + 24*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c))*sin
h(d*x + c)^2 + 6720*((9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^6 + (9*a
^8*b^2 + 10*a^4*b^6 + b^10)*sinh(d*x + c)^6 - 2*(9*a^10 + 10*a^6*b^4 + a^2*
b^8)*cosh(d*x + c)^5 - 2*(9*a^10 + 10*a^6*b^4 + a^2*b^8 - 3*(9*a^8*b^2 + 10
*a^4*b^6 + b^10)*cosh(d*x + c))*sinh(d*x + c)^5 - (9*a^8*b^2 + 10*a^4*b^6 +
 b^10)*cosh(d*x + c)^4 - (9*a^8*b^2 + 10*a^4*b^6 + b^10 - 15*(9*a^8*b^2 + 1
0*a^4*b^6 + b^10)*cosh(d*x + c)^2 + 10*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh
(d*x + c))*sinh(d*x + c)^4 + 4*(5*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x
+ c)^3 - 5*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh(d*x + c)^2 - (9*a^8*b^2 + 1
0*a^4*b^6 + b^10)*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(9*a^8*b^2 + 10*a^4*
b^6 + b^10)*cosh(d*x + c)^4 - 20*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh(d*x +
 c)^3 - 6*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 2*(3*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^5 - 5*(9*a^10 + 10*a^6
*b^4 + a^2*b^8)*cosh(d*x + c)^4 - 2*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*
x + c)^3)*sinh(d*x + c))*log(-(b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 +
2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) + 4*(
a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x +
 c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x +
 c) - a^2)*sinh(d*x + c))) + 6720*((9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x
 + c)^6 + (9*a^8*b^2 + 10*a^4*b^6 + b^10)*sinh(d*x + c)^6 - 2*(9*a^10 + 10*
a^6*b^4 + a^2*b^8)*cosh(d*x + c)^5 - 2*(9*a^10 + 10*a^6*b^4 + a^2*b^8 - 3*(
9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c))*sinh(d*x + c)^5 - (9*a^8*b^2
+ 10*a^4*b^6 + b^10)*cosh(d*x + c)^4 - (9*a^8*b^2 + 10*a^4*b^6 + b^10 - 15*
(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^2 + 10*(9*a^10 + 10*a^6*b^4 +
 a^2*b^8)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(5*(9*a^8*b^2 + 10*a^4*b^6 + b
^10)*cosh(d*x + c)^3 - 5*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh(d*x + c)^2 -
(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(9*a^8
*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^4 - 20*(9*a^10 + 10*a^6*b^4 + a^2*b
^8)*cosh(d*x + c)^3 - 6*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + 2*(3*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^5 - 5*(9

```
*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh(d*x + c)^4 - 2*(9*a^8*b^2 + 10*a^4*b^6 +
 b^10)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*(b^2*sinh(d*x + c) - a^2)/(cos
h(d*x + c) - sinh(d*x + c))) + 210*(5*b^10*cosh(d*x + c)^9 + 27*a^2*b^8*cos
h(d*x + c)^8 + 3*a^2*b^8 + 4*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^7 + 28*(1
8*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^6 - 12*(112*a^8*b^2 + 94*a^4*b^6 + 3*
b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6
+ b^10)*c)*cosh(d*x + c)^5 - 40*(16*a^10 + 60*a^6*b^4 + 37*a^2*b^8 - 8*(9*a
^10 + 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c)*cosh
(d*x + c)^4 + 8*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4
*b^6 + b^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^3 +
12*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^2 - (24*a^4*b^6 + 11*b^10)*cosh(
d*x + c))*sinh(d*x + c) - 32*(15*a*b^9*cosh(d*x + c)^9 + 15*a*b^9*sinh(d*x
+ c)^9 + 54*a^3*b^7*cosh(d*x + c)^8 - 54*a^3*b^7*cosh(d*x + c)^2 + 15*a*b^9
*cosh(d*x + c) + 27*(5*a*b^9*cosh(d*x + c) + 2*a^3*b^7)*sinh(d*x + c)^8 + 4
*(63*a^5*b^5 + 55*a*b^9)*cosh(d*x + c)^7 + 4*(135*a*b^9*cosh(d*x + c)^2 + 1
08*a^3*b^7*cosh(d*x + c) + 63*a^5*b^5 + 55*a*b^9)*sinh(d*x + c)^7 + 2*(1260
*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c)^6 + 2*(630*a*b^9*cosh(d*x + c)^3 + 7
56*a^3*b^7*cosh(d*x + c)^2 + 1260*a^7*b^3 + 1319*a^3*b^7 + 14*(63*a^5*b^5 +
 55*a*b^9)*cosh(d*x + c))*sinh(d*x + c)^6 - 2*(3780*a^9*b + 4452*a^5*b^5 +
655*a*b^9)*cosh(d*x + c)^5 + 2*(945*a*b^9*cosh(d*x + c)^4 + 1512*a^3*b^7*co
sh(d*x + c)^3 - 3780*a^9*b - 4452*a^5*b^5 - 655*a*b^9 + 42*(63*a^5*b^5 + 55
*a*b^9)*cosh(d*x + c)^2 + 6*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c))*si
nh(d*x + c)^5 - 2*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c)^4 + 2*(945*a*
b^9*cosh(d*x + c)^5 + 1890*a^3*b^7*cosh(d*x + c)^4 - 1260*a^7*b^3 - 1319*a^
3*b^7 + 70*(63*a^5*b^5 + 55*a*b^9)*cosh(d*x + c)^3 + 15*(1260*a^7*b^3 + 131
9*a^3*b^7)*cosh(d*x + c)^2 - 5*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9)*cosh
(d*x + c))*sinh(d*x + c)^4 + 4*(63*a^5*b^5 + 55*a*b^9)*cosh(d*x + c)^3 + 4*
(315*a*b^9*cosh(d*x + c)^6 + 756*a^3*b^7*cosh(d*x + c)^5 + 63*a^5*b^5 + 55*
a*b^9 + 35*(63*a^5*b^5 + 55*a*b^9)*cosh(d*x + c)^4 + 10*(1260*a^7*b^3 + 131
9*a^3*b^7)*cosh(d*x + c)^3 - 5*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9)*cosh
(d*x + c)^2 - 2*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c))*sinh(d*x + c)^
3 + 2*(270*a*b^9*cosh(d*x + c)^7 + 756*a^3*b^7*cosh(d*x + c)^6 - 27*a^3*b^7
 + 42*(63*a^5*b^5 + 55*a*b^9)*cosh(d*x + c)^5 + 15*(1260*a^7*b^3 + 1319*a^3
*b^7)*cosh(d*x + c)^4 - 10*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9)*cosh(d*x
 + c)^3 - 6*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c)^2 + 6*(63*a^5*b^5 +
 55*a*b^9)*cosh(d*x + c))*sinh(d*x + c)^2 + (135*a*b^9*cosh(d*x + c)^8 + 43
2*a^3*b^7*cosh(d*x + c)^7 - 108*a^3*b^7*cosh(d*x + c) + 15*a*b^9 + 28*(63*a
^5*b^5 + 55*a*b^9)*cosh(d*x + c)^6 + 12*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(
d*x + c)^5 - 10*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9)*cosh(d*x + c)^4 - 8
*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c)^3 + 12*(63*a^5*b^5 + 55*a*b^9)
*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^12*d*cosh(d*x + c)
^6 + b^12*d*sinh(d*x + c)^6 - 2*a^2*b^10*d*cosh(d*x + c)^5 - b^12*d*cosh(d*
x + c)^4 + 2*(3*b^12*d*cosh(d*x + c) - a^2*b^10*d)*sinh(d*x + c)^5 + (15*b^
12*d*cosh(d*x + c)^2 - 10*a^2*b^10*d*cosh(d*x + c) - b^12*d)*sinh(d*x + c)^
4 + 4*(5*b^12*d*cosh(d*x + c)^3 - 5*a^2*b^10*d*cosh(d*x + c)^2 - b^12*d*cos
h(d*x + c))*sinh(d*x + c)^3 + (15*b^12*d*cosh(d*x + c)^4 - 20*a^2*b^10*d*co
sh(d*x + c)^3 - 6*b^12*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*b^12*d*cos
h(d*x + c)^5 - 5*a^2*b^10*d*cosh(d*x + c)^4 - 2*b^12*d*cosh(d*x + c)^3)*sin
h(d*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
```

ostep)]Evaluation time: 7.79Unable to divide, perhaps due to rounding error

%%%{-1729382256910270464,[0,42,80,42,0]%%%}+%%%{15996785876420001792,[0,42,
80,40,0]%%%}+%%%{-68526771930069467136,[0,42,80,38,0]%%%}+%%%{180423208271716810752,[0,42,80,36,0]%%%}+%%%{-326747411964797911040,[0,42,80,34,0]%%%}+%%%{431598529639377534976,[0,42,80,32,0]%%%}+%%%{-430224087328099401728,[0,42,80,30,0]%%%}+%%%{330307894498327265280,[0,42,80,28,0]%%%}+%%%{-197600852044182192128,[0,42,80,26,0]%%%}+%%%{92585071508226834432,[0,42,80,24,0]%%%}+%%%{-33970440883023118336,[0,42,80,22,0]%%%}+%%%{9710760527558344704,[0,42,80,20,0]%%%}+%%%{-2140791573556756480,[0,42,80,18,0]%%%}+%%%{358224865469136896,[0,42,80,16,0]%%%}+%%%{-44470606778859520,[0,42,80,14,0]%%%}+%%%{3967229615931392,[0,42,80,12,0]%%%}+%%%{-243215139602432,[0,42,80,10,0]%%%}+%%%{9588782923776,[0,42,80,8,0]%%%}+%%%{-216560304128,[0,42,80,6,0]%%%}+%%%{2113929216,[0,42,80,4,0]%%%}+%%%{-630503947831869440,[0,38,84,38,0]%%%}+%%%{5201657569612922880,[0,38,84,36,0]%%%}+%%%{-19689737570863808512,[0,38,84,34,0]%%%}+%%%{45329293199437463552,[0,38,84,32,0]%%%}+%%%{-70927964587643895808,[0,38,84,30,0]%%%}+%%%{79847537163697651712,[0,38,84,28,0]%%%}+%%%{-66770948617534439424,[0,38,84,26,0]%%%}+%%%{42219726981528813568,[0,38,84,24,0]%%%}+%%%{-20353757381889359872,[0,38,84,22,0]%%%}+%%%{7488142989424852992,[0,38,84,20,0]%%%}+%%%{-2090414390313484288,[0,38,84,18,0]%%%}+%%%{437321867955535872,[0,38,84,16,0]%%%}+%%%{-67182577449959424,[0,38,84,14,0]%%%}+%%%{7352034554544128,[0,38,84,12,0]%%%}+%%%{-548079300247552,[0,38,84,10,0]%%%}+%%%{25997302824960,[0,38,84,8,0]%%%}+%%%{-697294651392,[0,38,84,6,0]%%%}+%%%{7969177600,[0,38,84,4,0]%%%}+%%%{-562949953421312,[0,36,68,36,1]%%%}+%%%{4503599627370496,[0,36,68,34,1]%%%}+%%%{-15621861207441408,[0,36,68,32,1]%%%}+%%%{30540034973106176,[0,36,68,30,1]%%%}+%%%{-35978219484086272,[0,36,68,28,1]%%%}+%%%{23762645299494912,[0,36,68,26,1]%%%}+%%%{-3508541604233216,[0,36,68,24,1]%%%}+%%%{-9089800065777664,[0,36,68,22,1]%%%}+%%%{9933383182319616,[0,36,68,20,1]%%%}+%%%{-5561664820740096,[0,36,68,18,1]%%%}+%%%{1991473255940096,[0,36,68,16,1]%%%}+%%%{-476780024561664,[0,36,68,14,1]%%%}+%%%{76036188405760,[0,36,68,12,1]%%%}+%%%{-7830362914816,[0,36,68,10,1]%%%}+%%%{492012830720,[0,36,68,8,1]%%%}+%%%{-17188257792,[0,36,68,6,1]%%%}+%%%{275775488,[0,36,68,4,1]%%%}+%%%{-1048576,[0,36,68,2,1]%%%}+%%%{-94575592174780416,[0,34,88,34,0]%%%}+%%%{686517468197289984,[0,34,88,32,0]%%%}+%%%{-2259821850521501696,[0,34,88,30,0]%%%}+%%%{4463612588491538432,[0,34,88,28,0]%%%}+%%%{-5899565978273972224,[0,34,88,26,0]%%%}+%%%{5508477388855443456,[0,34,88,24,0]%%%}+%%%{-3738978625572044800,[0,34,88,22,0]%%%}+%%%{1870074940166766592,[0,34,88,20,0]%%%}+%%%{-691108015142600704,[0,34,88,18,0]%%%}+%%%{187501856778354688,[0,34,88,16,0]%%%}+%%%{-36757706656186368,[0,34,88,14,0]%%%}+%%%{5067624664793088,[0,34,88,12,0]%%%}+%%%{-470798410186752,[0,34,88,10,0]%%%}+%%%{27533189840896,[0,34,88,8,0]%%%}+%%%{-898503802880,[0,34,88,6,0]%%%}+%%%{12297699328,[0,34,88,4,0]%%%}+%%%{19351404648576,[0,32,72,28,1]%%%}+%%%{-1162183790559232,[0,32,72,26,1]%%%}+%%%{3077258168238080,[0,32,72,24,1]%%%}+%%%{-4728243596820480,[0,32,72,22,1]%%%}+%%%{4669660242903040,[0,32,72,20,1]%%%}+%%%{-3102521165873152,[0,32,72,18,1]%%%}+%%%{1410863470739456,[0,32,72,16,1]%%%}+%%%{-438356441825280,[0,32,72,14,1]%%%}+%%%{91350129180672,[0,32,72,12,1]%%%}+%%%{-12302698938368,[0,32,72,10,1]%%%}+%%%{1006058340352,[0,32,72,8,1]%%%}+%%%{-45027950592,[0,32,72,6,1]%%%}+%%%{903872512,[0,32,72,4,1]%%%}+%%%{-4194304,[0,32,72,2,1]%%%}+%%%{-7564639999098880,[0,30,92,30,0]%%%}+%%%{47507698412945408,[0,30,92,28,0]%%%}+%%%{-133256411239940096,[0,30,92,26,0]%%%}+%%%{220312443392360448,[0,30,92,24,0]%%%}+%%%{-238605155782623232,[0,30,92,22,0]%%%}+%%%{177943827967901696,[0,30,92,20,0]%%%}+%%%{-93502168178360320,[0,30,92,18,0]%%%}+%%%{34825220481089536,[0,30,92,16,0]%%%}+%%%{-9124293485002752,[0,30,92,14,0]%%%}+%%%{1646232376180736,[0,30,92,12,0]%%%}+%%%{-196807145553920,[0,30,92,10,0]%%%}+%%%{14601882173440,[0,30,92,8,0]%%%}+%%%{-596228702208,[0,30,92,6,0]%%%}+%%%{10041163776,[0,30,92,4,0]%%%}+%%%{9895604649984,[0,30,56,30,2]%%%}+%%%{-66795331387392,[0,30,56,28,2]%%%}+%%%{200042396778496,[0,30,56,26,2]%%%}+%%%{-350091374231552,[0,30,56,24,2]%%%}+%%%{396906517757952,[0,30,56,22,2]%%%}+%%%{-305424788094976,[0,30,56,20,2]%%%}+%%%{162543305752576,[0,30,56,18,2]%%%}+%%%{-5985164433817

6,[0,30,56,16,2]%%%}+%%%{15024131145728,[0,30,56,14,2]%%%}+%%%{-24942980628
48,[0,30,56,12,2]%%%}+%%%{261026217984,[0,30,56,10,2]%%%}+%%%{-16062087168,
[0,30,56,8,2]%%%}+%%%{523763712,[0,30,56,6,2]%%%}+%%%{-6946816,[0,30,56,4,2
]%%%}+%%%{23089744183296,[0,28,76,28,1]%%%}+%%%{-140737488355328,[0,28,76,2
6,1]%%%}+%%%{397473453441024,[0,28,76,24,1]%%%}+%%%{-686370133639168,[0,28,
76,22,1]%%%}+%%%{799332068491264,[0,28,76,20,1]%%%}+%%%{-651106304655360,[0
,28,76,18,1]%%%}+%%%{373432105566208,[0,28,76,16,1]%%%}+%%%{-1491266332262
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40,[0,28,76,10,1]%%%}+%%%{767515688960,[0,28,76,8,1]%%%}+%%%{-45071466496,[
0,28,76,6,1]%%%}+%%%{1164967936,[0,28,76,4,1]%%%}+%%%{-6815744,[0,28,76,2,1
]%%%}+%%%{-351293965074432,[0,26,96,26,0]%%%}+%%%{1868207694544896,[0,26,96
,24,0]%%%}+%%%{-4347468976226304,[0,26,96,22,0]%%%}+%%%{5816459460608000,[0
,26,96,20,0]%%%}+%%%{-4943217447403520,[0,26,96,18,0]%%%}+%%%{2783137197195
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1295360,[0,26,60,16,2]%%%}+%%%{9797281775616,[0,26,60,14,2]%%%}+%%%{-239439
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0352,[0,26,60,8,2]%%%}+%%%{1316880384,[0,26,60,6,2]%%%}+%%%{-20774912,[0,26
,60,4,2]%%%}+%%%{2954937499648,[0,24,80,24,1]%%%}+%%%{-15238543966208,[0,24
,80,22,1]%%%}+%%%{34763465293824,[0,24,80,20,1]%%%}+%%%{-46071040442368,[0,
24,80,18,1]%%%}+%%%{39036957753344,[0,24,80,16,1]%%%}+%%%{-21876415922176,[
0,24,80,14,1]%%%}+%%%{8101700829184,[0,24,80,12,1]%%%}+%%%{-1927848919040,[
0,24,80,10,1]%%%}+%%%{278057189376,[0,24,80,8,1]%%%}+%%%{-21830041600,[0,24
,80,6,1]%%%}+%%%{749993984,[0,24,80,4,1]%%%}+%%%{-5767168,[0,24,80,2,1]%%%}
+%%%{-9663676416,[0,24,44,24,3]%%%}+%%%{53150220288,[0,24,44,22,3]%%%}+%%%{
-125829120000,[0,24,44,20,3]%%%}+%%%{167604387840,[0,24,44,18,3]%%%}+%%%{-1
37824829440,[0,24,44,16,3]%%%}+%%%{72284635136,[0,24,44,14,3]%%%}+%%%{-2415
6307456,[0,24,44,12,3]%%%}+%%%{5004853248,[0,24,44,10,3]%%%}+%%%{-608665600
,[0,24,44,8,3]%%%}+%%%{39649280,[0,24,44,6,3]%%%}+%%%{-1163264,[0,24,44,4,3
]%%%}+%%%{8192,[0,24,44,2,3]%%%}+%%%{-9740985827328,[0,22,100,22,0]%%%}+%%%
{42627550412800,[0,22,100,20,0]%%%}+%%%{-79274358865920,[0,22,100,18,0]%%%}
+%%%{81645180813312,[0,22,100,16,0]%%%}+%%%{-50860801392640,[0,22,100,14,0]
%%%}+%%%{19654357549056,[0,22,100,12,0]%%%}+%%%{-4649931243520,[0,22,100,10
,0]%%%}+%%%{640363266048,[0,22,100,8,0]%%%}+%%%{-45981630464,[0,22,100,6,0]
%%%}+%%%{1298661376,[0,22,100,4,0]%%%}+%%%{224412041216,[0,22,64,22,2]%%%}+
%%%{-1064346583040,[0,22,64,20,2]%%%}+%%%{2126210138112,[0,22,64,18,2]%%%}+
%%%{-2323812188160,[0,22,64,16,2]%%%}+%%%{1510704414720,[0,22,64,14,2]%%%}+
%%%{-595005014016,[0,22,64,12,2]%%%}+%%%{138570366976,[0,22,64,10,2]%%%}+%%
%{-17797087232,[0,22,64,8,2]%%%}+%%%{1090617344,[0,22,64,6,2]%%%}+%%%{-2254
4384,[0,22,64,4,2]%%%}+%%%{151934468096,[0,20,84,20,1]%%%}+%%%{-64343978803
2,[0,20,84,18,1]%%%}+%%%{1155010658304,[0,20,84,16,1]%%%}+%%%{-114096812851
2,[0,20,84,14,1]%%%}+%%%{672088981504,[0,20,84,12,1]%%%}+%%%{-238691549184,
[0,20,84,10,1]%%%}+%%%{49267081216,[0,20,84,8,1]%%%}+%%%{-5401411584,[0,20,
84,6,1]%%%}+%%%{255803392,[0,20,84,4,1]%%%}+%%%{-2719744,[0,20,84,2,1]%%%}+
%%%{-2080374784,[0,20,48,20,3]%%%}+%%%{9361686528,[0,20,48,18,3]%%%}+%%%{-1
7511219200,[0,20,48,16,3]%%%}+%%%{17600348160,[0,20,48,14,3]%%%}+%%%{-10259
529728,[0,20,48,12,3]%%%}+%%%{3493724160,[0,20,48,10,3]%%%}+%%%{-666894336,
[0,20,48,8,3]%%%}+%%%{64839680,[0,20,48,6,3]%%%}+%%%{-2629632,[0,20,48,4,3]
%%%}+%%%{24576,[0,20,48,2,3]%%%}+%%%{-161061273600,[0,18,104,18,0]%%%}+%%%{
556466700288,[0,18,104,16,0]%%%}+%%%{-780341870592,[0,18,104,14,0]%%%}+%%%{
568630181888,[0,18,104,12,0]%%%}+%%%{-228686036992,[0,18,104,10,0]%%%}+%%%{
49696210944,[0,18,104,8,0]%%%}+%%%{-5289017344,[0,18,104,6,0]%%%}+%%%{20997
7344,[0,18,104,4,0]%%%}+%%%{9563013120,[0,18,68,18,2]%%%}+%%%{-35643195392,
[0,18,68,16,2]%%%}+%%%{53273952256,[0,18,68,14,2]%%%}+%%%{-40687370240,[0,1
8,68,12,2]%%%}+%%%{16733962240,[0,18,68,10,2]%%%}+%%%{-3574857728,[0,18,68,

```
8,2]%%%}+%%%{347766784,[0,18,68,6,2]%%%}+%%%{-10797056,[0,18,68,4,2]%%%}+%%
%{524288,[0,18,32,18,4]%%%}+%%%{-2228224,[0,18,32,16,4]%%%}+%%%{3932160,[0,
18,32,14,4]%%%}+%%%{-3727360,[0,18,32,12,4]%%%}+%%%{2035712,[0,18,32,10,4]%
%%}+%%%{-626688,[0,18,32,8,4]%%%}+%%%{95232,[0,18,32,6,4]%%%}+%%%{-5120,[0,
18,32,4,4]%%%}+%%%{3758096384,[0,16,88,16,1]%%%}+%%%{-12515803136,[0,16,88,
14,1]%%%}+%%%{16601055232,[0,16,88,12,1]%%%}+%%%{-11094982656,[0,16,88,10,1
]%%%}+%%%{3874226176,[0,16,88,8,1]%%%}+%%%{-660668416,[0,16,88,6,1]%%%}+%%%
{46268416,[0,16,88,4,1]%%%}+%%%{-720896,[0,16,88,2,1]%%%}+%%%{-160432128,[0
,16,52,16,3]%%%}+%%%{561512448,[0,16,52,14,3]%%%}+%%%{-771162112,[0,16,52,1
2,3]%%%}+%%%{523862016,[0,16,52,10,3]%%%}+%%%{-181960704,[0,16,52,8,3]%%%}+
%%%{30070784,[0,16,52,6,3]%%%}+%%%{-1943552,[0,16,52,4,3]%%%}+%%%{26624,[0,
16,52,2,3]%%%}+%%%{-1543503872,[0,14,108,14,0]%%%}+%%%{3934257152,[0,14,108
,12,0]%%%}+%%%{-3739222016,[0,14,108,10,0]%%%}+%%%{1612185600,[0,14,108,8,0
]%%%}+%%%{-302383104,[0,14,108,6,0]%%%}+%%%{19005440,[0,14,108,4,0]%%%}+%%%
{192937984,[0,14,72,14,2]%%%}+%%%{-521928704,[0,14,72,12,2]%%%}+%%%{5172756
48,[0,14,72,10,2]%%%}+%%%{-226590720,[0,14,72,8,2]%%%}+%%%{41209856,[0,14,7
2,6,2]%%%}+%%%{-2230272,[0,14,72,4,2]%%%}+%%%{-49152,[0,14,36,14,4]%%%}+%%%
{159744,[0,14,36,12,4]%%%}+%%%{-174080,[0,14,36,10,4]%%%}+%%%{63488,[0,14,3
6,8,4]%%%}+%%%{3072,[0,14,36,6,4]%%%}+%%%{-3072,[0,14,36,4,4]%%%}+%%%{46137
344,[0,12,92,12,1]%%%}+%%%{-111935488,[0,12,92,10,1]%%%}+%%%{96403456,[0,12
,92,8,1]%%%}+%%%{-34291712,[0,12,92,6,1]%%%}+%%%{4206592,[0,12,92,4,1]%%%}+
%%%{-106496,[0,12,92,2,1]%%%}+%%%{-5242880,[0,12,56,12,3]%%%}+%%%{13156352,
[0,12,56,10,3]%%%}+%%%{-11575296,[0,12,56,8,3]%%%}+%%%{4152320,[0,12,56,6,3
]%%%}+%%%{-515072,[0,12,56,4,3]%%%}+%%%{12288,[0,12,56,2,3]%%%}+%%%{1536,[0
,12,20,12,5]%%%}+%%%{-4608,[0,12,20,10,5]%%%}+%%%{4928,[0,12,20,8,5]%%%}+%%
%{-2176,[0,12,20,6,5]%%%}+%%%{336,[0,12,20,4,5]%%%}+%%%{-16,[0,12,20,2,5]%%
%}+%%%{-7864320,[0,10,112,10,0]%%%}+%%%{12976128,[0,10,112,8,0]%%%}+%%%{-64
22528,[0,10,112,6,0]%%%}+%%%{843776,[0,10,112,4,0]%%%}+%%%{1703936,[0,10,76
,10,2]%%%}+%%%{-2875392,[0,10,76,8,2]%%%}+%%%{1423360,[0,10,76,6,2]%%%}+%%%
{-181248,[0,10,76,4,2]%%%}+%%%{-9216,[0,10,40,10,4]%%%}+%%%{20480,[0,10,40,
8,4]%%%}+%%%{-13440,[0,10,40,6,4]%%%}+%%%{2176,[0,10,40,4,4]%%%}+%%%{262144
,[0,8,96,8,1]%%%}+%%%{-393216,[0,8,96,6,1]%%%}+%%%{155648,[0,8,96,4,1]%%%}+
%%%{-8192,[0,8,96,2,1]%%%}+%%%{-63488,[0,8,60,8,3]%%%}+%%%{98560,[0,8,60,6,
3]%%%}+%%%{-39360,[0,8,60,4,3]%%%}+%%%{2176,[0,8,60,2,3]%%%}+%%%{320,[0,8,2
4,8,5]%%%}+%%%{-640,[0,8,24,6,5]%%%}+%%%{352,[0,8,24,4,5]%%%}+%%%{-32,[0,8,
24,2,5]%%%}+%%%{-16384,[0,6,116,6,0]%%%}+%%%{12288,[0,6,116,4,0]%%%}+%%%{51
20,[0,6,80,6,2]%%%}+%%%{-3584,[0,6,80,4,2]%%%}+%%%{-128,[0,6,44,6,4]%%%}+%%
%{128,[0,6,44,4,4]%%%}+%%%{512,[0,4,100,4,1]%%%}+%%%{-256,[0,4,100,2,1]%%%}
+%%%{-192,[0,4,64,4,3]%%%}+%%%{128,[0,4,64,2,3]%%%}+%%%{16,[0,4,28,4,5]%%%}
+%%%{-16,[0,4,28,2,5]%%%} / %%%{4194304,[0,16,32,16,0]%%%}+%%%{-14680064,[0
,16,32,14,0]%%%}+%%%{21233664,[0,16,32,12,0]%%%}+%%%{-16384000,[0,16,32,10,
0]%%%}+%%%{7241728,[0,16,32,8,0]%%%}+%%%{-1818624,[0,16,32,6,0]%%%}+%%%{238
592,[0,16,32,4,0]%%%}+%%%{-13312,[0,16,32,2,0]%%%}+%%%{256,[0,16,32,0,0]%%%
}+%%%{786432,[0,12,36,12,0]%%%}+%%%{-1966080,[0,12,36,10,0]%%%}+%%%{1884160
,[0,12,36,8,0]%%%}+%%%{-864256,[0,12,36,6,0]%%%}+%%%{191488,[0,12,36,4,0]%%
%}+%%%{-17920,[0,12,36,2,0]%%%}+%%%{512,[0,12,36,0,0]%%%}+%%%{8192,[0,10,20
,10,1]%%%}+%%%{-18432,[0,10,20,8,1]%%%}+%%%{13312,[0,10,20,6,1]%%%}+%%%{-32
00,[0,10,20,4,1]%%%}+%%%{128,[0,10,20,2,1]%%%}+%%%{45056,[0,8,40,8,0]%%%}+%
%%{-69632,[0,8,40,6,0]%%%}+%%%{35072,[0,8,40,4,0]%%%}+%%%{-6272,[0,8,40,2,0
]%%%}+%%%{320,[0,8,40,0,0]%%%}+%%%{768,[0,6,24,6,1]%%%}+%%%{-960,[0,6,24,4,
1]%%%}+%%%{224,[0,6,24,2,1]%%%}+%%%{768,[0,4,44,4,0]%%%}+%%%{-576,[0,4,44,2
,0]%%%}+%%%{64,[0,4,44,0,0]%%%}+%%%{-12,[0,4,8,4,2]%%%}+%%%{12,[0,4,8,2,2]%
%%}+%%%{-1,[0,4,8,0,2]%%%}+%%%{8,[0,2,28,2,1]%%%}+%%%{4,[0,0,48,0,0]%%%}+%%
%{-1,[0,0,12,0,2]%%%} Error: Bad Argument Value
```

maple [C] time = 0.39, size = 955, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

```
[Out] -1/d/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-4/d/b^
8*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-
a^2)*a^8-8/d/b^4*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(
1/2*d*x+1/2*c)-a^2)*a^4-10/d/b^6*ln(tanh(1/2*d*x+1/2*c)+1)*a^4-7/d/b^8/(tan
h(1/2*d*x+1/2*c)-1)*a^6+5/2/d/b^6/(tanh(1/2*d*x+1/2*c)-1)*a^4-6/d/b^4/(tanh
(1/2*d*x+1/2*c)-1)*a^2-1/d/b^4/(tanh(1/2*d*x+1/2*c)-1)^3*a^2+9/d/b^10*ln(a^
2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^8+10/d/b^6*ln(a^2*
tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^4+5/2/d/b^6/(tanh(1/
2*d*x+1/2*c)-1)^2*a^4-3/2/d/b^4/(tanh(1/2*d*x+1/2*c)-1)^2*a^2-9/d/b^10*ln(t
anh(1/2*d*x+1/2*c)-1)*a^8-10/d/b^6*ln(tanh(1/2*d*x+1/2*c)-1)*a^4-7/d/b^8/(t
anh(1/2*d*x+1/2*c)+1)*a^6-5/2/d/b^6/(tanh(1/2*d*x+1/2*c)+1)*a^4-6/d/b^4/(ta
nh(1/2*d*x+1/2*c)+1)*a^2-1/d/b^4/(tanh(1/2*d*x+1/2*c)+1)^3*a^2+5/2/d/b^6/(t
anh(1/2*d*x+1/2*c)+1)^2*a^4+3/2/d/b^4/(tanh(1/2*d*x+1/2*c)+1)^2*a^2-9/d/b^1
0*ln(tanh(1/2*d*x+1/2*c)+1)*a^8+7/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)-7/8/d/b^2
/(tanh(1/2*d*x+1/2*c)+1)+9/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2+9/8/d/b^2/(tan
h(1/2*d*x+1/2*c)+1)^2+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^3-4/d*tanh(1/2*d*x+
1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+1/4/d/b^2/
(tanh(1/2*d*x+1/2*c)+1)^4-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^3+1/d/b^2*ln(a^
2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+1/4/d/b^2/(tanh(1/2*
d*x+1/2*c)-1)^4+`int/indef0`(-2*cosh(d*x+c)^4*a*b*sinh(d*x+c)^(1/2)/(b^4*si
nh(d*x+c)^2-2*a^2*b^2*sinh(d*x+c)+a^4),sinh(d*x+c))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh\left(c+d\,x\right)^5}{{\left(a+b\,\sqrt{\mathrm{sinh}\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2))^2,x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^(1/2))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

```
[Out] Timed out
```

3.416 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{2a\left(a^4+b^4\right)}{b^6d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2\left(5a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}-\frac{8a^3\sqrt{\sinh(c+dx)}}{b^5d}+\frac{3a^2\sinh(c+dx)}{b^4d}-\frac{4a\,\mathrm{si}}{}$$

```
[Out] 2*(5*a^4+b^4)*ln(a+b*sinh(d*x+c)^(1/2))/b^6/d+3*a^2*sinh(d*x+c)/b^4/d-4/3*a
*sinh(d*x+c)^(3/2)/b^3/d+1/2*sinh(d*x+c)^2/b^2/d-8*a^3*sinh(d*x+c)^(1/2)/b^
5/d+2*a*(a^4+b^4)/b^6/d/(a+b*sinh(d*x+c)^(1/2))
```

Rubi [A] time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3223, 1890, 1620}

$$-\frac{8a^3\sqrt{\sinh(c+dx)}}{b^5d}+\frac{3a^2\sinh(c+dx)}{b^4d}+\frac{2a\left(a^4+b^4\right)}{b^6d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2\left(5a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}-\frac{4a\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*(5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^6*d) + (2*a*(a^4 + b^4)
)/(b^6*d*(a + b*Sqrt[Sinh[c + d*x]])) - (8*a^3*Sqrt[Sinh[c + d*x]])/(b^5*d)
 + (3*a^2*Sinh[c + d*x])/(b^4*d) - (4*a*Sinh[c + d*x]^(3/2))/(3*b^3*d) + Si
nh[c + d*x]^2/(2*b^2*d)
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1890

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b\sqrt{x}\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x(1+x^4)}{(a+bx)^2}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a(a^4+b^4)}{b^5(a+bx)^2}+\frac{5a^4+b^4}{b^5(a+bx)}\right)dx, x, \sqrt{\sinh(c+dx}\right.}{d}$$

$$= \frac{2\left(5a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d} + \frac{2a\left(a^4+b^4\right)}{b^6d\left(a+b\sqrt{\sinh(c+dx)}\right)} - \frac{8a^3\sqrt{s}}{}$$

Mathematica [A] time = 0.47, size = 123, normalized size = 0.87

$$\frac{12\left(\frac{a(a^4+b^4)}{a+b\sqrt{\sinh(c+dx)}}+\left(5a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)\right)-48a^3b\sqrt{\sinh(c+dx)}+18a^2b^2\sinh(c+dx)-8ab}{6b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (12*((5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]] + (a*(a^4 + b^4))/(a + b*
Sqrt[Sinh[c + d*x]])) - 48*a^3*b*Sqrt[Sinh[c + d*x]] + 18*a^2*b^2*Sinh[c +
d*x] - 8*a*b^3*Sinh[c + d*x]^(3/2) + 3*b^4*Sinh[c + d*x]^2)/(6*b^6*d)
```

fricas [B] time = 1.97, size = 2137, normalized size = 15.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^6*cosh(d*x + c)^6 + 3*b^6*sinh(d*x + c)^6 + 30*a^2*b^4*cosh(d*x +
 c)^5 + 30*a^2*b^4*cosh(d*x + c) - 3*b^6 + 6*(3*b^6*cosh(d*x + c) + 5*a^2*b
^4)*sinh(d*x + c)^5 - 3*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*
a^4*b^2 + b^6)*c)*cosh(d*x + c)^4 + 3*(15*b^6*cosh(d*x + c)^2 + 50*a^2*b^4*
cosh(d*x + c) - 24*a^4*b^2 - b^6 - 8*(5*a^4*b^2 + b^6)*d*x - 8*(5*a^4*b^2 +
 b^6)*c)*sinh(d*x + c)^4 - 24*(4*a^6 + 7*a^2*b^4 - 2*(5*a^6 + a^2*b^4)*d*x
- 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x + c)^3 + 12*(5*b^6*cosh(d*x + c)^3 + 25*a
^2*b^4*cosh(d*x + c)^2 - 8*a^6 - 14*a^2*b^4 + 4*(5*a^6 + a^2*b^4)*d*x + 4*(
5*a^6 + a^2*b^4)*c - (24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*a^4
*b^2 + b^6)*c)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(24*a^4*b^2 + b^6 + 8*(5*
a^4*b^2 + b^6)*d*x + 8*(5*a^4*b^2 + b^6)*c)*cosh(d*x + c)^2 + 3*(15*b^6*cos
h(d*x + c)^4 + 100*a^2*b^4*cosh(d*x + c)^3 + 24*a^4*b^2 + b^6 + 8*(5*a^4*b^
2 + b^6)*d*x - 6*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*a^4*b^2
 + b^6)*c)*cosh(d*x + c)^2 + 8*(5*a^4*b^2 + b^6)*c - 24*(4*a^6 + 7*a^2*b^4
- 2*(5*a^6 + a^2*b^4)*d*x - 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x + c))*sinh(d*x
+ c)^2 + 24*((5*a^4*b^2 + b^6)*cosh(d*x + c)^4 + (5*a^4*b^2 + b^6)*sinh(d*x
 + c)^4 - 2*(5*a^6 + a^2*b^4)*cosh(d*x + c)^3 - 2*(5*a^6 + a^2*b^4 - 2*(5*a
^4*b^2 + b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (5*a^4*b^2 + b^6)*cosh(d*x +
 c)^2 - (5*a^4*b^2 + b^6 - 6*(5*a^4*b^2 + b^6)*cosh(d*x + c)^2 + 6*(5*a^6 +
 a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(2*(5*a^4*b^2 + b^6)*cosh(d*x
+ c)^3 - 3*(5*a^6 + a^2*b^4)*cosh(d*x + c)^2 - (5*a^4*b^2 + b^6)*cosh(d*x +
```

```
c))*sinh(d*x + c))*log(-(b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2
*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) + 4*(a*b*c
osh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2
 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) -
 a^2)*sinh(d*x + c))) + 24*((5*a^4*b^2 + b^6)*cosh(d*x + c)^4 + (5*a^4*b^2
+ b^6)*sinh(d*x + c)^4 - 2*(5*a^6 + a^2*b^4)*cosh(d*x + c)^3 - 2*(5*a^6 + a
^2*b^4 - 2*(5*a^4*b^2 + b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (5*a^4*b^2 +
b^6)*cosh(d*x + c)^2 - (5*a^4*b^2 + b^6 - 6*(5*a^4*b^2 + b^6)*cosh(d*x + c)
^2 + 6*(5*a^6 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(2*(5*a^4*b^2 +
 b^6)*cosh(d*x + c)^3 - 3*(5*a^6 + a^2*b^4)*cosh(d*x + c)^2 - (5*a^4*b^2 +
b^6)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b^2*sinh(d*x + c) - a^2)/(cosh(d*
x + c) - sinh(d*x + c))) + 6*(3*b^6*cosh(d*x + c)^5 + 25*a^2*b^4*cosh(d*x +
 c)^4 + 5*a^2*b^4 - 2*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*a^
4*b^2 + b^6)*c)*cosh(d*x + c)^3 - 12*(4*a^6 + 7*a^2*b^4 - 2*(5*a^6 + a^2*b^
4)*d*x - 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x + c)^2 + (24*a^4*b^2 + b^6 + 8*(5*
a^4*b^2 + b^6)*d*x + 8*(5*a^4*b^2 + b^6)*c)*cosh(d*x + c))*sinh(d*x + c) -
16*(a*b^5*cosh(d*x + c)^5 + a*b^5*sinh(d*x + c)^5 + 10*a^3*b^3*cosh(d*x + c
)^4 - 10*a^3*b^3*cosh(d*x + c)^2 + a*b^5*cosh(d*x + c) + 5*(a*b^5*cosh(d*x
+ c) + 2*a^3*b^3)*sinh(d*x + c)^4 - 2*(15*a^5*b + 4*a*b^5)*cosh(d*x + c)^3
+ 2*(5*a*b^5*cosh(d*x + c)^2 + 20*a^3*b^3*cosh(d*x + c) - 15*a^5*b - 4*a*b^
5)*sinh(d*x + c)^3 + 2*(5*a*b^5*cosh(d*x + c)^3 + 30*a^3*b^3*cosh(d*x + c)^
2 - 5*a^3*b^3 - 3*(15*a^5*b + 4*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + (5*
a*b^5*cosh(d*x + c)^4 + 40*a^3*b^3*cosh(d*x + c)^3 - 20*a^3*b^3*cosh(d*x +
c) + a*b^5 - 6*(15*a^5*b + 4*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(si
nh(d*x + c)))/(b^8*d*cosh(d*x + c)^4 + b^8*d*sinh(d*x + c)^4 - 2*a^2*b^6*d*
cosh(d*x + c)^3 - b^8*d*cosh(d*x + c)^2 + 2*(2*b^8*d*cosh(d*x + c) - a^2*b^
6*d)*sinh(d*x + c)^3 + (6*b^8*d*cosh(d*x + c)^2 - 6*a^2*b^6*d*cosh(d*x + c)
 - b^8*d)*sinh(d*x + c)^2 + 2*(2*b^8*d*cosh(d*x + c)^3 - 3*a^2*b^6*d*cosh(d
*x + c)^2 - b^8*d*cosh(d*x + c))*sinh(d*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Undef/Unsigned Inf encountered in limitEvaluation time: 2.93Limit: M
ax order reached or unable to make series expansion Error: Bad Argument Val
ue
```

maple [C] time = 0.37, size = 481, normalized size = 3.39

$$\frac{1}{2d\,b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3a^2}{d\,b^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{1}{2d\,b^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^4}{d\,b^6}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

```
[Out] 1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2-3/d/b^4/(tanh(1/2*d*x+1/2*c)-1)*a^2+1/2
/d/b^2/(tanh(1/2*d*x+1/2*c)-1)-5/d/b^6*ln(tanh(1/2*d*x+1/2*c)-1)*a^4-1/d/b^
2*ln(tanh(1/2*d*x+1/2*c)-1)+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2-3/d/b^4/(ta
nh(1/2*d*x+1/2*c)+1)*a^2-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)-5/d/b^6*ln(tanh(
1/2*d*x+1/2*c)+1)*a^4-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-4/d/b^4*tanh(1/2*d*
```

```
x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^4-4/d*
tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^
2)+5/d/b^6*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^4+
1/d/b^2*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+`int/in
def0`(-2*cosh(d*x+c)^2*a*b*sinh(d*x+c)^(1/2)/(b^4*sinh(d*x+c)^2-2*a^2*b^2*s
inh(d*x+c)+a^4),sinh(d*x+c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(dx+c\right)^{3}}{\left(b\sqrt{\sinh\left(dx+c\right)}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^{3}}{\left(a+b\,\sqrt{\sinh\left(c+d\,x\right)}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2))^2,x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^(1/2))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

```
[Out] Timed out
```

3.417 $\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{2a}{b^2d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}$$

```
[Out] 2*ln(a+b*sinh(d*x+c)^(1/2))/b^2/d+2*a/b^2/d/(a+b*sinh(d*x+c)^(1/2))
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3223, 190, 43}

$$\frac{2a}{b^2d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^2*d) + (2*a)/(b^2*d*(a + b*Sqrt[Sinh[
c + d*x]]))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d} + \frac{2a}{b^2d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

Mathematica [A] time = 0.06, size = 42, normalized size = 0.86

$$\frac{2\left(\frac{a}{a+b\sqrt{\sinh(c+dx)}}+\log\left(a+b\sqrt{\sinh(c+dx)}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*(Log[a + b*Sqrt[Sinh[c + d*x]]] + a/(a + b*Sqrt[Sinh[c + d*x]])))/(b^2*d
)
```

fricas [B] time = 0.53, size = 564, normalized size = 11.51

$$\frac{b^2dx + b^2c - \left(b^2dx+b^2c\right)\cosh\left(dx+c\right)^2 - \left(b^2dx+b^2c\right)\sinh\left(dx+c\right)^2 + 2\left(a^2dx+a^2c-2\,a^2\right)\cosh\left(dx+c\right) +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cosh(d*x + c)^2 - (b^2*d*x + b^2*c)*si
nh(d*x + c)^2 + 2*(a^2*d*x + a^2*c - 2*a^2)*cosh(d*x + c) + (b^2*cosh(d*x +
 c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x +
 c) - a^2)*sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 +
2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) + 4*(
a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x +
 c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x +
 c) - a^2)*sinh(d*x + c))) + (b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 - 2
*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh(d*x + c))*log(2
*(b^2*sinh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(a^2*d*x +
a^2*c - 2*a^2 - (b^2*d*x + b^2*c)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cos
h(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^2
 + b^4*d*sinh(d*x + c)^2 - 2*a^2*b^2*d*cosh(d*x + c) - b^4*d + 2*(b^4*d*cos
h(d*x + c) - a^2*b^2*d)*sinh(d*x + c))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Undef/Unsigned Inf encountered in limitEvaluation time: 1.04Limit: M
ax order reached or unable to make series expansion Error: Bad Argument Val
ue
```

maple [B] time = 0.03, size = 144, normalized size = 2.94

$$-\frac{2a^2}{d\left(b^2\sinh(dx+c)-a^2\right)b^2}+\frac{\ln\left(b^2\sinh(dx+c)-a^2\right)}{d\,b^2}+\frac{a}{d\,b^2\left(b\left(\sqrt{\sinh}\,(dx+c)\right)-a\right)}-\frac{\ln\left(b\left(\sqrt{\sinh}\,(dx+c)\right)-\right.}{d\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

```
[Out] -2/d*a^2/(b^2*sinh(d*x+c)-a^2)/b^2+1/d*ln(b^2*sinh(d*x+c)-a^2)/b^2+1/d*a/b^
2/(b*sinh(d*x+c)^(1/2)-a)-1/d/b^2*ln(b*sinh(d*x+c)^(1/2)-a)+a/b^2/d/(a+b*si
nh(d*x+c)^(1/2))+ln(a+b*sinh(d*x+c)^(1/2))/b^2/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{\left(b\sqrt{\sinh(dx+c)}+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

mupad [B] time = 1.41, size = 45, normalized size = 0.92

$$\frac{2\,a}{b^2\,\left(a\,d+b\,d\,\sqrt{\sinh(c+d\,x)}\right)}+\frac{2\,\ln\left(a+b\,\sqrt{\sinh(c+d\,x)}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^(1/2))^2,x)
```

```
[Out] (2*a)/(b^2*(a*d + b*d*sinh(c + d*x)^(1/2))) + (2*log(a + b*sinh(c + d*x)^(1
/2)))/(b^2*d)
```

sympy [A] time = 4.36, size = 151, normalized size = 3.08

$$\begin{cases} \frac{x\cosh(c)}{a^2} & \text{for } b=0 \wedge d=0 \\ \frac{\sinh(c+dx)}{a^2 d} & \text{for } b=0 \\ \frac{x\cosh(c)}{\left(a+b\sqrt{\sinh(c)}\right)^2} & \text{for } d=0 \\ \frac{2a\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)}{ab^2d+b^3d\sqrt{\sinh(c+dx)}}+\frac{2a}{ab^2d+b^3d\sqrt{\sinh(c+dx)}}+\frac{2b\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)\sqrt{\sinh(c+dx)}}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((x*cosh(c)/a**2, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a**2*d), E
q(b, 0)), (x*cosh(c)/(a + b*sqrt(sinh(c)))**2, Eq(d, 0)), (2*a*log(a/b + sq
rt(sinh(c + d*x)))/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))) + 2*a/(a*b**2*d
+ b**3*d*sqrt(sinh(c + d*x))) + 2*b*log(a/b + sqrt(sinh(c + d*x)))*sqrt(sin
h(c + d*x))/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))), True))
```

3.418 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=384

$$\frac{2ab^2}{d\left(a^4+b^4\right)\left(a+b\sqrt{\sinh(c+dx)}\right)}-\frac{2b^2\left(3a^4-b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{d\left(a^4+b^4\right)^2}+\frac{b^2\left(3a^4-b^4\right)\log(\cosh(c+dx))}{d\left(a^4+b^4\right)^2}$$

```
[Out] a^2*(a^4-3*b^4)*arctan(sinh(d*x+c))/(a^4+b^4)^2/d+b^2*(3*a^4-b^4)*ln(cosh(d
*x+c))/(a^4+b^4)^2/d-2*b^2*(3*a^4-b^4)*ln(a+b*sinh(d*x+c)^(1/2))/(a^4+b^4)^
2/d-1/2*a*b*(a^4+2*a^2*b^2-b^4)*ln(1+sinh(d*x+c)-2^(1/2)*sinh(d*x+c)^(1/2))
/(a^4+b^4)^2/d*2^(1/2)+1/2*a*b*(a^4+2*a^2*b^2-b^4)*ln(1+sinh(d*x+c)+2^(1/2)
*sinh(d*x+c)^(1/2))/(a^4+b^4)^2/d*2^(1/2)-a*b*(a^4-2*a^2*b^2-b^4)*arctan(-1
+2^(1/2)*sinh(d*x+c)^(1/2))*2^(1/2)/(a^4+b^4)^2/d-a*b*(a^4-2*a^2*b^2-b^4)*a
rctan(1+2^(1/2)*sinh(d*x+c)^(1/2))*2^(1/2)/(a^4+b^4)^2/d+2*a*b^2/(a^4+b^4)/
d/(a+b*sinh(d*x+c)^(1/2))
```

Rubi [A] time = 0.63, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3223, 6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$\frac{2ab^2}{d\left(a^4+b^4\right)\left(a+b\sqrt{\sinh(c+dx)}\right)}-\frac{ab\left(2a^2b^2+a^4-b^4\right)\log\left(\sinh(c+dx)-\sqrt{2}\sqrt{\sinh(c+dx)}+1\right)}{\sqrt{2}d\left(a^4+b^4\right)^2}+\frac{ab\left(2a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]
])/((a^4 + b^4)^2*d) - (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 + Sqrt
[2]*Sqrt[Sinh[c + d*x]]])/((a^4 + b^4)^2*d) + (a^2*(a^4 - 3*b^4)*ArcTan[Sin
h[c + d*x]])/((a^4 + b^4)^2*d) + (b^2*(3*a^4 - b^4)*Log[Cosh[c + d*x]])/((a
^4 + b^4)^2*d) - (2*b^2*(3*a^4 - b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/((a^4
 + b^4)^2*d) - (a*b*(a^4 + 2*a^2*b^2 - b^4)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d
*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 + b^4)^2*d) + (a*b*(a^4 + 2*a^2*b^2 -
b^4)*Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 +
b^4)^2*d) + (2*a*b^2)/((a^4 + b^4)*d*(a + b*Sqrt[Sinh[c + d*x]]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
```

```
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(1+x^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x}{(a+bx)^2(1+x^4)}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{ab^3}{(a^4+b^4)(a+bx)^2} + \frac{-3a^4b^3+b^7}{(a^4+b^4)^2(a+bx)} + \frac{4a^3b^3+a^2(a^4-3b^4)x-2ab(a^4-b^4)x^2+b^2(3a^4\right.\right.}{d}$$

$$= -\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)\,d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

$$= -\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)\,d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

$$= -\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)\,d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

$$= -\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)\,d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

$$= -\frac{2b^2(3a^4-b^4)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)\,d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

$$= \frac{a^2(a^4-3b^4)\tan^{-1}(\sinh(c+dx))}{(a^4+b^4)^2 d} + \frac{b^2(3a^4-b^4)\log(\cosh(c+dx))}{(a^4+b^4)^2 d} - \frac{2b^2(3\ldots}{}$$

$$= \frac{\sqrt{2}\,ab\left(a^4-2a^2b^2-b^4\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{(a^4+b^4)^2 d} - \frac{\sqrt{2}\,ab\left(a^4-2a^2b^2-\ldots\right.}{}$$

Mathematica [C] time = 0.72, size = 280, normalized size = 0.73

$$\frac{-4ab(a^4-b^4)\sinh^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\sinh^2(c+dx)\right) + \frac{6ab^2(a^4+b^4)}{a+b\sqrt{\sinh(c+dx)}} + 6b^2(b^4-3a^4)\log\left(a+b\sqrt{\sinh(\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (-6*Sqrt[2]*a^3*b^3*(ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]] - ArcTan[1 + S
qrt[2]*Sqrt[Sinh[c + d*x]]]) + 3*a^2*(a^4 - 3*b^4)*ArcTan[Sinh[c + d*x]] -
3*b^2*(-3*a^4 + b^4)*Log[Cosh[c + d*x]] + 6*b^2*(-3*a^4 + b^4)*Log[a + b*Sq
rt[Sinh[c + d*x]]] - 3*Sqrt[2]*a^3*b^3*(Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]
 + Sinh[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]]) +
 (6*a*b^2*(a^4 + b^4))/(a + b*Sqrt[Sinh[c + d*x]]) - 4*a*b*(a^4 - b^4)*Hype
rgeometric2F1[3/4, 1, 7/4, -Sinh[c + d*x]^2]*Sinh[c + d*x]^(3/2))/(3*(a^4 +
 b^4)^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Undef/Unsigned Inf encountered in limitLimit: Max order reached or u
nable to make series expansion Error: Bad Argument Value
```

maple [C] time = 0.48, size = 567, normalized size = 1.48

$$-\frac{4b^4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a^4}{d\left(a^4+b^4\right)^2\left(a^2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a^2\right)}-\frac{4b^8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a^4+b^4\right)^2\left(a^2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b^2\tanh\left(\frac{dx}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

```
[Out] -4/d*b^4/(a^4+b^4)^2*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*t
anh(1/2*d*x+1/2*c)-a^2)*a^4-4/d*b^8/(a^4+b^4)^2*tanh(1/2*d*x+1/2*c)/(a^2*ta
nh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)-3/d*b^2/(a^4+b^4)^2*ln(a
^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^4+1/d*b^6/(a^4+b^
4)^2*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+3/d/(a^8+2
*a^4*b^4+b^8)*ln(tanh(1/2*d*x+1/2*c)^2+1)*a^4*b^2-1/d/(a^8+2*a^4*b^4+b^8)*l
n(tanh(1/2*d*x+1/2*c)^2+1)*b^6+2/d/(a^8+2*a^4*b^4+b^8)*arctan(tanh(1/2*d*x+
1/2*c))*a^6-6/d/(a^8+2*a^4*b^4+b^8)*arctan(tanh(1/2*d*x+1/2*c))*a^2*b^4+`in
t/indef0`(2*a*b*sinh(d*x+c)^(1/2)*(b^4*sinh(d*x+c)^2-2*a^2*b^2*sinh(d*x+c)+
a^4)/(4*a^2*b^6*sinh(d*x+c)*cosh(d*x+c)^4+(4*a^6*b^2-4*a^2*b^6)*cosh(d*x+c)
^2*sinh(d*x+c)-b^8*cosh(d*x+c)^6+(-6*a^4*b^4+2*b^8)*cosh(d*x+c)^4+(-a^8+6*a
^4*b^4-b^8)*cosh(d*x+c)^2),sinh(d*x+c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{sech}\left(dx+c\right)}{\left(b\sqrt{\sinh\left(dx+c\right)}+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cosh\left(c+d\,x\right)\,{\left(a+b\,\sqrt{\mathrm{sinh}\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))^2),x)
[Out] int(1/(cosh(c + d*x)*(a + b*sinh(c + d*x)^(1/2))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}{\left(c+dx \right)}}{\left(a+b\sqrt{\sinh{\left(c+dx \right)}}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**(1/2))**2,x)
[Out] Integral(sech(c + d*x)/(a + b*sqrt(sinh(c + d*x)))**2, x)
```

3.419 $\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}+\frac{\sinh^5(c+dx)\,{}_2F_1\left(1,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{5ad}+\frac{2\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n}\right.}{3ad}$$

```
[Out] hypergeom([1, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a/d+2/3*hypergeo
m([1, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a/d+1/5*hypergeom([1
, 5/n],[(5+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^5/a/d
```

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^5(c+dx)\,{}_2F_1\left(1,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{5ad}+\frac{2\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3ad}+\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d) + (2*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^
n)/a)]*Sinh[c + d*x]^3)/(3*a*d) + (Hypergeometric2F1[1, 5/n, (5 + n)/n, -((
b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a+bx^n} + \frac{2x^2}{a+bx^n} + \frac{x^4}{a+bx^n}\right) dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{x^4}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{2\,\text{Su}}{}$$

$$= \frac{{}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad} + \frac{2\,{}_2F_1\left(1, \frac{3}{n}; \frac{3+n}{n}; -\frac{b\sinh^n(c+dx)}{a}\right)\sinh}{3ad}$$

Mathematica [A] time = 0.14, size = 119, normalized size = 0.92

$$\frac{15\sinh(c+dx)\,{}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{b\sinh^n(c+dx)}{a}\right) + 3\sinh^5(c+dx)\,{}_2F_1\left(1, \frac{5}{n}; \frac{n+5}{n}; -\frac{b\sinh^n(c+dx)}{a}\right) + 10\sinh^3(c+dx}{15ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (15*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh
[c + d*x] + 10*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a
)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[1, 5/n, (5 + n)/n, -((b*Sinh[c + d
*x]^n)/a)]*Sinh[c + d*x]^5)/(15*a*d)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(dx+c)^5}{b\sinh(dx+c)^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)^5}{b\sinh(dx+c)^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\cosh^5(dx+c)}{a+b\left(\sinh^n(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)
```

```
[Out] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh (dx+c)^5}{b\sinh (dx+c)^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh (c+d\,x)^5}{a+b\sinh (c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n),x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**n),x)
```

```
[Out] Timed out
```

3.420 $\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}+\frac{\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3ad}$$

```
[Out] hypergeom([1, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a/d+1/3*hypergeo
m([1, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a/d
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3ad}+\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d) + (Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)
/a)]*Sinh[c + d*x]^3)/(3*a*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+bx^n}\,dx,x,\sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a+bx^n}+\frac{x^2}{a+bx^n}\right)dx,x,\sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^n}\,dx,x,\sinh(c+dx)\right)}{d}+\frac{\text{Subst}\left(\int \frac{x^2}{a+bx^n}\,dx,x,\sinh(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}+\frac{{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+}{3ad}$$

Mathematica [A] time = 0.06, size = 82, normalized size = 0.98

$$\frac{\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a}+\frac{\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};1+\frac{3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] ((Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/a + (Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*Sinh[c + d*x]^n)/a)]*
Sinh[c + d*x]^3)/(3*a))/d
```

fricas [F] time = 1.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(dx+c)^3}{b\sinh(dx+c)^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)^3}{b\sinh(dx+c)^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(dx+c)}{a+b\left(\sinh^n(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)
```

```
[Out] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)^3}{b\sinh(dx+c)^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c+d\,x)^3}{a+b\sinh(c+d\,x)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n),x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**n),x)
```

```
[Out] Timed out
```

3.421 $\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sinh(c+dx)\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

```
[Out] hypergeom([1, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a/d
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3223, 245}

$$\frac{\sinh(c+dx)\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^n}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{\sinh(c+dx)\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d)
```

fricas [F] time = 2.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(dx+c)}{b\sinh(dx+c)^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
[Out] integral(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{b\sinh(dx+c)^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{a+b\left(\sinh^n(dx+c)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x)
[Out] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)}{b\sinh(dx+c)^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

mupad [B] time = 1.13, size = 38, normalized size = 1.03

$$\frac{\sinh(c+d\,x)\;{}_2\mathrm{F}_1\left(1,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\sinh(c+d\,x)^n}{a}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^n),x)
[Out] (sinh(c + d*x)*hypergeom([1, 1/n], 1/n + 1, -(b*sinh(c + d*x)^n)/a))/(a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**n),x)
[Out] Timed out
```

3.422 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^n(c+dx)\right)^2}\,dx$

Optimal. Leaf size=130

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}+\frac{\sinh^5(c+dx)\,{}_2F_1\left(2,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{5a^2d}+\frac{2\sinh^3(c+dx)\,{}_2F_1\left(2,\right.}{3a^2}$$

```
[Out] hypergeom([2, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a^2/d+2/3*hyperg
eom([2, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a^2/d+1/5*hypergeo
m([2, 5/n],[(5+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^5(c+dx)\,{}_2F_1\left(2,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{5a^2d}+\frac{2\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2d}+\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};\right.}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d) + (2*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x
]^n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d) + (Hypergeometric2F1[2, 5/n, (5 + n)/n,
 -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a^2*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^n(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+bx^n)^2} + \frac{2x^2}{(a+bx^n)^2} + \frac{x^4}{(a+bx^n)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{x^4}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(2, \frac{1}{n}; 1+\frac{1}{n}; -\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2 d} + \frac{2\,{}_2F_1\left(2, \frac{3}{n}; \frac{3+n}{n}; -\frac{b\sinh^n(c+dx)}{a}\right)\text{si}}{3a^2 d}$$

Mathematica [A] time = 0.12, size = 119, normalized size = 0.92

$$\frac{15\sinh(c+dx)\,{}_2F_1\left(2, \frac{1}{n}; 1+\frac{1}{n}; -\frac{b\sinh^n(c+dx)}{a}\right) + 3\sinh^5(c+dx)\,{}_2F_1\left(2, \frac{5}{n}; \frac{n+5}{n}; -\frac{b\sinh^n(c+dx)}{a}\right) + 10\sinh^3(c+d}{15a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (15*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh
[c + d*x] + 10*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a
)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[2, 5/n, (5 + n)/n, -((b*Sinh[c + d
*x]^n)/a)]*Sinh[c + d*x]^5)/(15*a^2*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(dx+c)^5}{b^2\sinh(dx+c)^{2n} + 2ab\sinh(dx+c)^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^5/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n +
 a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)^5}{\left(b\sinh(dx+c)^n + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a)^2, x)
```

maple [F] time = 5.82, size = 0, normalized size = 0.00

$$\int \frac{\cosh^5(dx+c)}{\left(a+b\left(\sinh^n(dx+c)\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

```
[Out] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2^{n}e^{(cn+10\,dx+10\,c)}+3\cdot 2^{n}e^{(cn+8\,dx+8\,c)}+2^{n+1}e^{(cn+6\,dx+6\,c)}-2^{n+1}e^{(cn+4\,dx+4\,c)}-3\cdot 2^{n}e^{(cn+2\,dx+2\,c)}-2^{n}e^{(cn)}\right)e^{(dnx)}}{32\left(2^{n}a^{2}dne^{(dnx+cn+5\,dx+5\,c)}+abdne^{\left(5\,dx+n\log\left(e^{(dx+c)}+1\right)+n\log\left(e^{(dx+c)}-1\right)+5\,c\right)}\right)}+\frac{1}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(2^n*e^(c*n + 10*d*x + 10*c) + 3*2^n*e^(c*n + 8*d*x + 8*c) + 2^(n + 1)
*e^(c*n + 6*d*x + 6*c) - 2^(n + 1)*e^(c*n + 4*d*x + 4*c) - 3*2^n*e^(c*n + 2
*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 5*d*x +
5*c) + a*b*d*n*e^(5*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) +
 5*c)) + 1/32*integrate((2^n*n*e^(c*n) - 5*2^n*e^(c*n) + (2^n*n*e^(c*n) - 5
*2^n*e^(c*n))*e^(10*d*x + 10*c) + (5*2^n*n*e^(c*n) - 9*2^n*e^(c*n))*e^(8*d*
x + 8*c) + (5*2^(n + 1)*n*e^(c*n) - 2^(n + 1)*e^(c*n))*e^(6*d*x + 6*c) + (5
*2^(n + 1)*n*e^(c*n) - 2^(n + 1)*e^(c*n))*e^(4*d*x + 4*c) + (5*2^n*n*e^(c*n
) - 9*2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n*e^(d*n*x + c*n + 5
*d*x + 5*c) + a*b*n*e^(5*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) -
 1) + 5*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh\left(c+d\,x\right)^5}{{\left(a+b\,{\sinh\left(c+d\,x\right)}^n\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n)^2,x)
```

```
[Out] int(cosh(c + d*x)^5/(a + b*sinh(c + d*x)^n)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

3.423 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^n(c+dx)\right)^2} \, dx$

Optimal. Leaf size=84

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}+\frac{\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2d}$$

```
[Out] hypergeom([2, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a^2/d+1/3*hyperg
eom([2, 3/n],[(3+n)/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2d}+\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d) + (Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^
n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^n(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+bx^n)^2}+\frac{x^2}{(a+bx^n)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d} + \frac{{}_2F_1\left(2,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3}{3a^2d}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.98

$$\frac{\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2}+\frac{\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};1+\frac{3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] ((Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/a^2 + (Hypergeometric2F1[2, 3/n, 1 + 3/n, -((b*Sinh[c + d*x]^n)/a)
]*Sinh[c + d*x]^3)/(3*a^2))/d
```

fricas [F] time = 2.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh(dx+c)^3}{b^2\sinh(dx+c)^{2n}+2ab\sinh(dx+c)^n+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n +
 a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(dx+c)^3}{\left(b\sinh(dx+c)^n+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a)^2, x)
```

maple [F] time = 5.58, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(dx+c)}{\left(a+b\left(\sinh^n(dx+c)\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)
```

```
[Out] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2^{n}e^{(cn+6\,dx+6\,c)}+2^{n}e^{(cn+4\,dx+4\,c)}-2^{n}e^{(cn+2\,dx+2\,c)}-2^{n}e^{(cn)}\right)e^{(dnx)}}{8\left(2^{n}a^{2}dne^{(dnx+cn+3\,dx+3\,c)}+abdne^{\left(3\,dx+n\log\left(e^{(dx+c)}+1\right)+n\log\left(e^{(dx+c)}-1\right)+3\,c\right)}\right)}+\frac{1}{8}\int\frac{\left(2^{n}ne^{(cn)}-3\cdot 2^{n}e^{(cn)}+\left(2^{n}ne^{(cn)}\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2^n*e^(c*n + 6*d*x + 6*c) + 2^n*e^(c*n + 4*d*x + 4*c) - 2^n*e^(c*n + 2
*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 3*d*x +
3*c) + a*b*d*n*e^(3*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) +
 3*c)) + 1/8*integrate((2^n*n*e^(c*n) - 3*2^n*e^(c*n) + (2^n*n*e^(c*n) - 3*
2^n*e^(c*n))*e^(6*d*x + 6*c) + (3*2^n*n*e^(c*n) - 2^n*e^(c*n))*e^(4*d*x + 4
*c) + (3*2^n*n*e^(c*n) - 2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n
*e^(d*n*x + c*n + 3*d*x + 3*c) + a*b*n*e^(3*d*x + n*log(e^(d*x + c) + 1) +
n*log(e^(d*x + c) - 1) + 3*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cosh\left(c+d\,x\right)^{3}}{\left(a+b\sinh\left(c+d\,x\right)^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n)^2,x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*sinh(c + d*x)^n)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

3.424 $\int \frac{\cosh(c+dx)}{\left(a+b\sinh^n(c+dx)\right)^2}\,dx$

Optimal. Leaf size=37

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

```
[Out] hypergeom([2, 1/n],[1+1/n],-b*sinh(d*x+c)^n/a)*sinh(d*x+c)/a^2/d
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3223, 245}

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{\left(a+b\sinh^n(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^n)^2}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d)
```

fricas [F] time = 2.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)}{b^2\sinh\left(dx+c\right)^{2n}+2\,ab\sinh\left(dx+c\right)^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n + a
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^n+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a)^2, x)
```

maple [F] time = 6.03, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(dx+c\right)}{\left(a+b\left(\sinh^n\left(dx+c\right)\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)
```

```
[Out] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2^n e^{(cn+2\,dx+2\,c)}-2^n e^{(cn)}\right)e^{(dnx)}}{2\left(2^n a^2dne^{(dnx+cn+dx+c)}+abdne^{\left(dx+n\log\left(e^{(dx+c)}+1\right)+n\log\left(e^{(dx+c)}-1\right)+c\right)}\right)}+\frac{1}{2}\int\frac{\left(2^n ne^{(cn)}-2^n e^{(cn)}+\left(2^n ne^{(cn)}-2^n\right.\right.}{2^n a^2ne^{(dnx+cn+dx+c)}+abne^{\left(dx+n\log\left(e^{(d\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2^n*e^(c*n + 2*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n
*x + c*n + d*x + c) + a*b*d*n*e^(d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*
x + c) - 1) + c)) + 1/2*integrate((2^n*n*e^(c*n) - 2^n*e^(c*n) + (2^n*n*e^(
c*n) - 2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n*e^(d*n*x + c*n +
d*x + c) + a*b*n*e^(d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) +
 c)), x)
```

mupad [B] time = 0.95, size = 38, normalized size = 1.03

$$\frac{\sinh\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(2,\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{b\sinh(c+d\,x)^n}{a}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*sinh(c + d*x)^n)^2,x)
```

```
[Out] (sinh(c + d*x)*hypergeom([2, 1/n], 1/n + 1, -(b*sinh(c + d*x)^n)/a))/(a^2*d
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

3.425 $\int \frac{\coth(x)}{1-\sinh^2(x)}\, dx$

Optimal. Leaf size=17

$$\log(\sinh(x)) - \frac{1}{2}\log\left(1-\sinh^2(x)\right)$$

```
[Out] ln(sinh(x))-1/2*ln(1-sinh(x)^2)
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3194, 36, 31, 29}

$$\log(\sinh(x)) - \frac{1}{2}\log\left(1-\sinh^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(1 - Sinh[x]^2),x]
```

```
[Out] Log[Sinh[x]] - Log[1 - Sinh[x]^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\coth(x)}{1-\sinh^2(x)}\, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x}\, dx, x, \sinh^2(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x}\, dx, x, \sinh^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\, dx, x, \sinh^2(x)\right)\\ &= \log(\sinh(x)) - \frac{1}{2}\log\left(1-\sinh^2(x)\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.35

$$-2\left(\frac{1}{4}\log\left(1-\sinh^2(x)\right) - \frac{1}{2}\log(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 - Sinh[x]^2),x]
[Out] -2*(-1/2*Log[Sinh[x]] + Log[1 - Sinh[x]^2]/4)
```

fricas [B] time = 1.03, size = 47, normalized size = 2.76

$$-\frac{1}{2}\log\left(\frac{2\left(\cosh(x)^2+\sinh(x)^2-3\right)}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right)+\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="fricas")
[Out] -1/2*log(2*(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

giac [A] time = 0.13, size = 25, normalized size = 1.47

$$-\frac{1}{2}\log\left(\left|e^{(4x)}-6e^{(2x)}+1\right|\right)+\log\left(\left|e^{(2x)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="giac")
[Out] -1/2*log(abs(e^(4*x) - 6*e^(2*x) + 1)) + log(abs(e^(2*x) - 1))
```

maple [B] time = 0.09, size = 41, normalized size = 2.41

$$-\frac{\ln\left(\tanh^2\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{\ln\left(\tanh^2\left(\frac{x}{2}\right)+2\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1-sinh(x)^2),x)
[Out] -1/2*ln(tanh(1/2*x)^2-2*tanh(1/2*x)-1)-1/2*ln(tanh(1/2*x)^2+2*tanh(1/2*x)-1
)+ln(tanh(1/2*x))
```

maxima [B] time = 0.33, size = 45, normalized size = 2.65

$$-\frac{1}{2}\log\left(2e^{(-x)}+e^{(-2x)}-1\right)+\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)-\frac{1}{2}\log\left(-2e^{(-x)}+e^{(-2x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="maxima")
[Out] -1/2*log(2*e^(-x) + e^(-2*x) - 1) + log(e^(-x) + 1) + log(e^(-x) - 1) - 1/2
*log(-2*e^(-x) + e^(-2*x) - 1)
```

mupad [B] time = 0.08, size = 27, normalized size = 1.59

$$\ln\left(5184\,\mathrm{e}^{2x}-5184\right)-\frac{\ln\left(9\,\mathrm{e}^{4x}-54\,\mathrm{e}^{2x}+9\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-coth(x)/(sinh(x)^2 - 1),x)
[Out] log(5184*exp(2*x) - 5184) - log(9*exp(4*x) - 54*exp(2*x) + 9)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\coth(x)}{\sinh^2(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)**2),x)
```

```
[Out] -Integral(coth(x)/(sinh(x)**2 - 1), x)
```

3.426 $\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^5(e+fx)\,dx$

Optimal. Leaf size=63

$$-\frac{a^2}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}+\frac{2a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] -1/3*a^2/f/(a*cosh(f*x+e)^2)^(3/2)+2*a/f/(a*cosh(f*x+e)^2)^(1/2)+(a*cosh(f*
x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3205, 16, 43}

$$-\frac{a^2}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}+\frac{2a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] -a^2/(3*f*(a*Cosh[e + f*x]^2)^(3/2)) + (2*a)/(f*Sqrt[a*Cosh[e + f*x]^2]) +
Sqrt[a*Cosh[e + f*x]^2]/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\tanh^5(e+fx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2\sqrt{ax}}{x^3}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{5/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}} - \frac{2}{a(ax)^{3/2}} + \frac{1}{a^2\sqrt{ax}}\right)dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2}{3f\left(a\cosh^2(e+fx)\right)^{3/2}} + \frac{2a}{f\sqrt{a\cosh^2(e+fx)}} + \frac{\sqrt{a\cosh^2(e+}}{f}$$

Mathematica [A] time = 0.10, size = 51, normalized size = 0.81

$$\frac{\left(3\cosh^4(e+fx)+6\cosh^2(e+fx)-1\right)\text{sech}^4(e+fx)\sqrt{a\cosh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(-1 + 6*Cosh[e + f*x]^2 + 3*Cosh[e + f*x]^4)*Sech[
e + f*x]^4)/(3*f)
```

fricas [B] time = 2.95, size = 875, normalized size = 13.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(24*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + 3*e^(f*x + e)*sinh(f*x
+ e)^8 + 12*(7*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^6 + 24*(7*cos
h(f*x + e)^3 + 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 10*(21*cosh(f
*x + e)^4 + 54*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(21*cos
h(f*x + e)^5 + 90*cosh(f*x + e)^3 + 25*cosh(f*x + e))*e^(f*x + e)*sinh(f*x
+ e)^3 + 12*(7*cosh(f*x + e)^6 + 45*cosh(f*x + e)^4 + 25*cosh(f*x + e)^2 +
3)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(3*cosh(f*x + e)^7 + 27*cosh(f*x + e)^5
+ 25*cosh(f*x + e)^3 + 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (3*cosh
(f*x + e)^8 + 36*cosh(f*x + e)^6 + 50*cosh(f*x + e)^4 + 36*cosh(f*x + e)^2
+ 3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x
 - e)/(f*cosh(f*x + e)^7 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 7*(f*c
osh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^6 + 3*f*cosh(
f*x + e)^5 + 3*(7*f*cosh(f*x + e)^2 + (7*f*cosh(f*x + e)^2 + f)*e^(2*f*x +
2*e) + f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (7
*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 +
3*f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 + 30*f*cosh(f*x + e)^2 + (35*f*
cosh(f*x + e)^4 + 30*f*cosh(f*x + e)^2 + 3*f)*e^(2*f*x + 2*e) + 3*f)*sinh(f
*x + e)^3 + 3*(7*f*cosh(f*x + e)^5 + 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x +
e) + (7*f*cosh(f*x + e)^5 + 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*
f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^7 + 3*f*co
sh(f*x + e)^5 + 3*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (7
```

```
*f*cosh(f*x + e)^6 + 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + (7*f*cosh
(f*x + e)^6 + 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*
e) + f)*sinh(f*x + e))
```

giac [A] time = 0.21, size = 80, normalized size = 1.27

$$\frac{\sqrt{a}\left(\frac{8\left(3e^{(5fx+5e)}+4e^{(3fx+3e)}+3e^{(fx+e)}\right)}{\left(e^{(2fx+2e)}+1\right)^3}+3e^{(fx+e)}+3e^{(-fx-e)}\right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a)*(8*(3*e^(5*f*x + 5*e) + 4*e^(3*f*x + 3*e) + 3*e^(f*x + e))/(e^(
2*f*x + 2*e) + 1)^3 + 3*e^(f*x + e) + 3*e^(-f*x - e))/f
```

maple [C] time = 0.24, size = 42, normalized size = 0.67

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\left(\sinh^5\left(fx+e\right)\right)a}{\cosh\left(fx+e\right)^4\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5*a/cosh(f*x+e)^4/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

maxima [B] time = 0.88, size = 292, normalized size = 4.63

$$\frac{6\sqrt{a}e^{(-2fx-2e)}}{f\left(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)}\right)}+\frac{25\sqrt{a}e^{(-4fx-4e)}}{3f\left(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)}\right)}+\frac{}{f(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] 6*sqrt(a)*e^(-2*f*x - 2*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*
f*x - 5*e) + e^(-7*f*x - 7*e))) + 25/3*sqrt(a)*e^(-4*f*x - 4*e)/(f*(e^(-f*x
 - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 6*sq
rt(a)*e^(-6*f*x - 6*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x
- 5*e) + e^(-7*f*x - 7*e))) + 1/2*sqrt(a)*e^(-8*f*x - 8*e)/(f*(e^(-f*x - e)
 + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 1/2*sqrt(
a)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x -
 7*e)))
```

mupad [B] time = 0.94, size = 252, normalized size = 4.00

$$\frac{\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{f}+\frac{8e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{f\left(e^{2e+2fx}+1\right)\left(e^{e+fx}+e^{3e+3fx}\right)}-\frac{16e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{3f\left(e^{2e+2fx}+1\right)^2\left(e^{e+fx}+e^{3e+3fx}\right)}+\frac{16e^{3e+3}}{3f\left(e^{2e+2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] (a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)/f + (8*exp(3*e + 3*f*x)
*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x)
+ 1)*(exp(e + f*x) + exp(3*e + 3*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(
e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*f*(exp(2*e + 2*f*x) + 1)^2*(exp
(e + f*x) + exp(3*e + 3*f*x))) + (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/
2 - exp(- e - f*x)/2)^2)^(1/2))/(3*f*(exp(2*e + 2*f*x) + 1)^3*(exp(e + f*x)
 + exp(3*e + 3*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\tanh^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**5, x)
```

3.427 $\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^3(e+fx)\,dx$

Optimal. Leaf size=38

$$\frac{a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

[Out] a/f/(a*cosh(f*x+e)^2)^(1/2)+(a*cosh(f*x+e)^2)^(1/2)/f

Rubi [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]

[Out] a/(f*Sqrt[a*Cosh[e + f*x]^2]) + Sqrt[a*Cosh[e + f*x]^2]/f

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^3(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\,\tanh^3(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)\sqrt{ax}}{x^2}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{3/2}} - \frac{1}{a\sqrt{ax}}\right)dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a}{f\sqrt{a\cosh^2(e+fx)}} + \frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Mathematica [A] time = 0.09, size = 29, normalized size = 0.76

$$\frac{a\left(\cosh^2(e+fx)+1\right)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] (a*(1 + Cosh[e + f*x]^2))/(f*Sqrt[a*Cosh[e + f*x]^2])
```

fricas [B] time = 0.70, size = 311, normalized size = 8.18

$$\frac{\left(4\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)^{3} + e^{\left(fx+e\right)}\sinh\left(fx+e\right)^{4} + 6\left(\cosh\left(fx+e\right)^{2}+1\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right.}{2\left(f\cosh\left(fx+e\right)^{3} + \left(fe^{\left(2fx+2e\right)}+f\right)\sinh\left(fx+e\right)^{3} + 3\left(f\cosh\left(fx+e\right)e^{\left(2fx+2e\right)} + f\cosh\left(fx+e\right)\right)\sinh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e
)^4 + 6*(cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e
)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 + 6*cos
h(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e)
 + a)*e^(-f*x - e)/(f*cosh(f*x + e)^3 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x +
e)^3 + 3*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^
2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e)
 + (3*f*cosh(f*x + e)^2 + (3*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*si
nh(f*x + e))
```

giac [A] time = 0.19, size = 53, normalized size = 1.39

$$\frac{\sqrt{a}\left(\frac{\left(5e^{\left(2fx+2e\right)}+1\right)e^{\left(-e\right)}}{e^{\left(3fx+2e\right)}+e^{\left(fx\right)}} + e^{\left(fx+e\right)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*((5*e^(2*f*x + 2*e) + 1)*e^(-e)/(e^(3*f*x + 2*e) + e^(f*x)) + e
^(f*x + e))/f
```

maple [C] time = 0.22, size = 42, normalized size = 1.11

$$\frac{`int/indef0`\left(\frac{\left(\sinh^{3}\left(fx+e\right)\right)a}{\cosh\left(fx+e\right)^{2}\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3*a/cosh(f*x+e)^2/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

maxima [B] time = 1.19, size = 106, normalized size = 2.79

$$\frac{3\sqrt{a}e^{\left(-2fx-2e\right)}}{f\left(e^{\left(-fx-e\right)}+e^{\left(-3fx-3e\right)}\right)}+\frac{\sqrt{a}e^{\left(-4fx-4e\right)}}{2f\left(e^{\left(-fx-e\right)}+e^{\left(-3fx-3e\right)}\right)}+\frac{\sqrt{a}}{2f\left(e^{\left(-fx-e\right)}+e^{\left(-3fx-3e\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 3*sqrt(a)*e^(-2*f*x - 2*e)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e))) + 1/2*sqrt
(a)*e^(-4*f*x - 4*e)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e))) + 1/2*sqrt(a)/(f
*(e^(-f*x - e) + e^(-3*f*x - 3*e)))
```

mupad [B] time = 0.91, size = 67, normalized size = 1.76

$$\frac{\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}\left(6e^{2e+2fx}+e^{4e+4fx}+1\right)}{f\left(e^{2e+2fx}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] ((a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(6*exp(2*e + 2*f*x) +
exp(4*e + 4*f*x) + 1))/(f*(exp(2*e + 2*f*x) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\tanh^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**3, x)
```

3.428 $\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh(e+fx)\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

[Out] (a*cosh(f*x+e)^2)^(1/2)/f

Rubi [A] time = 0.07, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3176, 3205, 16, 32}

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x],x]

[Out] Sqrt[a*Cosh[e + f*x]^2]/f

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\,\tanh(e+fx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{x}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] Sqrt[a*Cosh[e + f*x]^2]/f
```

fricas [B] time = 1.02, size = 139, normalized size = 7.72

$$\frac{\left(2\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)+e^{\left(fx+e\right)}\sinh\left(fx+e\right)^2+\left(\cosh\left(fx+e\right)^2+1\right)e^{\left(fx+e\right)}\right)\sqrt{ae^{\left(4\,fx+4\,e\right)}+2\,ae^{\left(}}}{2\left(f\cosh\left(fx+e\right)e^{\left(2\,fx+2\,e\right)}+f\cosh\left(fx+e\right)+\left(fe^{\left(2\,fx+2\,e\right)}+f\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*
x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x +
e) + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

giac [A] time = 0.14, size = 26, normalized size = 1.44

$$\frac{\sqrt{a}\left(e^{\left(fx+e\right)}+e^{\left(-fx-e\right)}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*(e^(f*x + e) + e^(-f*x - e))/f
```

maple [A] time = 0.08, size = 19, normalized size = 1.06

$$\frac{\sqrt{a+a\left(\sinh^2\left(fx+e\right)\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x)
[Out] 1/f*(a+a*sinh(f*x+e)^2)^(1/2)
```

maxima [A] time = 1.76, size = 32, normalized size = 1.78

$$\frac{\sqrt{a}e^{(fx+e)}}{2f}+\frac{\sqrt{a}e^{(-fx-e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="maxima")
[Out] 1/2*sqrt(a)*e^(f*x + e)/f + 1/2*sqrt(a)*e^(-f*x - e)/f
```

mupad [B] time = 0.92, size = 18, normalized size = 1.00

$$\frac{\sqrt{a\sinh\left(e+fx\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)*(a + a*sinh(e + f*x)^2)^(1/2),x)
[Out] (a + a*sinh(e + f*x)^2)^(1/2)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\tanh\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e),x)
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x), x)
```

3.429 $\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a*cosh(f*x+e)^2)^(1/2)
/f
```

Rubi [A] time = 0.10, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3176, 3205, 50, 63, 206}

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2/Sqrt[a]])/f) + Sqrt[a*Cosh[e + f
*x]^2]/f
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
```

```
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx &= \int \sqrt{a\cosh^2(e+fx)}\coth(e+fx)\,dx\\ &= -\frac{\mathrm{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx,x,\cosh^2(e+fx)\right)}{2f}\\ &= \frac{\sqrt{a\cosh^2(e+fx)}}{f}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx,x,\cosh^2(e+fx)\right)}{2f}\\ &= \frac{\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\mathrm{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a\cosh^2(e+fx)}\right)}{f}\\ &= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.06, size = 42, normalized size = 0.84

$$\frac{\mathrm{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\cosh(e+fx)+\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(Cosh[e + f*x] + Log[Tanh[(e + f*x)/2]])*Sech[e +
f*x])/f
```

fricas [B] time = 2.39, size = 200, normalized size = 4.00

$$\frac{\left(2\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)+e^{\left(fx+e\right)}\sinh\left(fx+e\right)^2+\left(\cosh\left(fx+e\right)^2+1\right)e^{\left(fx+e\right)}+2\left(\cosh\left(fx+\right.\right.}{2\left(f\cosh\left(fx+e\right)e^{\left(2fx+2e\right)}+f\cosh\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e) + 2*(cosh(f*x + e)*e^(f*x + e) + e^(f
*x + e)*sinh(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x +
e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)
*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e) + (f*e^(2*
f*x + 2*e) + f)*sinh(f*x + e))
```

giac [A] time = 0.14, size = 51, normalized size = 1.02

$$\frac{\sqrt{a}\left(e^{\left(fx+e\right)}+e^{\left(-fx-e\right)}-2\log\left(e^{\left(fx+e\right)}+1\right)+2\log\left(\left|e^{\left(fx+e\right)}-1\right|\right)\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*(e^(f*x + e) + e^(-f*x - e) - 2*log(e^(f*x + e) + 1) + 2*log(ab
s(e^(f*x + e) - 1)))/f
```

maple [C] time = 0.18, size = 42, normalized size = 0.84

$$\frac{`int/indef0`\left(\frac{a\left(\cosh^{2}\left(fx+e\right)\right)}{\sinh\left(fx+e\right)\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(a*cosh(f*x+e)^2/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

maxima [A] time = 2.34, size = 68, normalized size = 1.36

$$\frac{\left(\sqrt{a}e^{\left(-2\,fx-2\,e\right)} + \sqrt{a}\right)e^{\left(fx+e\right)}}{2\,f} - \frac{\sqrt{a}\log\left(e^{\left(-fx-e\right)} + 1\right)}{f} + \frac{\sqrt{a}\log\left(e^{\left(-fx-e\right)} - 1\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a))*e^(f*x + e)/f - sqrt(a)*log(e^(-f*
x - e) + 1)/f + sqrt(a)*log(e^(-f*x - e) - 1)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth\left(e+f\,x\right)\,\sqrt{a\,{\sinh\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\coth\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x), x)
```

3.430 $\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=87

$$\frac{3\sqrt{a\cosh^2(e+fx)}}{2f}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\operatorname{csch}^2(e+fx)\left(a\cosh^2(e+fx)\right)^{3/2}}{2af}$$

```
[Out] -1/2*(a*cosh(f*x+e)^2)^(3/2)*csch(f*x+e)^2/a/f-3/2*arctanh((a*cosh(f*x+e)^2
)^(1/2)/a^(1/2))*a^(1/2)/f+3/2*(a*cosh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3176, 3205, 16, 47, 50, 63, 206}

$$\frac{3\sqrt{a\cosh^2(e+fx)}}{2f}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\operatorname{csch}^2(e+fx)\left(a\cosh^2(e+fx)\right)^{3/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-3*Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]])/(2*f) + (3*Sqrt[a*Cos
h[e + f*x]^2])/(2*f) - ((a*Cosh[e + f*x]^2)^(3/2)*Csch[e + f*x]^2)/(2*a*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx &= \int \sqrt{a\cosh^2(e+fx)}\coth^3(e+fx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x\sqrt{ax}}{(1-x)^2}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{\text{Subst}\left(\int \frac{(ax)^{3/2}}{(1-x)^2}\,dx, x, \cosh^2(e+fx)\right)}{2af}\\
&= -\frac{\left(a\cosh^2(e+fx)\right)^{3/2}\operatorname{csch}^2(e+fx)}{2af} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx, x, \cosh^2(\right.}{4f}\\
&= \frac{3\sqrt{a\cosh^2(e+fx)}}{2f} - \frac{\left(a\cosh^2(e+fx)\right)^{3/2}\operatorname{csch}^2(e+fx)}{2af} - \frac{(3a)\,\text{Sul}}{}\\
&= \frac{3\sqrt{a\cosh^2(e+fx)}}{2f} - \frac{\left(a\cosh^2(e+fx)\right)^{3/2}\operatorname{csch}^2(e+fx)}{2af} - \frac{3\,\text{Subst}}{}\\
&= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f} + \frac{3\sqrt{a\cosh^2(e+fx)}}{2f} - \frac{\left(a\cosh^2(e+\right.}{}\end{aligned}$$

Mathematica [A] time = 0.26, size = 77, normalized size = 0.89

$$\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(8\cosh(e+fx)-\operatorname{csch}^2\left(\frac{1}{2}(e+fx)\right)-\operatorname{sech}^2\left(\frac{1}{2}(e+fx)\right)+12\log\left(\tanh\left(\frac{1}{2}(e+f\right.\right.\right.}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(8*Cosh[e + f*x] - Csch[(e + f*x)/2]^2 + 12*Log[Ta
nh[(e + f*x)/2]] - Sech[(e + f*x)/2]^2)*Sech[e + f*x])/(8*f)
```

fricas [B] time = 1.39, size = 764, normalized size = 8.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^5 + e^(f*x + e)*sinh(f*x + e
)^6 + 3*(5*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^4 + 4*(5*cosh(f*x
 + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 3*(5*cosh(f*x + e)
^4 - 6*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 6*(cosh(f*x + e)^
5 - 2*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*
x + e)^6 - 3*cosh(f*x + e)^4 - 3*cosh(f*x + e)^2 + 1)*e^(f*x + e) + 3*(5*co
sh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + e^(f*x + e)*sinh(f*x + e)^5 + 2*(
5*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 2*(5*cosh(f*x + e)^3 -
 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (5*cosh(f*x + e)^4 - 6*cosh
(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^5 - 2*cosh(f*x
+ e)^3 + cosh(f*x + e))*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1
)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f
*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^5 + (f*e^(2*f*x + 2*e) + f)*si
nh(f*x + e)^5 + 5*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(
f*x + e)^4 - 2*f*cosh(f*x + e)^3 + 2*(5*f*cosh(f*x + e)^2 + (5*f*cosh(f*x +
 e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e)^3 + 2*(5*f*cosh(f*x + e)^3 -
3*f*cosh(f*x + e) + (5*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^5 - 2*f*cosh(f*x +
 e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (5*f*cosh(f*x + e)^4 - 6*f*cosh(
f*x + e)^2 + (5*f*cosh(f*x + e)^4 - 6*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e
) + f)*sinh(f*x + e))
```

giac [A] time = 0.15, size = 89, normalized size = 1.02

$$-\frac{\sqrt{a}\left(\frac{2\left(e^{\left(3\,fx+3\,e\right)}+e^{\left(fx+e\right)}\right)}{\left(e^{\left(2\,fx+2\,e\right)}-1\right)^{2}}-e^{\left(fx+e\right)}-e^{\left(-fx-e\right)}+3\,\log\left(e^{\left(fx+e\right)}+1\right)-3\,\log\left(\left|e^{\left(fx+e\right)}-1\right|\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(2*(e^(3*f*x + 3*e) + e^(f*x + e))/(e^(2*f*x + 2*e) - 1)^2 - e
^(f*x + e) - e^(-f*x - e) + 3*log(e^(f*x + e) + 1) - 3*log(abs(e^(f*x + e)
- 1)))/f
```

maple [C] time = 0.20, size = 54, normalized size = 0.62

$$\frac{`\mathit{int/indef0}`\left(\frac{a\left(\cosh^{4}\left(fx+e\right)\right)}{\sinh\left(fx+e\right)\left(\cosh^{2}\left(fx+e\right)-1\right)\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(a*cosh(f*x+e)^4/sinh(f*x+e)/(cosh(f*x+e)^2-1)/(a*cosh(f*x+e)^2
)^(1/2),sinh(f*x+e))/f
```

maxima [A] time = 1.61, size = 126, normalized size = 1.45

$$-\frac{3\,\sqrt{a}\,\log\left(e^{\left(-fx-e\right)}+1\right)}{2\,f}+\frac{3\,\sqrt{a}\,\log\left(e^{\left(-fx-e\right)}-1\right)}{2\,f}-\frac{3\,\sqrt{a}\,e^{\left(-2\,fx-2\,e\right)}+3\,\sqrt{a}\,e^{\left(-4\,fx-4\,e\right)}-\sqrt{a}\,e^{\left(-6\,fx-6\,e\right)}-\sqrt{a}}{2\,f\left(e^{\left(-fx-e\right)}-2\,e^{\left(-3\,fx-3\,e\right)}+e^{\left(-5\,fx-5\,e\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(a)*log(e^(-f*x - e) + 1)/f + 3/2*sqrt(a)*log(e^(-f*x - e) - 1)/f
- 1/2*(3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) - sqrt(a)*e^
(-6*f*x - 6*e) - sqrt(a))/(f*(e^(-f*x - e) - 2*e^(-3*f*x - 3*e) + e^(-5*f*x
 - 5*e)))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(e+f\,x\right)^3\,\sqrt{a\,\sinh\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^3*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\,\coth^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**3, x)
```

3.431 $\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^6(e+fx)\,dx$

Optimal. Leaf size=120

$$-\frac{\tanh^5(e+fx)\sqrt{a\cosh^2(e+fx)}}{4f}-\frac{5\tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}}{8f}+\frac{15\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{8f}-\text{1}$$

```
[Out] -15/8*arctan(sinh(f*x+e))*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f+15/8*(a*cos
h(f*x+e)^2)^(1/2)*tanh(f*x+e)/f-5/8*(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3/f
-1/4*(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5/f
```

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3176, 3207, 2592, 288, 321, 203}

$$-\frac{\tanh^5(e+fx)\sqrt{a\cosh^2(e+fx)}}{4f}-\frac{5\tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}}{8f}+\frac{15\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{8f}-\text{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^6,x]
```

```
[Out] (-15*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(8*f) + (
15*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(8*f) - (5*Sqrt[a*Cosh[e + f*x]^2
]*Tanh[e + f*x]^3)/(8*f) - (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x]^5)/(4*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\ \tanh^6(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\ \tanh^6(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\int \sinh(e+fx)\tanh^5(e+fx)\,dx$$
$$= \frac{\left(\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^3}\,dx, x, \sinh(e+f$$
$$f$$
$$= -\frac{\sqrt{a\cosh^2(e+fx)}\ \tanh^5(e+fx)}{4f} + \frac{\left(5\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+$$
$$= -\frac{5\sqrt{a\cosh^2(e+fx)}\ \tanh^3(e+fx)}{8f} - \frac{\sqrt{a\cosh^2(e+fx)}\ \tanh^5(e+$$
$$4f$$
$$= \frac{15\sqrt{a\cosh^2(e+fx)}\ \tanh(e+fx)}{8f} - \frac{5\sqrt{a\cosh^2(e+fx)}\ \tanh^3(e+$$
$$8f$$
$$= -\frac{15\tan^{-1}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)}{8f} + \frac{15\sqrt{a\,\text{co}$$

Mathematica [A] time = 0.36, size = 75, normalized size = 0.62

$$-\frac{\text{sech}^5(e+fx)\sqrt{a\cosh^2(e+fx)}\ \left(-5\sinh(e+fx)-15\sinh(3(e+fx))-2\sinh(5(e+fx))+60\cosh^4(e+fx)\,t$$
$$32f$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^6,x]
```

```
[Out] -1/32*(Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]^5*(60*ArcTan[Sinh[e + f*x]]*Co
sh[e + f*x]^4 - 5*Sinh[e + f*x] - 15*Sinh[3*(e + f*x)] - 2*Sinh[5*(e + f*x)
]))/f
```

fricas [B] time = 0.57, size = 1645, normalized size = 13.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="fricas")
```

[Out] 1/4*(20*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^9 + 2*e^(f*x + e)*sinh(f*x + e)^10 + 15*(6*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^8 + 120*(2*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^7 + 5*(84*cosh(f*x + e)^4 + 84*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^6 + 6*(84*cosh(f*x + e)^5 + 140*cosh(f*x + e)^3 + 5*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 5*(84*cosh(f*x + e)^6 + 210*cosh(f*x + e)^4 + 15*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^4 + 20*(12*cosh(f*x + e)^7 + 42*cosh(f*x + e)^5 + 5*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 15*(6*cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 5*cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 10*(2*cosh(f*x + e)^9 + 12*cosh(f*x + e)^7 + 3*cosh(f*x + e)^5 - 2*cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) - 15*(9*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + e^(f*x + e)*sinh(f*x + e)^9 + 4*(9*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^7 + 28*(3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 6*(21*cosh(f*x + e)^4 + 14*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^5 + 2*(63*cosh(f*x + e)^5 + 70*cosh(f*x + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 4*(21*cosh(f*x + e)^6 + 35*cosh(f*x + e)^4 + 15*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 12*(3*cosh(f*x + e)^7 + 7*cosh(f*x + e)^5 + 5*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (9*cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 30*cosh(f*x + e)^4 + 12*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^9 + 4*cosh(f*x + e)^7 + 6*cosh(f*x + e)^5 + 4*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (2*cosh(f*x + e)^10 + 15*cosh(f*x + e)^8 + 5*cosh(f*x + e)^6 - 5*cosh(f*x + e)^4 - 15*cosh(f*x + e)^2 - 2)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^9 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^9 + 9*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^8 + 4*f*cosh(f*x + e)^7 + 4*(9*f*cosh(f*x + e)^2 + (9*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 28*(3*f*cosh(f*x + e)^3 + f*cosh(f*x + e) + (3*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 6*f*cosh(f*x + e)^5 + 6*(21*f*cosh(f*x + e)^4 + 14*f*cosh(f*x + e)^2 + (21*f*cosh(f*x + e)^4 + 14*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^5 + 2*(63*f*cosh(f*x + e)^5 + 70*f*cosh(f*x + e)^3 + 15*f*cosh(f*x + e) + (63*f*cosh(f*x + e)^5 + 70*f*cosh(f*x + e)^3 + 15*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 4*f*cosh(f*x + e)^3 + 4*(21*f*cosh(f*x + e)^6 + 35*f*cosh(f*x + e)^4 + 15*f*cosh(f*x + e)^2 + (21*f*cosh(f*x + e)^6 + 35*f*cosh(f*x + e)^4 + 15*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^3 + 12*(3*f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)^3 + f*cosh(f*x + e) + (3*f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^9 + 4*f*cosh(f*x + e)^7 + 6*f*cosh(f*x + e)^5 + 4*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (9*f*cosh(f*x + e)^8 + 28*f*cosh(f*x + e)^6 + 30*f*cosh(f*x + e)^4 + 12*f*cosh(f*x + e)^2 + (9*f*cosh(f*x + e)^8 + 28*f*cosh(f*x + e)^6 + 30*f*cosh(f*x + e)^4 + 12*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e))

giac [A] time = 0.23, size = 99, normalized size = 0.82

$$\frac{\sqrt{a}\left(\frac{9\,e^{\left(7\,fx+7\,e\right)}+e^{\left(5\,fx+5\,e\right)}-e^{\left(3\,fx+3\,e\right)}-9\,e^{\left(fx+e\right)}}{\left(e^{\left(2\,fx+2\,e\right)}+1\right)^{4}}-15\,\arctan\left(e^{\left(fx+e\right)}\right)+2\,e^{\left(fx+e\right)}-2\,e^{\left(-fx-e\right)}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="giac")

[Out] 1/4*sqrt(a)*((9*e^(7*f*x + 7*e) + e^(5*f*x + 5*e) - e^(3*f*x + 3*e) - 9*e^(f*x + e))/(e^(2*f*x + 2*e) + 1)^4 - 15*arctan(e^(f*x + e)) + 2*e^(f*x + e) - 2*e^(-f*x - e))/f

maple [A] time = 0.26, size = 85, normalized size = 0.71

$$-\frac{a\left(15\arctan\left(\sinh\left(fx+e\right)\right)\left(\cosh^4\left(fx+e\right)\right)-8\left(\cosh^4\left(fx+e\right)\right)\sinh\left(fx+e\right)-9\left(\cosh^2\left(fx+e\right)\right)\sinh\left(fx\right.\right.}{8\cosh\left(fx+e\right)^3\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x)
```

```
[Out] -1/8*a*(15*arctan(sinh(f*x+e))*cosh(f*x+e)^4-8*cosh(f*x+e)^4*sinh(f*x+e)-9*
cosh(f*x+e)^2*sinh(f*x+e)+2*sinh(f*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1
/2)/f
```

maxima [B] time = 0.96, size = 891, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 315/128*sqrt(a)*arctan(e^(-f*x - e))/f + 1/128*(105*sqrt(a)*arctan(e^(-f*x
 - e)) + (279*sqrt(a)*e^(-f*x - e) + 511*sqrt(a)*e^(-3*f*x - 3*e) + 385*sqrt
(a)*e^(-5*f*x - 5*e) + 105*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e) +
6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f + 1/128*
(105*sqrt(a)*arctan(e^(-f*x - e)) - (105*sqrt(a)*e^(-f*x - e) + 385*sqrt(a)
*e^(-3*f*x - 3*e) + 511*sqrt(a)*e^(-5*f*x - 5*e) + 279*sqrt(a)*e^(-7*f*x -
7*e))/(4*e^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8
*f*x - 8*e) + 1))/f - 5/256*(15*sqrt(a)*arctan(e^(-f*x - e)) - (15*sqrt(a)*
e^(-f*x - e) + 55*sqrt(a)*e^(-3*f*x - 3*e) + 73*sqrt(a)*e^(-5*f*x - 5*e) -
15*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e
^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f - 5/256*(15*sqrt(a)*arctan(e^(-f
*x - e)) - (15*sqrt(a)*e^(-f*x - e) - 73*sqrt(a)*e^(-3*f*x - 3*e) - 55*sqrt
(a)*e^(-5*f*x - 5*e) - 15*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e) + 6
*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f + 5/64*(3
*sqrt(a)*arctan(e^(-f*x - e)) - (3*sqrt(a)*e^(-f*x - e) + 11*sqrt(a)*e^(-3*
f*x - 3*e) - 11*sqrt(a)*e^(-5*f*x - 5*e) - 3*sqrt(a)*e^(-7*f*x - 7*e))/(4*e
^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e
) + 1))/f + 1/256*(837*sqrt(a)*e^(-2*f*x - 2*e) + 1533*sqrt(a)*e^(-4*f*x -
4*e) + 1155*sqrt(a)*e^(-6*f*x - 6*e) + 315*sqrt(a)*e^(-8*f*x - 8*e) + 128*s
qrt(a))/(f*(e^(-f*x - e) + 4*e^(-3*f*x - 3*e) + 6*e^(-5*f*x - 5*e) + 4*e^(-
7*f*x - 7*e) + e^(-9*f*x - 9*e))) - 1/256*(315*sqrt(a)*e^(-f*x - e) + 1155*
sqrt(a)*e^(-3*f*x - 3*e) + 1533*sqrt(a)*e^(-5*f*x - 5*e) + 837*sqrt(a)*e^(-
7*f*x - 7*e) + 128*sqrt(a)*e^(-9*f*x - 9*e))/(f*(4*e^(-2*f*x - 2*e) + 6*e^(
-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e+f\,x\right)^6\,\sqrt{a\,\sinh\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^6*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^6*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\,\tanh^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**6,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**6, x)
```

3.432 $\int \sqrt{a + a\sinh^2(e + fx)} \tanh^4(e + fx)\, dx$

Optimal. Leaf size=91

$$-\frac{\tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}}{2f}+\frac{3\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{2f}-\frac{3\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\tan^{-1}(\text{si}}{2f}$$

```
[Out] -3/2*arctan(sinh(f*x+e))*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f+3/2*(a*cosh(
f*x+e)^2)^(1/2)*tanh(f*x+e)/f-1/2*(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3/f
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3176, 3207, 2592, 288, 321, 203}

$$-\frac{\tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}}{2f}+\frac{3\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{2f}-\frac{3\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\tan^{-1}(\text{si}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] (-3*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(2*f) + (3
*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(2*f) - (Sqrt[a*Cosh[e + f*x]^2]*Ta
nh[e + f*x]^3)/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\,\tanh^4(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\,\tanh^4(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\int \sinh(e+fx)\tanh^3(e+fx)$$
$$= \frac{\left(\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \sinh(e+\right.}{f}$$
$$= -\frac{\sqrt{a\cosh^2(e+fx)}\,\tanh^3(e+fx)}{2f} + \frac{\left(3\sqrt{a\cosh^2(e+fx)}\,\text{sech}(\right.}{}$$
$$= \frac{3\sqrt{a\cosh^2(e+fx)}\,\tanh(e+fx)}{2f} - \frac{\sqrt{a\cosh^2(e+fx)}\,\tanh^3(e+}{2f}$$
$$= -\frac{3\tan^{-1}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\text{sech}(e+fx)}{2f} + \frac{3\sqrt{a\,\text{c}}}{}$$

Mathematica [A] time = 0.20, size = 55, normalized size = 0.60

$$\frac{a\left((\cosh(2(e+fx))+2)\tanh(e+fx)-3\cosh(e+fx)\tan^{-1}(\sinh(e+fx))\right)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] (a*(-3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + (2 + Cosh[2*(e + f*x)])*Tanh[e
 + f*x]))/(2*f*Sqrt[a*Cosh[e + f*x]^2])
```

fricas [B] time = 2.12, size = 742, normalized size = 8.15

$$\frac{\left(6\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)^5+e^{\left(fx+e\right)}\sinh\left(fx+e\right)^6+3\left(5\cosh\left(fx+e\right)^2+1\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^5 + e^(f*x + e)*sinh(f*x + e
)^6 + 3*(5*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^4 + 4*(5*cosh(f*x
```

```
 + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 3*(5*cosh(f*x + e)
^4 + 6*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 6*(cosh(f*x + e)^
5 + 2*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) - 6*(5*cos
h(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + e^(f*x + e)*sinh(f*x + e)^5 + 2*(5
*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 2*(5*cosh(f*x + e)^3 +
3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (5*cosh(f*x + e)^4 + 6*cosh(
f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^5 + 2*cosh(f*x +
 e)^3 + cosh(f*x + e))*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) +
 (cosh(f*x + e)^6 + 3*cosh(f*x + e)^4 - 3*cosh(f*x + e)^2 - 1)*e^(f*x + e))
*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x
 + e)^5 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^5 + 5*(f*cosh(f*x + e)*e^(2
*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^4 + 2*f*cosh(f*x + e)^3 + 2*(5
*f*cosh(f*x + e)^2 + (5*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*
x + e)^3 + 2*(5*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (5*f*cosh(f*x + e)^
3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) +
 (f*cosh(f*x + e)^5 + 2*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e
) + (5*f*cosh(f*x + e)^4 + 6*f*cosh(f*x + e)^2 + (5*f*cosh(f*x + e)^4 + 6*f
*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

giac [A] time = 0.20, size = 74, normalized size = 0.81

$$\frac{\sqrt{a}\left(\frac{2\left(e^{(3fx+3e)}-e^{(fx+e)}\right)}{\left(e^{(2fx+2e)}+1\right)^2}-6\arctan\left(e^{(fx+e)}\right)+e^{(fx+e)}-e^{(-fx-e)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*(2*(e^(3*f*x + 3*e) - e^(f*x + e))/(e^(2*f*x + 2*e) + 1)^2 - 6*
arctan(e^(f*x + e)) + e^(f*x + e) - e^(-f*x - e))/f
```

maple [A] time = 0.21, size = 69, normalized size = 0.76

$$-\frac{a\left(3\arctan\left(\sinh\left(fx+e\right)\right)\left(\cosh^2\left(fx+e\right)\right)-2\left(\cosh^2\left(fx+e\right)\right)\sinh\left(fx+e\right)-\sinh\left(fx+e\right)\right)}{2\cosh\left(fx+e\right)\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x)
```

```
[Out] -1/2*a*(3*arctan(sinh(f*x+e))*cosh(f*x+e)^2-2*cosh(f*x+e)^2*sinh(f*x+e)-sin
h(f*x+e))/cosh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 1.22, size = 387, normalized size = 4.25

$$\frac{15\sqrt{a}\arctan\left(e^{(-fx-e)}\right)}{8f}+\frac{3\sqrt{a}\arctan\left(e^{(-fx-e)}\right)+\frac{5\sqrt{a}e^{(-fx-e)}+3\sqrt{a}e^{(-3fx-3e)}}{2e^{(-2fx-2e)}+e^{(-4fx-4e)}+1}}{4f}+\frac{3\sqrt{a}\arctan\left(e^{(-fx-e)}\right)-\frac{3\sqrt{a}e^{(-fx-e)}+}{2e^{(-2fx-2e)}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 15/8*sqrt(a)*arctan(e^(-f*x - e))/f + 1/4*(3*sqrt(a)*arctan(e^(-f*x - e)) +
 (5*sqrt(a)*e^(-f*x - e) + 3*sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e)
+ e^(-4*f*x - 4*e) + 1))/f + 1/4*(3*sqrt(a)*arctan(e^(-f*x - e)) - (3*sqrt(
a)*e^(-f*x - e) + 5*sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e) + e^(-4*f
*x - 4*e) + 1))/f - 3/8*(sqrt(a)*arctan(e^(-f*x - e)) - (sqrt(a)*e^(-f*x -
```

```
e) - sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))
/f + 1/16*(25*sqrt(a)*e^(-2*f*x - 2*e) + 15*sqrt(a)*e^(-4*f*x - 4*e) + 8*sq
rt(a))/(f*(e^(-f*x - e) + 2*e^(-3*f*x - 3*e) + e^(-5*f*x - 5*e))) - 1/16*(1
5*sqrt(a)*e^(-f*x - e) + 25*sqrt(a)*e^(-3*f*x - 3*e) + 8*sqrt(a)*e^(-5*f*x
- 5*e))/(f*(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e+f\,x\right)^4\,\sqrt{a\,{\sinh\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4*(a + a*sinh(e + f*x)^2)^(1/2),x)
[Out] int(tanh(e + f*x)^4*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\tanh^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**4,x)
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**4, x)
```

3.433 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\tan^{-1}(\sinh(e+fx))}{f}$$

```
[Out] -arctan(sinh(f*x+e))*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f+(a*cosh(f*x+e)^2
)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.11, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3176, 3207, 2592, 321, 203}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\tan^{-1}(\sinh(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] -((ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/f) + (Sqrt[
a*Cosh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\tanh^2(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\int \sinh(e+fx)\tanh(e+fx)\,d$$
$$= \frac{\left(\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(e+f\right.}{f}$$
$$= \frac{\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)}{f} - \frac{\left(\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+\right.}{}$$
$$= -\frac{\tan^{-1}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)}{f} + \frac{\sqrt{a\cosh}}{}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.70

$$\frac{\mathrm{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\sinh(e+fx)-\tan^{-1}(\sinh(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*(-ArcTan[Sinh[e + f*x]] + Sinh[e + f
*x]))/f
```

fricas [B] time = 0.92, size = 182, normalized size = 3.19

$$\frac{\left(2\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)+e^{\left(fx+e\right)}\sinh\left(fx+e\right)^2-4\left(\cosh\left(fx+e\right)e^{\left(fx+e\right)}+e^{\left(fx+e\right)}\sinh\left(fx+e\right.\right.\right.}{2\left(f\cosh\left(fx+e\right)e^{\left(2fx+2e\right)}+f\cosh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 - 4*(cosh(f*x + e)*e^(f*x + e) + e^(f*x + e)*sinh(f*x + e))*arctan(cosh(f
*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^2 - 1)*e^(f*x + e))*sqrt(a*e^(4*f
*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x
 + 2*e) + f*cosh(f*x + e) + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

giac [A] time = 0.14, size = 38, normalized size = 0.67

$$-\frac{\sqrt{a}\left(4\arctan\left(e^{\left(fx+e\right)}\right)-e^{\left(fx+e\right)}+e^{\left(-fx-e\right)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(4*arctan(e^(f*x + e)) - e^(f*x + e) + e^(-f*x - e))/f
```

maple [A] time = 0.20, size = 41, normalized size = 0.72

$$-\frac{a\cosh\left(fx+e\right)\left(-\sinh\left(fx+e\right)+\arctan\left(\sinh\left(fx+e\right)\right)\right)}{\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x)
```

```
[Out] -a*cosh(f*x+e)*(-sinh(f*x+e)+arctan(sinh(f*x+e)))/(a*cosh(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 1.42, size = 50, normalized size = 0.88

$$\frac{2\sqrt{a}\arctan\left(e^{\left(-fx-e\right)}\right)}{f}+\frac{\sqrt{a}e^{\left(fx+e\right)}}{2f}-\frac{\sqrt{a}e^{\left(-fx-e\right)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*arctan(e^(-f*x - e))/f + 1/2*sqrt(a)*e^(f*x + e)/f - 1/2*sqrt(a)*
e^(-f*x - e)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh\left(e+f\,x\right)^2\,\sqrt{a\,\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^2*(a + a*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\tanh^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**2, x)
```

3.434 $\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] -csch(f*x+e)*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f+(a*cosh(f*x+e)^2)^(1/2)*
tanh(f*x+e)/f
```

Rubi [A] time = 0.11, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3207, 2590, 14}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f) + (Sqrt[a*Cosh[e
 + f*x]^2]*Tanh[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\,\coth^2(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\int \cosh(e+fx)\coth^2(e+fx)\,dx$$
$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -i\sinh(e+\right.}{f}$$
$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, -i\sin\right.}{f}$$
$$= -\frac{\sqrt{a\cosh^2(e+fx)}\,\mathrm{csch}(e+fx)\mathrm{sech}(e+fx)}{f} + \frac{\sqrt{a\cosh^2(e+fx)}\ \mathrm{t}}{f}$$

Mathematica [A] time = 0.08, size = 35, normalized size = 0.62

$$-\frac{\tanh(e+fx)\left(\mathrm{csch}^2(e+fx)-1\right)\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a*Cosh[e + f*x]^2]*(-1 + Csch[e + f*x]^2)*Tanh[e + f*x])/f)
```

fricas [B] time = 1.13, size = 317, normalized size = 5.66

$$\frac{\left(4\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)^3 + e^{\left(fx+e\right)}\sinh\left(fx+e\right)^4 + 6\left(\cosh\left(fx+e\right)^2-1\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)^2\right.}{2\left(f\cosh\left(fx+e\right)^3 + \left(fe^{\left(2fx+2e\right)}+f\right)\sinh\left(fx+e\right)^3 + 3\left(f\cosh\left(fx+e\right)e^{\left(2fx+2e\right)} + f\cosh\left(fx+e\right)\right)\sinh\left(fx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e
)^4 + 6*(cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e
)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 - 6*cos
h(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e)
 + a)*e^(-f*x - e)/(f*cosh(f*x + e)^3 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x +
e)^3 + 3*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^
2 - f*cosh(f*x + e) + (f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x + 2*e)
 + (3*f*cosh(f*x + e)^2 + (3*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*e) - f)*si
nh(f*x + e))
```

giac [A] time = 0.15, size = 57, normalized size = 1.02

$$-\frac{\sqrt{a}\left(\frac{\left(5e^{\left(2fx+2e\right)}-1\right)e^{\left(-e\right)}}{e^{\left(3fx+2e\right)}-e^{\left(fx\right)}} - e^{\left(fx+e\right)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*((5*e^(2*f*x + 2*e) - 1)*e^(-e)/(e^(3*f*x + 2*e) - e^(f*x)) -
e^(f*x + e))/f
```

maple [A] time = 0.24, size = 42, normalized size = 0.75

$$\frac{\cosh\left(fx+e\right)a\left(-1+\sinh^{2}\left(fx+e\right)\right)}{\sinh\left(fx+e\right)\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] cosh(f*x+e)*a*(-1+sinh(f*x+e)^2)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 1.48, size = 125, normalized size = 2.23

$$\frac{\sqrt{a}e^{\left(-fx-e\right)}}{f\left(e^{\left(-2\,fx-2\,e\right)}-1\right)}-\frac{2\,\sqrt{a}e^{\left(-2\,fx-2\,e\right)}-\sqrt{a}}{2\,f\left(e^{\left(-fx-e\right)}-e^{\left(-3\,fx-3\,e\right)}\right)}+\frac{2\,\sqrt{a}e^{\left(-fx-e\right)}-\sqrt{a}e^{\left(-3\,fx-3\,e\right)}}{2\,f\left(e^{\left(-2\,fx-2\,e\right)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*e^(-f*x - e)/(f*(e^(-2*f*x - 2*e) - 1)) - 1/2*(2*sqrt(a)*e^(-2*f*x
- 2*e) - sqrt(a))/(f*(e^(-f*x - e) - e^(-3*f*x - 3*e))) + 1/2*(2*sqrt(a)*e^
(-f*x - e) - sqrt(a)*e^(-3*f*x - 3*e))/(f*(e^(-2*f*x - 2*e) - 1))
```

mupad [B] time = 0.93, size = 67, normalized size = 1.20

$$\frac{\sqrt{a+a\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)^{2}}\,\left(\mathrm{e}^{4\,e+4\,f\,x}-6\,\mathrm{e}^{2\,e+2\,f\,x}+1\right)}{f\,\left(\mathrm{e}^{4\,e+4\,f\,x}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] ((a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(exp(4*e + 4*f*x) - 6*
exp(2*e + 2*f*x) + 1))/(f*(exp(4*e + 4*f*x) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\coth^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**2, x)
```

3.435 $\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=91

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{3f}-\frac{2\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] -2*csch(f*x+e)*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f-1/3*csch(f*x+e)^3*sech
(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f+(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3207, 2590, 270}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{3f}-\frac{2\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-2*Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f - (Sqrt[a*Cosh[e
 + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x])/(3*f) + (Sqrt[a*Cosh[e + f*x]^2]*
Tanh[e + f*x])/f
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\coth^4(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\int \cosh(e+fx)\coth^4(e+fx)\,d$$
$$= \frac{\left(i\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, -i\sinh(\right.}{f}$$
$$= \frac{\left(i\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, -\right.}{f}$$
$$= -\frac{2\sqrt{a\cosh^2(e+fx)}\,\mathrm{csch}(e+fx)\mathrm{sech}(e+fx)}{f} - \sqrt{a\cosh^2(e+}$$

Mathematica [A] time = 0.08, size = 47, normalized size = 0.52

$$-\frac{\tanh(e+fx)\left(\mathrm{csch}^4(e+fx)+6\mathrm{csch}^2(e+fx)-3\right)\sqrt{a\cosh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -1/3*(Sqrt[a*Cosh[e + f*x]^2]*(-3 + 6*Csch[e + f*x]^2 + Csch[e + f*x]^4)*Ta
nh[e + f*x])/f
```

fricas [B] time = 0.68, size = 885, normalized size = 9.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(24*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + 3*e^(f*x + e)*sinh(f*x
+ e)^8 + 12*(7*cosh(f*x + e)^2 - 3)*e^(f*x + e)*sinh(f*x + e)^6 + 24*(7*cos
h(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 10*(21*cosh(f
*x + e)^4 - 54*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(21*cos
h(f*x + e)^5 - 90*cosh(f*x + e)^3 + 25*cosh(f*x + e))*e^(f*x + e)*sinh(f*x
+ e)^3 + 12*(7*cosh(f*x + e)^6 - 45*cosh(f*x + e)^4 + 25*cosh(f*x + e)^2 -
3)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(3*cosh(f*x + e)^7 - 27*cosh(f*x + e)^5
+ 25*cosh(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (3*cosh
(f*x + e)^8 - 36*cosh(f*x + e)^6 + 50*cosh(f*x + e)^4 - 36*cosh(f*x + e)^2
+ 3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x
 - e)/(f*cosh(f*x + e)^7 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 7*(f*c
osh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^6 - 3*f*cosh(
f*x + e)^5 + 3*(7*f*cosh(f*x + e)^2 + (7*f*cosh(f*x + e)^2 - f)*e^(2*f*x +
2*e) - f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e) + (7
*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 +
3*f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 30*f*cosh(f*x + e)^2 + (35*f*
cosh(f*x + e)^4 - 30*f*cosh(f*x + e)^2 + 3*f)*e^(2*f*x + 2*e) + 3*f)*sinh(f
*x + e)^3 + 3*(7*f*cosh(f*x + e)^5 - 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x +
e) + (7*f*cosh(f*x + e)^5 - 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*
f*x + 2*e))*sinh(f*x + e)^2 - f*cosh(f*x + e) + (f*cosh(f*x + e)^7 - 3*f*co
sh(f*x + e)^5 + 3*f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x + 2*e) + (7
*f*cosh(f*x + e)^6 - 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + (7*f*cosh
```

```
(f*x + e)^6 - 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*
e) - f)*sinh(f*x + e))
```

giac [A] time = 0.17, size = 80, normalized size = 0.88

$$-\frac{\sqrt{a}\left(\frac{8\left(3e^{(5fx+5e)}-4e^{(3fx+3e)}+3e^{(fx+e)}\right)}{\left(e^{(2fx+2e)}-1\right)^3}-3e^{(fx+e)}+3e^{(-fx-e)}\right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a)*(8*(3*e^(5*f*x + 5*e) - 4*e^(3*f*x + 3*e) + 3*e^(f*x + e))/(e^
(2*f*x + 2*e) - 1)^3 - 3*e^(f*x + e) + 3*e^(-f*x - e))/f
```

maple [A] time = 0.20, size = 55, normalized size = 0.60

$$\frac{\cosh(fx+e)\,a\left(3\left(\sinh^4(fx+e)\right)-6\left(\sinh^2(fx+e)\right)-1\right)}{3\sinh(fx+e)^3\sqrt{a\left(\cosh^2(fx+e)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*cosh(f*x+e)*a*(3*sinh(f*x+e)^4-6*sinh(f*x+e)^2-1)/sinh(f*x+e)^3/(a*cosh
(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 1.11, size = 487, normalized size = 5.35

$$-\frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)-3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)-\frac{2\left(9\sqrt{a}e^{(-fx-e)}-8\sqrt{a}e^{(-3fx-3e)}+3\sqrt{a}e^{(-5fx-5e)}\right)}{3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1}}{12f}+\frac{3\sqrt{a}\log\left(e^{(-fx-e)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*sqrt(a)*log(e^(-f*x - e) + 1) - 3*sqrt(a)*log(e^(-f*x - e) - 1) -
2*(9*sqrt(a)*e^(-f*x - e) - 8*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-5*f*
x - 5*e))/(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*e) + e^(-6*f*x - 6*e) - 1))
/f + 1/12*(3*sqrt(a)*log(e^(-f*x - e) + 1) - 3*sqrt(a)*log(e^(-f*x - e) - 1
) + 2*(3*sqrt(a)*e^(-f*x - e) - 8*sqrt(a)*e^(-3*f*x - 3*e) + 9*sqrt(a)*e^(-
5*f*x - 5*e))/(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*e) + e^(-6*f*x - 6*e) -
 1))/f + sqrt(a)*e^(-3*f*x - 3*e)/(f*(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*
e) + e^(-6*f*x - 6*e) - 1)) - 1/12*(33*sqrt(a)*e^(-2*f*x - 2*e) - 40*sqrt(a
)*e^(-4*f*x - 4*e) + 15*sqrt(a)*e^(-6*f*x - 6*e) - 6*sqrt(a))/(f*(e^(-f*x -
 e) - 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) - e^(-7*f*x - 7*e))) + 1/12*(
15*sqrt(a)*e^(-f*x - e) - 40*sqrt(a)*e^(-3*f*x - 3*e) + 33*sqrt(a)*e^(-5*f*
x - 5*e) - 6*sqrt(a)*e^(-7*f*x - 7*e))/(f*(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x
 - 4*e) + e^(-6*f*x - 6*e) - 1))
```

mupad [B] time = 0.94, size = 281, normalized size = 3.09

$$-\frac{\left(\frac{1}{f}-\frac{e^{2e+2fx}}{f}\right)\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{e^{2e+2fx}+1}-\frac{8e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{f\left(e^{2e+2fx}-1\right)\left(e^{e+fx}+e^{3e+3fx}\right)}-\frac{16e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right.}}{3f\left(e^{2e+2fx}-1\right)^2\left(e^{e+fx}+e^{3e+3\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] - ((1/f - exp(2*e + 2*f*x)/f)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)
^(1/2))/(exp(2*e + 2*f*x) + 1) - (8*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2
 - exp(- e - f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) + ex
p(3*e + 3*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e -
f*x)/2)^2)^(1/2))/(3*f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3
*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2
)^(1/2))/(3*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\coth^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**4, x)
```

3.436 $\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=124

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}^5(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{5f}-\frac{\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] -3*csch(f*x+e)*sech(f*x+e)*(a*cosh(f*x+e)^2)^(1/2)/f-csch(f*x+e)^3*sech(f*x
+e)*(a*cosh(f*x+e)^2)^(1/2)/f-1/5*csch(f*x+e)^5*sech(f*x+e)*(a*cosh(f*x+e)^
2)^(1/2)/f+(a*cosh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3207, 2590, 270}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}^5(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{5f}-\frac{\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^6*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-3*Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f - (Sqrt[a*Cosh[e
 + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x])/f - (Sqrt[a*Cosh[e + f*x]^2]*Csch
[e + f*x]^5*Sech[e + f*x])/(5*f) + (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/
f
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\coth^6(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\int \cosh(e+fx)\coth^6(e+fx)\,d$$
$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, -i\sinh\right.}{f}$$
$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)\right.}{f}$$
$$= -\frac{3\sqrt{a\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)}{f} - \frac{\sqrt{a\cosh^2(e+}}{}$$

Mathematica [A] time = 0.20, size = 67, normalized size = 0.54

$$\frac{(235\cosh(2(e+fx))-90\cosh(4(e+fx))+5\cosh(6(e+fx))-182)\operatorname{csch}^5(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+}}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^6*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(-182 + 235*Cosh[2*(e + f*x)] - 90*Cosh[4*(e + f*x
)] + 5*Cosh[6*(e + f*x)])*Csch[e + f*x]^5*Sech[e + f*x])/(160*f)
```

fricas [B] time = 0.62, size = 1696, normalized size = 13.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(60*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^11 + 5*e^(f*x + e)*sinh(f*
x + e)^12 + 30*(11*cosh(f*x + e)^2 - 3)*e^(f*x + e)*sinh(f*x + e)^10 + 100*
(11*cosh(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^9 + 5*(495
*cosh(f*x + e)^4 - 810*cosh(f*x + e)^2 + 47)*e^(f*x + e)*sinh(f*x + e)^8 +
40*(99*cosh(f*x + e)^5 - 270*cosh(f*x + e)^3 + 47*cosh(f*x + e))*e^(f*x + e
)*sinh(f*x + e)^7 + 28*(165*cosh(f*x + e)^6 - 675*cosh(f*x + e)^4 + 235*cos
h(f*x + e)^2 - 13)*e^(f*x + e)*sinh(f*x + e)^6 + 8*(495*cosh(f*x + e)^7 - 2
835*cosh(f*x + e)^5 + 1645*cosh(f*x + e)^3 - 273*cosh(f*x + e))*e^(f*x + e)
*sinh(f*x + e)^5 + 5*(495*cosh(f*x + e)^8 - 3780*cosh(f*x + e)^6 + 3290*cos
h(f*x + e)^4 - 1092*cosh(f*x + e)^2 + 47)*e^(f*x + e)*sinh(f*x + e)^4 + 20*
(55*cosh(f*x + e)^9 - 540*cosh(f*x + e)^7 + 658*cosh(f*x + e)^5 - 364*cosh(
f*x + e)^3 + 47*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 10*(33*cosh(f*
x + e)^10 - 405*cosh(f*x + e)^8 + 658*cosh(f*x + e)^6 - 546*cosh(f*x + e)^4
 + 141*cosh(f*x + e)^2 - 9)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(15*cosh(f*x +
e)^11 - 225*cosh(f*x + e)^9 + 470*cosh(f*x + e)^7 - 546*cosh(f*x + e)^5 + 2
35*cosh(f*x + e)^3 - 45*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (5*cosh(
f*x + e)^12 - 90*cosh(f*x + e)^10 + 235*cosh(f*x + e)^8 - 364*cosh(f*x + e)
^6 + 235*cosh(f*x + e)^4 - 90*cosh(f*x + e)^2 + 5)*e^(f*x + e))*sqrt(a*e^(4
*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^11 + (
f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^11 + 11*(f*cosh(f*x + e)*e^(2*f*x + 2*
e) + f*cosh(f*x + e))*sinh(f*x + e)^10 - 5*f*cosh(f*x + e)^9 + 5*(11*f*cosh
(f*x + e)^2 + (11*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e)
```

```
^9 + 15*(11*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e) + (11*f*cosh(f*x + e)^3 -
 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*f*cosh(f*x + e)^7
 + 10*(33*f*cosh(f*x + e)^4 - 18*f*cosh(f*x + e)^2 + (33*f*cosh(f*x + e)^4
- 18*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 14*(33*f
*cosh(f*x + e)^5 - 30*f*cosh(f*x + e)^3 + 5*f*cosh(f*x + e) + (33*f*cosh(f*
x + e)^5 - 30*f*cosh(f*x + e)^3 + 5*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(
f*x + e)^6 - 10*f*cosh(f*x + e)^5 + 2*(231*f*cosh(f*x + e)^6 - 315*f*cosh(f
*x + e)^4 + 105*f*cosh(f*x + e)^2 + (231*f*cosh(f*x + e)^6 - 315*f*cosh(f*x
 + e)^4 + 105*f*cosh(f*x + e)^2 - 5*f)*e^(2*f*x + 2*e) - 5*f)*sinh(f*x + e)
^5 + 10*(33*f*cosh(f*x + e)^7 - 63*f*cosh(f*x + e)^5 + 35*f*cosh(f*x + e)^3
 - 5*f*cosh(f*x + e) + (33*f*cosh(f*x + e)^7 - 63*f*cosh(f*x + e)^5 + 35*f*
cosh(f*x + e)^3 - 5*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 5*f
*cosh(f*x + e)^3 + 5*(33*f*cosh(f*x + e)^8 - 84*f*cosh(f*x + e)^6 + 70*f*co
sh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + (33*f*cosh(f*x + e)^8 - 84*f*cosh(f*
x + e)^6 + 70*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e)
 + f)*sinh(f*x + e)^3 + 5*(11*f*cosh(f*x + e)^9 - 36*f*cosh(f*x + e)^7 + 42
*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (11*f*cosh(
f*x + e)^9 - 36*f*cosh(f*x + e)^7 + 42*f*cosh(f*x + e)^5 - 20*f*cosh(f*x +
e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 - f*cosh(f*x + e
) + (f*cosh(f*x + e)^11 - 5*f*cosh(f*x + e)^9 + 10*f*cosh(f*x + e)^7 - 10*f
*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x + 2*e) +
 (11*f*cosh(f*x + e)^10 - 45*f*cosh(f*x + e)^8 + 70*f*cosh(f*x + e)^6 - 50*
f*cosh(f*x + e)^4 + 15*f*cosh(f*x + e)^2 + (11*f*cosh(f*x + e)^10 - 45*f*co
sh(f*x + e)^8 + 70*f*cosh(f*x + e)^6 - 50*f*cosh(f*x + e)^4 + 15*f*cosh(f*x
 + e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e))
```

giac [A] time = 0.20, size = 104, normalized size = 0.84

$$-\frac{\sqrt{a}\left(\frac{4\left(15e^{(9fx+9e)}-40e^{(7fx+7e)}+66e^{(5fx+5e)}-40e^{(3fx+3e)}+15e^{(fx+e)}\right)}{\left(e^{(2fx+2e)}-1\right)^5}-5e^{(fx+e)}+5e^{(-fx-e)}\right)}{10f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(a)*(4*(15*e^(9*f*x + 9*e) - 40*e^(7*f*x + 7*e) + 66*e^(5*f*x + 5
*e) - 40*e^(3*f*x + 3*e) + 15*e^(f*x + e))/(e^(2*f*x + 2*e) - 1)^5 - 5*e^(f
*x + e) + 5*e^(-f*x - e))/f
```

maple [A] time = 0.21, size = 65, normalized size = 0.52

$$\frac{\cosh\left(fx+e\right)a\left(5\left(\sinh^6\left(fx+e\right)\right)-15\left(\sinh^4\left(fx+e\right)\right)-5\left(\sinh^2\left(fx+e\right)\right)-1\right)}{5\sinh\left(fx+e\right)^5\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/5*cosh(f*x+e)*a*(5*sinh(f*x+e)^6-15*sinh(f*x+e)^4-5*sinh(f*x+e)^2-1)/sinh
(f*x+e)^5/(a*cosh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.74, size = 1050, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/320*(105*sqrt(a)*log(e^(-f*x - e) + 1) - 105*sqrt(a)*log(e^(-f*x - e) -
1) - 2*(375*sqrt(a)*e^(-f*x - e) - 790*sqrt(a)*e^(-3*f*x - 3*e) + 896*sqrt(
a)*e^(-5*f*x - 5*e) - 490*sqrt(a)*e^(-7*f*x - 7*e) + 105*sqrt(a)*e^(-9*f*x
- 9*e))/(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5
*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))/f + 1/320*(105*sqrt(a)*log(e^(
-f*x - e) + 1) - 105*sqrt(a)*log(e^(-f*x - e) - 1) + 2*(105*sqrt(a)*e^(-f*x
 - e) - 490*sqrt(a)*e^(-3*f*x - 3*e) + 896*sqrt(a)*e^(-5*f*x - 5*e) - 790*s
qrt(a)*e^(-7*f*x - 7*e) + 375*sqrt(a)*e^(-9*f*x - 9*e))/(5*e^(-2*f*x - 2*e)
 - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e) + e^(-10*
f*x - 10*e) - 1))/f + 1/256*(15*sqrt(a)*log(e^(-f*x - e) + 1) - 15*sqrt(a)*
log(e^(-f*x - e) - 1) + 2*(15*sqrt(a)*e^(-f*x - e) + 250*sqrt(a)*e^(-3*f*x
- 3*e) - 128*sqrt(a)*e^(-5*f*x - 5*e) + 70*sqrt(a)*e^(-7*f*x - 7*e) - 15*sq
rt(a)*e^(-9*f*x - 9*e))/(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-
6*f*x - 6*e) - 5*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))/f - 1/256*(15*
sqrt(a)*log(e^(-f*x - e) + 1) - 15*sqrt(a)*log(e^(-f*x - e) - 1) + 2*(15*sq
rt(a)*e^(-f*x - e) - 70*sqrt(a)*e^(-3*f*x - 3*e) + 128*sqrt(a)*e^(-5*f*x -
5*e) - 250*sqrt(a)*e^(-7*f*x - 7*e) - 15*sqrt(a)*e^(-9*f*x - 9*e))/(5*e^(-2
*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e
) + e^(-10*f*x - 10*e) - 1))/f + 2*sqrt(a)*e^(-5*f*x - 5*e)/(f*(5*e^(-2*f*x
 - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e) +
e^(-10*f*x - 10*e) - 1)) - 1/640*(2895*sqrt(a)*e^(-2*f*x - 2*e) - 7110*sqrt
(a)*e^(-4*f*x - 4*e) + 8064*sqrt(a)*e^(-6*f*x - 6*e) - 4410*sqrt(a)*e^(-8*f
*x - 8*e) + 945*sqrt(a)*e^(-10*f*x - 10*e) - 320*sqrt(a))/(f*(e^(-f*x - e)
- 5*e^(-3*f*x - 3*e) + 10*e^(-5*f*x - 5*e) - 10*e^(-7*f*x - 7*e) + 5*e^(-9*
f*x - 9*e) - e^(-11*f*x - 11*e))) + 1/640*(945*sqrt(a)*e^(-f*x - e) - 4410*
sqrt(a)*e^(-3*f*x - 3*e) + 8064*sqrt(a)*e^(-5*f*x - 5*e) - 7110*sqrt(a)*e^(
-7*f*x - 7*e) + 2895*sqrt(a)*e^(-9*f*x - 9*e) - 320*sqrt(a)*e^(-11*f*x - 11
*e))/(f*(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5
*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))
```

mupad [B] time = 0.91, size = 427, normalized size = 3.44

$$-\frac{\left(\frac{1}{f}-\frac{e^{2e+2fx}}{f}\right)\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{e^{2e+2fx}+1}-\frac{12\,e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{f\left(e^{2e+2fx}-1\right)\left(e^{e+fx}+e^{3e+3fx}\right)}-\frac{16\,e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-}}{}\right.}}{f\left(e^{2e+2fx}-1\right)^2\left(e^{e+fx}+e^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^6*(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] - ((1/f - exp(2*e + 2*f*x)/f)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)
^(1/2))/(exp(2*e + 2*f*x) + 1) - (12*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/
2 - exp(- e - f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) + e
xp(3*e + 3*f*x))) - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e -
 f*x)/2)^2)^(1/2))/(f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*
f*x))) - (144*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2
)^(1/2))/(5*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) -
 (128*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(5*f*(exp(2*e + 2*f*x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*exp
(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*f*(ex
p(2*e + 2*f*x) - 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**6*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.437 $\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}+\frac{2a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -1/5*a^2/f/(a*cosh(f*x+e)^2)^(5/2)+2/3*a/f/(a*cosh(f*x+e)^2)^(3/2)-1/f/(a*c
osh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3205, 16, 43}

$$-\frac{a^2}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}+\frac{2a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -a^2/(5*f*(a*Cosh[e + f*x]^2)^(5/2)) + (2*a)/(3*f*(a*Cosh[e + f*x]^2)^(3/2)
) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\int \frac{\tanh^5(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$=\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{ax}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=\frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{7/2}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}}-\frac{2}{a(ax)^{5/2}}+\frac{1}{a^2(ax)^{3/2}}\right)dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=-\frac{a^2}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}+\frac{2a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.09, size = 43, normalized size = 0.65

$$\frac{-3\text{sech}^4(e+fx)+10\text{sech}^2(e+fx)-15}{15f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-15 + 10*Sech[e + f*x]^2 - 3*Sech[e + f*x]^4)/(15*f*Sqrt[a*Cosh[e + f*x]^2
])
```

fricas [B] time = 0.49, size = 1387, normalized size = 21.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(135*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + 15*e^(f*x + e)*sinh(
f*x + e)^9 + 20*(27*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^7 + 140*
(9*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 2*(945*co
sh(f*x + e)^4 + 210*cosh(f*x + e)^2 + 29)*e^(f*x + e)*sinh(f*x + e)^5 + 10*
(189*cosh(f*x + e)^5 + 70*cosh(f*x + e)^3 + 29*cosh(f*x + e))*e^(f*x + e)*s
inh(f*x + e)^4 + 20*(63*cosh(f*x + e)^6 + 35*cosh(f*x + e)^4 + 29*cosh(f*x
+ e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 20*(27*cosh(f*x + e)^7 + 21*cosh(
f*x + e)^5 + 29*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(27*cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 58*cosh(f*x + e)^4 + 12*
cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e) + (15*cosh(f*x + e)^9 + 20*c
osh(f*x + e)^7 + 58*cosh(f*x + e)^5 + 20*cosh(f*x + e)^3 + 15*cosh(f*x + e)
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a*f*cosh(f*x + e)^10 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^10 + 5
*a*f*cosh(f*x + e)^8 + 10*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x
 + e))*sinh(f*x + e)^9 + 5*(9*a*f*cosh(f*x + e)^2 + a*f + (9*a*f*cosh(f*x +
 e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 + 40
*(3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e)^3 + a*f*
cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^7 + 10*(21*a*f*cosh(f*x + e)^
4 + 14*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)^4 + 14*a*f*cosh(f*
x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 10*a*f*cosh(f*x + e)^4 +
 4*(63*a*f*cosh(f*x + e)^5 + 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e)
```

```
+ (63*a*f*cosh(f*x + e)^5 + 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e))*
e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 10*(21*a*f*cosh(f*x + e)^6 + 35*a*f*cosh
(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)^6 + 35*a
*f*cosh(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*
x + e)^4 + 5*a*f*cosh(f*x + e)^2 + 40*(3*a*f*cosh(f*x + e)^7 + 7*a*f*cosh(f
*x + e)^5 + 5*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e
)^7 + 7*a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^
(2*f*x + 2*e))*sinh(f*x + e)^3 + 5*(9*a*f*cosh(f*x + e)^8 + 28*a*f*cosh(f*x
 + e)^6 + 30*a*f*cosh(f*x + e)^4 + 12*a*f*cosh(f*x + e)^2 + a*f + (9*a*f*co
sh(f*x + e)^8 + 28*a*f*cosh(f*x + e)^6 + 30*a*f*cosh(f*x + e)^4 + 12*a*f*co
sh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x
 + e)^10 + 5*a*f*cosh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 + 10*a*f*cosh(f*x
 + e)^4 + 5*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 10*(a*f*cosh(f*x +
 e)^9 + 4*a*f*cosh(f*x + e)^7 + 6*a*f*cosh(f*x + e)^5 + 4*a*f*cosh(f*x + e)
^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^9 + 4*a*f*cosh(f*x + e)^7 + 6*a
*f*cosh(f*x + e)^5 + 4*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e))
```

giac [A] time = 0.40, size = 95, normalized size = 1.44

$$-\frac{2\left(15\sqrt{a}e^{(9fx+9e)}+20\sqrt{a}e^{(7fx+7e)}+58\sqrt{a}e^{(5fx+5e)}+20\sqrt{a}e^{(3fx+3e)}+15\sqrt{a}e^{(fx+e)}\right)}{15af\left(e^{(2fx+2e)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(a)*e^(9*f*x + 9*e) + 20*sqrt(a)*e^(7*f*x + 7*e) + 58*sqrt(a)
*e^(5*f*x + 5*e) + 20*sqrt(a)*e^(3*f*x + 3*e) + 15*sqrt(a)*e^(f*x + e))/(a*
f*(e^(2*f*x + 2*e) + 1)^5)
```

maple [C] time = 0.23, size = 41, normalized size = 0.62

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\sinh^5(fx+e)}{\cosh(fx+e)^6\sqrt{a(\cosh^2(fx+e))}},\sinh(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5/cosh(f*x+e)^6/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

maxima [B] time = 1.97, size = 446, normalized size = 6.76

$$-\frac{2e^{(-fx-e)}}{\left(5\sqrt{a}e^{(-2fx-2e)}+10\sqrt{a}e^{(-4fx-4e)}+10\sqrt{a}e^{(-6fx-6e)}+5\sqrt{a}e^{(-8fx-8e)}+\sqrt{a}e^{(-10fx-10e)}+\sqrt{a}\right)f}-\frac{}{3\left(5\sqrt{a}e^{(-\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-f*x - e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e)
+ 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10
*f*x - 10*e) + sqrt(a))*f) - 8/3*e^(-3*f*x - 3*e)/((5*sqrt(a)*e^(-2*f*x - 2
*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)
*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e) + sqrt(a))*f) - 116/15*e^(-5
```

```
*f*x - 5*e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10
*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x
 - 10*e) + sqrt(a))*f) - 8/3*e^(-7*f*x - 7*e)/((5*sqrt(a)*e^(-2*f*x - 2*e)
+ 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(
-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e) + sqrt(a))*f) - 2*e^(-9*f*x - 9*
e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*
e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e)
+ sqrt(a))*f)
```

mupad [B] time = 0.89, size = 381, normalized size = 5.77

$$\frac{32\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^2\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{4\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{a\,f\,\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{352\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\right.}}{15\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}^3\,\left({\mathrm{e}}^{e+f\,x}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] (32*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(
3*a*f*(exp(2*e + 2*f*x) + 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (4*exp(
3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(a*f*(exp
(2*e + 2*f*x) + 1)*(exp(e + f*x) + exp(3*e + 3*f*x))) - (352*exp(3*e + 3*f*
x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(15*a*f*(exp(2*e +
2*f*x) + 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) + (128*exp(3*e + 3*f*x)*(a
 + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e + 2*f*x)
 + 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*exp(3*e + 3*f*x)*(a + a*(e
xp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e + 2*f*x) + 1)^5
*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{5}{\left(e + f x \right)}}{\sqrt{a \left(\sinh^{2}{\left(e + f x \right)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

3.438 $\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$\frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] 1/3*a/f/(a*cosh(f*x+e)^2)^(3/2)-1/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] a/(3*f*(a*Cosh[e + f*x]^2)^(3/2)) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{5/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}} - \frac{1}{a(ax)^{3/2}}\right)dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}} - \frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [**A**] time = 0.08, size = 31, normalized size = 0.74

$$\frac{\text{sech}^2(e+fx)-3}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-3 + Sech[e + f*x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])
```

fricas [**B**] time = 0.43, size = 641, normalized size = 15.26

$$-\frac{}{3\left(af\cosh\left(fx+e\right)^6+\left(afe^{(2fx+2e)}+af\right)\sinh\left(fx+e\right)^6+3\,af\cosh\left(fx+e\right)^4+6\left(af\cosh\left(fx+e\right)e^{(2fx+}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(15*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + 3*e^(f*x + e)*sinh(f*x
 + e)^5 + 2*(15*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 6*(5*cos
h(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 3*(5*cosh(f*x +
 e)^4 + 2*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (3*cosh(f*x + e)
^5 + 2*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*
e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^6 + (a*f*e^(2
*f*x + 2*e) + a*f)*sinh(f*x + e)^6 + 3*a*f*cosh(f*x + e)^4 + 6*(a*f*cosh(f*
x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 3*(5*a*f*cosh
(f*x + e)^2 + a*f + (5*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^4 + 3*a*f*cosh(f*x + e)^2 + 4*(5*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x
 + e) + (5*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh
(f*x + e)^3 + 3*(5*a*f*cosh(f*x + e)^4 + 6*a*f*cosh(f*x + e)^2 + a*f + (5*a
*f*cosh(f*x + e)^4 + 6*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^2 + a*f + (a*f*cosh(f*x + e)^6 + 3*a*f*cosh(f*x + e)^4 + 3*a*f*cosh(f
*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 6*(a*f*cosh(f*x + e)^5 + 2*a*f*cosh(f*x
+ e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^5 + 2*a*f*cosh(f*x + e)^3 +
 a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

giac [A] time = 0.31, size = 65, normalized size = 1.55

$$-\frac{2\left(3\sqrt{a}e^{(5fx+5e)}+2\sqrt{a}e^{(3fx+3e)}+3\sqrt{a}e^{(fx+e)}\right)}{3af\left(e^{(2fx+2e)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(a)*e^(5*f*x + 5*e) + 2*sqrt(a)*e^(3*f*x + 3*e) + 3*sqrt(a)*e^(
f*x + e))/(a*f*(e^(2*f*x + 2*e) + 1)^3)
```

maple [C] time = 0.19, size = 41, normalized size = 0.98

$$\frac{`int/indef0`\left(\frac{\sinh^3(fx+e)}{\cosh(fx+e)^4\sqrt{a(\cosh^2(fx+e))}},\sinh(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3/cosh(f*x+e)^4/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

maxima [B] time = 1.89, size = 184, normalized size = 4.38

$$-\frac{2e^{(-fx-e)}}{\left(3\sqrt{a}e^{(-2fx-2e)}+3\sqrt{a}e^{(-4fx-4e)}+\sqrt{a}e^{(-6fx-6e)}+\sqrt{a}\right)f}-\frac{4e^{(-3fx-3e)}}{3\left(3\sqrt{a}e^{(-2fx-2e)}+3\sqrt{a}e^{(-4fx-4e)}+\sqrt{a}e^{(-6fx-6e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-f*x - e)/((3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) +
 sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a))*f) - 4/3*e^(-3*f*x - 3*e)/((3*sqrt(a)*
e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a)*e^(-6*f*x - 6*e) +
sqrt(a))*f) - 2*e^(-5*f*x - 5*e)/((3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e
^(-4*f*x - 4*e) + sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a))*f)
```

mupad [B] time = 0.12, size = 82, normalized size = 1.95

$$-\frac{4e^{2e+2fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}\left(2e^{2e+2fx}+3e^{4e+4fx}+3\right)}{3af\left(e^{2e+2fx}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] -(4*exp(2*e + 2*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(2
*exp(2*e + 2*f*x) + 3*exp(4*e + 4*f*x) + 3))/(3*a*f*(exp(2*e + 2*f*x) + 1)^
4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh^3(e+fx)}{\sqrt{a\left(\sinh^2(e+fx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

3.439 $\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

[Out] -1/f/(a*cosh(f*x+e)^2)^(1/2)

Rubi [A] time = 0.07, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3176, 3205, 16, 32}

$$-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]

[Out] -(1/(f*Sqrt[a*Cosh[e + f*x]^2]))

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.03, size = 19, normalized size = 1.00

$$-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(1/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

fricas [B] time = 0.52, size = 168, normalized size = 8.84

$$-\frac{2\sqrt{ae^{(4fx+4e)}+2\,ae^{(2fx+2e)}+a}\left(\cosh\left(fx+e\right)e^{(fx+e)}+e^{(fx+e)}\sinh\left(\right.\right.}{af\cosh\left(fx+e\right)^2+\left(afe^{(2fx+2e)}+af\right)\sinh\left(fx+e\right)^2+af+\left(af\cosh\left(fx+e\right)^2+af\right)e^{(2fx+2e)}+2\left(af\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(cosh(f*x + e)*e^(f*x
+ e) + e^(f*x + e)*sinh(f*x + e))*e^(-f*x - e)/(a*f*cosh(f*x + e)^2 + (a*f*
e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^2 + a*f)*
e^(2*f*x + 2*e) + 2*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))
*sinh(f*x + e))
```

giac [A] time = 0.20, size = 29, normalized size = 1.53

$$-\frac{2\,e^{(fx+e)}}{\sqrt{a}\,f\left(e^{(2fx+2e)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*e^(f*x + e)/(sqrt(a)*f*(e^(2*f*x + 2*e) + 1))
```

maple [A] time = 0.09, size = 20, normalized size = 1.05

$$-\frac{1}{f\sqrt{a+a\left(\sinh^2\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/(a+a*sinh(f*x+e)^2)^(1/2)
```

maxima [A] time = 2.60, size = 33, normalized size = 1.74

$$-\frac{2 e^{\left(-f x-e\right)}}{\left(\sqrt{a} e^{\left(-2 f x-2 e\right)}+\sqrt{a}\right) f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-f*x - e)/((sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a))*f)
```

mupad [B] time = 0.85, size = 30, normalized size = 1.58

$$-\frac{\sqrt{a \sinh\left(e+f x\right)^2+a}}{a f \cosh\left(e+f x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] -(a + a*sinh(e + f*x)^2)^(1/2)/(a*f*cosh(e + f*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(e+fx\right)}{\sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

3.440 $\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3176, 3205, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{af}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Mathematica [A] time = 0.05, size = 49, normalized size = 1.58

$$\frac{\cosh(e+fx)\left(\log\left(\sinh\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*(-Log[Cosh[(e + f*x)/2]] + Log[Sinh[(e + f*x)/2]]))/(f*Sqrt[
a*Cosh[e + f*x]^2])
```

fricas [B] time = 0.53, size = 174, normalized size = 5.61

$$\left[\frac{\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\,\log\left(\frac{\cosh(fx+e)+\sinh(fx+e)-1}{\cosh(fx+e)+\sinh(fx+e)+1}\right)}{afe^{(2fx+2e)}+af}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}}{a\cosh(fx+e)e^{(2fx+2e)}+a\cosh(fx+e)+(ae^{(2fx}}\right.}{af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*log((cosh(f*x + e) + sin
h(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1))/(a*f*e^(2*f*x + 2*e) +
 a*f), 2*sqrt(-a)*arctan(sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*
sqrt(-a)/(a*cosh(f*x + e)*e^(2*f*x + 2*e) + a*cosh(f*x + e) + (a*e^(2*f*x +
 2*e) + a)*sinh(f*x + e)))/(a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
```

```
ostep^3*exp(exp(1))^3+t_nostep*exp(exp(1)))]index.cc index_m operator + Err
or: Bad Argument Value
```

maple [C] time = 0.16, size = 33, normalized size = 1.06

$$\frac{\text{`}int/indef0\text{`}\left(\frac{1}{\sinh\left(fx+e\right)\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x)
[Out] `int/indef0`(1/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [A] time = 3.00, size = 40, normalized size = 1.29

$$-\frac{\log\left(e^{\left(-fx-e\right)}+1\right)}{\sqrt{a}f}+\frac{\log\left(e^{\left(-fx-e\right)}-1\right)}{\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -log(e^(-f*x - e) + 1)/(sqrt(a)*f) + log(e^(-f*x - e) - 1)/(sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth\left(e+f\,x\right)}{\sqrt{a\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(1/2),x)
[Out] int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(e+fx\right)}{\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(coth(e + f*x)/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

3.441 $\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=66

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{2af}$$

```
[Out] -1/2*arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*csch(f*x+e)^2*(
a*cosh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3176, 3205, 16, 47, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*Sqrt[a]*f) - (Sqrt[a*Cosh[e +
f*x]^2]*Csch[e + f*x]^2)/(2*a*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\coth^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{(1-x)^2}\,dx, x, \cosh^2(e+fx)\right)}{2af}\\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2af} - \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{4f}\\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2af} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{2af}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f} - \frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2af}\end{aligned}$$

Mathematica [A] time = 0.12, size = 65, normalized size = 0.98

$$-\frac{\cosh(e+fx)\left(\text{csch}^2\left(\frac{1}{2}(e+fx)\right)+\text{sech}^2\left(\frac{1}{2}(e+fx)\right)-4\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -1/8*(Cosh[e + f*x]*(Csch[(e + f*x)/2]^2 - 4*Log[Tanh[(e + f*x)/2]] + Sech[
(e + f*x)/2]^2))/(f*Sqrt[a*Cosh[e + f*x]^2])
```

fricas [B] time = 0.92, size = 529, normalized size = 8.02

$$-\frac{\left(6\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)^2+2e^{(fx+e)}\sinh\left(fx+e\right)^3+2\left(3\cosh\left(fx+e\right)^2+1\right)e^{(fx+e)}\sinh\left(fx\right.\right.}{2\left(af\cosh\left(fx+e\right)^4+\left(afe^{(2fx+2e)}+af\right)\sinh\left(fx+e\right)^4-2af\cosh\left(fx+e\right)^2+4\left(af\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + 2*e^(f*x + e)*sinh(f*x
+ e)^3 + 2*(3*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + 2*(cosh(f*x
+ e)^3 + cosh(f*x + e))*e^(f*x + e) - (4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x
 + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*e^(f*x +
e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f
*x + e) + (cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(
f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*sqrt(a*
e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^
4 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 + 4
*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^3 +
2*(3*a*f*cosh(f*x + e)^2 - a*f + (3*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2
*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 +
 a*f)*e^(2*f*x + 2*e) + 4*(a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (a*f*c
osh(f*x + e)^3 - a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^3*exp(exp(1))^3+t_nostep*exp(exp(1)))]index.cc index_m operator + Err
or: Bad Argument Value
```

maple [C] time = 0.21, size = 42, normalized size = 0.64

$$\frac{`int/indef0`\left(\frac{\frac{1}{\sinh\left(fx+e\right)}+\frac{1}{\sinh\left(fx+e\right)^{3}}}{\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((1/sinh(f*x+e)+1/sinh(f*x+e)^3)/(a*cosh(f*x+e)^2)^(1/2),sinh(f
*x+e))/f
```

maxima [A] time = 2.62, size = 100, normalized size = 1.52

$$-\frac{\log\left(e^{\left(-fx-e\right)}+1\right)}{2\sqrt{a}f}+\frac{\log\left(e^{\left(-fx-e\right)}-1\right)}{2\sqrt{a}f}+\frac{e^{\left(-fx-e\right)}+e^{\left(-3\,fx-3\,e\right)}}{\left(2\sqrt{a}e^{\left(-2\,fx-2\,e\right)}-\sqrt{a}e^{\left(-4\,fx-4\,e\right)}-\sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-f*x - e) + 1)/(sqrt(a)*f) + 1/2*log(e^(-f*x - e) - 1)/(sqrt(a)
*f) + (e^(-f*x - e) + e^(-3*f*x - 3*e))/((2*sqrt(a)*e^(-2*f*x - 2*e) - sqrt
(a)*e^(-4*f*x - 4*e) - sqrt(a))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth\left(e+f\,x\right)^3}{\sqrt{a\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(e+fx)}{\sqrt{a\left(\sinh^2(e+fx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

3.442 $\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$-\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}}-\frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}}+\frac{3\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] 3/8*arctan(sinh(f*x+e))*cosh(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-3/8*tanh(f*x+
e)/f/(a*cosh(f*x+e)^2)^(1/2)-1/4*tanh(f*x+e)^3/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3207, 2611, 3770}

$$-\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}}-\frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}}+\frac{3\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - (3*
Tanh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[e + f*x]^3/(4*f*Sqrt[a*
Cosh[e + f*x]^2])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^4(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\tanh^4(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}} + \frac{(3\cosh(e+fx))\int \operatorname{sech}(e+fx)\tanh^2(e+fx)\,dx}{4\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}} - \frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}} + \frac{(3\cosh(e+fx))\int \operatorname{sech}(e+ \ldots}{8\sqrt{a\cosh^2(e+fx)}}$$
$$= \frac{3\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}} - \frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}} - \frac{\tanh^3(e+f\ldots}{4f\sqrt{a\cosh^2(e+\ldots}}$$

Mathematica [A] time = 0.14, size = 66, normalized size = 0.73

$$\frac{\tanh(e+fx)\left(-8\tanh^2(e+fx)-6\operatorname{sech}^2(e+fx)+3\right)+3\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + Tanh[e + f*x]*(3 - 6*Sech[e + f*x]
^2 - 8*Tanh[e + f*x]^2))/(8*f*Sqrt[a*Cosh[e + f*x]^2])
```

fricas [B] time = 0.47, size = 1328, normalized size = 14.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*x
 + e)^7 + 3*(35*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*co
sh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + (175*cosh(f*
x + e)^4 - 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^3 + 3*(35*cosh
(f*x + e)^5 - 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x +
e)^2 + (35*cosh(f*x + e)^6 - 15*cosh(f*x + e)^4 + 9*cosh(f*x + e)^2 - 5)*e^
(f*x + e)*sinh(f*x + e) - 3*(8*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 +
e^(f*x + e)*sinh(f*x + e)^8 + 4*(7*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*
x + e)^6 + 8*(7*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^5 + 2*(35*cosh(f*x + e)^4 + 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x
+ e)^4 + 8*(7*cosh(f*x + e)^5 + 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*
x + e)*sinh(f*x + e)^3 + 4*(7*cosh(f*x + e)^6 + 15*cosh(f*x + e)^4 + 9*cosh
(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(cosh(f*x + e)^7 + 3*cosh(
f*x + e)^5 + 3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) +
 (cosh(f*x + e)^8 + 4*cosh(f*x + e)^6 + 6*cosh(f*x + e)^4 + 4*cosh(f*x + e)
^2 + 1)*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (5*cosh(f*x +
e)^7 - 3*cosh(f*x + e)^5 + 3*cosh(f*x + e)^3 - 5*cosh(f*x + e))*e^(f*x + e)
)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(
f*x + e)^8 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^8 + 4*a*f*cosh(f*x +
 e)^6 + 8*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x
```

```
+ e)^7 + 4*(7*a*f*cosh(f*x + e)^2 + a*f + (7*a*f*cosh(f*x + e)^2 + a*f)*e^(
2*f*x + 2*e))*sinh(f*x + e)^6 + 6*a*f*cosh(f*x + e)^4 + 8*(7*a*f*cosh(f*x +
 e)^3 + 3*a*f*cosh(f*x + e) + (7*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))
*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 2*(35*a*f*cosh(f*x + e)^4 + 30*a*f*cosh
(f*x + e)^2 + 3*a*f + (35*a*f*cosh(f*x + e)^4 + 30*a*f*cosh(f*x + e)^2 + 3*
a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 4*a*f*cosh(f*x + e)^2 + 8*(7*a*f*co
sh(f*x + e)^5 + 10*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e) + (7*a*f*cosh(
f*x + e)^5 + 10*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^3 + 4*(7*a*f*cosh(f*x + e)^6 + 15*a*f*cosh(f*x + e)^4 + 9*a*
f*cosh(f*x + e)^2 + a*f + (7*a*f*cosh(f*x + e)^6 + 15*a*f*cosh(f*x + e)^4 +
 9*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f
*cosh(f*x + e)^8 + 4*a*f*cosh(f*x + e)^6 + 6*a*f*cosh(f*x + e)^4 + 4*a*f*co
sh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 8*(a*f*cosh(f*x + e)^7 + 3*a*f*cosh(
f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)
^7 + 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(
2*f*x + 2*e))*sinh(f*x + e))
```

giac [A] time = 0.36, size = 96, normalized size = 1.05

$$\frac{\frac{3\arctan\left(e^{(fx+e)}\right)}{\sqrt{a}}-\frac{5\sqrt{a}e^{(7fx+7e)}-3\sqrt{a}e^{(5fx+5e)}+3\sqrt{a}e^{(3fx+3e)}-5\sqrt{a}e^{(fx+e)}}{a\left(e^{(2fx+2e)}+1\right)^4}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*arctan(e^(f*x + e))/sqrt(a) - (5*sqrt(a)*e^(7*f*x + 7*e) - 3*sqrt(a)
*e^(5*f*x + 5*e) + 3*sqrt(a)*e^(3*f*x + 3*e) - 5*sqrt(a)*e^(f*x + e))/(a*(e
^(2*f*x + 2*e) + 1)^4))/f
```

maple [A] time = 0.24, size = 68, normalized size = 0.75

$$\frac{3\arctan\left(\sinh\left(fx+e\right)\right)\left(\cosh^4\left(fx+e\right)\right)-5\left(\cosh^2\left(fx+e\right)\right)\sinh\left(fx+e\right)+2\sinh\left(fx+e\right)}{8\cosh\left(fx+e\right)^3\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8*(3*arctan(sinh(f*x+e))*cosh(f*x+e)^4-5*cosh(f*x+e)^2*sinh(f*x+e)+2*sinh
(f*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.72, size = 628, normalized size = 6.90

$$\frac{\frac{15\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}-\frac{15e^{(-fx-e)}+55e^{(-3fx-3e)}+73e^{(-5fx-5e)}-15e^{(-7fx-7e)}}{4\sqrt{a}e^{(-2fx-2e)}+6\sqrt{a}e^{(-4fx-4e)}+4\sqrt{a}e^{(-6fx-6e)}+\sqrt{a}e^{(-8fx-8e)}+\sqrt{a}}}{48f}+\frac{\frac{15\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}-\frac{15e^{(-fx-e)}-73e^{(-\cdots}}{4\sqrt{a}e^{(-2fx-2e)}+6\sqrt{a}e^{(-4\cdots}}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/48*(15*arctan(e^(-f*x - e))/sqrt(a) - (15*e^(-f*x - e) + 55*e^(-3*f*x - 3
*e) + 73*e^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e))/(4*sqrt(a)*e^(-2*f*x - 2*e
) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a)*e^(-8
*f*x - 8*e) + sqrt(a)))/f + 1/48*(15*arctan(e^(-f*x - e))/sqrt(a) - (15*e^(
-f*x - e) - 73*e^(-3*f*x - 3*e) - 55*e^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e)
```

```
)/(4*sqrt(a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-
6*f*x - 6*e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)))/f - 3/32*(3*arctan(e^(-
f*x - e))/sqrt(a) - (3*e^(-f*x - e) + 11*e^(-3*f*x - 3*e) - 11*e^(-5*f*x -
5*e) - 3*e^(-7*f*x - 7*e))/(4*sqrt(a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*
x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))
)/f - 35/32*arctan(e^(-f*x - e))/(sqrt(a)*f) - 1/192*(279*e^(-f*x - e) + 51
1*e^(-3*f*x - 3*e) + 385*e^(-5*f*x - 5*e) + 105*e^(-7*f*x - 7*e))/((4*sqrt(
a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*
e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))*f) + 1/192*(105*e^(-f*x - e) + 385
*e^(-3*f*x - 3*e) + 511*e^(-5*f*x - 5*e) + 279*e^(-7*f*x - 7*e))/((4*sqrt(a
)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e
) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh\left(e+fx\right)^4}{\sqrt{a\sinh\left(e+fx\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(1/2),x)
[Out] int(tanh(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4\left(e+fx\right)}{\sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(tanh(e + f*x)**4/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

3.443 $\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{2f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] 1/2*arctan(sinh(f*x+e))*cosh(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-1/2*tanh(f*x+
e)/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3207, 2611, 3770}

$$\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{2f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(2*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[
e + f*x]/(2*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\tanh^2(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}} + \frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\,dx}{2\sqrt{a\cosh^2(e+fx)}}$$
$$= \frac{\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}} - \frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 0.71

$$\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx)) - \tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] - Tanh[e + f*x])/(2*f*Sqrt[a*Cosh[e +
f*x]^2])
```

fricas [B] time = 0.58, size = 504, normalized size = 8.13

$$-\frac{\left(3\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)^2 + e^{\left(fx+e\right)}\sinh\left(fx+e\right)^3 + \left(3\cosh\left(fx+e\right)^2 - 1\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right.\right.}{af\cosh\left(fx+e\right)^4 + \left(afe^{\left(2fx+2e\right)} + af\right)\sinh\left(fx+e\right)^4 + 2af\cosh\left(fx+e\right)^2 + 4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x + e)^3
 + (3*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e) - (4*cosh(f*x + e)*e^(
f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)
^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*e
^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 + 2*cosh(f*x + e)^2 + 1)*e^(f*x
 + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^3 - cosh(f*x
+ e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*
x - e)/(a*f*cosh(f*x + e)^4 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^4 +
 2*a*f*cosh(f*x + e)^2 + 4*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*
x + e))*sinh(f*x + e)^3 + 2*(3*a*f*cosh(f*x + e)^2 + a*f + (3*a*f*cosh(f*x
+ e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^4
 + 2*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 4*(a*f*cosh(f*x + e)^3 +
a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e))
```

giac [A] time = 0.27, size = 63, normalized size = 1.02

$$\frac{\frac{\arctan\left(e^{\left(fx+e\right)}\right)}{\sqrt{a}} - \frac{\sqrt{a}e^{\left(3fx+3e\right)} - \sqrt{a}e^{\left(fx+e\right)}}{a\left(e^{\left(2fx+2e\right)}+1\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (arctan(e^(f*x + e))/sqrt(a) - (sqrt(a)*e^(3*f*x + 3*e) - sqrt(a)*e^(f*x +
e))/(a*(e^(2*f*x + 2*e) + 1)^2))/f
```

maple [A] time = 0.22, size = 51, normalized size = 0.82

$$\frac{\frac{\arctan\left(\sinh\left(fx+e\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}{2}-\frac{\sinh\left(fx+e\right)}{2}}{\cosh\left(fx+e\right)\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] (1/2*arctan(sinh(f*x+e))*cosh(f*x+e)^2-1/2*sinh(f*x+e))/cosh(f*x+e)/(a*cosh
(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.49, size = 217, normalized size = 3.50

$$\frac{\frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}-\frac{e^{(-fx-e)}-e^{(-3fx-3e)}}{2\sqrt{a}e^{(-2fx-2e)}+\sqrt{a}e^{(-4fx-4e)}+\sqrt{a}}}{2f}-\frac{3\arctan\left(e^{(-fx-e)}\right)}{2\sqrt{a}f}-\frac{5e^{(-fx-e)}+3e^{(-3fx-3e)}}{4\left(2\sqrt{a}e^{(-2fx-2e)}+\sqrt{a}e^{(-4fx-4e)}+\sqrt{a}\right)f}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(arctan(e^(-f*x - e))/sqrt(a) - (e^(-f*x - e) - e^(-3*f*x - 3*e))/(2*sq
rt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a)))/f - 3/2*arcta
n(e^(-f*x - e))/(sqrt(a)*f) - 1/4*(5*e^(-f*x - e) + 3*e^(-3*f*x - 3*e))/((2
*sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a))*f) + 1/4*(3
*e^(-f*x - e) + 5*e^(-3*f*x - 3*e))/((2*sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*
e^(-4*f*x - 4*e) + sqrt(a))*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh\left(e+f\,x\right)^2}{\sqrt{a\,\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{2}\left(e+fx\right)}{\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

3.444 $\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=25

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -coth(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3176, 3207, 2606, 8}

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\cosh(e+fx)\int \coth(e+fx)\text{csch}(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{(i\cosh(e+fx))\,\text{Subst}(\int 1\,dx, x, -i\text{csch}(e+fx))}{f\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.04, size = 25, normalized size = 1.00

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

fricas [B] time = 0.51, size = 170, normalized size = 6.80

$$-\frac{2\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\left(\cosh\left(fx+e\right)e^{(fx+e)}+e^{(fx+e)}\sinh\left(fx\right.\right.}{af\cosh\left(fx+e\right)^2+\left(afe^{(2fx+2e)}+af\right)\sinh\left(fx+e\right)^2-af+\left(af\cosh\left(fx+e\right)^2-af\right)e^{(2fx+2e)}+2\left(af\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(cosh(f*x + e)*e^(f*x
+ e) + e^(f*x + e)*sinh(f*x + e))*e^(-f*x - e)/(a*f*cosh(f*x + e)^2 + (a*f*
e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^2 - a*f + (a*f*cosh(f*x + e)^2 - a*f)*
e^(2*f*x + 2*e) + 2*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))
*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^3*exp(exp(1))^3+t_nostep*exp(exp(1)))]index.cc index_m operator + Err
or: Bad Argument Value
```

maple [A] time = 0.16, size = 32, normalized size = 1.28

$$-\frac{\cosh\left(fx+e\right)}{\sinh\left(fx+e\right)\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -cosh(f*x+e)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.53, size = 101, normalized size = 4.04

$$\frac{\frac{\arctan\left(e^{\left(-fx-e\right)}\right)}{\sqrt{a}} + \frac{\sqrt{a}\,e^{\left(-fx-e\right)}}{ae^{\left(-2\,fx-2\,e\right)}-a}}{f} - \frac{\arctan\left(e^{\left(-fx-e\right)}\right)}{\sqrt{a}\,f} + \frac{\sqrt{a}\,e^{\left(-fx-e\right)}}{\left(ae^{\left(-2\,fx-2\,e\right)} - a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (arctan(e^(-f*x - e))/sqrt(a) + sqrt(a)*e^(-f*x - e)/(a*e^(-2*f*x - 2*e) -
a))/f - arctan(e^(-f*x - e))/(sqrt(a)*f) + sqrt(a)*e^(-f*x - e)/((a*e^(-2*f
*x - 2*e) - a)*f)
```

mupad [B] time = 0.11, size = 76, normalized size = 3.04

$$-\frac{4\,\mathrm{e}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{a\,f\,\left(\mathrm{e}^{2\,e+2\,f\,x}-1\right)\,\left(\mathrm{e}^{e+f\,x}+\mathrm{e}^{3\,e+3\,f\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] -(4*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(
a*f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}\left(e + fx\right)}{\sqrt{a\left(\sinh^{2}\left(e + fx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**2/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

3.445 $\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=61

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -coth(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-1/3*coth(f*x+e)*csch(f*x+e)^2/f/(a*c
osh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3176, 3207, 2606}

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (Coth[e + f*x]*Csch[e + f*x]
^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth^4(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\cosh(e+fx)\int \coth^3(e+fx)\text{csch}(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}$$
$$= \frac{(i\cosh(e+fx))\,\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, -i\text{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)\text{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 0.61

$$-\frac{\coth(e+fx)\left(\text{csch}^2(e+fx)+3\right)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
[Out] -1/3*(Coth[e + f*x]*(3 + Csch[e + f*x]^2))/(f*Sqrt[a*Cosh[e + f*x]^2])
```

fricas [B] time = 0.43, size = 647, normalized size = 10.61

$$-\frac{}{3\left(af\cosh\left(fx+e\right)^6+\left(afe^{(2fx+2e)}+af\right)\sinh\left(fx+e\right)^6-3\,af\cosh\left(fx+e\right)^4+6\left(af\cosh\left(fx+e\right)e^{(2fx+}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] -2/3*(15*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + 3*e^(f*x + e)*sinh(f*x
 + e)^5 + 2*(15*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 6*(5*cos
h(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 3*(5*cosh(f*x +
 e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (3*cosh(f*x + e)
^5 - 2*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*
e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^6 + (a*f*e^(2
*f*x + 2*e) + a*f)*sinh(f*x + e)^6 - 3*a*f*cosh(f*x + e)^4 + 6*(a*f*cosh(f*
x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 3*(5*a*f*cosh
(f*x + e)^2 - a*f + (5*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^4 + 3*a*f*cosh(f*x + e)^2 + 4*(5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x
 + e) + (5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh
(f*x + e)^3 + 3*(5*a*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f + (5*a
*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^2 - a*f + (a*f*cosh(f*x + e)^6 - 3*a*f*cosh(f*x + e)^4 + 3*a*f*cosh(f
*x + e)^2 - a*f)*e^(2*f*x + 2*e) + 6*(a*f*cosh(f*x + e)^5 - 2*a*f*cosh(f*x
+ e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^5 - 2*a*f*cosh(f*x + e)^3 +
 a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^3*exp(exp(1))^3+t_nostep*exp(exp(1)))]index.cc index_m operator + Err
or: Bad Argument Value
```

maple [A] time = 0.21, size = 44, normalized size = 0.72

$$-\frac{\cosh\left(fx+e\right)\left(3\left(\sinh^{2}\left(fx+e\right)\right)+1\right)}{3\sinh\left(fx+e\right)^{3}\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*cosh(f*x+e)*(3*sinh(f*x+e)^2+1)/sinh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/
f
```

maxima [B] time = 0.73, size = 556, normalized size = 9.11

$$\frac{\frac{6\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}+\frac{3\log\left(e^{(-fx-e)}+1\right)}{\sqrt{a}}-\frac{3\log\left(e^{(-fx-e)}-1\right)}{\sqrt{a}}+\frac{4\left(3\sqrt{a}e^{(-fx-e)}-\sqrt{a}e^{(-3fx-3e)}\right)}{3ae^{(-2fx-2e)}-3ae^{(-4fx-4e)}+ae^{(-6fx-6e)}-a}}{12f}+\frac{\frac{6\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}}-\frac{3\log\left(e^{(-f}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*arctan(e^(-f*x - e))/sqrt(a) + 3*log(e^(-f*x - e) + 1)/sqrt(a) - 3*
log(e^(-f*x - e) - 1)/sqrt(a) + 4*(3*sqrt(a)*e^(-f*x - e) - sqrt(a)*e^(-3*f
*x - 3*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e
) - a))/f + 1/12*(6*arctan(e^(-f*x - e))/sqrt(a) - 3*log(e^(-f*x - e) + 1)/
sqrt(a) + 3*log(e^(-f*x - e) - 1)/sqrt(a) - 4*(sqrt(a)*e^(-3*f*x - 3*e) - 3
*sqrt(a)*e^(-5*f*x - 5*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a
*e^(-6*f*x - 6*e) - a))/f - 1/4*(3*arctan(e^(-f*x - e))/sqrt(a) + (3*sqrt(a
)*e^(-f*x - e) - 10*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-5*f*x - 5*e))/
(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a))/f -
 1/4*arctan(e^(-f*x - e))/(sqrt(a)*f) + 1/24*(27*sqrt(a)*e^(-f*x - e) - 38*
sqrt(a)*e^(-3*f*x - 3*e) + 15*sqrt(a)*e^(-5*f*x - 5*e))/((3*a*e^(-2*f*x - 2
*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a)*f) + 1/24*(15*sqrt(a)*
e^(-f*x - e) - 38*sqrt(a)*e^(-3*f*x - 3*e) + 27*sqrt(a)*e^(-5*f*x - 5*e))/(
(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a)*f)
```

mupad [B] time = 0.89, size = 95, normalized size = 1.56

$$-\frac{4\,e^{2\,e+2\,f\,x}\,\sqrt{a+a\,{\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)}^2}\,\left(3\,e^{4\,e+4\,f\,x}-2\,e^{2\,e+2\,f\,x}+3\right)}{3\,a\,f\,{\left(e^{2\,e+2\,f\,x}-1\right)}^3\,\left(e^{2\,e+2\,f\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] -(4*exp(2*e + 2*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2)*(3
*exp(4*e + 4*f*x) - 2*exp(2*e + 2*f*x) + 3))/(3*a*f*(exp(2*e + 2*f*x) - 1)^
3*(exp(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(e+fx)}{\sqrt{a(\sinh^2(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(coth(e + f*x)**4/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

3.446 $\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=96

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5f\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -coth(f*x+e)/f/(a*cosh(f*x+e)^2)^(1/2)-2/3*coth(f*x+e)*csch(f*x+e)^2/f/(a*c
osh(f*x+e)^2)^(1/2)-1/5*coth(f*x+e)*csch(f*x+e)^4/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3207, 2606, 194}

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5f\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^6/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (2*Coth[e + f*x]*Csch[e + f*
x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x]^4)/(5*f*
Sqrt[a*Cosh[e + f*x]^2])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth^6(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\cosh(e+fx)\int \coth^5(e+fx)\operatorname{csch}(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{(i\cosh(e+fx))\operatorname{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, -i\operatorname{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{(i\cosh(e+fx))\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -i\operatorname{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}} - \frac{2\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^4(e+}{5f\sqrt{a\cosh^2(e+fx}$$

Mathematica [**A**] time = 0.09, size = 49, normalized size = 0.51

$$-\frac{\coth(e+fx)\left(3\operatorname{csch}^4(e+fx)+10\operatorname{csch}^2(e+fx)+15\right)}{15f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^6/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -1/15*(Coth[e + f*x]*(15 + 10*Csch[e + f*x]^2 + 3*Csch[e + f*x]^4))/(f*Sqrt
[a*Cosh[e + f*x]^2])
```

fricas [**B**] time = 0.53, size = 1399, normalized size = 14.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(135*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + 15*e^(f*x + e)*sinh(
f*x + e)^9 + 20*(27*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^7 + 140*
(9*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 2*(945*co
sh(f*x + e)^4 - 210*cosh(f*x + e)^2 + 29)*e^(f*x + e)*sinh(f*x + e)^5 + 10*
(189*cosh(f*x + e)^5 - 70*cosh(f*x + e)^3 + 29*cosh(f*x + e))*e^(f*x + e)*s
inh(f*x + e)^4 + 20*(63*cosh(f*x + e)^6 - 35*cosh(f*x + e)^4 + 29*cosh(f*x
+ e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 20*(27*cosh(f*x + e)^7 - 21*cosh(
f*x + e)^5 + 29*cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(27*cosh(f*x + e)^8 - 28*cosh(f*x + e)^6 + 58*cosh(f*x + e)^4 - 12*
cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e) + (15*cosh(f*x + e)^9 - 20*c
osh(f*x + e)^7 + 58*cosh(f*x + e)^5 - 20*cosh(f*x + e)^3 + 15*cosh(f*x + e)
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a*f*cosh(f*x + e)^10 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^10 - 5
*a*f*cosh(f*x + e)^8 + 10*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x
 + e))*sinh(f*x + e)^9 + 5*(9*a*f*cosh(f*x + e)^2 - a*f + (9*a*f*cosh(f*x +
 e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 + 40
*(3*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e)^3 - a*f*
cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^7 + 10*(21*a*f*cosh(f*x + e)^
4 - 14*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)^4 - 14*a*f*cosh(f*
```

```
x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 - 10*a*f*cosh(f*x + e)^4 +
 4*(63*a*f*cosh(f*x + e)^5 - 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e)
+ (63*a*f*cosh(f*x + e)^5 - 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e))*
e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 10*(21*a*f*cosh(f*x + e)^6 - 35*a*f*cosh
(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 - a*f + (21*a*f*cosh(f*x + e)^6 - 35*a
*f*cosh(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*
x + e)^4 + 5*a*f*cosh(f*x + e)^2 + 40*(3*a*f*cosh(f*x + e)^7 - 7*a*f*cosh(f
*x + e)^5 + 5*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e
)^7 - 7*a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e))*e^
(2*f*x + 2*e))*sinh(f*x + e)^3 + 5*(9*a*f*cosh(f*x + e)^8 - 28*a*f*cosh(f*x
 + e)^6 + 30*a*f*cosh(f*x + e)^4 - 12*a*f*cosh(f*x + e)^2 + a*f + (9*a*f*co
sh(f*x + e)^8 - 28*a*f*cosh(f*x + e)^6 + 30*a*f*cosh(f*x + e)^4 - 12*a*f*co
sh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 - a*f + (a*f*cosh(f*x
 + e)^10 - 5*a*f*cosh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 - 10*a*f*cosh(f*x
 + e)^4 + 5*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e) + 10*(a*f*cosh(f*x +
 e)^9 - 4*a*f*cosh(f*x + e)^7 + 6*a*f*cosh(f*x + e)^5 - 4*a*f*cosh(f*x + e)
^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^9 - 4*a*f*cosh(f*x + e)^7 + 6*a
*f*cosh(f*x + e)^5 - 4*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^3*exp(exp(1))^3+t_nostep*exp(exp(1)))]index.cc index_m operator + Err
or: Bad Argument Value
```

maple [A] time = 0.25, size = 54, normalized size = 0.56

$$-\frac{\cosh\left(fx+e\right)\left(15\left(\sinh^{4}\left(fx+e\right)\right)+10\left(\sinh^{2}\left(fx+e\right)\right)+3\right)}{15\sinh\left(fx+e\right)^{5}\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15*cosh(f*x+e)*(15*sinh(f*x+e)^4+10*sinh(f*x+e)^2+3)/sinh(f*x+e)^5/(a*co
sh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.65, size = 1231, normalized size = 12.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/256*(120*arctan(e^(-f*x - e))/sqrt(a) + 45*log(e^(-f*x - e) + 1)/sqrt(a)
 - 45*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(105*sqrt(a)*e^(-f*x - e) - 530*sqr
t(a)*e^(-3*f*x - 3*e) + 328*sqrt(a)*e^(-5*f*x - 5*e) - 110*sqrt(a)*e^(-7*f*
x - 7*e) + 15*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*
f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x -
10*e) - a))/f - 1/256*(120*arctan(e^(-f*x - e))/sqrt(a) - 45*log(e^(-f*x -
e) + 1)/sqrt(a) + 45*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(15*sqrt(a)*e^(-f*x
```

```
- e) - 110*sqrt(a)*e^(-3*f*x - 3*e) + 328*sqrt(a)*e^(-5*f*x - 5*e) - 530*sq
rt(a)*e^(-7*f*x - 7*e) + 105*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e
) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) +
a*e^(-10*f*x - 10*e) - a))/f + 1/320*(60*arctan(e^(-f*x - e))/sqrt(a) + 75*
log(e^(-f*x - e) + 1)/sqrt(a) - 75*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(105*s
qrt(a)*e^(-f*x - e) + 130*sqrt(a)*e^(-3*f*x - 3*e) - 284*sqrt(a)*e^(-5*f*x
- 5*e) + 190*sqrt(a)*e^(-7*f*x - 7*e) - 45*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e
^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8
*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f + 1/320*(60*arctan(e^(-f*x - e))
/sqrt(a) - 75*log(e^(-f*x - e) + 1)/sqrt(a) + 75*log(e^(-f*x - e) - 1)/sqrt
(a) - 2*(45*sqrt(a)*e^(-f*x - e) - 190*sqrt(a)*e^(-3*f*x - 3*e) + 284*sqrt(
a)*e^(-5*f*x - 5*e) - 130*sqrt(a)*e^(-7*f*x - 7*e) - 105*sqrt(a)*e^(-9*f*x
- 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*
e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f + 1/24*(15*arctan(
e^(-f*x - e))/sqrt(a) + (15*sqrt(a)*e^(-f*x - e) - 80*sqrt(a)*e^(-3*f*x - 3
*e) + 178*sqrt(a)*e^(-5*f*x - 5*e) - 80*sqrt(a)*e^(-7*f*x - 7*e) + 15*sqrt(
a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e
^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f - 1/1
6*arctan(e^(-f*x - e))/(sqrt(a)*f) + 1/1920*(2685*sqrt(a)*e^(-f*x - e) - 73
70*sqrt(a)*e^(-3*f*x - 3*e) + 8632*sqrt(a)*e^(-5*f*x - 5*e) - 4790*sqrt(a)*
e^(-7*f*x - 7*e) + 1035*sqrt(a)*e^(-9*f*x - 9*e))/((5*a*e^(-2*f*x - 2*e) -
10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^
(-10*f*x - 10*e) - a)*f) + 1/1920*(1035*sqrt(a)*e^(-f*x - e) - 4790*sqrt(a)
*e^(-3*f*x - 3*e) + 8632*sqrt(a)*e^(-5*f*x - 5*e) - 7370*sqrt(a)*e^(-7*f*x
- 7*e) + 2685*sqrt(a)*e^(-9*f*x - 9*e))/((5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4
*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x -
 10*e) - a)*f)
```

mupad [B] time = 0.90, size = 381, normalized size = 3.97

$$-\frac{4\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{a\,f\,\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{32\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\frac{{\mathrm{e}}^{-e-f\,x}}{2}\right)}^2}}{3\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^2\,\left({\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}\right)}-\frac{352\,{\mathrm{e}}^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{e+f\,x}}{2}-\right.}}{15\,a\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^3\,\left({\mathrm{e}}^{e+f\,x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^6/(a + a*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] - (4*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(a*f*(exp(2*e + 2*f*x) - 1)*(exp(e + f*x) + exp(3*e + 3*f*x))) - (32*exp(3*
e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*a*f*(exp
(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (352*exp(3*e + 3*
f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(15*a*f*(exp(2*e
+ 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (128*exp(3*e + 3*f*x)*
(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e + 2*f*
x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*exp(3*e + 3*f*x)*(a + a*
(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a*f*(exp(2*e + 2*f*x) - 1)
^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{6}{\left(e + f x \right)}}{\sqrt{a \left(\sinh^{2}{\left(e + f x \right)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**6/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**6/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

3.447 $\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{a^2}{7f\left(a\cosh^2(e+fx)\right)^{7/2}}+\frac{2a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

```
[Out] -1/7*a^2/f/(a*cosh(f*x+e)^2)^(7/2)+2/5*a/f/(a*cosh(f*x+e)^2)^(5/2)-1/3/f/(a
*cosh(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.14, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3205, 16, 43}

$$-\frac{a^2}{7f\left(a\cosh^2(e+fx)\right)^{7/2}}+\frac{2a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -a^2/(7*f*(a*Cosh[e + f*x]^2)^(7/2)) + (2*a)/(5*f*(a*Cosh[e + f*x]^2)^(5/2)
) - 1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tanh^5(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{9/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{9/2}} - \frac{2}{a(ax)^{7/2}} + \frac{1}{a^2(ax)^{5/2}}\right)dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2}{7f\left(a\cosh^2(e+fx)\right)^{7/2}} + \frac{2a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}} - \frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.12, size = 51, normalized size = 0.75

$$\frac{\left(-35\cosh^4(e+fx)+42\cosh^2(e+fx)-15\right)\operatorname{sech}^4(e+fx)}{105f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-15 + 42*Cosh[e + f*x]^2 - 35*Cosh[e + f*x]^4)*Sech[e + f*x]^4)/(105*f*(a
*Cosh[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.56, size = 2507, normalized size = 36.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/105*(385*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^10 + 35*e^(f*x + e)*sin
h(f*x + e)^11 + 7*(275*cosh(f*x + e)^2 - 4)*e^(f*x + e)*sinh(f*x + e)^9 + 2
1*(275*cosh(f*x + e)^3 - 12*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^8 + 6*
(1925*cosh(f*x + e)^4 - 168*cosh(f*x + e)^2 + 19)*e^(f*x + e)*sinh(f*x + e)
^7 + 42*(385*cosh(f*x + e)^5 - 56*cosh(f*x + e)^3 + 19*cosh(f*x + e))*e^(f*
x + e)*sinh(f*x + e)^6 + 14*(1155*cosh(f*x + e)^6 - 252*cosh(f*x + e)^4 + 1
71*cosh(f*x + e)^2 - 2)*e^(f*x + e)*sinh(f*x + e)^5 + 14*(825*cosh(f*x + e)
^7 - 252*cosh(f*x + e)^5 + 285*cosh(f*x + e)^3 - 10*cosh(f*x + e))*e^(f*x +
 e)*sinh(f*x + e)^4 + 7*(825*cosh(f*x + e)^8 - 336*cosh(f*x + e)^6 + 570*co
sh(f*x + e)^4 - 40*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^3 + 7*(27
5*cosh(f*x + e)^9 - 144*cosh(f*x + e)^7 + 342*cosh(f*x + e)^5 - 40*cosh(f*x
 + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 7*(55*cosh(f*x +
e)^10 - 36*cosh(f*x + e)^8 + 114*cosh(f*x + e)^6 - 20*cosh(f*x + e)^4 + 15*
cosh(f*x + e)^2)*e^(f*x + e)*sinh(f*x + e) + (35*cosh(f*x + e)^11 - 28*cosh
(f*x + e)^9 + 114*cosh(f*x + e)^7 - 28*cosh(f*x + e)^5 + 35*cosh(f*x + e)^3
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a^2*f*cosh(f*x + e)^14 + 7*a^2*f*cosh(f*x + e)^12 + (a^2*f*e^(2*f*x + 2
*e) + a^2*f)*sinh(f*x + e)^14 + 14*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a
^2*f*cosh(f*x + e))*sinh(f*x + e)^13 + 21*a^2*f*cosh(f*x + e)^10 + 7*(13*a^
2*f*cosh(f*x + e)^2 + a^2*f + (13*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x +
 2*e))*sinh(f*x + e)^12 + 28*(13*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x +
```

```
e) + (13*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*s
inh(f*x + e)^11 + 35*a^2*f*cosh(f*x + e)^8 + 7*(143*a^2*f*cosh(f*x + e)^4 +
 66*a^2*f*cosh(f*x + e)^2 + 3*a^2*f + (143*a^2*f*cosh(f*x + e)^4 + 66*a^2*f
*cosh(f*x + e)^2 + 3*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^10 + 14*(143*a^2
*f*cosh(f*x + e)^5 + 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (
143*a^2*f*cosh(f*x + e)^5 + 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x +
 e))*e^(2*f*x + 2*e))*sinh(f*x + e)^9 + 35*a^2*f*cosh(f*x + e)^6 + 7*(429*a
^2*f*cosh(f*x + e)^6 + 495*a^2*f*cosh(f*x + e)^4 + 135*a^2*f*cosh(f*x + e)^
2 + 5*a^2*f + (429*a^2*f*cosh(f*x + e)^6 + 495*a^2*f*cosh(f*x + e)^4 + 135*
a^2*f*cosh(f*x + e)^2 + 5*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 8*(429*
a^2*f*cosh(f*x + e)^7 + 693*a^2*f*cosh(f*x + e)^5 + 315*a^2*f*cosh(f*x + e)
^3 + 35*a^2*f*cosh(f*x + e) + (429*a^2*f*cosh(f*x + e)^7 + 693*a^2*f*cosh(f
*x + e)^5 + 315*a^2*f*cosh(f*x + e)^3 + 35*a^2*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e)^7 + 21*a^2*f*cosh(f*x + e)^4 + 7*(429*a^2*f*cosh(f*x +
e)^8 + 924*a^2*f*cosh(f*x + e)^6 + 630*a^2*f*cosh(f*x + e)^4 + 140*a^2*f*co
sh(f*x + e)^2 + 5*a^2*f + (429*a^2*f*cosh(f*x + e)^8 + 924*a^2*f*cosh(f*x +
 e)^6 + 630*a^2*f*cosh(f*x + e)^4 + 140*a^2*f*cosh(f*x + e)^2 + 5*a^2*f)*e^
(2*f*x + 2*e))*sinh(f*x + e)^6 + 14*(143*a^2*f*cosh(f*x + e)^9 + 396*a^2*f*
cosh(f*x + e)^7 + 378*a^2*f*cosh(f*x + e)^5 + 140*a^2*f*cosh(f*x + e)^3 + 1
5*a^2*f*cosh(f*x + e) + (143*a^2*f*cosh(f*x + e)^9 + 396*a^2*f*cosh(f*x + e
)^7 + 378*a^2*f*cosh(f*x + e)^5 + 140*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh
(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 7*a^2*f*cosh(f*x + e)^2 + 7*(
143*a^2*f*cosh(f*x + e)^10 + 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*x
 + e)^6 + 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 + 3*a^2*f +
(143*a^2*f*cosh(f*x + e)^10 + 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*
x + e)^6 + 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 + 3*a^2*f)*
e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 28*(13*a^2*f*cosh(f*x + e)^11 + 55*a^2*f
*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 + 70*a^2*f*cosh(f*x + e)^5 + 25
*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e) + (13*a^2*f*cosh(f*x + e)^11
 + 55*a^2*f*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 + 70*a^2*f*cosh(f*x
+ e)^5 + 25*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^3 + a^2*f + 7*(13*a^2*f*cosh(f*x + e)^12 + 66*a^2*f*cosh(f*x
 + e)^10 + 135*a^2*f*cosh(f*x + e)^8 + 140*a^2*f*cosh(f*x + e)^6 + 75*a^2*f
*cosh(f*x + e)^4 + 18*a^2*f*cosh(f*x + e)^2 + a^2*f + (13*a^2*f*cosh(f*x +
e)^12 + 66*a^2*f*cosh(f*x + e)^10 + 135*a^2*f*cosh(f*x + e)^8 + 140*a^2*f*c
osh(f*x + e)^6 + 75*a^2*f*cosh(f*x + e)^4 + 18*a^2*f*cosh(f*x + e)^2 + a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^14 + 7*a^2*f*cos
h(f*x + e)^12 + 21*a^2*f*cosh(f*x + e)^10 + 35*a^2*f*cosh(f*x + e)^8 + 35*a
^2*f*cosh(f*x + e)^6 + 21*a^2*f*cosh(f*x + e)^4 + 7*a^2*f*cosh(f*x + e)^2 +
 a^2*f)*e^(2*f*x + 2*e) + 14*(a^2*f*cosh(f*x + e)^13 + 6*a^2*f*cosh(f*x + e
)^11 + 15*a^2*f*cosh(f*x + e)^9 + 20*a^2*f*cosh(f*x + e)^7 + 15*a^2*f*cosh(
f*x + e)^5 + 6*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*
x + e)^13 + 6*a^2*f*cosh(f*x + e)^11 + 15*a^2*f*cosh(f*x + e)^9 + 20*a^2*f*
cosh(f*x + e)^7 + 15*a^2*f*cosh(f*x + e)^5 + 6*a^2*f*cosh(f*x + e)^3 + a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

giac [A] time = 0.54, size = 98, normalized size = 1.44

$$-\frac{8\left(35\sqrt{a}e^{\left(11fx+11e\right)}-28\sqrt{a}e^{\left(9fx+9e\right)}+114\sqrt{a}e^{\left(7fx+7e\right)}-28\sqrt{a}e^{\left(5fx+5e\right)}+35\sqrt{a}e^{\left(3fx+3e\right)}\right)}{105a^2f\left(e^{\left(2fx+2e\right)}+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] -8/105*(35*sqrt(a)*e^(11*f*x + 11*e) - 28*sqrt(a)*e^(9*f*x + 9*e) + 114*sqr
t(a)*e^(7*f*x + 7*e) - 28*sqrt(a)*e^(5*f*x + 5*e) + 35*sqrt(a)*e^(3*f*x + 3
*e))/(a^2*f*(e^(2*f*x + 2*e) + 1)^7)
```

maple [C] time = 0.29, size = 44, normalized size = 0.65

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\sinh^5(fx+e)}{\cosh(fx+e)^8 a\sqrt{a(\cosh^2(fx+e))}}, \sinh(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5/cosh(f*x+e)^8/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

maxima [B] time = 0.70, size = 586, normalized size = 8.62

$$-\frac{8\,e^{(-3\,fx-3\,e)}}{3\left(7\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+21\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+35\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+35\,a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+21\,a^{\frac{3}{2}}e^{(-10\,fx-10\,e)}+7\,a^{\frac{3}{2}}e^{(-12\,fx-12\,e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*f*x - 3*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x -
 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3
/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x
- 14*e) + a^(3/2))*f) + 32/15*e^(-5*f*x - 5*e)/((7*a^(3/2)*e^(-2*f*x - 2*e)
 + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e
^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12
*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) - 304/35*e^(-7*f*x - 7*e)/((
7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6
*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) +
 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) +
32/15*e^(-9*f*x - 9*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x
- 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(
3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x
 - 14*e) + a^(3/2))*f) - 8/3*e^(-11*f*x - 11*e)/((7*a^(3/2)*e^(-2*f*x - 2*e
) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*
e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 1
2*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f)
```

mupad [B] time = 0.16, size = 457, normalized size = 6.72

$$\frac{464\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{15\,a^2\,f\left(e^{2\,e+2\,f\,x}+1\right)^3\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}-\frac{16\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)^2}}{3\,a^2\,f\left(e^{2\,e+2\,f\,x}+1\right)^2\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}-\frac{3072\,e^{3\,e+3\,f\,x}\sqrt{a+a\left(\frac{e}{}\right.}}{35\,a^2\,f\left(e^{2\,e+2\,f\,x}+1\right)^4\left(e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] (464*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(15*a^2*f*(exp(2*e + 2*f*x) + 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (16
*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*a
^2*f*(exp(2*e + 2*f*x) + 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (3072*ex
p(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(35*a^2
*f*(exp(2*e + 2*f*x) + 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) + (4736*exp(
3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(35*a^2*f
*(exp(2*e + 2*f*x) + 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x))) - (768*exp(3*e
 + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*a^2*f*(ex
```

```
p(2*e + 2*f*x) + 1)^6*(exp(e + f*x) + exp(3*e + 3*f*x))) + (256*exp(3*e + 3
*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*a^2*f*(exp(2*
e + 2*f*x) + 1)^7*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

3.448 $\int \frac{\tanh^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=44

$$\frac{a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/5*a/f/(a*cosh(f*x+e)^2)^(5/2)-1/3/f/(a*cosh(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] a/(5*f*(a*Cosh[e + f*x]^2)^(5/2)) - 1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tanh^3(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{7/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}} - \frac{1}{a(ax)^{5/2}}\right)dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}} - \frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.13, size = 34, normalized size = 0.77

$$\frac{a\left(3-5\cosh^2(e+fx)\right)}{15f\left(a\cosh^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
[Out] (a*(3 - 5*Cosh[e + f*x]^2))/(15*f*(a*Cosh[e + f*x]^2)^(5/2))
```

fricas [B] time = 0.45, size = 1400, normalized size = 31.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] -8/15*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*
x + e)^7 + (105*cosh(f*x + e)^2 - 2)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*co
sh(f*x + e)^3 - 2*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 5*(35*cosh(f
*x + e)^4 - 4*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 5*(21*cosh
(f*x + e)^5 - 4*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(7*cosh(f*x + e)^6 - 2*cosh(f*x + e)^4 + 3*cosh(f*x + e)^2)*e^(f*x
+ e)*sinh(f*x + e) + (5*cosh(f*x + e)^7 - 2*cosh(f*x + e)^5 + 5*cosh(f*x +
e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*
x - e)/(a^2*f*cosh(f*x + e)^10 + 5*a^2*f*cosh(f*x + e)^8 + (a^2*f*e^(2*f*x
+ 2*e) + a^2*f)*sinh(f*x + e)^10 + 10*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e)
+ a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^6 + 5*(9*a^
2*f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x +
2*e))*sinh(f*x + e)^8 + 40*(3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) +
 (3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x
+ e)^7 + 10*a^2*f*cosh(f*x + e)^4 + 10*(21*a^2*f*cosh(f*x + e)^4 + 14*a^2*f
*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*cosh(f*x + e)^4 + 14*a^2*f*cosh(f*x +
e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 4*(63*a^2*f*cosh(f*x + e)^
5 + 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (63*a^2*f*cosh(f*x
+ e)^5 + 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e)
)*sinh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^2 + 10*(21*a^2*f*cosh(f*x + e)^6
+ 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*c
osh(f*x + e)^6 + 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 + a^2*
```

```
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 40*(3*a^2*f*cosh(f*x + e)^7 + 7*a^2*f
*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (3*a^2*f
*cosh(f*x + e)^7 + 7*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 + a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*f + 5*(9*a^2*f*cosh
(f*x + e)^8 + 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f*cosh(f*x + e)^4 + 12*a^2*
f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^8 + 28*a^2*f*cosh(f*x +
e)^6 + 30*a^2*f*cosh(f*x + e)^4 + 12*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*
x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^10 + 5*a^2*f*cosh(f*x + e)
^8 + 10*a^2*f*cosh(f*x + e)^6 + 10*a^2*f*cosh(f*x + e)^4 + 5*a^2*f*cosh(f*x
 + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 10*(a^2*f*cosh(f*x + e)^9 + 4*a^2*f*cosh
(f*x + e)^7 + 6*a^2*f*cosh(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cos
h(f*x + e) + (a^2*f*cosh(f*x + e)^9 + 4*a^2*f*cosh(f*x + e)^7 + 6*a^2*f*cos
h(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e))
```

giac [A] time = 0.44, size = 68, normalized size = 1.55

$$-\frac{8\left(5\sqrt{a}e^{(7fx+7e)}-2\sqrt{a}e^{(5fx+5e)}+5\sqrt{a}e^{(3fx+3e)}\right)}{15a^2f\left(e^{(2fx+2e)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/15*(5*sqrt(a)*e^(7*f*x + 7*e) - 2*sqrt(a)*e^(5*f*x + 5*e) + 5*sqrt(a)*e^
(3*f*x + 3*e))/(a^2*f*(e^(2*f*x + 2*e) + 1)^5)
```

maple [C] time = 0.24, size = 44, normalized size = 1.00

$$\frac{\text{`int/indef0`}\left(\frac{\sinh^3(fx+e)}{\cosh(fx+e)^6a\sqrt{a(\cosh^2(fx+e))}},\sinh(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3/cosh(f*x+e)^6/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

maxima [B] time = 0.71, size = 268, normalized size = 6.09

$$-\frac{8e^{(-3fx-3e)}}{3\left(5a^{\frac{3}{2}}e^{(-2fx-2e)}+10a^{\frac{3}{2}}e^{(-4fx-4e)}+10a^{\frac{3}{2}}e^{(-6fx-6e)}+5a^{\frac{3}{2}}e^{(-8fx-8e)}+a^{\frac{3}{2}}e^{(-10fx-10e)}+a^{\frac{3}{2}}\right)f}+\frac{}{15\left(5a^{\frac{3}{2}}e^{(-2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*f*x - 3*e)/((5*a^(3/2)*e^(-2*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x -
 4*e) + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*
e^(-10*f*x - 10*e) + a^(3/2))*f) + 16/15*e^(-5*f*x - 5*e)/((5*a^(3/2)*e^(-2
*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x - 4*e) + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5
*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) + a^(3/2))*f) - 8/3*
e^(-7*f*x - 7*e)/((5*a^(3/2)*e^(-2*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x - 4*e)
 + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-1
0*f*x - 10*e) + a^(3/2))*f)
```

mupad [B] time = 0.93, size = 305, normalized size = 6.93

$$\frac{272\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)}^2}}{15\,a^2\,f\,{\left(e^{2\,e+2\,f\,x}+1\right)}^3\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}-\frac{16\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left(e^{2\,e+2\,f\,x}+1\right)}^2\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}-\frac{128\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+f\,x}}{2}\right.}}{5\,a^2\,f\,{\left(e^{2\,e+2\,f\,x}+1\right)}^4\,\left(e^{e+f\,x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] (272*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(15*a^2*f*(exp(2*e + 2*f*x) + 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (16
*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(3*a
^2*f*(exp(2*e + 2*f*x) + 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (128*exp
(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a^2*f
*(exp(2*e + 2*f*x) + 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) + (64*exp(3*e
+ 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a^2*f*(exp
(2*e + 2*f*x) + 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

3.449 $\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

```
[Out] -1/3/f/(a*cosh(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3176, 3205, 16, 32}

$$-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tanh(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(ax)^{5/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.04, size = 21, normalized size = 1.00

$$-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
[Out] -1/3*1/(f*(a*Cosh[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.50, size = 608, normalized size = 28.95

$$-\frac{}{3\left(a^2 f\cosh\left(fx+e\right)^6+3\,a^2 f\cosh\left(fx+e\right)^4+\left(a^2 f e^{(2\,fx+2\,e)}+a^2 f\right)\sinh\left(fx+e\right)^6+6\left(a^2 f\cosh\left(fx+e\right)e^{(2\,f}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] -8/3*(cosh(f*x + e)^3*e^(f*x + e) + 3*cosh(f*x + e)^2*e^(f*x + e)*sinh(f*x
+ e) + 3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x +
 e)^3)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*
f*cosh(f*x + e)^6 + 3*a^2*f*cosh(f*x + e)^4 + (a^2*f*e^(2*f*x + 2*e) + a^2*
f)*sinh(f*x + e)^6 + 6*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*
x + e))*sinh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^2 + 3*(5*a^2*f*cosh(f*x + e
)^2 + a^2*f + (5*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x +
 e)^4 + 4*(5*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e) + (5*a^2*f*cosh(
f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*
f + 3*(5*a^2*f*cosh(f*x + e)^4 + 6*a^2*f*cosh(f*x + e)^2 + a^2*f + (5*a^2*f
*cosh(f*x + e)^4 + 6*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f
*x + e)^2 + (a^2*f*cosh(f*x + e)^6 + 3*a^2*f*cosh(f*x + e)^4 + 3*a^2*f*cosh
(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 6*(a^2*f*cosh(f*x + e)^5 + 2*a^2*f*c
osh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^5 + 2*a^2*f*cos
h(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

giac [A] time = 0.30, size = 32, normalized size = 1.52

$$-\frac{8\,e^{(3\,fx+3\,e)}}{3\,a^{\frac{3}{2}} f\left(e^{(2\,fx+2\,e)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/3*e^(3*f*x + 3*e)/(a^(3/2)*f*(e^(2*f*x + 2*e) + 1)^3)
```

maple [A] time = 0.08, size = 20, normalized size = 0.95

$$-\frac{1}{3f\left(a+a\left(\sinh^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3/f/(a+a*sinh(f*x+e)^2)^(3/2)
```

maxima [B] time = 0.60, size = 61, normalized size = 2.90

$$-\frac{8\,e^{\left(-3\,fx-3\,e\right)}}{3\left(3\,a^{\frac{3}{2}}e^{\left(-2\,fx-2\,e\right)}+3\,a^{\frac{3}{2}}e^{\left(-4\,fx-4\,e\right)}+a^{\frac{3}{2}}e^{\left(-6\,fx-6\,e\right)}+a^{\frac{3}{2}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*f*x - 3*e)/((3*a^(3/2)*e^(-2*f*x - 2*e) + 3*a^(3/2)*e^(-4*f*x -
4*e) + a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2))*f)
```

mupad [B] time = 0.88, size = 58, normalized size = 2.76

$$-\frac{16\,\mathrm{e}^{4\,e+4\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left(\mathrm{e}^{2\,e+2\,f\,x}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] -(16*exp(4*e + 4*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(3*a^2*f*(exp(2*e + 2*f*x) + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

3.450 $\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[Out] -arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a*cosh(f*x+e)^2)
^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3176, 3205, 51, 63, 206}

$$\frac{1}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a*Cos
h[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)(ax)^{3/2}}\,dx,x,\cosh^2(e+fx)\right)}{2f}\\
&= \frac{1}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx,x,\cosh^2(e+fx)\right)}{2af}\\
&= \frac{1}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a\cosh^2(e+fx)}\right)}{a^2f}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}+\frac{1}{af\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 41, normalized size = 0.77

$$\frac{\cosh(e+fx)\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)+1}{af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (1 + Cosh[e + f*x]*Log[Tanh[(e + f*x)/2]])/(a*f*Sqrt[a*Cosh[e + f*x]^2])
```

fricas [B] time = 0.47, size = 271, normalized size = 5.11

$$\frac{\sqrt{ae^{(4fx+4e)}+2\,ae^{(2fx+2e)}+a}\left(2\cosh\left(fx+e\right)e^{(fx+e)}+\left(2\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\right.\right.}{a^2f\cosh\left(fx+e\right)^2+a^2f+\left(a^2fe^{(2fx+2e)}+a^2f\right)\sinh\left(fx+e\right)^2+\left(a^2f\cosh\left(fx+e\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(2*cosh(f*x + e)*e^(f*x +
 e) + (2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e
)^2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e)
 - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)) + 2*e^(f*x + e)*sinh(f*x + e))*e
^(-f*x - e)/(a^2*f*cosh(f*x + e)^2 + a^2*f + (a^2*f*e^(2*f*x + 2*e) + a^2*f
)*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 2*(a^
2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^3*exp(exp(1))^3+t_nostep*exp(exp(1)))]index.cc index_m operator + Err
or: Bad Argument Value
```

maple [C] time = 0.16, size = 44, normalized size = 0.83

$$\frac{\text{`}int/indef0\text{`}\left(\frac{1}{\cosh\left(fx+e\right)^{2}\sinh\left(fx+e\right)a\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^2/sinh(f*x+e)/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

maxima [A] time = 1.02, size = 76, normalized size = 1.43

$$\frac{2\sqrt{a}e^{\left(-fx-e\right)}}{\left(a^{2}e^{\left(-2fx-2e\right)}+a^{2}\right)f}-\frac{\log\left(e^{\left(-fx-e\right)}+1\right)}{a^{\frac{3}{2}}f}+\frac{\log\left(e^{\left(-fx-e\right)}-1\right)}{a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*e^(-f*x - e)/((a^2*e^(-2*f*x - 2*e) + a^2)*f) - log(e^(-f*x - e)
+ 1)/(a^(3/2)*f) + log(e^(-f*x - e) - 1)/(a^(3/2)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth\left(e+f\,x\right)}{{\left(a\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(e+fx\right)}{\left(a\left(\sinh^{2}\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

3.451 $\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{2a^2f}$$

```
[Out] 1/2*arctanh((a*cosh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csch(f*x+e)^2*(a
*cosh(f*x+e)^2)^(1/2)/a^2/f
```

Rubi [A] time = 0.14, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3176, 3205, 16, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*a^(3/2)*f) - (Sqrt[a*Cosh[e + f
*x]^2]*Csch[e + f*x]^2)/(2*a^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth^3(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$=\frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(ax)^{3/2}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=\frac{\text{Subst}\left(\int \frac{1}{(1-x)^2\sqrt{ax}}\,dx,x,\cosh^2(e+fx)\right)}{2af}$$
$$=-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2a^2f}+\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx,x,\cosh^2(e+fx)\right)}{4af}$$
$$=-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2a^2f}+\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a\cosh^2(e+fx)}\right)}{2a^2f}$$
$$=\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2a^2f}$$

Mathematica [A] time = 0.13, size = 67, normalized size = 1.02

$$-\frac{\cosh^3(e+fx)\left(\text{csch}^2\left(\frac{1}{2}(e+fx)\right)+\text{sech}^2\left(\frac{1}{2}(e+fx)\right)+4\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*(Cosh[e + f*x]^3*(Csch[(e + f*x)/2]^2 + 4*Log[Tanh[(e + f*x)/2]] + Sec
h[(e + f*x)/2]^2))/(f*(a*Cosh[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.52, size = 565, normalized size = 8.56

$$-\frac{\left(6\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)^2+2e^{\left(fx+e\right)}\sinh\left(fx+e\right)^3+2\left(3\cosh\left(fx+e\right)^2+1\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right.}{2\left(a^2f\cosh\left(fx+e\right)^4-2a^2f\cosh\left(fx+e\right)^2+\left(a^2fe^{\left(2fx+2e\right)}+a^2f\right)\sinh\left(fx+e\right)^4+4\left(a^2f\cosh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + 2*e^(f*x + e)*sinh(f*x
+ e)^3 + 2*(3*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + 2*(cosh(f*x
+ e)^3 + cosh(f*x + e))*e^(f*x + e) - (4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x
 + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*e^(f*x +
e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f
*x + e) + (cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(
f*x + e) + sinh(f*x + e) + 1)/(cosh(f*x + e) + sinh(f*x + e) - 1)))*sqrt(a*
e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e
)^4 - 2*a^2*f*cosh(f*x + e)^2 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x +
e)^4 + 4*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f
*x + e)^3 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2 - a^2*f + (3*a^2*f*cosh(f*x
+ e)^2 - a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^4 -
 2*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 4*(a^2*f*cosh(f*x + e)^
3 - a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*e^(
2*f*x + 2*e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^3*exp(exp(1))^3+t_nostep*exp(exp(1)))]index.cc index_m operator + Err
or: Bad Argument Value
```

maple [C] time = 0.18, size = 36, normalized size = 0.55

$$\frac{\text{`}int/indef0\text{`}\left(\frac{1}{\sinh\left(fx+e\right)^{3} a\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)^3/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [A] time = 0.86, size = 100, normalized size = 1.52

$$\frac{e^{(-fx-e)} + e^{(-3fx-3e)}}{\left(2a^{\frac{3}{2}}e^{(-2fx-2e)} - a^{\frac{3}{2}}e^{(-4fx-4e)} - a^{\frac{3}{2}}\right)f} + \frac{\log\left(e^{(-fx-e)} + 1\right)}{2a^{\frac{3}{2}}f} - \frac{\log\left(e^{(-fx-e)} - 1\right)}{2a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (e^(-f*x - e) + e^(-3*f*x - 3*e))/((2*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^
(-4*f*x - 4*e) - a^(3/2))*f) + 1/2*log(e^(-f*x - e) + 1)/(a^(3/2)*f) - 1/2*
log(e^(-f*x - e) - 1)/(a^(3/2)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth\left(e+f\,x\right)^3}{\left(a\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^3/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

3.452 $\int \frac{\tanh^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)\text{sech}^2(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] 1/8*arctan(sinh(f*x+e))*cosh(f*x+e)/a/f/(a*cosh(f*x+e)^2)^(1/2)+1/8*tanh(f*
x+e)/a/f/(a*cosh(f*x+e)^2)^(1/2)-1/4*sech(f*x+e)^2*tanh(f*x+e)/a/f/(a*cosh(
f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3176, 3207, 2611, 3768, 3770}

$$\frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)\text{sech}^2(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) + Tan
h[e + f*x]/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Sech[e + f*x]^2*Tanh[e + f*x]
)/(4*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tanh^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\tanh^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\ &= \frac{\cosh(e+fx)\int \operatorname{sech}^3(e+fx)\tanh^2(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}\\ &= -\frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\int \operatorname{sech}^3(e+fx)\,dx}{4a\sqrt{a\cosh^2(e+fx)}}\\ &= \frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\int \operatorname{sech}(e-}{8a\sqrt{a\cosh^2(e+f}\\ &= \frac{\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}+\frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\operatorname{sech}^2(e+fx)\tan}{4af\sqrt{a\cosh^2(}\end{aligned}$$

Mathematica [A] time = 0.10, size = 58, normalized size = 0.55

$$\frac{\tanh(e+fx)\left(1-2\operatorname{sech}^2(e+fx)\right)+\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + (1 - 2*Sech[e + f*x]^2)*Tanh[e + f*x
])/(8*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

fricas [B] time = 0.53, size = 1423, normalized size = 13.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(7*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + e^(f*x + e)*sinh(f*x + e
)^7 + 7*(3*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^5 + 35*(cosh(f*x
+ e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 7*(5*cosh(f*x + e)^4
- 10*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 7*(3*cosh(f*x + e)^
5 - 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (7*
cosh(f*x + e)^6 - 35*cosh(f*x + e)^4 + 21*cosh(f*x + e)^2 - 1)*e^(f*x + e)*
sinh(f*x + e) + (8*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + e^(f*x + e)*
sinh(f*x + e)^8 + 4*(7*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^6 + 8
*(7*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 2*(35*
cosh(f*x + e)^4 + 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(
7*cosh(f*x + e)^5 + 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(
f*x + e)^3 + 4*(7*cosh(f*x + e)^6 + 15*cosh(f*x + e)^4 + 9*cosh(f*x + e)^2
+ 1)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(cosh(f*x + e)^7 + 3*cosh(f*x + e)^5 +
 3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x +
 e)^8 + 4*cosh(f*x + e)^6 + 6*cosh(f*x + e)^4 + 4*cosh(f*x + e)^2 + 1)*e^(f
```

```
*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^7 - 7*cosh(
f*x + e)^5 + 7*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*
x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^8 + 4
*a^2*f*cosh(f*x + e)^6 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^8 +
8*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e)
^7 + 6*a^2*f*cosh(f*x + e)^4 + 4*(7*a^2*f*cosh(f*x + e)^2 + a^2*f + (7*a^2*
f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 8*(7*a^2*f*co
sh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e) + (7*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*
cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^2 +
 2*(35*a^2*f*cosh(f*x + e)^4 + 30*a^2*f*cosh(f*x + e)^2 + 3*a^2*f + (35*a^2
*f*cosh(f*x + e)^4 + 30*a^2*f*cosh(f*x + e)^2 + 3*a^2*f)*e^(2*f*x + 2*e))*s
inh(f*x + e)^4 + 8*(7*a^2*f*cosh(f*x + e)^5 + 10*a^2*f*cosh(f*x + e)^3 + 3*
a^2*f*cosh(f*x + e) + (7*a^2*f*cosh(f*x + e)^5 + 10*a^2*f*cosh(f*x + e)^3 +
 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*f + 4*(7*a^2
*f*cosh(f*x + e)^6 + 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f*x + e)^2 + a
^2*f + (7*a^2*f*cosh(f*x + e)^6 + 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f
*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^
8 + 4*a^2*f*cosh(f*x + e)^6 + 6*a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x +
e)^2 + a^2*f)*e^(2*f*x + 2*e) + 8*(a^2*f*cosh(f*x + e)^7 + 3*a^2*f*cosh(f*x
 + e)^5 + 3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x +
 e)^7 + 3*a^2*f*cosh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x
+ e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

giac [A] time = 0.37, size = 93, normalized size = 0.88

$$\frac{\frac{\arctan\left(e^{(fx+e)}\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{a}e^{(7fx+7e)} - 7\sqrt{a}e^{(5fx+5e)} + 7\sqrt{a}e^{(3fx+3e)} - \sqrt{a}e^{(fx+e)}}{a^2\left(e^{(2fx+2e)}+1\right)^4}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(arctan(e^(f*x + e))/a^(3/2) + (sqrt(a)*e^(7*f*x + 7*e) - 7*sqrt(a)*e^(
5*f*x + 5*e) + 7*sqrt(a)*e^(3*f*x + 3*e) - sqrt(a)*e^(f*x + e))/(a^2*(e^(2*
f*x + 2*e) + 1)^4))/f
```

maple [A] time = 0.22, size = 69, normalized size = 0.65

$$\frac{\arctan\left(\sinh\left(fx+e\right)\right)\left(\cosh^4\left(fx+e\right)\right)+\left(\cosh^2\left(fx+e\right)\right)\sinh\left(fx+e\right)-2\sinh\left(fx+e\right)}{8a\cosh\left(fx+e\right)^3\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/a*(arctan(sinh(f*x+e))*cosh(f*x+e)^4+cosh(f*x+e)^2*sinh(f*x+e)-2*sinh(f
*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.57, size = 369, normalized size = 3.48

$$-\frac{\frac{3e^{(-fx-e)}+11e^{(-3fx-3e)}-11e^{(-5fx-5e)}-3e^{(-7fx-7e)}}{4a^{\frac{3}{2}}e^{(-2fx-2e)}+6a^{\frac{3}{2}}e^{(-4fx-4e)}+4a^{\frac{3}{2}}e^{(-6fx-6e)}+a^{\frac{3}{2}}e^{(-8fx-8e)}+a^{\frac{3}{2}}}-\frac{3\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}}{8f}+\frac{15e^{(-fx-e)}+55e^{(-3fx-3e)}+}{48\left(4a^{\frac{3}{2}}e^{(-2fx-2e)}+6a^{\frac{3}{2}}e^{(-4fx-4e)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*((3*e^(-f*x - e) + 11*e^(-3*f*x - 3*e) - 11*e^(-5*f*x - 5*e) - 3*e^(-7
*f*x - 7*e))/(4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(3/2)*e^(-4*f*x - 4*e) + 4*a
^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)) - 3*arctan(e^
(-f*x - e))/a^(3/2))/f + 1/48*(15*e^(-f*x - e) + 55*e^(-3*f*x - 3*e) + 73*e
^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e))/((4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(
3/2)*e^(-4*f*x - 4*e) + 4*a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x - 8*
e) + a^(3/2))*f) + 1/48*(15*e^(-f*x - e) - 73*e^(-3*f*x - 3*e) - 55*e^(-5*f
*x - 5*e) - 15*e^(-7*f*x - 7*e))/((4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(3/2)*e
^(-4*f*x - 4*e) + 4*a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x - 8*e) + a
^(3/2))*f) - 5/8*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh\left(e+f\,x\right)^2}{{\left(a\,\sinh\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{2}{\left(e+fx \right)}}{\left(a \left(\sinh^{2}{\left(e+fx \right)}+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

3.453 $\int \frac{\coth^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -arctan(sinh(f*x+e))*cosh(f*x+e)/a/f/(a*cosh(f*x+e)^2)^(1/2)-coth(f*x+e)/a/
f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3176, 3207, 2621, 321, 207}

$$-\frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(a*f*Sqrt[a*Cosh[e + f*x]^2])) - Co
th[e + f*x]/(a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\ &= \frac{\cosh(e+fx)\int \operatorname{csch}^2(e+fx)\operatorname{sech}(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}\\ &= -\frac{(i\cosh(e+fx))\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,-i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}\\ &= -\frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}-\frac{(i\cosh(e+fx))\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,-i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}\\ &= -\frac{\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}\end{aligned}$$

Mathematica [C] time = 0.08, size = 46, normalized size = 0.72

$$-\frac{\coth(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\sinh^2(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Coth[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[e + f*x]^2])/(a*f*Sq
rt[a*Cosh[e + f*x]^2]))
```

fricas [B] time = 0.56, size = 254, normalized size = 3.97

$$-\frac{2\left(\left(2\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)+e^{\left(fx+e\right)}\sinh\left(fx+e\right)^2+\left(\cosh\left(fx+e\right)^2-1\right)e^{\left(fx+e\right)}\right)\arctan\left(\cosh\left(\right.\right.}{a^2f\cosh\left(fx+e\right)^2-a^2f+\left(a^2fe^{\left(2fx+2e\right)}+a^2f\right)\sinh\left(fx+e\right)^2+\left(a^2f\cosh\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 + (cosh(f*x + e)^2 - 1)*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)
) + cosh(f*x + e)*e^(f*x + e) + e^(f*x + e)*sinh(f*x + e))*sqrt(a*e^(4*f*x
+ 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^2 - a^2
*f + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)
^2 - a^2*f)*e^(2*f*x + 2*e) + 2*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*
f*cosh(f*x + e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^3*exp(exp(1))^3+t_nostep*exp(exp(1)))]index.cc index_m operator + Err
or: Bad Argument Value
```

maple [A] time = 0.21, size = 51, normalized size = 0.80

$$-\frac{\cosh\left(fx+e\right)\left(\arctan\left(\sinh\left(fx+e\right)\right)\sinh\left(fx+e\right)+1\right)}{a\sinh\left(fx+e\right)\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/a*cosh(f*x+e)*(arctan(sinh(f*x+e))*sinh(f*x+e)+1)/sinh(f*x+e)/(a*cosh(f*
x+e)^2)^(1/2)/f
```

maxima [B] time = 0.52, size = 321, normalized size = 5.02

$$-\frac{\frac{3\sqrt{a}e^{(-fx-e)}+2\sqrt{a}e^{(-3fx-3e)}+3\sqrt{a}e^{(-5fx-5e)}}{a^2e^{(-2fx-2e)}-a^2e^{(-4fx-4e)}-a^2e^{(-6fx-6e)}+a^2}-\frac{3\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}}{2f}-\frac{5\sqrt{a}e^{(-fx-e)}+6\sqrt{a}e^{(-3fx-3e)}-3\sqrt{a}e^{(-5fx-5e)}}{4\left(a^2e^{(-2fx-2e)}-a^2e^{(-4fx-4e)}-a^2e^{(-6fx-6e)}+a^2\right)f}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((3*sqrt(a)*e^(-f*x - e) + 2*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-
5*f*x - 5*e))/(a^2*e^(-2*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x
- 6*e) + a^2) - 3*arctan(e^(-f*x - e))/a^(3/2))/f - 1/4*(5*sqrt(a)*e^(-f*x
- e) + 6*sqrt(a)*e^(-3*f*x - 3*e) - 3*sqrt(a)*e^(-5*f*x - 5*e))/((a^2*e^(-2
*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x - 6*e) + a^2)*f) + 1/4*(
3*sqrt(a)*e^(-f*x - e) - 6*sqrt(a)*e^(-3*f*x - 3*e) - 5*sqrt(a)*e^(-5*f*x -
 5*e))/((a^2*e^(-2*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x - 6*e)
 + a^2)*f) + 1/2*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\mathrm{coth}\left(e+f\,x\right)}^2}{{\left(a\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^2/(a + a*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}\left(e+fx\right)}{\left(a\left(\sinh^{2}\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(3/2),x)
[Out] Integral(coth(e + f*x)**2/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

3.454 $\int \frac{\coth^4(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -1/3*coth(f*x+e)*csch(f*x+e)^2/a/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3176, 3207, 2606, 30}

$$-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\coth^4(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$= \frac{\cosh(e+fx)\int \coth(e+fx)\operatorname{csch}^3(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}$$
$$= \frac{(i\cosh(e+fx))\,\text{Subst}\left(\int x^2\,dx, x, -i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.05, size = 29, normalized size = 0.76

$$-\frac{\coth^3(e+fx)}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*Coth[e + f*x]^3/(f*(a*Cosh[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.60, size = 612, normalized size = 16.11

$$-\frac{}{3\left(a^2f\cosh\left(fx+e\right)^6-3\,a^2f\cosh\left(fx+e\right)^4+\left(a^2fe^{(2\,fx+2\,e)}+a^2f\right)\sinh\left(fx+e\right)^6+6\left(a^2f\cosh\left(fx+e\right)e^{(2\,f}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(f*x + e)^3*e^(f*x + e) + 3*cosh(f*x + e)^2*e^(f*x + e)*sinh(f*x
+ e) + 3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x +
 e)^3)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*
f*cosh(f*x + e)^6 - 3*a^2*f*cosh(f*x + e)^4 + (a^2*f*e^(2*f*x + 2*e) + a^2*
f)*sinh(f*x + e)^6 + 6*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*
x + e))*sinh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^2 + 3*(5*a^2*f*cosh(f*x + e
)^2 - a^2*f + (5*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e))*sinh(f*x +
 e)^4 + 4*(5*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e) + (5*a^2*f*cosh(
f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 - a^2*
f + 3*(5*a^2*f*cosh(f*x + e)^4 - 6*a^2*f*cosh(f*x + e)^2 + a^2*f + (5*a^2*f
*cosh(f*x + e)^4 - 6*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f
*x + e)^2 + (a^2*f*cosh(f*x + e)^6 - 3*a^2*f*cosh(f*x + e)^4 + 3*a^2*f*cosh
(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e) + 6*(a^2*f*cosh(f*x + e)^5 - 2*a^2*f*c
osh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^5 - 2*a^2*f*cos
h(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep^3*exp(exp(1))^3+t_nostep*exp(exp(1)))]index.cc index_m operator + Err
or: Bad Argument Value
```

maple [A] time = 0.19, size = 35, normalized size = 0.92

$$-\frac{\cosh\left(fx+e\right)}{3a\sinh\left(fx+e\right)^{3}\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*cosh(f*x+e)/a/sinh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.80, size = 823, normalized size = 21.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*((21*e^(-f*x - e) - 16*e^(-3*f*x - 3*e) + 34*e^(-5*f*x - 5*e) + 8*e^(-
7*f*x - 7*e) - 15*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e
^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a
^(3/2)*e^(-10*f*x - 10*e) - a^(3/2)) + 3*arctan(e^(-f*x - e))/a^(3/2) + 9*l
og(e^(-f*x - e) + 1)/a^(3/2) - 9*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/12*((
15*e^(-f*x - e) - 8*e^(-3*f*x - 3*e) - 34*e^(-5*f*x - 5*e) + 16*e^(-7*f*x -
 7*e) - 21*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*
x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*
e^(-10*f*x - 10*e) - a^(3/2)) - 3*arctan(e^(-f*x - e))/a^(3/2) + 9*log(e^(-
f*x - e) + 1)/a^(3/2) - 9*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/8*((15*e^(-f
*x - e) - 20*e^(-3*f*x - 3*e) - 22*e^(-5*f*x - 5*e) - 20*e^(-7*f*x - 7*e) +
 15*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e
) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*
f*x - 10*e) - a^(3/2)) + 15*arctan(e^(-f*x - e))/a^(3/2))/f + 1/48*(45*e^(-
f*x - e) - 52*e^(-3*f*x - 3*e) - 74*e^(-5*f*x - 5*e) + 92*e^(-7*f*x - 7*e)
+ 21*e^(-9*f*x - 9*e))/((a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4
*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-1
0*f*x - 10*e) - a^(3/2))*f) + 1/48*(21*e^(-f*x - e) + 92*e^(-3*f*x - 3*e) -
 74*e^(-5*f*x - 5*e) - 52*e^(-7*f*x - 7*e) + 45*e^(-9*f*x - 9*e))/((a^(3/2)
*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e)
 - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2))*f) + 11
/8*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

mupad [B] time = 0.90, size = 71, normalized size = 1.87

$$-\frac{16\,\mathrm{e}^{4\,e+4\,f\,x}\,\sqrt{a+a\,{\left(\frac{\mathrm{e}^{e+f\,x}}{2}-\frac{\mathrm{e}^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left({\mathrm{e}}^{2\,e+2\,f\,x}-1\right)}^3\,\left({\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] -(16*exp(4*e + 4*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/
(3*a^2*f*(exp(2*e + 2*f*x) - 1)^3*(exp(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(e+fx)}{\left(a\left(\sinh^2(e+fx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**4/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

3.455 $\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -1/3*coth(f*x+e)*csch(f*x+e)^2/a/f/(a*cosh(f*x+e)^2)^(1/2)-1/5*coth(f*x+e)*
csch(f*x+e)^4/a/f/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3207, 2606, 14}

$$-\frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^6/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e
+ f*x]*Csch[e + f*x]^4)/(5*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\coth^6(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$=\frac{\cosh(e+fx)\int \coth^3(e+fx)\operatorname{csch}^3(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}$$
$$=-\frac{(i\cosh(e+fx))\,\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,-i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$
$$=-\frac{(i\cosh(e+fx))\,\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,-i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$
$$=-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.11, size = 41, normalized size = 0.53

$$-\frac{\coth^3(e+fx)\left(3\operatorname{csch}^2(e+fx)+5\right)}{15f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^6/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*(Coth[e + f*x]^3*(5 + 3*Csch[e + f*x]^2))/(f*(a*Cosh[e + f*x]^2)^(3/2
))
```

fricas [B] time = 0.50, size = 1410, normalized size = 18.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/15*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*
x + e)^7 + (105*cosh(f*x + e)^2 + 2)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*co
sh(f*x + e)^3 + 2*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 5*(35*cosh(f
*x + e)^4 + 4*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 5*(21*cosh
(f*x + e)^5 + 4*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(7*cosh(f*x + e)^6 + 2*cosh(f*x + e)^4 + 3*cosh(f*x + e)^2)*e^(f*x
+ e)*sinh(f*x + e) + (5*cosh(f*x + e)^7 + 2*cosh(f*x + e)^5 + 5*cosh(f*x +
e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*
x - e)/(a^2*f*cosh(f*x + e)^10 - 5*a^2*f*cosh(f*x + e)^8 + (a^2*f*e^(2*f*x
+ 2*e) + a^2*f)*sinh(f*x + e)^10 + 10*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e)
+ a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^6 + 5*(9*a^
2*f*cosh(f*x + e)^2 - a^2*f + (9*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x +
2*e))*sinh(f*x + e)^8 + 40*(3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e) +
 (3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x
+ e)^7 - 10*a^2*f*cosh(f*x + e)^4 + 10*(21*a^2*f*cosh(f*x + e)^4 - 14*a^2*f
*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*cosh(f*x + e)^4 - 14*a^2*f*cosh(f*x +
e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 4*(63*a^2*f*cosh(f*x + e)^
5 - 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (63*a^2*f*cosh(f*x
+ e)^5 - 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e)
)*sinh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^2 + 10*(21*a^2*f*cosh(f*x + e)^6
```

```
- 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 - a^2*f + (21*a^2*f*c
osh(f*x + e)^6 - 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 - a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 40*(3*a^2*f*cosh(f*x + e)^7 - 7*a^2*f
*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e) + (3*a^2*f
*cosh(f*x + e)^7 - 7*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 - a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 - a^2*f + 5*(9*a^2*f*cosh
(f*x + e)^8 - 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f*cosh(f*x + e)^4 - 12*a^2*
f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^8 - 28*a^2*f*cosh(f*x +
e)^6 + 30*a^2*f*cosh(f*x + e)^4 - 12*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*
x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^10 - 5*a^2*f*cosh(f*x + e)
^8 + 10*a^2*f*cosh(f*x + e)^6 - 10*a^2*f*cosh(f*x + e)^4 + 5*a^2*f*cosh(f*x
 + e)^2 - a^2*f)*e^(2*f*x + 2*e) + 10*(a^2*f*cosh(f*x + e)^9 - 4*a^2*f*cosh
(f*x + e)^7 + 6*a^2*f*cosh(f*x + e)^5 - 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cos
h(f*x + e) + (a^2*f*cosh(f*x + e)^9 - 4*a^2*f*cosh(f*x + e)^7 + 6*a^2*f*cos
h(f*x + e)^5 - 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e))
```

giac [A] time = 0.59, size = 68, normalized size = 0.88

$$-\frac{8\left(5\sqrt{a}e^{\left(7fx+7e\right)}+2\sqrt{a}e^{\left(5fx+5e\right)}+5\sqrt{a}e^{\left(3fx+3e\right)}\right)}{15a^2f\left(e^{\left(2fx+2e\right)}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/15*(5*sqrt(a)*e^(7*f*x + 7*e) + 2*sqrt(a)*e^(5*f*x + 5*e) + 5*sqrt(a)*e^
(3*f*x + 3*e))/(a^2*f*(e^(2*f*x + 2*e) - 1)^5)
```

maple [A] time = 0.29, size = 67, normalized size = 0.87

$$-\frac{\cosh\left(fx+e\right)\left(5\left(\cosh^2\left(fx+e\right)\right)-2\right)}{15\left(-1+\cosh\left(fx+e\right)\right)^2\left(\cosh\left(fx+e\right)+1\right)^2a\sinh\left(fx+e\right)\sqrt{a\left(\cosh^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15*cosh(f*x+e)*(5*cosh(f*x+e)^2-2)/(-1+cosh(f*x+e))^2/(cosh(f*x+e)+1)^2/
a/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.62, size = 1531, normalized size = 19.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/256*(2*(105*e^(-f*x - e) - 300*e^(-3*f*x - 3*e) + 81*e^(-5*f*x - 5*e) -
248*e^(-7*f*x - 7*e) + 51*e^(-9*f*x - 9*e) + 100*e^(-11*f*x - 11*e) - 45*e^
(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) -
5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*
x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(
3/2)) + 60*arctan(e^(-f*x - e))/a^(3/2) + 75*log(e^(-f*x - e) + 1)/a^(3/2)
- 75*log(e^(-f*x - e) - 1)/a^(3/2))/f + 1/48*((105*e^(-f*x - e) - 350*e^(-3
*f*x - 3*e) + 231*e^(-5*f*x - 5*e) + 412*e^(-7*f*x - 7*e) + 231*e^(-9*f*x -
 9*e) - 350*e^(-11*f*x - 11*e) + 105*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f
*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/
```

```
2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 1
2*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2)) + 105*arctan(e^(-f*x - e))/a^(
3/2))/f + 3/256*(2*(45*e^(-f*x - e) - 100*e^(-3*f*x - 3*e) - 51*e^(-5*f*x -
 5*e) + 248*e^(-7*f*x - 7*e) - 81*e^(-9*f*x - 9*e) + 300*e^(-11*f*x - 11*e)
 - 105*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x
- 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*
e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14
*e) - a^(3/2)) - 60*arctan(e^(-f*x - e))/a^(3/2) + 75*log(e^(-f*x - e) + 1)
/a^(3/2) - 75*log(e^(-f*x - e) - 1)/a^(3/2))/f - 3/320*(4*(45*e^(-f*x - e)
- 135*e^(-3*f*x - 3*e) + 54*e^(-5*f*x - 5*e) + 198*e^(-7*f*x - 7*e) - 211*e
^(-9*f*x - 9*e) - 15*e^(-11*f*x - 11*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3
/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8
*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e
^(-14*f*x - 14*e) - a^(3/2)) + 90*arctan(e^(-f*x - e))/a^(3/2) + 45*log(e^(
-f*x - e) + 1)/a^(3/2) - 45*log(e^(-f*x - e) - 1)/a^(3/2))/f + 3/320*(4*(15
*e^(-3*f*x - 3*e) + 211*e^(-5*f*x - 5*e) - 198*e^(-7*f*x - 7*e) - 54*e^(-9*
f*x - 9*e) + 135*e^(-11*f*x - 11*e) - 45*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(
-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a
^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x
 - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2)) - 90*arctan(e^(-f*x - e))/
a^(3/2) + 45*log(e^(-f*x - e) + 1)/a^(3/2) - 45*log(e^(-f*x - e) - 1)/a^(3/
2))/f + 1/1920*(1155*e^(-f*x - e) + 1460*e^(-3*f*x - 3*e) - 4173*e^(-5*f*x
- 5*e) + 2024*e^(-7*f*x - 7*e) + 1857*e^(-9*f*x - 9*e) - 2140*e^(-11*f*x -
11*e) + 585*e^(-13*f*x - 13*e))/((3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-
4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^
(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*
x - 14*e) - a^(3/2))*f) + 1/1920*(585*e^(-f*x - e) - 2140*e^(-3*f*x - 3*e)
+ 1857*e^(-5*f*x - 5*e) + 2024*e^(-7*f*x - 7*e) - 4173*e^(-9*f*x - 9*e) + 1
460*e^(-11*f*x - 11*e) + 1155*e^(-13*f*x - 13*e))/((3*a^(3/2)*e^(-2*f*x - 2
*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(
-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) +
 a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2))*f) + 29/32*arctan(e^(-f*x - e))/(a^(
3/2)*f)
```

mupad [B] time = 0.16, size = 305, normalized size = 3.96

$$-\frac{16\,e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{3\,a^2\,f\left(e^{2e+2fx}-1\right)^2\left(e^{e+fx}+e^{3e+3fx}\right)}-\frac{272\,e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+fx}}{2}-\frac{e^{-e-fx}}{2}\right)^2}}{15\,a^2\,f\left(e^{2e+2fx}-1\right)^3\left(e^{e+fx}+e^{3e+3fx}\right)}-\frac{128\,e^{3e+3fx}\sqrt{a+a\left(\frac{e^{e+f}}{2}\right.}}{5\,a^2\,f\left(e^{2e+2fx}-1\right)^4\left(e^{e+f}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^6/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(3*a^2*f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (27
2*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(15
*a^2*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (128*e
xp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a^2
*f*(exp(2*e + 2*f*x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (64*exp(3*
e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(5*a^2*f*(e
xp(2*e + 2*f*x) - 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^6\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**6/(a+a*sinh(f*x+e)**2)**(3/2),x)
[Out] Integral(coth(e + f*x)**6/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

3.456 $\int \frac{\coth^8(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{\coth(e+fx)\operatorname{csch}^6(e+fx)}{7af\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -1/3*coth(f*x+e)*csch(f*x+e)^2/a/f/(a*cosh(f*x+e)^2)^(1/2)-2/5*coth(f*x+e)*
csch(f*x+e)^4/a/f/(a*cosh(f*x+e)^2)^(1/2)-1/7*coth(f*x+e)*csch(f*x+e)^6/a/f
/(a*cosh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3176, 3207, 2606, 270}

$$-\frac{\coth(e+fx)\operatorname{csch}^6(e+fx)}{7af\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^8/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (2*Coth[
e + f*x]*Csch[e + f*x]^4)/(5*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*
Csch[e + f*x]^6)/(7*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\coth^8(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\coth^8(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$= \frac{\cosh(e+fx)\int \coth^5(e+fx)\operatorname{csch}^3(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}$$
$$= \frac{(i\cosh(e+fx))\operatorname{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, -i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$
$$= \frac{(i\cosh(e+fx))\operatorname{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, -i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}} - \frac{2\coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)c}{7af\sqrt{a\cosh}}$$

Mathematica [A] time = 0.14, size = 51, normalized size = 0.44

$$-\frac{\coth^3(e+fx)\left(15\operatorname{csch}^4(e+fx)+42\operatorname{csch}^2(e+fx)+35\right)}{105f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^8/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/105*(Coth[e + f*x]^3*(35 + 42*Csch[e + f*x]^2 + 15*Csch[e + f*x]^4))/(f*
(a*Cosh[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.57, size = 2511, normalized size = 21.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/105*(385*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^10 + 35*e^(f*x + e)*sin
h(f*x + e)^11 + 7*(275*cosh(f*x + e)^2 + 4)*e^(f*x + e)*sinh(f*x + e)^9 + 2
1*(275*cosh(f*x + e)^3 + 12*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^8 + 6*
(1925*cosh(f*x + e)^4 + 168*cosh(f*x + e)^2 + 19)*e^(f*x + e)*sinh(f*x + e)
^7 + 42*(385*cosh(f*x + e)^5 + 56*cosh(f*x + e)^3 + 19*cosh(f*x + e))*e^(f*
x + e)*sinh(f*x + e)^6 + 14*(1155*cosh(f*x + e)^6 + 252*cosh(f*x + e)^4 + 1
71*cosh(f*x + e)^2 + 2)*e^(f*x + e)*sinh(f*x + e)^5 + 14*(825*cosh(f*x + e)
^7 + 252*cosh(f*x + e)^5 + 285*cosh(f*x + e)^3 + 10*cosh(f*x + e))*e^(f*x +
 e)*sinh(f*x + e)^4 + 7*(825*cosh(f*x + e)^8 + 336*cosh(f*x + e)^6 + 570*co
sh(f*x + e)^4 + 40*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^3 + 7*(27
5*cosh(f*x + e)^9 + 144*cosh(f*x + e)^7 + 342*cosh(f*x + e)^5 + 40*cosh(f*x
 + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 7*(55*cosh(f*x +
e)^10 + 36*cosh(f*x + e)^8 + 114*cosh(f*x + e)^6 + 20*cosh(f*x + e)^4 + 15*
cosh(f*x + e)^2)*e^(f*x + e)*sinh(f*x + e) + (35*cosh(f*x + e)^11 + 28*cosh
(f*x + e)^9 + 114*cosh(f*x + e)^7 + 28*cosh(f*x + e)^5 + 35*cosh(f*x + e)^3
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a^2*f*cosh(f*x + e)^14 - 7*a^2*f*cosh(f*x + e)^12 + (a^2*f*e^(2*f*x + 2
*e) + a^2*f)*sinh(f*x + e)^14 + 14*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a
^2*f*cosh(f*x + e))*sinh(f*x + e)^13 + 21*a^2*f*cosh(f*x + e)^10 + 7*(13*a^
2*f*cosh(f*x + e)^2 - a^2*f + (13*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x +
```

```
 2*e))*sinh(f*x + e)^12 + 28*(13*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x +
 e) + (13*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*s
inh(f*x + e)^11 - 35*a^2*f*cosh(f*x + e)^8 + 7*(143*a^2*f*cosh(f*x + e)^4 -
 66*a^2*f*cosh(f*x + e)^2 + 3*a^2*f + (143*a^2*f*cosh(f*x + e)^4 - 66*a^2*f
*cosh(f*x + e)^2 + 3*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^10 + 14*(143*a^2
*f*cosh(f*x + e)^5 - 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (
143*a^2*f*cosh(f*x + e)^5 - 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x +
 e))*e^(2*f*x + 2*e))*sinh(f*x + e)^9 + 35*a^2*f*cosh(f*x + e)^6 + 7*(429*a
^2*f*cosh(f*x + e)^6 - 495*a^2*f*cosh(f*x + e)^4 + 135*a^2*f*cosh(f*x + e)^
2 - 5*a^2*f + (429*a^2*f*cosh(f*x + e)^6 - 495*a^2*f*cosh(f*x + e)^4 + 135*
a^2*f*cosh(f*x + e)^2 - 5*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 8*(429*
a^2*f*cosh(f*x + e)^7 - 693*a^2*f*cosh(f*x + e)^5 + 315*a^2*f*cosh(f*x + e)
^3 - 35*a^2*f*cosh(f*x + e) + (429*a^2*f*cosh(f*x + e)^7 - 693*a^2*f*cosh(f
*x + e)^5 + 315*a^2*f*cosh(f*x + e)^3 - 35*a^2*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e)^7 - 21*a^2*f*cosh(f*x + e)^4 + 7*(429*a^2*f*cosh(f*x +
e)^8 - 924*a^2*f*cosh(f*x + e)^6 + 630*a^2*f*cosh(f*x + e)^4 - 140*a^2*f*co
sh(f*x + e)^2 + 5*a^2*f + (429*a^2*f*cosh(f*x + e)^8 - 924*a^2*f*cosh(f*x +
 e)^6 + 630*a^2*f*cosh(f*x + e)^4 - 140*a^2*f*cosh(f*x + e)^2 + 5*a^2*f)*e^
(2*f*x + 2*e))*sinh(f*x + e)^6 + 14*(143*a^2*f*cosh(f*x + e)^9 - 396*a^2*f*
cosh(f*x + e)^7 + 378*a^2*f*cosh(f*x + e)^5 - 140*a^2*f*cosh(f*x + e)^3 + 1
5*a^2*f*cosh(f*x + e) + (143*a^2*f*cosh(f*x + e)^9 - 396*a^2*f*cosh(f*x + e
)^7 + 378*a^2*f*cosh(f*x + e)^5 - 140*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh
(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 7*a^2*f*cosh(f*x + e)^2 + 7*(
143*a^2*f*cosh(f*x + e)^10 - 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*x
 + e)^6 - 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 - 3*a^2*f +
(143*a^2*f*cosh(f*x + e)^10 - 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*
x + e)^6 - 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 - 3*a^2*f)*
e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 28*(13*a^2*f*cosh(f*x + e)^11 - 55*a^2*f
*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 - 70*a^2*f*cosh(f*x + e)^5 + 25
*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e) + (13*a^2*f*cosh(f*x + e)^11
 - 55*a^2*f*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 - 70*a^2*f*cosh(f*x
+ e)^5 + 25*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^3 - a^2*f + 7*(13*a^2*f*cosh(f*x + e)^12 - 66*a^2*f*cosh(f*x
 + e)^10 + 135*a^2*f*cosh(f*x + e)^8 - 140*a^2*f*cosh(f*x + e)^6 + 75*a^2*f
*cosh(f*x + e)^4 - 18*a^2*f*cosh(f*x + e)^2 + a^2*f + (13*a^2*f*cosh(f*x +
e)^12 - 66*a^2*f*cosh(f*x + e)^10 + 135*a^2*f*cosh(f*x + e)^8 - 140*a^2*f*c
osh(f*x + e)^6 + 75*a^2*f*cosh(f*x + e)^4 - 18*a^2*f*cosh(f*x + e)^2 + a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^14 - 7*a^2*f*cos
h(f*x + e)^12 + 21*a^2*f*cosh(f*x + e)^10 - 35*a^2*f*cosh(f*x + e)^8 + 35*a
^2*f*cosh(f*x + e)^6 - 21*a^2*f*cosh(f*x + e)^4 + 7*a^2*f*cosh(f*x + e)^2 -
 a^2*f)*e^(2*f*x + 2*e) + 14*(a^2*f*cosh(f*x + e)^13 - 6*a^2*f*cosh(f*x + e
)^11 + 15*a^2*f*cosh(f*x + e)^9 - 20*a^2*f*cosh(f*x + e)^7 + 15*a^2*f*cosh(
f*x + e)^5 - 6*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*
x + e)^13 - 6*a^2*f*cosh(f*x + e)^11 + 15*a^2*f*cosh(f*x + e)^9 - 20*a^2*f*
cosh(f*x + e)^7 + 15*a^2*f*cosh(f*x + e)^5 - 6*a^2*f*cosh(f*x + e)^3 + a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

giac [A] time = 0.66, size = 98, normalized size = 0.85

$$-\frac{8\left(35\sqrt{a}e^{(11fx+11e)}+28\sqrt{a}e^{(9fx+9e)}+114\sqrt{a}e^{(7fx+7e)}+28\sqrt{a}e^{(5fx+5e)}+35\sqrt{a}e^{(3fx+3e)}\right)}{105a^2f\left(e^{(2fx+2e)}-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/105*(35*sqrt(a)*e^(11*f*x + 11*e) + 28*sqrt(a)*e^(9*f*x + 9*e) + 114*sqr
t(a)*e^(7*f*x + 7*e) + 28*sqrt(a)*e^(5*f*x + 5*e) + 35*sqrt(a)*e^(3*f*x + 3
*e))/(a^2*f*(e^(2*f*x + 2*e) - 1)^7)
```

maple [A] time = 0.26, size = 57, normalized size = 0.50

$$-\frac{\cosh\left(fx+e\right)\left(35\left(\cosh^{4}\left(fx+e\right)\right)-28\left(\cosh^{2}\left(fx+e\right)\right)+8\right)}{105a\sinh\left(fx+e\right)^{7}\sqrt{a\left(\cosh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/105*cosh(f*x+e)*(35*cosh(f*x+e)^4-28*cosh(f*x+e)^2+8)/a/sinh(f*x+e)^7/(a
*cosh(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.68, size = 2216, normalized size = 19.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(4095*e^(-f*x - e) - 20090*e^(-3*f*x - 3*e) + 31654*e^(-5*f*x - 5
*e) - 850*e^(-7*f*x - 7*e) - 51148*e^(-9*f*x - 9*e) + 51090*e^(-11*f*x - 11
*e) - 2646*e^(-13*f*x - 13*e) + 4410*e^(-15*f*x - 15*e) - 1155*e^(-17*f*x -
 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/
2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x
- 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/
2)) + 2940*arctan(e^(-f*x - e))/a^(3/2) + 2625*log(e^(-f*x - e) + 1)/a^(3/2
) - 2625*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/8960*(2*(4095*e^(-f*x - e) -
21630*e^(-3*f*x - 3*e) + 39354*e^(-5*f*x - 5*e) - 13830*e^(-7*f*x - 7*e) -
47848*e^(-9*f*x - 9*e) + 66950*e^(-11*f*x - 11*e) - 22106*e^(-13*f*x - 13*e
) - 18690*e^(-15*f*x - 15*e) + 3465*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*
x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^
(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*
f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/2)) + 7560*arctan(e^(-f*x -
 e))/a^(3/2) + 315*log(e^(-f*x - e) + 1)/a^(3/2) - 315*log(e^(-f*x - e) - 1
)/a^(3/2))/f - 1/8960*(2*(3465*e^(-f*x - e) - 18690*e^(-3*f*x - 3*e) - 2210
6*e^(-5*f*x - 5*e) + 66950*e^(-7*f*x - 7*e) - 47848*e^(-9*f*x - 9*e) - 1383
0*e^(-11*f*x - 11*e) + 39354*e^(-13*f*x - 13*e) - 21630*e^(-15*f*x - 15*e)
+ 4095*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*
x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*
a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*
f*x - 18*e) - a^(3/2)) + 7560*arctan(e^(-f*x - e))/a^(3/2) - 315*log(e^(-f*
x - e) + 1)/a^(3/2) + 315*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/3840*(2*(115
5*e^(-f*x - e) - 4410*e^(-3*f*x - 3*e) + 2646*e^(-5*f*x - 5*e) - 51090*e^(-
7*f*x - 7*e) + 51148*e^(-9*f*x - 9*e) + 850*e^(-11*f*x - 11*e) - 31654*e^(-
13*f*x - 13*e) + 20090*e^(-15*f*x - 15*e) - 4095*e^(-17*f*x - 17*e))/(5*a^(
3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x -
 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^
(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/2)) - 2940*arc
tan(e^(-f*x - e))/a^(3/2) + 2625*log(e^(-f*x - e) + 1)/a^(3/2) - 2625*log(e
^(-f*x - e) - 1)/a^(3/2))/f + 1/768*(2*(1155*e^(-f*x - e) - 5670*e^(-3*f*x
- 3*e) + 8946*e^(-5*f*x - 5*e) - 270*e^(-7*f*x - 7*e) + 4696*e^(-9*f*x - 9*
e) - 2930*e^(-11*f*x - 11*e) - 658*e^(-13*f*x - 13*e) + 1190*e^(-15*f*x - 1
5*e) - 315*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-
4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e)
+ 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(
-18*f*x - 18*e) - a^(3/2)) + 840*arctan(e^(-f*x - e))/a^(3/2) + 735*log(e^(
-f*x - e) + 1)/a^(3/2) - 735*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/768*(2*(3
15*e^(-f*x - e) - 1190*e^(-3*f*x - 3*e) + 658*e^(-5*f*x - 5*e) + 2930*e^(-7
*f*x - 7*e) - 4696*e^(-9*f*x - 9*e) + 270*e^(-11*f*x - 11*e) - 8946*e^(-13*
```

```
f*x - 13*e) + 5670*e^(-15*f*x - 15*e) - 1155*e^(-17*f*x - 17*e))/(5*a^(3/2)
*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e
) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2
)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/2)) - 840*arctan(e
^(-f*x - e))/a^(3/2) + 735*log(e^(-f*x - e) + 1)/a^(3/2) - 735*log(e^(-f*x
- e) - 1)/a^(3/2))/f - 1/128*((315*e^(-f*x - e) - 1680*e^(-3*f*x - 3*e) + 3
108*e^(-5*f*x - 5*e) - 1200*e^(-7*f*x - 7*e) - 3646*e^(-9*f*x - 9*e) - 1200
*e^(-11*f*x - 11*e) + 3108*e^(-13*f*x - 13*e) - 1680*e^(-15*f*x - 15*e) + 3
15*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x -
4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3
/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x
- 18*e) - a^(3/2)) + 315*arctan(e^(-f*x - e))/a^(3/2))/f + 1/2688*(1155*e^(
-f*x - e) + 1393*e^(-3*f*x - 3*e) - 4865*e^(-5*f*x - 5*e) + 3965*e^(-7*f*x
- 7*e) + 825*e^(-9*f*x - 9*e) - 3245*e^(-11*f*x - 11*e) + 1925*e^(-13*f*x -
 13*e) - 385*e^(-15*f*x - 15*e))/((5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e
^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*
e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*
e^(-18*f*x - 18*e) - a^(3/2))*f) - 1/2688*(385*e^(-3*f*x - 3*e) - 1925*e^(-
5*f*x - 5*e) + 3245*e^(-7*f*x - 7*e) - 825*e^(-9*f*x - 9*e) - 3965*e^(-11*f
*x - 11*e) + 4865*e^(-13*f*x - 13*e) - 1393*e^(-15*f*x - 15*e) - 1155*e^(-1
7*f*x - 17*e))/((5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) +
14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(
-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e)
 - a^(3/2))*f) + 55/128*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

mupad [B] time = 0.95, size = 457, normalized size = 3.97

$$-\frac{16\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)}^2}}{3\,a^2\,f\,{\left(e^{2\,e+2\,f\,x}-1\right)}^2\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}-\frac{464\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,{\left(\frac{e^{e+f\,x}}{2}-\frac{e^{-e-f\,x}}{2}\right)}^2}}{15\,a^2\,f\,{\left(e^{2\,e+2\,f\,x}-1\right)}^3\,\left(e^{e+f\,x}+e^{3\,e+3\,f\,x}\right)}-\frac{3072\,e^{3\,e+3\,f\,x}\,\sqrt{a+a\,\left(\frac{e^{e+}}{2}\right.}}{35\,a^2\,f\,{\left(e^{2\,e+2\,f\,x}-1\right)}^4\,\left(e^{e+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^8/(a + a*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] - (16*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))
/(3*a^2*f*(exp(2*e + 2*f*x) - 1)^2*(exp(e + f*x) + exp(3*e + 3*f*x))) - (46
4*exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(15
*a^2*f*(exp(2*e + 2*f*x) - 1)^3*(exp(e + f*x) + exp(3*e + 3*f*x))) - (3072*
exp(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(35*a
^2*f*(exp(2*e + 2*f*x) - 1)^4*(exp(e + f*x) + exp(3*e + 3*f*x))) - (4736*ex
p(3*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(35*a^2
*f*(exp(2*e + 2*f*x) - 1)^5*(exp(e + f*x) + exp(3*e + 3*f*x))) - (768*exp(3
*e + 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*a^2*f*(
exp(2*e + 2*f*x) - 1)^6*(exp(e + f*x) + exp(3*e + 3*f*x))) - (256*exp(3*e +
 3*f*x)*(a + a*(exp(e + f*x)/2 - exp(- e - f*x)/2)^2)^(1/2))/(7*a^2*f*(exp(
2*e + 2*f*x) - 1)^7*(exp(e + f*x) + exp(3*e + 3*f*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**8/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.457 $\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^5(e+fx)\,dx$

Optimal. Leaf size=187

$$\frac{(8a^2-24ab+15b^2)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)^2}-\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{3/2}}-\frac{\operatorname{sech}^4(e+fx)\,(a\ \cdots}{4f(\cdots}$$

```
[Out] -1/8*(8*a^2-24*a*b+15*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(
a-b)^(3/2)/f+1/8*(8*a-7*b)*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/
f-1/4*sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)/f+1/8*(8*a^2-24*a*b+15*
b^2)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/f
```

Rubi [A] time = 0.22, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{(8a^2-24ab+15b^2)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)^2}-\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{3/2}}-\frac{\operatorname{sech}^4(e+fx)\,(a\ \cdots}{4f(\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] -((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]
])/(8*(a - b)^(3/2)*f) + ((8*a^2 - 24*a*b + 15*b^2)*Sqrt[a + b*Sinh[e + f*x
]^2])/(8*(a - b)^2*f) + ((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
)^(3/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/
(4*(a - b)*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{(1+x)^3}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4(a-b)f} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(-4a+3b)+2(a-b\cdots}{(1+x)^2}\right.}{4(\cdots}$$

$$= \frac{(8a-7b)\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{8(a-b)^2f} - \frac{\text{sech}^4(e+fx)\left(a\cdots\right.}{4(\cdots}$$

$$= \frac{\left(8a^2-24ab+15b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} + \frac{(8a-7b)\text{sech}^2(e+f\cdots}{8(a\cdots}$$

$$= \frac{\left(8a^2-24ab+15b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} + \frac{(8a-7b)\text{sech}^2(e+f\cdots}{8(a\cdots}$$

$$= -\frac{\left(8a^2-24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{3/2}f} + \frac{\left(8a^2-24ab+15\cdots\right.}{8\cdots}$$

Mathematica [A] time = 0.56, size = 151, normalized size = 0.81

$$-\frac{\left(8a^2-24ab+15b^2\right)\left(\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)-\sqrt{a+b\sinh^2(e+fx)}\right)+2(a-b)\text{sech}^4(e+fx)\left(a+b\cdots\right.}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] -1/8*(-((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2)) + 2*(a -
 b)*Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2) + (8*a^2 - 24*a*b + 15*b^
2)*(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] - Sqrt[a +
 b*Sinh[e + f*x]^2]))/((a - b)^2*f)
```

fricas [B] time = 1.61, size = 4704, normalized size = 25.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^9 + 9*(8*a^2 - 24*a*b + 15
*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 - 24*a*b + 15*b^2)*sinh(f*x +
e)^9 + 4*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 - 24*a*b +
 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^7 + 28*(3
*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh
(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 +
6*(21*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 14*(8*a^2 - 24*a*b + 15*b
^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^5 + 2*(63*(8*a
^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 70*(8*a^2 - 24*a*b + 15*b^2)*cosh(f
*x + e)^3 + 15*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + 4
*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 - 24*a*b + 15*b^2
)*cosh(f*x + e)^6 + 35*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 15*(8*a^
2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x +
e)^3 + 12*(3*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 7*(8*a^2 - 24*a*b
+ 15*b^2)*cosh(f*x + e)^5 + 5*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (
8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 24*a*b +
 15*b^2)*cosh(f*x + e) + (9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^8 + 28*
(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^6 + 30*(8*a^2 - 24*a*b + 15*b^2)*co
sh(f*x + e)^4 + 12*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a
*b + 15*b^2)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f
*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2
 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a -
 b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(
f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)
 + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4
+ 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f
*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*(2*(a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^8 + 16*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)
^7 + 2*(a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 + (16*a^2 - 33*a*b + 17*b^2)*cos
h(f*x + e)^6 + (56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 16*a^2 - 33*a*b +
17*b^2)*sinh(f*x + e)^6 + 2*(56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 + 3*(16
*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(10*a^2 - 21*a*b
 + 11*b^2)*cosh(f*x + e)^4 + (140*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 15*
(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + 20*a^2 - 42*a*b + 22*b^2)*sinh
(f*x + e)^4 + 4*(28*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 5*(16*a^2 - 33*a*
b + 17*b^2)*cosh(f*x + e)^3 + 2*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e))*s
inh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + (56*(a^2 - 2*
a*b + b^2)*cosh(f*x + e)^6 + 15*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^4
+ 12*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^2 + 16*a^2 - 33*a*b + 17*b^2)
*sinh(f*x + e)^2 + 2*a^2 - 4*a*b + 2*b^2 + 2*(8*(a^2 - 2*a*b + b^2)*cosh(f*
x + e)^7 + 3*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^5 + 4*(10*a^2 - 21*a*
b + 11*b^2)*cosh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b
 + b^2)*f*cosh(f*x + e)^9 + 9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x
+ e)^8 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^9 + 4*(a^2 - 2*a*b + b^2)*f*co
sh(f*x + e)^7 + 4*(9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b +
```

b^2)*f)*sinh(f*x + e)^7 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 28*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(21*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 14*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^5 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 2*(63*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(21*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e) + 12*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 28*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 30*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 12*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)), -1/8*(((8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^9 + 9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^9 + 4*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 14*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^5 + 2*(63*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 70*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + 15*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^6 + 35*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 15*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^3 + 12*(3*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 7*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 5*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e) + (9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^8 + 28*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^6 + 30*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 12*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*(2*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^8 + 16*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + 2*(a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^6 + (56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 16*a^2 - 33*a*b + 17*b^2)*sinh(f*x + e)^6 + 2*(56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 + 3*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^4 + (140*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 15*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + 20*a^2 - 42*a*b + 22*b^2)*sinh(f*x + e)^4 + 4*(28*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 5*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^3 + 2*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + (56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 15*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^4 + 12*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^2 + 16*a^2 - 33*a*b + 17*b^2)*sinh(f*x + e)^2 + 2*a^2 - 4*a*b + 2*b^2 + 2*(8*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^7 + 3*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^5 + 4*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^9 + 9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^8 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^9 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 4*(9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^7 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 28*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b

```
^2)*f*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(21*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)^4 + 14*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)
*sinh(f*x + e)^5 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 2*(63*(a^2 - 2
*a*b + b^2)*f*cosh(f*x + e)^5 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 +
15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(21*(a^2 - 2*a*
b + b^2)*f*cosh(f*x + e)^6 + 35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e
)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e) + 12*(3*(a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^7 + 7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b
^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^
2 + (9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 28*(a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^6 + 30*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 12*(a^2 - 2*a*b +
 b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.95Unable to divide, perhaps due to rou
nding error%%%{%%{[262144,0]:[1,0,%%%{-1,[1]%%%}]%%},[10,13,13]%%%}+%%%{%%{
[%%%{-1572864,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,13,12]%%%}+%%%{%%{[%%%
{3932160,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,13,11]%%%}+%%%{%%{[%%%{-524
2880,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,13,10]%%%}+%%%{%%{[%%%{3932160,
[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,13,9]%%%}+%%%{%%{[%%%{-1572864,[5]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,13,8]%%%}+%%%{%%{[%%%{262144,[6]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[10,13,7]%%%}+%%%{%%%{-2621440,[1]%%%},[9,13,13]%%%}
+%%%{%%%{15728640,[2]%%%},[9,13,12]%%%}+%%%{%%%{-39321600,[3]%%%},[9,13,11]
%%%}+%%%{%%%{52428800,[4]%%%},[9,13,10]%%%}+%%%{%%%{-39321600,[5]%%%},[9,13
,9]%%%}+%%%{%%%{15728640,[6]%%%},[9,13,8]%%%}+%%%{%%%{-2621440,[7]%%%},[9,1
3,7]%%%}+%%%{%%{[5242880,0]:[1,0,%%%{-1,[1]%%%}]%%},[8,13,14]%%%}+%%%{%%{[%
%%{-24903680,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,13,13]%%%}+%%%{%%{[%%%{3
9321600,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,13,12]%%%}+%%%{%%{[%%%{-65536
00,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,13,11]%%%}+%%%{%%{[%%%{-52428800,[
4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,13,10]%%%}+%%%{%%{[%%%{66846720,[5]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,13,9]%%%}+%%%{%%{[%%%{-34078720,[6]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[8,13,8]%%%}+%%%{%%{[%%%{6553600,[7]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[8,13,7]%%%}+%%%{%%%{-41943040,[1]%%%},[7,13,14]%%%}+%%%{%
%%{262144000,[2]%%%},[7,13,13]%%%}+%%%{%%%{-692060160,[3]%%%},[7,13,12]%%%}
+%%%{%%%{996147200,[4]%%%},[7,13,11]%%%}+%%%{%%%{-838860800,[5]%%%},[7,13,1
0]%%%}+%%%{%%%{408944640,[6]%%%},[7,13,9]%%%}+%%%{%%%{-104857600,[7]%%%},[7
,13,8]%%%}+%%%{%%%{10485760,[8]%%%},[7,13,7]%%%}+%%%{%%{[41943040,0]:[1,0,%
%%{-1,[1]%%%}]%%},[6,13,15]%%%}+%%%{%%{[%%%{-188743680,[1]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[6,13,14]%%%}+%%%{%%{[%%%{201850880,[2]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[6,13,13]%%%}+%%%{%%{[%%%{403701760,[3]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[6,13,12]%%%}+%%%{%%{[%%%{-1376256000,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[6,13,11]%%%}+%%%{%%{[%%%{1688207360,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[6,13,10]%%%}+%%%{%%{[%%%{-1082654720,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[6,13,9]%%%}+%%%{%%{[%%%{361758720,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,13
,8]%%%}+%%%{%%{[%%%{-49807360,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,13,7]%%
%}+%%%{%%%{-251658240,[1]%%%},[5,13,15]%%%}+%%%{%%%{1719664640,[2]%%%},[5,1
3,14]%%%}+%%%{%%%{-5057282048,[3]%%%},[5,13,13]%%%}+%%%{%%%{8323596288,[4]%
%%},[5,13,12]%%%}+%%%{%%%{-8330936320,[5]%%%},[5,13,11]%%%}+%%%{%%%{5138022
400,[6]%%%},[5,13,10]%%%}+%%%{%%%{-1871708160,[7]%%%},[5,13,9]%%%}+%%%{%%%{
354418688,[8]%%%},[5,13,8]%%%}+%%%{%%%{-24117248,[9]%%%},[5,13,7]%%%}+%%%{%
%{[167772160,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,13,16]%%%}+%%%{%%{[%%%{-88080384
0,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,13,15]%%%}+%%%{%%{[%%%{1373634560,[
```

2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,13,14]%%%}+%%%{%%{[%%%{1009254400,[3]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,13,13]%%%}+%%%{%%{[%%%{-6716129280,[4]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,13,12]%%%}+%%%{%%{[%%%{10881597440,[5]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[4,13,11]%%%}+%%%{%%{[%%%{-9395240960,[6]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[4,13,10]%%%}+%%%{%%{[%%%{4694999040,[7]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[4,13,9]%%%}+%%%{%%{[%%%{-1284505600,[8]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[4,13,8]%%%}+%%%{%%{[%%%{149422080,[9]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[4,13,7]%%%}+%%%{%%%{-671088640,[1]%%%},[3,13,16]%%%}+%%%{%%%
{5200936960,[2]%%%},[3,13,15]%%%}+%%%{%%%{-17741905920,[3]%%%},[3,13,14]%%%
}+%%%{%%%{34907095040,[4]%%%},[3,13,13]%%%}+%%%{%%%{-43557847040,[5]%%%},[3
,13,12]%%%}+%%%{%%%{35641098240,[6]%%%},[3,13,11]%%%}+%%%{%%%{-19042140160,
[7]%%%},[3,13,10]%%%}+%%%{%%%{6364856320,[8]%%%},[3,13,9]%%%}+%%%{%%%{-1195
376640,[9]%%%},[3,13,8]%%%}+%%%{%%%{94371840,[10]%%%},[3,13,7]%%%}+%%%{%%{[
335544320,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,13,17]%%%}+%%%{%%{[%%%{-2348810240,
[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,13,16]%%%}+%%%{%%{[%%%{6668943360,[2]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,13,15]%%%}+%%%{%%{[%%%{-8912896000,[3]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,13,14]%%%}+%%%{%%{[%%%{2507407360,[4]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[2,13,13]%%%}+%%%{%%{[%%%{10058465280,[5]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[2,13,12]%%%}+%%%{%%{[%%%{-17292328960,[6]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[2,13,11]%%%}+%%%{%%{[%%%{13961789440,[7]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[2,13,10]%%%}+%%%{%%{[%%%{-6429081600,[8]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[2,13,9]%%%}+%%%{%%{[%%%{1627914240,[9]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[2,13,8]%%%}+%%%{%%{[%%%{-176947200,[10]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[2,13,7]%%%}+%%%{%%%{-671088640,[1]%%%},[1,13,17]%%%}+%%%{%%%{
6039797760,[2]%%%},[1,13,16]%%%}+%%%{%%%{-24410849280,[3]%%%},[1,13,15]%%%}
+%%%{%%%{58342768640,[4]%%%},[1,13,14]%%%}+%%%{%%%{-91312619520,[5]%%%},[1,
13,13]%%%}+%%%{%%%{97784954880,[6]%%%},[1,13,12]%%%}+%%%{%%%{-72558837760,[
7]%%%},[1,13,11]%%%}+%%%{%%%{36836474880,[8]%%%},[1,13,10]%%%}+%%%{%%%{-122
44746240,[9]%%%},[1,13,9]%%%}+%%%{%%%{2406481920,[10]%%%},[1,13,8]%%%}+%%%{
%%%{-212336640,[11]%%%},[1,13,7]%%%}+%%%{%%{[268435456,0]:[1,0,%%%{-1,[1]%%
%}]%%},[0,13,18]%%%}+%%%{%%{[%%%{-2617245696,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[0,13,17]%%%}+%%%{%%{[%%%{11576279040,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[0,13,16]%%%}+%%%{%%{[%%%{-30660362240,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[0,13,15]%%%}+%%%{%%{[%%%{54027878400,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[0,13,14]%%%}+%%%{%%{[%%%{-66507767808,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
0,13,13]%%%}+%%%{%%{[%%%{58359021568,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,
13,12]%%%}+%%%{%%{[%%%{-36502241280,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,1
3,11]%%%}+%%%{%%{[%%%{15948840960,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,13,
10]%%%}+%%%{%%{[%%%{-4636016640,[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,13,9]
%%%}+%%%{%%{[%%%{806879232,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,13,8]%%%}
+%%%{%%{[%%%{-63700992,[11]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,13,7]%%%} / %
%%{%%{poly1[%%%{1,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,0,0]%%%}+%%%{%%%{-
10,[3]%%%},[9,0,0]%%%}+%%%{%%{[%%%{20,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8
,0,1]%%%}+%%%{%%{poly1[%%%{25,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,0,0]%%%
}+%%%{%%%{-160,[3]%%%},[7,0,1]%%%}+%%%{%%%{40,[4]%%%},[7,0,0]%%%}+%%%{%%{po
ly1[%%%{160,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,2]%%%}+%%%{%%{[%%%{240,
[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,1]%%%}+%%%{%%{poly1[%%%{-190,[4]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,0]%%%}+%%%{%%%{-960,[3]%%%},[5,0,2]%%%}+%
%%{%%%{800,[4]%%%},[5,0,1]%%%}+%%%{%%%{-92,[5]%%%},[5,0,0]%%%}+%%%{%%{[%%%{
640,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,3]%%%}+%%%{%%{poly1[%%%{480,[3]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,2]%%%}+%%%{%%{[%%%{-1480,[4]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[4,0,1]%%%}+%%%{%%{poly1[%%%{570,[5]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[4,0,0]%%%}+%%%{%%%{-2560,[3]%%%},[3,0,3]%%%}+%%%{%%%{4480,[
4]%%%},[3,0,2]%%%}+%%%{%%%{-2400,[5]%%%},[3,0,1]%%%}+%%%{%%%{360,[6]%%%},[3
,0,0]%%%}+%%%{%%{[%%%{1280,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,4]%%%}+%
%%{%%{[%%%{-1280,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,3]%%%}+%%%{%%{poly
1[%%%{-1440,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,2]%%%}+%%%{%%{[%%%{2160
,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,1]%%%}+%%%{%%{poly1[%%%{-675,[6]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0]%%%}+%%%{%%%{-2560,[3]%%%},[1,0,4]%%%}

```
+%%%{%%%{7680,[4]%%%},[1,0,3]%%%}+%%%{%%%{-8640,[5]%%%},[1,0,2]%%%}+%%%{%%%
{4320,[6]%%%},[1,0,1]%%%}+%%%{%%%{-810,[7]%%%},[1,0,0]%%%}+%%%{%%{[%%%{1024
,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,5]%%%}+%%%{%%{[%%%{-3840,[3]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,4]%%%}+%%%{%%{[%%%{5760,[4]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[0,0,3]%%%}+%%%{%%{poly1[%%%{-4320,[5]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[0,0,2]%%%}+%%%{%%{[%%%{1620,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
0,0,1]%%%}+%%%{%%{poly1[%%%{-243,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,0]
%%%} Error: Bad Argument Value
```

maple [C] time = 0.25, size = 43, normalized size = 0.23

$$\frac{`int/indef0`\left(\frac{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,\left(\sinh^{5}\left(fx+e\right)\right)}{\cosh\left(fx+e\right)^{6}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x)
```

```
[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)^5/cosh(f*x+e)^6,sinh(f*x
+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\tanh\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e+f\,x\right)^{5}\,\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^{2}\left(e+fx\right)}\,\tanh^{5}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**5, x)
```

3.458 $\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^3(e+fx)\,dx$

Optimal. Leaf size=126

$$\frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f\sqrt{a-b}}+\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f(a-b)}$$

```
[Out] 1/2*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)/f-1/2*(2*a-3*b)*arctanh((
a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)+1/2*(2*a-3*b)*(a+b*sinh
(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A] time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f\sqrt{a-b}}+\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*Sqrt[a -
 b]*f) + ((2*a - 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a - b)*f) + (Sech[e
+ f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(2*(a - b)*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{(1+x)^2}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)f} + \frac{(2a-3b)\,\text{Subst}\left(\int \frac{\sqrt{a+}}{1+}\right.}{4(a}$$

$$= \frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+\right.}{2(a-b)f}$$

$$= \frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+\right.}{2(a-b)f}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2\sqrt{a-b}\,f} + \frac{(2a-3b)\sqrt{a+b\sinh^2(e}}{2(a-b)f}$$

Mathematica [A] time = 0.42, size = 88, normalized size = 0.70

$$\frac{(\cosh(2(e+fx))+2)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)} - \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] (-(((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a -
b]) + (2 + Cosh[2*(e + f*x)])*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/
(2*f)
```

fricas [B] time = 1.56, size = 1670, normalized size = 13.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a - 3*b)*cosh(f*x + e)^5 + 5*(2*a - 3*b)*cosh(f*x + e)*sinh(f*x
+ e)^4 + (2*a - 3*b)*sinh(f*x + e)^5 + 2*(2*a - 3*b)*cosh(f*x + e)^3 + 2*(5
*(2*a - 3*b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e)^3 + 2*(5*(2*a - 3*b
)*cosh(f*x + e)^3 + 3*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a - 3
*b)*cosh(f*x + e) + (5*(2*a - 3*b)*cosh(f*x + e)^4 + 6*(2*a - 3*b)*cosh(f*x
 + e)^2 + 2*a - 3*b)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*
b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*
```

```
x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*
sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e)
 + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(
f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x
 + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4
*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((a - b)
*cosh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f
*x + e)^4 + 4*(a - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 + 2*a
- 2*b)*sinh(f*x + e)^2 + 4*((a - b)*cosh(f*x + e)^3 + 2*(a - b)*cosh(f*x +
e))*sinh(f*x + e) + a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
)/((a - b)*f*cosh(f*x + e)^5 + 5*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^4 +
(a - b)*f*sinh(f*x + e)^5 + 2*(a - b)*f*cosh(f*x + e)^3 + 2*(5*(a - b)*f*co
sh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^3 + (a - b)*f*cosh(f*x + e) + 2*(5
*(a - b)*f*cosh(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (
5*(a - b)*f*cosh(f*x + e)^4 + 6*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh
(f*x + e)), -1/2*(((2*a - 3*b)*cosh(f*x + e)^5 + 5*(2*a - 3*b)*cosh(f*x + e
)*sinh(f*x + e)^4 + (2*a - 3*b)*sinh(f*x + e)^5 + 2*(2*a - 3*b)*cosh(f*x +
e)^3 + 2*(5*(2*a - 3*b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e)^3 + 2*(5
*(2*a - 3*b)*cosh(f*x + e)^3 + 3*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^2
 + (2*a - 3*b)*cosh(f*x + e) + (5*(2*a - 3*b)*cosh(f*x + e)^4 + 6*(2*a - 3*
b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqr
t(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)
*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*((a - b)*cosh(f*x + e)^4
 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f*x + e)^4 + 4*(a
 - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 + 2*a - 2*b)*sinh(f*x
+ e)^2 + 4*((a - b)*cosh(f*x + e)^3 + 2*(a - b)*cosh(f*x + e))*sinh(f*x + e
) + a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cos
h(f*x + e)^5 + 5*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^4 + (a - b)*f*sinh(f
*x + e)^5 + 2*(a - b)*f*cosh(f*x + e)^3 + 2*(5*(a - b)*f*cosh(f*x + e)^2 +
(a - b)*f)*sinh(f*x + e)^3 + (a - b)*f*cosh(f*x + e) + 2*(5*(a - b)*f*cosh(
f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (5*(a - b)*f*cosh
(f*x + e)^4 + 6*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.06Unable to divide, perhaps due to rou
nding error%%%{%%{[16384,0]:[1,0,%%%{-1,[1]%%%}]%%},[6,9,9]%%%}+%%%{%%{[%%%
{-65536,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,9,8]%%%}+%%%{%%{[%%%{98304,[2
]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,9,7]%%%}+%%%{%%{[%%%{-65536,[3]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[6,9,6]%%%}+%%%{%%{[%%%{16384,[4]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[6,9,5]%%%}+%%%{%%%{-98304,[1]%%%},[5,9,9]%%%}+%%%{%%%{393216
,[2]%%%},[5,9,8]%%%}+%%%{%%%{-589824,[3]%%%},[5,9,7]%%%}+%%%{%%%{393216,[4]
%%%},[5,9,6]%%%}+%%%{%%%{-98304,[5]%%%},[5,9,5]%%%}+%%%{%%{[196608,0]:[1,0,
%%%{-1,[1]%%%}]%%},[4,9,10]%%%}+%%%{%%{[%%%{-737280,[1]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[4,9,9]%%%}+%%%{%%{[%%%{983040,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[4,9,8]%%%}+%%%{%%{[%%%{-491520,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,9,
7]%%%}+%%%{%%{[%%%{49152,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,9,5]%%%}+%%%
{%%%{-786432,[1]%%%},[3,9,10]%%%}+%%%{%%%{3604480,[2]%%%},[3,9,9]%%%}+%%%{%
%%{-6553600,[3]%%%},[3,9,8]%%%}+%%%{%%%{5898240,[4]%%%},[3,9,7]%%%}+%%%{%%%
{-2621440,[5]%%%},[3,9,6]%%%}+%%%{%%%{458752,[6]%%%},[3,9,5]%%%}+%%%{%%{[78
```

```
6432,0]:[1,0,%%%{-1,[1]%%%}]%%},[2,9,11]%%%}+%%%{%%{[%%%{-3538944,[1]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[2,9,10]%%%}+%%%{%%{[%%%{6144000,[2]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[2,9,9]%%%}+%%%{%%{[%%%{-4915200,[3]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[2,9,8]%%%}+%%%{%%{[%%%{1474560,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[2,9,7]%%%}+%%%{%%{[%%%{196608,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,9,
6]%%%}+%%%{%%{[%%%{-147456,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,9,5]%%%}+%
%%{%%%{-1572864,[1]%%%},[1,9,11]%%%}+%%%{%%%{8650752,[2]%%%},[1,9,10]%%%}+%
%%{%%%{-19759104,[3]%%%},[1,9,9]%%%}+%%%{%%%{23986176,[4]%%%},[1,9,8]%%%}+%
%%{%%%{-16318464,[5]%%%},[1,9,7]%%%}+%%%{%%%{5898240,[6]%%%},[1,9,6]%%%}+%%
%{%%%{-884736,[7]%%%},[1,9,5]%%%}+%%%{%%{[1048576,0]:[1,0,%%%{-1,[1]%%%}]%%
},[0,9,12]%%%}+%%%{%%{[%%%{-6553600,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,9
,11]%%%}+%%%{%%{[%%%{17498112,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,9,10]%%
%}+%%%{%%{[%%%{-25870336,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,9,9]%%%}+%%%
{%%{[%%%{22872064,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,9,8]%%%}+%%%{%%{[%%
%{-12091392,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,9,7]%%%}+%%%{%%{[%%%{3538
944,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,9,6]%%%}+%%%{%%{[%%%{-442368,[7]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,9,5]%%%} / %%%{%%{poly1[%%%{1,[1]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[6,0,0]%%%}+%%%{%%%{-6,[2]%%%},[5,0,0]%%%}+%%%{%%{[
%%%{12,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,1]%%%}+%%%{%%{poly1[%%%{3,[2
]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0]%%%}+%%%{%%%{-48,[2]%%%},[3,0,1]%%%
}+%%%{%%%{28,[3]%%%},[3,0,0]%%%}+%%%{%%{[%%%{48,[1]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[2,0,2]%%%}+%%%{%%{[%%%{-24,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0
,1]%%%}+%%%{%%{poly1[%%%{-9,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0]%%%}+
%%%{%%%{-96,[2]%%%},[1,0,2]%%%}+%%%{%%%{144,[3]%%%},[1,0,1]%%%}+%%%{%%%{-54
,[4]%%%},[1,0,0]%%%}+%%%{%%{[%%%{64,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0
,3]%%%}+%%%{%%{[%%%{-144,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,2]%%%}+%%%
{%%{[%%%{108,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,1]%%%}+%%%{%%{poly1[%%
%{-27,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,0]%%%} Error: Bad Argument Va
lue
```

maple [C] time = 0.22, size = 43, normalized size = 0.34

$$\frac{`int/indef0`\left(\frac{\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\,\left(\sinh^3\left(fx+e\right)\right)}{\cosh\left(fx+e\right)^4}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x)
```

```
[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)^3/cosh(f*x+e)^4,sinh(f*x
+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a}\, \tanh\left(fx+e\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e+f\,x\right)^3\, \sqrt{b\sinh\left(e+f\,x\right)^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\tanh^3\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**3, x)
```

3.459 $\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/f+(a+b*sinh(f*x
+e)^2)^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[
a + b*Sinh[e + f*x]^2]/f
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \sinh^2(e+\right.}{2f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2}\right.}{bf}$$

$$= -\frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 1.05

$$\frac{\sqrt{a+b\cosh^2(e+fx)-b} - \sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(e+fx)-b}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] (-(Sqrt[a - b]*ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]]) + Sqrt
[a - b + b*Cosh[e + f*x]^2])/f
```

fricas [B] time = 1.29, size = 624, normalized size = 10.06

$$\left[\frac{\sqrt{a-b}\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\log\left(\frac{b\cosh\left(fx+e\right)^4+4\,b\cosh\left(fx+e\right)\sinh\left(fx+e\right)^3+b\sinh\left(fx+e\right)^4+2\,(4\,a-3\,b)\cosh\left(fx+e\right)^2+2}{\cosh\left(fx+e\right)^4+4\,\cosh\left(fx+e\right)}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*(cosh(f*x + e) + sinh(f*x + e))*log((b*cosh(f*x + e)^4 +
4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(
f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2
)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(
f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x +
e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sin
h(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f
*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 +
 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e
)), -1/2*(2*sqrt(-a + b)*(cosh(f*x + e) + sinh(f*x + e))*arctan(-1/2*sqrt(2
)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
```

```
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*co
sh(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.Non regular value [0
] was discarded and replaced randomly by 0=[-27]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.Non
 regular value [0] was discarded and replaced randomly by 0=[14]Warning, ne
ed to choose a branch for the root of a polynomial with parameters. This mi
ght be wrong.Non regular value [0] was discarded and replaced randomly by 0
=[-85]Warning, need to choose a branch for the root of a polynomial with pa
rameters. This might be wrong.Non regular value [0] was discarded and repla
ced randomly by 0=[-68]Precision problem choosing root in common_EXT, curre
nt precision 14Warning, need to choose a branch for the root of a polynomia
l with parameters. This might be wrong.Non regular value [0] was discarded
and replaced randomly by 0=[-96]Warning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.Non regular value [
0] was discarded and replaced randomly by 0=[86]Warning, need to choose a b
ranch for the root of a polynomial with parameters. This might be wrong.Non
 regular value [0] was discarded and replaced randomly by 0=[22]Warning, ne
ed to choose a branch for the root of a polynomial with parameters. This mi
ght be wrong.Non regular value [0] was discarded and replaced randomly by 0
=[98]Evaluation time: 0.57index.cc index_m operator + Error: Bad Argument V
alue
```

maple [C] time = 0.17, size = 41, normalized size = 0.66

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\,\sinh\left(fx+e\right)}{\cosh\left(fx+e\right)^2},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x)
```

```
[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)/cosh(f*x+e)^2,sinh(f*x+e
))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,\tanh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh\left(e+f\,x\right)\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\,\tanh\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x), x)
```

3.460 $\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Si
nh[e + f*x]^2]/f
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx}\right.}{bf}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.98

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right) - \sqrt{a+b\sinh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] - Sqrt[a + b*Sinh[e
 + f*x]^2])/f)
```

fricas [B] time = 0.82, size = 605, normalized size = 11.20

$$\left[\frac{\sqrt{a}\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\log\left(\frac{b\cosh\left(fx+e\right)^4+4\,b\cosh\left(fx+e\right)\sinh\left(fx+e\right)^3+b\sinh\left(fx+e\right)^4+2\,(4\,a-b)\cosh\left(fx+e\right)^2+2\,\left(3\,b\cos\cdots\right.}{\cosh\left(fx+e\right)^4+4\,\cosh\left(fx+e\right)\sinh\left(fx\cdots\right.}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cosh(f*x + e) + sinh(f*x + e))*log((b*cosh(f*x + e)^4 + 4*b*
cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x +
e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*
x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b
)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*
(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x +
 e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e)), 1/2*(2*sqr
t(-a)*(cosh(f*x + e) + sinh(f*x + e))*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
```

```
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))
) + sqrt(2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x +
 e) + f*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.Non regular value [0
] was discarded and replaced randomly by 0=[32]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.Non
regular value [0] was discarded and replaced randomly by 0=[62]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.Non regular value [0] was discarded and replaced randomly by 0=
[89]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.Non regular value [0] was discarded and replace
d randomly by 0=[-30]Warning, need to choose a branch for the root of a pol
ynomial with parameters. This might be wrong.Non regular value [0] was disc
arded and replaced randomly by 0=[10]Warning, need to choose a branch for t
he root of a polynomial with parameters. This might be wrong.Non regular va
lue [0] was discarded and replaced randomly by 0=[37]Evaluation time: 0.73i
ndex.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.16, size = 46, normalized size = 0.85

$$\frac{\text{`}int/indef0\text{`}\left(\frac{b\sinh\left(fx+e\right)+\frac{a}{\sinh\left(fx+e\right)}}{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((b*sinh(f*x+e)+a/sinh(f*x+e))/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f
*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\coth\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth\left(e+f\,x\right)\,\sqrt{b\,{\sinh\left(e+f\,x\right)}^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\,\coth\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x), x)
```

3.461 $\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=106

$$\frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af}$$

```
[Out] -1/2*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2)/a/f-1/2*(2*a+b)*arctanh((a+b*s
inh(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)+1/2*(2*a+b)*(a+b*sinh(f*x+e)^2)^(1/2
)/a/f
```

Rubi [A] time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*Sqrt[a]*f) + (
(2*a + b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*a*f) - (Csch[e + f*x]^2*(a + b*Si
nh[e + f*x]^2)^(3/2))/(2*a*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)\sqrt{a+bx}}{x^2}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af} + \frac{(2a+b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\right.}{4af}$$

$$= \frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af} - \frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{2af}$$

$$= \frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af} - \frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{2af}$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f} + \frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af}$$

Mathematica [A] time = 0.38, size = 69, normalized size = 0.65

$$-\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \left(\operatorname{csch}^2(e+fx)-2\right)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*(((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (-
2 + Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2])/f
```

fricas [B] time = 0.92, size = 1445, normalized size = 13.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a + b)*cosh(f*x + e)^5 + 5*(2*a + b)*cosh(f*x + e)*sinh(f*x + e)^
4 + (2*a + b)*sinh(f*x + e)^5 - 2*(2*a + b)*cosh(f*x + e)^3 + 2*(5*(2*a + b
)*cosh(f*x + e)^2 - 2*a - b)*sinh(f*x + e)^3 + 2*(5*(2*a + b)*cosh(f*x + e)
^3 - 3*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a + b)*cosh(f*x + e) +
 (5*(2*a + b)*cosh(f*x + e)^4 - 6*(2*a + b)*cosh(f*x + e)^2 + 2*a + b)*sinh
(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)
^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)
^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 +
```

```
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e
)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh
(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sin
h(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh
(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x
 + e)^3 + a*sinh(f*x + e)^4 - 4*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2
- 2*a)*sinh(f*x + e)^2 + 4*(a*cosh(f*x + e)^3 - 2*a*cosh(f*x + e))*sinh(f*x
 + e) + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x
+ e)^5 + 5*a*f*cosh(f*x + e)*sinh(f*x + e)^4 + a*f*sinh(f*x + e)^5 - 2*a*f*
cosh(f*x + e)^3 + 2*(5*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^3 + a*f*cos
h(f*x + e) + 2*(5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*sinh(f*x + e)^
2 + (5*a*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f)*sinh(f*x + e)), 1
/2*(((2*a + b)*cosh(f*x + e)^5 + 5*(2*a + b)*cosh(f*x + e)*sinh(f*x + e)^4
+ (2*a + b)*sinh(f*x + e)^5 - 2*(2*a + b)*cosh(f*x + e)^3 + 2*(5*(2*a + b)*
cosh(f*x + e)^2 - 2*a - b)*sinh(f*x + e)^3 + 2*(5*(2*a + b)*cosh(f*x + e)^3
 - 3*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a + b)*cosh(f*x + e) + (
5*(2*a + b)*cosh(f*x + e)^4 - 6*(2*a + b)*cosh(f*x + e)^2 + 2*a + b)*sinh(f
*x + e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + sqrt(2)*(a*cosh(
f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh(f*x + e)^4 - 4*a*co
sh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2 - 2*a)*sinh(f*x + e)^2 + 4*(a*cosh(f
*x + e)^3 - 2*a*cosh(f*x + e))*sinh(f*x + e) + a)*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)*sinh(f*
x + e)^4 + a*f*sinh(f*x + e)^5 - 2*a*f*cosh(f*x + e)^3 + 2*(5*a*f*cosh(f*x
+ e)^2 - a*f)*sinh(f*x + e)^3 + a*f*cosh(f*x + e) + 2*(5*a*f*cosh(f*x + e)^
3 - 3*a*f*cosh(f*x + e))*sinh(f*x + e)^2 + (5*a*f*cosh(f*x + e)^4 - 6*a*f*c
osh(f*x + e)^2 + a*f)*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.64Unable to divide, perha
ps due to rounding error%%%{128,[6,12,6]%%%}+%%%{%%%{-384,[1]%%%},[6,12,5]%
%%}+%%%{%%%{384,[2]%%%},[6,12,4]%%%}+%%%{%%%{-128,[3]%%%},[6,12,3]%%%}+%%%{
%%{[768,0]:[1,0,%%%{-1,[1]%%%}]%%},[5,12,6]%%%}+%%%{%%{[%%%{-2304,[1]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[5,12,5]%%%}+%%%{%%{[%%%{2304,[2]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[5,12,4]%%%}+%%%{%%{[%%%{-768,[3]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[5,12,3]%%%}+%%%{-1536,[4,12,7]%%%}+%%%{%%%{6528,[1]%%%},[4,12,6]%%%}
+%%%{%%%{-10368,[2]%%%},[4,12,5]%%%}+%%%{%%%{7296,[3]%%%},[4,12,4]%%%}+%%%{
%%%{-1920,[4]%%%},[4,12,3]%%%}+%%%{%%{[-6144,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,
12,7]%%%}+%%%{%%{[%%%{20992,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,6]%%%}
+%%%{%%{[%%%{-26112,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,5]%%%}+%%%{%%{
[%%%{13824,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,4]%%%}+%%%{%%{[%%%{-256
0,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,3]%%%}+%%%{6144,[2,12,8]%%%}+%%%
{%%%{-27648,[1]%%%},[2,12,7]%%%}+%%%{%%%{48000,[2]%%%},[2,12,6]%%%}+%%%{%%%
{-39552,[3]%%%},[2,12,5]%%%}+%%%{%%%{14976,[4]%%%},[2,12,4]%%%}+%%%{%%%{-19
20,[5]%%%},[2,12,3]%%%}+%%%{%%{[12288,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,8]%%
%}+%%%{%%{[%%%{-43008,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,7]%%%}+%%%{%
%{[%%%{56064,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,6]%%%}+%%%{%%{[%%%{-3
3024,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,5]%%%}+%%%{%%{[%%%{8448,[4]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,4]%%%}+%%%{%%{[%%%{-768,[5]%%%},0]:[1,0
```

,%%%{-1,[1]%%%}]%%},[1,12,3]%%%}+%%%{-8192,[0,12,9]%%%}+%%%{%%%{30720,[1]%%
%},[0,12,8]%%%}+%%%{%%%{-44544,[2]%%%},[0,12,7]%%%}+%%%{%%%{31360,[3]%%%},[
0,12,6]%%%}+%%%{%%%{-11136,[4]%%%},[0,12,5]%%%}+%%%{%%%{1920,[5]%%%},[0,12,
4]%%%}+%%%{%%%{-128,[6]%%%},[0,12,3]%%%} / %%%{%%{poly1[%%%{1,[1]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[6,0,0]%%%}+%%%{%%%{6,[2]%%%},[5,0,0]%%%}+%%%{%%{[%%%
{-12,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,1]%%%}+%%%{%%{poly1[%%%{15,[2]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0]%%%}+%%%{%%%{-48,[2]%%%},[3,0,1]%%%}
+%%%{%%%{20,[3]%%%},[3,0,0]%%%}+%%%{%%{[%%%{48,[1]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[2,0,2]%%%}+%%%{%%{[%%%{-72,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,
1]%%%}+%%%{%%{poly1[%%%{15,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0]%%%}+%
%%{%%%{96,[2]%%%},[1,0,2]%%%}+%%%{%%%{-48,[3]%%%},[1,0,1]%%%}+%%%{%%%{6,[4]
%%%},[1,0,0]%%%}+%%%{%%{[%%%{-64,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,3]
%%%}+%%%{%%{[%%%{48,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,2]%%%}+%%%{%%{[
%%%{-12,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,1]%%%}+%%%{%%{poly1[%%%{1,[
4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,0]%%%} Error: Bad Argument Value

maple [C] time = 0.22, size = 58, normalized size = 0.55

$$\frac{\text{`}int/indef0\text{`}\left(\frac{b\sinh\left(fx+e\right)+\frac{a+b}{\sinh\left(fx+e\right)}+\frac{a}{\sinh\left(fx+e\right)^{3}}}{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((b*sinh(f*x+e)+(a+b)/sinh(f*x+e)+a/sinh(f*x+e)^3)/(a+b*sinh(f*
x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\coth\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(e+f\,x\right)^{3}\,\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^{2}\left(e+fx\right)}\,\coth^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**3, x)
```

3.462 $\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=167

$$\frac{(8a^2+8ab-b^2)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{(8a-b)\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{8a^2f}-\frac{(8a^2+8ab-b^2)\tanh}{8a^3}$$

```
[Out] -1/8*(8*a^2+8*a*b-b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f
-1/8*(8*a-b)*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2)/a^2/f-1/4*csch(f*x+e)^
4*(a+b*sinh(f*x+e)^2)^(3/2)/a/f+1/8*(8*a^2+8*a*b-b^2)*(a+b*sinh(f*x+e)^2)^(
1/2)/a^2/f
```

Rubi [A] time = 0.18, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{(8a^2+8ab-b^2)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{(8a^2+8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}-\frac{(8a-b)\text{csch}^2(e+fx)\left(a\right.}{8a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*a^
(3/2)*f) + ((8*a^2 + 8*a*b - b^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a^2*f) -
((8*a - b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(8*a^2*f) - (Csch
[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*a*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2\sqrt{a+bx}}{x^3}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4af} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(8a-b)+2ax\right)\sqrt{a+}}{x^2}\right.}{4a}$$

$$= -\frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{8a^2f} - \frac{\operatorname{csch}^4(e+fx)\left(a\right.}{}$$

$$= \frac{\left(8a^2+8ab-b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+\right.}{8a^2f}$$

$$= \frac{\left(8a^2+8ab-b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+\right.}{8a^2f}$$

$$= -\frac{\left(8a^2+8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{\left(8a^2+8ab-b^2\right)\sqrt{a}}{8a^2f}$$

Mathematica [A] time = 0.58, size = 102, normalized size = 0.61

$$\frac{\left(-8a^2-8ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a}\sqrt{a+b\sinh^2(e+fx)}\left((8a+b)\operatorname{csch}^2(e+fx)+2a\operatorname{csch}^4(e+fx\right.}{8a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 - 8*a*b + b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] - Sqrt
[a]*(-8*a + (8*a + b)*Csch[e + f*x]^2 + 2*a*Csch[e + f*x]^4)*Sqrt[a + b*Sin
h[e + f*x]^2])/(8*a^(3/2)*f)
```

fricas [B] time = 1.36, size = 3880, normalized size = 23.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^9 + 9*(8*a^2 + 8*a*b - b^2)*co
sh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 + 8*a*b - b^2)*sinh(f*x + e)^9 - 4*(8*
a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 + 8*a*b - b^2)*cosh(f*x +
e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 + 8*a*b - b^2)*c
osh(f*x + e)^3 - (8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(
8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 + 8*a*b - b^2)*cosh(f*x
 + e)^4 - 14*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*s
inh(f*x + e)^5 + 2*(63*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 - 70*(8*a^2 +
8*a*b - b^2)*cosh(f*x + e)^3 + 15*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh
(f*x + e)^4 - 4*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 + 8*a*
b - b^2)*cosh(f*x + e)^6 - 35*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 + 15*(8
*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^3
 + 12*(3*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 - 7*(8*a^2 + 8*a*b - b^2)*cos
h(f*x + e)^5 + 5*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 - (8*a^2 + 8*a*b - b
^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 + 8*a*b - b^2)*cosh(f*x + e) +
(9*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^8 - 28*(8*a^2 + 8*a*b - b^2)*cosh(f*
x + e)^6 + 30*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 - 12*(8*a^2 + 8*a*b - b
^2)*cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*sinh(f*x + e))*sqrt(a)*log((b*co
sh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(
4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^
2 + 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cos
h(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e
))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 +
sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x +
e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*
(2*a^2*cosh(f*x + e)^8 + 16*a^2*cosh(f*x + e)*sinh(f*x + e)^7 + 2*a^2*sinh(
f*x + e)^8 - (16*a^2 + a*b)*cosh(f*x + e)^6 + (56*a^2*cosh(f*x + e)^2 - 16*
a^2 - a*b)*sinh(f*x + e)^6 + 2*(56*a^2*cosh(f*x + e)^3 - 3*(16*a^2 + a*b)*c
osh(f*x + e))*sinh(f*x + e)^5 + 2*(10*a^2 + a*b)*cosh(f*x + e)^4 + (140*a^2
*cosh(f*x + e)^4 - 15*(16*a^2 + a*b)*cosh(f*x + e)^2 + 20*a^2 + 2*a*b)*sinh
(f*x + e)^4 + 4*(28*a^2*cosh(f*x + e)^5 - 5*(16*a^2 + a*b)*cosh(f*x + e)^3
 + 2*(10*a^2 + a*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (16*a^2 + a*b)*cosh(f*x
 + e)^2 + (56*a^2*cosh(f*x + e)^6 - 15*(16*a^2 + a*b)*cosh(f*x + e)^4 + 12*
(10*a^2 + a*b)*cosh(f*x + e)^2 - 16*a^2 - a*b)*sinh(f*x + e)^2 + 2*a^2 + 2*
(8*a^2*cosh(f*x + e)^7 - 3*(16*a^2 + a*b)*cosh(f*x + e)^5 + 4*(10*a^2 + a*b
)*cosh(f*x + e)^3 - (16*a^2 + a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*co
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^9 + 9*a^2*f*co
sh(f*x + e)*sinh(f*x + e)^8 + a^2*f*sinh(f*x + e)^9 - 4*a^2*f*cosh(f*x + e)
^7 + 6*a^2*f*cosh(f*x + e)^5 + 4*(9*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x
 + e)^7 + 28*(3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)^
6 - 4*a^2*f*cosh(f*x + e)^3 + 6*(21*a^2*f*cosh(f*x + e)^4 - 14*a^2*f*cosh(f
*x + e)^2 + a^2*f)*sinh(f*x + e)^5 + 2*(63*a^2*f*cosh(f*x + e)^5 - 70*a^2*f
*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e))*sinh(f*x + e)^4 + a^2*f*cosh(f*x
 + e) + 4*(21*a^2*f*cosh(f*x + e)^6 - 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*c
osh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^3 + 12*(3*a^2*f*cosh(f*x + e)^7 - 7*a
^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(
f*x + e)^2 + (9*a^2*f*cosh(f*x + e)^8 - 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f
```

```
*cosh(f*x + e)^4 - 12*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)), 1/8*((
(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^9 + 9*(8*a^2 + 8*a*b - b^2)*cosh(f*x +
e)*sinh(f*x + e)^8 + (8*a^2 + 8*a*b - b^2)*sinh(f*x + e)^9 - 4*(8*a^2 + 8*a
*b - b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 - 8*
a^2 - 8*a*b + b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 + 8*a*b - b^2)*cosh(f*x +
 e)^3 - (8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 + 8
*a*b - b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 -
 14*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*sinh(f*x +
 e)^5 + 2*(63*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 - 70*(8*a^2 + 8*a*b - b
^2)*cosh(f*x + e)^3 + 15*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)
^4 - 4*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 + 8*a*b - b^2)*
cosh(f*x + e)^6 - 35*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 + 15*(8*a^2 + 8*
a*b - b^2)*cosh(f*x + e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^3 + 12*(3*(
8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 - 7*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e
)^5 + 5*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 - (8*a^2 + 8*a*b - b^2)*cosh(
f*x + e))*sinh(f*x + e)^2 + (8*a^2 + 8*a*b - b^2)*cosh(f*x + e) + (9*(8*a^2
 + 8*a*b - b^2)*cosh(f*x + e)^8 - 28*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^6
+ 30*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 - 12*(8*a^2 + 8*a*b - b^2)*cosh(
f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*sinh(f*x + e))*sqrt(-a)*arctan(1/2*sqrt(2
)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e
) + a*sinh(f*x + e))) + 2*sqrt(2)*(2*a^2*cosh(f*x + e)^8 + 16*a^2*cosh(f*x
+ e)*sinh(f*x + e)^7 + 2*a^2*sinh(f*x + e)^8 - (16*a^2 + a*b)*cosh(f*x + e)
^6 + (56*a^2*cosh(f*x + e)^2 - 16*a^2 - a*b)*sinh(f*x + e)^6 + 2*(56*a^2*co
sh(f*x + e)^3 - 3*(16*a^2 + a*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(10*a^2
 + a*b)*cosh(f*x + e)^4 + (140*a^2*cosh(f*x + e)^4 - 15*(16*a^2 + a*b)*cosh
(f*x + e)^2 + 20*a^2 + 2*a*b)*sinh(f*x + e)^4 + 4*(28*a^2*cosh(f*x + e)^5 -
 5*(16*a^2 + a*b)*cosh(f*x + e)^3 + 2*(10*a^2 + a*b)*cosh(f*x + e))*sinh(f*
x + e)^3 - (16*a^2 + a*b)*cosh(f*x + e)^2 + (56*a^2*cosh(f*x + e)^6 - 15*(1
6*a^2 + a*b)*cosh(f*x + e)^4 + 12*(10*a^2 + a*b)*cosh(f*x + e)^2 - 16*a^2 -
 a*b)*sinh(f*x + e)^2 + 2*a^2 + 2*(8*a^2*cosh(f*x + e)^7 - 3*(16*a^2 + a*b)
*cosh(f*x + e)^5 + 4*(10*a^2 + a*b)*cosh(f*x + e)^3 - (16*a^2 + a*b)*cosh(f
*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(
a^2*f*cosh(f*x + e)^9 + 9*a^2*f*cosh(f*x + e)*sinh(f*x + e)^8 + a^2*f*sinh(
f*x + e)^9 - 4*a^2*f*cosh(f*x + e)^7 + 6*a^2*f*cosh(f*x + e)^5 + 4*(9*a^2*f
*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^7 + 28*(3*a^2*f*cosh(f*x + e)^3 - a
^2*f*cosh(f*x + e))*sinh(f*x + e)^6 - 4*a^2*f*cosh(f*x + e)^3 + 6*(21*a^2*f
*cosh(f*x + e)^4 - 14*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)^5 + 2*(6
3*a^2*f*cosh(f*x + e)^5 - 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e)
)*sinh(f*x + e)^4 + a^2*f*cosh(f*x + e) + 4*(21*a^2*f*cosh(f*x + e)^6 - 35*
a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^3 +
 12*(3*a^2*f*cosh(f*x + e)^7 - 7*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x +
 e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)^2 + (9*a^2*f*cosh(f*x + e)^8 - 2
8*a^2*f*cosh(f*x + e)^6 + 30*a^2*f*cosh(f*x + e)^4 - 12*a^2*f*cosh(f*x + e)
^2 + a^2*f)*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.78Unable to divide, perha
ps due to rounding error%%%{2048,[10,20,10]%%%}+%%%{%%%{-10240,[1]%%%},[10,
20,9]%%%}+%%%{%%%{20480,[2]%%%},[10,20,8]%%%}+%%%{%%%{-20480,[3]%%%},[10,20
,7]%%%}+%%%{%%%{10240,[4]%%%},[10,20,6]%%%}+%%%{%%%{-2048,[5]%%%},[10,20,5]
%%%}+%%%{%%{[20480,0]:[1,0,%%%{-1,[1]%%%}]%%},[9,20,10]%%%}+%%%{%%{[%%%{-10
```

```
2400,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,20,9]%%%}+%%%{%%{[%%%{204800,[2]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,20,8]%%%}+%%%{%%{[%%%{-204800,[3]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[9,20,7]%%%}+%%%{%%{[%%%{102400,[4]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[9,20,6]%%%}+%%%{%%{[%%%{-20480,[5]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[9,20,5]%%%}+%%%{-40960,[8,20,11]%%%}+%%%{%%%{296960,[1]%%%},[8,20
,10]%%%}+%%%{%%%{-870400,[2]%%%},[8,20,9]%%%}+%%%{%%%{1331200,[3]%%%},[8,20
,8]%%%}+%%%{%%%{-1126400,[4]%%%},[8,20,7]%%%}+%%%{%%%{501760,[5]%%%},[8,20,
6]%%%}+%%%{%%%{-92160,[6]%%%},[8,20,5]%%%}+%%%{%%{[-327680,0]:[1,0,%%%{-1,[
1]%%%}]%%},[7,20,11]%%%}+%%%{%%{[%%%{1884160,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[7,20,10]%%%}+%%%{%%{[%%%{-4505600,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[7,20,9]%%%}+%%%{%%{[%%%{5734400,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,20,8
]%%%}+%%%{%%{[%%%{-4096000,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,20,7]%%%}+
%%%{%%{[%%%{1556480,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,20,6]%%%}+%%%{%%{
[%%%{-245760,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,20,5]%%%}+%%%{327680,[6,
20,12]%%%}+%%%{%%%{-2785280,[1]%%%},[6,20,11]%%%}+%%%{%%%{9441280,[2]%%%},[
6,20,10]%%%}+%%%{%%%{-16896000,[3]%%%},[6,20,9]%%%}+%%%{%%%{17408000,[4]%%%
},[6,20,8]%%%}+%%%{%%%{-10362880,[5]%%%},[6,20,7]%%%}+%%%{%%%{3297280,[6]%%
%},[6,20,6]%%%}+%%%{%%%{-430080,[7]%%%},[6,20,5]%%%}+%%%{%%{[1966080,0]:[1,
0,%%%{-1,[1]%%%}]%%},[5,20,12]%%%}+%%%{%%{[%%%{-12124160,[1]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[5,20,11]%%%}+%%%{%%{[%%%{31645696,[2]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[5,20,10]%%%}+%%%{%%{[%%%{-45178880,[3]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[5,20,9]%%%}+%%%{%%{[%%%{37928960,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[5,20,8]%%%}+%%%{%%{[%%%{-18595840,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,
20,7]%%%}+%%%{%%{[%%%{4874240,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,20,6]%%
%}+%%%{%%{[%%%{-516096,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,20,5]%%%}+%%%{
-1310720,[4,20,13]%%%}+%%%{%%%{11468800,[1]%%%},[4,20,12]%%%}+%%%{%%%{-4055
0400,[2]%%%},[4,20,11]%%%}+%%%{%%%{77025280,[3]%%%},[4,20,10]%%%}+%%%{%%%{-
86528000,[4]%%%},[4,20,9]%%%}+%%%{%%%{58859520,[5]%%%},[4,20,8]%%%}+%%%{%%%
{-23552000,[6]%%%},[4,20,7]%%%}+%%%{%%%{5017600,[7]%%%},[4,20,6]%%%}+%%%{%%
%{-430080,[8]%%%},[4,20,5]%%%}+%%%{%%{[-5242880,0]:[1,0,%%%{-1,[1]%%%}]%%},
[3,20,13]%%%}+%%%{%%{[%%%{32768000,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,20
,12]%%%}+%%%{%%{[%%%{-87490560,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,20,11]
%%%}+%%%{%%{[%%%{129679360,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,20,10]%%%}
+%%%{%%{[%%%{-115916800,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,20,9]%%%}+%%%
{%%{[%%%{63406080,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,20,8]%%%}+%%%{%%{[%
%%{-20480000,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,20,7]%%%}+%%%{%%{[%%%{35
22560,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,20,6]%%%}+%%%{%%{[%%%{-245760,[
8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,20,5]%%%}+%%%{2621440,[2,20,14]%%%}+%%
%{%%%{-20971520,[1]%%%},[2,20,13]%%%}+%%%{%%%{70451200,[2]%%%},[2,20,12]%%%
}+%%%{%%%{-130580480,[3]%%%},[2,20,11]%%%}+%%%{%%%{146728960,[4]%%%},[2,20,
10]%%%}+%%%{%%%{-103024640,[5]%%%},[2,20,9]%%%}+%%%{%%%{44830720,[6]%%%},[2
,20,8]%%%}+%%%{%%%{-11571200,[7]%%%},[2,20,7]%%%}+%%%{%%%{1607680,[8]%%%},[
2,20,6]%%%}+%%%{%%%{-92160,[9]%%%},[2,20,5]%%%}+%%%{%%{[5242880,0]:[1,0,%%%
{-1,[1]%%%}]%%},[1,20,14]%%%}+%%%{%%{[%%%{-31457280,[1]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[1,20,13]%%%}+%%%{%%{[%%%{80609280,[2]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[1,20,12]%%%}+%%%{%%{[%%%{-115015680,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[1,20,11]%%%}+%%%{%%{[%%%{99962880,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
1,20,10]%%%}+%%%{%%{[%%%{-54497280,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,20
,9]%%%}+%%%{%%{[%%%{18554880,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,20,8]%%%
}+%%%{%%{[%%%{-3809280,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,20,7]%%%}+%%%{
%%{[%%%{430080,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,20,6]%%%}+%%%{%%{[%%%{
-20480,[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,20,5]%%%}+%%%{-2097152,[0,20,1
5]%%%}+%%%{%%%{13107200,[1]%%%},[0,20,14]%%%}+%%%{%%%{-35389440,[2]%%%},[0,
20,13]%%%}+%%%{%%%{54067200,[3]%%%},[0,20,12]%%%}+%%%{%%%{-51486720,[4]%%%}
,[0,20,11]%%%}+%%%{%%%{31795200,[5]%%%},[0,20,10]%%%}+%%%{%%%{-12871680,[6]
%%%},[0,20,9]%%%}+%%%{%%%{3379200,[7]%%%},[0,20,8]%%%}+%%%{%%%{-552960,[8]%
%%},[0,20,7]%%%}+%%%{%%%{51200,[9]%%%},[0,20,6]%%%}+%%%{%%%{-2048,[10]%%%},
[0,20,5]%%%} / %%%{%%{poly1[%%%{1,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,0,
0]%%%}+%%%{%%%{10,[3]%%%},[9,0,0]%%%}+%%%{%%{[%%%{-20,[2]%%%},0]:[1,0,%%%{-
```

```
1,[1]%%%}]%%},[8,0,1]%%%}+%%%{%%{poly1[%%%{45,[3]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[8,0,0]%%%}+%%%{%%%{-160,[3]%%%},[7,0,1]%%%}+%%%{%%%{120,[4]%%%},[7,0
,0]%%%}+%%%{%%{poly1[%%%{160,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,2]%%%}
+%%%{%%{[%%%{-560,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,1]%%%}+%%%{%%{pol
y1[%%%{210,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,0]%%%}+%%%{%%%{960,[3]%%
%},[5,0,2]%%%}+%%%{%%%{-1120,[4]%%%},[5,0,1]%%%}+%%%{%%%{252,[5]%%%},[5,0,0
]%%%}+%%%{%%{[%%%{-640,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,3]%%%}+%%%{%
%{poly1[%%%{2400,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,2]%%%}+%%%{%%{[%%%
{-1400,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,1]%%%}+%%%{%%{poly1[%%%{210,
[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0]%%%}+%%%{%%%{-2560,[3]%%%},[3,0,3
]%%%}+%%%{%%%{3200,[4]%%%},[3,0,2]%%%}+%%%{%%%{-1120,[5]%%%},[3,0,1]%%%}+%%
%{%%%{120,[6]%%%},[3,0,0]%%%}+%%%{%%{[%%%{1280,[2]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[2,0,4]%%%}+%%%{%%{[%%%{-3840,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,
0,3]%%%}+%%%{%%{poly1[%%%{2400,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,2]%%
%}+%%%{%%{[%%%{-560,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,1]%%%}+%%%{%%{p
oly1[%%%{45,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0]%%%}+%%%{%%%{2560,[3]
%%%},[1,0,4]%%%}+%%%{%%%{-2560,[4]%%%},[1,0,3]%%%}+%%%{%%%{960,[5]%%%},[1,0
,2]%%%}+%%%{%%%{-160,[6]%%%},[1,0,1]%%%}+%%%{%%%{10,[7]%%%},[1,0,0]%%%}+%%%
{%%{[%%%{-1024,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,5]%%%}+%%%{%%{[%%%{1
280,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,4]%%%}+%%%{%%{[%%%{-640,[4]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,3]%%%}+%%%{%%{poly1[%%%{160,[5]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[0,0,2]%%%}+%%%{%%{[%%%{-20,[6]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[0,0,1]%%%}+%%%{%%{poly1[%%%{1,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[0,0,0]%%%} Error: Bad Argument Value
```

maple **[C]** time = 0.24, size = 80, normalized size = 0.48

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\left(\cosh^4\left(fx+e\right)\right)\left(a-b+b\left(\cosh^2\left(fx+e\right)\right)\right)}{\sinh\left(fx+e\right)\left(\cosh^4\left(fx+e\right)-2\left(\cosh^2\left(fx+e\right)\right)+1\right)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(cosh(f*x+e)^4-2*cosh(f*x+e)^2+1)*cosh(f*x+e)^4*
(a-b+b*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,\coth\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^5, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(e+f\,x\right)^5\,\sqrt{b\,\sinh\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.463 $\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^4(e+fx)\,dx$

Optimal. Leaf size=292

$$-\frac{\tanh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}-\frac{(3a-4b)\tanh(e+fx)\sqrt{a}}{3f(a-b)}$$

```
[Out] -1/3*(7*a-8*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(
f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*
a-4*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2
)^(1/2)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(7*a-8*b)*(
a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f-1/3*(3*a-4*b)*(a+b*sinh(f*x+e)
^2)^(1/2)*tanh(f*x+e)/(a-b)/f-1/3*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3/f
```

Rubi [A] time = 0.31, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3196, 467, 578, 531, 418, 492, 411}

$$-\frac{\tanh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}-\frac{(3a-4b)\tanh(e+fx)\sqrt{a}}{3f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] -((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[
e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a
 + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[
e + f*x])/(3*(a - b)*f) - ((3*a - 4*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e +
 f*x])/(3*(a - b)*f) - (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3)/(3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
```

```
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{(1+x^2)^{5/2}}\,dx, x, \sinh(e+f\right.}{f}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)}{3f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+\right.}{}$$

$$= -\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f} - \frac{\sqrt{a+b\sinh^2(e+}}{3}$$

$$= -\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f} - \frac{\sqrt{a+b\sinh^2(e+}}{3}$$

$$= \frac{(3a-4b)F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

$$= -\frac{(7a-8b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 2.03, size = 214, normalized size = 0.73

$$\frac{-\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\left(4\left(4a^2-6ab+b^2\right)\cosh(2(e+fx))+8a^2+b(4a-5b)\cosh(4(e+fx))-12ab+b^2\right)}{2\sqrt{2}} + 8ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e\right.}{6f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] ((-2*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(
e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*El
lipticF[I*(e + f*x), b/a] - ((8*a^2 - 12*a*b + b^2 + 4*(4*a^2 - 6*a*b + b^2
)*Cosh[2*(e + f*x)] + (4*a - 5*b)*b*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh
[e + f*x])/(2*Sqrt[2]))/(6*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\tanh\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.36Unable to divide, perhaps due to rou
nding error%%%{65536,[8,11,11]%%%}+%%%{%%%{-327680,[1]%%%},[8,11,10]%%%}+%%
%{%%%{655360,[2]%%%},[8,11,9]%%%}+%%%{%%%{-655360,[3]%%%},[8,11,8]%%%}+%%%{
%%%{327680,[4]%%%},[8,11,7]%%%}+%%%{%%%{-65536,[5]%%%},[8,11,6]%%%}+%%%{%%{
[-524288,0]:[1,0,%%%{-1,[1]%%%}]%%},[7,11,11]%%%}+%%%{%%{[%%%{2621440,[1]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,11,10]%%%}+%%%{%%{[%%%{-5242880,[2]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[7,11,9]%%%}+%%%{%%{[%%%{5242880,[3]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[7,11,8]%%%}+%%%{%%{[%%%{-2621440,[4]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[7,11,7]%%%}+%%%{%%{[%%%{524288,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[7,11,6]%%%}+%%%{1048576,[6,11,12]%%%}+%%%{%%%{-4456448,[1]%%%},[6,11,1
1]%%%}+%%%{%%%{6553600,[2]%%%},[6,11,10]%%%}+%%%{%%%{-2621440,[3]%%%},[6,11
,9]%%%}+%%%{%%%{-2621440,[4]%%%},[6,11,8]%%%}+%%%{%%%{2883584,[5]%%%},[6,11
,7]%%%}+%%%{%%%{-786432,[6]%%%},[6,11,6]%%%}+%%%{%%{[-6291456,0]:[1,0,%%%{-
1,[1]%%%}]%%},[5,11,12]%%%}+%%%{%%{[%%%{34078720,[1]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[5,11,11]%%%}+%%%{%%{[%%%{-76021760,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[5,11,10]%%%}+%%%{%%{[%%%{89128960,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[5,11,9]%%%}+%%%{%%{[%%%{-57671680,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,11
,8]%%%}+%%%{%%{[%%%{19398656,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,11,7]%%%
}+%%%{%%{[%%%{-2621440,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,11,6]%%%}+%%%{
6291456,[4,11,13]%%%}+%%%{%%%{-28311552,[1]%%%},[4,11,12]%%%}+%%%{%%%{42336
256,[2]%%%},[4,11,11]%%%}+%%%{%%%{-7208960,[3]%%%},[4,11,10]%%%}+%%%{%%%{-4
8496640,[4]%%%},[4,11,9]%%%}+%%%{%%%{57933824,[5]%%%},[4,11,8]%%%}+%%%{%%%{
-27394048,[6]%%%},[4,11,7]%%%}+%%%{%%%{4849664,[7]%%%},[4,11,6]%%%}+%%%{%%{
[-25165824,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,11,13]%%%}+%%%{%%{[%%%{155189248,[
1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,11,12]%%%}+%%%{%%{[%%%{-406323200,[2]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,11,11]%%%}+%%%{%%{[%%%{584581120,[3]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[3,11,10]%%%}+%%%{%%{[%%%{-498073600,[4]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[3,11,9]%%%}+%%%{%%{[%%%{250609664,[5]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[3,11,8]%%%}+%%%{%%{[%%%{-68681728,[6]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[3,11,7]%%%}+%%%{%%{[%%%{7864320,[7]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[3,11,6]%%%}+%%%{16777216,[2,11,14]%%%}+%%%{%%%{-96468992,[1]%%%},[2
,11,13]%%%}+%%%{%%%{221249536,[2]%%%},[2,11,12]%%%}+%%%{%%%{-239337472,[3]%
%%},[2,11,11]%%%}+%%%{%%%{79953920,[4]%%%},[2,11,10]%%%}+%%%{%%%{85458944,[
5]%%%},[2,11,9]%%%}+%%%{%%%{-105381888,[6]%%%},[2,11,8]%%%}+%%%{%%%{4482662
4,[7]%%%},[2,11,7]%%%}+%%%{%%%{-7077888,[8]%%%},[2,11,6]%%%}+%%%{%%{[-33554
432,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,11,14]%%%}+%%%{%%{[%%%{243269632,[1]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[1,11,13]%%%}+%%%{%%{[%%%{-769654784,[2]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[1,11,12]%%%}+%%%{%%{[%%%{1387790336,[3]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[1,11,11]%%%}+%%%{%%{[%%%{-1559756800,[4]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[1,11,10]%%%}+%%%{%%{[%%%{1118830592,[5]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[1,11,9]%%%}+%%%{%%{[%%%{-500170752,[6]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[1,11,8]%%%}+%%%{%%{[%%%{127401984,[7]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[1,11,7]%%%}+%%%{%%{[%%%{-14155776,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[1,11,6]%%%}+%%%{16777216,[0,11,15]%%%}+%%%{%%%{-134217728,[1]%%%},[0,11,
14]%%%}+%%%{%%%{476053504,[2]%%%},[0,11,13]%%%}+%%%{%%%{-982515712,[3]%%%},
[0,11,12]%%%}+%%%{%%%{1300299776,[4]%%%},[0,11,11]%%%}+%%%{%%%{-1144324096,
[5]%%%},[0,11,10]%%%}+%%%{%%%{669646848,[6]%%%},[0,11,9]%%%}+%%%{%%%{-25126
5024,[7]%%%},[0,11,8]%%%}+%%%{%%%{54853632,[8]%%%},[0,11,7]%%%}+%%%{%%%{-53
08416,[9]%%%},[0,11,6]%%%} / %%%{%%%{1,[2]%%%},[8,0,0]%%%}+%%%{%%{poly1[%%%
{-8,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,0,0]%%%}+%%%{%%%{16,[2]%%%},[6,0,
1]%%%}+%%%{%%%{12,[3]%%%},[6,0,0]%%%}+%%%{%%{[%%%{-96,[2]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[5,0,1]%%%}+%%%{%%{poly1[%%%{40,[3]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[5,0,0]%%%}+%%%{%%%{96,[2]%%%},[4,0,2]%%%}+%%%{%%%{48,[3]%%%},[4,0,1]
%%%}+%%%{%%%{-74,[4]%%%},[4,0,0]%%%}+%%%{%%{poly1[%%%{-384,[2]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[3,0,2]%%%}+%%%{%%{[%%%{448,[3]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[3,0,1]%%%}+%%%{%%{poly1[%%%{-120,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[3,0,0]%%%}+%%%{%%%{256,[2]%%%},[2,0,3]%%%}+%%%{%%%{-192,[3]%%%},[2,0,2]%%
```

```
%}+%%%{%%%{-144,[4]%%%},[2,0,1]%%%}+%%%{%%%{108,[5]%%%},[2,0,0]%%%}+%%%{%%{
[%%%{-512,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,3]%%%}+%%%{%%{poly1[%%%{1
152,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,2]%%%}+%%%{%%{[%%%{-864,[4]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1]%%%}+%%%{%%{poly1[%%%{216,[5]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[1,0,0]%%%}+%%%{%%%{256,[2]%%%},[0,0,4]%%%}+%%%{%%%{-7
68,[3]%%%},[0,0,3]%%%}+%%%{%%%{864,[4]%%%},[0,0,2]%%%}+%%%{%%%{-432,[5]%%%}
,[0,0,1]%%%}+%%%{%%%{81,[6]%%%},[0,0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.31, size = 369, normalized size = 1.26

$$-\frac{\left(4\sqrt{-\frac{b}{a}}\,ab-5\sqrt{-\frac{b}{a}}\,b^{2}\right)\sinh\left(fx+e\right)\left(\cosh^{4}\left(fx+e\right)\right)+\left(4\sqrt{-\frac{b}{a}}\,a^{2}-10\sqrt{-\frac{b}{a}}\,ab+6\sqrt{-\frac{b}{a}}\,b^{2}\right)\left(\cosh^{2}\left(fx+e\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x)
```

```
[Out] -1/3*((4*(-1/a*b)^(1/2)*a*b-5*(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4
+(4*(-1/a*b)^(1/2)*a^2-10*(-1/a*b)^(1/2)*a*b+6*(-1/a*b)^(1/2)*b^2)*cosh(f*x
+e)^2*sinh(f*x+e)+(-(-1/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*
b^2)*sinh(f*x+e)-(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(3
*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-11*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8
*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2)/cosh
(f*x+e)^3/(a-b)/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\tanh\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tanh\left(e+f\,x\right)^{4}\,\sqrt{b\,\sinh\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^{2}\left(e+fx\right)}\,\tanh^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**4, x)
```

3.464 $\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^2(e+fx)\,dx$

Optimal. Leaf size=168

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2\operatorname{sech}}{}$$

```
[Out] -2*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e
)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1
/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))^(1/
2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1
-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*si
nh(f*x+e)^2)/a)^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.18, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3196, 467, 531, 418, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2\operatorname{sech}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] (-2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Elli
pticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x
]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*S
inh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\,\tanh^2(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{\left(1+x^2\right)^{3/2}}\,dx, x, \sinh(e+f}{f}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+\right.}{}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{f} + \frac{\left(a\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+\right.}{}$$

$$= \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} +$$

$$= -\frac{2E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 0.52, size = 150, normalized size = 0.89

$$\frac{\tanh(e+fx)(-2a-b\cosh(2(e+fx))+b)+i\sqrt{2}\,a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right)-2i\sqrt{2}\,a\sqrt{\frac{2a+b\cosh(2(e+fx)}{a}}}{f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
f*x), b/a] + I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[
```

```
I*(e + f*x), b/a] + (-2*a + b - b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(f*Sqrt
[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\tanh\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.48Error: Bad Argument Type
```

maple [A] time = 0.29, size = 233, normalized size = 1.39

$$-\frac{\sqrt{-\frac{b}{a}}\,b\left(\sinh^{3}\left(fx+e\right)\right)-a\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\;\text{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+2b\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x)
```

```
[Out] -((-1/a*b)^(1/2)*b*sinh(f*x+e)^3-a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))-2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
llipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+(-1/a*b)^(1/2)*a*sinh(f*x+
e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\tanh\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tanh\left(e+f\,x\right)^{2}\,\sqrt{b\,\sinh\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^2\left(e+fx\right)}\,\tanh^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**2,x)
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**2, x)
```

3.465 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(
1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\\ &= -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 69, normalized size = 1.15

$$-\frac{ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,E\left(i(e+fx)\middle|\frac{b}{a}\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a]
)/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.22, size = 140, normalized size = 2.33

$$\frac{\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\left(a\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{}\right.\right.}{\sqrt{-\frac{b}{a}}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{b\sinh\left(e+fx\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

3.466 $\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=202

$$\frac{2\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+}}{af\sqrt{\frac{\text{sech}^2(e+fx)(a}{}}}$$

```
[Out] -coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+s
inh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(
1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+
e)^2)/a)^(1/2)+(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*El
lipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+2*(a+
b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.20, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3196, 473, 531, 418, 492, 411}

$$\frac{2\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+}}{af\sqrt{\frac{\text{sech}^2(e+fx)(a}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (2*EllipticE[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Se
ch[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*EllipticF[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/f
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}\,\sqrt{a+bx^2}}{x^2}\,dx, x, \text{sinh}\right.}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\,\text{sech}(e\right.}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(2b\sqrt{\cosh^2(e+fx)}\,\text{sech}(\right.}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a+b)F\left(\tan^{-1}(\sinh(e+\right.}{af\sqrt{}}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{2E\left(\tan^{-1}(\sinh(e+fx))|1\right.}{f\sqrt{\frac{\text{sec}}{}}}$$

Mathematica [C] time = 0.58, size = 154, normalized size = 0.76

$$\frac{\coth(e+fx)(-2a-b\cosh(2(e+fx))+b)+i\sqrt{2}\,(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)-2i\sqrt{2}\,a\sqrt{\frac{2a+b\cos}{}}}{f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*Sqrt[2]*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*(a -
```

```
 b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(f
*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\coth\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{64,[4,8,4]%%%}+%%%{%%%{-128,[1]%%%},[4,8,3]%%%}+%%%{%%%{64,[2]%%%},[
4,8,2]%%%}+%%%{%%{[256,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8,4]%%%}+%%%{%%{[%%%{-
512,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8,3]%%%}+%%%{%%{[%%%{256,[2]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8,2]%%%}+%%%{-512,[2,8,5]%%%}+%%%{%%%{1408,[1
]%%%},[2,8,4]%%%}+%%%{%%%{-1280,[2]%%%},[2,8,3]%%%}+%%%{%%%{384,[3]%%%},[2,
8,2]%%%}+%%%{%%{[-1024,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,8,5]%%%}+%%%{%%{[%%%{2
304,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,8,4]%%%}+%%%{%%{[%%%{-1536,[2]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,8,3]%%%}+%%%{%%{[%%%{256,[3]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[1,8,2]%%%}+%%%{1024,[0,8,6]%%%}+%%%{%%%{-2560,[1]%%%},[0,
8,5]%%%}+%%%{%%%{2112,[2]%%%},[0,8,4]%%%}+%%%{%%%{-640,[3]%%%},[0,8,3]%%%}+
%%%{%%%{64,[4]%%%},[0,8,2]%%%} / %%%{%%%{1,[1]%%%},[4,0,0]%%%}+%%%{%%{poly1
[%%%{4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,0]%%%}+%%%{%%%{-8,[1]%%%},[2
,0,1]%%%}+%%%{%%%{6,[2]%%%},[2,0,0]%%%}+%%%{%%{poly1[%%%{-16,[1]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[1,0,1]%%%}+%%%{%%{poly1[%%%{4,[2]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[1,0,0]%%%}+%%%{%%%{16,[1]%%%},[0,0,2]%%%}+%%%{%%%{-8,[2]%%%},[
0,0,1]%%%}+%%%{%%%{1,[3]%%%},[0,0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.27, size = 215, normalized size = 1.06

$$-\frac{-\sinh\left(fx+e\right)\sqrt{\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\left(a\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\,\mathrm{EllipticF}\left(\right.\right.}{\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -(-sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*E
llipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2)))+(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*co
sh(f*x+e)^2)/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/
2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\coth\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \coth\left(e+f\,x\right)^2\,\sqrt{b\,\mathrm{sinh}\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh^{2}{\left(e + f x \right)}} \coth^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**2, x)
```

3.467 $\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=270

$$\frac{(7a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{(3a+b)\coth(e+fx)\sqrt{a+b\,\mathrm{s}}}{3af}$$

```
[Out] -1/3*(3*a+b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-1/3*coth(f*x+e)^3*(a
+b*sinh(f*x+e)^2)^(1/2)/f-1/3*(7*a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))
*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^
2)/a)^(1/2)+1/3*(3*a+5*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/
2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)
*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+
1/3*(7*a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/f
```

Rubi [A] time = 0.30, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3196, 473, 580, 531, 418, 492, 411}

$$\frac{(7a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{(3a+b)\coth(e+fx)\sqrt{a+b\,\mathrm{s}}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((3*a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e +
f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a + b)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqr
t[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[Ar
cTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3
*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a
 + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{(1+x^2)^{3/2}\sqrt{a+bx^2}}{x^4}dx, x, \sinh(\ldots\right.}{f}$$

$$= -\frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+\ldots\right.}{\ldots}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+fx)\sqrt{a+\ldots}}{3f}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+fx)\sqrt{a+\ldots}}{3f}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+fx)\sqrt{a+\ldots}}{3f}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+fx)\sqrt{a+\ldots}}{3f}$$

Mathematica [C] time = 3.10, size = 210, normalized size = 0.78

$$\frac{-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\left(4\left(4a^2-2ab-b^2\right)\cosh(2(e+fx))-8a^2+b(4a+b)\cosh(4(e+fx))+4ab+3b^2\right)}{2\sqrt{2}} + 8ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e\ldots\right.}{6af\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-1/2*((-8*a^2 + 4*a*b + 3*b^2 + 4*(4*a^2 - 2*a*b - b^2)*Cosh[2*(e + f*x)]
+ b*(4*a + b)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] - (
2*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f
*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cF[I*(e + f*x), b/a])/(6*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\coth\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.95Unable to divide, perha
ps due to rounding error%%%{1024,[8,16,8]%%%}+%%%{%%%{-4096,[1]%%%},[8,16,7
]%%%}+%%%{%%%{6144,[2]%%%},[8,16,6]%%%}+%%%{%%%{-4096,[3]%%%},[8,16,5]%%%}+
%%%{%%%{1024,[4]%%%},[8,16,4]%%%}+%%%{%%{[8192,0]:[1,0,%%%{-1,[1]%%%}]%%},[
7,16,8]%%%}+%%%{%%{[%%%{-32768,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,16,7]%
%%}+%%%{%%{[%%%{49152,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,16,6]%%%}+%%%{%
%{[%%%{-32768,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,16,5]%%%}+%%%{%%{[%%%{8
192,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,16,4]%%%}+%%%{-16384,[6,16,9]%%%}
+%%%{%%%{94208,[1]%%%},[6,16,8]%%%}+%%%{%%%{-212992,[2]%%%},[6,16,7]%%%}+%%
%{%%%{237568,[3]%%%},[6,16,6]%%%}+%%%{%%%{-131072,[4]%%%},[6,16,5]%%%}+%%%{
%%%{28672,[5]%%%},[6,16,4]%%%}+%%%{%%{[-98304,0]:[1,0,%%%{-1,[1]%%%}]%%},[5
,16,9]%%%}+%%%{%%{[%%%{450560,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,16,8]%%
%}+%%%{%%{[%%%{-819200,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,16,7]%%%}+%%%{
%%{[%%%{737280,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,16,6]%%%}+%%%{%%{[%%%{
-327680,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,16,5]%%%}+%%%{%%{[%%%{57344,[
5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,16,4]%%%}+%%%{98304,[4,16,10]%%%}+%%%{
%%%{-638976,[1]%%%},[4,16,9]%%%}+%%%{%%%{1644544,[2]%%%},[4,16,8]%%%}+%%%{%
%%{-2154496,[3]%%%},[4,16,7]%%%}+%%%{%%%{1511424,[4]%%%},[4,16,6]%%%}+%%%{%
%%{-532480,[5]%%%},[4,16,5]%%%}+%%%{%%%{71680,[6]%%%},[4,16,4]%%%}+%%%{%%{[
393216,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,16,10]%%%}+%%%{%%{[%%%{-1900544,[1]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,16,9]%%%}+%%%{%%{[%%%{3727360,[2]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[3,16,8]%%%}+%%%{%%{[%%%{-3768320,[3]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[3,16,7]%%%}+%%%{%%{[%%%{2048000,[4]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[3,16,6]%%%}+%%%{%%{[%%%{-557056,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[3,16,5]%%%}+%%%{%%{[%%%{57344,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,16,4
]%%%}+%%%{-262144,[2,16,11]%%%}+%%%{%%%{1638400,[1]%%%},[2,16,10]%%%}+%%%{%
%%{-4177920,[2]%%%},[2,16,9]%%%}+%%%{%%%{5599232,[3]%%%},[2,16,8]%%%}+%%%{%
%%{-4210688,[4]%%%},[2,16,7]%%%}+%%%{%%%{1744896,[5]%%%},[2,16,6]%%%}+%%%{%
%%{-360448,[6]%%%},[2,16,5]%%%}+%%%{%%%{28672,[7]%%%},[2,16,4]%%%}+%%%{%%{[
-524288,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,16,11]%%%}+%%%{%%{[%%%{2490368,[1]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,16,10]%%%}+%%%{%%{[%%%{-4816896,[2]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[1,16,9]%%%}+%%%{%%{[%%%{4857856,[3]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[1,16,8]%%%}+%%%{%%{[%%%{-2719744,[4]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[1,16,7]%%%}+%%%{%%{[%%%{835584,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[1,16,6]%%%}+%%%{%%{[%%%{-131072,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1
6,5]%%%}+%%%{%%{[%%%{8192,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,16,4]%%%}+%
%%{262144,[0,16,12]%%%}+%%%{%%%{-1310720,[1]%%%},[0,16,11]%%%}+%%%{%%%{2719
744,[2]%%%},[0,16,10]%%%}+%%%{%%%{-3031040,[3]%%%},[0,16,9]%%%}+%%%{%%%{196
7104,[4]%%%},[0,16,8]%%%}+%%%{%%%{-757760,[5]%%%},[0,16,7]%%%}+%%%{%%%{1699
84,[6]%%%},[0,16,6]%%%}+%%%{%%%{-20480,[7]%%%},[0,16,5]%%%}+%%%{%%%{1024,[8
]%%%},[0,16,4]%%%} / %%%{%%%{1,[2]%%%},[8,0,0]%%%}+%%%{%%{poly1[%%%{8,[2]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,0,0]%%%}+%%%{%%%{-16,[2]%%%},[6,0,1]%%%}+%
%%{%%%{28,[3]%%%},[6,0,0]%%%}+%%%{%%{[%%%{-96,[2]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[5,0,1]%%%}+%%%{%%{poly1[%%%{56,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5
,0,0]%%%}+%%%{%%%{96,[2]%%%},[4,0,2]%%%}+%%%{%%%{-240,[3]%%%},[4,0,1]%%%}+%
%%{%%%{70,[4]%%%},[4,0,0]%%%}+%%%{%%{poly1[%%%{384,[2]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[3,0,2]%%%}+%%%{%%{[%%%{-320,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[3,0,1]%%%}+%%%{%%{poly1[%%%{56,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,0]%
%%}+%%%{%%%{-256,[2]%%%},[2,0,3]%%%}+%%%{%%%{576,[3]%%%},[2,0,2]%%%}+%%%{%%
%{-240,[4]%%%},[2,0,1]%%%}+%%%{%%%{28,[5]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-512,
[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,3]%%%}+%%%{%%{poly1[%%%{384,[3]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,2]%%%}+%%%{%%{[%%%{-96,[4]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[1,0,1]%%%}+%%%{%%{poly1[%%%{8,[5]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[1,0,0]%%%}+%%%{%%%{256,[2]%%%},[0,0,4]%%%}+%%%{%%%{-256,[3]%%%},[0,
0,3]%%%}+%%%{%%%{96,[4]%%%},[0,0,2]%%%}+%%%{%%%{-16,[5]%%%},[0,0,1]%%%}+%%%
{%%%{1,[6]%%%},[0,0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.31, size = 522, normalized size = 1.93

$$-4\sqrt{-\frac{b}{a}}\, ab\left(\sinh^{6}\left(fx+e\right)\right)-\sqrt{-\frac{b}{a}}\, b^{2}\left(\sinh^{6}\left(fx+e\right)\right)+3a^{2}\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+3
*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x
+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*x+e)^3-2*b*((a+b*sinh(f*x+e)^2)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)
)*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+7*((a+b*sinh
(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1
/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+
e)^3-4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-6*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^4-
(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4-5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2-2*(-1/a
*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2)/a/sinh(f*x+e)^3/(-1/a*b)^(1
/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\coth\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \coth\left(e+f\,x\right)^{4}\,\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sinh^{2}\left(e+fx\right)}\,\coth^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**4, x)
```

3.468 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx$

Optimal. Leaf size=232

$$\frac{\left(8a^2-40ab+35b^2\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24f(a-b)^2}+\frac{\left(8a^2-40ab+35b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)}-\frac{\left(8a^2-40ab+35b\right.}{8}$$

```
[Out] 1/24*(8*a^2-40*a*b+35*b^2)*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/f+1/8*(8*a-9*b
)*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(5/2)/(a-b)^2/f-1/4*sech(f*x+e)^4*(a+b*
sinh(f*x+e)^2)^(5/2)/(a-b)/f-1/8*(8*a^2-40*a*b+35*b^2)*arctanh((a+b*sinh(f*
x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)+1/8*(8*a^2-40*a*b+35*b^2)*(a+b*sin
h(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A] time = 0.29, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{\left(8a^2-40ab+35b^2\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24f(a-b)^2}+\frac{\left(8a^2-40ab+35b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)}-\frac{\left(8a^2-40ab+35b\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^5,x]
```

```
[Out] -((8*a^2 - 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]
])/(8*Sqrt[a - b]*f) + ((8*a^2 - 40*a*b + 35*b^2)*Sqrt[a + b*Sinh[e + f*x]^
2])/(8*(a - b)*f) + ((8*a^2 - 40*a*b + 35*b^2)*(a + b*Sinh[e + f*x]^2)^(3/2
))/(24*(a - b)^2*f) + ((8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^
(5/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4
*(a - b)*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{x^2(a+bx)^{3/2}}{(1+x)^3}\,dx,x,\sinh^2(e+fx)\right)}{2f}$$

$$=-\frac{\text{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{4(a-b)f}+\frac{\text{Subst}\left(\int\frac{\left(\frac{1}{2}(-4a+5b)+2(a\right.}{(1+x}\right.}{}$$

$$=\frac{(8a-9b)\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{8(a-b)^2f}-\frac{\text{sech}^4(e+fx)\left(\right.}{4}$$

$$=\frac{\left(8a^2-40ab+35b^2\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24(a-b)^2f}+\frac{(8a-9b)\text{sech}^2(e}{}$$

$$=\frac{\left(8a^2-40ab+35b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)f}+\frac{\left(8a^2-40ab+35b^2\right.}{24(}$$

$$=\frac{\left(8a^2-40ab+35b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)f}+\frac{\left(8a^2-40ab+35b^2\right.}{24(}$$

$$=-\frac{\left(8a^2-40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8\sqrt{a-b}\,f}+\frac{\left(8a^2-40ab+3\right.}{}$$

Mathematica [A] time = 1.64, size = 169, normalized size = 0.73

$$-\frac{-\left(8a^2-40ab+35b^2\right)\left(\sqrt{a+b\sinh^2(e+fx)}\left(4a+b\sinh^2(e+fx)-3b\right)-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right.\right.}{24f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^5,x]
```

```
[Out] -1/24*(-3*(8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2) + 6*(a
- b)*Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2) - (8*a^2 - 40*a*b + 35*b
^2)*(-3*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + Sq
rt[a + b*Sinh[e + f*x]^2]*(4*a - 3*b + b*Sinh[e + f*x]^2)))/((a - b)^2*f)
```

fricas [B] time = 1.29, size = 6380, normalized size = 27.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 - 40*a*b +
 35*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 - 40*a*b + 35*b^2)*sinh(f*
x + e)^11 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 - 40*a
*b + 35*b^2)*cosh(f*x + e)^2 + 32*a^2 - 160*a*b + 140*b^2)*sinh(f*x + e)^9
+ 3*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + 12*(8*a^2 - 40*a*b + 35
*b^2)*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x
 + e)^7 + 6*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 24*(8*a^2 - 40*
a*b + 35*b^2)*cosh(f*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^7 +
42*(11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 8*(8*a^2 - 40*a*b + 35*b
^2)*cosh(f*x + e)^3 + (8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e
)^6 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 2*(231*(8*a^2 - 40*a*b
+ 35*b^2)*cosh(f*x + e)^6 + 252*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 +
 63*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2 + 16*a^2 - 80*a*b + 70*b^2)*s
inh(f*x + e)^5 + 2*(165*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 + 252*(8*
a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 105*(8*a^2 - 40*a*b + 35*b^2)*cosh
(f*x + e)^3 + 10*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 +
 (8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + (165*(8*a^2 - 40*a*b + 35*b^2)
*cosh(f*x + e)^8 + 336*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 + 210*(8*a
^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 40*(8*a^2 - 40*a*b + 35*b^2)*cosh(f
*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^3 + (55*(8*a^2 - 40*a*b
+ 35*b^2)*cosh(f*x + e)^9 + 144*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 +
 126*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 40*(8*a^2 - 40*a*b + 35*b^
2)*cosh(f*x + e)^3 + 3*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x +
e)^2 + (11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^10 + 36*(8*a^2 - 40*a*b
+ 35*b^2)*cosh(f*x + e)^8 + 42*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 +
20*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 40*a*b + 35*b^2)*
cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*co
sh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x +
e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt
(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + s
inh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x
+ e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e
)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(co
sh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*((a*b - b^2)
*cosh(f*x + e)^12 + 12*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^11 + (a*b -
b^2)*sinh(f*x + e)^12 + 2*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^10 + 2*(3
3*(a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 25*a*b + 17*b^2)*sinh(f*x + e)^10 +
 20*(11*(a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^2)*cosh(f*x +
e))*sinh(f*x + e)^9 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^8 + (495*
(a*b - b^2)*cosh(f*x + e)^4 + 90*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^2
+ 112*a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^8 + 8*(99*(a*b - b^2)*cosh(f*x
 + e)^5 + 30*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^3 + (112*a^2 - 335*a*b
 + 223*b^2)*cosh(f*x + e))*sinh(f*x + e)^7 + 8*(18*a^2 - 59*a*b + 41*b^2)*c
```

osh(f*x + e)^6 + 4*(231*(a*b - b^2)*cosh(f*x + e)^6 + 105*(8*a^2 - 25*a*b +
 17*b^2)*cosh(f*x + e)^4 + 7*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^2
+ 36*a^2 - 118*a*b + 82*b^2)*sinh(f*x + e)^6 + 8*(99*(a*b - b^2)*cosh(f*x +
 e)^7 + 63*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^5 + 7*(112*a^2 - 335*a*b
 + 223*b^2)*cosh(f*x + e)^3 + 6*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e))*s
inh(f*x + e)^5 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^4 + (495*(a*b
- b^2)*cosh(f*x + e)^8 + 420*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^6 + 70
*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^4 + 120*(18*a^2 - 59*a*b + 41*
b^2)*cosh(f*x + e)^2 + 112*a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^4 + 4*(55
*(a*b - b^2)*cosh(f*x + e)^9 + 60*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^7
 + 14*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^5 + 40*(18*a^2 - 59*a*b +
 41*b^2)*cosh(f*x + e)^3 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e))*sin
h(f*x + e)^3 + 2*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^2 + 2*(33*(a*b - b
^2)*cosh(f*x + e)^10 + 45*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^8 + 14*(1
12*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^6 + 60*(18*a^2 - 59*a*b + 41*b^2)
*cosh(f*x + e)^4 + 3*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^2 + 8*a^2
- 25*a*b + 17*b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*(3*(a*b - b^2)*cosh(f*x
+ e)^11 + 5*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^9 + 2*(112*a^2 - 335*a*
b + 223*b^2)*cosh(f*x + e)^7 + 12*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^
5 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^
2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)))/((a - b)*f*cosh(f*x + e)^11 + 11*(a - b)*f*cosh(f*x + e)*sinh(f*x +
 e)^10 + (a - b)*f*sinh(f*x + e)^11 + 4*(a - b)*f*cosh(f*x + e)^9 + (55*(a
- b)*f*cosh(f*x + e)^2 + 4*(a - b)*f)*sinh(f*x + e)^9 + 6*(a - b)*f*cosh(f*
x + e)^7 + 3*(55*(a - b)*f*cosh(f*x + e)^3 + 12*(a - b)*f*cosh(f*x + e))*si
nh(f*x + e)^8 + 6*(55*(a - b)*f*cosh(f*x + e)^4 + 24*(a - b)*f*cosh(f*x + e
)^2 + (a - b)*f)*sinh(f*x + e)^7 + 4*(a - b)*f*cosh(f*x + e)^5 + 42*(11*(a
- b)*f*cosh(f*x + e)^5 + 8*(a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x +
 e))*sinh(f*x + e)^6 + 2*(231*(a - b)*f*cosh(f*x + e)^6 + 252*(a - b)*f*cos
h(f*x + e)^4 + 63*(a - b)*f*cosh(f*x + e)^2 + 2*(a - b)*f)*sinh(f*x + e)^5
+ (a - b)*f*cosh(f*x + e)^3 + 2*(165*(a - b)*f*cosh(f*x + e)^7 + 252*(a - b
)*f*cosh(f*x + e)^5 + 105*(a - b)*f*cosh(f*x + e)^3 + 10*(a - b)*f*cosh(f*x
 + e))*sinh(f*x + e)^4 + (165*(a - b)*f*cosh(f*x + e)^8 + 336*(a - b)*f*cos
h(f*x + e)^6 + 210*(a - b)*f*cosh(f*x + e)^4 + 40*(a - b)*f*cosh(f*x + e)^2
 + (a - b)*f)*sinh(f*x + e)^3 + (55*(a - b)*f*cosh(f*x + e)^9 + 144*(a - b)
*f*cosh(f*x + e)^7 + 126*(a - b)*f*cosh(f*x + e)^5 + 40*(a - b)*f*cosh(f*x
+ e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (11*(a - b)*f*cosh(f*
x + e)^10 + 36*(a - b)*f*cosh(f*x + e)^8 + 42*(a - b)*f*cosh(f*x + e)^6 + 2
0*(a - b)*f*cosh(f*x + e)^4 + 3*(a - b)*f*cosh(f*x + e)^2)*sinh(f*x + e)),
-1/24*(3*((8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 - 40*a*b +
 35*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 - 40*a*b + 35*b^2)*sinh(f*
x + e)^11 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 - 40*a
*b + 35*b^2)*cosh(f*x + e)^2 + 32*a^2 - 160*a*b + 140*b^2)*sinh(f*x + e)^9
+ 3*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + 12*(8*a^2 - 40*a*b + 35
*b^2)*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x
 + e)^7 + 6*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 24*(8*a^2 - 40*
a*b + 35*b^2)*cosh(f*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^7 +
42*(11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 8*(8*a^2 - 40*a*b + 35*b
^2)*cosh(f*x + e)^3 + (8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e
)^6 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 2*(231*(8*a^2 - 40*a*b
+ 35*b^2)*cosh(f*x + e)^6 + 252*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 +
 63*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2 + 16*a^2 - 80*a*b + 70*b^2)*s
inh(f*x + e)^5 + 2*(165*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 + 252*(8*
a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 105*(8*a^2 - 40*a*b + 35*b^2)*cosh
(f*x + e)^3 + 10*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 +
 (8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + (165*(8*a^2 - 40*a*b + 35*b^2)
*cosh(f*x + e)^8 + 336*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 + 210*(8*a
^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 40*(8*a^2 - 40*a*b + 35*b^2)*cosh(f

*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^3 + (55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^9 + 144*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 + 126*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 40*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^10 + 36*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^8 + 42*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 + 20*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*((a*b - b^2)*cosh(f*x + e)^12 + 12*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^11 + (a*b - b^2)*sinh(f*x + e)^12 + 2*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^10 + 2*(33*(a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 25*a*b + 17*b^2)*sinh(f*x + e)^10 + 20*(11*(a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e)^9 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^8 + (495*(a*b - b^2)*cosh(f*x + e)^4 + 90*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^2 + 112*a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^8 + 8*(99*(a*b - b^2)*cosh(f*x + e)^5 + 30*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^3 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e))*sinh(f*x + e)^7 + 8*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^6 + 4*(231*(a*b - b^2)*cosh(f*x + e)^6 + 105*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^4 + 7*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^2 + 36*a^2 - 118*a*b + 82*b^2)*sinh(f*x + e)^6 + 8*(99*(a*b - b^2)*cosh(f*x + e)^7 + 63*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^5 + 7*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^3 + 6*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^4 + (495*(a*b - b^2)*cosh(f*x + e)^8 + 420*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^6 + 70*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^4 + 120*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^2 + 112*a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^4 + 4*(55*(a*b - b^2)*cosh(f*x + e)^9 + 60*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^7 + 14*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^5 + 40*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^3 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^2 + 2*(33*(a*b - b^2)*cosh(f*x + e)^10 + 45*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^8 + 14*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^6 + 60*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^4 + 3*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^2 + 8*a^2 - 25*a*b + 17*b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*(3*(a*b - b^2)*cosh(f*x + e)^11 + 5*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^9 + 2*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^7 + 12*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^5 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)^11 + 11*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^10 + (a - b)*f*sinh(f*x + e)^11 + 4*(a - b)*f*cosh(f*x + e)^9 + (55*(a - b)*f*cosh(f*x + e)^2 + 4*(a - b)*f)*sinh(f*x + e)^9 + 6*(a - b)*f*cosh(f*x + e)^7 + 3*(55*(a - b)*f*cosh(f*x + e)^3 + 12*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(55*(a - b)*f*cosh(f*x + e)^4 + 24*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^7 + 4*(a - b)*f*cosh(f*x + e)^5 + 42*(11*(a - b)*f*cosh(f*x + e)^5 + 8*(a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(231*(a - b)*f*cosh(f*x + e)^6 + 252*(a - b)*f*cosh(f*x + e)^4 + 63*(a - b)*f*cosh(f*x + e)^2 + 2*(a - b)*f)*sinh(f*x + e)^5 + (a - b)*f*cosh(f*x + e)^3 + 2*(165*(a - b)*f*cosh(f*x + e)^7 + 252*(a - b)*f*cosh(f*x + e)^5 + 105*(a - b)*f*cosh(f*x + e)^3 + 10*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^4 + (165*(a - b)*f*cosh(f*x + e)^8 + 336*(a - b)*f*cosh(f*x + e)^6 + 210*(a - b)*f*cosh(f*x + e)^4 + 40*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^3 + (55*(a - b)*f*cosh(f*x + e)^9 + 144*(a - b)*f*cosh(f*x + e)^7 + 126*(a - b)*f*cosh(f*x + e)^5 + 40*(a - b)*f*cosh(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (11*(a - b)*f*cosh(f*x + e)^10 + 36*(a - b)*f*cosh(f*x + e)^8 + 42*(a - b)*f*cosh(f*x + e)^6 + 20*(a - b)*f*cosh(f*x + e)^4

```
+ 3*(a - b)*f*cosh(f*x + e)^2)*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 4.09Error: Bad Argument Typ
e
```

maple [C] time = 0.26, size = 71, normalized size = 0.31

$$\frac{`int/indef0`\left(\frac{\left(\sinh^5\left(fx+e\right)\right)\left(b^2\left(\sinh^4\left(fx+e\right)\right)+2ab\left(\sinh^2\left(fx+e\right)\right)+a^2\right)}{\cosh\left(fx+e\right)^6\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x)
[Out] `int/indef0`(sinh(f*x+e)^5*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh
(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tanh\left(e+f\,x\right)^5\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(tanh(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**5,x)
[Out] Timed out
```

3.469 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx$

Optimal. Leaf size=156

$$\frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6f(a-b)}+\frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f}-\frac{(2a-5b)\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f}$$

```
[Out] 1/6*(2*a-5*b)*(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)/f+1/2*sech(f*x+e)^2*(a+b*sinh
(f*x+e)^2)^(5/2)/(a-b)/f-1/2*(2*a-5*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a
-b)^(1/2))*(a-b)^(1/2)/f+1/2*(2*a-5*b)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6f(a-b)}+\frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f}-\frac{(2a-5b)\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3,x]
```

```
[Out] -((2*a - 5*b)*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])
/(2*f) + ((2*a - 5*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a - 5*b)*(a
+ b*Sinh[e + f*x]^2)^(3/2))/(6*(a - b)*f) + (Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2)^(5/2))/(2*(a - b)*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{(1+x)^2}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2(a-b)f} + \frac{(2a-5b)\,\text{Subst}\left(\int \frac{(a+bx}{1+x}\right.}{4(a-}$$

$$= \frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6(a-b)f} + \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e\right.}{2(a-b)f}$$

$$= \frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3}}{6(a-b)f}$$

$$= \frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3}}{6(a-b)f}$$

$$= -\frac{(2a-5b)\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f} + \frac{(2a-5b)\sqrt{a+b\sin}}{2f}$$

Mathematica [A] time = 0.56, size = 122, normalized size = 0.78

$$\frac{(2a-5b)\left(\sqrt{a+b\sinh^2(e+fx)}\left(4a+b\sinh^2(e+fx)-3b\right)-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)\right)+3\text{sech}^2(e+}{6f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3,x]
```

```
[Out] (3*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2) + (2*a - 5*b)*(-3*(a - b)^
(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a + b*Sinh[e
+ f*x]^2]*(4*a - 3*b + b*Sinh[e + f*x]^2)))/(6*(a - b)*f)
```

fricas [B] time = 1.18, size = 2454, normalized size = 15.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(6*((2*a - 5*b)*cosh(f*x + e)^7 + 7*(2*a - 5*b)*cosh(f*x + e)*sinh(f
*x + e)^6 + (2*a - 5*b)*sinh(f*x + e)^7 + 2*(2*a - 5*b)*cosh(f*x + e)^5 + (
21*(2*a - 5*b)*cosh(f*x + e)^2 + 4*a - 10*b)*sinh(f*x + e)^5 + 5*(7*(2*a -
5*b)*cosh(f*x + e)^3 + 2*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a
```

```
- 5*b)*cosh(f*x + e)^3 + (35*(2*a - 5*b)*cosh(f*x + e)^4 + 20*(2*a - 5*b)*c
osh(f*x + e)^2 + 2*a - 5*b)*sinh(f*x + e)^3 + (21*(2*a - 5*b)*cosh(f*x + e)
^5 + 20*(2*a - 5*b)*cosh(f*x + e)^3 + 3*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x
 + e)^2 + (7*(2*a - 5*b)*cosh(f*x + e)^6 + 10*(2*a - 5*b)*cosh(f*x + e)^4 +
 3*(2*a - 5*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x
+ e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3
*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 +
 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(co
sh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x
+ e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3
 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x
 + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - sqrt(2)
*(b*cosh(f*x + e)^8 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8
 + 8*(2*a - 3*b)*cosh(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^2 + 4*a - 6*b)*sinh
(f*x + e)^6 + 8*(7*b*cosh(f*x + e)^3 + 6*(2*a - 3*b)*cosh(f*x + e))*sinh(f*
x + e)^5 + 2*(28*a - 37*b)*cosh(f*x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(
2*a - 3*b)*cosh(f*x + e)^2 + 28*a - 37*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x
 + e)^5 + 20*(2*a - 3*b)*cosh(f*x + e)^3 + (28*a - 37*b)*cosh(f*x + e))*sin
h(f*x + e)^3 + 8*(2*a - 3*b)*cosh(f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*
(2*a - 3*b)*cosh(f*x + e)^4 + 3*(28*a - 37*b)*cosh(f*x + e)^2 + 4*a - 6*b)*
sinh(f*x + e)^2 + 8*(b*cosh(f*x + e)^7 + 6*(2*a - 3*b)*cosh(f*x + e)^5 + (2
8*a - 37*b)*cosh(f*x + e)^3 + 2*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) +
b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^7 + 7
*f*cosh(f*x + e)*sinh(f*x + e)^6 + f*sinh(f*x + e)^7 + 2*f*cosh(f*x + e)^5
+ (21*f*cosh(f*x + e)^2 + 2*f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 2
*f*cosh(f*x + e))*sinh(f*x + e)^4 + f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)
^4 + 20*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^3 + (21*f*cosh(f*x + e)^5 + 20
*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e
)^6 + 10*f*cosh(f*x + e)^4 + 3*f*cosh(f*x + e)^2)*sinh(f*x + e)), -1/24*(12
*((2*a - 5*b)*cosh(f*x + e)^7 + 7*(2*a - 5*b)*cosh(f*x + e)*sinh(f*x + e)^6
 + (2*a - 5*b)*sinh(f*x + e)^7 + 2*(2*a - 5*b)*cosh(f*x + e)^5 + (21*(2*a -
 5*b)*cosh(f*x + e)^2 + 4*a - 10*b)*sinh(f*x + e)^5 + 5*(7*(2*a - 5*b)*cosh
(f*x + e)^3 + 2*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a - 5*b)*co
sh(f*x + e)^3 + (35*(2*a - 5*b)*cosh(f*x + e)^4 + 20*(2*a - 5*b)*cosh(f*x +
 e)^2 + 2*a - 5*b)*sinh(f*x + e)^3 + (21*(2*a - 5*b)*cosh(f*x + e)^5 + 20*(
2*a - 5*b)*cosh(f*x + e)^3 + 3*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^2 +
 (7*(2*a - 5*b)*cosh(f*x + e)^6 + 10*(2*a - 5*b)*cosh(f*x + e)^4 + 3*(2*a -
 5*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt
(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x
 + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*(b*cosh(f*x + e)^8 + 8*b*cosh(f*x
 + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8 + 8*(2*a - 3*b)*cosh(f*x + e)^6 +
 4*(7*b*cosh(f*x + e)^2 + 4*a - 6*b)*sinh(f*x + e)^6 + 8*(7*b*cosh(f*x + e)
^3 + 6*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(28*a - 37*b)*cosh(f*
x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(2*a - 3*b)*cosh(f*x + e)^2 + 28*a
- 37*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x + e)^5 + 20*(2*a - 3*b)*cosh(f*x
+ e)^3 + (28*a - 37*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*(2*a - 3*b)*cosh(
f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*(2*a - 3*b)*cosh(f*x + e)^4 + 3*(2
8*a - 37*b)*cosh(f*x + e)^2 + 4*a - 6*b)*sinh(f*x + e)^2 + 8*(b*cosh(f*x +
e)^7 + 6*(2*a - 3*b)*cosh(f*x + e)^5 + (28*a - 37*b)*cosh(f*x + e)^3 + 2*(2
*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sin
h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2)))/(f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)*sinh(f*x + e)^6 +
 f*sinh(f*x + e)^7 + 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 + 2*f)*sin
h(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 2*f*cosh(f*x + e))*sinh(f*x + e)^4
+ f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 + 20*f*cosh(f*x + e)^2 + f)*sin
h(f*x + e)^3 + (21*f*cosh(f*x + e)^5 + 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x
```

```
+ e))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 + 10*f*cosh(f*x + e)^4 + 3*f*c
osh(f*x + e)^2)*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.96Error: Bad Argument Typ
e
```

maple [C] time = 0.23, size = 71, normalized size = 0.46

$$\frac{`int/indef0`\left(\frac{\left(\sinh^{3}\left(fx+e\right)\right)\left(b^{2}\left(\sinh^{4}\left(fx+e\right)\right)+2ab\left(\sinh^{2}\left(fx+e\right)\right)+a^{2}\right)}{\cosh\left(fx+e\right)^{4}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh
(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e+f\,x\right)^{3}\,\left(b\,\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**3,x)
```

```
[Out] Timed out
```

3.470 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx$

Optimal. Leaf size=90

$$\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -(a-b)^(3/2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f+1/3*(a+b*sinh
(f*x+e)^2)^(3/2)/f+(a-b)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x],x]
```

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
 - b)*Sqrt[a + b*Sinh[e + f*x]^2])/f + (a + b*Sinh[e + f*x]^2)^(3/2)/(3*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \sinh^2(e+\right.}{2f}$$
$$= \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)^2}{}$$
$$= \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)^2}{}$$
$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{(a-b)\sqrt{a+b\sinh^2(e+fx}}{f}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.96

$$\frac{\left(4a+b\cosh^2(e+fx)-4b\right)\sqrt{a+b\cosh^2(e+fx)-b} - 3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(e+fx)-b}}{\sqrt{a-b}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x],x]
```

```
[Out] (-3*(a - b)^(3/2)*ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]] + (4
*a - 4*b + b*Cosh[e + f*x]^2)*Sqrt[a - b + b*Cosh[e + f*x]^2])/(3*f)
```

fricas [B] time = 1.11, size = 1052, normalized size = 11.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] [-1/24*(12*((a - b)*cosh(f*x + e)^3 + 3*(a - b)*cosh(f*x + e)^2*sinh(f*x +
e) + 3*(a - b)*cosh(f*x + e)*sinh(f*x + e)^2 + (a - b)*sinh(f*x + e)^3)*sqr
t(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sin
h(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a
 - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^
3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh
(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sin
h(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh
(f*x + e) + 1)) - sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
 e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - 7*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x
 + e)^2 + 8*a - 7*b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (8*a - 7*b)*c
osh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f*x + e) + 3*f*cosh(f
*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3), -1/24*(24*((a - b)*cosh(f*x +
 e)^3 + 3*(a - b)*cosh(f*x + e)^2*sinh(f*x + e) + 3*(a - b)*cosh(f*x + e)*s
```

```
inh(f*x + e)^2 + (a - b)*sinh(f*x + e)^3)*sqrt(-a + b)*arctan(-1/2*sqrt(2)*
sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh
(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh
(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - 7*b)*cosh(f*x + e)
^2 + 2*(3*b*cosh(f*x + e)^2 + 8*a - 7*b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
e)^3 + (8*a - 7*b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f
*x + e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.Non regular value [0
] was discarded and replaced randomly by 0=[91]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.Non
regular value [0] was discarded and replaced randomly by 0=[74]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.Non regular value [0] was discarded and replaced randomly by 0=
[-62]Warning, need to choose a branch for the root of a polynomial with par
ameters. This might be wrong.Non regular value [0] was discarded and replac
ed randomly by 0=[-44]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.Non regular value [0] was dis
carded and replaced randomly by 0=[-3]Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.Non regular v
alue [0] was discarded and replaced randomly by 0=[-6]Warning, need to choo
se a branch for the root of a polynomial with parameters. This might be wro
ng.Non regular value [0] was discarded and replaced randomly by 0=[-77]Warn
ing, need to choose a branch for the root of a polynomial with parameters.
This might be wrong.Non regular value [0] was discarded and replaced random
ly by 0=[-10]Evaluation time: 0.81index.cc index_m operator + Error: Bad Ar
gument Value
```

maple [C] time = 0.20, size = 69, normalized size = 0.77

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\sinh\left(fx+e\right)\left(b^{2}\left(\sinh^{4}\left(fx+e\right)\right)+2ab\left(\sinh^{2}\left(fx+e\right)\right)+a^{2}\right)}{\cosh\left(fx+e\right)^{2}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f
*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="maxima")
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e+f\,x\right)\,{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(tanh(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sinh^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}} \tanh{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e),x)
[Out] Integral((a + b*sinh(e + f*x)**2)**(3/2)*tanh(e + f*x), x)
```

3.471 $\int \coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sinh(f*x+e)^
2)^(3/2)/f+a*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b
*Sinh[e + f*x]^2])/f + (a + b*Sinh[e + f*x]^2)^(3/2)/(3*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{a\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{a\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\right.}{}$$

Mathematica [A] time = 0.14, size = 69, normalized size = 0.88

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(4a+b\sinh^2(e+fx)\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sinh[
e + f*x]^2]*(4*a + b*Sinh[e + f*x]^2))/(3*f)
```

fricas [B] time = 0.75, size = 1000, normalized size = 12.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(12*(a*cosh(f*x + e)^3 + 3*a*cosh(f*x + e)^2*sinh(f*x + e) + 3*a*cosh
(f*x + e)*sinh(f*x + e)^2 + a*sinh(f*x + e)^3)*sqrt(a)*log((b*cosh(f*x + e)
^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*co
sh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(
2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e)
+ sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x
 + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x +
e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(c
osh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*(b*cosh(f*x +
 e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - b)
*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 8*a - b)*sinh(f*x + e)^2 + 4*(b
*cosh(f*x + e)^3 + (8*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e
)^2*sinh(f*x + e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3),
 1/24*(24*(a*cosh(f*x + e)^3 + 3*a*cosh(f*x + e)^2*sinh(f*x + e) + 3*a*cosh
(f*x + e)*sinh(f*x + e)^2 + a*sinh(f*x + e)^3)*sqrt(-a)*arctan(1/2*sqrt(2)*
```

```
sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e)
+ a*sinh(f*x + e))) + sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f
*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f
*x + e)^2 + 8*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (8*a - b)*cos
h(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f*x + e) + 3*f*cosh(f*x
 + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.Non regular value [0
] was discarded and replaced randomly by 0=[45]Warning, need to choose a br
anch for the root of a polynomial with parameters. This might be wrong.Non
regular value [0] was discarded and replaced randomly by 0=[-8]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.Non regular value [0] was discarded and replaced randomly by 0=
[87]Warning, need to choose a branch for the root of a polynomial with para
meters. This might be wrong.Non regular value [0] was discarded and replace
d randomly by 0=[51]Warning, need to choose a branch for the root of a poly
nomial with parameters. This might be wrong.Non regular value [0] was disca
rded and replaced randomly by 0=[-90]Warning, need to choose a branch for t
he root of a polynomial with parameters. This might be wrong.Non regular va
lue [0] was discarded and replaced randomly by 0=[-27]Evaluation time: 3.63
index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.17, size = 62, normalized size = 0.79

$$\frac{\text{`}int/indef0\text{`}\left(\frac{b^{2}\left(\sinh^{3}\left(fx+e\right)\right)+2ab\sinh\left(fx+e\right)+\frac{a^{2}}{\sinh\left(fx+e\right)}}{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^3+2*a*b*sinh(f*x+e)+a^2/sinh(f*x+e))/(a+b*sin
h(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}\left(e+f\,x\right)\,{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.472 $\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af}+\frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f}-\frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\text{c}$$

```
[Out] 1/6*(2*a+3*b)*(a+b*sinh(f*x+e)^2)^(3/2)/a/f-1/2*csch(f*x+e)^2*(a+b*sinh(f*x
+e)^2)^(5/2)/a/f-1/2*(2*a+3*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))*a
^(1/2)/f+1/2*(2*a+3*b)*(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af}+\frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f}-\frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\text{c}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*f) +
 ((2*a + 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a + 3*b)*(a + b*Sinh
[e + f*x]^2)^(3/2))/(6*a*f) - (Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2
))/(2*a*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(1+x)(a+bx)^{3/2}}{x^2}dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2af} + \frac{(2a+3b)\,\text{Subst}\left(\int \frac{(a+\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af} - \frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e\cdots\right.}{2af}$$

$$= \frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)\cdots}{6af}$$

$$= \frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)\cdots}{6af}$$

$$= -\frac{\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f} + \frac{(2a+3b)\sqrt{a+b\sinh^2\cdots}}{2f}$$

Mathematica [A] time = 0.45, size = 90, normalized size = 0.64

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(-3a\operatorname{csch}^2(e+fx)+8a+b\cosh(2(e+fx))+5b\right)-3\sqrt{a}\,(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\cdots\right.}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + (8*a
 + 5*b + b*Cosh[2*(e + f*x)] - 3*a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x
]^2])/(6*f)
```

fricas [B] time = 1.02, size = 2406, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(6*((2*a + 3*b)*cosh(f*x + e)^7 + 7*(2*a + 3*b)*cosh(f*x + e)*sinh(f*
x + e)^6 + (2*a + 3*b)*sinh(f*x + e)^7 - 2*(2*a + 3*b)*cosh(f*x + e)^5 + (2
1*(2*a + 3*b)*cosh(f*x + e)^2 - 4*a - 6*b)*sinh(f*x + e)^5 + 5*(7*(2*a + 3*
b)*cosh(f*x + e)^3 - 2*(2*a + 3*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a +
```

```
3*b)*cosh(f*x + e)^3 + (35*(2*a + 3*b)*cosh(f*x + e)^4 - 20*(2*a + 3*b)*cos
h(f*x + e)^2 + 2*a + 3*b)*sinh(f*x + e)^3 + (21*(2*a + 3*b)*cosh(f*x + e)^5
 - 20*(2*a + 3*b)*cosh(f*x + e)^3 + 3*(2*a + 3*b)*cosh(f*x + e))*sinh(f*x +
 e)^2 + (7*(2*a + 3*b)*cosh(f*x + e)^6 - 10*(2*a + 3*b)*cosh(f*x + e)^4 + 3
*(2*a + 3*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4
 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh
(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)
*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) +
sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x +
 e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)
^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cos
h(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*(b*cosh(f*x + e
)^8 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8 + 8*(2*a + b)*c
osh(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^6 + 8*(7
*b*cosh(f*x + e)^3 + 6*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(28*a +
 9*b)*cosh(f*x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(2*a + b)*cosh(f*x + e
)^2 - 28*a - 9*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x + e)^5 + 20*(2*a + b)*c
osh(f*x + e)^3 - (28*a + 9*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*(2*a + b)*
cosh(f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*(2*a + b)*cosh(f*x + e)^4 - 3
*(28*a + 9*b)*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^2 + 8*(b*cosh(f*x
+ e)^7 + 6*(2*a + b)*cosh(f*x + e)^5 - (28*a + 9*b)*cosh(f*x + e)^3 + 2*(2*
a + b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f
*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sin
h(f*x + e)^2)))/(f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)*sinh(f*x + e)^6 + f*
sinh(f*x + e)^7 - 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 - 2*f)*sinh(f
*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 - 2*f*cosh(f*x + e))*sinh(f*x + e)^4 + f
*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*sinh(f
*x + e)^3 + (21*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e
))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 - 10*f*cosh(f*x + e)^4 + 3*f*cosh
(f*x + e)^2)*sinh(f*x + e)), 1/24*(12*((2*a + 3*b)*cosh(f*x + e)^7 + 7*(2*a
 + 3*b)*cosh(f*x + e)*sinh(f*x + e)^6 + (2*a + 3*b)*sinh(f*x + e)^7 - 2*(2*
a + 3*b)*cosh(f*x + e)^5 + (21*(2*a + 3*b)*cosh(f*x + e)^2 - 4*a - 6*b)*sin
h(f*x + e)^5 + 5*(7*(2*a + 3*b)*cosh(f*x + e)^3 - 2*(2*a + 3*b)*cosh(f*x +
e))*sinh(f*x + e)^4 + (2*a + 3*b)*cosh(f*x + e)^3 + (35*(2*a + 3*b)*cosh(f*
x + e)^4 - 20*(2*a + 3*b)*cosh(f*x + e)^2 + 2*a + 3*b)*sinh(f*x + e)^3 + (2
1*(2*a + 3*b)*cosh(f*x + e)^5 - 20*(2*a + 3*b)*cosh(f*x + e)^3 + 3*(2*a + 3
*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (7*(2*a + 3*b)*cosh(f*x + e)^6 - 10*(2
*a + 3*b)*cosh(f*x + e)^4 + 3*(2*a + 3*b)*cosh(f*x + e)^2)*sinh(f*x + e))*s
qrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + sqrt(2)*(b*cosh(f*x + e)^8
 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8 + 8*(2*a + b)*cosh
(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^6 + 8*(7*b*
cosh(f*x + e)^3 + 6*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(28*a + 9*
b)*cosh(f*x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(2*a + b)*cosh(f*x + e)^2
 - 28*a - 9*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x + e)^5 + 20*(2*a + b)*cosh
(f*x + e)^3 - (28*a + 9*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*(2*a + b)*cos
h(f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*(2*a + b)*cosh(f*x + e)^4 - 3*(2
8*a + 9*b)*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^2 + 8*(b*cosh(f*x + e
)^7 + 6*(2*a + b)*cosh(f*x + e)^5 - (28*a + 9*b)*cosh(f*x + e)^3 + 2*(2*a +
 b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)))/(f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)*sinh(f*x + e)^6 + f*sin
h(f*x + e)^7 - 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 - 2*f)*sinh(f*x
+ e)^5 + 5*(7*f*cosh(f*x + e)^3 - 2*f*cosh(f*x + e))*sinh(f*x + e)^4 + f*co
sh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*sinh(f*x
+ e)^3 + (21*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*
sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 - 10*f*cosh(f*x + e)^4 + 3*f*cosh(f*
```

```
x + e)^2)*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.46Unable to divide, perha
ps due to rounding error%%%{2,[6,0,8]%%%}+%%%{%%{[12,0]:[1,0,%%%{-1,[1]%%%}
]%%},[5,0,8]%%%}+%%%{-24,[4,1,8]%%%}+%%%{%%%{30,[1]%%%},[4,0,8]%%%}+%%%{%%{
[-96,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,8]%%%}+%%%{%%{[%%%{40,[1]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[3,0,8]%%%}+%%%{96,[2,2,8]%%%}+%%%{%%%{-144,[1]%%%},[2,1
,8]%%%}+%%%{%%%{30,[2]%%%},[2,0,8]%%%}+%%%{%%{[192,0]:[1,0,%%%{-1,[1]%%%}]%
%},[1,2,8]%%%}+%%%{%%{[%%%{-96,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,8]%%
%}+%%%{%%{[%%%{12,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,8]%%%}+%%%{-128,[
0,3,8]%%%}+%%%{%%%{96,[1]%%%},[0,2,8]%%%}+%%%{%%%{-24,[2]%%%},[0,1,8]%%%}+%
%%{%%%{2,[3]%%%},[0,0,8]%%%} / %%%{%%{poly1[%%%{1,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[6,0,0]%%%}+%%%{%%%{6,[2]%%%},[5,0,0]%%%}+%%%{%%{[%%%{-12,[1]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,1,0]%%%}+%%%{%%{poly1[%%%{15,[2]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[4,0,0]%%%}+%%%{%%%{-48,[2]%%%},[3,1,0]%%%}+%%%{%%%{20,
[3]%%%},[3,0,0]%%%}+%%%{%%{[%%%{48,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,
0]%%%}+%%%{%%{[%%%{-72,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,1,0]%%%}+%%%{%
%{poly1[%%%{15,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0]%%%}+%%%{%%%{96,[2
]%%%},[1,2,0]%%%}+%%%{%%%{-48,[3]%%%},[1,1,0]%%%}+%%%{%%%{6,[4]%%%},[1,0,0]
%%%}+%%%{%%{[%%%{-64,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,3,0]%%%}+%%%{%%{
[%%%{48,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,2,0]%%%}+%%%{%%{[%%%{-12,[3]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,1,0]%%%}+%%%{%%{poly1[%%%{1,[4]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[0,0,0]%%%} Error: Bad Argument Value
```

maple [C] time = 0.21, size = 84, normalized size = 0.60

$$\frac{\text{`}int/indef0\text{`}\left(\frac{b^{2}\left(\sinh^{3}\left(fx+e\right)\right)+\left(2ab+b^{2}\right)\sinh\left(fx+e\right)+\frac{a^{2}+2ab}{\sinh\left(fx+e\right)}+\frac{a^{2}}{\sinh\left(fx+e\right)^{3}}}{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^3+(2*a*b+b^2)*sinh(f*x+e)+(a^2+2*a*b)/sinh(f*
x+e)+a^2/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{coth}\left(e+f\,x\right)^3\,{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(coth(e + f*x)^3*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.473 $\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=203

$$\frac{\left(8a^2+3b(8a+b)\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24a^2f}+\frac{\left(8a^2+3b(8a+b)\right)\sqrt{a+b\sinh^2(e+fx)}}{8af}-\frac{\left(8a^2+3b(8a+b)\right)\tanh^{-1}}{8\sqrt{a}f}$$

```
[Out] 1/24*(8*a^2+3*b*(8*a+b))*(a+b*sinh(f*x+e)^2)^(3/2)/a^2/f-1/8*(8*a+b)*csch(f
*x+e)^2*(a+b*sinh(f*x+e)^2)^(5/2)/a^2/f-1/4*csch(f*x+e)^4*(a+b*sinh(f*x+e)^
2)^(5/2)/a/f-1/8*(8*a^2+3*b*(8*a+b))*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1
/2))/f/a^(1/2)+1/8*(8*a^2+3*b*(8*a+b))*(a+b*sinh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.23, antiderivative size = 199, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{\left(\frac{3b(8a+b)}{a^2}+8\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24f}+\frac{\left(8a^2+3b(8a+b)\right)\sqrt{a+b\sinh^2(e+fx)}}{8af}-\frac{\left(8a^2+3b(8a+b)\right)\tanh^{-1}\left(\sqrt{\cdot}\right.}{8\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 + 3*b*(8*a + b))*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*
Sqrt[a]*f) + ((8*a^2 + 3*b*(8*a + b))*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a*f)
+ ((8 + (3*b*(8*a + b))/a^2)*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*f) - ((8*a
+ b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2))/(8*a^2*f) - (Csch[e + f
*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4*a*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2(a+bx)^{3/2}}{x^3}dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{4af} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(8a+b)+2a\right.}{x}\right.}{}$$

$$= -\frac{(8a+b)\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{8a^2f} - \frac{\text{csch}^4(e+f}{}$$

$$= \frac{\left(8a^2+3b(8a+b)\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24a^2f} - \frac{(8a+b)\text{csch}^2(e}{}$$

$$= \frac{\left(8a^2+3b(8a+b)\right)\sqrt{a+b\sinh^2(e+fx)}}{8af} + \frac{\left(8a^2+3b(8a+b)\right)}{}$$

$$= \frac{\left(8a^2+3b(8a+b)\right)\sqrt{a+b\sinh^2(e+fx)}}{8af} + \frac{\left(8a^2+3b(8a+b)\right)}{}$$

$$= -\frac{\left(8a^2+3b(8a+b)\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f} + \frac{\left(8a^2+3b(8a\right.}{}$$

Mathematica [A] time = 0.81, size = 123, normalized size = 0.61

$$\frac{\sqrt{a}\sqrt{a+b\sinh^2(e+fx)}\left(8\left(4a+b\sinh^2(e+fx)+6b\right)-3(8a+5b)\text{csch}^2(e+fx)-6a\text{csch}^4(e+fx)\right)-3\left(8\right.}{24\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] +
 Sqrt[a]*Sqrt[a + b*Sinh[e + f*x]^2]*(-3*(8*a + 5*b)*Csch[e + f*x]^2 - 6*a*
Csch[e + f*x]^4 + 8*(4*a + 6*b + b*Sinh[e + f*x]^2)))/(24*Sqrt[a]*f)
```

fricas [B] time = 1.21, size = 5509, normalized size = 27.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 + 24*a*b +
3*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 + 24*a*b + 3*b^2)*sinh(f*x +
 e)^11 - 4*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 + 24*a*b +
 3*b^2)*cosh(f*x + e)^2 - 32*a^2 - 96*a*b - 12*b^2)*sinh(f*x + e)^9 + 3*(55
*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 12*(8*a^2 + 24*a*b + 3*b^2)*cos
h(f*x + e))*sinh(f*x + e)^8 + 6*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 +
6*(55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^4 - 24*(8*a^2 + 24*a*b + 3*b^2
)*cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3*b^2)*sinh(f*x + e)^7 + 42*(11*(8*a^2
 + 24*a*b + 3*b^2)*cosh(f*x + e)^5 - 8*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x +
e)^3 + (8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 - 4*(8*a^2 +
 24*a*b + 3*b^2)*cosh(f*x + e)^5 + 2*(231*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x
 + e)^6 - 252*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^4 + 63*(8*a^2 + 24*a*b
 + 3*b^2)*cosh(f*x + e)^2 - 16*a^2 - 48*a*b - 6*b^2)*sinh(f*x + e)^5 + 2*(1
65*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 - 252*(8*a^2 + 24*a*b + 3*b^2)*
cosh(f*x + e)^5 + 105*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 10*(8*a^2
+ 24*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + (8*a^2 + 24*a*b + 3*b^2)
*cosh(f*x + e)^3 + (165*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^8 - 336*(8*a
^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^6 + 210*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*
x + e)^4 - 40*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3
*b^2)*sinh(f*x + e)^3 + (55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 - 144*
(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 + 126*(8*a^2 + 24*a*b + 3*b^2)*cos
h(f*x + e)^5 - 40*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 + 24*
a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (11*(8*a^2 + 24*a*b + 3*b^2)*
cosh(f*x + e)^10 - 36*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^8 + 42*(8*a^2
+ 24*a*b + 3*b^2)*cosh(f*x + e)^6 - 20*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x +
e)^4 + 3*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a)*l
og((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)
^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f
*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh
(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x +
 e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cos
h(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*
sqrt(2)*(a*b*cosh(f*x + e)^12 + 12*a*b*cosh(f*x + e)*sinh(f*x + e)^11 + a*b
*sinh(f*x + e)^12 + 2*(8*a^2 + 9*a*b)*cosh(f*x + e)^10 + 2*(33*a*b*cosh(f*x
 + e)^2 + 8*a^2 + 9*a*b)*sinh(f*x + e)^10 + 20*(11*a*b*cosh(f*x + e)^3 + (8
*a^2 + 9*a*b)*cosh(f*x + e))*sinh(f*x + e)^9 - (112*a^2 + 111*a*b)*cosh(f*x
 + e)^8 + (495*a*b*cosh(f*x + e)^4 + 90*(8*a^2 + 9*a*b)*cosh(f*x + e)^2 - 1
12*a^2 - 111*a*b)*sinh(f*x + e)^8 + 8*(99*a*b*cosh(f*x + e)^5 + 30*(8*a^2 +
 9*a*b)*cosh(f*x + e)^3 - (112*a^2 + 111*a*b)*cosh(f*x + e))*sinh(f*x + e)^
7 + 8*(18*a^2 + 23*a*b)*cosh(f*x + e)^6 + 4*(231*a*b*cosh(f*x + e)^6 + 105*
(8*a^2 + 9*a*b)*cosh(f*x + e)^4 - 7*(112*a^2 + 111*a*b)*cosh(f*x + e)^2 + 3
6*a^2 + 46*a*b)*sinh(f*x + e)^6 + 8*(99*a*b*cosh(f*x + e)^7 + 63*(8*a^2 + 9
*a*b)*cosh(f*x + e)^5 - 7*(112*a^2 + 111*a*b)*cosh(f*x + e)^3 + 6*(18*a^2 +
 23*a*b)*cosh(f*x + e))*sinh(f*x + e)^5 - (112*a^2 + 111*a*b)*cosh(f*x + e)
^4 + (495*a*b*cosh(f*x + e)^8 + 420*(8*a^2 + 9*a*b)*cosh(f*x + e)^6 - 70*(1
```

12*a^2 + 111*a*b)*cosh(f*x + e)^4 + 120*(18*a^2 + 23*a*b)*cosh(f*x + e)^2 -
112*a^2 - 111*a*b)*sinh(f*x + e)^4 + 4*(55*a*b*cosh(f*x + e)^9 + 60*(8*a^2
+ 9*a*b)*cosh(f*x + e)^7 - 14*(112*a^2 + 111*a*b)*cosh(f*x + e)^5 + 40*(18
*a^2 + 23*a*b)*cosh(f*x + e)^3 - (112*a^2 + 111*a*b)*cosh(f*x + e))*sinh(f*
x + e)^3 + 2*(8*a^2 + 9*a*b)*cosh(f*x + e)^2 + 2*(33*a*b*cosh(f*x + e)^10 +
45*(8*a^2 + 9*a*b)*cosh(f*x + e)^8 - 14*(112*a^2 + 111*a*b)*cosh(f*x + e)^
6 + 60*(18*a^2 + 23*a*b)*cosh(f*x + e)^4 - 3*(112*a^2 + 111*a*b)*cosh(f*x +
e)^2 + 8*a^2 + 9*a*b)*sinh(f*x + e)^2 + a*b + 4*(3*a*b*cosh(f*x + e)^11 +
5*(8*a^2 + 9*a*b)*cosh(f*x + e)^9 - 2*(112*a^2 + 111*a*b)*cosh(f*x + e)^7 +
12*(18*a^2 + 23*a*b)*cosh(f*x + e)^5 - (112*a^2 + 111*a*b)*cosh(f*x + e)^3
+ (8*a^2 + 9*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^11 + 11*a*f*cosh(f*x + e)*sinh(f
*x + e)^10 + a*f*sinh(f*x + e)^11 - 4*a*f*cosh(f*x + e)^9 + (55*a*f*cosh(f*
x + e)^2 - 4*a*f)*sinh(f*x + e)^9 + 6*a*f*cosh(f*x + e)^7 + 3*(55*a*f*cosh(
f*x + e)^3 - 12*a*f*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(55*a*f*cosh(f*x + e
)^4 - 24*a*f*cosh(f*x + e)^2 + a*f)*sinh(f*x + e)^7 - 4*a*f*cosh(f*x + e)^5
+ 42*(11*a*f*cosh(f*x + e)^5 - 8*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*
sinh(f*x + e)^6 + 2*(231*a*f*cosh(f*x + e)^6 - 252*a*f*cosh(f*x + e)^4 + 63
*a*f*cosh(f*x + e)^2 - 2*a*f)*sinh(f*x + e)^5 + a*f*cosh(f*x + e)^3 + 2*(16
5*a*f*cosh(f*x + e)^7 - 252*a*f*cosh(f*x + e)^5 + 105*a*f*cosh(f*x + e)^3 -
10*a*f*cosh(f*x + e))*sinh(f*x + e)^4 + (165*a*f*cosh(f*x + e)^8 - 336*a*f
*cosh(f*x + e)^6 + 210*a*f*cosh(f*x + e)^4 - 40*a*f*cosh(f*x + e)^2 + a*f)*
sinh(f*x + e)^3 + (55*a*f*cosh(f*x + e)^9 - 144*a*f*cosh(f*x + e)^7 + 126*a
*f*cosh(f*x + e)^5 - 40*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*sinh(f*x
+ e)^2 + (11*a*f*cosh(f*x + e)^10 - 36*a*f*cosh(f*x + e)^8 + 42*a*f*cosh(f
*x + e)^6 - 20*a*f*cosh(f*x + e)^4 + 3*a*f*cosh(f*x + e)^2)*sinh(f*x + e)),
1/24*(3*((8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 + 24*a*b +
3*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 + 24*a*b + 3*b^2)*sinh(f*x +
e)^11 - 4*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 + 24*a*b +
3*b^2)*cosh(f*x + e)^2 - 32*a^2 - 96*a*b - 12*b^2)*sinh(f*x + e)^9 + 3*(55
*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 12*(8*a^2 + 24*a*b + 3*b^2)*cos
h(f*x + e))*sinh(f*x + e)^8 + 6*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 +
6*(55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^4 - 24*(8*a^2 + 24*a*b + 3*b^2
)*cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3*b^2)*sinh(f*x + e)^7 + 42*(11*(8*a^2
+ 24*a*b + 3*b^2)*cosh(f*x + e)^5 - 8*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x +
e)^3 + (8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 - 4*(8*a^2 +
24*a*b + 3*b^2)*cosh(f*x + e)^5 + 2*(231*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x
+ e)^6 - 252*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^4 + 63*(8*a^2 + 24*a*b
+ 3*b^2)*cosh(f*x + e)^2 - 16*a^2 - 48*a*b - 6*b^2)*sinh(f*x + e)^5 + 2*(1
65*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 - 252*(8*a^2 + 24*a*b + 3*b^2)*
cosh(f*x + e)^5 + 105*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 10*(8*a^2
+ 24*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + (8*a^2 + 24*a*b + 3*b^2)
*cosh(f*x + e)^3 + (165*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^8 - 336*(8*a
^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^6 + 210*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*
x + e)^4 - 40*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3
*b^2)*sinh(f*x + e)^3 + (55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 - 144*
(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 + 126*(8*a^2 + 24*a*b + 3*b^2)*cos
h(f*x + e)^5 - 40*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 + 24*
a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (11*(8*a^2 + 24*a*b + 3*b^2)*
cosh(f*x + e)^10 - 36*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^8 + 42*(8*a^2
+ 24*a*b + 3*b^2)*cosh(f*x + e)^6 - 20*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x +
e)^4 + 3*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(-a)*
arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2
*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)
)/(a*cosh(f*x + e) + a*sinh(f*x + e))) + sqrt(2)*(a*b*cosh(f*x + e)^12 + 12
*a*b*cosh(f*x + e)*sinh(f*x + e)^11 + a*b*sinh(f*x + e)^12 + 2*(8*a^2 + 9*a
*b)*cosh(f*x + e)^10 + 2*(33*a*b*cosh(f*x + e)^2 + 8*a^2 + 9*a*b)*sinh(f*x
+ e)^10 + 20*(11*a*b*cosh(f*x + e)^3 + (8*a^2 + 9*a*b)*cosh(f*x + e))*sinh(

```
f*x + e)^9 - (112*a^2 + 111*a*b)*cosh(f*x + e)^8 + (495*a*b*cosh(f*x + e)^4
 + 90*(8*a^2 + 9*a*b)*cosh(f*x + e)^2 - 112*a^2 - 111*a*b)*sinh(f*x + e)^8
+ 8*(99*a*b*cosh(f*x + e)^5 + 30*(8*a^2 + 9*a*b)*cosh(f*x + e)^3 - (112*a^2
 + 111*a*b)*cosh(f*x + e))*sinh(f*x + e)^7 + 8*(18*a^2 + 23*a*b)*cosh(f*x +
 e)^6 + 4*(231*a*b*cosh(f*x + e)^6 + 105*(8*a^2 + 9*a*b)*cosh(f*x + e)^4 -
7*(112*a^2 + 111*a*b)*cosh(f*x + e)^2 + 36*a^2 + 46*a*b)*sinh(f*x + e)^6 +
8*(99*a*b*cosh(f*x + e)^7 + 63*(8*a^2 + 9*a*b)*cosh(f*x + e)^5 - 7*(112*a^2
 + 111*a*b)*cosh(f*x + e)^3 + 6*(18*a^2 + 23*a*b)*cosh(f*x + e))*sinh(f*x +
 e)^5 - (112*a^2 + 111*a*b)*cosh(f*x + e)^4 + (495*a*b*cosh(f*x + e)^8 + 42
0*(8*a^2 + 9*a*b)*cosh(f*x + e)^6 - 70*(112*a^2 + 111*a*b)*cosh(f*x + e)^4
+ 120*(18*a^2 + 23*a*b)*cosh(f*x + e)^2 - 112*a^2 - 111*a*b)*sinh(f*x + e)^
4 + 4*(55*a*b*cosh(f*x + e)^9 + 60*(8*a^2 + 9*a*b)*cosh(f*x + e)^7 - 14*(11
2*a^2 + 111*a*b)*cosh(f*x + e)^5 + 40*(18*a^2 + 23*a*b)*cosh(f*x + e)^3 - (
112*a^2 + 111*a*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*(8*a^2 + 9*a*b)*cosh(
f*x + e)^2 + 2*(33*a*b*cosh(f*x + e)^10 + 45*(8*a^2 + 9*a*b)*cosh(f*x + e)^
8 - 14*(112*a^2 + 111*a*b)*cosh(f*x + e)^6 + 60*(18*a^2 + 23*a*b)*cosh(f*x
+ e)^4 - 3*(112*a^2 + 111*a*b)*cosh(f*x + e)^2 + 8*a^2 + 9*a*b)*sinh(f*x +
e)^2 + a*b + 4*(3*a*b*cosh(f*x + e)^11 + 5*(8*a^2 + 9*a*b)*cosh(f*x + e)^9
- 2*(112*a^2 + 111*a*b)*cosh(f*x + e)^7 + 12*(18*a^2 + 23*a*b)*cosh(f*x + e
)^5 - (112*a^2 + 111*a*b)*cosh(f*x + e)^3 + (8*a^2 + 9*a*b)*cosh(f*x + e))*
sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(
f*x + e)^11 + 11*a*f*cosh(f*x + e)*sinh(f*x + e)^10 + a*f*sinh(f*x + e)^11
- 4*a*f*cosh(f*x + e)^9 + (55*a*f*cosh(f*x + e)^2 - 4*a*f)*sinh(f*x + e)^9
+ 6*a*f*cosh(f*x + e)^7 + 3*(55*a*f*cosh(f*x + e)^3 - 12*a*f*cosh(f*x + e))
*sinh(f*x + e)^8 + 6*(55*a*f*cosh(f*x + e)^4 - 24*a*f*cosh(f*x + e)^2 + a*f
)*sinh(f*x + e)^7 - 4*a*f*cosh(f*x + e)^5 + 42*(11*a*f*cosh(f*x + e)^5 - 8*
a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(231*a*f*cosh(
f*x + e)^6 - 252*a*f*cosh(f*x + e)^4 + 63*a*f*cosh(f*x + e)^2 - 2*a*f)*sinh
(f*x + e)^5 + a*f*cosh(f*x + e)^3 + 2*(165*a*f*cosh(f*x + e)^7 - 252*a*f*co
sh(f*x + e)^5 + 105*a*f*cosh(f*x + e)^3 - 10*a*f*cosh(f*x + e))*sinh(f*x +
e)^4 + (165*a*f*cosh(f*x + e)^8 - 336*a*f*cosh(f*x + e)^6 + 210*a*f*cosh(f*
x + e)^4 - 40*a*f*cosh(f*x + e)^2 + a*f)*sinh(f*x + e)^3 + (55*a*f*cosh(f*x
 + e)^9 - 144*a*f*cosh(f*x + e)^7 + 126*a*f*cosh(f*x + e)^5 - 40*a*f*cosh(f
*x + e)^3 + 3*a*f*cosh(f*x + e))*sinh(f*x + e)^2 + (11*a*f*cosh(f*x + e)^10
 - 36*a*f*cosh(f*x + e)^8 + 42*a*f*cosh(f*x + e)^6 - 20*a*f*cosh(f*x + e)^4
 + 3*a*f*cosh(f*x + e)^2)*sinh(f*x + e))]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 4.59Unable to divide, perha
ps due to rounding error%%%{1,[10,0,12]%%%}+%%%{%%{[10,0]:[1,0,%%%{-1,[1]%%
%}]%%},[9,0,12]%%%}+%%%{-20,[8,1,12]%%%}+%%%{%%%{45,[1]%%%},[8,0,12]%%%}+%%
%{%%{[-160,0]:[1,0,%%%{-1,[1]%%%}]%%},[7,1,12]%%%}+%%%{%%{[%%%{120,[1]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[7,0,12]%%%}+%%%{160,[6,2,12]%%%}+%%%{%%%{-560,[
1]%%%},[6,1,12]%%%}+%%%{%%%{210,[2]%%%},[6,0,12]%%%}+%%%{%%{[960,0]:[1,0,%%
%{-1,[1]%%%}]%%},[5,2,12]%%%}+%%%{%%{[%%%{-1120,[1]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[5,1,12]%%%}+%%%{%%{[%%%{252,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,
0,12]%%%}+%%%{-640,[4,3,12]%%%}+%%%{%%%{2400,[1]%%%},[4,2,12]%%%}+%%%{%%%{-
1400,[2]%%%},[4,1,12]%%%}+%%%{%%%{210,[3]%%%},[4,0,12]%%%}+%%%{%%{[-2560,0]
:[1,0,%%%{-1,[1]%%%}]%%},[3,3,12]%%%}+%%%{%%{[%%%{3200,[1]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[3,2,12]%%%}+%%%{%%{[%%%{-1120,[2]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[3,1,12]%%%}+%%%{%%{[%%%{120,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,
12]%%%}+%%%{1280,[2,4,12]%%%}+%%%{%%%{-3840,[1]%%%},[2,3,12]%%%}+%%%{%%%{24
```

```
00,[2]%%%},[2,2,12]%%%}+%%%{%%%{-560,[3]%%%},[2,1,12]%%%}+%%%{%%%{45,[4]%%%
},[2,0,12]%%%}+%%%{%%{[2560,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,4,12]%%%}+%%%{%%{
[%%%{-2560,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,3,12]%%%}+%%%{%%{[%%%{960,
[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,2,12]%%%}+%%%{%%{[%%%{-160,[3]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[1,1,12]%%%}+%%%{%%{[%%%{10,[4]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[1,0,12]%%%}+%%%{-1024,[0,5,12]%%%}+%%%{%%%{1280,[1]%%%},[0,4,
12]%%%}+%%%{%%%{-640,[2]%%%},[0,3,12]%%%}+%%%{%%%{160,[3]%%%},[0,2,12]%%%}+
%%%{%%%{-20,[4]%%%},[0,1,12]%%%}+%%%{%%%{1,[5]%%%},[0,0,12]%%%} / %%%{%%{po
ly1[%%%{1,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,0,0]%%%}+%%%{%%%{10,[3]%%%
},[9,0,0]%%%}+%%%{%%{[%%%{-20,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,1,0]%%%
}+%%%{%%{poly1[%%%{45,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[8,0,0]%%%}+%%%{%%
%{-160,[3]%%%},[7,1,0]%%%}+%%%{%%%{120,[4]%%%},[7,0,0]%%%}+%%%{%%{poly1[%%%
{160,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,2,0]%%%}+%%%{%%{[%%%{-560,[3]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,1,0]%%%}+%%%{%%{poly1[%%%{210,[4]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[6,0,0]%%%}+%%%{%%%{960,[3]%%%},[5,2,0]%%%}+%%%{%%%{-
1120,[4]%%%},[5,1,0]%%%}+%%%{%%%{252,[5]%%%},[5,0,0]%%%}+%%%{%%{[%%%{-640,[
2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,3,0]%%%}+%%%{%%{poly1[%%%{2400,[3]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[4,2,0]%%%}+%%%{%%{[%%%{-1400,[4]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[4,1,0]%%%}+%%%{%%{poly1[%%%{210,[5]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[4,0,0]%%%}+%%%{%%%{-2560,[3]%%%},[3,3,0]%%%}+%%%{%%%{3200,[4]%%
%},[3,2,0]%%%}+%%%{%%%{-1120,[5]%%%},[3,1,0]%%%}+%%%{%%%{120,[6]%%%},[3,0,0
]%%%}+%%%{%%{[%%%{1280,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,4,0]%%%}+%%%{%
%{[%%%{-3840,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,3,0]%%%}+%%%{%%{poly1[%%
%{2400,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,2,0]%%%}+%%%{%%{[%%%{-560,[5]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,1,0]%%%}+%%%{%%{poly1[%%%{45,[6]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[2,0,0]%%%}+%%%{%%%{2560,[3]%%%},[1,4,0]%%%}+%%%{%%%
{-2560,[4]%%%},[1,3,0]%%%}+%%%{%%%{960,[5]%%%},[1,2,0]%%%}+%%%{%%%{-160,[6]
%%%},[1,1,0]%%%}+%%%{%%%{10,[7]%%%},[1,0,0]%%%}+%%%{%%{[%%%{-1024,[2]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[0,5,0]%%%}+%%%{%%{[%%%{1280,[3]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[0,4,0]%%%}+%%%{%%{[%%%{-640,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[0,3,0]%%%}+%%%{%%{poly1[%%%{160,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,
2,0]%%%}+%%%{%%{[%%%{-20,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,1,0]%%%}+%%%
{%%{poly1[%%%{1,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,0]%%%} Error: Bad A
rgument Value
```

maple [C] time = 0.25, size = 113, normalized size = 0.56

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\left(\cosh^{4}\left(fx+e\right)\right)\left(b^{2}\left(\cosh^{4}\left(fx+e\right)\right)+2ab\left(\cosh^{2}\left(fx+e\right)\right)-2b^{2}\left(\cosh^{2}\left(fx+e\right)\right)+a^{2}-2ab+b^{2}\right)}{\sinh\left(fx+e\right)\left(\cosh^{4}\left(fx+e\right)-2\left(\cosh^{2}\left(fx+e\right)\right)+1\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(cosh(f*x+e)^4-2*cosh(f*x+e)^2+1)*cosh(f*x+e)^4*
(b^2*cosh(f*x+e)^4+2*a*b*cosh(f*x+e)^2-2*b^2*cosh(f*x+e)^2+a^2-2*a*b+b^2)/(
a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \coth\left(e+f\,x\right)^5\,{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(coth(e + f*x)^5*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.474 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx$

Optimal. Leaf size=305

$$-\frac{\tanh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-2b)\sinh^2(e+fx)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{8(a-2b}{}$$

```
[Out] -1/3*(3*a-8*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-8/3*(a-2
*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+
e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(
1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a-8*b)*(1/(1+sinh
(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x
+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f
*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+8/3*(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*
tanh(f*x+e)/f+(a-2*b)*sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
-1/3*(a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3/f
```

Rubi [A] time = 0.38, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 467, 577, 582, 531, 418, 492, 411}

$$-\frac{\tanh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-2b)\sinh^2(e+fx)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{8(a-2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^4,x]
```

```
[Out] -((3*a - 8*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f
) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sq
rt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^
2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*
f) + ((a - 2*b)*Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/
f - ((a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3)/(3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
```

```
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^{5/2}}\,dx, x, \sinh\right.}{f}$$

$$= -\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)}{3f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{se}\right.}{}$$

$$= \frac{(a-2b)\sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{f} - \frac{(a}{}$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} +$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} +$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} +$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} -$$

Mathematica [C] time = 2.81, size = 224, normalized size = 0.73

$$\frac{-\frac{\tanh(e+fx)\mathrm{sech}^2(e+fx)\left(\left(64a^2-160ab+17b^2\right)\cosh(2(e+fx))+32a^2+2b(6a-17b)\cosh(4(e+fx))-108ab-b^2\cosh(6(e+fx))+18b^2\right)}{4\sqrt{2}} + 4ia(5a-}{12f\sqrt{2a+b\cosh(2(e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^4,x]
```

```
[Out] ((-32*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] + (4*I)*a*(5*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticF[I*(e + f*x), b/a] - ((32*a^2 - 108*a*b + 18*b^2 + (64*a^2 - 160*
a*b + 17*b^2)*Cosh[2*(e + f*x)] + 2*(6*a - 17*b)*b*Cosh[4*(e + f*x)] - b^2*
Cosh[6*(e + f*x)])*Sech[e + f*x]^2*Tanh[e + f*x])/(4*Sqrt[2]))/(12*f*Sqrt[2
*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 2.27Unable to divide, perhaps due to rou
nding error%%%{-786432,[8,13,12]%%%}+%%%{%%%{3932160,[1]%%%},[8,13,11]%%%}+
%%%{%%%{-7864320,[2]%%%},[8,13,10]%%%}+%%%{%%%{7864320,[3]%%%},[8,13,9]%%%}
+%%%{%%%{-3932160,[4]%%%},[8,13,8]%%%}+%%%{%%%{786432,[5]%%%},[8,13,7]%%%}+
%%%{%%{[6291456,0]:[1,0,%%%{-1,[1]%%%}]%%},[7,13,12]%%%}+%%%{%%{[%%%{-31457
280,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,13,11]%%%}+%%%{%%{[%%%{62914560,[
2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,13,10]%%%}+%%%{%%{[%%%{-62914560,[3]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,13,9]%%%}+%%%{%%{[%%%{31457280,[4]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[7,13,8]%%%}+%%%{%%{[%%%{-6291456,[5]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[7,13,7]%%%}+%%%{-12582912,[6,13,13]%%%}+%%%{%%%{53477376
,[1]%%%},[6,13,12]%%%}+%%%{%%%{-78643200,[2]%%%},[6,13,11]%%%}+%%%{%%%{3145
7280,[3]%%%},[6,13,10]%%%}+%%%{%%%{31457280,[4]%%%},[6,13,9]%%%}+%%%{%%%{-3
4603008,[5]%%%},[6,13,8]%%%}+%%%{%%%{9437184,[6]%%%},[6,13,7]%%%}+%%%{%%{[7
5497472,0]:[1,0,%%%{-1,[1]%%%}]%%},[5,13,13]%%%}+%%%{%%{[%%%{-408944640,[1]
%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,13,12]%%%}+%%%{%%{[%%%{912261120,[2]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[5,13,11]%%%}+%%%{%%{[%%%{-1069547520,[3]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[5,13,10]%%%}+%%%{%%{[%%%{692060160,[4]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[5,13,9]%%%}+%%%{%%{[%%%{-232783872,[5]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[5,13,8]%%%}+%%%{%%{[%%%{31457280,[6]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[5,13,7]%%%}+%%%{-75497472,[4,13,14]%%%}+%%%{%%%{339738624,[1]%
%%},[4,13,13]%%%}+%%%{%%%{-508035072,[2]%%%},[4,13,12]%%%}+%%%{%%%{86507520
,[3]%%%},[4,13,11]%%%}+%%%{%%%{581959680,[4]%%%},[4,13,10]%%%}+%%%{%%%{-695
205888,[5]%%%},[4,13,9]%%%}+%%%{%%%{328728576,[6]%%%},[4,13,8]%%%}+%%%{%%%{
-58195968,[7]%%%},[4,13,7]%%%}+%%%{%%{[301989888,0]:[1,0,%%%{-1,[1]%%%}]%%}
,[3,13,14]%%%}+%%%{%%{[%%%{-1862270976,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
3,13,13]%%%}+%%%{%%{[%%%{4875878400,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1
3,12]%%%}+%%%{%%{[%%%{-7014973440,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,13,
11]%%%}+%%%{%%{[%%%{5976883200,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,13,10]
%%%}+%%%{%%{[%%%{-3007315968,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,13,9]%%%
}+%%%{%%{[%%%{824180736,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,13,8]%%%}+%%%
{%%{[%%%{-94371840,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,13,7]%%%}+%%%{-201
326592,[2,13,15]%%%}+%%%{%%%{1157627904,[1]%%%},[2,13,14]%%%}+%%%{%%%{-2654
994432,[2]%%%},[2,13,13]%%%}+%%%{%%%{2872049664,[3]%%%},[2,13,12]%%%}+%%%{%
%%{-959447040,[4]%%%},[2,13,11]%%%}+%%%{%%%{-1025507328,[5]%%%},[2,13,10]%%
%}+%%%{%%%{1264582656,[6]%%%},[2,13,9]%%%}+%%%{%%%{-537919488,[7]%%%},[2,13
,8]%%%}+%%%{%%%{84934656,[8]%%%},[2,13,7]%%%}+%%%{%%{[402653184,0]:[1,0,%%%
{-1,[1]%%%}]%%},[1,13,15]%%%}+%%%{%%{[%%%{-2919235584,[1]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[1,13,14]%%%}+%%%{%%{[%%%{9235857408,[2]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[1,13,13]%%%}+%%%{%%{[%%%{-16653484032,[3]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[1,13,12]%%%}+%%%{%%{[%%%{18717081600,[4]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[1,13,11]%%%}+%%%{%%{[%%%{-13425967104,[5]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[1,13,10]%%%}+%%%{%%{[%%%{6002049024,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[1,13,9]%%%}+%%%{%%{[%%%{-1528823808,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[1,13,8]%%%}+%%%{%%{[%%%{169869312,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1
3,7]%%%}+%%%{-201326592,[0,13,16]%%%}+%%%{%%%{1610612736,[1]%%%},[0,13,15]%
%%}+%%%{%%%{-5712642048,[2]%%%},[0,13,14]%%%}+%%%{%%%{11790188544,[3]%%%},[
0,13,13]%%%}+%%%{%%%{-15603597312,[4]%%%},[0,13,12]%%%}+%%%{%%%{13731889152
,[5]%%%},[0,13,11]%%%}+%%%{%%%{-8035762176,[6]%%%},[0,13,10]%%%}+%%%{%%%{30
15180288,[7]%%%},[0,13,9]%%%}+%%%{%%%{-658243584,[8]%%%},[0,13,8]%%%}+%%%{%
%%{63700992,[9]%%%},[0,13,7]%%%} / %%%{%%%{1,[2]%%%},[8,0,0]%%%}+%%%{%%{pol
y1[%%%{-8,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,0,0]%%%}+%%%{%%%{16,[2]%%%}
,[6,0,1]%%%}+%%%{%%%{12,[3]%%%},[6,0,0]%%%}+%%%{%%{[%%%{-96,[2]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[5,0,1]%%%}+%%%{%%{poly1[%%%{40,[3]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[5,0,0]%%%}+%%%{%%%{96,[2]%%%},[4,0,2]%%%}+%%%{%%%{48,[3]%%%},[
4,0,1]%%%}+%%%{%%%{-74,[4]%%%},[4,0,0]%%%}+%%%{%%{poly1[%%%{-384,[2]%%%},0]
```

```
:[1,0,%%%{-1,[1]%%%}]%%},[3,0,2]%%%}+%%%{%%{[%%%{448,[3]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[3,0,1]%%%}+%%%{%%{poly1[%%%{-120,[4]%%%},0]:[1,0,%%%{-1,[1]%%
%}]%%},[3,0,0]%%%}+%%%{%%%{256,[2]%%%},[2,0,3]%%%}+%%%{%%%{-192,[3]%%%},[2,
0,2]%%%}+%%%{%%%{-144,[4]%%%},[2,0,1]%%%}+%%%{%%%{108,[5]%%%},[2,0,0]%%%}+%
%%{%%{[%%%{-512,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,3]%%%}+%%%{%%{poly1
[%%%{1152,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,2]%%%}+%%%{%%{[%%%{-864,[
4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1]%%%}+%%%{%%{poly1[%%%{216,[5]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,0]%%%}+%%%{%%%{256,[2]%%%},[0,0,4]%%%}+%%%{
%%%{-768,[3]%%%},[0,0,3]%%%}+%%%{%%%{864,[4]%%%},[0,0,2]%%%}+%%%{%%%{-432,[
5]%%%},[0,0,1]%%%}+%%%{%%%{81,[6]%%%},[0,0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.31, size = 385, normalized size = 1.26

$$\sqrt{-\frac{b}{a}}\, b^2 \sinh\left(fx+e\right)\left(\cosh^6\left(fx+e\right)\right)+\left(-3\sqrt{-\frac{b}{a}}\, ab+7\sqrt{-\frac{b}{a}}\, b^2\right)\left(\cosh^4\left(fx+e\right)\right)\sinh\left(fx+e\right)+\left(-4\sqrt{-\frac{b}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^6+(-3*(-1/a*b)^(1/2)*a*b+7*
(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(-4*(-1/a*b)^(1/2)*a^2+13*(-1
/a*b)^(1/2)*a*b-9*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+((-1/a*b)^(
1/2)*a^2-2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)+(b/a*cosh(f*x
+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*a^2-16*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))*a*b+16*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+8*EllipticE
(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-16*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2)/(-1/a*b)^(1/2)/cosh(f*x+e)^3/(a+
b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tanh\left(e+f\,x\right)^4\,\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**4,x)
```

```
[Out] Timed out
```

3.475 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx$

Optimal. Leaf size=260

$$-\frac{\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{4b\sinh(e+fx)\cosh(e+}{3}$$

```
[Out] 4/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-1/3*(7*a-8*b)*(1/
(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+s
inh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/
(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a-4*b)*(1/(1+sinh(f*x+e)
^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^
(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2
*(a+b*sinh(f*x+e)^2)/a)^(1/2)-(a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)/f+1/3*(
7*a-8*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.25, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3196, 467, 528, 531, 418, 492, 411}

$$-\frac{\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{4b\sinh(e+fx)\cosh(e+}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^2,x]
```

```
[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a
 - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*
Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) +
 ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f) - ((a
+ b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/f
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
```

```
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^{3/2}}\,dx,x,\sinh(e\right.}{f}$$

$$= -\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(\right.}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\left(a+b\sinh\right.}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\left(a+b\sinh\right.}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{(3a-4b)F(}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{(7a-8b)E(}{}$$

Mathematica [C] time = 2.91, size = 188, normalized size = 0.72

$$\frac{\sqrt{2}\tanh(e+fx)\left(-24a^2-4b(2a-3b)\cosh(2(e+fx))+40ab+b^2\cosh(4(e+fx))-13b^2\right)+32ia(a-b)\sqrt{\frac{2a+b}{}}}{24f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^2,x]
```

```
[Out] ((-8*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(
e + f*x), b/a] + (32*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticF[I*(e + f*x), b/a] + Sqrt[2]*(-24*a^2 + 40*a*b - 13*b^2 - 4*(2*a -
3*b)*b*Cosh[2*(e + f*x)] + b^2*Cosh[4*(e + f*x)])*Tanh[e + f*x])/(24*f*Sqrt
[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.39Error: Bad Argument Typ
e
```

maple [A] time = 0.28, size = 414, normalized size = 1.59

$$-\frac{-\sqrt{-\frac{b}{a}}\, b^2 \left(\sinh^5\left(fx+e\right)\right) + 2\sqrt{-\frac{b}{a}}\, ab \left(\sinh^3\left(fx+e\right)\right) - 4\sqrt{-\frac{b}{a}}\, b^2 \left(\sinh^3\left(fx+e\right)\right) - 3a^2\sqrt{\frac{a+b\left(\sinh^2\left(fx+e\right)\right)}{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x)
```

```
[Out] -1/3*(-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^3-
4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+11*a*((
a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1
/a*b)^(1/2),(a/b)^(1/2))*b-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^
(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-7*((a+b*sinh(f*
x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*a*b+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ell
ipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+3*(-1/a*b)^(1/2)*a^2*sin
h(f*x+e)-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*
sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2 + a\right)^{\frac{3}{2}} \tanh\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tanh\left(e+f\,x\right)^2 \left(b\sinh\left(e+f\,x\right)^2 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}} \tanh^2\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**2,x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(3/2)*tanh(e + f*x)**2, x)
```

3.476 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=174

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\text{s}}}{3f\sqrt{}}$$

```
[Out] 1/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f-2/3*I*(2*a-b)*(co
s(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*
(a+b*sinh(f*x+e)^2)^(1/2)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)+1/3*I*a*(a-b)*(cos(
I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(1/2))*(1
+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\text{s}}}{3f\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{1}{3}\int \frac{a(3a-b)+2(2a-}{\sqrt{a+b\sinh}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{1}{3}(a(a-b))\int \frac{}{\sqrt{a+b\,\mathrm{si}}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2(2a-b)\sqrt{a+b\sinh^2}\right.}{3\sqrt{1}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{2i(2a-b)E\left(ie+ifx\left|\frac{b}{a}\right.\right)}{3f\sqrt{1+\frac{b}{}}}$$

Mathematica [A] time = 0.76, size = 169, normalized size = 0.97

$$\frac{b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)+2i\sqrt{2}\,a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\left|\frac{b}{a}\right.\right)-4i\sqrt{2}\,a(2a}{6f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.24, size = 416, normalized size = 2.39

$$\frac{\sqrt{-\frac{b}{a}}\, b^{2} \sinh\left(f x+e\right)\left(\cosh^{4}\left(f x+e\right)\right)+\left(\sqrt{-\frac{b}{a}}\, a b-\sqrt{-\frac{b}{a}}\, b^{2}\right)\left(\cosh^{2}\left(f x+e\right)\right)\sinh\left(f x+e\right)+3 a^{2}\sqrt{\frac{b\left(\cosh^{2}\left(f x+\right.\right.}{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a*b-(-1/a
*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^
2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sinh\left(f x+e\right)^{2}+a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.477 $\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=256

$$\frac{(7a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{4b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth(e+f\ldots}{}$$

```
[Out] -coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+4/3*b*cosh(f*x+e)*sinh(f*x+e)*(a+b
*sinh(f*x+e)^2)^(1/2)/f-1/3*(7*a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x
+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*s
ech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a
)^(1/2)+1/3*(3*a+5*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+
b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(7
*a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.27, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3196, 473, 528, 531, 418, 492, 411}

$$\frac{(7a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{4b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth(e+f\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (Coth
[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/f - ((7*a + b)*EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*
f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a +
b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
```

```
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int\frac{\sqrt{1+x^2}\left(a+bx^2\right)^{3/2}}{x^2}dx,x,\right.}{f}$$

$$= -\frac{\coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\,\text{se}\right.}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\coth(e+}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\coth(e+}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\coth(e+}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\coth(e+}{}$$

Mathematica [C] time = 2.15, size = 184, normalized size = 0.72

$$\frac{\sqrt{2}\coth(e+fx)\left(-24a^2-4b(2a+b)\cosh(2(e+fx))+8ab+b^2\cosh(4(e+fx))+3b^2\right)+32ia(a-b)\sqrt{\frac{2a+b}{}}}{24f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*(-24*a^2 + 8*a*b + 3*b^2 - 4*b*(2*a + b)*Cosh[2*(e + f*x)] + b^2*C
osh[4*(e + f*x)])*Coth[e + f*x] - (8*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[
2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (32*I)*a*(a - b)*Sqrt[(2*a -
 b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(24*f*Sqrt[2*a -
b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\coth\left(fx+e\right)^2\sinh\left(fx+e\right)^2+a\coth\left(fx+e\right)^2\right)\sqrt{b\sinh\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*coth(f*x + e)^2*sinh(f*x + e)^2 + a*coth(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.74Unable to divide, perha
ps due to rounding error%%%{64,[4,8,4]%%%}+%%%{%%%{-128,[1]%%%},[4,8,3]%%%}
+%%%{%%%{64,[2]%%%},[4,8,2]%%%}+%%%{%%{[256,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8
,4]%%%}+%%%{%%{[%%%{-512,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8,3]%%%}+%%%
{%%{[%%%{256,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8,2]%%%}+%%%{-512,[2,8,5
]%%%}+%%%{%%%{1408,[1]%%%},[2,8,4]%%%}+%%%{%%%{-1280,[2]%%%},[2,8,3]%%%}+%%
%{%%%{384,[3]%%%},[2,8,2]%%%}+%%%{%%{[-1024,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,8
,5]%%%}+%%%{%%{[%%%{2304,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,8,4]%%%}+%%%
{%%{[%%%{-1536,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,8,3]%%%}+%%%{%%{[%%%{2
56,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,8,2]%%%}+%%%{1024,[0,8,6]%%%}+%%%{
%%%{-2560,[1]%%%},[0,8,5]%%%}+%%%{%%%{2112,[2]%%%},[0,8,4]%%%}+%%%{%%%{-640
,[3]%%%},[0,8,3]%%%}+%%%{%%%{64,[4]%%%},[0,8,2]%%%} / %%%{%%%{1,[1]%%%},[4,
0,0]%%%}+%%%{%%{poly1[%%%{4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,0]%%%}+
%%%{%%%{-8,[1]%%%},[2,0,1]%%%}+%%%{%%%{6,[2]%%%},[2,0,0]%%%}+%%%{%%{poly1[%
%%{-16,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1]%%%}+%%%{%%{poly1[%%%{4,[2
]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,0]%%%}+%%%{%%%{16,[1]%%%},[0,0,2]%%%}
+%%%{%%%{-8,[2]%%%},[0,0,1]%%%}+%%%{%%%{1,[3]%%%},[0,0,0]%%%} Error: Bad Ar
gument Value
```

maple [A] time = 0.30, size = 330, normalized size = 1.29

$$-\frac{-\sinh\left(fx+e\right)\sqrt{\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\left(3\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)a^2-2\,\mathrm{Elliptic}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(-sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
(3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-2*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),
(a/b)^(1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+El
lipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)-(-1/a*b)^(1/2)*b^2*cos
h(f*x+e)^6+(2*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4+(3*(-1
/a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2)/sin
h(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{coth}\left(e+f\,x\right)^2\,{\left(b\,\mathrm{sinh}\left(e+f\,x\right)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^2*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.478 $\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=306

$$\frac{8(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{(3a+5b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth^3(e}{}$$

```
[Out] -1/3*coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2)/f-(a+b)*cosh(f*x+e)^2*coth(f*x
+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/3*(3*a+5*b)*cosh(f*x+e)*sinh(f*x+e)*(a+b*
sinh(f*x+e)^2)^(1/2)/f-8/3*(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech
(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(
1/2)+1/3*(3*a+b)*(a+3*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2
)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*
(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+8
/3*(a+b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/f
```

Rubi [A] time = 0.36, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 473, 580, 528, 531, 418, 492, 411}

$$\frac{8(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{(3a+5b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth^3(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + b)*Cosh[e + f*x]^2*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) +
((3*a + 5*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f)
 - (Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2))/(3*f) - (8*(a + b)*Ellip
ticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]
^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + b)*(
a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b
*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
) + (8*(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
```

```
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}}{x^4}dx,x,\mathrm{s}\right.}{f}$$

$$= -\frac{\coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\operatorname{sec}\right.}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{\coth}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a}{}$$

Mathematica [C] time = 4.71, size = 229, normalized size = 0.75

$$\frac{4i\left(5a^2-2ab-3b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\left(\left(64a^2+32ab-79b^2\right)\cosh(2(e+fx))-32a^2+2b(6a\right.}{4\sqrt{2}}}{12f\sqrt{2a+b\cosh(2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-1/4*((-32*a^2 - 44*a*b + 58*b^2 + (64*a^2 + 32*a*b - 79*b^2)*Cosh[2*(e +
f*x)] + 2*b*(6*a + 11*b)*Cosh[4*(e + f*x)] - b^2*Cosh[6*(e + f*x)])*Coth[e
+ f*x]*Csch[e + f*x]^2)/Sqrt[2] - (32*I)*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2
*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (4*I)*(5*a^2 - 2*a*b - 3*b^2)
*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(12*f
*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\coth\left(fx+e\right)^4\sinh\left(fx+e\right)^2+a\coth\left(fx+e\right)^4\right)\sqrt{b\sinh\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*coth(f*x + e)^4*sinh(f*x + e)^2 + a*coth(f*x + e)^4)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.99Unable to divide, perha
ps due to rounding error%%%{1,[8,0,10]%%%}+%%%{%%{[8,0]:[1,0,%%%{-1,[1]%%%}
]%%},[7,0,10]%%%}+%%%{-16,[6,1,10]%%%}+%%%{%%%{28,[1]%%%},[6,0,10]%%%}+%%%{
%%{[-96,0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,10]%%%}+%%%{%%{[%%%{56,[1]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[5,0,10]%%%}+%%%{96,[4,2,10]%%%}+%%%{%%%{-240,[1]%%%
},[4,1,10]%%%}+%%%{%%%{70,[2]%%%},[4,0,10]%%%}+%%%{%%{[384,0]:[1,0,%%%{-1,[
1]%%%}]%%},[3,2,10]%%%}+%%%{%%{[%%%{-320,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[3,1,10]%%%}+%%%{%%{[%%%{56,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,10]%%%
}+%%%{-256,[2,3,10]%%%}+%%%{%%%{576,[1]%%%},[2,2,10]%%%}+%%%{%%%{-240,[2]%%
%},[2,1,10]%%%}+%%%{%%%{28,[3]%%%},[2,0,10]%%%}+%%%{%%{[-512,0]:[1,0,%%%{-1
,[1]%%%}]%%},[1,3,10]%%%}+%%%{%%{[%%%{384,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%
},[1,2,10]%%%}+%%%{%%{[%%%{-96,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,10]%
%%}+%%%{%%{[%%%{8,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,10]%%%}+%%%{256,[
0,4,10]%%%}+%%%{%%%{-256,[1]%%%},[0,3,10]%%%}+%%%{%%%{96,[2]%%%},[0,2,10]%%
%}+%%%{%%%{-16,[3]%%%},[0,1,10]%%%}+%%%{%%%{1,[4]%%%},[0,0,10]%%%} / %%%{%%
%{1,[2]%%%},[8,0,0]%%%}+%%%{%%{poly1[%%%{8,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[7,0,0]%%%}+%%%{%%%{-16,[2]%%%},[6,1,0]%%%}+%%%{%%%{28,[3]%%%},[6,0,0]%%
%}+%%%{%%{[%%%{-96,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1,0]%%%}+%%%{%%{po
ly1[%%%{56,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,0,0]%%%}+%%%{%%%{96,[2]%%%
},[4,2,0]%%%}+%%%{%%%{-240,[3]%%%},[4,1,0]%%%}+%%%{%%%{70,[4]%%%},[4,0,0]%%
%}+%%%{%%{poly1[%%%{384,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,2,0]%%%}+%%%{
%%{[%%%{-320,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,0]%%%}+%%%{%%{poly1[%%
%{56,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,0]%%%}+%%%{%%%{-256,[2]%%%},[2
,3,0]%%%}+%%%{%%%{576,[3]%%%},[2,2,0]%%%}+%%%{%%%{-240,[4]%%%},[2,1,0]%%%}+
%%%{%%%{28,[5]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-512,[2]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[1,3,0]%%%}+%%%{%%{poly1[%%%{384,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[1,2,0]%%%}+%%%{%%{[%%%{-96,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,0]%%%}
+%%%{%%{poly1[%%%{8,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,0]%%%}+%%%{%%%{
256,[2]%%%},[0,4,0]%%%}+%%%{%%%{-256,[3]%%%},[0,3,0]%%%}+%%%{%%%{96,[4]%%%}
,[0,2,0]%%%}+%%%{%%%{-16,[5]%%%},[0,1,0]%%%}+%%%{%%%{1,[6]%%%},[0,0,0]%%%}
Error: Bad Argument Value
```

maple [A] time = 0.30, size = 540, normalized size = 1.76

$$-\frac{-\sqrt{-\frac{b}{a}}\, b^2 \left(\sinh^8\left(fx+e\right)\right) + 3\sqrt{-\frac{b}{a}}\, ab \left(\sinh^6\left(fx+e\right)\right) + 3\sqrt{-\frac{b}{a}}\, b^2 \left(\sinh^6\left(fx+e\right)\right) - 3a^2\sqrt{\frac{a+b\left(\sinh^2\left(fx+e\right)\right)}{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^8+3*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+
3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*
x+e)^3-2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3+5*((a+b*sinh(f*x+e)^2
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*b^2*sinh(f*x+e)^3-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(
1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3-8*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
```

```
-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+
e)^4+8*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^4+4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+
5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2+5*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a
*b)^(1/2)*a^2)/(-1/a*b)^(1/2)/sinh(f*x+e)^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \coth\left(e+f\,x\right)^{4}\,\left(b\,\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^4*(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.479 $\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=142

$$-\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{5/2}}-\frac{\operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4f(a-b)}+\frac{(8a-5b)\operatorname{sech}^2(e+fx)\sqrt{\cdots}}{8f(a-\cdots}$$

```
[Out] -1/8*(8*a^2-8*a*b+3*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-
b)^(5/2)/f+1/8*(8*a-5*b)*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^2/f-
1/4*sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A] time = 0.19, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 89, 78, 63, 208}

$$-\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{5/2}}-\frac{\operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4f(a-b)}+\frac{(8a-5b)\operatorname{sech}^2(e+fx)\sqrt{\cdots}}{8f(a-\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])
/(8*(a - b)^(5/2)*f) + ((8*a - 5*b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x
]^2])/(8*(a - b)^2*f) - (Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/(4*(a
 - b)*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4(a-b)f} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a+b)+2(a-b)x}{(1+x)^2\sqrt{a+bx}}\,dx, x, \sinh^2(e+\right.}{4(a-b)f}$$

$$= \frac{(8a-5b)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} - \frac{\operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+f}}{4(a-b)f}$$

$$= \frac{(8a-5b)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} - \frac{\operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+f}}{4(a-b)f}$$

$$= -\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}f} + \frac{(8a-5b)\operatorname{sech}^2(e+fx)\sqrt{a+b\sin}}{8(a-b)^2f}$$

Mathematica [A] time = 0.47, size = 116, normalized size = 0.82

$$\frac{\left(-8a^2+8ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)+\sqrt{a-b}\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\left(-2(a-b)\operatorname{sech}^2(e+f\right.}{8f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 8*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]
+ Sqrt[a - b]*Sech[e + f*x]^2*(8*a - 5*b - 2*(a - b)*Sech[e + f*x]^2)*Sqrt[
a + b*Sinh[e + f*x]^2])/(8*(a - b)^(5/2)*f)
```

fricas [B] time = 1.00, size = 4100, normalized size = 28.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)
*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 +
```

```
4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*co
sh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(
f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 + 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 +
24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cos
h(f*x + e)^5 + 10*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(
f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 15*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 8
*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 -
 8*a*b + 3*b^2)*cosh(f*x + e)^7 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5
 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x
 + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 +
 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*
x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) +
 b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 +
2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x
 + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*((8*a^2 - 13*a*b +
 5*b^2)*cosh(f*x + e)^5 + 5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)*sinh(f*x
 + e)^4 + (8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2
)*cosh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 + 4*a^2 -
 5*a*b + b^2)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)
^3 + 3*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 13*a
*b + 5*b^2)*cosh(f*x + e) + (5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^4 + 6
*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sinh(f*x +
 e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*f*cosh(f*x + e)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f
*x + e)*sinh(f*x + e)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*sinh(f*x + e)^8
 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*s
inh(f*x + e)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^4 + 8*(7
*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^3 + 3*(a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*f*cosh(f*x + e)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x
 + e)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*sinh(f*x + e)^4 + 4*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*f*cosh(f*x + e)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e
)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)^3 +
4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^6 + 15*(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*f*cosh(f*x + e)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*co
sh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*sinh(f*x + e)^2 + (a^3 -
 3*a^2*b + 3*a*b^2 - b^3)*f + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x
 + e)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^5 + 3*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*
f*cosh(f*x + e))*sinh(f*x + e)), -1/8*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x +
e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8
*a*b + 3*b^2)*sinh(f*x + e)^8 + 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 +
 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh
(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh
(f*x + e)^4 + 2*(35*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 30*(8*a^2 - 8
*a*b + 3*b^2)*cosh(f*x + e)^2 + 24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 +
8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5 + 10*(8*a^2 - 8*a*b + 3*b^2)*c
osh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
```

```
4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*c
osh(f*x + e)^6 + 15*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*
a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^2 + 8*a
^2 - 8*a*b + 3*b^2 + 8*((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^7 + 3*(8*a^2
- 8*a*b + 3*b^2)*cosh(f*x + e)^5 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^
3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*arct
an(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((8*a^2 - 1
3*a*b + 5*b^2)*cosh(f*x + e)^5 + 5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)*s
inh(f*x + e)^4 + (8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*
b + b^2)*cosh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 +
4*a^2 - 5*a*b + b^2)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f
*x + e)^3 + 3*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2
 - 13*a*b + 5*b^2)*cosh(f*x + e) + (5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e
)^4 + 6*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f
*cosh(f*x + e)*sinh(f*x + e)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*sinh(f*x
 + e)^8 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^3 -
 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*f)*sinh(f*x + e)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^4
 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^3 + 3*(a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*f*cosh(f*x + e)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*c
osh(f*x + e)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*sinh(f*x + e)^4 + 4*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 8*(7*(a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*f*cosh(f*x + e)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(
f*x + e)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x +
e)^3 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^6 + 15*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*f*cosh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*sinh(f*x + e)^2 +
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*c
osh(f*x + e)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^5 + 3*(a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^3 + (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage3:=type(sage2):;OUTPUT:Evaluation time: 0.9Error: Bad Argument Type
```

maple [C] time = 0.25, size = 43, normalized size = 0.30

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\sinh^5(fx+e)}{\cosh(fx+e)^6\sqrt{a+b(\sinh^2(fx+e))}}, \sinh(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5/cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^{5}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh\left(e+f\,x\right)^{5}}{\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{5}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(tanh(e + f*x)**5/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.480 $\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{3/2}}$$

```
[Out] -1/2*(2*a-b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1
/2*sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3194, 78, 63, 208}

$$\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^(3
/2)*f) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a - b)*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \sinh\right.}{4(a-b)f}$$

$$= \frac{\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\right.}}{2(a-b)bf}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}f} + \frac{\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f}$$

Mathematica [A] time = 0.12, size = 85, normalized size = 0.96

$$-\frac{\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} - \frac{\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a - b)^
(3/2) - (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(a - b))/f
```

fricas [B] time = 0.65, size = 1320, normalized size = 14.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)^
3 + (2*a - b)*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a - b
)*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^3
 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(a - b)*log((b*cos
h(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4
*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x +
e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cos
h(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
+ e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*co
sh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4
*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f
*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^
2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*
b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((2*a - b)*cosh(f*x + e)^4
```

```
+ 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a - b)*sinh(f*x + e)^4 + 2
*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a - b)*cosh(f*x + e)^2 + 2*a - b)*sinh
(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(
f*x + e) + 2*a - b)*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*si
nh(f*x + e))) - 2*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2
 - 2*a*b + b^2)*f*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2
 + 2*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh
(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e
)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [B] time = 5.15, size = 754, normalized size = 8.47

$$-\frac{\frac{2\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}{\sqrt{-b}}\right)e^{e}}{\sqrt{-b}}-\frac{(3ae^{e}-2be^{e})\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+\cdots}}{2\sqrt{a-b}}\cdots\right)}{(a-b)^{\frac{3}{2}}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*
x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))*e^e/sqrt(-b) - (3*a*e^e - 2*
b*e^e)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/(a - b)^
(3/2) + 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
 + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*e^e + 7*(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*
a*sqrt(b)*e^e - 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(3/2)*e^e + 12*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))*a^2*e^e - 17*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b*e^e + 8*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))*b^2*e^e - 4*a^2*sqrt(b)*e^e + 9*a*b^(3/2)*e^e - 4*b^(5/2)*e^e)/
(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3
*b)^2*(a - b)))/f^2
```

maple [C] time = 0.22, size = 43, normalized size = 0.48

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\sinh^{3}(fx+e)}{\cosh(fx+e)^{4}\sqrt{a+b(\sinh^{2}(fx+e))}},\sinh(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^{3}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh\left(e+f\,x\right)^{3}}{\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{3}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(tanh(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.481 $\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/f/(a-b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3194, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx=\frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx,x,\sinh^2(e+fx)\right)}{2f}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^2(e+fx)}\right)}{bf}$$

$$=-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,f}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 1.07

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(e+fx)-b}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
[Out] -(ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

fricas [B] time = 0.83, size = 433, normalized size = 10.56

$$\left[\frac{\log\left(\frac{b\cosh(fx+e)^4+4\,b\cosh(fx+e)\sinh(fx+e)^3+b\sinh(fx+e)^4+2\,(4\,a-3\,b)\cosh(fx+e)^2+2\left(3\,b\cosh(fx+e)^2+4\,a-3\,b\right)\sinh(fx+e)^2-4\,\sqrt{2}\,\sqrt{}}{\cosh(fx+e)^4+4\,\cosh(fx+e)\sinh(fx+e)^3+\sinh(fx+e)^4+2\left(3\,\cosh(fx+e)^2+1\right.}\right)}{2\sqrt{a-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*
x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3
*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sin
h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 +
(4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x
 + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*
x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
 + e) + 1))/(sqrt(a - b)*f), -sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) +
(a - b)*sinh(f*x + e)))/((a - b)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [C] time = 0.18, size = 41, normalized size = 1.00

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\sinh(fx+e)}{\cosh(fx+e)^2\sqrt{a+b(\sinh^2(fx+e))}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh\left(e+f\,x\right)}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.482 $\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3194, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

fricas [B] time = 0.55, size = 410, normalized size = 12.42

$$\left[\frac{\log\left(\frac{b\cosh(fx+e)^4+4\,b\cosh(fx+e)\sinh(fx+e)^3+b\sinh(fx+e)^4+2\,(4\,a-b)\cosh(fx+e)^2+2\left(3\,b\cosh(fx+e)^2+4\,a-b\right)\sinh(fx+e)^2-4\,\sqrt{2}\,\sqrt{a}\,\sqrt{\cdots}}{\cosh(fx+e)^4+4\,\cosh(fx+e)\sinh(fx+e)^3+\sinh(fx+e)^4+2\left(3\,\cosh(fx+e)^2-1\right)\sinh(\cdots}\right)}{2\,\sqrt{a}\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*
x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*
sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b
)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh
(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 -
 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)
)/(sqrt(a)*f), sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e)))/(a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Error: Bad Argument Type
```

maple [C] time = 0.15, size = 35, normalized size = 1.06

$$\frac{\text{`}int/indef0\text{`}\left(\frac{1}{\sinh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth\left(e+f\,x\right)}{\sqrt{b\,\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.483 $\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af}$$

```
[Out] -1/2*(2*a-b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csch(
f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3194, 78, 63, 208}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(3/2)*f) - (
Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(2*a*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x}{x^2\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(\right.}{4af}$$

$$= -\frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{2abf}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af}$$

Mathematica [**A**] time = 0.20, size = 72, normalized size = 0.94

$$-\frac{\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -1/2*(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/a^(3/2) + (C
sch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/a)/f
```

fricas [**B**] time = 0.80, size = 1144, normalized size = 14.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)
^3 + (2*a - b)*sinh(f*x + e)^4 - 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a -
b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^
3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(a)*log((b*cosh(f
*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a
- b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 +
4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*
x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*s
inh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh
(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2
 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(a*c
osh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)))/(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2
*f*sinh(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x
+ e)^2 - a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x
 + e))*sinh(f*x + e)), 1/2*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f
*x + e)*sinh(f*x + e)^3 + (2*a - b)*sinh(f*x + e)^4 - 2*(2*a - b)*cosh(f*x
+ e)^2 + 2*(3*(2*a - b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 4*((2*
```

```
a - b)*cosh(f*x + e)^3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*
sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) - 2*sqrt(2)*(a*cosh(f*x + e
) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2
*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*f*sinh(f*x
 + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2 - a^
2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh
(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.82Error: Bad Argument Type
```

maple [C] time = 0.20, size = 44, normalized size = 0.57

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\frac{1}{\sinh(fx+e)}+\frac{1}{\sinh(fx+e)^{3}}}{\sqrt{a+b\left(\sinh^{2}(fx+e)\right)}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((1/sinh(f*x+e)+1/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sinh
(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^{3}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth\left(e+f\,x\right)^{3}}{\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.484 $\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{(8a-3b)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}-\frac{\operatorname{csch}^4(e+fx)\sqrt{a+}}{4af}$$

```
[Out] -1/8*(8*a^2-8*a*b+3*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)
/f-1/8*(8*a-3*b)*csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f-1/4*csch(f*x
+e)^4*(a+b*sinh(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.15, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 89, 78, 63, 208}

$$-\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}-\frac{(8a-3b)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{\operatorname{csch}^4(e+fx)\sqrt{a+}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*
a^(5/2)*f) - ((8*a - 3*b)*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a
^2*f) - (Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/(4*a*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-3b)+2ax}{x^2\sqrt{a+bx}}\,dx, x, \sinh^2(e+\right.}{4af}$$

$$= -\frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e}}{4af}$$

$$= -\frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e}}{4af}$$

$$= -\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} - \frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b}}{8a^2f}$$

Mathematica [A] time = 0.37, size = 100, normalized size = 0.79

$$\frac{\left(-8a^2+8ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\,\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\left(-2a\text{csch}^2(e+fx)-8\right.}{8a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 8*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + Sq
rt[a]*Csch[e + f*x]^2*(-8*a + 3*b - 2*a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e
+ f*x]^2])/(8*a^(5/2)*f)
```

fricas [B] time = 0.77, size = 3086, normalized size = 24.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)
*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 -
4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*co
```

```
sh(f*x + e)^2 - 8*a^2 + 8*a*b - 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e)^3 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(
f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 - 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 +
24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cos
h(f*x + e)^5 - 10*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(
f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 - 15*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 - 8
*a^2 + 8*a*b - 3*b^2)*sinh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 -
 8*a*b + 3*b^2)*cosh(f*x + e)^7 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5
 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 - (8*a^2 - 8*a*b + 3*b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e
)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*
b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*co
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*
(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x
+ e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x
+ e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh
(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((8*a^2 - 3*a*b)*cosh(f*x + e)^5
 + 5*(8*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (8*a^2 - 3*a*b)*sinh(f
*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(5*(8*a^2 - 3*a*b)*cosh(f
*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 3*a*b)*cosh(f*x
+ e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 3*a*b)
*cosh(f*x + e) + (5*(8*a^2 - 3*a*b)*cosh(f*x + e)^4 - 6*(4*a^2 - 3*a*b)*cos
h(f*x + e)^2 + 8*a^2 - 3*a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)))/(a^3*f*cosh(f*x + e)^8 + 8*a^3*f*cosh(f*x + e)*sinh(f*x
 + e)^7 + a^3*f*sinh(f*x + e)^8 - 4*a^3*f*cosh(f*x + e)^6 + 6*a^3*f*cosh(f*
x + e)^4 + 4*(7*a^3*f*cosh(f*x + e)^2 - a^3*f)*sinh(f*x + e)^6 - 4*a^3*f*co
sh(f*x + e)^2 + 8*(7*a^3*f*cosh(f*x + e)^3 - 3*a^3*f*cosh(f*x + e))*sinh(f*
x + e)^5 + 2*(35*a^3*f*cosh(f*x + e)^4 - 30*a^3*f*cosh(f*x + e)^2 + 3*a^3*f
)*sinh(f*x + e)^4 + a^3*f + 8*(7*a^3*f*cosh(f*x + e)^5 - 10*a^3*f*cosh(f*x
+ e)^3 + 3*a^3*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*f*cosh(f*x + e)^
6 - 15*a^3*f*cosh(f*x + e)^4 + 9*a^3*f*cosh(f*x + e)^2 - a^3*f)*sinh(f*x +
e)^2 + 8*(a^3*f*cosh(f*x + e)^7 - 3*a^3*f*cosh(f*x + e)^5 + 3*a^3*f*cosh(f*
x + e)^3 - a^3*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((8*a^2 - 8*a*b + 3*b^
2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^
7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 - 4*(8*a^2 - 8*a*b + 3*b^2)*cos
h(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 8*a*b
 - 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 -
3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(8*a^2 - 8*a*b
 + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 -
 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 24*a^2 - 24*a*b + 9*b^2)*sinh
(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5 - 10*(8*a^2 - 8*
a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sin
h(f*x + e)^3 - 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(8*a^2 - 8*
a*b + 3*b^2)*cosh(f*x + e)^6 - 15*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 +
 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 8*a*b - 3*b^2)*sinh(f*
x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)
^7 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5 + 3*(8*a^2 - 8*a*b + 3*b^2)*
cosh(f*x + e)^3 - (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqr
t(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) - 2*sqrt(2)*((8*a^2 - 3*a*b)*c
osh(f*x + e)^5 + 5*(8*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (8*a^2 -
 3*a*b)*sinh(f*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(5*(8*a^2 -
 3*a*b)*cosh(f*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 3*
```

```
a*b)*cosh(f*x + e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (
8*a^2 - 3*a*b)*cosh(f*x + e) + (5*(8*a^2 - 3*a*b)*cosh(f*x + e)^4 - 6*(4*a^
2 - 3*a*b)*cosh(f*x + e)^2 + 8*a^2 - 3*a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2)))/(a^3*f*cosh(f*x + e)^8 + 8*a^3*f*cosh(f*x
 + e)*sinh(f*x + e)^7 + a^3*f*sinh(f*x + e)^8 - 4*a^3*f*cosh(f*x + e)^6 + 6
*a^3*f*cosh(f*x + e)^4 + 4*(7*a^3*f*cosh(f*x + e)^2 - a^3*f)*sinh(f*x + e)^
6 - 4*a^3*f*cosh(f*x + e)^2 + 8*(7*a^3*f*cosh(f*x + e)^3 - 3*a^3*f*cosh(f*x
 + e))*sinh(f*x + e)^5 + 2*(35*a^3*f*cosh(f*x + e)^4 - 30*a^3*f*cosh(f*x +
e)^2 + 3*a^3*f)*sinh(f*x + e)^4 + a^3*f + 8*(7*a^3*f*cosh(f*x + e)^5 - 10*a
^3*f*cosh(f*x + e)^3 + 3*a^3*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*f*
cosh(f*x + e)^6 - 15*a^3*f*cosh(f*x + e)^4 + 9*a^3*f*cosh(f*x + e)^2 - a^3*
f)*sinh(f*x + e)^2 + 8*(a^3*f*cosh(f*x + e)^7 - 3*a^3*f*cosh(f*x + e)^5 + 3
*a^3*f*cosh(f*x + e)^3 - a^3*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 3.83Unable to divide, perhaps due to rounding error
%%%{4096,[10,12,10]%%%}+%%%{%%%{-20480,[1]%%%},[10,12,9]%%%}+%%%{%%%{40960,
[2]%%%},[10,12,8]%%%}+%%%{%%%{-40960,[3]%%%},[10,12,7]%%%}+%%%{%%%{20480,[4
]%%%},[10,12,6]%%%}+%%%{%%%{-4096,[5]%%%},[10,12,5]%%%}+%%%{%%{[40960,0]:[1
,0,%%%{-1,[1]%%%}]%%},[9,12,10]%%%}+%%%{%%{[%%%{-204800,[1]%%%},0]:[1,0,%%%
{-1,[1]%%%}]%%},[9,12,9]%%%}+%%%{%%{[%%%{409600,[2]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[9,12,8]%%%}+%%%{%%{[%%%{-409600,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[9,12,7]%%%}+%%%{%%{[%%%{204800,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,12,6
]%%%}+%%%{%%{[%%%{-40960,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[9,12,5]%%%}+%%
%{-81920,[8,12,11]%%%}+%%%{%%%{593920,[1]%%%},[8,12,10]%%%}+%%%{%%%{-174080
0,[2]%%%},[8,12,9]%%%}+%%%{%%%{2662400,[3]%%%},[8,12,8]%%%}+%%%{%%%{-225280
0,[4]%%%},[8,12,7]%%%}+%%%{%%%{1003520,[5]%%%},[8,12,6]%%%}+%%%{%%%{-184320
,[6]%%%},[8,12,5]%%%}+%%%{%%{[-655360,0]:[1,0,%%%{-1,[1]%%%}]%%},[7,12,11]%
%%}+%%%{%%{[%%%{3768320,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,12,10]%%%}+%%
%{%%{[%%%{-9011200,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,12,9]%%%}+%%%{%%{[
%%%{11468800,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,12,8]%%%}+%%%{%%{[%%%{-8
192000,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,12,7]%%%}+%%%{%%{[%%%{3112960,
[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,12,6]%%%}+%%%{%%{[%%%{-491520,[6]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[7,12,5]%%%}+%%%{655360,[6,12,12]%%%}+%%%{%%%{-
5570560,[1]%%%},[6,12,11]%%%}+%%%{%%%{18882560,[2]%%%},[6,12,10]%%%}+%%%{%%
%{-33792000,[3]%%%},[6,12,9]%%%}+%%%{%%%{34816000,[4]%%%},[6,12,8]%%%}+%%%{
%%%{-20725760,[5]%%%},[6,12,7]%%%}+%%%{%%%{6594560,[6]%%%},[6,12,6]%%%}+%%%
{%%%{-860160,[7]%%%},[6,12,5]%%%}+%%%{%%{[3932160,0]:[1,0,%%%{-1,[1]%%%}]%%
},[5,12,12]%%%}+%%%{%%{[%%%{-24248320,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5
,12,11]%%%}+%%%{%%{[%%%{63291392,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,12,1
0]%%%}+%%%{%%{[%%%{-90357760,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,12,9]%%%
}+%%%{%%{[%%%{75857920,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,12,8]%%%}+%%%{
%%{[%%%{-37191680,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,12,7]%%%}+%%%{%%{[%
%%{9748480,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,12,6]%%%}+%%%{%%{[%%%{-103
2192,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,12,5]%%%}+%%%{-2621440,[4,12,13]
%%%}+%%%{%%%{22937600,[1]%%%},[4,12,12]%%%}+%%%{%%%{-81100800,[2]%%%},[4,12
,11]%%%}+%%%{%%%{154050560,[3]%%%},[4,12,10]%%%}+%%%{%%%{-173056000,[4]%%%}
,[4,12,9]%%%}+%%%{%%%{117719040,[5]%%%},[4,12,8]%%%}+%%%{%%%{-47104000,[6]%
%%},[4,12,7]%%%}+%%%{%%%{10035200,[7]%%%},[4,12,6]%%%}+%%%{%%%{-860160,[8]%
%%},[4,12,5]%%%}+%%%{%%{[-10485760,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,13]%%%}
```

```
+%%%{%%{[%%%{65536000,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,12]%%%}+%%%{
%%{[%%%{-174981120,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,11]%%%}+%%%{%%{
[%%%{259358720,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,10]%%%}+%%%{%%{[%%%
{-231833600,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,9]%%%}+%%%{%%{[%%%{126
812160,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,8]%%%}+%%%{%%{[%%%{-4096000
0,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,7]%%%}+%%%{%%{[%%%{7045120,[7]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12,6]%%%}+%%%{%%{[%%%{-491520,[8]%%%},0]:[
1,0,%%%{-1,[1]%%%}]%%},[3,12,5]%%%}+%%%{5242880,[2,12,14]%%%}+%%%{%%%{-4194
3040,[1]%%%},[2,12,13]%%%}+%%%{%%%{140902400,[2]%%%},[2,12,12]%%%}+%%%{%%%{
-261160960,[3]%%%},[2,12,11]%%%}+%%%{%%%{293457920,[4]%%%},[2,12,10]%%%}+%%
%{%%%{-206049280,[5]%%%},[2,12,9]%%%}+%%%{%%%{89661440,[6]%%%},[2,12,8]%%%}
+%%%{%%%{-23142400,[7]%%%},[2,12,7]%%%}+%%%{%%%{3215360,[8]%%%},[2,12,6]%%%
}+%%%{%%%{-184320,[9]%%%},[2,12,5]%%%}+%%%{%%{[10485760,0]:[1,0,%%%{-1,[1]%
%%}]%%},[1,12,14]%%%}+%%%{%%{[%%%{-62914560,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[1,12,13]%%%}+%%%{%%{[%%%{161218560,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[1,12,12]%%%}+%%%{%%{[%%%{-230031360,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,
12,11]%%%}+%%%{%%{[%%%{199925760,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,1
0]%%%}+%%%{%%{[%%%{-108994560,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,9]%%
%}+%%%{%%{[%%%{37109760,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,8]%%%}+%%%
{%%{[%%%{-7618560,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,7]%%%}+%%%{%%{[%
%%{860160,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,6]%%%}+%%%{%%{[%%%{-4096
0,[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12,5]%%%}+%%%{-4194304,[0,12,15]%%%
}+%%%{%%%{26214400,[1]%%%},[0,12,14]%%%}+%%%{%%%{-70778880,[2]%%%},[0,12,13
]%%%}+%%%{%%%{108134400,[3]%%%},[0,12,12]%%%}+%%%{%%%{-102973440,[4]%%%},[0
,12,11]%%%}+%%%{%%%{63590400,[5]%%%},[0,12,10]%%%}+%%%{%%%{-25743360,[6]%%%
},[0,12,9]%%%}+%%%{%%%{6758400,[7]%%%},[0,12,8]%%%}+%%%{%%%{-1105920,[8]%%%
},[0,12,7]%%%}+%%%{%%%{102400,[9]%%%},[0,12,6]%%%}+%%%{%%%{-4096,[10]%%%},[
0,12,5]%%%} / %%%{%%{poly1[%%%{1,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[10,0,0
]%%%}+%%%{%%%{10,[3]%%%},[9,0,0]%%%}+%%%{%%{[%%%{-20,[2]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[8,0,1]%%%}+%%%{%%{poly1[%%%{45,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[8,0,0]%%%}+%%%{%%%{-160,[3]%%%},[7,0,1]%%%}+%%%{%%%{120,[4]%%%},[7,0,
0]%%%}+%%%{%%{poly1[%%%{160,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,2]%%%}+
%%%{%%{[%%%{-560,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,1]%%%}+%%%{%%{poly
1[%%%{210,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[6,0,0]%%%}+%%%{%%%{960,[3]%%%
},[5,0,2]%%%}+%%%{%%%{-1120,[4]%%%},[5,0,1]%%%}+%%%{%%%{252,[5]%%%},[5,0,0]
%%%}+%%%{%%{[%%%{-640,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,3]%%%}+%%%{%%
{poly1[%%%{2400,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,2]%%%}+%%%{%%{[%%%{
-1400,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,1]%%%}+%%%{%%{poly1[%%%{210,[
5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[4,0,0]%%%}+%%%{%%%{-2560,[3]%%%},[3,0,3]
%%%}+%%%{%%%{3200,[4]%%%},[3,0,2]%%%}+%%%{%%%{-1120,[5]%%%},[3,0,1]%%%}+%%%
{%%%{120,[6]%%%},[3,0,0]%%%}+%%%{%%{[%%%{1280,[2]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[2,0,4]%%%}+%%%{%%{[%%%{-3840,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0
,3]%%%}+%%%{%%{poly1[%%%{2400,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,2]%%%
}+%%%{%%{[%%%{-560,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,1]%%%}+%%%{%%{po
ly1[%%%{45,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[2,0,0]%%%}+%%%{%%%{2560,[3]%
%%},[1,0,4]%%%}+%%%{%%%{-2560,[4]%%%},[1,0,3]%%%}+%%%{%%%{960,[5]%%%},[1,0,
2]%%%}+%%%{%%%{-160,[6]%%%},[1,0,1]%%%}+%%%{%%%{10,[7]%%%},[1,0,0]%%%}+%%%{
%%{[%%%{-1024,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,5]%%%}+%%%{%%{[%%%{12
80,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,4]%%%}+%%%{%%{[%%%{-640,[4]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[0,0,3]%%%}+%%%{%%{poly1[%%%{160,[5]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[0,0,2]%%%}+%%%{%%{[%%%{-20,[6]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[0,0,1]%%%}+%%%{%%{poly1[%%%{1,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
0,0,0]%%%} Error: Bad Argument Value
```


maple [C] time = 0.22, size = 54, normalized size = 0.43

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\frac{1}{\sinh\left(fx+e\right)}+\frac{2}{\sinh\left(fx+e\right)^{3}}+\frac{1}{\sinh\left(fx+e\right)^{5}}}{\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((1/sinh(f*x+e)+2/sinh(f*x+e)^3+1/sinh(f*x+e)^5)/(a+b*sinh(f*x+
e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^{5}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth\left(e+f\,x\right)^{5}}{\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{5}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**5/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.485 $\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=219

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}+\frac{(3a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+\ldots\right.}{3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] -2/3*(2*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*
x+e)^2)^(1/2)/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*
a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x
+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^
(1/2)/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*sech(f*x+e)
^2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/(a-b)/f
```

Rubi [A] time = 0.20, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3196, 470, 525, 418, 411}

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}+\frac{(3a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+\ldots\right.}{3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-2*(2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((3*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[
e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*
(a + b*Sinh[e + f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]
*Tanh[e + f*x])/(3*(a - b)*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
```

```
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^{5/2}\sqrt{a+bx^2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+\right.}{}$$

$$= \frac{\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} - \frac{\left(2(2a-b)\sqrt{\cosh^2(e+fx}\right.}{}$$

$$= -\frac{2(2a-b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2 f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} +$$

Mathematica [C] time = 2.13, size = 206, normalized size = 0.94

$$\frac{-\frac{\tanh(e+fx)\text{sech}^2(e+fx)\left(2\left(4a^2-3ab+b^2\right)\cosh(2(e+fx))+(2a-b)(2a+b\cosh(4(e+fx))+b)\right)}{\sqrt{2}} + 2ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+\right.}{6f(a-b)^2\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-4*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (2*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] - ((2*(4*a^2 - 3*a*b + b^2)*Cosh[2*(e + f*x)] + (2*
a - b)*(2*a + b + b*Cosh[4*(e + f*x)]))*Sech[e + f*x]^2*Tanh[e + f*x])/Sqrt
[2])/(6*(a - b)^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tanh\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [B] time = 9.52, size = 1320, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(6*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))*e^e/sqrt(-b) - 3*(3*a*e^e
 - 2*b*e^e)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/(a
- b)^(3/2) + 2*(9*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*e^e - 6*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^5*b*e^e + 21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a*sqrt(b)*e^e - 6*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b))^4*b^(3/2)*e^e + 64*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*e^e - 38*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))^3*a*b*e^e + 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^2*e^e + 19
2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) -
 2*b*e^(2*f*x + 2*e) + b))^2*a^2*sqrt(b)*e^e - 246*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
^2*a*b^(3/2)*e^e + 84*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(5/2)*e^e + 240*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))*a^3*e^e - 576*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b*e^e + 477*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))*a*b^2*e^e - 126*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^3*e^e - 14
4*a^3*sqrt(b)*e^e + 320*a^2*b^(3/2)*e^e - 223*a*b^(5/2)*e^e + 50*b^(7/2)*e^
e)/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a
- 3*b)^3*(a - b)))/f^2
```

maple [A] time = 0.33, size = 366, normalized size = 1.67

$$\frac{\left(-4\sqrt{-\frac{b}{a}}\,ab+2\sqrt{-\frac{b}{a}}\,b^{2}\right)\sinh\left(fx+e\right)\left(\cosh^{4}\left(fx+e\right)\right)+\left(-4\sqrt{-\frac{b}{a}}\,a^{2}+7\sqrt{-\frac{b}{a}}\,ab-3\sqrt{-\frac{b}{a}}\,b^{2}\right)\left(\cosh^{2}\left(fx+e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-4*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4
+(-4*(-1/a*b)^(1/2)*a^2+7*(-1/a*b)^(1/2)*a*b-3*(-1/a*b)^(1/2)*b^2)*cosh(f*x
+e)^2*sinh(f*x+e)+((-1/a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b
^2)*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*
EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-5*EllipticF(sinh(f*x+
```

```
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(
a/b)^(1/2))*b^2+4*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*E
llipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2)/cosh(f
*x+e)^3/(a-b)^2/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh\left(e+f\,x\right)^{4}}{\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{4}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(tanh(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.486 $\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=156

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\operatorname{si}\right.}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+\right.}{a}}}$$

```
[Out] -(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/
(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2
)/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))
^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2
),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)/f/(sech(f*x+e)
^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3196, 471, 422, 418, 492, 411}

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\operatorname{si}\right.}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]))
 + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
```

```
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}}{(-a+b)}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{(a-b)f} - \frac{\left(a\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subs}}{(-a+}$$

$$= \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{\left(\sqrt{\cosh^2(\right.}{}$$

$$= -\frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{F\left(\tan^{-1}\right.}{}$$

Mathematica [C] time = 0.41, size = 109, normalized size = 0.70

$$\frac{\sqrt{2}\tanh(e+fx)(-2a-b\cosh(2(e+fx))+b)-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{2f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] + Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*(a - b)*f*S
qrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tanh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [B] time = 2.66, size = 374, normalized size = 2.40

$$-\frac{2\left(\frac{\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}+\sqrt{b}}{2\sqrt{a-b}}\right)e^{e}}{\sqrt{a-b}}-\frac{\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}{\sqrt{-b}}\right)e^{e}}{\sqrt{-b}}\cdots\right.}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*(arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))*e^e/sqrt(a
- b) - arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))*e^e/sqrt(-b) - 2*((sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))*e^e - sqrt(b)*e^e)/((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(
2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b))/f^2
```

maple [A] time = 0.30, size = 239, normalized size = 1.53

$$\frac{-\sqrt{-\frac{b}{a}}\,b\left(\sinh^3\left(fx+e\right)\right)+a\sqrt{\frac{a+b\left(\sinh^2\left(fx+e\right)\right)}{a}}\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\sqrt{\frac{a+b(\cdots}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] (-(-1/a*b)^(1/2)*b*sinh(f*x+e)^3+a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(-1/a*b)^(1/2)*a*sinh(f*x+e))/
(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tanh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh\left(e+f\,x\right)^2}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(tanh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.487 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x),(b/a)^(
1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sinh^2(e+fx)}}\\ &= -\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 68, normalized size = 1.13

$$-\frac{i\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/
(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.16, size = 86, normalized size = 1.43

$$\frac{\sqrt{\frac{a+b\left(\sinh^2\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)}{\sqrt{-\frac{b}{a}}\,\cosh\left(fx+e\right)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\sinh\left(e+fx\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sinh^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.488 $\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=207

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b}{a}}}$$

```
[Out] -coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-(1/(1+sinh(f*x+e)^2))^(1/2)*(1+s
inh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(
1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*
x+e)^2)/a)^(1/2)+(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sin
h(f*x+e)^2)^(1/2)/a/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(a+b*sinh
(f*x+e)^2)^(1/2)*tanh(f*x+e)/a/f
```

Rubi [A] time = 0.20, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3196, 475, 422, 418, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e +
 f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tan
h[e + f*x])/(a*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \right.}{af}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \right.}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\text{sech}(e+fx}{af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e\right.}{a}}}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} - \frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\text{sech}(e+fx}{af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e\right.}{a}}}$$

Mathematica [C] time = 0.40, size = 105, normalized size = 0.51

$$\frac{\sqrt{2}\coth(e+fx)(-2a-b\cosh(2(e+fx))+b)-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{2af\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a])/(2*a*f*Sqrt[2*a
 - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\coth\left(fx+e\right)^{2}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 0.59Unable to divide, perhaps due to rounding error
%%%{64,[4,6,4]%%%}+%%%{%%%{-128,[1]%%%},[4,6,3]%%%}+%%%{%%%{64,[2]%%%},[4,6
,2]%%%}+%%%{%%{[256,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,6,4]%%%}+%%%{%%{[%%%{-512
,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,6,3]%%%}+%%%{%%{[%%%{256,[2]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[3,6,2]%%%}+%%%{-512,[2,6,5]%%%}+%%%{%%%{1408,[1]%%
%},[2,6,4]%%%}+%%%{%%%{-1280,[2]%%%},[2,6,3]%%%}+%%%{%%%{384,[3]%%%},[2,6,2
]%%%}+%%%{%%{[-1024,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,6,5]%%%}+%%%{%%{[%%%{2304
,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,6,4]%%%}+%%%{%%{[%%%{-1536,[2]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[1,6,3]%%%}+%%%{%%{[%%%{256,[3]%%%},0]:[1,0,%%%{-
1,[1]%%%}]%%},[1,6,2]%%%}+%%%{1024,[0,6,6]%%%}+%%%{%%%{-2560,[1]%%%},[0,6,5
]%%%}+%%%{%%%{2112,[2]%%%},[0,6,4]%%%}+%%%{%%%{-640,[3]%%%},[0,6,3]%%%}+%%%
{%%%{64,[4]%%%},[0,6,2]%%%} / %%%{%%%{1,[1]%%%},[4,0,0]%%%}+%%%{%%{poly1[%%
%{4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0,0]%%%}+%%%{%%%{-8,[1]%%%},[2,0,
1]%%%}+%%%{%%%{6,[2]%%%},[2,0,0]%%%}+%%%{%%{poly1[%%%{-16,[1]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[1,0,1]%%%}+%%%{%%{poly1[%%%{4,[2]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[1,0,0]%%%}+%%%{%%%{16,[1]%%%},[0,0,2]%%%}+%%%{%%%{-8,[2]%%%},[0,0
,1]%%%}+%%%{%%%{1,[3]%%%},[0,0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 0.27, size = 217, normalized size = 1.05

$$-\frac{-\sinh\left(fx+e\right)\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\left(a\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\,\mathrm{Elliptic}\right.}{\sqrt{-\frac{b}{a}}\,a\sinh\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -(-sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(a*E
llipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
)))+(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*cosh
(f*x+e)^2)/(-1/a*b)^(1/2)/a/sinh(f*x+e)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/
2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth\left(e+f\,x\right)^2}{\sqrt{b\sinh\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2),x)
[Out] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(coth(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.489 $\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=285

$$\frac{2(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}+\frac{(3a-b)\text{sech}(e+$$

```
[Out] -2/3*(2*a-b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f-1/3*coth(f*x+e)*cs
ch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a/f-2/3*(2*a-b)*(1/(1+sinh(f*x+e)^2))
^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2
),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2
*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a-b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+si
nh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1
/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh(f
*x+e)^2)/a)^(1/2)+2/3*(2*a-b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^2/f
```

Rubi [A] time = 0.30, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3196, 474, 583, 531, 418, 492, 411}

$$\frac{2(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}+\frac{(3a-b)\text{sech}(e+$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-2*(2*a - b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f) - (Coth[
e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (2*(2*a - b
)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + (2*(2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
```

```
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e\right.}{}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\sqrt{a}}{3af}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\sqrt{a}}{3af}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\sqrt{a}}{3af}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\sqrt{a}}{3af}$$

Mathematica [C] time = 3.54, size = 208, normalized size = 0.73

$$\frac{-\frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\left(2\left(4a^2-5ab+2b^2\right)\cosh(2(e+fx))-(2a-b)(2a-b\cosh(4(e+fx))-3b)\right)}{\sqrt{2}} + 2ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+\right.}{6a^2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-(((2*(4*a^2 - 5*a*b + 2*b^2)*Cosh[2*(e + f*x)] - (2*a - b)*(2*a - 3*b - b
*Cosh[4*(e + f*x)]))*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2]) - (4*I)*a*(2*a
 - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] +
 (2*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f
*x), b/a])/(6*a^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\coth\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 1.66Error: Bad Argument Type
```

maple [A] time = 0.34, size = 522, normalized size = 1.83

$$-4\sqrt{-\frac{b}{a}}\,ab\left(\sinh^{6}\left(fx+e\right)\right)+2\sqrt{-\frac{b}{a}}\,b^{2}\left(\sinh^{6}\left(fx+e\right)\right)+3a^{2}\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\mathrm{EllipticF}\left(\mathrm{si}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6
+3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*x+e)^3-5*b*((a+b*sinh(f*x+e)^2)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*sinh(f*x+e)^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
llipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+4*((a+b*
sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sin
h(f*x+e)^3-4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-3*(-1/a*b)^(1/2)*a*b*sinh(f*x
+e)^4+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4-5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2
+(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2)/(-1/a*b)^(1/2)/a^2/si
nh(f*x+e)^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth\left(e+f\,x\right)^{4}}{\sqrt{b\sinh\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2),x)
```

```
[Out] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{4}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
[Out] Integral(coth(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

3.490 $\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{8a^2+8ab-b^2}{8f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}-\frac{\left(8a^2+8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{7/2}}-\frac{\operatorname{sech}^4(e+fx)}{4f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+$$

```
[Out] -1/8*(8*a^2+8*a*b-b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)
^(7/2)/f+1/8*(8*a^2+8*a*b-b^2)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/8*(8*a
-3*b)*sech(f*x+e)^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/4*sech(f*x+e)^4/(
a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2+8ab-b^2}{8f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}-\frac{\left(8a^2+8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{7/2}}-\frac{\operatorname{sech}^4(e+fx)}{4f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(
8*(a - b)^(7/2)*f) + (8*a^2 + 8*a*b - b^2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh[e
 + f*x]^2]) + ((8*a - 3*b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*Sqrt[a + b*Sinh[
e + f*x]^2]) - Sech[e + f*x]^4/(4*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a-b)+2(a-b)x}{(1+x)^2(a+bx)^{3/2}}\,dx, x, \sinh^2(e+\right.}{4(a-b)f}$$

$$= \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(8a^2+\right.}{}$$

$$= \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{}{4(a-}$$

$$= \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{}{4(a-}$$

$$= -\frac{\left(8a^2+8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f} + \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+f}}$$

Mathematica [C] time = 0.47, size = 113, normalized size = 0.60

$$\frac{\left(8a^2+8ab-b^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sinh^2(e+fx)+a}{a-b}\right)+\frac{1}{2}(a-b)\text{sech}^4(e+fx)((8a-3b)\cosh(2(e+fx))+4a+b)}{8f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((8*a^2 + 8*a*b - b^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sinh[e + f*x]
^2)/(a - b)] + ((a - b)*(4*a + b + (8*a - 3*b)*Cosh[2*(e + f*x)])*Sech[e +
f*x]^4)/2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

fricas [B] time = 1.15, size = 10168, normalized size = 54.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^12 + 12*(8*a^2*b + 8*a*b^2
- b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (8*a^2*b + 8*a*b^2 - b^3)*sinh(f*x
+ e)^12 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^10 + 2*(16*a^
3 + 24*a^2*b + 6*a*b^2 - b^3 + 33*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^10 + 20*(11*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^3 + (16
*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e)^9 + (128*a^3
+ 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^8 + (495*(8*a^2*b + 8*a*b^2 - b
^3)*cosh(f*x + e)^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 90*(16*a^3 + 2
4*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(8*a^2*b
+ 8*a*b^2 - b^3)*cosh(f*x + e)^5 + 30*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*c
osh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e))*sinh
(f*x + e)^7 + 4*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^6 + 4*(2
31*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^6 + 105*(16*a^3 + 24*a^2*b + 6*a
*b^2 - b^3)*cosh(f*x + e)^4 + 48*a^3 + 40*a^2*b - 14*a*b^2 + b^3 + 7*(128*a
^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(
8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^7 + 63*(16*a^3 + 24*a^2*b + 6*a*b^2
- b^3)*cosh(f*x + e)^5 + 7*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x
+ e)^3 + 3*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e
)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^4 + (495*(8*a^2*
b + 8*a*b^2 - b^3)*cosh(f*x + e)^8 + 420*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3
)*cosh(f*x + e)^6 + 70*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)
^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 60*(48*a^3 + 40*a^2*b - 14*a*b^
2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(8*a^2*b + 8*a*b^2 - b^3)
*cosh(f*x + e)^9 + 60*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^7 +
 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^5 + 20*(48*a^3 + 4
0*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2
 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*a^2*b + 8*a*b^2 - b^3 + 2*(16*a^
3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(33*(8*a^2*b + 8*a*b^2 -
b^3)*cosh(f*x + e)^10 + 45*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e
)^8 + 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^6 + 30*(48*a^
3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^4 + 16*a^3 + 24*a^2*b + 6*a*b^
2 - b^3 + 3*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^2 + 4*(3*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^11 + 5*(16*a^3 + 24
*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^9 + 2*(128*a^3 + 120*a^2*b - 24*a*b^2
 + b^3)*cosh(f*x + e)^7 + 6*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x +
 e)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^3 + (16*a^3 +
24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*
cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2
*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x
 + e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*
cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f
*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2
*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1))
```

```
+ 4*sqrt(2)*((8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^9 + 9*(8*a^3 - 9*a*b^2 +
 b^3)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^3 - 9*a*b^2 + b^3)*sinh(f*x + e)
^9 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^7 + 4*(16*a^3 -
19*a^2*b + 5*a*b^2 - 2*b^3 + 9*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^7 + 28*(3*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^3 + (16*a^3 - 19
*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(40*a^3 - 28*a
^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*co
sh(f*x + e)^4 + 40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3 + 42*(16*a^3 - 19*a^2*
b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*
b^2 + b^3)*cosh(f*x + e)^5 + 70*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(
f*x + e)^3 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e))*sinh(f
*x + e)^4 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + 4*(21
*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^6 + 35*(16*a^3 - 19*a^2*b + 5*a*b^2
 - 2*b^3)*cosh(f*x + e)^4 + 16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3 + 5*(40*a^3
 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(9*(8*a
^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^7 + 21*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b
^3)*cosh(f*x + e)^5 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e
)^3 + 3*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^
2 + (8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e) + (9*(8*a^3 - 9*a*b^2 + b^3)*cosh
(f*x + e)^8 + 28*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 10
*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^4 + 8*a^3 - 9*a*b^2 +
 b^3 + 12*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x +
 e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - 4*a^3*b^2
+ 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^12 + 12*(a^4*b - 4*a^3*b^2 + 6
*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)*sinh(f*x + e)^11 + (a^4*b - 4*a^3
*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*sinh(f*x + e)^12 + 2*(2*a^5 - 7*a^4*b +
 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^10 + 2*(33*(a^4*b -
 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^2 + (2*a^5 - 7*a^4*
b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f)*sinh(f*x + e)^10 + (16*a^5 -
65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^8 + 2
0*(11*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (
2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e))*s
inh(f*x + e)^9 + (495*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos
h(f*x + e)^4 + 90*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)
*f*cosh(f*x + e)^2 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b
^4 - b^5)*f)*sinh(f*x + e)^8 + 4*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^
3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^6 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^
3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^5 + 30*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*
a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (16*a^5 - 65*a^4*b + 100*a^3*b
^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231
*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^6 + 105*(2
*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^4 +
 7*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f
*x + e)^2 + (6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f
)*sinh(f*x + e)^6 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^
4 - b^5)*f*cosh(f*x + e)^4 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4
 + b^5)*f*cosh(f*x + e)^7 + 63*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2
*a*b^4 + b^5)*f*cosh(f*x + e)^5 + 7*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a
^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^3 + 3*(6*a^5 - 25*a^4*b + 40*a^3*b
^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*(
a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^8 + 420*(2*a
^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^6 + 7
0*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*
x + e)^4 + 60*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)
*f*cosh(f*x + e)^2 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b
^4 - b^5)*f)*sinh(f*x + e)^4 + 2*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 -
 2*a*b^4 + b^5)*f*cosh(f*x + e)^2 + 4*(55*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 -
4*a*b^4 + b^5)*f*cosh(f*x + e)^9 + 60*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*
```

b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^7 + 14*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^5 + 20*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^3 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 2*(33*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^10 + 45*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^8 + 14*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^6 + 30*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^4 + 3*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^2 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f)*sinh(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f + 4*(3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^11 + 5*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^9 + 2*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^7 + 6*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^5 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^3 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e)), -1/8*(((8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^12 + 12*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (8*a^2*b + 8*a*b^2 - b^3)*sinh(f*x + e)^12 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^10 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3 + 33*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^3 + (16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e)^9 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^8 + (495*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 90*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^5 + 30*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^6 + 4*(231*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^6 + 105*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^4 + 48*a^3 + 40*a^2*b - 14*a*b^2 + b^3 + 7*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^7 + 63*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^5 + 7*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^4 + (495*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^8 + 420*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^6 + 70*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 60*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^9 + 60*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^7 + 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^5 + 20*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*a^2*b + 8*a*b^2 - b^3 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(33*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^10 + 45*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^8 + 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^6 + 30*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^4 + 16*a^3 + 24*a^2*b + 6*a*b^2 - b^3 + 3*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^11 + 5*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^9 + 2*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^7 + 6*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^3 + (16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^9 + 9*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)*

sinh(f*x + e)^8 + (8*a^3 - 9*a*b^2 + b^3)*sinh(f*x + e)^9 + 4*(16*a^3 - 19*
a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^7 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 -
2*b^3 + 9*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(3
*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^3 + (16*a^3 - 19*a^2*b + 5*a*b^2 - 2
*b^3)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*
b^3)*cosh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^4 + 40*a
^3 - 28*a^2*b - 19*a*b^2 + 7*b^3 + 42*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)
*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x
+ e)^5 + 70*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + 5*(40*a
^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(16*a^
3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + 4*(21*(8*a^3 - 9*a*b^2 +
b^3)*cosh(f*x + e)^6 + 35*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x +
e)^4 + 16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3 + 5*(40*a^3 - 28*a^2*b - 19*a*b^
2 + 7*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(9*(8*a^3 - 9*a*b^2 + b^3)*
cosh(f*x + e)^7 + 21*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5
+ 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^3 + 3*(16*a^3 - 19
*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^3 - 9*a*b^2
+ b^3)*cosh(f*x + e) + (9*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^8 + 28*(16
*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 10*(40*a^3 - 28*a^2*b
- 19*a*b^2 + 7*b^3)*cosh(f*x + e)^4 + 8*a^3 - 9*a*b^2 + b^3 + 12*(16*a^3 -
19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^
4 + b^5)*f*cosh(f*x + e)^12 + 12*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 +
b^5)*f*cosh(f*x + e)*sinh(f*x + e)^11 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4
*a*b^4 + b^5)*f*sinh(f*x + e)^12 + 2*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b
^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^10 + 2*(33*(a^4*b - 4*a^3*b^2 + 6*a^2*b
^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^2 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^
2*b^3 - 2*a*b^4 + b^5)*f)*sinh(f*x + e)^10 + (16*a^5 - 65*a^4*b + 100*a^3*b
^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^8 + 20*(11*(a^4*b - 4*a^3
*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (2*a^5 - 7*a^4*b + 8*
a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e)^9 + (49
5*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^4 + 90*(2
*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^2 +
(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f)*sinh(f*
x + e)^8 + 4*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*
f*cosh(f*x + e)^6 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f
*cosh(f*x + e)^5 + 30*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 +
b^5)*f*cosh(f*x + e)^3 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20
*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231*(a^4*b - 4*a^3*b^2
+ 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^6 + 105*(2*a^5 - 7*a^4*b + 8*a
^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^4 + 7*(16*a^5 - 65*a^4*
b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^2 + (6*a^5 -
25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f)*sinh(f*x + e)^6 +
(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x
+ e)^4 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x +
e)^7 + 63*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh
(f*x + e)^5 + 7*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 -
b^5)*f*cosh(f*x + e)^3 + 3*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10
*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*(a^4*b - 4*a^3*b^2 +
6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^8 + 420*(2*a^5 - 7*a^4*b + 8*a^3
*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^6 + 70*(16*a^5 - 65*a^4*b
+ 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^4 + 60*(6*a^5
- 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^2 +
(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f)*sinh(f*
x + e)^4 + 2*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*co
sh(f*x + e)^2 + 4*(55*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos
h(f*x + e)^9 + 60*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)
*f*cosh(f*x + e)^7 + 14*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*

```
a*b^4 - b^5)*f*cosh(f*x + e)^5 + 20*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2
*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^3 + (16*a^5 - 65*a^4*b + 100*a^3*b^2
 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 2*(33*(a
^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^10 + 45*(2*a^
5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^8 + 14
*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x
 + e)^6 + 30*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*
f*cosh(f*x + e)^4 + 3*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*
b^4 - b^5)*f*cosh(f*x + e)^2 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2
*a*b^4 + b^5)*f)*sinh(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4
 + b^5)*f + 4*(3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x
 + e)^11 + 5*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*co
sh(f*x + e)^9 + 2*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4
- b^5)*f*cosh(f*x + e)^7 + 6*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 +
10*a*b^4 - b^5)*f*cosh(f*x + e)^5 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a
^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^3 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 -
 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [B] time = 51.39, size = 2634, normalized size = 14.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(a^9*e^(5*e) - 5*a^8*b*e^(5*e) + 10*a^7*b^2*e^(5*e) - 10*a^6*b^3*e^(5*e)
+ 5*a^5*b^4*e^(5*e) - a^4*b^5*e^(5*e))*e^(f*x)/((a^10*e^(4*e) - 8*a^9*b*e^(
4*e) + 28*a^8*b^2*e^(4*e) - 56*a^7*b^3*e^(4*e) + 70*a^6*b^4*e^(4*e) - 56*a^
5*b^5*e^(4*e) + 28*a^4*b^6*e^(4*e) - 8*a^3*b^7*e^(4*e) + a^2*b^8*e^(4*e))*s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)*f) +
 1/12*(45*a^2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))*e
^e/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a - b)) - 24*a^2*arctan(-(sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))/sqrt(-b))*e^e/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(-b)) -
2*(21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*
e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2*e^e + 243*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a
^2*sqrt(b)*e^e - 144*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(3/2)*e^e + 48*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))^6*b^(5/2)*e^e + 436*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3*e^e - 1
23*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
- 2*b*e^(2*f*x + 2*e) + b))^5*a^2*b*e^e + 288*(sqrt(b)*e^(2*f*x + 2*e) - sq
rt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*
b^2*e^e - 160*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*b^3*e^e + 1796*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
+ b))^4*a^3*sqrt(b)*e^e - 1029*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2)*e^e -
 240*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^4*a*b^(5/2)*e^e + 208*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)^4*b^(7/2)*e^e + 1840*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^4*e^e + 168*(sqrt(b)*e^
(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))^3*a^3*b*e^e - 2553*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*b^2*e^e + 1
472*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
```

```
 - 2*b*e^(2*f*x + 2*e) + b))^3*a*b^3*e^e - 192*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b
^4*e^e + 7056*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^4*sqrt(b)*e^e - 14872*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^2*a^3*b^(3/2)*e^e + 11745*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^2*b^(
5/2)*e^e - 3696*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(7/2)*e^e + 208*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b))^2*b^(9/2)*e^e + 4800*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^5*e^e - 15024*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))*a^4*b*e^e + 19876*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(
b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^3*b^2
*e^e - 12705*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b^3*e^e + 3360*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))*a*b^4*e^e - 160*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^5*e^e - 1344*a^5*sqrt(b)*
e^e + 5360*a^4*b^(3/2)*e^e - 7404*a^3*b^(5/2)*e^e + 4401*a^2*b^(7/2)*e^e -
1040*a*b^(9/2)*e^e + 48*b^(11/2)*e^e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*((sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*
e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^4
))/f^2
```

maple [C] time = 0.40, size = 103, normalized size = 0.55

$$\frac{\text{`}int/indef0\text{`}\left(-\frac{\left(\sinh^{5}\left(fx+e\right)\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}\,\left(\cosh^{4}\left(fx+e\right)\right)}{-b^{2}\left(\cosh^{14}\left(fx+e\right)\right)+\left(-2ab+2b^{2}\right)\left(\cosh^{12}\left(fx+e\right)\right)+\left(-a^{2}+2ab-b^{2}\right)\left(\cosh^{10}\left(fx+e\right)\right)},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^4/(-b^2*c
osh(f*x+e)^14+(-2*a*b+2*b^2)*cosh(f*x+e)^12+(-a^2+2*a*b-b^2)*cosh(f*x+e)^10
),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^{5}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.491 $\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{2a+b}{2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}}+\frac{\operatorname{sech}^2(e+fx)}{2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -1/2*(2*a+b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1
/2*(2*a+b)/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)+1/2*sech(f*x+e)^2/(a-b)/f/(a
+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a+b}{2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}}+\frac{\operatorname{sech}^2(e+fx)}{2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^(5
/2)*f) + (2*a + b)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + Sech[e + f
*x]^2/(2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \sinh^2(e+\right.}{4(a-b)f}$$

$$= \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a+b)\,\text{S}}{}$$

$$= \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a+b)\,\text{S}}{}$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}f} + \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{}{2(a-}$$

Mathematica [C] time = 0.12, size = 79, normalized size = 0.65

$$\frac{(2a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sinh^2(e+fx)+a}{a-b}\right)+(a-b)\text{sech}^2(e+fx)}{2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sinh[e + f*x]^2)/(a - b)]
 + (a - b)*Sech[e + f*x]^2)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

fricas [B] time = 0.74, size = 4050, normalized size = 33.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a*b + b^2)*cosh(f*x + e)^8 + 8*(2*a*b + b^2)*cosh(f*x + e)*sinh(f
*x + e)^7 + (2*a*b + b^2)*sinh(f*x + e)^8 + 4*(2*a^2 + a*b)*cosh(f*x + e)^6
```

```
 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*sinh(f*x + e)^6 + 8*(7
*(2*a*b + b^2)*cosh(f*x + e)^3 + 3*(2*a^2 + a*b)*cosh(f*x + e))*sinh(f*x +
e)^5 + 2*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(2*a*b + b^2)*cosh(f
*x + e)^4 + 30*(2*a^2 + a*b)*cosh(f*x + e)^2 + 8*a^2 + 2*a*b - b^2)*sinh(f*
x + e)^4 + 8*(7*(2*a*b + b^2)*cosh(f*x + e)^5 + 10*(2*a^2 + a*b)*cosh(f*x +
 e)^3 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 + a
*b)*cosh(f*x + e)^2 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^6 + 15*(2*a^2 + a*b)
*cosh(f*x + e)^4 + 3*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*s
inh(f*x + e)^2 + 2*a*b + b^2 + 8*((2*a*b + b^2)*cosh(f*x + e)^7 + 3*(2*a^2
+ a*b)*cosh(f*x + e)^5 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 + a
*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*
cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x
+ e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sq
rt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) +
 sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*
x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x +
 e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(
cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*((2*a^2 -
a*b - b^2)*cosh(f*x + e)^5 + 5*(2*a^2 - a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e)^4 + (2*a^2 - a*b - b^2)*sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2)*cosh(
f*x + e)^3 + 2*(5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^2 + 4*a^2 - 5*a*b + b^2
)*sinh(f*x + e)^3 + 2*(5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^3 + 3*(4*a^2 - 5
*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a^2 - a*b - b^2)*cosh(f*x +
 e) + (5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^4 + 6*(4*a^2 - 5*a*b + b^2)*cosh
(f*x + e)^2 + 2*a^2 - a*b - b^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)
^8 + 8*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7
+ (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*sinh(f*x + e)^8 + 4*(a^4 - 3*a^3*b
+ 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3
- b^4)*f*cosh(f*x + e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*sinh(f*x
+ e)^6 + 2*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x + e)^
4 + 8*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^3 + 3*(a^4 - 3
*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3*b
 - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^4 + 30*(a^4 - 3*a^3*b + 3*a^2
*b^2 - a*b^3)*f*cosh(f*x + e)^2 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3
+ b^4)*f)*sinh(f*x + e)^4 + 4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*
x + e)^2 + 8*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^5 + 10*
(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^3 + (4*a^4 - 13*a^3*b +
 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^3*b
 - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^6 + 15*(a^4 - 3*a^3*b + 3*a^2
*b^2 - a*b^3)*f*cosh(f*x + e)^4 + 3*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^
3 + b^4)*f*cosh(f*x + e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*sinh(f*
x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f + 8*((a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*f*cosh(f*x + e)^7 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*
cosh(f*x + e)^5 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*
x + e)^3 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x +
e)), -1/2*(((2*a*b + b^2)*cosh(f*x + e)^8 + 8*(2*a*b + b^2)*cosh(f*x + e)*s
inh(f*x + e)^7 + (2*a*b + b^2)*sinh(f*x + e)^8 + 4*(2*a^2 + a*b)*cosh(f*x +
 e)^6 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*sinh(f*x + e)^6 +
 8*(7*(2*a*b + b^2)*cosh(f*x + e)^3 + 3*(2*a^2 + a*b)*cosh(f*x + e))*sinh(f
*x + e)^5 + 2*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(2*a*b + b^2)*c
osh(f*x + e)^4 + 30*(2*a^2 + a*b)*cosh(f*x + e)^2 + 8*a^2 + 2*a*b - b^2)*si
nh(f*x + e)^4 + 8*(7*(2*a*b + b^2)*cosh(f*x + e)^5 + 10*(2*a^2 + a*b)*cosh(
f*x + e)^3 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^
2 + a*b)*cosh(f*x + e)^2 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^6 + 15*(2*a^2 +
 a*b)*cosh(f*x + e)^4 + 3*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a
*b)*sinh(f*x + e)^2 + 2*a*b + b^2 + 8*((2*a*b + b^2)*cosh(f*x + e)^7 + 3*(2
```

```
*a^2 + a*b)*cosh(f*x + e)^5 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^3 + (2*a^
2 + a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqr
t(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*
x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((2*a^2 - a*b - b^2)*cosh(f*x
+ e)^5 + 5*(2*a^2 - a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + (2*a^2 - a*b
 - b^2)*sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e)^3 + 2*(5*(2
*a^2 - a*b - b^2)*cosh(f*x + e)^2 + 4*a^2 - 5*a*b + b^2)*sinh(f*x + e)^3 +
2*(5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^3 + 3*(4*a^2 - 5*a*b + b^2)*cosh(f*x
 + e))*sinh(f*x + e)^2 + (2*a^2 - a*b - b^2)*cosh(f*x + e) + (5*(2*a^2 - a*
b - b^2)*cosh(f*x + e)^4 + 6*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e)^2 + 2*a^2
- a*b - b^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2
*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)
))/((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^8 + 8*(a^3*b - 3*a^
2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^3*b - 3*a^2*b^2
 + 3*a*b^3 - b^4)*f*sinh(f*x + e)^8 + 4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)
*f*cosh(f*x + e)^6 + 4*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x +
e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^6 + 2*(4*a^4 -
13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^3*b - 3*
a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^3 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 -
 a*b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3*b - 3*a^2*b^2 + 3*a*b
^3 - b^4)*f*cosh(f*x + e)^4 + 30*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh
(f*x + e)^2 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f)*sinh(f*x +
 e)^4 + 4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^2 + 8*(7*(a^3
*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^5 + 10*(a^4 - 3*a^3*b + 3*a
^2*b^2 - a*b^3)*f*cosh(f*x + e)^3 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^
3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^3*b - 3*a^2*b^2 + 3*a*b
^3 - b^4)*f*cosh(f*x + e)^6 + 15*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh
(f*x + e)^4 + 3*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x
+ e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 + (a^3*b -
3*a^2*b^2 + 3*a*b^3 - b^4)*f + 8*((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cos
h(f*x + e)^7 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^5 + (4
*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x + e)^3 + (a^4 - 3*
a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [B] time = 15.04, size = 975, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(a^7*e^(5*e) - 4*a^6*b*e^(5*e) + 6*a^5*b^2*e^(5*e) - 4*a^4*b^3*e^(5*e) +
a^3*b^4*e^(5*e))*e^(f*x)/((a^8*e^(4*e) - 6*a^7*b*e^(4*e) + 15*a^6*b^2*e^(4*
e) - 20*a^5*b^3*e^(4*e) + 15*a^4*b^4*e^(4*e) - 6*a^3*b^5*e^(4*e) + a^2*b^6*
e^(4*e))*sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b)*f) + (3*a*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*
e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))
*e^e/((a^2 - 2*a*b + b^2)*sqrt(a - b)) - 2*a*arctan(-(sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
))/sqrt(-b))*e^e/((a^2 - 2*a*b + b^2)*sqrt(-b)) - 2*((sqrt(b)*e^(2*f*x + 2*
e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b
))^3*a*e^e + 7*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2
*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*sqrt(b)*e^e - 4*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^2*b^(3/2)*e^e + 12*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*e^e - 17*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))*a*b*e^e + 8*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^2*e^e - 4*a^2*sqrt(
```

```
b)*e^e + 9*a*b^(3/2)*e^e - 4*b^(5/2)*e^e)/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt
(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^2*(a^2 - 2*a*b + b^2)))/f^2
```

maple [C] time = 0.40, size = 103, normalized size = 0.84

$$\frac{\text{`}int/indef0\text{`}\left(-\frac{\left(\sinh^3\left(fx+e\right)\right)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}\left(\cosh^2\left(fx+e\right)\right)}{-b^2\left(\cosh^{10}\left(fx+e\right)\right)+\left(-2ab+2b^2\right)\left(\cosh^8\left(fx+e\right)\right)+\left(-a^2+2ab-b^2\right)\left(\cosh^6\left(fx+e\right)\right)},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2/(-b^2*c
osh(f*x+e)^10+(-2*a*b+2*b^2)*cosh(f*x+e)^8+(-a^2+2*a*b-b^2)*cosh(f*x+e)^6),
sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh\left(e+f\,x\right)^3}{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.492 $\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/f+1/(a-b)/f/(a+
b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 51, 63, 208}

$$\frac{1}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + 1/(
(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2(a-b)f}$$

$$= \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{(a-b)bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] time = 0.08, size = 58, normalized size = 0.84

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\cosh^2(e+fx)}{a-b}+1\right)}{f(b-a)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Cosh[e + f*x]^2)/(a - b)]/((-a + b
)*f*Sqrt[a - b + b*Cosh[e + f*x]^2]))
```

fricas [B] time = 0.66, size = 1370, normalized size = 19.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x
+ e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*si
nh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x +
e) + b)*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e
)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
 e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cos
h(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)
^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)
^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x
 + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2*b - 2*a
*b^2 + b^3)*f*cosh(f*x + e)^4 + 4*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)*s
inh(f*x + e)^3 + (a^2*b - 2*a*b^2 + b^3)*f*sinh(f*x + e)^4 + 2*(2*a^3 - 5*a
^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^2*b - 2*a*b^2 + b^3)*f*co
```

```
sh(f*x + e)^2 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f)*sinh(f*x + e)^2 + (a^2
*b - 2*a*b^2 + b^3)*f + 4*((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^3 + (2*a
^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)), -((b*cosh(f*
x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a -
 b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4
*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-a +
 b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((a
 - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)))/((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^4 + 4*(a^2*b -
 2*a*b^2 + b^3)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2*b - 2*a*b^2 + b^3)*f
*sinh(f*x + e)^4 + 2*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e)^2 +
2*(3*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^2 + (2*a^3 - 5*a^2*b + 4*a*b^2
 - b^3)*f)*sinh(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f + 4*((a^2*b - 2*a*b^
2 + b^3)*f*cosh(f*x + e)^3 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x +
 e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.78Error: Bad Argument Type
```

maple [C] time = 0.23, size = 93, normalized size = 1.35

$$\frac{`int/indef0`\left(-\frac{\sinh\left(fx+e\right)\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}}{-b^{2}\left(\sinh^{6}\left(fx+e\right)\right)+\left(-2ab-b^{2}\right)\left(\sinh^{4}\left(fx+e\right)\right)+\left(-a^{2}-2ab\right)\left(\sinh^{2}\left(fx+e\right)\right)-a^{2}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(-b^2*sinh(f*x+e)^6+(-2
*a*b-b^2)*sinh(f*x+e)^4+(-a^2-2*a*b)*sinh(f*x+e)^2-a^2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh\left(e+f\,x\right)}{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(e + fx\right)}{\left(a + b\sinh^2\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.493 $\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{1}{af\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sinh(f*x+e
)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{af\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
 + b*Sinh[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{1}{af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2af}$$
$$= \frac{1}{af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{abf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] time = 0.06, size = 46, normalized size = 0.81

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sinh[e + f*x]^2)/a]/(a*f*Sqrt[a + b*
Sinh[e + f*x]^2])
```

fricas [B] time = 0.55, size = 1137, normalized size = 19.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
 e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sin
h(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e
) + b)*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
 b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3
+ (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x
 + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*
x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x
 + e) + 1)) + 4*sqrt(2)*(a*cosh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b*f*cosh(f*x + e)^4 + 4*a^2*b*f*cos
h(f*x + e)*sinh(f*x + e)^3 + a^2*b*f*sinh(f*x + e)^4 + a^2*b*f + 2*(2*a^3 -
 a^2*b)*f*cosh(f*x + e)^2 + 2*(3*a^2*b*f*cosh(f*x + e)^2 + (2*a^3 - a^2*b)*
f)*sinh(f*x + e)^2 + 4*(a^2*b*f*cosh(f*x + e)^3 + (2*a^3 - a^2*b)*f*cosh(f*
```

```
x + e))*sinh(f*x + e)), ((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x
 + e))*sinh(f*x + e) + b)*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) +
 2*sqrt(2)*(a*cosh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2)))/(a^2*b*f*cosh(f*x + e)^4 + 4*a^2*b*f*cosh(f*x + e)*si
nh(f*x + e)^3 + a^2*b*f*sinh(f*x + e)^4 + a^2*b*f + 2*(2*a^3 - a^2*b)*f*cos
h(f*x + e)^2 + 2*(3*a^2*b*f*cosh(f*x + e)^2 + (2*a^3 - a^2*b)*f)*sinh(f*x +
 e)^2 + 4*(a^2*b*f*cosh(f*x + e)^3 + (2*a^3 - a^2*b)*f*cosh(f*x + e))*sinh(
f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.49Error: Bad Argument Type
```

maple [C] time = 0.14, size = 35, normalized size = 0.61

$$\frac{\text{`}int/indef0\text{`}\left(\frac{1}{\sinh\left(fx+e\right)\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
[Out] `int/indef0`(1/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth\left(e+f\,x\right)}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.494 $\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}+\frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -1/2*(2*a-3*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(2*
a-3*b)/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/2*csch(f*x+e)^2/a/f/(a+b*sinh(f*x+
e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) +
 (2*a - 3*b)/(2*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - Csch[e + f*x]^2/(2*a*f
*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x}{x^2(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sinh^2(e+f\right.}{4af}$$

$$= \frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a-3b)\,\text{Subst}\left(\right.}{}$$

$$= \frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a-3b)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} + \frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{cs}}{2af\sqrt{a}}$$

Mathematica [C] time = 0.11, size = 69, normalized size = 0.63

$$\frac{(2a-3b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)-a\operatorname{csch}^2(e+fx)}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a*Csch[e + f*x]^2) + (2*a - 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*
Sinh[e + f*x]^2)/a])/(2*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

fricas [B] time = 0.68, size = 3228, normalized size = 29.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a*b - 3*b^2)*cosh(f*x + e)^8 + 8*(2*a*b - 3*b^2)*cosh(f*x + e)*s
inh(f*x + e)^7 + (2*a*b - 3*b^2)*sinh(f*x + e)^8 + 4*(2*a^2 - 5*a*b + 3*b^2
```

)*cosh(f*x + e)^6 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^2 + 2*a^2 - 5*a*b + 3*b^2)*sinh(f*x + e)^6 + 8*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^3 + 3*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^4 + 2*(35*(2*a*b - 3*b^2)*cosh(f*x + e)^4 + 30*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 18*a*b - 9*b^2)*sinh(f*x + e)^4 + 8*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^5 + 10*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^3 - (8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^6 + 15*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^4 - 3*(8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^2 + 2*a^2 - 5*a*b + 3*b^2)*sinh(f*x + e)^2 + 2*a*b - 3*b^2 + 8*((2*a*b - 3*b^2)*cosh(f*x + e)^7 + 3*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^5 - (8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^3 + (2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((2*a^2 - 3*a*b)*cosh(f*x + e)^5 + 5*(2*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (2*a^2 - 3*a*b)*sinh(f*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a^2 - 3*a*b)*cosh(f*x + e) + (5*(2*a^2 - 3*a*b)*cosh(f*x + e)^4 - 6*(4*a^2 - 3*a*b)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^3*b*f*cosh(f*x + e)^8 + 8*a^3*b*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^3*b*f*sinh(f*x + e)^8 + 4*(a^4 - a^3*b)*f*cosh(f*x + e)^6 + 4*(7*a^3*b*f*cosh(f*x + e)^2 + (a^4 - a^3*b)*f)*sinh(f*x + e)^6 - 2*(4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^4 + 8*(7*a^3*b*f*cosh(f*x + e)^3 + 3*(a^4 - a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)^5 + a^3*b*f + 2*(35*a^3*b*f*cosh(f*x + e)^4 + 30*(a^4 - a^3*b)*f*cosh(f*x + e)^2 - (4*a^4 - 3*a^3*b)*f)*sinh(f*x + e)^4 + 4*(a^4 - a^3*b)*f*cosh(f*x + e)^2 + 8*(7*a^3*b*f*cosh(f*x + e)^5 + 10*(a^4 - a^3*b)*f*cosh(f*x + e)^3 - (4*a^4 - 3*a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*b*f*cosh(f*x + e)^6 + 15*(a^4 - a^3*b)*f*cosh(f*x + e)^4 - 3*(4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^2 + (a^4 - a^3*b)*f)*sinh(f*x + e)^2 + 8*(a^3*b*f*cosh(f*x + e)^7 + 3*(a^4 - a^3*b)*f*cosh(f*x + e)^5 - (4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^3 + (a^4 - a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)), 1/2*(((2*a*b - 3*b^2)*cosh(f*x + e)^8 + 8*(2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (2*a*b - 3*b^2)*sinh(f*x + e)^8 + 4*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^6 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^2 + 2*a^2 - 5*a*b + 3*b^2)*sinh(f*x + e)^6 + 8*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^3 + 3*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^4 + 2*(35*(2*a*b - 3*b^2)*cosh(f*x + e)^4 + 30*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 18*a*b - 9*b^2)*sinh(f*x + e)^4 + 8*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^5 + 10*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^3 - (8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^6 + 15*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^4 - 3*(8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^2 + 2*a^2 - 5*a*b + 3*b^2)*sinh(f*x + e)^2 + 2*a*b - 3*b^2 + 8*((2*a*b - 3*b^2)*cosh(f*x + e)^7 + 3*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^5 - (8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^3 + (2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt(2)*((2*a^2 - 3*a*b)*cosh(f*x + e)^5 + 5*(2*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (2*a^2 - 3*a*b)*sinh(f*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x

```
+ e)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e
)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e
))*sinh(f*x + e)^2 + (2*a^2 - 3*a*b)*cosh(f*x + e) + (5*(2*a^2 - 3*a*b)*cos
h(f*x + e)^4 - 6*(4*a^2 - 3*a*b)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b)*sinh(f*x
+ e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^3*b*f*cosh(f*x +
 e)^8 + 8*a^3*b*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^3*b*f*sinh(f*x + e)^8 +
 4*(a^4 - a^3*b)*f*cosh(f*x + e)^6 + 4*(7*a^3*b*f*cosh(f*x + e)^2 + (a^4 -
a^3*b)*f)*sinh(f*x + e)^6 - 2*(4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^4 + 8*(7*a^
3*b*f*cosh(f*x + e)^3 + 3*(a^4 - a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)^5 +
a^3*b*f + 2*(35*a^3*b*f*cosh(f*x + e)^4 + 30*(a^4 - a^3*b)*f*cosh(f*x + e)^
2 - (4*a^4 - 3*a^3*b)*f)*sinh(f*x + e)^4 + 4*(a^4 - a^3*b)*f*cosh(f*x + e)^
2 + 8*(7*a^3*b*f*cosh(f*x + e)^5 + 10*(a^4 - a^3*b)*f*cosh(f*x + e)^3 - (4*
a^4 - 3*a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*b*f*cosh(f*x + e
)^6 + 15*(a^4 - a^3*b)*f*cosh(f*x + e)^4 - 3*(4*a^4 - 3*a^3*b)*f*cosh(f*x +
 e)^2 + (a^4 - a^3*b)*f)*sinh(f*x + e)^2 + 8*(a^3*b*f*cosh(f*x + e)^7 + 3*(
a^4 - a^3*b)*f*cosh(f*x + e)^5 - (4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^3 + (a^4
 - a^3*b)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 1.35sym2poly/r2sym(const gen & e,const index_m & i,
const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.19, size = 43, normalized size = 0.39

$$\frac{`int/indef0`\left(\frac{\cosh^2\left(fx+e\right)}{\sinh\left(fx+e\right)^3\left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}, \sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x
+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth\left(e+f\,x\right)^3}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.495 $\int \frac{\coth^5(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -1/8*(8*a^2-24*a*b+15*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/
2)/f+1/8*(8*a^2-24*a*b+15*b^2)/a^3/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/8*(8*a-5*b
)*csch(f*x+e)^2/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/4*csch(f*x+e)^4/a/f/(a+b*
sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}-\frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(
8*a^(7/2)*f) + (8*a^2 - 24*a*b + 15*b^2)/(8*a^3*f*Sqrt[a + b*Sinh[e + f*x]^
2]) - ((8*a - 5*b)*Csch[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) -
 Csch[e + f*x]^4/(4*a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-5b)+2ax}{x^2(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{4af}$$

$$= -\frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(8a^2-24ab+15b^2\right.}{}$$

$$= \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e}}$$

$$= \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e}}$$

$$= -\frac{\left(8a^2-24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} -$$

Mathematica [C] time = 0.33, size = 94, normalized size = 0.56

$$\frac{\left(8a^2-24ab+15b^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)+a\,\text{csch}^2(e+fx)\left(-2a\,\text{csch}^2(e+fx)-8a+5b\right)}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Csch[e + f*x]^2*(-8*a + 5*b - 2*a*Csch[e + f*x]^2) + (8*a^2 - 24*a*b + 1
5*b^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sinh[e + f*x]^2)/a])/(8*a^3*f
*Sqrt[a + b*Sinh[e + f*x]^2])
```

fricas [B] time = 0.87, size = 7562, normalized size = 45.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^12 + 12*(8*a^2*b - 24*a
*b^2 + 15*b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (8*a^2*b - 24*a*b^2 + 15*b^
3)*sinh(f*x + e)^12 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x +
 e)^10 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3 + 33*(8*a^2*b - 24*a*b^2
 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(8*a^2*b - 24*a*b^2 +
 15*b^3)*cosh(f*x + e)^3 + (16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*
x + e))*sinh(f*x + e)^9 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(
f*x + e)^8 + (495*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^4 - 128*a^3 +
 504*a^2*b - 600*a*b^2 + 225*b^3 + 90*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3)*
cosh(f*x + e)^5 + 30*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)
^3 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e))*sinh(f*x +
e)^7 + 4*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^6 + 4*(231
*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^6 + 105*(16*a^3 - 72*a^2*b + 1
02*a*b^2 - 45*b^3)*cosh(f*x + e)^4 + 48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^
3 - 7*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^2)*sinh(f*x
 + e)^6 + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^7 + 63*(16*a^3
- 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^5 - 7*(128*a^3 - 504*a^2*b +
 600*a*b^2 - 225*b^3)*cosh(f*x + e)^3 + 3*(48*a^3 - 184*a^2*b + 210*a*b^2 -
 75*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - (128*a^3 - 504*a^2*b + 600*a*b^2
- 225*b^3)*cosh(f*x + e)^4 + (495*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x +
e)^8 + 420*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^6 - 70*(1
28*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^4 - 128*a^3 + 504*a
^2*b - 600*a*b^2 + 225*b^3 + 60*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*c
osh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(
f*x + e)^9 + 60*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^7 -
14*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^5 + 20*(48*a^3
 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^3 - (128*a^3 - 504*a^2*b +
 600*a*b^2 - 225*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*a^2*b - 24*a*b^2 +
 15*b^3 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^2 + 2*(3
3*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^10 + 45*(16*a^3 - 72*a^2*b +
102*a*b^2 - 45*b^3)*cosh(f*x + e)^8 - 14*(128*a^3 - 504*a^2*b + 600*a*b^2 -
 225*b^3)*cosh(f*x + e)^6 + 30*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*co
sh(f*x + e)^4 + 16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3 - 3*(128*a^3 - 504*a
^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(8*a^2*
b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^11 + 5*(16*a^3 - 72*a^2*b + 102*a*b^2
- 45*b^3)*cosh(f*x + e)^9 - 2*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*c
osh(f*x + e)^7 + 6*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^
5 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^3 + (16*a^3 -
 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log((
b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x +
 e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*
(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x
```

+ e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^
3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*
x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt
(2)*((8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^9 + 9*(8*a^3 - 24*a^2*b +
15*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^3 - 24*a^2*b + 15*a*b^2)*sin
h(f*x + e)^9 - 4*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 - 4*(16*a^3
 - 29*a^2*b + 15*a*b^2 - 9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*s
inh(f*x + e)^7 + 28*(3*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 - (16*
a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(40*a^3 - 92*
a^2*b + 45*a*b^2)*cosh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b^2)*cos
h(f*x + e)^4 + 40*a^3 - 92*a^2*b + 45*a*b^2 - 42*(16*a^3 - 29*a^2*b + 15*a*
b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b^2)
*cosh(f*x + e)^5 - 70*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 5*(4
0*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 - 4*(16*a^3 - 2
9*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^3 - 24*a^2*b + 15*a*b^2)*c
osh(f*x + e)^6 - 35*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^4 - 16*a^3
 + 29*a^2*b - 15*a*b^2 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^2)*
sinh(f*x + e)^3 + 4*(9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 - 21*(
16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^5 + 5*(40*a^3 - 92*a^2*b + 45*a
*b^2)*cosh(f*x + e)^3 - 3*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e))*sin
h(f*x + e)^2 + (8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e) + (9*(8*a^3 - 24
*a^2*b + 15*a*b^2)*cosh(f*x + e)^8 - 28*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh
(f*x + e)^6 + 10*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^4 + 8*a^3 - 2
4*a^2*b + 15*a*b^2 - 12*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^4*b*f*cosh
(f*x + e)^12 + 12*a^4*b*f*cosh(f*x + e)*sinh(f*x + e)^11 + a^4*b*f*sinh(f*x
 + e)^12 + 2*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^10 + 2*(33*a^4*b*f*cosh(f*x
+ e)^2 + (2*a^5 - 3*a^4*b)*f)*sinh(f*x + e)^10 - (16*a^5 - 15*a^4*b)*f*cosh
(f*x + e)^8 + 20*(11*a^4*b*f*cosh(f*x + e)^3 + (2*a^5 - 3*a^4*b)*f*cosh(f*x
 + e))*sinh(f*x + e)^9 + (495*a^4*b*f*cosh(f*x + e)^4 + 90*(2*a^5 - 3*a^4*b
)*f*cosh(f*x + e)^2 - (16*a^5 - 15*a^4*b)*f)*sinh(f*x + e)^8 + 4*(6*a^5 - 5
*a^4*b)*f*cosh(f*x + e)^6 + 8*(99*a^4*b*f*cosh(f*x + e)^5 + 30*(2*a^5 - 3*a
^4*b)*f*cosh(f*x + e)^3 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e))*sinh(f*x + e
)^7 + 4*(231*a^4*b*f*cosh(f*x + e)^6 + 105*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e
)^4 - 7*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^2 + (6*a^5 - 5*a^4*b)*f)*sinh(f
*x + e)^6 + a^4*b*f - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^4 + 8*(99*a^4*b*f
*cosh(f*x + e)^7 + 63*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^5 - 7*(16*a^5 - 15*
a^4*b)*f*cosh(f*x + e)^3 + 3*(6*a^5 - 5*a^4*b)*f*cosh(f*x + e))*sinh(f*x +
e)^5 + (495*a^4*b*f*cosh(f*x + e)^8 + 420*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)
^6 - 70*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^4 + 60*(6*a^5 - 5*a^4*b)*f*cosh
(f*x + e)^2 - (16*a^5 - 15*a^4*b)*f)*sinh(f*x + e)^4 + 2*(2*a^5 - 3*a^4*b)*
f*cosh(f*x + e)^2 + 4*(55*a^4*b*f*cosh(f*x + e)^9 + 60*(2*a^5 - 3*a^4*b)*f*
cosh(f*x + e)^7 - 14*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^5 + 20*(6*a^5 - 5*
a^4*b)*f*cosh(f*x + e)^3 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e))*sinh(f*x +
e)^3 + 2*(33*a^4*b*f*cosh(f*x + e)^10 + 45*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e
)^8 - 14*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^6 + 30*(6*a^5 - 5*a^4*b)*f*cos
h(f*x + e)^4 - 3*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^2 + (2*a^5 - 3*a^4*b)*
f)*sinh(f*x + e)^2 + 4*(3*a^4*b*f*cosh(f*x + e)^11 + 5*(2*a^5 - 3*a^4*b)*f*
cosh(f*x + e)^9 - 2*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^7 + 6*(6*a^5 - 5*a^
4*b)*f*cosh(f*x + e)^5 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^3 + (2*a^5 - 3
*a^4*b)*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((8*a^2*b - 24*a*b^2 + 15*b^3
)*cosh(f*x + e)^12 + 12*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)*sinh(f*
x + e)^11 + (8*a^2*b - 24*a*b^2 + 15*b^3)*sinh(f*x + e)^12 + 2*(16*a^3 - 72
*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^10 + 2*(16*a^3 - 72*a^2*b + 102*
a*b^2 - 45*b^3 + 33*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x
 + e)^10 + 20*(11*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^3 + (16*a^3 -
 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e))*sinh(f*x + e)^9 - (128*a^3 -
 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^8 + (495*(8*a^2*b - 24*a*b^

2 + 15*b^3)*cosh(f*x + e)^4 - 128*a^3 + 504*a^2*b - 600*a*b^2 + 225*b^3 + 9
0*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8
 + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^5 + 30*(16*a^3 - 72*a^
2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^3 - (128*a^3 - 504*a^2*b + 600*a*b^
2 - 225*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(48*a^3 - 184*a^2*b + 210*a
*b^2 - 75*b^3)*cosh(f*x + e)^6 + 4*(231*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(
f*x + e)^6 + 105*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^4 +
 48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3 - 7*(128*a^3 - 504*a^2*b + 600*a*b
^2 - 225*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(8*a^2*b - 24*a*b^2
+ 15*b^3)*cosh(f*x + e)^7 + 63*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cos
h(f*x + e)^5 - 7*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^
3 + 3*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e))*sinh(f*x + e
)^5 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^4 + (495*(8
*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^8 + 420*(16*a^3 - 72*a^2*b + 102*
a*b^2 - 45*b^3)*cosh(f*x + e)^6 - 70*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225
*b^3)*cosh(f*x + e)^4 - 128*a^3 + 504*a^2*b - 600*a*b^2 + 225*b^3 + 60*(48*
a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*
(55*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^9 + 60*(16*a^3 - 72*a^2*b +
 102*a*b^2 - 45*b^3)*cosh(f*x + e)^7 - 14*(128*a^3 - 504*a^2*b + 600*a*b^2
- 225*b^3)*cosh(f*x + e)^5 + 20*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*c
osh(f*x + e)^3 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e))
*sinh(f*x + e)^3 + 8*a^2*b - 24*a*b^2 + 15*b^3 + 2*(16*a^3 - 72*a^2*b + 102
*a*b^2 - 45*b^3)*cosh(f*x + e)^2 + 2*(33*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh
(f*x + e)^10 + 45*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^8
- 14*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^6 + 30*(48*a
^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + 16*a^3 - 72*a^2*b +
102*a*b^2 - 45*b^3 - 3*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x
 + e)^2)*sinh(f*x + e)^2 + 4*(3*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)
^11 + 5*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^9 - 2*(128*a
^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^7 + 6*(48*a^3 - 184*a^2
*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^5 - (128*a^3 - 504*a^2*b + 600*a*b^2
 - 225*b^3)*cosh(f*x + e)^3 + (16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) +
 2*sqrt(2)*((8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^9 + 9*(8*a^3 - 24*a
^2*b + 15*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^3 - 24*a^2*b + 15*a*b
^2)*sinh(f*x + e)^9 - 4*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 - 4*
(16*a^3 - 29*a^2*b + 15*a*b^2 - 9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x +
e)^2)*sinh(f*x + e)^7 + 28*(3*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^3
 - (16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(40*a^
3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b
^2)*cosh(f*x + e)^4 + 40*a^3 - 92*a^2*b + 45*a*b^2 - 42*(16*a^3 - 29*a^2*b
+ 15*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15
*a*b^2)*cosh(f*x + e)^5 - 70*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^3
 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 - 4*(16*
a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^3 - 24*a^2*b + 15*a
*b^2)*cosh(f*x + e)^6 - 35*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^4 -
 16*a^3 + 29*a^2*b - 15*a*b^2 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x +
 e)^2)*sinh(f*x + e)^3 + 4*(9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^7
 - 21*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^5 + 5*(40*a^3 - 92*a^2*b
 + 45*a*b^2)*cosh(f*x + e)^3 - 3*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x +
e))*sinh(f*x + e)^2 + (8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e) + (9*(8*a
^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^8 - 28*(16*a^3 - 29*a^2*b + 15*a*b^
2)*cosh(f*x + e)^6 + 10*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^4 + 8*
a^3 - 24*a^2*b + 15*a*b^2 - 12*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)
^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^4*b
*f*cosh(f*x + e)^12 + 12*a^4*b*f*cosh(f*x + e)*sinh(f*x + e)^11 + a^4*b*f*s

```
inh(f*x + e)^12 + 2*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^10 + 2*(33*a^4*b*f*co
sh(f*x + e)^2 + (2*a^5 - 3*a^4*b)*f)*sinh(f*x + e)^10 - (16*a^5 - 15*a^4*b)
*f*cosh(f*x + e)^8 + 20*(11*a^4*b*f*cosh(f*x + e)^3 + (2*a^5 - 3*a^4*b)*f*c
osh(f*x + e))*sinh(f*x + e)^9 + (495*a^4*b*f*cosh(f*x + e)^4 + 90*(2*a^5 -
3*a^4*b)*f*cosh(f*x + e)^2 - (16*a^5 - 15*a^4*b)*f)*sinh(f*x + e)^8 + 4*(6*
a^5 - 5*a^4*b)*f*cosh(f*x + e)^6 + 8*(99*a^4*b*f*cosh(f*x + e)^5 + 30*(2*a^
5 - 3*a^4*b)*f*cosh(f*x + e)^3 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e))*sinh(
f*x + e)^7 + 4*(231*a^4*b*f*cosh(f*x + e)^6 + 105*(2*a^5 - 3*a^4*b)*f*cosh(
f*x + e)^4 - 7*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^2 + (6*a^5 - 5*a^4*b)*f)
*sinh(f*x + e)^6 + a^4*b*f - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^4 + 8*(99*
a^4*b*f*cosh(f*x + e)^7 + 63*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^5 - 7*(16*a^
5 - 15*a^4*b)*f*cosh(f*x + e)^3 + 3*(6*a^5 - 5*a^4*b)*f*cosh(f*x + e))*sinh
(f*x + e)^5 + (495*a^4*b*f*cosh(f*x + e)^8 + 420*(2*a^5 - 3*a^4*b)*f*cosh(f
*x + e)^6 - 70*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^4 + 60*(6*a^5 - 5*a^4*b)
*f*cosh(f*x + e)^2 - (16*a^5 - 15*a^4*b)*f)*sinh(f*x + e)^4 + 2*(2*a^5 - 3*
a^4*b)*f*cosh(f*x + e)^2 + 4*(55*a^4*b*f*cosh(f*x + e)^9 + 60*(2*a^5 - 3*a^
4*b)*f*cosh(f*x + e)^7 - 14*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^5 + 20*(6*a
^5 - 5*a^4*b)*f*cosh(f*x + e)^3 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e))*sinh
(f*x + e)^3 + 2*(33*a^4*b*f*cosh(f*x + e)^10 + 45*(2*a^5 - 3*a^4*b)*f*cosh(
f*x + e)^8 - 14*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^6 + 30*(6*a^5 - 5*a^4*b
)*f*cosh(f*x + e)^4 - 3*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^2 + (2*a^5 - 3*
a^4*b)*f)*sinh(f*x + e)^2 + 4*(3*a^4*b*f*cosh(f*x + e)^11 + 5*(2*a^5 - 3*a^
4*b)*f*cosh(f*x + e)^9 - 2*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^7 + 6*(6*a^5
 - 5*a^4*b)*f*cosh(f*x + e)^5 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^3 + (2*
a^5 - 3*a^4*b)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 2.08sym2poly/r2sym(const gen & e,const index_m & i,
const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.22, size = 43, normalized size = 0.26

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\cosh^{4}\left(fx+e\right)}{\sinh\left(fx+e\right)^{5}\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^4/sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x
+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^{5}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^5\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Integral(coth(e + f*x)**5/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.496 $\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=275

$$-\frac{4a\tanh(e+fx)}{3f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)}{3f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\sqrt{a}\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sin}{}\right.\right.}{3f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\,c}{a+b}}}$$

```
[Out] -1/3*(7*a+b)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2
/a)^(1/2)*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2)
,(1-a/b)^(1/2))*a^(1/2)*b^(1/2)/(a-b)^3/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)
^2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(3*a+5*b)*(1/(1+sinh(f*x+e)^2))^(1
/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(
1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^3/f/(sech(f*x+e)^
2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-4/3*a*tanh(f*x+e)/(a-b)^2/f/(a+b*sinh(f*x+e)
^2)^(1/2)+1/3*sech(f*x+e)^2*tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3196, 470, 527, 525, 418, 411}

$$-\frac{4a\tanh(e+fx)}{3f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)}{3f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\sqrt{a}\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sin}{}\right.\right.}{3f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\,c}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[a]*Sqrt[b]*(7*a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e
+ f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^3*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*
Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + 5*b)*EllipticF[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*
(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (4*a*Tanh[
e + f*x])/(3*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (Sech[e + f*x]^2*Ta
nh[e + f*x])/(3*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
```

```
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\text{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \right.}{3(a-b)}$$

$$= -\frac{4a\tanh(e+fx)}{3(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\text{co}\right.}{}$$

$$= -\frac{4a\tanh(e+fx)}{3(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(ab(7\right.}{}$$

$$= -\frac{\sqrt{a}\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{(3a+5b)F\left(\text{t}\right.}{}$$

Mathematica [C] time = 2.25, size = 212, normalized size = 0.77

$$\frac{-\frac{\tanh(e+fx)\text{sech}^2(e+fx)\left(4\left(4a^2+3ab+b^2\right)\cosh(2(e+fx))+8a^2+b(7a+b)\cosh(4(e+fx))+21ab-5b^2\right)}{2\sqrt{2}}+8ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\Big(i(e}{6f(a-b)^3\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] - ((8*a^2 + 21*a*b - 5*b^2 + 4*(4*a^2 + 3*a*b + b^2
)*Cosh[2*(e + f*x)] + b*(7*a + b)*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh[e
 + f*x])/(2*Sqrt[2]))/(6*(a - b)^3*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\tanh\left(fx+e\right)^4}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 3.29Error: Bad Argument Type
```

maple [A] time = 0.36, size = 354, normalized size = 1.29

$$-\frac{\left(7\sqrt{-\frac{b}{a}}\,ab+\sqrt{-\frac{b}{a}}\,b^2\right)\sinh\left(fx+e\right)\left(\cosh^4\left(fx+e\right)\right)+\left(4\sqrt{-\frac{b}{a}}\,a^2-4\sqrt{-\frac{b}{a}}\,ab\right)\left(\cosh^2\left(fx+e\right)\right)\sinh\left(fx+e\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*((7*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+(
4*(-1/a*b)^(1/2)*a^2-4*(-1/a*b)^(1/2)*a*b)*cosh(f*x+e)^2*sinh(f*x+e)+(-(-1/
a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)-(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a^2-2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*a*b-EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+7*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+EllipticE(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2)/(-1/a*b)^(1/2)/cosh(f*x+e)^3/(a-b
)^3/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh\left(e+f\,x\right)^4}{{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
[Out] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{4}{\left(e+fx\right)}}{\left(a+b\sinh^{2}{\left(e+fx\right)}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
[Out] Integral(tanh(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.497 $\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{(a+b)\text{sech}(e+fx)\sqrt{a+}}{af(a-}$$

```
[Out] -2*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)*
EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^(
1/2))*a^(1/2)*b^(1/2)/(a-b)^2/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2)
/(a+b*sinh(f*x+e)^2)^(1/2)+(a+b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)
^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech
(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+
e)^2)/a)^(1/2)-tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3196, 471, 525, 418, 411}

$$-\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{(a+b)\text{sech}(e+fx)\sqrt{a+}}{af(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[b]*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/
Sqrt[a]], 1 - a/b])/((a - b)^2*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f
*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((a + b)*EllipticF[ArcTan[Sinh[e + f
*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)^2*f*Sq
rt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - Tanh[e + f*x]/((a - b)*f
*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right.}{f}$$

$$= -\frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int\right.}{(-a+b)}$$

$$= -\frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2ab\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}}{(a-b)(-a+}$$

$$= -\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a+b)F\left(\tan^{-1}(\sin\right.}{a(}$$

Mathematica [C] time = 1.33, size = 158, normalized size = 0.73

$$\frac{-2\tanh(e+fx)(a+b\cosh(2(e+fx)))+i\sqrt{2}(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}F\left(i(e+fx)\middle|\frac{b}{a}\right)-2i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2}{a}}}{f(a-b)^2\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
f*x), b/a] + I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] - 2*(a + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/((a -
b)^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\tanh\left(fx+e\right)^2}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.04Error: Bad Argument Type
```

maple [A] time = 0.32, size = 256, normalized size = 1.18

$$-\frac{2\sqrt{-\frac{b}{a}}\, b\left(\sinh^{3}\left(fx+e\right)\right)-a\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\operatorname{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+b\sqrt{\frac{a+\cdots}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(2*(-1/a*b)^(1/2)*b*sinh(f*x+e)^3-a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*
x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+b*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))-2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
llipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+(-1/a*b)^(1/2)*a*sinh(f*x+
e)+b*sinh(f*x+e)*(-1/a*b)^(1/2))/(a-b)^2/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh\left(e+f\,x\right)^{2}}{\left(b\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.498 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] -b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-I*(cos(I*e+I
*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(b/a)^(1/2))*(a+b*si
nh(f*x+e)^2)^(1/2)/a/(a-b)/f/(1+b*sinh(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3184, 21, 3178, 3177}

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{-a-b\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{a(a-b)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.16, size = 100, normalized size = 0.87

$$\frac{-\sqrt{2}\,b\sinh(2(e+fx)) - 2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,E\left(i(e+fx)\middle|\frac{b}{a}\right)}{2af(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x
+ e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Error: Bad Argument Type
```

maple [A] time = 0.26, size = 252, normalized size = 2.19

$$-\sqrt{-\frac{b}{a}}\,b\sinh\left(fx+e\right)\left(\cosh^2\left(fx+e\right)\right)+a\sqrt{\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\text{EllipticF}\left(\sinh\left(fx+e\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^2+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/a/(a-
b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sinh^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-3/2), x)
```

3.499 $\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f}-\frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+f}}{a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)}{}}}$$

```
[Out] coth(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)-2*coth(f*x+e)*(a+b*sinh(f*x+e)^2)
^(1/2)/a^2/f-2*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(
f*x+e)^2)^(1/2)/a^2/f/(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+(1/(1+sin
h(f*x+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*
x+e)^2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/f/(s
ech(f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+2*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(
f*x+e)/a^2/f
```

Rubi [A] time = 0.27, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3196, 469, 583, 531, 418, 492, 411}

$$\frac{2\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f}-\frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+f}}{a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Coth[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*Coth[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(a^2*f) - (2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]
*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a
 + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sec
h[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2
*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
```

```
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{-2\cdots}{x^2\sqrt{1+x^2}\cdots}\right.}{af}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \frac{\left(\sqrt{\cosh^2\cdots}\right.}{\cdots}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \frac{\left(\sqrt{\cosh^2\cdots}\right.}{\cdots}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \frac{F\left(\tan^{-1}(\cdots\right.}{\cdots}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} - \frac{2E\left(\tan^{-1}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.80, size = 153, normalized size = 0.65

$$\frac{-2\coth(e+fx)(a+b\cosh(2(e+fx))-b) + i\sqrt{2}\,a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,F\left(i(e+fx)\middle|\frac{b}{a}\right) - 2i\sqrt{2}\,a\sqrt{\frac{2a+b\cosh(2(e+\cdots}{a}}\cdots}{a^2f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - b + b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*Sqrt[2]*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*a*Sq
rt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(a^2*f*S
qrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\coth\left(fx+e\right)^2}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command: INPUT:sage2OUTPUT:Evaluation time: 0.96Error: Bad Argument Type

maple [A] time = 0.29, size = 219, normalized size = 0.92

$$-\frac{-\sinh\left(fx+e\right)\sqrt{\frac{b\left(\cosh^{2}\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\left(a\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-2b\,\mathrm{EllipticF}\right.}{a^{2}\sinh\left(fx+e\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] -(-sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))+2*(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-2*(-1/a*b)^(1/2)*b)*cosh(f*x+e)^2)/a^2/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth\left(e+f\,x\right)^{2}}{\left(b\sinh\left(e+f\,x\right)^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2),x)

[Out] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{2}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Integral(coth(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)

3.500 $\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=341

$$\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f}-\frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f}+\frac{(3a-4b)\text{sech}(e\cdot}{}$$

```
[Out] -(a-b)*coth(f*x+e)*csch(f*x+e)^2/a/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(7*a-8
*b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/f+1/3*(3*a-4*b)*coth(f*x+e)*c
sch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/b/f-1/3*(7*a-8*b)*(1/(1+sinh(f*x
+e)^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^
2)^(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/f/(sech(f
*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a-4*b)*(1/(1+sinh(f*x+e)^2))^(1
/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(
1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/f/(sech(f*x+e)^2*(a
+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(7*a-8*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x
+e)/a^3/f
```

Rubi [A] time = 0.41, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3196, 468, 583, 531, 418, 492, 411}

$$\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f}-\frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f}+\frac{(3a-4b)\coth(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) - ((7*a - 8*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f) + (
(3*a - 4*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a
^2*b*f) - ((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*
Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(
a + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh
[e + f*x])/(3*a^3*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}}{ab}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(3a-4b)\coth(e+fx)\text{csch}^2(e+fx)\sqrt{}}{3a^2bf}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e}}{3a^3f}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e}}{3a^3f}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e}}{3a^3f}$$

$$= -\frac{(a-b)\coth(e+fx)\text{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e}}{3a^3f}$$

Mathematica [C] time = 3.29, size = 214, normalized size = 0.63

$$\frac{-\frac{\coth(e+fx)\text{csch}^2(e+fx)\left(4\left(4a^2-11ab+8b^2\right)\cosh(2(e+fx))-8a^2+b(7a-8b)\cosh(4(e+fx))+37ab-24b^2\right)}{2\sqrt{2}} + 8ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}{6a^3f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-1/2*((-8*a^2 + 37*a*b - 24*b^2 + 4*(4*a^2 - 11*a*b + 8*b^2)*Cosh[2*(e + f
*x)] + (7*a - 8*b)*b*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt
[2] - (2*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE
[I*(e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a
]*EllipticF[I*(e + f*x), b/a])/(6*a^3*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\coth\left(fx+e\right)^4}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 1.64Error: Bad Argument Type
```

maple [A] time = 0.34, size = 522, normalized size = 1.53

$$-\frac{7\sqrt{-\frac{b}{a}}\, ab\left(\sinh^{6}\left(fx+e\right)\right)-8\sqrt{-\frac{b}{a}}\, b^{2}\left(\sinh^{6}\left(fx+e\right)\right)-3a^{2}\sqrt{\frac{a+b\left(\sinh^{2}\left(fx+e\right)\right)}{a}}\,\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \text{EllipticF}\left(\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(7*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-8*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6
-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*x+e)^3+11*b*((a+b*sinh(f*x+e)^2)/a
)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a*sinh(f*x+e)^3-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3-7*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*si
nh(f*x+e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+3*(-1/a*b)^(1/2)*a*b*sinh(f*
x+e)^4-8*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^
2-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2)/a^3/sinh(f*x+e)^3/
(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{coth}\left(e+f\,x\right)}^4}{{\left(b\,{\mathrm{sinh}\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2),x)
```

```
[Out] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

3.501 $\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=232

$$\frac{8a^2+24ab+3b^2}{8f(a-b)^4\sqrt{a+b\sinh^2(e+fx)}}+\frac{8a^2+24ab+3b^2}{24f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+}}{\sqrt{a-b}}\right.}{8f(a-b)^{9/2}}$$

```
[Out] -1/8*(8*a^2+24*a*b+3*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a
-b)^(9/2)/f+1/24*(8*a^2+24*a*b+3*b^2)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(3/2)+1
/8*(8*a-b)*sech(f*x+e)^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/4*sech(f*x+e
)^4/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/8*(8*a^2+24*a*b+3*b^2)/(a-b)^4/f/(a
+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2+24ab+3b^2}{8f(a-b)^4\sqrt{a+b\sinh^2(e+fx)}}+\frac{8a^2+24ab+3b^2}{24f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+}}{\sqrt{a-b}}\right.}{8f(a-b)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -((8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]
)/(8*(a - b)^(9/2)*f) + (8*a^2 + 24*a*b + 3*b^2)/(24*(a - b)^3*f*(a + b*Sin
h[e + f*x]^2)^(3/2)) + ((8*a - b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*(a + b*Si
nh[e + f*x]^2)^(3/2)) - Sech[e + f*x]^4/(4*(a - b)*f*(a + b*Sinh[e + f*x]^2
)^(3/2)) + (8*a^2 + 24*a*b + 3*b^2)/(8*(a - b)^4*f*Sqrt[a + b*Sinh[e + f*x]
^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{sech}^4(e+fx)}{4(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a-3b)+2(a-b)x}{(1+x)^2(a+bx)^{5/2}}\,dx, x, \sinh^2(\right.}{4(a-b)f}$$

$$= \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{sech}^4(e+fx)}{4(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8}{}$$

$$= \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} -$$

$$= \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} -$$

$$= \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} -$$

$$= -\frac{\left(8a^2+24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{9/2}f} + \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+\right.}$$

Mathematica [C] time = 0.60, size = 114, normalized size = 0.49

$$\frac{2\left(8a^2+24ab+3b^2\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sinh^2(e+fx)+a}{a-b}\right)+3(a-b)\text{sech}^4(e+fx)((8a-b)\cosh(2(e+fx))+4a+3b)}{48f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*(8*a^2 + 24*a*b + 3*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sinh[e
+ f*x]^2)/(a - b)] + 3*(a - b)*(4*a + 3*b + (8*a - b)*Cosh[2*(e + f*x)])*Se
ch[e + f*x]^4)/(48*(a - b)^3*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 104.39, size = 3972, normalized size = 17.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*((3*(a^22*b^3*e^(21*e) - 14*a^21*b^4*e^(21*e) + 88*a^20*b^5*e^(21*e) -
320*a^19*b^6*e^(21*e) + 700*a^18*b^7*e^(21*e) - 728*a^17*b^8*e^(21*e) - 728
*a^16*b^9*e^(21*e) + 4576*a^15*b^10*e^(21*e) - 10010*a^14*b^11*e^(21*e) + 1
4300*a^13*b^12*e^(21*e) - 14872*a^12*b^13*e^(21*e) + 11648*a^11*b^14*e^(21*
e) - 6916*a^10*b^15*e^(21*e) + 3080*a^9*b^16*e^(21*e) - 1000*a^8*b^17*e^(21
*e) + 224*a^7*b^18*e^(21*e) - 31*a^6*b^19*e^(21*e) + 2*a^5*b^20*e^(21*e))*e
^(2*f*x)/(a^24*b^2*e^(16*e) - 20*a^23*b^3*e^(16*e) + 190*a^22*b^4*e^(16*e)
- 1140*a^21*b^5*e^(16*e) + 4845*a^20*b^6*e^(16*e) - 15504*a^19*b^7*e^(16*e)
 + 38760*a^18*b^8*e^(16*e) - 77520*a^17*b^9*e^(16*e) + 125970*a^16*b^10*e^(
16*e) - 167960*a^15*b^11*e^(16*e) + 184756*a^14*b^12*e^(16*e) - 167960*a^13
*b^13*e^(16*e) + 125970*a^12*b^14*e^(16*e) - 77520*a^11*b^15*e^(16*e) + 387
60*a^10*b^16*e^(16*e) - 15504*a^9*b^17*e^(16*e) + 4845*a^8*b^18*e^(16*e) -
1140*a^7*b^19*e^(16*e) + 190*a^6*b^20*e^(16*e) - 20*a^5*b^21*e^(16*e) + a^4
*b^22*e^(16*e)) + 2*(8*a^23*b^2*e^(19*e) - 121*a^22*b^3*e^(19*e) + 842*a^21
*b^4*e^(19*e) - 3544*a^20*b^5*e^(19*e) + 9920*a^19*b^6*e^(19*e) - 18844*a^1
8*b^7*e^(19*e) + 22568*a^17*b^8*e^(19*e) - 9256*a^16*b^9*e^(19*e) - 25168*a
^15*b^10*e^(19*e) + 67210*a^14*b^11*e^(19*e) - 93236*a^13*b^12*e^(19*e) + 8
9752*a^12*b^13*e^(19*e) - 64064*a^11*b^14*e^(19*e) + 34468*a^10*b^15*e^(19*
e) - 13880*a^9*b^16*e^(19*e) + 4072*a^8*b^17*e^(19*e) - 824*a^7*b^18*e^(19*
e) + 103*a^6*b^19*e^(19*e) - 6*a^5*b^20*e^(19*e))/(a^24*b^2*e^(16*e) - 20*a
^23*b^3*e^(16*e) + 190*a^22*b^4*e^(16*e) - 1140*a^21*b^5*e^(16*e) + 4845*a^
20*b^6*e^(16*e) - 15504*a^19*b^7*e^(16*e) + 38760*a^18*b^8*e^(16*e) - 77520
*a^17*b^9*e^(16*e) + 125970*a^16*b^10*e^(16*e) - 167960*a^15*b^11*e^(16*e)
+ 184756*a^14*b^12*e^(16*e) - 167960*a^13*b^13*e^(16*e) + 125970*a^12*b^14*
e^(16*e) - 77520*a^11*b^15*e^(16*e) + 38760*a^10*b^16*e^(16*e) - 15504*a^9*
b^17*e^(16*e) + 4845*a^8*b^18*e^(16*e) - 1140*a^7*b^19*e^(16*e) + 190*a^6*b
^20*e^(16*e) - 20*a^5*b^21*e^(16*e) + a^4*b^22*e^(16*e)))*e^(2*f*x) + 3*(a^
22*b^3*e^(17*e) - 14*a^21*b^4*e^(17*e) + 88*a^20*b^5*e^(17*e) - 320*a^19*b^
6*e^(17*e) + 700*a^18*b^7*e^(17*e) - 728*a^17*b^8*e^(17*e) - 728*a^16*b^9*e
```

^(17*e) + 4576*a^15*b^10*e^(17*e) - 10010*a^14*b^11*e^(17*e) + 14300*a^13*b
^12*e^(17*e) - 14872*a^12*b^13*e^(17*e) + 11648*a^11*b^14*e^(17*e) - 6916*a
^10*b^15*e^(17*e) + 3080*a^9*b^16*e^(17*e) - 1000*a^8*b^17*e^(17*e) + 224*a
^7*b^18*e^(17*e) - 31*a^6*b^19*e^(17*e) + 2*a^5*b^20*e^(17*e))/(a^24*b^2*e^
(16*e) - 20*a^23*b^3*e^(16*e) + 190*a^22*b^4*e^(16*e) - 1140*a^21*b^5*e^(16
*e) + 4845*a^20*b^6*e^(16*e) - 15504*a^19*b^7*e^(16*e) + 38760*a^18*b^8*e^(
16*e) - 77520*a^17*b^9*e^(16*e) + 125970*a^16*b^10*e^(16*e) - 167960*a^15*b
^11*e^(16*e) + 184756*a^14*b^12*e^(16*e) - 167960*a^13*b^13*e^(16*e) + 1259
70*a^12*b^14*e^(16*e) - 77520*a^11*b^15*e^(16*e) + 38760*a^10*b^16*e^(16*e)
 - 15504*a^9*b^17*e^(16*e) + 4845*a^8*b^18*e^(16*e) - 1140*a^7*b^19*e^(16*e
) + 190*a^6*b^20*e^(16*e) - 20*a^5*b^21*e^(16*e) + a^4*b^22*e^(16*e)))*e^(f
*x)/((b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3
/2)*f) + 1/12*(15*(3*a^2*e^e + 4*a*b*e^e)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b) + sqrt(b))/sqrt(a - b))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqr
t(a - b)) - 24*(a^2*e^e + 2*a*b*e^e)*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqr
t(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(
-b))/((a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*sqrt(-b)) - 2*(21*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))^7*a^2*e^e + 12*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a*b*e^e + 243*(
sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))^6*a^2*sqrt(b)*e^e - 12*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a
*b^(3/2)*e^e + 436*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3*e^e + 117*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^5*a^2*b*e^e + 396*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*
e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2*e^e - 256*(sqr
t(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e
^(2*f*x + 2*e) + b))^5*b^3*e^e + 1796*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^3*sqrt(b
)*e^e + 363*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*
x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2)*e^e - 1644*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))^4*a*b^(5/2)*e^e + 640*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*b^(7/2)*e^e +
1840*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^3*a^4*e^e + 1512*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a
^3*b*e^e - 3609*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(
2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*b^2*e^e + 1412*(sqrt(b)*e^(2
*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b))^3*a*b^3*e^e + 7056*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^4*sqrt(b)*e^e -
 11608*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^3*b^(3/2)*e^e + 4929*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))^2*a^2*b^(5/2)*e^e + 1596*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(7/2)*e^e -
1280*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(9/2)*e^e + 4800*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
*a^5*e^e - 12720*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^4*b*e^e + 12388*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))*a^3*b^2*e^e - 2673*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b^3*e^e - 2844*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b

```
*e^(2*f*x + 2*e) + b))*a*b^4*e^e + 1280*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^5*e^e -
1344*a^5*sqrt(b)*e^e + 4592*a^4*b^(3/2)*e^e - 4524*a^3*b^(5/2)*e^e + 609*a^
2*b^(7/2)*e^e + 1084*a*b^(9/2)*e^e - 384*b^(11/2)*e^e)/((a^4 - 4*a^3*b + 6*
a^2*b^2 - 4*a*b^3 + b^4)*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
 + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x
+ 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b))*sqrt(b) + 4*a - 3*b)^4))/f^2
```

maple [C] time = 0.45, size = 213, normalized size = 0.92

$$\frac{\text{`}int/indef0\text{`}\left(-\frac{\left(\sinh^5\left(fx+e\right)\right)\left(b^2\left(\sinh^4\left(fx+e\right)\right)+2ab\left(\sinh^2\left(fx+e\right)\right)+a^2\right)\left(\cos\right.}{\left(-b^4\left(\cosh^{18}\left(fx+e\right)\right)+\left(-4a\,b^3+4b^4\right)\left(\cosh^{16}\left(fx+e\right)\right)+\left(-6a^2b^2+12a\,b^3-6b^4\right)\left(\cosh^{14}\left(fx+e\right)\right)+\left(-4a^3b+12a^2b^2-12a\,b^3+4b^4\right)\left(\cosh^1\right.\right.}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)^5*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)*cos
h(f*x+e)^4/(-b^4*cosh(f*x+e)^18+(-4*a*b^3+4*b^4)*cosh(f*x+e)^16+(-6*a^2*b^2
+12*a*b^3-6*b^4)*cosh(f*x+e)^14+(-4*a^3*b+12*a^2*b^2-12*a*b^3+4*b^4)*cosh(f
*x+e)^12+(-a^4+4*a^3*b-6*a^2*b^2+4*a*b^3-b^4)*cosh(f*x+e)^10)/(a+b*sinh(f*x
+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.502 $\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=163

$$\frac{2a+3b}{2f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}+\frac{2a+3b}{6f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}}+\frac{}{2f}$$

```
[Out] -1/2*(2*a+3*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(7/2)/f
+1/6*(2*a+3*b)/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/2*sech(f*x+e)^2/(a-b)/
f/(a+b*sinh(f*x+e)^2)^(3/2)+1/2*(2*a+3*b)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(1/
2)
```

Rubi [A] time = 0.17, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a+3b}{2f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}+\frac{2a+3b}{6f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}}+\frac{}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^
(7/2)*f) + (2*a + 3*b)/(6*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + Sech
[e + f*x]^2/(2*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*a + 3*b)/(2*(a
 - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\,\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x, \sinh^2\right.}{4(a-b)f}$$

$$= \frac{2a+3b}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(2a+}{}$$

$$= \frac{2a+3b}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{}{2(a-}$$

$$= \frac{2a+3b}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{}{2(a-}$$

$$= -\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f} + \frac{2a+3b}{6(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{}{2(a}$$

Mathematica [C] time = 0.13, size = 82, normalized size = 0.50

$$\frac{(2a+3b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sinh^2(e+fx)+a}{a-b}\right)+3(a-b)\text{sech}^2(e+fx)}{6f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*a + 3*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sinh[e + f*x]^2)/(a -
b)] + 3*(a - b)*Sech[e + f*x]^2)/(6*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/
2))
```

fricas [B] time = 1.39, size = 10506, normalized size = 64.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a*b^2 + 3*b^3)*cosh(f*x + e)^12 + 12*(2*a*b^2 + 3*b^3)*cosh(f
*x + e)*sinh(f*x + e)^11 + (2*a*b^2 + 3*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b
+ 10*a*b^2 - 3*b^3)*cosh(f*x + e)^10 + 2*(8*a^2*b + 10*a*b^2 - 3*b^3 + 33*(
2*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(2*a*b^2 + 3*b^
3)*cosh(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x +
 e)^9 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^8 + (495*(2*a*b
^2 + 3*b^3)*cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3 + 90*(8*a
^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(2*a*b^2
+ 3*b^3)*cosh(f*x + e)^5 + 30*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^3
+ (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 4*
(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 4*(231*(2*a*b^2 +
3*b^3)*cosh(f*x + e)^6 + 105*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^4 +
 16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3 + 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*
b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(2*a*b^2 + 3*b^3)*cosh(f*x +
e)^7 + 63*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 7*(32*a^3 + 48*a^2
*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3
*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3
)*cosh(f*x + e)^4 + (495*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^8 + 420*(8*a^2*b +
 10*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 70*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^
3)*cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3 + 60*(16*a^3 + 16*
a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(2*a*b^2
 + 3*b^3)*cosh(f*x + e)^9 + 60*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^7
 + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 20*(16*a^3 +
16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + (32*a^3 + 48*a^2*b - 2*a*b^2
 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*a*b^2 + 3*b^3 + 2*(8*a^2*b + 1
0*a*b^2 - 3*b^3)*cosh(f*x + e)^2 + 2*(33*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^10
 + 45*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 14*(32*a^3 + 48*a^2*b
- 2*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 30*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b
^3)*cosh(f*x + e)^4 + 8*a^2*b + 10*a*b^2 - 3*b^3 + 3*(32*a^3 + 48*a^2*b - 2
*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(2*a*b^2 + 3*b^3)*c
osh(f*x + e)^11 + 5*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^9 + 2*(32*a^
3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 6*(16*a^3 + 16*a^2*b - 10
*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cos
h(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e))*s
qrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*s
inh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4
*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e
)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*co
sh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*s
inh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*si
nh(f*x + e) + 1)) - 4*sqrt(2)*(3*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^9
+ 27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^8 + 3*(2*a^2*b +
 a*b^2 - 3*b^3)*sinh(f*x + e)^9 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*co
sh(f*x + e)^7 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 27*(2*a^2*b + a*b^2
 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(9*(2*a^2*b + a*b^2 - 3*b^3
)*cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e))*sin
h(f*x + e)^6 + 2*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 2
*(189*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 56*a^3 - 70*a^2*b + 17*a*
b^2 - 3*b^3 + 42*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh
(f*x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 70*(8*a^3
+ 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + 5*(56*a^3 - 70*a^2*b + 17*a
*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(8*a^3 + 2*a^2*b - 13*a*b^
2 + 3*b^3)*cosh(f*x + e)^3 + 4*(63*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^
6 + 35*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + 8*a^3 + 2*a^2
```

*b - 13*a*b^2 + 3*b^3 + 5*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x +
e)^2)*sinh(f*x + e)^3 + 4*(27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^7 +
21*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + 5*(56*a^3 - 70*a^
2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3
*b^3)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x
+ e) + (27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 28*(8*a^3 + 2*a^2*b
- 13*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 10*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3
*b^3)*cosh(f*x + e)^4 + 6*a^2*b + 3*a*b^2 - 9*b^3 + 12*(8*a^3 + 2*a^2*b - 1
3*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)))/((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*
f*cosh(f*x + e)^12 + 12*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f
*cosh(f*x + e)*sinh(f*x + e)^11 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^
5 + b^6)*f*sinh(f*x + e)^12 + 2*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2
*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^10 + 2*(33*(a^4*b^2 - 4*a^3*b^3 + 6*a
^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*
b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f)*sinh(f*x + e)^10 + (16*a^6 - 64*a^5*b
+ 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^8 +
20*(11*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^3
+ (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(
f*x + e))*sinh(f*x + e)^9 + (495*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5
+ b^6)*f*cosh(f*x + e)^4 + 90*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*
b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 -
60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*sinh(f*x + e)^8 + 4*(8*a^6 - 36
*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x +
e)^6 + 8*(99*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x
+ e)^5 + 30*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6
)*f*cosh(f*x + e)^3 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2
*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231*(a^4*b^2 -
4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^6 + 105*(4*a^5*b - 1
7*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^4 + 7*
(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*
f*cosh(f*x + e)^2 + (8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^
4 - 8*a*b^5 + b^6)*f)*sinh(f*x + e)^6 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 6
0*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^4 + 8*(99*(a^4*b^2
- 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^7 + 63*(4*a^5*b -
17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^5 + 7
*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)
*f*cosh(f*x + e)^3 + 3*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2
*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*(a^4*b^2 - 4*
a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^8 + 420*(4*a^5*b - 17*
a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^6 + 70*(
16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f
*cosh(f*x + e)^4 + 60*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*
b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 -
60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*sinh(f*x + e)^4 + 2*(4*a^5*b -
17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^2 + 4
(55(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^9 +
60*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(
f*x + e)^7 + 14*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 +
4*a*b^5 - b^6)*f*cosh(f*x + e)^5 + 20*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*
a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^3 + (16*a^6 - 64*a^5*
b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e))*
sinh(f*x + e)^3 + 2*(33*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f
*cosh(f*x + e)^10 + 45*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*
a*b^5 - b^6)*f*cosh(f*x + e)^8 + 14*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^
3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^6 + 30*(8*a^6 - 36*a^5*
b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^4
+ 3*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 -

```
b^6)*f*cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 +
8*a*b^5 - b^6)*f)*sinh(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*
b^5 + b^6)*f + 4*(3*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cos
h(f*x + e)^11 + 5*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5
 - b^6)*f*cosh(f*x + e)^9 + 2*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3
+ 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^7 + 6*(8*a^6 - 36*a^5*b + 65*
a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^5 + (16*
a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*co
sh(f*x + e)^3 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 -
 b^6)*f*cosh(f*x + e))*sinh(f*x + e)), -1/6*(3*((2*a*b^2 + 3*b^3)*cosh(f*x
+ e)^12 + 12*(2*a*b^2 + 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (2*a*b^2 +
3*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^10 +
 2*(8*a^2*b + 10*a*b^2 - 3*b^3 + 33*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh
(f*x + e)^10 + 20*(11*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + (8*a^2*b + 10*a*b
^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^9 + (32*a^3 + 48*a^2*b - 2*a*b^2 -
 3*b^3)*cosh(f*x + e)^8 + (495*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + 32*a^3 +
 48*a^2*b - 2*a*b^2 - 3*b^3 + 90*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)
^2)*sinh(f*x + e)^8 + 8*(99*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + 30*(8*a^2*b
 + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3
)*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)
*cosh(f*x + e)^6 + 4*(231*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 105*(8*a^2*b
+ 10*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3
+ 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6
+ 8*(99*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^7 + 63*(8*a^2*b + 10*a*b^2 - 3*b^3)
*cosh(f*x + e)^5 + 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3
+ 3*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 +
 (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + (495*(2*a*b^2 + 3*
b^3)*cosh(f*x + e)^8 + 420*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 7
0*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b
 - 2*a*b^2 - 3*b^3 + 60*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e
)^2)*sinh(f*x + e)^4 + 4*(55*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^9 + 60*(8*a^2*
b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3
*b^3)*cosh(f*x + e)^5 + 20*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x
+ e)^3 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)
^3 + 2*a*b^2 + 3*b^3 + 2*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^2 + 2*(
33*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^10 + 45*(8*a^2*b + 10*a*b^2 - 3*b^3)*cos
h(f*x + e)^8 + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 3
0*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + 8*a^2*b + 10*a*b
^2 - 3*b^3 + 3*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(
f*x + e)^2 + 4*(3*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^11 + 5*(8*a^2*b + 10*a*b^
2 - 3*b^3)*cosh(f*x + e)^9 + 2*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f
*x + e)^7 + 6*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + (32*
a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3
*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-
a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x +
 e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*(3*(2*a^2*b + a*b^2 - 3*b^3)*cosh
(f*x + e)^9 + 27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^8 +
3*(2*a^2*b + a*b^2 - 3*b^3)*sinh(f*x + e)^9 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2
 + 3*b^3)*cosh(f*x + e)^7 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 27*(2*a
^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(9*(2*a^2*b + a
*b^2 - 3*b^3)*cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f
*x + e))*sinh(f*x + e)^6 + 2*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*
x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 56*a^3 - 70*a
^2*b + 17*a*b^2 - 3*b^3 + 42*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^5
+ 70*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + 5*(56*a^3 - 70*
a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(8*a^3 + 2*a^2
*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + 4*(63*(2*a^2*b + a*b^2 - 3*b^3)*co
```

sh(f*x + e)^6 + 35*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + 8
*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 5*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3
)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f
x + e)^7 + 21(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + 5*(56
*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(8*a^3 + 2*a^2*b -
13*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(2*a^2*b + a*b^2 - 3*b
^3)*cosh(f*x + e) + (27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 28*(8*a
^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 10*(56*a^3 - 70*a^2*b +
17*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 6*a^2*b + 3*a*b^2 - 9*b^3 + 12*(8*a^3 +
2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a
*b^5 + b^6)*f*cosh(f*x + e)^12 + 12*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*
b^5 + b^6)*f*cosh(f*x + e)*sinh(f*x + e)^11 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*
b^4 - 4*a*b^5 + b^6)*f*sinh(f*x + e)^12 + 2*(4*a^5*b - 17*a^4*b^2 + 28*a^3*
b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^10 + 2*(33*(a^4*b^2 - 4*a
^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b
^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f)*sinh(f*x + e)^10 + (16*a^6
- 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(
f*x + e)^8 + 20*(11*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cos
h(f*x + e)^3 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 -
b^6)*f*cosh(f*x + e))*sinh(f*x + e)^9 + (495*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b
^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^4 + 90*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b
^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 9
5*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*sinh(f*x + e)^8 + 4
*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*
f*cosh(f*x + e)^6 + 8*(99*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)
*f*cosh(f*x + e)^5 + 30*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8
*a*b^5 - b^6)*f*cosh(f*x + e)^3 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*
b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231
*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^6 + 105*
(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x
+ e)^4 + 7*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a
*b^5 - b^6)*f*cosh(f*x + e)^2 + (8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3
+ 30*a^2*b^4 - 8*a*b^5 + b^6)*f)*sinh(f*x + e)^6 + (16*a^6 - 64*a^5*b + 95
*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^4 + 8*(
99*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^7 + 63
*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*
x + e)^5 + 7*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*
a*b^5 - b^6)*f*cosh(f*x + e)^3 + 3*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*
b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*(
a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^8 + 420*(4
*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x +
e)^6 + 70*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*
b^5 - b^6)*f*cosh(f*x + e)^4 + 60*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b
^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 9
5*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*sinh(f*x + e)^4 + 2
*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*
x + e)^2 + 4*(55*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f
x + e)^9 + 60(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 -
b^6)*f*cosh(f*x + e)^7 + 14*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 +
10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^5 + 20*(8*a^6 - 36*a^5*b + 65*a
^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^3 + (16*a
^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cos
h(f*x + e))*sinh(f*x + e)^3 + 2*(33*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*
b^5 + b^6)*f*cosh(f*x + e)^10 + 45*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*
a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^8 + 14*(16*a^6 - 64*a^5*b + 95*a^4
*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^6 + 30*(8*a
^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cos

```
h(f*x + e)^4 + 3*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4
+ 4*a*b^5 - b^6)*f*cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 2
2*a^2*b^4 + 8*a*b^5 - b^6)*f)*sinh(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^
2*b^4 - 4*a*b^5 + b^6)*f + 4*(3*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5
+ b^6)*f*cosh(f*x + e)^11 + 5*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b
^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^9 + 2*(16*a^6 - 64*a^5*b + 95*a^4*b^2 -
 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^7 + 6*(8*a^6 - 36
*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x +
 e)^5 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5
 - b^6)*f*cosh(f*x + e)^3 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4
 + 8*a*b^5 - b^6)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [B] time = 40.90, size = 2132, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*((3*(a^18*b^3*e^(21*e) - 12*a^17*b^4*e^(21*e) + 65*a^16*b^5*e^(21*e) -
208*a^15*b^6*e^(21*e) + 429*a^14*b^7*e^(21*e) - 572*a^13*b^8*e^(21*e) + 429
*a^12*b^9*e^(21*e) - 429*a^10*b^11*e^(21*e) + 572*a^9*b^12*e^(21*e) - 429*a
^8*b^13*e^(21*e) + 208*a^7*b^14*e^(21*e) - 65*a^6*b^15*e^(21*e) + 12*a^5*b^
16*e^(21*e) - a^4*b^17*e^(21*e))*e^(2*f*x)/(a^20*b^2*e^(16*e) - 16*a^19*b^3
*e^(16*e) + 120*a^18*b^4*e^(16*e) - 560*a^17*b^5*e^(16*e) + 1820*a^16*b^6*e
^(16*e) - 4368*a^15*b^7*e^(16*e) + 8008*a^14*b^8*e^(16*e) - 11440*a^13*b^9*
e^(16*e) + 12870*a^12*b^10*e^(16*e) - 11440*a^11*b^11*e^(16*e) + 8008*a^10*
b^12*e^(16*e) - 4368*a^9*b^13*e^(16*e) + 1820*a^8*b^14*e^(16*e) - 560*a^7*b
^15*e^(16*e) + 120*a^6*b^16*e^(16*e) - 16*a^5*b^17*e^(16*e) + a^4*b^18*e^(1
6*e)) + 2*(8*a^19*b^2*e^(19*e) - 103*a^18*b^3*e^(19*e) + 608*a^17*b^4*e^(19
*e) - 2171*a^16*b^5*e^(19*e) + 5200*a^15*b^6*e^(19*e) - 8723*a^14*b^7*e^(19
*e) + 10296*a^13*b^8*e^(19*e) - 8151*a^12*b^9*e^(19*e) + 3432*a^11*b^10*e^(
19*e) + 715*a^10*b^11*e^(19*e) - 2288*a^9*b^12*e^(19*e) + 1807*a^8*b^13*e^(
19*e) - 832*a^7*b^14*e^(19*e) + 239*a^6*b^15*e^(19*e) - 40*a^5*b^16*e^(19*e
) + 3*a^4*b^17*e^(19*e))/(a^20*b^2*e^(16*e) - 16*a^19*b^3*e^(16*e) + 120*a^
18*b^4*e^(16*e) - 560*a^17*b^5*e^(16*e) + 1820*a^16*b^6*e^(16*e) - 4368*a^1
5*b^7*e^(16*e) + 8008*a^14*b^8*e^(16*e) - 11440*a^13*b^9*e^(16*e) + 12870*a
^12*b^10*e^(16*e) - 11440*a^11*b^11*e^(16*e) + 8008*a^10*b^12*e^(16*e) - 43
68*a^9*b^13*e^(16*e) + 1820*a^8*b^14*e^(16*e) - 560*a^7*b^15*e^(16*e) + 120
*a^6*b^16*e^(16*e) - 16*a^5*b^17*e^(16*e) + a^4*b^18*e^(16*e)))*e^(2*f*x) +
 3*(a^18*b^3*e^(17*e) - 12*a^17*b^4*e^(17*e) + 65*a^16*b^5*e^(17*e) - 208*a
^15*b^6*e^(17*e) + 429*a^14*b^7*e^(17*e) - 572*a^13*b^8*e^(17*e) + 429*a^12
*b^9*e^(17*e) - 429*a^10*b^11*e^(17*e) + 572*a^9*b^12*e^(17*e) - 429*a^8*b^
13*e^(17*e) + 208*a^7*b^14*e^(17*e) - 65*a^6*b^15*e^(17*e) + 12*a^5*b^16*e^
(17*e) - a^4*b^17*e^(17*e))/(a^20*b^2*e^(16*e) - 16*a^19*b^3*e^(16*e) + 120
*a^18*b^4*e^(16*e) - 560*a^17*b^5*e^(16*e) + 1820*a^16*b^6*e^(16*e) - 4368*
a^15*b^7*e^(16*e) + 8008*a^14*b^8*e^(16*e) - 11440*a^13*b^9*e^(16*e) + 1287
0*a^12*b^10*e^(16*e) - 11440*a^11*b^11*e^(16*e) + 8008*a^10*b^12*e^(16*e) -
 4368*a^9*b^13*e^(16*e) + 1820*a^8*b^14*e^(16*e) - 560*a^7*b^15*e^(16*e) +
120*a^6*b^16*e^(16*e) - 16*a^5*b^17*e^(16*e) + a^4*b^18*e^(16*e)))*e^(f*x)/
((b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3/2)*
f) + ((3*a*e^e + 2*b*e^e)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sq
rt(a - b))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sqrt(a - b)) - 2*(a*e^e + b*e^e
)*arctan(-(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x
+ 2*e) - 2*b*e^(2*f*x + 2*e) + b))/sqrt(-b))/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*sqrt(-b)) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*e^e + 7*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))^2*a*sqrt(b)*e^e - 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
```

```
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(3/2)*e^e + 12*(sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(
2*f*x + 2*e) + b))*a^2*e^e - 17*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b*e^e + 8*(sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(
2*f*x + 2*e) + b))*b^2*e^e - 4*a^2*sqrt(b)*e^e + 9*a*b^(3/2)*e^e - 4*b^(5/2
)*e^e)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^2))/f^2
```

maple [C] time = 0.39, size = 213, normalized size = 1.31

$$\frac{\text{`}int/indef0\text{`}\left(-\frac{\left(\sinh^3\left(fx+e\right)\right)\left(b^2\left(\sinh^4\left(fx+e\right)\right)+2ab\left(\sinh^2\left(fx+e\right)\right)+a^2\right)\left(\cosh\right.}{\left(-b^4\left(\cosh^{14}\left(fx+e\right)\right)+\left(-4a\,b^3+4b^4\right)\left(\cosh^{12}\left(fx+e\right)\right)+\left(-6a^2b^2+12a\,b^3-6b^4\right)\left(\cosh^{10}\left(fx+e\right)\right)+\left(-4a^3b+12a^2b^2-12a\,b^3+4b^4\right)\left(\cosh^8\right.}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)^3*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)*cos
h(f*x+e)^2/(-b^4*cosh(f*x+e)^14+(-4*a*b^3+4*b^4)*cosh(f*x+e)^12+(-6*a^2*b^2
+12*a*b^3-6*b^4)*cosh(f*x+e)^10+(-4*a^3*b+12*a^2*b^2-12*a*b^3+4*b^4)*cosh(f
*x+e)^8+(-a^4+4*a^3*b-6*a^2*b^2+4*a*b^3-b^4)*cosh(f*x+e)^6)/(a+b*sinh(f*x+e
)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh\left(e+f\,x\right)^3}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tanh(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.503 $\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=99

$$\frac{1}{f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{1}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/f+1/3/(a-b)/f/(
a+b*sinh(f*x+e)^2)^(3/2)+1/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{1}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + 1/(
3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + 1/((a - b)^2*f*Sqrt[a + b*Sinh
[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2(a-b)f}$$

$$= \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{}{(a-b)^2f\sqrt{a}}$$

Mathematica [C] time = 0.09, size = 60, normalized size = 0.61

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\cosh^2(e+fx)}{a-b}+1\right)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Cosh[e + f*x]^2)/(a - b)]/(3*(a - b
)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.70, size = 4200, normalized size = 42.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cosh(f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*si
nh(f*x + e)^8 + 4*(2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2
+ 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)
*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4
 + 2*(35*b^2*cosh(f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8
*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)
*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
 4*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b
^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b
^2)*sinh(f*x + e)^2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f
*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*
x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*
(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b)*s
```

qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x +
 e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)
/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(
3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x +
e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(3*(a*b - b^2)*cosh(f
x + e)^5 + 15(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + 3*(a*b - b^2)*si
nh(f*x + e)^5 + 2*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^3 + 2*(15*(a*b - b
^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^3 + 6*(5*(a*b -
 b^2)*cosh(f*x + e)^3 + (8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e))*sinh(f*x +
e)^2 + 3*(a*b - b^2)*cosh(f*x + e) + 3*(5*(a*b - b^2)*cosh(f*x + e)^4 + 2*(
8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 + a*b - b^2)*sinh(f*x + e))*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4
- b^5)*f*cosh(f*x + e)^8 + 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f
*x + e)*sinh(f*x + e)^7 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*sinh(f*x
+ e)^8 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e
)^6 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^2 + (2*a^4
*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^5 -
 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^4 +
 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^3 + 3*(2*a^4*b
- 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 +
 2*(35*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^4 + 30*(2*a^4*
b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^2 + (8*a^5 - 32*
a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f)*sinh(f*x + e)^4 + 4*
(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^2 + 8*(7*
(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^5 + 10*(2*a^4*b - 7*a
^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (8*a^5 - 32*a^4*b +
 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e))*sinh(f*x + e)
^3 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^6 + 15*(2*a
^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^4 + 3*(8*a^5
- 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^2
+ (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(f*x + e)^2 + (a
^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f + 8*((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 -
 b^5)*f*cosh(f*x + e)^7 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^
5)*f*cosh(f*x + e)^5 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b
^4 - 3*b^5)*f*cosh(f*x + e)^3 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4
+ b^5)*f*cosh(f*x + e))*sinh(f*x + e)), -1/3*(3*(b^2*cosh(f*x + e)^8 + 8*b^
2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*sinh(f*x + e)^8 + 4*(2*a*b - b^2)*cos
h(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^6 + 8*
(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2
*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*b^2*cosh(f*x + e)^4 + 30*(
2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(
7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 8*a*b +
 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a*b - b^2)*cosh(f*x + e)^2 +
4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 8*
a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 8*(b^2*
cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2)
*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)
*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*(3*(a*
b - b^2)*cosh(f*x + e)^5 + 15*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + 3
*(a*b - b^2)*sinh(f*x + e)^5 + 2*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^3 +
 2*(15*(a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^
3 + 6*(5*(a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 13*a*b + 5*b^2)*cosh(f*x +
e))*sinh(f*x + e)^2 + 3*(a*b - b^2)*cosh(f*x + e) + 3*(5*(a*b - b^2)*cosh(f
x + e)^4 + 2(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 + a*b - b^2)*sinh(f*
x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +

```
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b^2 - 3*a^2
*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^8 + 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4
- b^5)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b
^5)*f*sinh(f*x + e)^8 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)
*f*cosh(f*x + e)^6 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x
+ e)^2 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(f*x + e)
^6 + 2*(8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*co
sh(f*x + e)^4 + 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^
3 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e))*si
nh(f*x + e)^5 + 2*(35*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)
^4 + 30*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^2
 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f)*sinh(
f*x + e)^4 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x
 + e)^2 + 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^5 + 10
*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (8*a
^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)
)*sinh(f*x + e)^3 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x +
 e)^6 + 15*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e
)^4 + 3*(8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*c
osh(f*x + e)^2 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(
f*x + e)^2 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f + 8*((a^3*b^2 - 3*a^2*
b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^7 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3
 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^5 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a
^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^3 + (2*a^4*b - 7*a^3*b^2 + 9*a^2
*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 2.85Error: Bad Argument Type
```

maple [C] time = 0.32, size = 173, normalized size = 1.75

$$\frac{\text{`}int/indef0\text{`}\left(-\frac{\sinh\left(fx+e\right)\left(b^{2}\left(\sinh^{4}\left(fx+e\right)\right)+2ab\left(\sinh^{2}\left(fx+e\right)\right)+a^{2}\right)}{\left(-b^{4}\left(\sinh^{10}\left(fx+e\right)\right)+\left(-4a\,b^{3}-b^{4}\right)\left(\sinh^{8}\left(fx+e\right)\right)+\left(-6a^{2}b^{2}-4a\,b^{3}\right)\left(\sinh^{6}\left(fx+e\right)\right)+\left(-4a^{3}b-6a^{2}b^{2}\right)\left(\sinh^{4}\left(fx+e\right)\right)+\left(-a^{4}-4a^{3}b\right)\left(\sinh\right.\right.}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(-b^4
*sinh(f*x+e)^10+(-4*a*b^3-b^4)*sinh(f*x+e)^8+(-6*a^2*b^2-4*a*b^3)*sinh(f*x+
e)^6+(-4*a^3*b-6*a^2*b^2)*sinh(f*x+e)^4+(-a^4-4*a^3*b)*sinh(f*x+e)^2-a^4)/(
a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh\left(e+f\,x\right)}{{\left(b\,\sinh\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(tanh(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Integral(tanh(e + f*x)/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.504 $\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

```
[Out] -arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sinh(f*x
+e)^2)^(3/2)+1/a^2/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
 b*Sinh[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2af}$$

$$= \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{}$$

$$= \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sinh}}$$

Mathematica [C] time = 0.07, size = 49, normalized size = 0.59

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sinh[e + f*x]^2)/a]/(3*a*f*(a + b*S
inh[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.62, size = 3084, normalized size = 37.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cosh(f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*si
nh(f*x + e)^8 + 4*(2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2
+ 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)
*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4
 + 2*(35*b^2*cosh(f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8
*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)
*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
 4*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b
^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b
^2)*sinh(f*x + e)^2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f
*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*
x + e))*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*c
osh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(
```

```
f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*
cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e
)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e
)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*
x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(3*a*b*cosh(f*x + e)^5 + 15*a*b*cos
h(f*x + e)*sinh(f*x + e)^4 + 3*a*b*sinh(f*x + e)^5 + 2*(8*a^2 - 3*a*b)*cosh
(f*x + e)^3 + 2*(15*a*b*cosh(f*x + e)^2 + 8*a^2 - 3*a*b)*sinh(f*x + e)^3 +
3*a*b*cosh(f*x + e) + 6*(5*a*b*cosh(f*x + e)^3 + (8*a^2 - 3*a*b)*cosh(f*x +
 e))*sinh(f*x + e)^2 + 3*(5*a*b*cosh(f*x + e)^4 + 2*(8*a^2 - 3*a*b)*cosh(f*
x + e)^2 + a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/(a^3*b^2*f*cosh(f*x + e)^8 + 8*a^3*b^2*f*cosh(f*x + e)*sinh(f*x + e)^
7 + a^3*b^2*f*sinh(f*x + e)^8 + 4*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^6 + 4
*(7*a^3*b^2*f*cosh(f*x + e)^2 + (2*a^4*b - a^3*b^2)*f)*sinh(f*x + e)^6 + a^
3*b^2*f + 2*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^4 + 8*(7*a^3*b^2*
f*cosh(f*x + e)^3 + 3*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5
+ 2*(35*a^3*b^2*f*cosh(f*x + e)^4 + 30*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^
2 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f)*sinh(f*x + e)^4 + 4*(2*a^4*b - a^3*b^2
)*f*cosh(f*x + e)^2 + 8*(7*a^3*b^2*f*cosh(f*x + e)^5 + 10*(2*a^4*b - a^3*b^
2)*f*cosh(f*x + e)^3 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e))*sinh(
f*x + e)^3 + 4*(7*a^3*b^2*f*cosh(f*x + e)^6 + 15*(2*a^4*b - a^3*b^2)*f*cosh
(f*x + e)^4 + 3*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^2 + (2*a^4*b
- a^3*b^2)*f)*sinh(f*x + e)^2 + 8*(a^3*b^2*f*cosh(f*x + e)^7 + 3*(2*a^4*b -
 a^3*b^2)*f*cosh(f*x + e)^5 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)
^3 + (2*a^4*b - a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/3*(3*(b^2*cosh(
f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*sinh(f*x + e)^8 + 4*
(2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sin
h(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e))*si
nh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*b^2*cosh(
f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh
(f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)*cosh(f*x + e)^3 +
 (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a*b - b^2)*c
osh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b^2)*cosh(f*x + e)^
4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^
2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f*x + e)^5 + (8*a^2
 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt(2)*(3*a*b*cos
h(f*x + e)^5 + 15*a*b*cosh(f*x + e)*sinh(f*x + e)^4 + 3*a*b*sinh(f*x + e)^5
 + 2*(8*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(15*a*b*cosh(f*x + e)^2 + 8*a^2 -
3*a*b)*sinh(f*x + e)^3 + 3*a*b*cosh(f*x + e) + 6*(5*a*b*cosh(f*x + e)^3 + (
8*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(5*a*b*cosh(f*x + e)^4 +
2*(8*a^2 - 3*a*b)*cosh(f*x + e)^2 + a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2)))/(a^3*b^2*f*cosh(f*x + e)^8 + 8*a^3*b^2*f*cos
h(f*x + e)*sinh(f*x + e)^7 + a^3*b^2*f*sinh(f*x + e)^8 + 4*(2*a^4*b - a^3*b
^2)*f*cosh(f*x + e)^6 + 4*(7*a^3*b^2*f*cosh(f*x + e)^2 + (2*a^4*b - a^3*b^2
)*f)*sinh(f*x + e)^6 + a^3*b^2*f + 2*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f
*x + e)^4 + 8*(7*a^3*b^2*f*cosh(f*x + e)^3 + 3*(2*a^4*b - a^3*b^2)*f*cosh(f
*x + e))*sinh(f*x + e)^5 + 2*(35*a^3*b^2*f*cosh(f*x + e)^4 + 30*(2*a^4*b -
a^3*b^2)*f*cosh(f*x + e)^2 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f)*sinh(f*x + e)
^4 + 4*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^2 + 8*(7*a^3*b^2*f*cosh(f*x + e)
^5 + 10*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^3 + (8*a^5 - 8*a^4*b + 3*a^3*b^
2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*b^2*f*cosh(f*x + e)^6 + 15*(
2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^4 + 3*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*co
sh(f*x + e)^2 + (2*a^4*b - a^3*b^2)*f)*sinh(f*x + e)^2 + 8*(a^3*b^2*f*cosh(
f*x + e)^7 + 3*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^5 + (8*a^5 - 8*a^4*b + 3
```

```
*a^3*b^2)*f*cosh(f*x + e)^3 + (2*a^4*b - a^3*b^2)*f*cosh(f*x + e))*sinh(f*x
 + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 0.73Error: Bad Argument Type
```

maple [C] time = 0.17, size = 65, normalized size = 0.78

$$\frac{\text{`}int/indef0\text{`}\left(\frac{1}{\left(b^2\left(\sinh^4\left(fx+e\right)\right)+2ab\left(\sinh^2\left(fx+e\right)\right)+a^2\right)\sinh\left(fx+e\right)\sqrt{a+b\left(\sinh^2\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(1/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh(f*x+e)/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth\left(e+f\,x\right)}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(coth(e + f*x)/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(coth(e + f*x)/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.505 $\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}+\frac{2a-5b}{2a^3f\sqrt{a+b\sinh^2(e+fx)}}+\frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\operatorname{csch}^2(e\ldots}{2af\left(a+b\sinh\ldots\right.}$$

```
[Out] -1/2*(2*a-5*b)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2)/f+1/6*(2*
a-5*b)/a^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/2*csch(f*x+e)^2/a/f/(a+b*sinh(f*x+
e)^2)^(3/2)+1/2*(2*a-5*b)/a^3/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a-5b}{2a^3f\sqrt{a+b\sinh^2(e+fx)}}+\frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{\operatorname{csch}^2(e+\ldots}{2af\left(a+b\sinh^2\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -((2*a - 5*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) +
 (2*a - 5*b)/(6*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - Csch[e + f*x]^2/(2*a
*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*a - 5*b)/(2*a^3*f*Sqrt[a + b*Sinh[e
+ f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x}{x^2(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(2a-5b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sinh^2(e+\right.}{4af}$$

$$= \frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\operatorname{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(2a-5b)\,\text{Sub}}{}$$

$$= \frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\operatorname{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2a}{2a^3f\sqrt{a+b}}$$

$$= \frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\operatorname{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2a}{2a^3f\sqrt{a+b}}$$

$$= -\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} + \frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{}{2af\left(\right.}$$

Mathematica [C] time = 0.31, size = 69, normalized size = 0.48

$$-\frac{(5b-2a)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)+3a\operatorname{csch}^2(e+fx)}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*(3*a*Csch[e + f*x]^2 + (-2*a + 5*b)*Hypergeometric2F1[-3/2, 1, -1/2, 1
 + (b*Sinh[e + f*x]^2)/a])/(a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.81, size = 7594, normalized size = 53.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a*b^2 - 5*b^3)*cosh(f*x + e)^12 + 12*(2*a*b^2 - 5*b^3)*cosh(f
*x + e)*sinh(f*x + e)^11 + (2*a*b^2 - 5*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b
- 26*a*b^2 + 15*b^3)*cosh(f*x + e)^10 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3 + 33
*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(2*a*b^2 - 5*
b^3)*cosh(f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f*
x + e)^9 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^8 + (495
*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^4 + 32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^
3 + 90*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(
99*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^5 + 30*(8*a^2*b - 26*a*b^2 + 15*b^3)*cos
h(f*x + e)^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e))*sin
h(f*x + e)^7 - 4*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e)^6 +
4*(231*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^6 + 105*(8*a^2*b - 26*a*b^2 + 15*b^3
)*cosh(f*x + e)^4 - 16*a^3 + 64*a^2*b - 70*a*b^2 + 25*b^3 + 7*(32*a^3 - 144
*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(2*a*
b^2 - 5*b^3)*cosh(f*x + e)^7 + 63*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x +
e)^5 + 7*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^3 - 3*(16*
a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (32*a^
3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + (495*(2*a*b^2 - 5*b^3
)*cosh(f*x + e)^8 + 420*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^6 + 70*
(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + 32*a^3 - 144*a^
2*b + 190*a*b^2 - 75*b^3 - 60*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(
f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^9 + 60*
(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^7 + 14*(32*a^3 - 144*a^2*b + 19
0*a*b^2 - 75*b^3)*cosh(f*x + e)^5 - 20*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b
^3)*cosh(f*x + e)^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x +
e))*sinh(f*x + e)^3 + 2*a*b^2 - 5*b^3 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3)*cos
h(f*x + e)^2 + 2*(33*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^10 + 45*(8*a^2*b - 26*
a*b^2 + 15*b^3)*cosh(f*x + e)^8 + 14*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b
^3)*cosh(f*x + e)^6 - 30*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x +
 e)^4 + 8*a^2*b - 26*a*b^2 + 15*b^3 + 3*(32*a^3 - 144*a^2*b + 190*a*b^2 - 7
5*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(2*a*b^2 - 5*b^3)*cosh(f*x +
 e)^11 + 5*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^9 + 2*(32*a^3 - 144*
a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^7 - 6*(16*a^3 - 64*a^2*b + 70*a*b
^2 - 25*b^3)*cosh(f*x + e)^5 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*co
sh(f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f*x + e))
*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sin
h(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a -
 b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*
x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a
 - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*
sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)
^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e)
+ 1)) - 4*sqrt(2)*(3*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^9 + 27*(2*a^2*b - 5*
a*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + 3*(2*a^2*b - 5*a*b^2)*sinh(f*x + e)^
9 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 + 4*(8*a^3 - 26*a^2*b +
 15*a*b^2 + 27*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(9
*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^3 + (8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f
*x + e))*sinh(f*x + e)^6 - 2*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^5
 + 2*(189*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^4 - 56*a^3 + 98*a^2*b - 45*a*b^
2 + 42*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(
189*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^5 + 70*(8*a^3 - 26*a^2*b + 15*a*b^2)*
cosh(f*x + e)^3 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e))*sinh(f*x
+ e)^4 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 4*(63*(2*a^2*b -
 5*a*b^2)*cosh(f*x + e)^6 + 35*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^
4 + 8*a^3 - 26*a^2*b + 15*a*b^2 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x
 + e)^2)*sinh(f*x + e)^3 + 4*(27*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^7 + 21*(
8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^5 - 5*(56*a^3 - 98*a^2*b + 45*a*
b^2)*cosh(f*x + e)^3 + 3*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(
```

```
f*x + e)^2 + 3*(2*a^2*b - 5*a*b^2)*cosh(f*x + e) + (27*(2*a^2*b - 5*a*b^2)*
cosh(f*x + e)^8 + 28*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^6 - 10*(56
*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^4 + 6*a^2*b - 15*a*b^2 + 12*(8*a^
3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/(a^4*b^2*f*cosh(f*x + e)^12 + 12*a^4*b^2*f*
cosh(f*x + e)*sinh(f*x + e)^11 + a^4*b^2*f*sinh(f*x + e)^12 + 2*(4*a^5*b -
3*a^4*b^2)*f*cosh(f*x + e)^10 + 2*(33*a^4*b^2*f*cosh(f*x + e)^2 + (4*a^5*b
- 3*a^4*b^2)*f)*sinh(f*x + e)^10 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(
f*x + e)^8 + 20*(11*a^4*b^2*f*cosh(f*x + e)^3 + (4*a^5*b - 3*a^4*b^2)*f*cos
h(f*x + e))*sinh(f*x + e)^9 + (495*a^4*b^2*f*cosh(f*x + e)^4 + 90*(4*a^5*b
- 3*a^4*b^2)*f*cosh(f*x + e)^2 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f)*sinh(f
*x + e)^8 - 4*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e)^6 + 8*(99*a^4*
b^2*f*cosh(f*x + e)^5 + 30*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^3 + (16*a^
6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^7 + a^4*b^2*f + 4
*(231*a^4*b^2*f*cosh(f*x + e)^6 + 105*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)
^4 + 7*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^2 - (8*a^6 - 12*a^5
*b + 5*a^4*b^2)*f)*sinh(f*x + e)^6 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cos
h(f*x + e)^4 + 8*(99*a^4*b^2*f*cosh(f*x + e)^7 + 63*(4*a^5*b - 3*a^4*b^2)*f
*cosh(f*x + e)^5 + 7*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^3 - 3
*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*a^4
*b^2*f*cosh(f*x + e)^8 + 420*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^6 + 70*(
16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^4 - 60*(8*a^6 - 12*a^5*b +
5*a^4*b^2)*f*cosh(f*x + e)^2 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f)*sinh(f*x
 + e)^4 + 2*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^2 + 4*(55*a^4*b^2*f*cosh(
f*x + e)^9 + 60*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^7 + 14*(16*a^6 - 32*a
^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^5 - 20*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*
cosh(f*x + e)^3 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e))*sinh(f*
x + e)^3 + 2*(33*a^4*b^2*f*cosh(f*x + e)^10 + 45*(4*a^5*b - 3*a^4*b^2)*f*co
sh(f*x + e)^8 + 14*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^6 - 30*
(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e)^4 + 3*(16*a^6 - 32*a^5*b + 1
5*a^4*b^2)*f*cosh(f*x + e)^2 + (4*a^5*b - 3*a^4*b^2)*f)*sinh(f*x + e)^2 + 4
*(3*a^4*b^2*f*cosh(f*x + e)^11 + 5*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^9
+ 2*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^7 - 6*(8*a^6 - 12*a^5*
b + 5*a^4*b^2)*f*cosh(f*x + e)^5 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(
f*x + e)^3 + (4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/6*(3*
((2*a*b^2 - 5*b^3)*cosh(f*x + e)^12 + 12*(2*a*b^2 - 5*b^3)*cosh(f*x + e)*si
nh(f*x + e)^11 + (2*a*b^2 - 5*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b - 26*a*b^2
 + 15*b^3)*cosh(f*x + e)^10 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3 + 33*(2*a*b^2
- 5*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(2*a*b^2 - 5*b^3)*cosh(
f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f*x + e)^9 +
 (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^8 + (495*(2*a*b^2
- 5*b^3)*cosh(f*x + e)^4 + 32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3 + 90*(8*
a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(2*a*b^
2 - 5*b^3)*cosh(f*x + e)^5 + 30*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)
^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e))*sinh(f*x + e)
^7 - 4*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e)^6 + 4*(231*(2*
a*b^2 - 5*b^3)*cosh(f*x + e)^6 + 105*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x
 + e)^4 - 16*a^3 + 64*a^2*b - 70*a*b^2 + 25*b^3 + 7*(32*a^3 - 144*a^2*b + 1
90*a*b^2 - 75*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(2*a*b^2 - 5*b^
3)*cosh(f*x + e)^7 + 63*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^5 + 7*(
32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^3 - 3*(16*a^3 - 64*a
^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (32*a^3 - 144*a^
2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + (495*(2*a*b^2 - 5*b^3)*cosh(f*x
 + e)^8 + 420*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^6 + 70*(32*a^3 -
144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + 32*a^3 - 144*a^2*b + 190*
a*b^2 - 75*b^3 - 60*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^4 + 4*(55*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^9 + 60*(8*a^2*b -
 26*a*b^2 + 15*b^3)*cosh(f*x + e)^7 + 14*(32*a^3 - 144*a^2*b + 190*a*b^2 -
```

75*b^3)*cosh(f*x + e)^5 - 20*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f
*x + e)^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e))*sinh(f
*x + e)^3 + 2*a*b^2 - 5*b^3 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)
^2 + 2*(33*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^10 + 45*(8*a^2*b - 26*a*b^2 + 15
*b^3)*cosh(f*x + e)^8 + 14*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f
x + e)^6 - 30(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e)^4 + 8*
a^2*b - 26*a*b^2 + 15*b^3 + 3*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cos
h(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^11 + 5
*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^9 + 2*(32*a^3 - 144*a^2*b + 19
0*a*b^2 - 75*b^3)*cosh(f*x + e)^7 - 6*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^
3)*cosh(f*x + e)^5 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e
)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*
arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2
*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)
)/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt(2)*(3*(2*a^2*b - 5*a*b^2)*c
osh(f*x + e)^9 + 27*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + 3*(
2*a^2*b - 5*a*b^2)*sinh(f*x + e)^9 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f
x + e)^7 + 4(8*a^3 - 26*a^2*b + 15*a*b^2 + 27*(2*a^2*b - 5*a*b^2)*cosh(f*
x + e)^2)*sinh(f*x + e)^7 + 28*(9*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^3 + (8*
a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 - 2*(56*a^3 - 98*
a^2*b + 45*a*b^2)*cosh(f*x + e)^5 + 2*(189*(2*a^2*b - 5*a*b^2)*cosh(f*x + e
)^4 - 56*a^3 + 98*a^2*b - 45*a*b^2 + 42*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(
f*x + e)^2)*sinh(f*x + e)^5 + 2*(189*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^5 +
70*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 - 5*(56*a^3 - 98*a^2*b + 4
5*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*c
osh(f*x + e)^3 + 4*(63*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^6 + 35*(8*a^3 - 26
*a^2*b + 15*a*b^2)*cosh(f*x + e)^4 + 8*a^3 - 26*a^2*b + 15*a*b^2 - 5*(56*a^
3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(27*(2*a^2*b
- 5*a*b^2)*cosh(f*x + e)^7 + 21*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)
^5 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^3 + 3*(8*a^3 - 26*a^2*b
 + 15*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(2*a^2*b - 5*a*b^2)*cosh(f*
x + e) + (27*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^8 + 28*(8*a^3 - 26*a^2*b + 1
5*a*b^2)*cosh(f*x + e)^6 - 10*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^
4 + 6*a^2*b - 15*a*b^2 + 12*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*
sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^4*b^2*f
*cosh(f*x + e)^12 + 12*a^4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^11 + a^4*b^2*f
*sinh(f*x + e)^12 + 2*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^10 + 2*(33*a^4*
b^2*f*cosh(f*x + e)^2 + (4*a^5*b - 3*a^4*b^2)*f)*sinh(f*x + e)^10 + (16*a^6
 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^8 + 20*(11*a^4*b^2*f*cosh(f*x + e
)^3 + (4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^9 + (495*a^4*b^2
*f*cosh(f*x + e)^4 + 90*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^2 + (16*a^6 -
 32*a^5*b + 15*a^4*b^2)*f)*sinh(f*x + e)^8 - 4*(8*a^6 - 12*a^5*b + 5*a^4*b^
2)*f*cosh(f*x + e)^6 + 8*(99*a^4*b^2*f*cosh(f*x + e)^5 + 30*(4*a^5*b - 3*a^
4*b^2)*f*cosh(f*x + e)^3 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)
)*sinh(f*x + e)^7 + a^4*b^2*f + 4*(231*a^4*b^2*f*cosh(f*x + e)^6 + 105*(4*a
^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^4 + 7*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*
cosh(f*x + e)^2 - (8*a^6 - 12*a^5*b + 5*a^4*b^2)*f)*sinh(f*x + e)^6 + (16*a
^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^4 + 8*(99*a^4*b^2*f*cosh(f*x +
e)^7 + 63*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^5 + 7*(16*a^6 - 32*a^5*b +
15*a^4*b^2)*f*cosh(f*x + e)^3 - 3*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x
 + e))*sinh(f*x + e)^5 + (495*a^4*b^2*f*cosh(f*x + e)^8 + 420*(4*a^5*b - 3*
a^4*b^2)*f*cosh(f*x + e)^6 + 70*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x
 + e)^4 - 60*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e)^2 + (16*a^6 - 3
2*a^5*b + 15*a^4*b^2)*f)*sinh(f*x + e)^4 + 2*(4*a^5*b - 3*a^4*b^2)*f*cosh(f
x + e)^2 + 4(55*a^4*b^2*f*cosh(f*x + e)^9 + 60*(4*a^5*b - 3*a^4*b^2)*f*co
sh(f*x + e)^7 + 14*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^5 - 20*
(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e)^3 + (16*a^6 - 32*a^5*b + 15*
a^4*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 2*(33*a^4*b^2*f*cosh(f*x + e)^1

```
0 + 45*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^8 + 14*(16*a^6 - 32*a^5*b + 15
*a^4*b^2)*f*cosh(f*x + e)^6 - 30*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x
+ e)^4 + 3*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^2 + (4*a^5*b -
3*a^4*b^2)*f)*sinh(f*x + e)^2 + 4*(3*a^4*b^2*f*cosh(f*x + e)^11 + 5*(4*a^5*
b - 3*a^4*b^2)*f*cosh(f*x + e)^9 + 2*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cos
h(f*x + e)^7 - 6*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e)^5 + (16*a^6
 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^3 + (4*a^5*b - 3*a^4*b^2)*f*cosh(
f*x + e))*sinh(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 2.74Error: Bad Argument Type
```

maple [C] time = 0.22, size = 73, normalized size = 0.51

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\cosh^{2}\left(fx+e\right)}{\left(b^{2}\left(\sinh^{4}\left(fx+e\right)\right)+2ab\left(\sinh^{2}\left(fx+e\right)\right)+a^{2}\right)\sinh\left(fx+e\right)^{3}\sqrt{a+b\left(\sinh^{2}\left(fx+e\right)\right)}},\sinh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh
(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth\left(e+f\,x\right)^{3}}{{\left(b\,\sinh\left(e+f\,x\right)^{2}+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(coth(e + f*x)^3/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Integral(coth(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.506 $\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=208

$$-\frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2-40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{8a^2-40ab+35b^2}{8a^4f\sqrt{a+b\sinh^2(e+fx)}}+$$

```
[Out] -1/8*(8*a^2-40*a*b+35*b^2)*arctanh((a+b*sinh(f*x+e)^2)^(1/2)/a^(1/2))/a^(9/
2)/f+1/24*(8*a^2-40*a*b+35*b^2)/a^3/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/8*(8*a-7*
b)*csch(f*x+e)^2/a^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/4*csch(f*x+e)^4/a/f/(a+b
*sinh(f*x+e)^2)^(3/2)+1/8*(8*a^2-40*a*b+35*b^2)/a^4/f/(a+b*sinh(f*x+e)^2)^(
1/2)
```

Rubi [A] time = 0.22, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2-40ab+35b^2}{8a^4f\sqrt{a+b\sinh^2(e+fx)}}+\frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2-40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}-$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -((8*a^2 - 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(
8*a^(9/2)*f) + (8*a^2 - 40*a*b + 35*b^2)/(24*a^3*f*(a + b*Sinh[e + f*x]^2)^
(3/2)) - ((8*a - 7*b)*Csch[e + f*x]^2)/(8*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - Csch[e + f*x]^4/(4*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (8*a^2 - 40*a
*b + 35*b^2)/(8*a^4*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^4(e+fx)}{4af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-7b)+2ax}{x^2(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{4af}$$

$$= -\frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^4(e+fx)}{4af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(8a^2-40ab+\right.}{}$$

$$= \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^4(}{4af\left(a+b\sin\right.}$$

$$= \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^4(}{4af\left(a+b\sin\right.}$$

$$= \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^4(}{4af\left(a+b\sin\right.}$$

$$= -\frac{\left(8a^2-40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/}}$$

Mathematica [C] time = 0.43, size = 117, normalized size = 0.56

$$-\frac{\operatorname{csch}^2(e+fx)\left(\left(-8a^2+40ab-35b^2\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)+3a\operatorname{csch}^2(e+fx)\left(2a\operatorname{csch}^2(e+fx)+\right.}{24a^3f\sqrt{a+b\sinh^2(e+fx)}\left(a\operatorname{csch}^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/24*(Csch[e + f*x]^2*(3*a*Csch[e + f*x]^2*(8*a - 7*b + 2*a*Csch[e + f*x]^
2) + (-8*a^2 + 40*a*b - 35*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sin
h[e + f*x]^2)/a]))/(a^3*f*(b + a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^
2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Evaluation time: 9.86Error: Bad Argument Type
```

maple [C] time = 0.26, size = 73, normalized size = 0.35

$$\frac{\text{`}int/indef0\text{`}\left(\frac{\cosh^4(fx+e)}{\left(b^2\left(\sinh^4(fx+e)\right)+2ab\left(\sinh^2(fx+e)\right)+a^2\right)\sinh(fx+e)^5\sqrt{a+b\left(\sinh^2(fx+e)\right)}},\sinh(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^4/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh
(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\coth(fx+e)^5}{\left(b\sinh(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^5/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

3.507 $\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=333

$$-\frac{2(2a+b)\tanh(e+fx)}{3f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\sinh(e+fx)\cosh(e+fx)}{3f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\tanh(e+fx)\text{sech}^2(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/}}$$

```
[Out] -1/3*b*(5*a+3*b)*cosh(f*x+e)*sinh(f*x+e)/(a-b)^3/f/(a+b*sinh(f*x+e)^2)^(3/2
)-8/3*(a+b)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/
a)^(1/2)*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),
(1-a/b)^(1/2))*a^(1/2)*b^(1/2)/(a-b)^4/f/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^
2))^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(3*a+b)*(a+3*b)*(1/(1+sinh(f*x+e)^2
))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1
/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^4/f/(sech(
f*x+e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)-2/3*(2*a+b)*tanh(f*x+e)/(a-b)^2/f/(a+
b*sinh(f*x+e)^2)^(3/2)+1/3*sech(f*x+e)^2*tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+
e)^2)^(3/2)
```

Rubi [A] time = 0.40, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3196, 470, 527, 525, 418, 411}

$$-\frac{2(2a+b)\tanh(e+fx)}{3f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\sinh(e+fx)\cosh(e+fx)}{3f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\tanh(e+fx)\text{sech}^2(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*(5*a + 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^3*f*(a + b*Sinh[e +
 f*x]^2)^(3/2)) - (8*Sqrt[a]*Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan
[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^4*f*Sqrt[(a*Cosh[e
+ f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + b
)*(a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(3*a*(a - b)^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) - (2*(2*a + b)*Tanh[e + f*x])/(3*(a - b)^2*f*(a + b*Sinh[e +
 f*x]^2)^(3/2)) + (Sech[e + f*x]^2*Tanh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[
e + f*x]^2)^(3/2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx,x,\sinh(e+fx)\right.}{f}$$

$$= \frac{\text{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\right.}{3(a-}$$

$$= -\frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - $$

$$= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

$$= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{8\sqrt{a}\sqrt{b}\,(a+b)\cosh(e+fx)E\left(\tan\right.}{3(a-b)^4f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a}}$$

Mathematica [C] time = 3.46, size = 252, normalized size = 0.76

$$-\frac{i\left(2ab\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\left(\left(-5a^2+2ab+3b^2\right)F\left(i(e+fx)\middle|\frac{b}{a}\right)+8a(a+b)E\left(i(e+fx)\middle|\frac{b}{a}\right)\right)-i\sqrt{2}\,b\left(2ab(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-1/6*I)*(2*a*b*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*(8*a*(a + b)*Ell
ipticE[I*(e + f*x), b/a] + (-5*a^2 + 2*a*b + 3*b^2)*EllipticF[I*(e + f*x),
b/a]) - I*Sqrt[2]*b*(2*a*(a - b)*b*Sinh[2*(e + f*x)] + 4*b*(a + b)*(2*a - b
 + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)] + 4*(a + b)*(2*a - b + b*Cosh[2*(
e + f*x)])^2*Tanh[e + f*x] - (a - b)*(2*a - b + b*Cosh[2*(e + f*x)])^2*Sech
[e + f*x]^2*Tanh[e + f*x])))/((a - b)^4*b*f*(2*a - b + b*Cosh[2*(e + f*x)])
^(3/2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\tanh\left(fx+e\right)^4}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 7.71Error: Bad Argument Type
```

maple [A] time = 0.40, size = 663, normalized size = 1.99

$$-\frac{\left(8\sqrt{-\frac{b}{a}}\,a\,b^{2}+8\sqrt{-\frac{b}{a}}\,b^{3}\right)\sinh\left(fx+e\right)\left(\cosh^{6}\left(fx+e\right)\right)+\left(13\sqrt{-\frac{b}{a}}\,a^{2}b-2\sqrt{-\frac{b}{a}}\,a\,b^{2}-11\sqrt{-\frac{b}{a}}\,b^{3}\right)\left(\cosh^{4}\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((8*(-1/a*b)^(1/2)*a*b^2+8*(-1/a*b)^(1/2)*b^3)*sinh(f*x+e)*cosh(f*x+e)
^6+(13*(-1/a*b)^(1/2)*a^2*b-2*(-1/a*b)^(1/2)*a*b^2-11*(-1/a*b)^(1/2)*b^3)*c
osh(f*x+e)^4*sinh(f*x+e)+(4*(-1/a*b)^(1/2)*a^3-6*(-1/a*b)^(1/2)*a^2*b+2*(-1
/a*b)^(1/2)*b^3)*cosh(f*x+e)^2*sinh(f*x+e)+(-(-1/a*b)^(1/2)*a^3+3*(-1/a*b)^
(1/2)*a^2*b-3*(-1/a*b)^(1/2)*a*b^2+(-1/a*b)^(1/2)*b^3)*sinh(f*x+e)-(cosh(f*
x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(3*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b
)^(1/2))*a*b-5*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+8*Elli
pticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^4-(cosh(f*x+e)^2)^(1/2)*(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a^3-EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-7*Elliptic
F(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+5*EllipticF(sinh(f*x+e)*(-1
/a*b)^(1/2),(a/b)^(1/2))*b^3+8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*a^2*b-8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)*cosh(f
*x+e)^2)/cosh(f*x+e)^3/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^4/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\tanh\left(e+f\,x\right)}^4}{{\left(b\,{\sinh\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tanh(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(e+fx)}{(a+b\sinh^2(e+fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tanh(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.508 $\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{\tanh(e+fx)}{f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{4b\sinh(e+fx)\cosh(e+fx)}{3f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sin}{}\right.\right.}{3\sqrt{a}\,f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{}}$$

```
[Out] -4/3*b*cosh(f*x+e)*sinh(f*x+e)/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(3/2)-1/3*(7*a
+b)*cosh(f*x+e)*(1/(1+b*sinh(f*x+e)^2/a))^(1/2)*(1+b*sinh(f*x+e)^2/a)^(1/2)
*EllipticE(sinh(f*x+e)*b^(1/2)/a^(1/2)/(1+b*sinh(f*x+e)^2/a)^(1/2),(1-a/b)^
(1/2))*b^(1/2)/(a-b)^3/f/a^(1/2)/(a*cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2))^(1/2
)/(a+b*sinh(f*x+e)^2)^(1/2)+1/3*(3*a+5*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+si
nh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1
/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a/(a-b)^3/f/(sech(f*x+e)^2*(a+b*
sinh(f*x+e)^2)/a)^(1/2)-tanh(f*x+e)/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.28, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3196, 471, 527, 525, 418, 411}

$$-\frac{\tanh(e+fx)}{f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{4b\sinh(e+fx)\cosh(e+fx)}{3f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sin}{}\right.\right.}{3\sqrt{a}\,f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (-4*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (Sqrt[b]*(7*a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e +
 f*x])/Sqrt[a]], 1 - a/b])/(3*Sqrt[a]*(a - b)^3*f*Sqrt[(a*Cosh[e + f*x]^2)/
(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + 5*b)*Ellipt
icF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - T
anh[e + f*x]/((a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
```

```
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right.}{f}$$

$$= -\frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\right.}{(-a+}$$

$$= -\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\right.}{}$$

$$= -\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(b\right.}{}$$

$$= -\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b}\,(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}}{}\right.\right.}{3\sqrt{a}\,(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\,\sqrt{a+}}$$

Mathematica [C] time = 2.71, size = 215, normalized size = 0.78

$$\frac{8ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}F\left(i(e+fx)\left|\frac{b}{a}\right.\right)-2ia^2(7a+b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)-\frac{\tanh(e+f\ldots}{\ldots}}{6af(a-b)^3(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-2*I)*a^2*(7*a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + (8*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3
/2)*EllipticF[I*(e + f*x), b/a] - ((24*a^3 - 4*a^2*b + 5*a*b^2 - b^3 + 4*a*
(11*a - 3*b)*b*Cosh[2*(e + f*x)] + b^2*(7*a + b)*Cosh[4*(e + f*x)])*Tanh[e
+ f*x])/Sqrt[2])/(6*a*(a - b)^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)^2}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 4.24Error: Bad Argument Type
```

maple [B] time = 0.36, size = 799, normalized size = 2.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(-7*(-1/a*b)^(1/2)*a*b^2*sinh(f*x+e)^5-(-1/a*b)^(1/2)*b^3*sinh(f*x+e)^5
+3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b*sinh(f*x+e)^2-2*((a+b*sinh(f*x+e)^2)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b^2*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)
*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3*sinh(f*x+e)^2+7*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x+e)^2+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3*s
inh(f*x+e)^2-11*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)^3-4*(-1/a*b)^(1/2)*a*b^2*s
inh(f*x+e)^3-(-1/a*b)^(1/2)*b^3*sinh(f*x+e)^3+3*((a+b*sinh(f*x+e)^2)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(
```

```
f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+7*(
(a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-3*(-1/a*b)
^(1/2)*a^3*sinh(f*x+e)-5*sinh(f*x+e)*b*a^2*(-1/a*b)^(1/2))/(-1/a*b)^(1/2)/(
a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^3/a/cosh(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tanh\left(e+f\,x\right)^{2}}{\left(b\sinh\left(e+f\,x\right)^{2}+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(tanh(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^{2}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Integral(tanh(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.509 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(}{3af(a-b)\left(a+b\sinh^2(\right.}$$

```
[Out] -1/3*b*cosh(f*x+e)*sinh(f*x+e)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(2*a
-b)*b*cosh(f*x+e)*sinh(f*x+e)/a^2/(a-b)^2/f/(a+b*sinh(f*x+e)^2)^(1/2)-2/3*I
*(2*a-b)*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticE(sin(I*e+I*f*x),(
b/a)^(1/2))*(a+b*sinh(f*x+e)^2)^(1/2)/a^2/(a-b)^2/f/(1+b*sinh(f*x+e)^2/a)^(
1/2)+1/3*I*(cos(I*e+I*f*x)^2)^(1/2)/cos(I*e+I*f*x)*EllipticF(sin(I*e+I*f*x)
,(b/a)^(1/2))*(1+b*sinh(f*x+e)^2/a)^(1/2)/a/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/
2)
```

Rubi [A] time = 0.29, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(}{3af(a-b)\left(a+b\sinh^2(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] -(b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*S
qrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a
*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a+2b+b\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A] time = 0.96, size = 190, normalized size = 0.76

$$\frac{\sqrt{2}\,b\sinh(2(e+fx))\left(-5a^2+b(b-2a)\cosh(2(e+fx))+5ab-b^2\right)+ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}F\left(i(e+\right.}{3a^2f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*
EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b
)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cos
h[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*
x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.45Error: Bad Argument Typ
e
```

maple [A] time = 0.58, size = 406, normalized size = 1.62

$$\frac{\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}\left(-\frac{\sinh\left(fx+e\right)\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)\left(\cosh^{2}\left(fx+e\right)\right)}}{3ab(a-b)\left(\sinh^{2}\left(fx+e\right)+\frac{a}{b}\right)^{2}}-\frac{2b\left(\cosh^{2}\left(fx+e\right)\right)\sinh\left(fx\right.}{3a^{2}(a-b)^{2}\sqrt{\left(a+b\left(\sinh^{2}\left(fx+e\right)\right)\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a/b/(a-b)*sinh(f*x+e)*((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+a/b)^2-2/3*b*cosh(f*x+e
)^2/a^2/(a-b)^2*sinh(f*x+e)*(2*a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/
2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2/3*b*(2*a-b)/a^2/(a-b)^2/(-1/a
*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*
x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))))/cosh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b \sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sinh(e + f*x)**2)**(-5/2), x)
```

3.510 $\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=351

$$\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)}-\frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)}+\frac{(3a-4b)\mathrm{sech}(e+f}{3a}$$

```
[Out] 1/3*coth(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(3/2)+1/3*(3*a-4*b)*coth(f*x+e)/a^2
/(a-b)/f/(a+b*sinh(f*x+e)^2)^(1/2)-1/3*(7*a-8*b)*coth(f*x+e)*(a+b*sinh(f*x+
e)^2)^(1/2)/a^3/(a-b)/f-1/3*(7*a-8*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))
*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh
(f*x+e)^2)/a)^(1/2)+1/3*(3*a-4*b)*(1/(1+sinh(f*x+e)^2))^(1/2)*(1+sinh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b/a)^(1/2))*sec
h(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/(a-b)/f/(sech(f*x+e)^2*(a+b*sinh(f*x
+e)^2)/a)^(1/2)+1/3*(7*a-8*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a^3/(a-
b)/f
```

Rubi [A] time = 0.41, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 469, 579, 583, 531, 418, 492, 411}

$$\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)}-\frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)}+\frac{(3a-4b)\,\mathrm{coth}}{3a^2f(a-b)\sqrt{a+b\,\mathrm{s}}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Coth[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((3*a - 4*b)*Coth[e +
 f*x])/(3*a^2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((7*a - 8*b)*Coth[e
+ f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f) - ((7*a - 8*b)*Ellipt
icE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3
*a^3*(a - b)*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{-}{x^2\sqrt{1+x}}\right.}{3af}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e\right.}{}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\,\mathrm{co}}{}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\,\mathrm{co}}{}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\,\mathrm{co}}{}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\,\mathrm{co}}{}$$

Mathematica [C] time = 2.70, size = 226, normalized size = 0.64

$$\frac{8ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2} F\left(i(e+fx)\middle|\frac{b}{a}\right) - 2ia^2(7a-8b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2} E\left(i(e+fx)\middle|\frac{b}{a}\right) - \frac{\coth(e+}{}}{6a^3f(a-b)(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(((24*a^3 - 68*a^2*b + 69*a*b^2 - 24*b^3 + 4*b*(11*a^2 - 19*a*b + 8*b^2)*
Cosh[2*(e + f*x)] + (7*a - 8*b)*b^2*Cosh[4*(e + f*x)])*Coth[e + f*x])/Sqrt[
2]) - (2*I)*a^2*(7*a - 8*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*Ellip
ticE[I*(e + f*x), b/a] + (8*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])
/a)^(3/2)*EllipticF[I*(e + f*x), b/a])/(6*a^3*(a - b)*f*(2*a - b + b*Cosh[2
*(e + f*x)])^(3/2))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\coth\left(fx+e\right)^2}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.89Error: Bad Argument Type
```

maple [A] time = 0.38, size = 642, normalized size = 1.83

$$-\frac{\sqrt{\frac{b\left(\cosh^2\left(fx+e\right)\right)}{a}+\frac{a-b}{a}}\sqrt{\frac{\cosh\left(2fx+2e\right)}{2}+\frac{1}{2}}\, b\left(3\,\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)a^2-11\,\mathrm{EllipticF}\left(\sinh\left(f\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(3*Ellipti
cF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-11*EllipticF(sinh(f*x+e)*(-1
/a*b)^(1/2),(a/b)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*sinh(f*x+e)*cosh(f*x+e)^2-(
b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3-14*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2
),(a/b)^(1/2))*a^2*b+19*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a
*b^2-8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3+7*EllipticE(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-15*EllipticE(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*a*b^2+8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*b^3)*sinh(f*x+e)+(7*(-1/a*b)^(1/2)*a*b^2-8*(-1/a*b)^(1/2)*b^3)*cosh(f*
x+e)^6+(11*(-1/a*b)^(1/2)*a^2*b-26*(-1/a*b)^(1/2)*a*b^2+16*(-1/a*b)^(1/2)*b
^3)*cosh(f*x+e)^4+(3*(-1/a*b)^(1/2)*a^3-14*(-1/a*b)^(1/2)*a^2*b+19*(-1/a*b)
^(1/2)*a*b^2-8*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^2)/a^3/(a-b)/(a+b*sinh(f*x+e
)^2)^(3/2)/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth\left(e+f\,x\right)^2}{\left(b\sinh\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2),x)
```

```
[Out] int(coth(e + f*x)^2/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(e + fx)}{(a + b\sinh^2(e + fx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(coth(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.511 $\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=385

$$\frac{8(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^4f}-\frac{8(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^4f}+\frac{(3a-8b)\text{sech}(e\cdot}{}$$

```
[Out] -1/3*(a-b)*coth(f*x+e)*csch(f*x+e)^2/a/b/f/(a+b*sinh(f*x+e)^2)^(3/2)-2/3*(a
-3*b)*coth(f*x+e)*csch(f*x+e)^2/a^2/b/f/(a+b*sinh(f*x+e)^2)^(1/2)-8/3*(a-2*
b)*coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^4/f+1/3*(3*a-8*b)*coth(f*x+e)*cs
ch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2)/a^3/b/f-8/3*(a-2*b)*(1/(1+sinh(f*x+e)
^2))^(1/2)*(1+sinh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)/(1+sinh(f*x+e)^2)^
(1/2),(1-b/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^4/f/(sech(f*x+
e)^2*(a+b*sinh(f*x+e)^2)/a)^(1/2)+1/3*(3*a-8*b)*(1/(1+sinh(f*x+e)^2))^(1/2)
*(1+sinh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)/(1+sinh(f*x+e)^2)^(1/2),(1-b
/a)^(1/2))*sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/a^4/f/(sech(f*x+e)^2*(a+b*
sinh(f*x+e)^2)/a)^(1/2)+8/3*(a-2*b)*(a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)/a
^4/f
```

Rubi [A] time = 0.56, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 468, 579, 583, 531, 418, 492, 411}

$$\frac{8(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^4f}-\frac{8(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^4f}+\frac{(3a-8b)\coth(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*(a - 3*b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a^2*b*f*Sqrt[a + b*S
inh[e + f*x]^2]) - (8*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/
(3*a^4*f) + ((3*a - 8*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e +
f*x]^2])/(3*a^3*b*f) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/
a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech[e + f*x]^
2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x
]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e +
 f*x]^2]*Tanh[e + f*x])/(3*a^4*f)
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}}{3a}$$

$$= -\frac{(a-b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{(a-b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= -\frac{(a-b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{(a-b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{(a-b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{(a-b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\mathrm{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} -$$

Mathematica [C] time = 2.89, size = 247, normalized size = 0.64

$$-\frac{i\left(2a^2b\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\left((8b-5a)F\left(i(e+fx)\middle|\frac{b}{a}\right)+8(a-2b)E\left(i(e+fx)\middle|\frac{b}{a}\right)\right)+\frac{ib\coth(e+fx)\mathrm{csch}^2(e+fx)(8}{}\right.}{6a^4bf(2a+b\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-1/6*I)*((I*b*(8*a^3 - 63*a^2*b + 92*a*b^2 - 40*b^3 - 2*(8*a^3 - 38*a^2*b
 + 63*a*b^2 - 30*b^3)*Cosh[2*(e + f*x)] - b*(13*a^2 - 36*a*b + 24*b^2)*Cosh
[4*(e + f*x)] - 2*a*b^2*Cosh[6*(e + f*x)] + 4*b^3*Cosh[6*(e + f*x)])*Coth[e
 + f*x]*Csch[e + f*x]^2)/Sqrt[2] + 2*a^2*b*((2*a - b + b*Cosh[2*(e + f*x)])
/a)^(3/2)*(8*(a - 2*b)*EllipticE[I*(e + f*x), b/a] + (-5*a + 8*b)*EllipticF
[I*(e + f*x), b/a])))/(a^4*b*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\coth\left(fx+e\right)^4}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 2.35Error: Bad Argument Type
```

maple [B] time = 0.37, size = 923, normalized size = 2.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(8*(-1/a*b)^(1/2)*a*b^2*sinh(f*x+e)^8-16*(-1/a*b)^(1/2)*b^3*sinh(f*x+e
)^8-3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b*sinh(f*x+e)^5+16*((a+b*sinh(f*x+e)^2
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*a*b^2*sinh(f*x+e)^5-16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3*sinh(f*x+e)^5
-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x+e)^5+16*((a+b*sinh(f*x+e)^2)/a
)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*b^3*sinh(f*x+e)^5+13*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)^6-16*(-1/a*b)^(1
/2)*a*b^2*sinh(f*x+e)^6-16*(-1/a*b)^(1/2)*b^3*sinh(f*x+e)^6-3*((a+b*sinh(f*
x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*a^3*sinh(f*x+e)^3+16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b*sinh(f*
x+e)^3-16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sin
h(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x+e)^3-8*((a+b*sinh(f*x+e
)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a
/b)^(1/2))*a^2*b*sinh(f*x+e)^3+16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x
+e)^3+4*(-1/a*b)^(1/2)*a^3*sinh(f*x+e)^4+7*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)
^4-24*(-1/a*b)^(1/2)*a*b^2*sinh(f*x+e)^4+5*(-1/a*b)^(1/2)*a^3*sinh(f*x+e)^2
-6*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^3)/a^4/(a+b*sinh(f*x
+e)^2)^(3/2)/(-1/a*b)^(1/2)/sinh(f*x+e)^3/cosh(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth\left(e+f\,x\right)^4}{{\left(b\,\sinh\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2),x)
[Out] int(coth(e + f*x)^4/(a + b*sinh(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^{4}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
[Out] Integral(coth(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(5/2), x)
```

3.512 $\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m \, dx$

Optimal. Leaf size=122

$$\frac{\cosh^2(e+fx)^{\frac{m+1}{2}}(d\tanh(e+fx))^{m+1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};-\sinh^2(e\right.}{df(m+1)}$$

```
[Out] AppellF1(1/2+1/2*m,1/2+1/2*m,-p,3/2+1/2*m,-sinh(f*x+e)^2,-b*sinh(f*x+e)^2/a
)*(cosh(f*x+e)^2)^(1/2+1/2*m)*(a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^(1+m)/d
/f/(1+m)/((1+b*sinh(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3197, 511, 510}

$$\frac{\cosh^2(e+fx)^{\frac{m+1}{2}}(d\tanh(e+fx))^{m+1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};-\sinh^2(e\right.}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, -Sinh[e + f*x]^2, -((b*Sinh[
e + f*x]^2)/a)]*(Cosh[e + f*x]^2)^((1 + m)/2)*(a + b*Sinh[e + f*x]^2)^p*(d*
Tanh[e + f*x])^(1 + m))/(d*f*(1 + m)*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3197

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(
ff*(d*Tan[e + f*x])^(m + 1)*(Cos[e + f*x]^2)^((m + 1)/2))/(d*f*Sin[e + f*x]
^(m + 1)), Subst[Int[((ff*x)^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/
2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !In
tegerQ[m]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx = \frac{\left(i\cosh^2(e+fx)\right)^{\frac{1+m}{2}}(i\sinh(e+fx))^{-1-m}(d\tanh(e+fx))^{1+m}}{}$$

$$= \frac{\left(i\cosh^2(e+fx)\right)^{\frac{1+m}{2}}(i\sinh(e+fx))^{-1-m}\left(a+b\sinh^2(e+fx\right.}{}$$

$$= \frac{F_1\left(\frac{1+m}{2};\frac{1+m}{2},-p;\frac{3+m}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\cosh^2(}{}$$

Mathematica [F] time = 10.59, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m,x]
[Out] Integrate[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m, x]
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\left(d\tanh\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="fricas")
[Out] integral((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\left(d\tanh\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="giac")
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2\left(fx+e\right)\right)\right)^p\left(d\tanh\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)
[Out] int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\left(d\tanh\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d \tanh\left(e+f x\right)\right)^m \left(b \sinh\left(e+f x\right)^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tanh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p,x)
```

```
[Out] int((d*tanh(e + f*x))^m*(a + b*sinh(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**p*(d*tanh(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.513 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{\operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{2d(a-b)}-\frac{(a-b(p+1))\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a-b}\right)}{2d(p+1)(a-b)^2}$$

```
[Out] -1/2*(a-b*(1+p))*hypergeom([1, 1+p],[2+p],(a+b*sinh(d*x+c)^2)/(a-b))*(a+b*s
inh(d*x+c)^2)^(1+p)/(a-b)^2/d/(1+p)+1/2*sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^(
1+p)/(a-b)/d
```

Rubi [A] time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3194, 78, 68}

$$\frac{\operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{2d(a-b)}-\frac{(a-b(p+1))\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a-b}\right)}{2d(p+1)(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^3,x]
```

```
[Out] -((a - b*(1 + p))*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2
)/(a - b)]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)^2*d*(1 + p)) + (Sech
[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^p}{(1+x)^2}\,dx, x, \sinh^2(c+dx)\right)}{2d}$$
$$= \frac{\text{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)d} - \frac{(a-b(1+p))\,\text{Subst}\left(\int \ldots\right.}{2(-a\ldots}$$
$$= -\frac{(a-b(1+p))\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sinh^2(c+dx)}{a-b}\right)\left(a+b\sinh^2(c+d\ldots\right.}{2(a-b)^2d(1+p)}$$

Mathematica [A] time = 0.26, size = 90, normalized size = 0.82

$$\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}\left((-a+bp+b)\,{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)+a}{a-b}\right)+(p+1)(a-b)\text{sech}^2(c+dx)\right)}{2d(p+1)(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^3,x]
```

```
[Out] (((-a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)
/(a - b)] + (a - b)*(1 + p)*Sech[c + d*x]^2)*(a + b*Sinh[c + d*x]^2)^(1 + p
))/(2*(a - b)^2*d*(1 + p))
```

fricas [F] time = 2.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh(dx+c)^2+a\right)^p\tanh(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh(dx+c)^2+a\right)^p\tanh(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2(dx+c)\right)\right)^p\left(\tanh^3(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)
```

```
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh(dx+c)^2+a\right)^p\tanh(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(c+d\,x\right)^3\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(tanh(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.514 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh(c+dx)\,dx$

Optimal. Leaf size=63

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],(a+b*sinh(d*x+c)^2)/(a-b))*(a+b*sinh(d*x+c)^2
)^(1+p)/(a-b)/d/(1+p)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3194, 68}

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x],x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)/(a - b)]*(a +
b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^2(c+dx)\right)^p \tanh(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx,x,\sinh^2(c+dx)\right)}{2d}\\ &= -\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sinh^2(c+dx)}{a-b}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)d(1+p)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 1.03

$$-\frac{\left(a+b\cosh^2(c+dx)-b\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\cosh^2(c+dx)}{a-b}+1\right)}{2d(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x],x]
```

```
[Out] -1/2*((a - b + b*Cosh[c + d*x]^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p
, 1 + (b*Cosh[c + d*x]^2)/(a - b)])/((a - b)*d*(1 + p))
```

fricas [F] time = 2.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\tanh\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\tanh\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sinh^{2}\left(dx+c\right)\right)\right)^{p}\tanh\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x)
```

```
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\tanh\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\tanh\left(c+d\,x\right)\,\left(b\,\sinh\left(c+d\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(tanh(c + d*x)*(a + b*sinh(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c),x)
```

```
[Out] Timed out
```

3.515 $\int \coth(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],1+b*sinh(d*x+c)^2/a)*(a+b*sinh(d*x+c)^2)^(1+p
)/a/d/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3194, 65}

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh
[c + d*x]^2)^(1 + p))/(2*a*d*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \coth(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,\sinh^2(c+dx)\right)}{2d}\\ &= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sinh^2(c+dx)}{a}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad(1+p)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 1.00

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]*(a + b*Sinh[c + d*x]^2)^p,x]

[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(a*d*(1 + p))

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\coth\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")

[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\coth\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int\coth\left(dx+c\right)\left(a+b\left(\sinh^{2}\left(dx+c\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)

[Out] int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\coth\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")

[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\coth\left(c+d\,x\right)\,\left(b\,\sinh\left(c+d\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)*(a + b*sinh(c + d*x)^2)^p,x)

[Out] int(coth(c + d*x)*(a + b*sinh(c + d*x)^2)^p, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)**2)**p,x)

[Out] Timed out

3.516 $\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=94

$$-\frac{(a+bp)\left(a+b\sinh^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2a^2d(p+1)}-\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{2ad}$$

```
[Out] -1/2*csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^(1+p)/a/d-1/2*(b*p+a)*hypergeom([1,
1+p],[2+p],1+b*sinh(d*x+c)^2/a)*(a+b*sinh(d*x+c)^2)^(1+p)/a^2/d/(1+p)
```

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3194, 78, 65}

$$-\frac{(a+bp)\left(a+b\sinh^2(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2a^2d(p+1)}-\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -(Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a*d) - ((a + b*p)*Hyp
ergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c +
d*x]^2)^(1 + p))/(2*a^2*d*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)(a+bx)^p}{x^2}\,dx, x, \sinh^2(c+dx)\right)}{2d}$$

$$= -\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad} + \frac{(a+bp)\,\text{Subst}\left(\int \frac{(a+\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad} - \frac{(a+bp)\,{}_2F_1\left(1,1+p;\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.38, size = 71, normalized size = 0.76

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}\left(\frac{(a+bp)\,{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{p+1}+a\operatorname{csch}^2(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^p,x]
[Out] -1/2*((a*Csch[c + d*x]^2 + ((a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
+ (b*Sinh[c + d*x]^2)/a])/(1 + p))*(a + b*Sinh[c + d*x]^2)^(1 + p))/(a^2*d)
```

fricas [F] time = 1.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int\left(\coth^3\left(dx+c\right)\right)\left(a+b\left(\sinh^2\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)
[Out] int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth (c+d\,x)^3\,{\left(b\,\sinh (c+d\,x)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(coth(c + d*x)^3*(a + b*sinh(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sinh(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

3.517 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{\sinh^4(c+dx)\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};-\sinh^2(\ldots\right.}{5d}$$

```
[Out] 1/5*AppellF1(5/2,5/2,-p,7/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*sinh(d*x+c)^
4*(a+b*sinh(d*x+c)^2)^p*(cosh(d*x+c)^2)^(1/2)*tanh(d*x+c)/d/((1+b*sinh(d*x+
c)^2/a)^p)
```

Rubi [A] time = 0.12, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3196, 511, 510}

$$\frac{\sinh^4(c+dx)\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};-\sinh^2(\ldots\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4,x]
```

```
[Out] (AppellF1[5/2, 5/2, -p, 7/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sq
rt[Cosh[c + d*x]^2]*Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]
)/(5*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\,\mathrm{sech}(c+dx)\right)\mathrm{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^p}{\left(1+x^2\right)^{5/2}}dx, x, \sinh(c+d\right.}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\,\mathrm{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b\sinh^2(c+d}{a}\right.\right.}{d}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\,\sinh}{5d}$$

Mathematica [F] time = 36.30, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4,x]
[Out] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4, x]
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh\left(dx+c\right)^2+a\right)^p\tanh\left(dx+c\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\tanh\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2\left(dx+c\right)\right)\right)^p\left(\tanh^4\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\tanh\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(c+d\,x\right)^4\,{\left(b\,\sinh\left(c+d\,x\right)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p,x)
[Out] int(tanh(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**4,x)
[Out] Timed out
```

3.518 $\int \left(a+b\sinh^2(c+dx)\right)^p\tanh^2(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{\sinh^2(c+dx)\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};-\sinh^2(c+\right.}{3d}$$

```
[Out] 1/3*AppellF1(3/2,3/2,-p,5/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*sinh(d*x+c)^
2*(a+b*sinh(d*x+c)^2)^p*(cosh(d*x+c)^2)^(1/2)*tanh(d*x+c)/d/((1+b*sinh(d*x+
c)^2/a)^p)
```

Rubi [A] time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3196, 511, 510}

$$\frac{\sinh^2(c+dx)\sqrt{\cosh^2(c+dx)}\,\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};-\sinh^2(c+\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2,x]
```

```
[Out] (AppellF1[3/2, 3/2, -p, 5/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sq
rt[Cosh[c + d*x]^2]*Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]
)/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^2(c+dx)\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\,\mathrm{sech}(c+dx)\right)\mathrm{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^p}{\left(1+x^2\right)^{3/2}}\,dx, x, \sinh(c}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\,\mathrm{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b\sinh^2}{a}\right.\right.}{d}$$

$$= \frac{F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\,\mathrm{si}}{}$$

Mathematica [F] time = 5.65, size = 0, normalized size = 0.00

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2,x]
[Out] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2, x]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh(dx+c)^2+a\right)^p \tanh(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh(dx+c)^2+a\right)^p \tanh(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sinh^2(dx+c)\right)\right)^p \left(\tanh^2(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh(dx+c)^2+a\right)^p \tanh(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(c+dx)^2 \left(b\sinh(c+dx)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(tanh(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.519 $\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{\cosh^2(c+dx)}\,\mathrm{csch}(c+dx)\mathrm{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};-\sinh$$
$$d$$

```
[Out] -AppellF1(-1/2,-1/2,-p,1/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*csch(d*x+c)*s
ech(d*x+c)*(a+b*sinh(d*x+c)^2)^p*(cosh(d*x+c)^2)^(1/2)/d/((1+b*sinh(d*x+c)^
2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3196, 511, 510}

$$-\frac{\sqrt{\cosh^2(c+dx)}\,\mathrm{csch}(c+dx)\mathrm{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};-\sinh$$
$$d$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)
]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]*Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)
^p)/(d*(1 + (b*Sinh[c + d*x]^2)/a)^p))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\,\mathrm{sech}(c+dx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}\left(a+bx^2\right)^p}{x^2}dx, x, \sinh(c$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\,\mathrm{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b\sinh^2(c+d}{a}\right.\right.}{d}$$

$$= -\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\,c}{}$$

Mathematica [F] time = 6.71, size = 0, normalized size = 0.00

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p,x]
[Out] Integrate[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p, x]
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \left(\coth^2\left(dx+c\right)\right)\left(a+b\left(\sinh^2\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)
[Out] int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(c+d\,x\right)^2\,{\left(b\,{\mathrm{sinh}\left(c+d\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p,x)
[Out] int(coth(c + d*x)^2*(a + b*sinh(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sinh(d*x+c)**2)**p,x)
[Out] Timed out
```

3.520 $\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{\cosh^2(c+dx)}\,\text{csch}^3(c+dx)\text{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};-\sinh\right.}{3d}$$

```
[Out] -1/3*AppellF1(-3/2,-3/2,-p,-1/2,-sinh(d*x+c)^2,-b*sinh(d*x+c)^2/a)*csch(d*x
+c)^3*sech(d*x+c)*(a+b*sinh(d*x+c)^2)^p*(cosh(d*x+c)^2)^(1/2)/d/((1+b*sinh(
d*x+c)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3196, 511, 510}

$$-\frac{\sqrt{\cosh^2(c+dx)}\,\text{csch}^3(c+dx)\text{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};-\sinh\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, -3/2, -p, -1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)
]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]^3*Sech[c + d*x]*(a + b*Sinh[c + d*x]^
2)^p)/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\,\mathrm{sech}(c+dx)\right)\mathrm{Subst}\left(\int \frac{(1+x^2)^{3/2}(a+bx^2)^p}{x^4}\,dx, x, \mathrm{si}\right.}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\,\mathrm{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b\sinh^2}{a}\right.\right.}{d}$$

$$= -\frac{F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+}}{}$$

Mathematica [F] time = 38.40, size = 0, normalized size = 0.00

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p,x]
[Out] Integrate[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p, x]
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sinh(dx+c)^2+a\right)^p\coth(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh(dx+c)^2+a\right)^p\coth(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \left(\coth^4(dx+c)\right)\left(a+b\left(\sinh^2(dx+c)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)
[Out] int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sinh(dx+c)^2+a\right)^p\coth(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth\left(c+d\,x\right)^4\,{\left(b\,{\sinh\left(c+d\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p,x)
```

```
[Out] int(coth(c + d*x)^4*(a + b*sinh(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sinh(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

3.521 $\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx$

Optimal. Leaf size=152

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sinh(x)+b^{2/3}\sinh^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sinh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\log\ldots}{\ldots}$$

```
[Out] -1/2*csch(x)^2/a+ln(sinh(x))/a-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*sinh(x))/a^(5
/3)+1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sinh(x)+b^(2/3)*sinh(x)^2)/a^(5/
3)-1/3*ln(a+b*sinh(x)^3)/a+1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sinh(x
))/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3230, 1834, 1871, 12, 200, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sinh(x)+b^{2/3}\sinh^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sinh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\log\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sinh[x]^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sinh[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(5/3)) - Csch[x]^2/(2*a) + Log[Sinh[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3
)*Sinh[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sinh[x] +
b^(2/3)*Sinh[x]^2])/(6*a^(5/3)) - Log[a + b*Sinh[x]^3]/(3*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx = \text{Subst}\left(\int \frac{1+x^2}{x^3\left(a+bx^3\right)}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{ax^3}+\frac{1}{ax}+\frac{-b-bx^2}{a\left(a+bx^3\right)}\right)dx, x, \sinh(x)\right)$$
$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}+\frac{\text{Subst}\left(\int \frac{-b-bx^2}{a+bx^3}\,dx, x, \sinh(x)\right)}{a}$$
$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{\text{Subst}\left(\int \frac{b}{a+bx^3}\,dx, x, \sinh(x)\right)}{a}-\frac{b\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \text{s}\right.}{a}$$
$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sinh(x)\right)}{a}$$
$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}-\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sinh(x)\right.}{3a^{5/3}}$$
$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}+\frac{b^{2/}}{}$$
$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin\right.}{6a^{5/3}}$$
$$= \frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sinh(x)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}+\frac{b}{}$$

Mathematica [A] time = 0.35, size = 136, normalized size = 0.89

$$-\frac{\left(a^{2/3}+(-1)^{2/3}b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sinh(x)\right)+\left(a^{2/3}+b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)+\left(a^{2/3}-\sqrt[3]{-1}\,b^{2/3}\right.}{3a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x]^3),x]
```

```
[Out] -1/2*Csch[x]^2/a + Log[Sinh[x]]/a - ((a^(2/3) + (-1)^(2/3)*b^(2/3))*Log[-((
-1)^(2/3)*a^(1/3)) - b^(1/3)*Sinh[x]] + (a^(2/3) + b^(2/3))*Log[a^(1/3) + b
^(1/3)*Sinh[x]] + (a^(2/3) - (-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b
^(1/3)*Sinh[x]])/(3*a^(5/3))
```

fricas [C] time = 4.05, size = 1115, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(1/3)*(a*e^(4*x) - 2*a*e^(2*x) + a)*sqrt(((((1/2)^(1/3)*(I*sqr
t(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a +
 4)/a^2)*arctan(-1/8*(2*sqrt(1/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3
+ b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^4*e^(2*x) + b^2*e^(4*x) + 2*a
*b*e^(3*x) - 2*a*b*e^x - (a^2*b*e^(3*x) + 4*a^3*e^(2*x) - a^2*b*e^x)*((1/2)
```

```
^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a) + b
^2 + 2*(2*a^2 - b^2)*e^(2*x))*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^
5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a^3 - 2*a^2)*sqrt((((1/2)^(1/3)*(I*sqrt(3
) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1
/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a + 4)
/a^2) + sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b
^2)/a^5)^(1/3) + 2/a)^2*a^5*e^x + 4*a^2*b*e^(2*x) + 4*a^3*e^x - 4*a^2*b - 2
*(a^3*b*e^(2*x) + 2*a^4*e^x - a^3*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 +
b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)
*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2))
*e^(-x)/b^2) + 2*(a*e^(4*x) - 2*a*e^(2*x) + a)*((1/2)^(1/3)*(I*sqrt(3) + 1)
*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*log(((1/2)^(1/3)*(I*sqrt(
3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a^2*e^x + b*e^(2*x
) - 2*a*e^x - b) - ((a*e^(4*x) - 2*a*e^(2*x) + a)*((1/2)^(1/3)*(I*sqrt(3) +
 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a) - 6*e^(4*x) + 12*e^(2*
x) - 6)*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5
)^(1/3) + 2/a)^2*a^4*e^(2*x) + b^2*e^(4*x) + 2*a*b*e^(3*x) - 2*a*b*e^x - (a
^2*b*e^(3*x) + 4*a^3*e^(2*x) - a^2*b*e^x)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a
^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a) + b^2 + 2*(2*a^2 - b^2)*e^(2*x
)) - 12*(e^(4*x) - 2*e^(2*x) + 1)*log(e^(2*x) - 1) + 24*e^(2*x))/(a*e^(4*x)
 - 2*a*e^(2*x) + a)
```

giac [A] time = 0.17, size = 209, normalized size = 1.38

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-e^{(-x)}+e^{x}\right|\right)}{3a^2}-\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)^3+8a\right|\right)}{3a}+\frac{\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{a}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}}{\ldots}\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="giac")
```

```
[Out] 1/3*b*(-a/b)^(1/3)*log(abs(-2*(-a/b)^(1/3) - e^(-x) + e^x))/a^2 - 1/3*log(a
bs(-b*(e^(-x) - e^x)^3 + 8*a))/a + log(abs(-e^(-x) + e^x))/a - 1/3*sqrt(3)*
(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) - e^(-x) + e^x)/(-a/b)^(1/3
))/a^2 - 1/6*(-a*b^2)^(1/3)*log((e^(-x) - e^x)^2 - 2*(-a/b)^(1/3)*(e^(-x) -
 e^x) + 4*(-a/b)^(2/3))/a^2 - 1/2*(3*(e^(-x) - e^x)^2 + 4)/(a*(e^(-x) - e^x
)^2)
```

maple [C] time = 0.15, size = 132, normalized size = 0.87

$$-\frac{\tanh^2\left(\frac{x}{2}\right)}{8a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\left(-\_R^5a-\_R^4b+2\_R^3a+4\_R^2b-\_Ra+b\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}}{3a}-\frac{1}{8a\tanh\left(\frac{x}{2}\right)^2}+\frac{\ln(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sinh(x)^3),x)
```

```
[Out] -1/8*tanh(1/2*x)^2/a+1/3/a*sum((-_R^5*a-_R^4*b+2*_R^3*a+4*_R^2*b-_R*a+b)/(_
R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-
8*_Z^3*b+3*_Z^2*a-a))-1/8/a/tanh(1/2*x)^2+1/a*ln(tanh(1/2*x))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 0.92, size = 1129, normalized size = 7.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a + b*sinh(x)^3),x)
```

```
[Out] 2/(a - a*exp(2*x)) - 2/(a - 2*a*exp(2*x) + a*exp(4*x)) + symsum(log((503316
48*a^6*exp(2*x) + 786432*b^6*exp(2*x) - 452984832*root(27*a^5*z^3 + 27*a^4*
z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^7 - 50331648*a^6 - 786432*b^6 - 13589544
96*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^2*a^8 - 135895
4496*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^9 - 5059
3792*a^2*b^4 - 102498304*a^4*b^2 + 1358954496*root(27*a^5*z^3 + 27*a^4*z^2
+ 9*a^3*z + b^2 + a^2, z, k)^2*a^8*exp(2*x) + 1358954496*root(27*a^5*z^3 +
27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^9*exp(2*x) + 50593792*a^2*b^4*e
xp(2*x) + 102498304*a^4*b^2*exp(2*x) - 7602176*root(27*a^5*z^3 + 27*a^4*z^2
 + 9*a^3*z + b^2 + a^2, z, k)*a^3*b^4 - 465305600*root(27*a^5*z^3 + 27*a^4*
z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^5*b^2 + 524288*a*b^5*exp(x) - 24379392*r
oot(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^2*a^4*b^4 - 138333
3888*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^2*a^6*b^2 -
18874368*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^5*b^
4 - 1370750976*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*
a^7*b^2 + 452984832*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z,
k)*a^7*exp(2*x) + 5242880*a^3*b^3*exp(x) - 524288*root(27*a^5*z^3 + 27*a^4*
z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^2*b^5*exp(x) + 8912896*root(27*a^5*z^3 +
 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^4*b^3*exp(x) + 7602176*root(27*a
^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^3*b^4*exp(2*x) + 4653056
00*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)*a^5*b^2*exp(2*
x) - 14155776*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a
^6*b^3*exp(x) + 24379392*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2
, z, k)^2*a^4*b^4*exp(2*x) + 1383333888*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^
3*z + b^2 + a^2, z, k)^2*a^6*b^2*exp(2*x) + 18874368*root(27*a^5*z^3 + 27*a
^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^5*b^4*exp(2*x) + 1370750976*root(27
*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k)^3*a^7*b^2*exp(2*x))/(a^6
*b^6))*root(27*a^5*z^3 + 27*a^4*z^2 + 9*a^3*z + b^2 + a^2, z, k), k, 1, 3)
+ log(3221225472*a^6*exp(2*x) + 786432*b^6*exp(2*x) - 3221225472*a^6 - 7864
32*b^6 - 101449728*a^2*b^4 - 3321888768*a^4*b^2 + 101449728*a^2*b^4*exp(2*x
) + 3321888768*a^4*b^2*exp(2*x))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{a + b\sinh^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sinh(x)**3),x)
```

```
[Out] Integral(coth(x)**3/(a + b*sinh(x)**3), x)
```

3.522 $\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]^3],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^3}}\,dx, x, \sinh(x)\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^3(x)\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^3(x)}\right)}{3b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]^3],x]
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(Expression(Integer)),failed) can
not be coerced to mode SparseUnivariatePolynomial(Expression(Integer))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(coth(x)/sqrt(b*sinh(x)^3 + a), x)
```

maple [A] time = 0.58, size = 21, normalized size = 0.75

$$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^3(x)\right)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x)^3)^(1/2),x)
```

```
[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)/sqrt(b*sinh(x)^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*sinh(x)^3)^(1/2),x)
[Out] int(coth(x)/(a + b*sinh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a + b\sinh^3(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)**3)**(1/2),x)
[Out] Integral(coth(x)/sqrt(a + b*sinh(x)**3), x)
```

3.523 $\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx$

Optimal. Leaf size=45

$$\frac{2}{3}\sqrt{a+b\sinh^3(x)}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)$$

```
[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sinh(x)^3)^(1/
2)
```

Rubi [A] time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2}{3}\sqrt{a+b\sinh^3(x)}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sinh[x]^3],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sinh[
x]^3])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
```

```
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x}\,dx, x, \sinh(x)\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sinh^3(x)\right)\\ &= \frac{2}{3}\sqrt{a+b\sinh^3(x)}+\frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^3(x)\right)\\ &= \frac{2}{3}\sqrt{a+b\sinh^3(x)}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^3(x)}\right)}{3b}\\ &= -\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{a+b\sinh^3(x)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{2}{3}\sqrt{a+b\sinh^3(x)}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]^3],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sinh[
x]^3])/3
```

fricas [B] time = 6.28, size = 1663, normalized size = 36.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*(cosh(x) + sinh(x))*log(-(b^2*cosh(x)^12 + 12*b^2*cosh(x)*sin
h(x)^11 + b^2*sinh(x)^12 - 6*b^2*cosh(x)^10 + 64*a*b*cosh(x)^9 + 6*(11*b^2*
cosh(x)^2 - b^2)*sinh(x)^10 + 15*b^2*cosh(x)^8 + 4*(55*b^2*cosh(x)^3 - 15*b
^2*cosh(x) + 16*a*b)*sinh(x)^9 - 192*a*b*cosh(x)^7 + 3*(165*b^2*cosh(x)^4 -
 90*b^2*cosh(x)^2 + 192*a*b*cosh(x) + 5*b^2)*sinh(x)^8 + 24*(33*b^2*cosh(x)
^5 - 30*b^2*cosh(x)^3 + 96*a*b*cosh(x)^2 + 5*b^2*cosh(x) - 8*a*b)*sinh(x)^7
 + 192*a*b*cosh(x)^5 + 4*(128*a^2 - 5*b^2)*cosh(x)^6 + 4*(231*b^2*cosh(x)^6
 - 315*b^2*cosh(x)^4 + 1344*a*b*cosh(x)^3 + 105*b^2*cosh(x)^2 - 336*a*b*cos
h(x) + 128*a^2 - 5*b^2)*sinh(x)^6 + 15*b^2*cosh(x)^4 + 24*(33*b^2*cosh(x)^7
 - 63*b^2*cosh(x)^5 + 336*a*b*cosh(x)^4 + 35*b^2*cosh(x)^3 - 168*a*b*cosh(x
)^2 + 8*a*b + (128*a^2 - 5*b^2)*cosh(x))*sinh(x)^5 - 64*a*b*cosh(x)^3 + 3*(
165*b^2*cosh(x)^8 - 420*b^2*cosh(x)^6 + 2688*a*b*cosh(x)^5 + 350*b^2*cosh(x
)^4 - 2240*a*b*cosh(x)^3 + 320*a*b*cosh(x) + 20*(128*a^2 - 5*b^2)*cosh(x)^2
 + 5*b^2)*sinh(x)^4 - 6*b^2*cosh(x)^2 + 4*(55*b^2*cosh(x)^9 - 180*b^2*cosh(
x)^7 + 1344*a*b*cosh(x)^6 + 210*b^2*cosh(x)^5 - 1680*a*b*cosh(x)^4 + 480*a*
```

```
b*cosh(x)^2 + 20*(128*a^2 - 5*b^2)*cosh(x)^3 + 15*b^2*cosh(x) - 16*a*b)*sin
h(x)^3 + 6*(11*b^2*cosh(x)^10 - 45*b^2*cosh(x)^8 + 384*a*b*cosh(x)^7 + 70*b
^2*cosh(x)^6 - 672*a*b*cosh(x)^5 + 320*a*b*cosh(x)^3 + 10*(128*a^2 - 5*b^2)
*cosh(x)^4 + 15*b^2*cosh(x)^2 - 32*a*b*cosh(x) - b^2)*sinh(x)^2 + b^2 - 16*
(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 - 3*b*cosh(x)^6 + (28*b*
cosh(x)^2 - 3*b)*sinh(x)^6 + 16*a*cosh(x)^5 + 2*(28*b*cosh(x)^3 - 9*b*cosh(
x) + 8*a)*sinh(x)^5 + 3*b*cosh(x)^4 + (70*b*cosh(x)^4 - 45*b*cosh(x)^2 + 80
*a*cosh(x) + 3*b)*sinh(x)^4 + 4*(14*b*cosh(x)^5 - 15*b*cosh(x)^3 + 40*a*cos
h(x)^2 + 3*b*cosh(x))*sinh(x)^3 - b*cosh(x)^2 + (28*b*cosh(x)^6 - 45*b*cosh
(x)^4 + 160*a*cosh(x)^3 + 18*b*cosh(x)^2 - b)*sinh(x)^2 + 2*(4*b*cosh(x)^7
- 9*b*cosh(x)^5 + 40*a*cosh(x)^4 + 6*b*cosh(x)^3 - b*cosh(x))*sinh(x))*sqrt
(a)*sqrt((b*sinh(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 12*(b^2*cosh(x)^11 - 5*b^2*cosh(x)^9 + 48*a*
b*cosh(x)^8 + 10*b^2*cosh(x)^7 - 112*a*b*cosh(x)^6 + 80*a*b*cosh(x)^4 + 2*(
128*a^2 - 5*b^2)*cosh(x)^5 + 5*b^2*cosh(x)^3 - 16*a*b*cosh(x)^2 - b^2*cosh(
x))*sinh(x))/(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(
x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)
^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33
*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*c
osh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 -
 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84
*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20
*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*si
nh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 1
5*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 1
0*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*sqrt(
(b*sinh(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)))/(cosh(x) + sinh(x)), 1/3*(sqrt(-a)*(cosh(x) + sinh(x))
*arctan(8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(-a)*sqrt((b*sinh
(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 - 3*b*cosh(
x)^4 + 3*(5*b*cosh(x)^2 - b)*sinh(x)^4 + 16*a*cosh(x)^3 + 4*(5*b*cosh(x)^3
- 3*b*cosh(x) + 4*a)*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 - 6*b*cos
h(x)^2 + 16*a*cosh(x) + b)*sinh(x)^2 + 6*(b*cosh(x)^5 - 2*b*cosh(x)^3 + 8*a
*cosh(x)^2 + b*cosh(x))*sinh(x) - b)) + sqrt((b*sinh(x)^3 + 3*(b*cosh(x)^2
- b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)
+ sinh(x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh(x)^3 + a}\, \coth(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^3 + a)*coth(x), x)
```

maple [A] time = 0.12, size = 34, normalized size = 0.76

$$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^3(x)\right)}}{\sqrt{a}}\right)\sqrt{a}}{3} + \frac{2\sqrt{a+b\left(\sinh^3(x)\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sinh(x)^3)^(1/2),x)
```

```
[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sinh(x)^3)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sinh(x)^3 + a} \coth(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sinh(x)^3 + a)*coth(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)\sqrt{b \sinh(x)^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b*sinh(x)^3)^(1/2),x)
[Out] int(coth(x)*(a + b*sinh(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sinh^3(x)} \coth(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)**3)**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(x)**3)*coth(x), x)
```

3.524 $\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

```
[Out] -2*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, \sinh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^n(x)\right)}{n}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^n(x)}\right)}{bn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

fricas [A] time = 2.25, size = 113, normalized size = 3.90

$$\left[\frac{\log\left(\frac{b\cosh\left(n\log(\sinh(x))\right)+b\sinh\left(n\log(\sinh(x))\right)-2\sqrt{b\cosh\left(n\log(\sinh(x))\right)+b\sinh\left(n\log(\sinh(x))\right)+a}\sqrt{a}+2a}{\cosh\left(n\log(\sinh(x))\right)+\sinh\left(n\log(\sinh(x))\right)}\right)}{\sqrt{a}\,n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{b\cosh\ldots}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) - 2*sqrt(b*cosh(n*log
(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*log(sinh(x)
)) + sinh(n*log(sinh(x)))))/(sqrt(a)*n), 2*sqrt(-a)*arctan(sqrt(b*cosh(n*lo
g(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(-a)/a)/(a*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x)^n + a), x)
```

maple [A] time = 0.07, size = 24, normalized size = 0.83

$$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^n(x)\right)}}{\sqrt{a}}\right)}{n\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x)^n)^(1/2),x)
[Out] -2*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{b \sinh(x)^n + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)/sqrt(b*sinh(x)^n + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\coth(x)}{\sqrt{a + b \sinh(x)^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*sinh(x)^n)^(1/2),x)
[Out] int(coth(x)/(a + b*sinh(x)^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a + b \sinh^n(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)**n)**(1/2),x)
[Out] Integral(coth(x)/sqrt(a + b*sinh(x)**n), x)
```

3.525 $\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{a+b\sinh^n(x)}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*sinh(x)^n)^(1/2)
/n
```

Rubi [A] time = 0.09, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2\sqrt{a+b\sinh^n(x)}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/n + (2*Sqrt[a + b*Sinh[
x]^n])/n
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
```

```
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx, x, \sinh(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sinh^n(x)\right)}{n}\\
&= \frac{2\sqrt{a+b\sinh^n(x)}}{n} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^n(x)\right)}{n}\\
&= \frac{2\sqrt{a+b\sinh^n(x)}}{n} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^n(x)}\right)}{bn}\\
&= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n} + \frac{2\sqrt{a+b\sinh^n(x)}}{n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.96

$$\frac{2\sqrt{a+b\sinh^n(x)} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]^n
])/n
```

fricas [A] time = 0.48, size = 156, normalized size = 3.32

$$\left[\frac{\sqrt{a}\log\left(\frac{b\cosh\left(n\log(\sinh(x))\right)+b\sinh\left(n\log(\sinh(x))\right)-2\sqrt{b\cosh\left(n\log(\sinh(x))\right)+b\sinh\left(n\log(\sinh(x))\right)+a}\sqrt{a}+2a}{\cosh\left(n\log(\sinh(x))\right)+\sinh\left(n\log(\sinh(x))\right)}\right)+2\sqrt{b\cosh\left(n\right.}}{n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) - 2*sqrt(b*c
osh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*lo
g(sinh(x))) + sinh(n*log(sinh(x))))) + 2*sqrt(b*cosh(n*log(sinh(x))) + b*si
nh(n*log(sinh(x))) + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*cosh(n*log(sinh(x)))
+ b*sinh(n*log(sinh(x))) + a)*sqrt(-a)/a) + sqrt(b*cosh(n*log(sinh(x))) + b
*sinh(n*log(sinh(x))) + a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sinh(x)^n + a}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^n + a)*coth(x), x)
```

maple [A] time = 0.05, size = 38, normalized size = 0.81

$$\frac{2\sqrt{a+b\left(\sinh^{n}(x)\right)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(\sinh^{n}(x)\right)}}{\sqrt{a}}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sinh(x)^n)^(1/2),x)
```

```
[Out] 1/n*(2*(a+b*sinh(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sinh(x)^{n}+a}\,\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x)^n + a)*coth(x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\coth(x)\,\sqrt{a+b\sinh(x)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a + b*sinh(x)^n)^(1/2),x)
```

```
[Out] int(coth(x)*(a + b*sinh(x)^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sinh^{n}{(x)}}\coth{(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x)**n)*coth(x), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```